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Teachers' Conceptions of Mathematics and Intelligent Tutoring System Use

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TEACHERS' CONCEPTIONS OF MATHEMATICS AND INTELLIGENT TUTORING SYSTEM USE

by

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ABSTRACT

Teachers' Conceptions of Mathematics and Intelligent Tutoring System Use

by

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This mixed-methods study was used to investigate the relationship between teachers' conceptions of mathematics and their use of intelligent tutoring systems for instruction. The participants were 93 junior high school mathematics teachers from three school districts in the Midwest. Data were gathered using a two-part online survey. The first part contained questions about teachers' use of intelligent tutoring systems and other mathematics-focused technology. The second part contained Likert questions from the teachers' version of the Conceptions of Mathematics Inventory.

The quantitative analysis examined the relationship between teachers' conceptions and their use or non-use of intelligent tutoring systems and other mathematics-specific technologies using eight separate 2x5 mixed ANOVAS. The five-level within-subject factors were the yes/no responses to questions pertaining to use of intelligent tutoring systems, graphing calculators, dynamic geometry software, and Desmos. Four yes/no questions addressed whether the technologies were used for

teaching. Four yes/no questions addressed how intelligent tutoring systems were used. Teachers using intelligent tutoring systems were asked if they used them to teach concepts, teach procedures, practice procedures, or fill-gaps in student knowledge. The dependent variable was each dimension's average of eight 5-point Likert items from the Conceptions of Mathematics Inventory. The quantitative analysis revealed no statistically significant interactions between teachers' conception scores and intelligent tutoring system use, or between teachers' conception scores and how they were used. There were statistically significant interactions between teachers' conception scores and their use of graphing calculators, Desmos, and dynamic geometry software.

The qualitative analysis examined teachers' written responses on their use of technology using a constant comparative method. The analysis revealed that teachers used intelligent tutoring systems for differentiation. Teachers used graphing calculators, dynamic geometry software, and Desmos for visual, computational, and exploratory purposes.

An overarching pattern of technology use demonstrated that teachers used intelligent tutoring systems mostly for procedural practice and filling gaps. Graphing calculators were employed mostly for computation and visualization. Desmos was used for exploratory activities. A subset of teachers selected and employed multiple technologies to address instructional and pedagogical needs. Teachers exclusively using intelligent tutoring systems to incorporate technology should also incorporate technology which promotes student exploration.

(171 pages)

PUBLIC ABSTRACT

Teachers’ Conceptions of Mathematics and Intelligent Tutoring System Use

Andrew R. Glaze

The purpose of this mixed-methods study was to investigate the relationship between teachers’ conceptions of mathematics and their use of intelligent tutoring systems for mathematics instruction. Intelligent tutoring systems are adaptive computer programs which administer mathematics instruction to students based on their cognitive state. A conception is a mixture of beliefs and knowledge. The participants in this study were 93 junior high school mathematics teachers from three school districts in the Midwest. Data were gathered using a two-part online survey. The first part of the survey contained questions about their use of intelligent tutoring systems, graphing calculators, Desmos and dynamic geometry software. The second part of the survey contained Likert questions from the teachers’ version of the Conceptions of Mathematics Inventory. Desmos is a website providing interactive classroom activities and a user-friendly graphing calculator. Dynamic geometry software is a class of interactive geometry programs.

The quantitative analysis revealed no statistically significant interactions between teachers’ conception scores and intelligent tutoring system use, or between teachers’ conception scores and how intelligent tutoring systems were used. There were statistically significant interactions between teachers’ conception scores and their use of graphing calculators, Desmos, and dynamic geometry software. The qualitative analysis

revealed that teachers used intelligent tutoring systems for differentiation. Teachers used graphing calculators, Desmos, and dynamic geometry software for visual, computational, and exploratory purposes. Teachers exclusively using intelligent tutoring systems to incorporate technology should also incorporate technology which promotes student exploration.

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Thank you to my Moms. Your examples meant more than you know. Finally, I offer a special thanks to Redell. You were with me every step of the way.

Andrew R. Glaze

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CHAPTER I

INTRODUCTION

It is a typical day in a junior high school mathematics classroom. The teacher uses a PowerPoint to review homework before using an interactive white board to instruct students on rates and proportions. During the lesson, various students wonder what variety of rates could be found at the local grocery store, so they use their smart phones to search advertisements. Encouraged by the National Council of Teachers of Mathematics (NCTM, 2000), and facilitated by increased availability and decreased cost, technology use in mathematics classrooms is omnipresent, but varies in both form and use. Graphing calculators, smart phone apps, interactive white board technologies, dynamic geometry software, and intelligent tutoring systems are examples of common technologies used in mathematics classrooms today. While research has explored teacher use of mathematics-specific technologies (Brown et al., 2007; Lee & McDougall, 2010), there is a void in in research on teachers' use of intelligent tutoring systems. Additionally, there is very little research on how teachers use technologies like intelligent tutoring systems and how their conceptions of mathematics might influence that use. The purpose of this study is to understand how a teacher's conception of mathematics is related to his or her use of intelligent tutoring systems.

Intelligent tutoring systems (ITSs) are a form of computer or internet learning that is adaptable, "encompassing all forms of teaching and learning that are electronically supported, through the internet or not, in the form of texts, images, animations, audios, or videos" (Steenbergen-Hu & Cooper, 2013, p. 971). ITSs are characterized by the self-

paced structure of the program that asks questions or assigns tasks, and assists when needed according to a mapped multidimensional model of the cognitive state of a student (Shute & Zapata-Rivera, 2007; Sottilare, Graesser, Hu, & Holden, 2013).

It is unclear how many teachers use ITSs to engage students in mathematics. Based on database searches, contacts with publishers and ITS experts, it is evident that there exists no such published information. What is evident is that ITSs have increased in number and efficiency since their inception over thirty years ago (C. Koedinger, Koedinger, & Anderson, 1997). Preliminary results from an unpublished survey in the state where this research will be conducted, indicates that 93% teachers using ITS under a state grant were not using this type of software four years ago (C. Ames, personal communication, June 4, 2018).

To date, much of the ITS research has focused on student outcomes and the overall efficacy of ITS instruction via different learning programs (Chu, Yang, Tseng, & Yang, 2014; Ma, Adesope, Nesbit, & Qing, 2014; Steenbergen-Hu & Cooper, 2013) and ITS design (Arevalillo-Herráez, Arnau, & Marco-Giménez, 2013; Baker et al., 2006; K. R. Koedinger, Anderson, Hadley, & Mark, 1997). Though Erümit and Vagifoglu Nabiyev (2015) reported on teachers' opinions of ITSs, no studies exist which address teachers' conceptions of mathematics and their use of ITSs.

Background of the Study

Teachers are the gatekeepers of technology implementation for learning in their classrooms (Aran, Derman, & Yagci, 2016). What technology students use to engage

with mathematics, the frequency of technology use, and the type of learning which accompanies technology use are all mediated by the mathematics teacher. Note the following statement by the NCTM (2000):

> Technology does not replace the mathematics teacher. When students are using technological tools, they often spend time working in ways that appear somewhat independent of the teacher, but this impression is misleading. The teacher plays several important roles in a technology-rich classroom, making decisions that affect students' learning in important ways. Initially, the teacher must decide if, when, and how technology will be used. (p. 26)

The influence of the teacher cannot be understated when promoting student learning of mathematics with the use of computer technology. Even when the teacher is not present, the teacher's influence guides students' use of technology. When students use technology for mathematics, they will use technology about which they are aware. That awareness stems in part from the student's teacher. Therefore, teacher's conceptions, practices and beliefs are important to students' technology implementation.

The term *teacher conceptions* is used varyingly in the literature (Golafshani, 2002). This study draws upon the definition by Steele and Widman (1997), which states that a conception is composed of two components–beliefs and knowledge. The advantage of this distinction is that it eases the burden of distinguishing between the two historically, highly interconnected components (Chappell, 2013).

The intelligent tutoring system (ITS) is one piece of technology which teachers employ in mathematics classrooms. An ITS is a class of software which enables students to learn at their own pace. This type of software may be appealing for mathematics teachers for many reasons. For example, the individual pace at which students learn can address the need to individualize instruction for students with advanced mathematical

knowledge and support students with deficient mathematical knowledge in the same classroom. Because some ITS programs have multiple language abilities, ITSs may also serve students with language barriers.

Though features of ITSs are not standard across programs, four generally accepted conceptual components of these program are: (1) the user interface for communicating with the computer, (2) the domain model representing what a student needs to learn, (3) a cognitive map of student knowledge based on answers to questions, and (4) a tutoring feature with instructional strategies (Sottilare et al., 2013).

Implementation of ITS instruction is relatively easy. Program designs allow for individualized instruction and easy implementation even by those who are not mathematics educators or skilled with computers. One of the components of ITS implementation, which is still not fully understood, is teacher conceptions and use when an ITS is selected for instructional use in a mathematics classroom. For example, with the increasing number of technology options available, such as dynamic geometry software, excel spreadsheets, and computer algebra systems, why do teachers choose to implement ITSs? And once the ITS is chosen for implementation, what are the methods of implementation?

Employing ITSs for instruction does not guarantee student gains in mathematical knowledge beyond what might be expected in a nontechnologically infused mathematics classroom (Ma et al., 2014; Steenbergen-Hu & Cooper, 2013). Indeed, researchers focusing on comparisons between ITSs and traditional instruction find that ITSs may improve student knowledge in mathematics above that of their traditionally instructed

classmates (Mendicino, Razzaq, & Heffernan, 2009), but they also find that students show either no statistically significant differences on tests of their mathematics knowledge (Huang, Craig, Xie, Graesser, & Hu, 2016) or a decrease in mathematics proficiency (Calhoun, 2011).

The current research on ITSs primarily uses experimental designs to examine its benefits. There are no studies that explicitly investigate teacher use of ITSs during mathematics instruction. Additionally, it is unclear whether teachers are using ITSs because they are mandated, convenient, perceivably effective at producing mathematical proficiency or for some other reason.

Teachers play an important role in setting the tone for technology implementation in their classrooms. Consequently, the way teachers employ ITSs may promote or impede student learning. Some strong factors in teacher use of technology are beliefs, knowledge of pedagogy, knowledge of technology, and knowledge of the subject (Mishra & Koehler, 2006). These factors are part of a teacher's conceptions of mathematics.

Statement of the Problem

A focus on student learning outcomes is a common approach to ITS studies (Steenbergen-Hu & Cooper, 2013). While student learning outcomes are important, it is also important to note that students use ITSs under the guidance or influence of teachers. Furthermore, teachers' conceptions of mathematics affect their technology choices in the classroom (Lee, 2007). Thus, it follows that teachers' conceptions of mathematics would influence their use of ITSs, yet the literature is silent on the influence of teachers'

conceptions and ITS use. To better understand intelligent tutoring systems and how teachers use them would be an important contribution to the field. For example, there is no research on how teachers' conceptions of mathematics are related to their use of ITSs, or if those conceptions contribute to their choice to use or not to use ITSs. This study seeks to address these gaps in the literature.

Motivation for this study also arose from the observation that ITSs are gaining prominence in an era of scripted curricula. The modern testing environment promotes a scripted approach to mathematics curricula (Au, 2011), as does the need to assist underprepared or inexperienced teachers (Milner, 2013). ITSs may present a way to circumvent inexperienced or under-qualified teachers in instructing students.

Research Questions

This mixed methods study used qualitative and quantitative data to answer the following research questions.

Over-arching research question: What is the relationship between teachers' conceptions of mathematics and their use of ITSs for mathematics instruction?

Questions answered using quantitative data were as follows.

1. What is the relationship between teachers' conceptions of mathematics and their use or non-use of ITSs?
2. What is the relationship between teachers' conceptions of mathematics and their use of non-ITS math-focused technologies?
3. Among those teachers who use ITSs, what is the relationship between their conceptions of mathematics and how they use ITSs?

Questions answered using qualitative data were as follows.

1. Why do teachers use or not use ITSs?
2. How do teachers use different technologies to teach mathematics?

Importance of the Study

This study was important because teachers' knowledge and beliefs about mathematics and mathematics learning are important to students' learning. Teachers' conceptions of mathematics affect their teaching of mathematics (Ernest, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001), and their use of technology in the classroom (Kim, Kim, Lee, Spector, & DeMeester, 2013; Lee, 2007).

Summary of the Research Design

This exploratory study used a convergent mixed-methods design. A convergent mixed-methods design includes the collection of qualitative and quantitative data during the same phase of research (Creswell & Plano Clark, 2018). In this study junior high school mathematics teachers completed a survey that measured teachers' conceptions of mathematics, using five dimensions of the Conceptions of Mathematics Inventory (CMI) for teachers by Grouws, Howald, and Colangelo (1996), and gathered information on teachers' reported teaching practices.

Scope of the Study

This study focused on junior high school mathematics teachers. The choice to study junior high school mathematics teachers was both deliberate and pragmatic. Junior

high school mathematics teachers come from a wide variety of educational backgrounds with varying degrees of mathematical knowledge and pedagogical practice (Schmidt et al., 2007). Because of the structure of the educational system, high school mathematics teachers are more likely to have degrees in mathematics or mathematics-related fields. Whereas junior high school mathematics teachers may also have mathematics degrees, they may have entered the field with an alternate degree and a mathematics teaching endorsement, thereby providing a more varied population of participants.

Making junior high school teachers the focus of the study was pragmatic to the extent that the author was a mathematics department head at a junior high. This positionality offered him the opportunity to collaborate with other department chairs within one of the districts in the study, which was instrumental in forming relationships of trust. This preexisting relationship of trust was invaluable for promoting teacher participation and eliciting sincere responses during data collection (Mertens & Wilson, 2012).

Definitions

Conceptions of mathematics inventory: The CMI is an instrument designed by Grouws et al. (1996) to measure conceptions of mathematics. This research project uses five of the seven dimensions contained in the teachers' version of the CMI.

Differentiation: Tomlinson (2005) describes differentiation as altering an approach to learning to change one (or more) of three curricular elements. The first element, *content*, describes what a student learns. The second element, process, describes

how students "go about making sense of ideas and information" (p. 4). The third element, *product,* describes the different ways in which student learning can be demonstrated.

Dynamic geometry software: DGS is a class of computer programs that facilitate the creation and manipulation of geometric objects. The clicking and dragging feature of the program allows students to alter the properties objects. For example, a user may alter two sides of a polygon by grabbing and moving a vertex to elongating two sides. DGS also allows for simultaneous measurement and manipulation of objects such as segment length or polygon area thereby allowing students to explore and conjecture. Though various DGS programs exist, three of the most prominent are Cabri, Geogebra, and Geometer's Sketchpad.

Desmos: Desmos is used to describe both the graphing calculator and interactive classroom activities created the developers under the same name.

Fill gaps: This research draws upon the definition of gap as "an incomplete or deficient area" (Merriam-Webster, 2018). Filling gaps, therefore, means a reparation of incomplete or deficient knowledge.

Intelligent tutoring system: An intelligent tutoring system in mathematics is a program which includes the following three criteria: (1) Performs tutoring functions such as presenting information, asking questions, assigning learning tasks, supplying feedback, or supplying prompts to promote motivational or cognitive change; (2) Constructs a cognitive model of a student's psychological state, or locates the psychological state in a previously defined domain model; and (3) Uses information from item number two to adjust an element from item number one (Ma et al., 2014). Examples of ITSs used in

education are Carnegie’s MATHia, ALEKS, and iReady.

Teacher conception: A conception is a combination of knowledge and beliefs (Steele & Widman, 1997), and for mathematics, it is combination of knowledge and beliefs about mathematics itself. For example, Skemp (2006) described two conceptions of mathematics as *relational* and *instrumental*. An instrumental conception is a procedural view of mathematics, whereas a relational conception describes a network of understanding allowing for the creation of multiple solution paths (Thompson, 1992).

Researcher: Personal Background

At the time of this dissertation, the researcher possessed approximately 20 years’ teaching experience in both public and private institutions ranging from early childhood education to university environments. This research was conducted in three school districts in the western United States. The first was a large school district in the western United States where the researcher taught for 12 years in both junior high and high schools. The researcher’s experience using ITSs in the classroom motivated an increased understanding of better methods of ITS implementation.

Summary

The purpose of this research was to explore the relationship between teachers’ conceptions of mathematics and their use of ITSs for mathematics instruction. While the literature on ITSs is replete with experimental designs which highlight student outcomes after ITS use, the literature does not address teachers’ use of ITSs. Because teachers’

conceptions of mathematics affect their choice and use of non-ITS technology (Kim et al., 2013; Lee & McDougall, 2010), teachers' conceptions of mathematics were explored.

CHAPTER II

LITERATURE REVIEW

The purpose of this chapter is to present literature relevant to the overarching research question: What is the relationship between teachers' conceptions of mathematics and their use of Intelligent Tutoring Systems (ITSs) for mathematics instruction? Intelligent Tutoring Systems are education tools which offer individualized instruction to students based on a student's readiness to learn.

The format of the chapter is as follows. Definitions of the two components of the research question are addressed first: (1) conceptions of mathematics, and (2) ITSs. After describing conceptions of mathematics and ITSs, the theoretical framework supporting this study is described, followed by an overview of teacher use of mathematics technology, and teacher use of ITSs.

Conceptions of Mathematics

While multiple definitions of conceptions of mathematics exist (Thompson, 1992), this dissertation draws upon the definition of conceptions advanced by Steele and Widman (1997) who claim that a conception is a mixture of beliefs and knowledge. Defining conceptions as beliefs and knowledge acknowledges the difficulty in distinguishing between knowledge and beliefs (Pajares, 1992). Despite decades of debate and refinement on definitional aspects of knowledge and belief, the field of education still lacks a solitary definition distinguishing belief from knowledge (Savasci-Acikalin, 2009).

This intertwining of knowledge and beliefs is important. Distinguishing

knowledge from beliefs is a centuries-old exercise dating back to at least 400 BC when Plato attempted to define knowledge as a belief justified by argument (Chappell, 2013). Savasci-Acikalin (2009) summarized general trends in research to form guiding principles about beliefs and knowledge (see Table 1).

Table 1

Beliefs and Knowledge as Generalized by Savasci-Acikalin (2009)

Beliefs	Knowledge
Refer to suppositions, commitments, and ideologies	Refers to factual propositions and the understandings that inform skillful action
Do not require a truth condition	Must satisfy a “truth condition”
Based on evaluation judgment	Based on objective fact
Cannot be evaluated	Can be evaluated or judged
Episodically stored material influenced by personal experiences or cultural and institutional sources	Stored in semantic networks
Static	Often changes

This research draws upon the CMI for teachers created by Grouws and Howald (Grouws et al., 1996; Howald, 1998). The CMI was designed to measure students’ conceptions and contained seven dimensions formed from several National Assessment of Educational Progress (NAEP) items (NAEP, 1983), Indiana Mathematics Belief Scales (Kloosterman & Stage, 1992), and original items. The conceptions measured by the CMI are: (1) the composition of mathematical knowledge, (2) the structure of mathematical knowledge, (3) the status of mathematical knowledge, (4) doing mathematics, (5) validating ideas in mathematics, (6) learning mathematics, and (7) usefulness of mathematics.

The CMI was used by Grouws et al. (1996) to measure the conceptions of mathematically gifted students. It was also used by Star and Hoffman (2005) and Walker (1999) to measure the effects of curriculum implementation.

The CMI for teachers was created from the student inventory by rewording the questions (Howald, 1998). It has been employed in dissertations addressing teachers' conceptions of mathematics (Howald, 1998; Lee, 2007) as well as an NSF-funded project addressing teachers' use of assessment (Online Evaluation Resource Library [OERL], 2018).

Intelligent Tutoring Systems

Anderson, Boyle, and Reiser (1985) delineated a distinction between ITS and non-ITS instruction when they broadly defined two different types of computer instruction: computer-assisted instruction and ITS instruction. Computer-assisted instruction is a broad term for instruction which is supported by a computer. ITS instruction, on the other hand, describes a system that responds to problem-solving strategies of the student. In other words, ITS instruction is a specialized subset of computer-assisted instruction considered "intelligent" because of its adaptive and individualized approach.

A basic ITS model contains modules for knowledge of the domain, knowledge of a tutor, and knowledge of a student (Chen, Yunus, Ali, & Bakar, 2008). The domain module contains knowledge of the subject matter. One might think of the domain module as the content knowledge of a teacher. The tutorial module contains knowledge of human

tutorial interactions and the student module contains knowledge of the student. These two modules are discussed in the sections below.

Tutorial Module

The tutorial module represents the methods of instruction students receive from ITSs. The following examples demonstrate the way that ITSs may individualize instruction similar to the way that a teacher might individualize instruction. Teachers recognize that students have diverse learning styles and may employ multiple modes of instruction to address multiple forms of intelligence (Gardner, 2011). Similarly, ITSs employ multiple modes of instruction. They may employ video tutorials, worked out examples, or written text to convey mathematical knowledge (Beal, Walles, Arroyo, & Woolf, 2007; Steenbergen-Hu & Cooper, 2013). When students need assistance with a problem, the ITSs can offer step-by-step problem-solving instructions, or simply offer hints (Burch & Kuo, 2010). Just as a teacher might decrease the frequency of hints or suggestions over time to improve student proficiency, ITSs can function similarly (Salden, Aleven, Renkl, & Schwonke, 2009).

Though ITS tutorial modules emulate aspects of teacher interactions, they do not, in fact, replace the need for teachers. Teachers still need to monitor and assist students in their learning. For example, ALEKS program designers recommend that teachers monitor student reports to assure that students are using the program for a specified amount of time each week while progressing through content (McGraw Hill Education, 2018). They also recommend that teachers communicate with students about their progress either in person or through an electronic medium. Carnegie program designers recommend that

teachers monitor students by physically roaming the room while learners are engaged with the program and by accessing student reports (Carnegie Learning, 2017). Teachers can still interact with students to help with their mathematics learning while students are engaged with the programs (Carnegie Learning, 2017; Mc Graw Hill Education, 2018).

Student Module

There are three types of ITS student modules (Chu et al., 2014). A *model tracing* ITS compares student answers against a set of rules which reflect common student misconceptions. This comparison allows the computer to assess a student's misconception (Kodaganallur, Weitz, & Rosenthal, 2005). For example, if a student is attempting to solve a multi-step linear equation such as $2(x+1)-3=4$, and begins by performing the mis-operation $2(x)-2=4$, the ITS would recognize that the student has a misconception about the distribution property.

A *constraint based* ITS compares student inputs to a set of correct solution methods (Chu et al., 2014). If an input violates a constraint, the ITS knows that the problem was incorrect. Continuing with the previous example, the constraint based ITS would recognize that $2(x)-2=4$ is not a correct solution path for solving $2(x+1)-3=4$, but would not recognize it as a violation of the distributive property. The ITS would not consider the mistake to be conceptually different than $2(x+1)=1$, which is a misconception of additive inverses.

Whereas in model tracing and constraint based ITSs, the inputs or lack of inputs of a student indicate to the computer the cognitive state of the student (Kodaganallur et al., 2005; Mitrovic, Koedinger, & Martin, 2003), *example tracing* ITSs compare student

work to generalized problem solving behavior (Aleven, McLaren, Sewall, & Koeding, 2009). What makes example tracing ITSs unique is their recognition of multiple and varied solution paths in problem solving and their ability to modify instruction based upon a chosen solution path. Returning to the problem $2(x + 1) - 3 = 4$, the example tracing ITS would recognize an intermediate input of $2(x + 1) = 7$ and $2x + 2 - 3 = 4$ equally and adjust subsequent student instruction based on the chosen solution path.

ITS Learning Structure

To understand the uniqueness of the ITS, it is helpful to consider the learning structure of ITSs. An initial ITS model was envisaged over forty years ago as a system that has knowledge of the domain (subject-matter knowledge), knowledge of the learner, and knowledge of teaching strategies (Hartley & Sleeman, 1973). Shute and Psotka (1996) identified four characterizing components of ITS: (1) An initial assessment of student knowledge, (2) a computer-directed learning path, (3) computer selected problems, and (4) a diagnosis of student knowledge based on answers to selected problems. A brief description of those components follows.

Initial knowledge check. ITSs may start by assessing current student knowledge. This is comparable to a pretest that a student might take at the beginning of a school year or learning unit. However, the ITS analyzes the pretest more acutely than a teacher might (VanLehn, 2011). As a student takes the pretest, the ITS creates a multidimensional model of the cognitive state of the student, or identifies the location of student knowledge in a previously defined cognitive model (Shute & Zapata-Rivera, 2007; Sottilare et al., 2013). This model, or map, of student knowledge allows the ITS to determine what a

student already knows, what a student needs to know, and what a student is ready to learn (Shute & Psotka, 1996).

Computer directed learning path and problem selection. ITSs are characterized by the self-paced structure of the program which asks questions, assigns tasks, or aids students when needed based upon student responses to a predetermined computer model of an appropriate solution.

Diagnosis of knowledge. The student modeling of knowledge and adaptive instruction are the most essential elements of ITSs (Shute & Psotka, 1996). After an initial knowledge check, the ITS continually assesses student knowledge and adapts the student's learning trajectory accordingly.

The projected learning path and the knowledge checks are complementary features which continually readjust. Loops in the program allow for the adaptive nature of instruction by repeating a series of commands multiple times. ITSs may have both an inner loop and an outer loop. An outer loop selects a learning task, but the inner loop elicits steps or gives guidance (VanLehn, 2011). Stated differently, the outer loop might represent the learning of a mathematical topic such as solving a system of linear equations. The inner loop directs the specific steps which a learner would practice, such as finding an opposite integer coefficient by employing a system of linear combinations.

Defining Intelligent Tutoring Systems

Two meta-analyses addressing the effects of ITS instruction defined ITSs using similar inclusion criteria for their analyses (see Table 2). Steenbergen-Hu and Cooper (2013) drew upon Shute and Zapata-Rivera's (2007) criteria that ITSs be adaptive and

Table 2

Defining Features of Intelligent Tutoring Systems

Ma et al. (2014) definitions	Steenberg-Hu and Cooper (2013) definition	Dissertation definition
1. Presents information to be learned	1. Self-paced and learner led, or instructor-directed.	1. Encompasses all forms of electronic teaching and learning.
2. Asks questions or assigns learning tasks	2. Highly adaptive and adjusts to individual's characteristics, needs, or pace of learning	2. Self-paced and learner led, or instructor-directed.
3. Provides feedback or hints	3. Encompasses all forms of electronic teaching and learning.	3. Asks questions or assigns learning tasks
4. Answers questions posed by students or offers prompts to provoke cognitive, motivational or metacognitive change	4. Tracks student knowledge, skills, learning strategies, emotions, or motivation	4. Provides feedback or hints
5. Adapts instruction according to a constructed multidimensional model of student's psychological state or a student's location in a preexisting model.	5. Uses outer loops to select learning tasks and inner loops to elicit steps or give guidance and feedback	5. Answers questions posed by students or offers prompts to provoke cognitive, motivational or metacognitive change
		6. Adapts instruction according to a constructed multidimensional model of student's psychological state or a student's location in a preexisting model.

respond to an individual's characteristics and needs with an individual learning pace. Graesser, Conley, and Olney (2011) definition requires that ITSs track student knowledge, learning skills, strategies, and emotions in fine detail. VanLehn (2006) cited the need for the inclusion of inner and outer loops. Similarly, Ma et al. (2014) meta-analysis drew upon Shute and Psotka (1996), as well as Sottilare et al. (2013), to require

that ITS perform tutoring functions such as presenting information, assigning learning tasks, and providing hints. They also state that ITSs must adapt instruction according to the student's psychological state or the student's location in a preexisting cognitive model.

While this dissertation drew upon components of both definitions, it primarily used the criteria described by Ma et al. (2014) for two reasons. First, the meta-analysis is more recent. Second, their meta-analysis is purposefully inclusive and draws from a wider range of literature than does the meta-analysis by Steenbergen-Hu and Cooper (2013). The broader inclusion criteria allowed for a more informed literature review as well as a broader spectrum of programs. The criteria drawn from Steenbergen-Hu and Cooper is that ITS instruction encompasses all forms of electronic teaching and learning. This allows for a wide array of instructional practices available in an internet environment.

This dissertation used the definition that ITSs are computer or internet-based programs encompassing all forms of electronic teaching and learning that:

1. Perform tutoring functions in any electronic format such as presenting information, asking questions, assigning learning tasks, supply feedback, or supply prompts to promote motivational or cognitive change.

2. Construct a cognitive model of a student's psychological state or locate the psychological state in a previously defined domain model.

3. Use information from item number two to adjust elements from item one.

Prominent Intelligent Tutoring Systems

While a variety of ITSs are currently used in mathematics instruction, the majority of the research articles on ITSs reference *Assessment and Learning in*

Knowledge Spaces (ALEKS) and Carnegie's Cognitive Tutor. They were also the two most prominent programs used in the largest school district that was included in the study. For those reasons, they are described here for the reader's benefit.

ALEKS designers define the program as a "Web-based, artificially intelligent assessment and learning system" (McGraw Hill Education, 2017) designed using knowledge space theory. Knowledge space theory posits that the current state of student knowledge is ascertainable, and that various multidimensional paths to a full knowledge state exist (Falmagne, Koppen, Villano, Doignon, & Johannesen, 1990). ALEKS students experience a personalized learning path or follow a teacher-directed one. It uses initial and intermittent assessments to monitor student progress. While implementation varies from teacher to teacher, ALEKS recommends three to five hours of student use per week (McGraw Hill Education, 2018).

Carnegie's *Cognitive Tutor* is based on ACT-R computational theory of thought (Mitrovic et al., 2003; Pane, McCaffrey, Slaughter, Steele, & Ikemoto, 2010) whose components or modules employ a model-tracing design (Chu et al., 2014) that allow the software to trace student progress and give targeted-feedback. It is the ITS component of Carnegie's curriculum package and is designed to complement classroom instruction. While Cognitive Tutor does not employ an initial knowledge check, students using it follow either a teacher-directed learning path or a predetermined learning path based on the student's enrolled course. Carnegie's designers recommend a blended approach (Horn & Staker, 2015) to instruction with face-to-face instruction 3 days per week and ITS instruction 2 days per week (Carnegie Learning, Inc., 2012). At the time of this

dissertation Cognitive Tutor was transitioning to the name *MATHia*. Describing the literature review requires that this section employ the name Cognitive Tutor. The methods section and discussion will necessarily use the term MATHia.

Two other prominent homework based ITSs in the literature are Pearson's MathXL, and ASSISTments. Their presence in the research literature and their intended purpose make them relevant for this literature review. Pearson's MathXL is an ITS which allows teachers to generate homework assignments to complement classwork. MathXL allows students to receive immediate feedback and hints, or similar questions to those selected by the instructor. ASSISTments is a program created through government grants which turns textbook assignments into ITS assignments. Through ASSISTments, the students complete textbook-based homework assignments then submit them through a web portal which gives immediate feedback. Unlike other ITSs, MathXL and ASSISTments do not construct a cognitive model of the students' psychological states. They meet criteria two of the ITS definition because they use student course and problem selection as indicators of a student's cognitive state. Having defined ITSs and introduced prominent ITSs for this dissertation, I now introduce a theoretical framework.

Theoretical Framework

This theoretical framework details the influences which shape teachers' conceptions, as well as, impediments to teachers' use of technology. The following sections describe sources of teachers' technological knowledge and mathematical conceptions. The section concludes with a discussion of impediments to technology

implementation. As this dissertation sought to determine relationships between teachers' conceptions of mathematics and technology use, it is appropriate to begin with a well-established framework, which addresses both teacher knowledge and technology use: Technological pedagogical content knowledge (TPACK).

Technological Pedagogical Content Knowledge

TPACK is an advancement of Shulman's (1986) framework establishing pedagogical content knowledge (PCK) as an essential form of teacher knowledge. Shulman's original framework established the important interplay between content knowledge (CK) and pedagogy knowledge (PK). CK is knowledge of the subject taught, and PK is knowledge of teaching and teaching practices. PCK is a specialized type of knowledge represented in the intersection of CK and PK (Shulman, 1986). It includes forms of representations, powerful analogies, illustrations, examples, demonstration, and a knowledge of what makes the subject matter difficult or easy for students to learn.

In their seminal framework, Mishra and Koehler (2006) advance Shulman's fusion of content and pedagogy by adding a technology component. Technology knowledge (TK) is general knowledge about information technology that would allow a person to use it at home or at work, and understand when it would assist or impede in a goal (Koehler & Mishra, 2009; Mishra & Koehler, 2006). TPACK combines PK, CK and TK and is a knowledge of how technology can be used to teach content. Whereas PCK, represented as a Venn diagram, has three subsections (PK, CK and PCK), TPACK has seven subsections (see Figure 1).

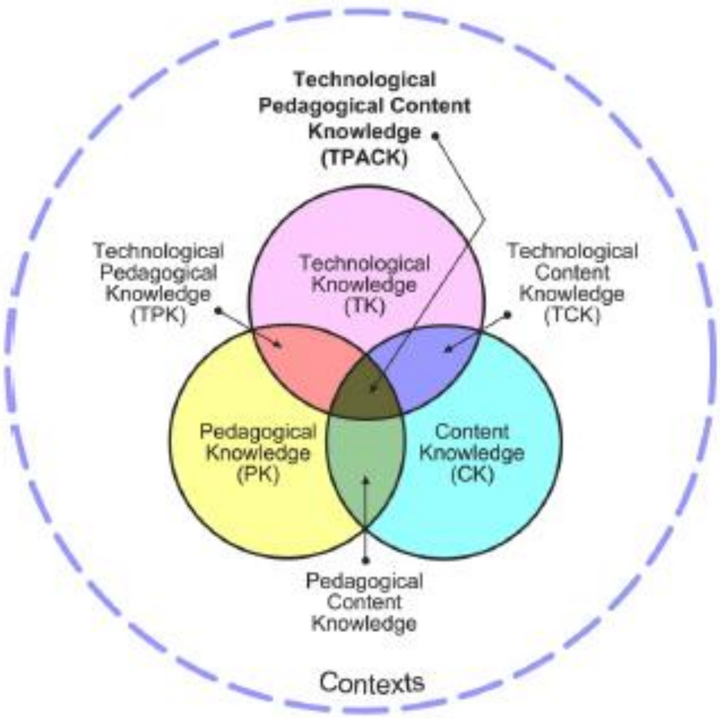

Figure 1. Technological pedagogical content knowledge (TPACK) diagram (Koehler & Mishra, 2009).

The addition of TK to Shulman's PCK introduces three additional subsets of knowledge. Technological pedagogical knowledge (TPK) is an understanding of how the infusion of certain technologies affect teaching and learning (Koehler & Mishra, 2009). For example, the use of calculators in a science class can reduce the time needed for trivial calculations, but it also reduces a student's opportunity to review basic mathematics facts. Similarly, technological content knowledge (TCK) is an understanding of how technology and content can influence or constrain one another. For example, a seasoned algebra teacher will recognize that students with weak algebraic skills find the graphing feature on calculators invaluable for solving systems of two equations. Yet the teacher will also recognize that the students' weak mathematical

knowledge also constrains their ability to understand and use key graphical features of a calculator.

The final subset of knowledge is known as TPACK. TPACK is an understanding of how teaching and learning are affected or influenced with the inclusion of certain technologies. It is an understanding of the interplay of TK, CK and PK (Koehler & Mishra, 2009). According to Grandgenett and Kiewit (2008), a teacher with TPACK possesses six defining characteristics: (1) The teacher is open to experimenting with new computer technology tools in lessons; (2) The teacher stays on track and is not sidetracked when using technology; (3) The teacher is aware of students’ current state of knowledge, what the students need to learn and how a lesson should flow with technology; (4) Teachers help students understand why technology is important; (5) Teachers use technology for teaching, assessment, and classroom management; and finally, (6) Teachers with TPACK are comfortable and optimistic about changes in technology.

Grandgenett and Kiewit’s (2008) six defining characteristics of TPACK hold implications for teachers using ITSs. Teachers using ITSs may find it easy to use the technology with fidelity. Unlike other mathematics specific technologies, ITSs do not offer easy opportunities for teachers to sidetrack their classes. Similarly, teachers may easily know the students’ cognitive state or their state of preparedness for mathematical topics by monitoring student progress through ITS generated reports.

Items four and five in Grandgenett and Kiewit’s (2008) list also address important elements for TPACK in mathematics education. While technology may allow for deeper

understanding of a subject through making multiple visual representations and demonstrating the interconnectedness of topics, care should be taken not to use technology for technology's sake or to study things which are not central to the curriculum simply because technology makes it possible (Garofalo, Drier, Harper, Timmerman, & Shockey, 2000).

Drawing upon the example of solving systems of equations, an algebra teacher with TPACK could approach a lesson on systems of equations by thoughtfully considering when to introduce graphing calculator functions. The teacher would take into consideration the affordances and limitations of the technology, student understanding of algebra and multiple representations, student comfort with technology, and time constraints incidental to teaching technology. But a teacher with TPACK would not demonstrate, for example, how to solve systems of equations using matrices when students are not yet aware of what a matrix is.

Koehler and Mishra's (2009) conceptualization of TPACK explicitly refers to teacher knowledge. Recall that teachers' conceptions of mathematics are a mixture of both knowledge and beliefs (Steele & Widman, 1997). While research has not yet fully explored the relationship between beliefs and TPACK in mathematics education, there does appear to be a correlational relationship between the two. The following sections describe sources of teachers' conceptions which would contribute to TPACK, as well as some impediments that teachers may have to properly employ TPACK in the classroom.

Sources of Teacher Conceptions

Figure 2 presents a framework for the sources and targets of teacher knowledge.

The first box in Figure 2 represents the sources of conceptions, while the arrow acknowledges the centrality of constructivism. The TPACK framework represents the teacher's acquired knowledge. The instructional buffers represent the influences affecting teacher implementation of technology which, in turn affect instructional practices.

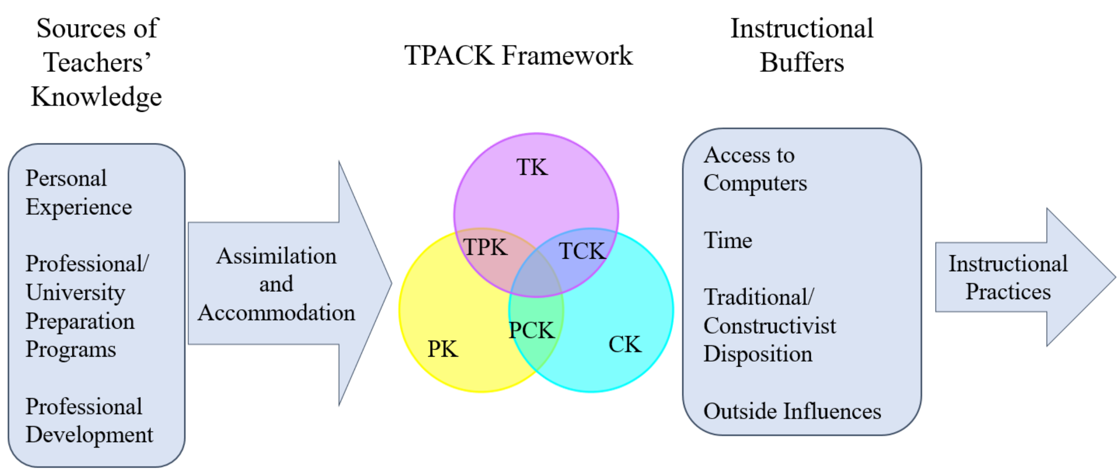

Figure 2. Sources and targets of teacher knowledge.

Personal experience. Personal experience is a source for both mathematical and computer knowledge. Personal experience in mathematics here is defined as mathematics learned in out-of-school situations. This type of mathematical knowledge is often referred to as ethnomathematics as it incorporates both cultural and mathematical knowledge (D'Ambrosio, 2004). A thorough exploration of this topic is beyond the scope of this dissertation. It suffices to note that teachers may acquire mathematical knowledge outside of traditional classroom experiences.

Teachers acquire knowledge about computers through personal experience. Personal experience may include knowledge of smart devices, phones, tablets, laptops or other technology knowledge which one acquires through experiences outside of

education. Teachers who are comfortable using computers for personal use are also more comfortable using computers for instruction (Cox, Preston, & Cox, 1999)**.**

Preparation programs. Professional preparation or university preparation programs are important sources of teachers' knowledge of mathematics and technology. While mathematical knowledge is acquired through university programs, it is noteworthy that university programs are not altogether effective at creating the types of knowledge or conceptions of mathematics which are important for reform-based mathematics instruction (Ball, Lubienski, & Mewborn, 2001).

It is in teacher preparation programs where preservice teachers might experience various mathematics-specific technology for the first time. Though teacher preparation programs can address TPACK by preparing preservice teachers to thoughtfully incorporate technology in student-centered classrooms (Mistretta, 2005), many teacher preparation programs focus their effort on making teachers the primary users of technology in the classrooms instead of making students the primary users of technology (Ledermann & Niess, 2000). Consequently, preservice programs do not always promote TPACK.

Professional development. Professional development is on-the-job training. Teachers attending professional development will generally receive training on instructional materials, technology, or monitoring students for understanding (Banilower et al., 2013). However, while the material covered during a professional development may be necessary and essential for promoting TPACK, the experience is often quick and not sustained over time (Ball et al., 2001; Banilower et al., 2013). While sustainability is

an essential component for promoting lasting change in teaching practices (Loucks-Horsley, Hewson, Love, & Stiles, 2010), other factors are important for ensuring successful technology implementation. Unger and Tracey (2013) suggested that programs which promote TPACK and lasting implementation ensure access to resources, provide administrative resources, allow the teachers involved to direct their own learning, promote activities that change attitudes, and promote collaborative learning environments.

While personal experience, preparation programs, and professional development provide opportunities to gain TPACK, teachers process and use the provided information individually. The TPACK theoretical framework used in this study draws upon the idea of assimilation and accommodation to explain the impact of beliefs.

Assimilation and Accommodation

The arrow indicating assimilation and accommodation in the framework represents the mental process of the teacher acquiring new knowledge. It is in this component of the framework where beliefs have the most impact on knowledge because individuals may, for various reasons, "create an ideal, or alternative situation that may differ from reality" (Pajares, 1992, p. 309). Knowledge is not passively received, but is actively attained (von Glasersfeld, 1989).

From a constructivist perspective, learning takes place when a person confronts or experiences new knowledge. Piaget (1948) describes the two processes of schema building as assimilation and accommodation. Assimilation occurs when one perceives a new object in terms of an existing object (Driscoll, 2005). This does not mean that the

differences are not perceived. It could be that the differences are actually disregarded (von Glasersfeld, 1995). For example, suppose that a teacher attends a professional development intended to instruct on a new type of ITS software. The ITS software is unique insofar that it is specifically created for assistance with homework and not for classroom learning. A teacher assimilating the new knowledge might perceive that the software is created for a different purpose but does not understand the magnitude of the differences and assumes that the software is "just like all the other software packages."

Accommodation occurs when existing schemes or operations must be modified to account for a new experience (Driscoll, 2005). A teacher examining the software more closely may realize that it is unique in ways previously disregarded. The new understanding results in an accommodation (von Glasersfeld, 1995). The accommodation is a permanent modification to a person's mental schema (Steffe, Thompson, & von Glaserfeld, 2000). Consider the previous example of an ITS. Suppose a teacher encounters an ITS for the first time and perceives that it is unique from other software because of its ability to offer adaptive and individualized instruction based on the student's mental schema. This new knowledge acquisition by the teacher is considered an accommodation because it requires a modification to the teacher's mental schema.

The assimilation and accommodation processes involve teachers' beliefs. Indeed, beliefs are closely tied to affective influences (Fiedler & Bless, 2000). Therefore, beliefs influentially impact the amount and type of teacher knowledge that teachers attain. Teachers who believe that technology is important for constructing mathematical knowledge may have more TPACK than teachers who believe that technology in

mathematics classes should be reserved for checking answers (R. C. Smith, Kim, & McIntyre, 2016).

Instructional Buffers

After teachers assimilate and accommodate the presented information, and before dissemination in the classroom, the newly attained knowledge encounters *instructional buffers*. Instructional buffers are factors which may alter or impede the implementation of the intended technology. Common instructional buffers include access to computers, time, teacher disposition, and outside influences. These buffers are discussed below.

Access to computers. Despite a steady push for increased technological resources in schools, basic access to computers is still a limiting factor to technology implementation. In the most recent national survey on educational technology in U.S. public schools, Gray, Thomas, and Lewis (2010) reported that a typical student to computer ratio for classrooms is 5.3 to 1. Similarly, Hutchison (2009) conducted a national survey on teacher perception and uses of information and communication technology revealing that only 12% of teachers had laptop computers for each student. The U.S. Census Bureau (2017) reports that individuals’ lack of computer ownership or home internet availability varies from 12% to 30% based on race. Based on these surveys, internet or computer availability remains a limiting factor both in and out of school.

Time. Time constraints to teachers’ use of technology take two forms, classroom time and teacher preparation time. Classroom time or time with students is a fixed quantity, but how classroom time is used is malleable. In the high-stakes testing

environment of schooling, time is an essential consideration for the implementation of any new technology or material (Hutchison, 2009). Implementing new mathematics technology into a classroom requires extra instructional time (Ruthven, Deaney, & Hennessy, 2009). Classroom time devoted to the implementation of new technology is time not available for other instruction. Not only is time a consideration in the classroom, but the time required to familiarize oneself with the technology is also a constraint (Prieto-Rodriguez, 2016). While this may be considered a factor for teacher knowledge, it is designated as an instructional buffer because teachers need personal time to plan the implementation of technology.

Teacher disposition. Teachers' conceptions of mathematics influence their implementation of technology. While teachers who support student-centered learning are more likely to use technology in the classroom (Wozney, Venkatesh, & Abrami, 2006), teachers who have a constructivist disposition are more likely to use it for activities which promote higher order thinking (Matzen & Edmunds, 2007). This means that a mathematics teacher with a constructivist disposition is likely to implement more technology, such as dynamic geometry software, to engage students in a process of exploration and discovery. This is in contrast to one common practice of using technology for practicing basic skills (Prieto-Rodriguez, 2016).

While certain technologies such as interactive apps, graphing calculators, or dynamic geometry software, afford uses which are constructivist in nature, a teacher employing them may not necessarily do so in a constructivist manner (Richter et al., 2013; Windschitl & Sahl, 2002). Because technology does not promote a change in

teaching disposition, one can surmise that teachers typically choose technologies which match their pedagogical dispositions. In order for teachers to implement technology, the technology needs to match the teacher's conceptions (Zhao, Pugh, Sheldon, & Byers, 2002). Therefore, like teacher disposition, the type of technology available to teachers has the potential to hinder or promote technology use.

Outside influences. Even if computers are available and teachers choose to use them, certain barriers still impede use and effective implementation. One barrier is lack of technical support (Hutchison, 2009). When computers or programs do not work properly, teachers will stop using them. School culture is also an instructional buffer. If computer use for mathematics teaching does not match the culture of the mathematics department, then it may not be used (Zhao et al., 2002). For example, teachers on a team can negatively influence technology use (Zhao et al., 2002) as a solitary teacher will find it hard to implement technology which a team does not support.

Instructional Practices

Taken together, the sources of teacher knowledge, assimilation and accommodation, TPACK, and instructional buffers can all impact the instructional practices used by teachers. In terms of the impact to instruction, ITSs are unique pieces of software because the technical demands on teachers implementing the software can be minimal. There are minimum requirements for teachers using ITSs including: creating student accounts, giving students access to the software, and knowing how to generate and read reports. Thus, teachers with relatively little TPACK can use ITSs. The minimal requirements for ITS use may afford teachers with differing conceptions of mathematics

to use the software. Since previous research has not explored the relationship between teachers' conceptions and their ITS use, it is informative to consider similar research with other mathematics education technology.

Teacher Conception and Use of Technology in Mathematics Education

This dissertation builds upon previous research establishing the relationships between teachers' conceptions of mathematics and their instructional practices with technology (Lee, 2007; Tondeur, Braak, Ertmer, & Ottenbreit-Leftwich, 2016; Wachira, Keengwe, & Onchwari, 2008). Therefore, this section considers studies involving both beliefs and knowledge with technology use in mathematics education.

One of the first considerations with conceptions and technology is whether teachers choose to use technology. Teachers choose not to use technology when they do not have sufficient knowledge about the technology, the available technology does not match their pedagogical beliefs or instructional practices (Ertmer & Ottenbreit-Leftwich, 2010; Zhao et al., 2002), or the change in practice requires too much effort (Joglar Prieto, Sordo Juanena, & Star, 2014). Knowledge of and about the technology must exist for teachers to effectively implement it, but beliefs are also important. Teachers' beliefs about employing constructivist teaching practices may be an additional factor influencing the adoption of technology (Judson, 2006; Tondeur et al., 2016).

To date only one published study exists which explores teachers' conceptions of mathematics and use of ITS. Erümit and Vagifoglu Nabiyev (2015) published a study exploring teachers' opinions about an ITS prepared to improve the problem-solving skills

of students. While the study did not directly address teachers' knowledge, their exploration of teachers' opinions illuminated portions of their beliefs. Teachers in the study revealed that they valued the ITS because it gave students a process-oriented approach to solving problems, clarified and simplified problems, and improved student motivation by offering students success and instant feedback. Based upon these results, it is likely that some of the teachers in the study possessed an instrumental conception of mathematics because they valued the facility with which one can achieve an answer to a problem with a set of predetermined rules (Thompson, 1992).

To better understand the influence of teachers' conceptions of mathematics on technology use in the mathematics classroom, and because of the limited studies on teachers' conceptions and ITSs specifically, the next section considers teacher's use of other technologies to hypothesize on the potential relationship between teachers' conceptions and ITS use.

Graphing Calculator Use

Lee (2007) investigated teachers' conceptions of mathematics and their teaching practices using graphing calculators through a collective case study. In a separate classroom-based observational case study, Doer and Zangor (2000) describe how a teacher's knowledge and beliefs about the graphing calculator were reflected in her practice. The teachers in Lee's study viewed mathematics as a dynamic field where mathematics is about understanding concepts rather than knowing mechanical procedures. While Doer and Zangor's study did not specifically investigate the teacher's conception of mathematics, the teacher's conception of the graphing calculator hints at

her conception of mathematics. The fact that the teacher valued student explorations on the calculator may reflect her value of student explorations in general and hints at a relational conception of mathematics. Results from these studies show that teachers with a relational conception of mathematics who use graphing calculators, value them as tools to increase mathematical understanding.

In addition to using the calculator as a tool for computations and data analysis, teachers in the studies also used them to turn routine calculations into exploratory and sense-making activities. Teachers used the calculators to lay a foundation of exploration and further mathematical investigation, often using the calculator to form a common entry point for the entire class (Lee & McDougall, 2010). These findings are notable because, in general, secondary mathematics teachers tend to use calculators as computational tools (Brown et al., 2007), or instruments to improve the accuracy and appearance of student work (Ruthven et al., 2009).

The previous examples highlight how the teachers' beliefs affected their calculator practices, but their knowledge of mathematics and pedagogy were also powerful factors in guiding their instruction. The participant in Doerr and Zangor's (2000) study understood the limitations of a graphing calculator to give contextual meaning to problems and encouraged her students to think critically about the results of regression analyses rather than accept them wholeheartedly.

While the studies by Lee (2007) and Doerr and Zangor (2000) highlight the use of technology by constructivist teachers, the studies do not address how conceptions of mathematics affect calculator use for teachers inclined towards more traditional forms of

instruction. This is likely due to sampling bias. An alternative method for finding participants in the present dissertation proposal was to avoid the same sampling bias by including all junior high school mathematics teachers within three school districts. Searching for ITS users and non-users of differing conceptions offered a richer comparison of instructional practices.

This section explored research relating teachers' conceptions of mathematics to technology use. Based on these studies, it would follow that there may exist a relationship between teachers' conceptions of mathematics and ITS use. This topic, however, is not addressed in literature.

Mathematics Teacher Use of Intelligent Tutoring Systems

While teacher use of ITSs is still a relatively unexplored domain, research on ITSs in general can lend understanding about teachers' ITS use. This section presents an overview of two mathematics specific ITS meta-analyses, then details findings from individual ITS research articles that have implications for teacher use. The meta-analyses were mentioned previously while defining ITS, but their results are discussed here in more detail.

Meta-Analyses of Intelligent Tutoring Systems

Steenbergen-Hu and Cooper (2013) and Ma et al. (2014) conducted meta-analyses on the effects of ITS instruction in mathematics education. Steenbergen-Hu and Cooper's meta-analysis for K-12 had strict inclusion criteria that yielded 26 reports containing 34 independent studies and 61 effect sizes. Based on their meta-analyses, they formed the

following conclusions: (1) The effectiveness for ITSs did not differ for different mathematical topics under a fixed-effect model; (2) The advantage of ITSs, compared with regular classroom instruction, was significant only for basic math under the fixed-effects model; (3) The effect sizes were greater when the intervention lasted less than 1 year; (4) Helping general-achieving students had a greater effect than helping low-achieving learners; and (5) The effects were greater for elementary school than for high school.

Ma et al. (2014) used broader inclusion criteria for their meta-analysis. In total, they found 107 effect sizes involving 14,321 participants. With the broader inclusion criteria, they reported the following outcomes; however, they caution that the results lack statistical power.

1. Students who used ITSs learned significantly more than those who used other modes of instruction. The only exception to this was when comparisons were made with small group teaching experiments with eight or fewer participants.
2. Studies which used ITSs for separate in-class activities or homework had larger effect sizes than those which used ITSs as the principal form of instruction.
3. Effect sizes were not moderated by whether the ITS provided feedback.
4. Students in secondary schools had higher weighted mean averages than those in elementary school.
5. Classroom based studies had a higher effect than laboratory studies
6. Higher effect sizes were associated with longer study duration.

The results from the two studies show a stark contrast with respect to grade-level studies and duration of study. Steenberger-Hu and Cooper (2013) indicate that the largest gains in teaching mathematics occur when using ITSs for elementary school for basic

arithmetic and for studies of shorter duration. Ma et al. (2014) found that ITS instruction produced stronger effects in secondary school and for studies of longer duration. The focus of these meta-analyses was on student achievement. Consequently, neither meta-analysis directly addressed teachers' conceptions of mathematics or teachers' use of ITSs. Indirectly, however, one can assess a variety of teacher uses of ITSs by observing patterns in the research on student use. The next section addresses various approaches to ITS use by teachers.

Patterns of Research and Notable Findings in Intelligent Tutoring Systems

This section reports on the teaching trends towards using ITSs instead of traditional instruction and ITSs as a supplement to classroom instruction. It also reports on comparisons of individual ITSs and ITS instructional strategies.

Intelligent tutoring systems vs. traditional instruction. For mathematics instructors, explicit use of ITSs in mathematics teaching is appealing for a variety of pragmatic reasons. In a secondary school, ITSs instruction facilitates credit recovery or remediation when an instructor works with students of varying individual learning needs and abilities. In a university setting, ITSs may be used to facilitate instruction of pre-collegiate mathematics topics when student to instructor ratios are large. At all grade levels, ITSs may be used for remediation purposes when students are not performing at grade level.

ITS instruction may produce learning gains for students functioning at grade-level (Chu et al., 2014), as well as students who are functioning below grade level in

mathematics (Graff, Mayer, & Lebens, 2008). Even when ITS instruction is beneficial, teachers may still opt to not use it based on student access to technology (Hagerty & Smith, 2005).

Explicit ITS instruction may be more enticing for secondary schools and teachers because it can fill a niche. For example, Beal et al. (2007) used ITSs to prepare students for the ACT. In the quasi-experimental design, 153 high school students used Wayang Outpost (recently renamed MathSpring), while the control group received classroom instruction. Whereas there was no significant difference between pre- and posttests for the control group, the experimental group showed significant overall improvement, $M = 4.13$, $F(1,125) = 12.977$, $p < .001$. Beal et al. noted that it was evident from the pretest scores that teachers from both schools selected students with the lowest mathematics proficiencies to participate in the experimental group. While the study demonstrated that students with the lowest initial mathematics ability made the highest gains, it also demonstrated the propensity for teachers to view ITS technology as an instrument for remediation—even among college-bound students.

When university professors introduce ITSs as an alternative to classroom instruction the results can be beneficial to students. For example, when instructors implemented ITS instruction with a multiple solution path capability for 38 Spanish students in their third year of a college, the experimental group showed significant gains in a pre and posttest design, $F(1, 36) = 2.10$, $p < .001$, while the control group showed no difference (Arevalillo-Herráez et al., 2013). Similarly, Taylor (2008) implemented an ALEKS course in intermediate algebra and found that the experimental group showed

greater gains after the semester course (16.56 to 20.56, $d = .611$) than the control group. Taylor also found that the anxiety levels for individuals in the experimental group decreased by a larger amount than that of the control group. Therefore, teachers might not only implement ITS instruction for the positive mathematical effects, but also for the increased emotional effects. In contrast, Hrubik-Vulanovik (2013) found no differences between students in an ALEKS course and their contemporaries in a traditional course after entering their subsequent paper and pencil math classes together.

Comparison of intelligent tutoring systems. To date, one published study exists comparing tutoring systems. Sabo, Atkinson, Barrus, Joseph, and Perez (2013) placed 31 students in a summer mathematics remediation program on the ALEKS or Carnegie systems for 4 hours per day for 14 days. The two groups of students studied arithmetic and algebra. The pre- and posttest experimental design showed significant gains for both groups of students but produced no significant difference between the groups. It is noteworthy that Carnegie's Cognitive Tutor is not intended as a standalone program. While the study by Sabo et al. suggests that students would benefit equally from ALEKS or Cognitive Tutor, teachers' conceptions of mathematics might produce a preference towards one or the other.

Supplementary instruction. This section describes teacher use of ITS as supplemental instruction. There are various methods of implementing supplemental instruction. Supplemental use of ITSs can be built into a school day as part of a mathematics class or in an additional lab. ITSs can be used as an after-school program where students receive additional tutoring. ITSs may also be used as homework for

mathematics practice outside the supervision of a teacher. Each of these uses is discussed in further detail below.

***Supplemental instruction at school*.** During-school programs have the distinct advantage of allowing for greater (and even mandatory) participation while also affording teachers the opportunity to monitor students. In a university study, Buzzetto-More and Ukoha (2009) found that students were unlikely to complete required ITS assignments until researchers added a mandatory lab to the remedial algebra course because the majority of the students indicated that they were more likely to access the program on campus. Once supplemental ITS use was required and monitored, Buzzetto-More and Ukoha found that student dropout rates decreased, and student pass rates increased.

In various studies of secondary teachers of mathematics, teachers responded favorably to the supplemental program Cognitive Tutor (Carnegie Learning, 2001; Morgan & Ritter, 2002; J. E. Smith, 2001). It is noteworthy that even when student outcomes for the Cognitive Tutor produced no significant gains over the IMP mathematics curriculum, teachers still preferred the use of the Cognitive Tutor (Carnegie Learning, 2001). What makes this noteworthy is that both IMP and Cognitive Tutor address conceptual understanding through inquiry and exploration (Carnegie Learning, 2017; It’s About Time Interactive, 2012). It is possible, therefore, that there may have been something particularly appealing about the ITS component of Cognitive Tutor that appealed to teachers. This seems especially likely when considering that some teachers find the textbook component of the curricula for Carnegie unengaging (Pane et al., 2010).

Required computer time in a K-12 setting may be less appealing or affordable for

teachers. If a teacher does not have a classroom set of computers, moving to an alternate location in the school requires coordination with other teachers and may interfere with the teaching progression (Horn & Staker, 2015). Because ITS programs suggest a set number of user hours per week (Carnegie Learning, 2017; McGraw Hill Education, 2017), the availability of a computer lab could also cause teachers to choose not to implement the ITS curriculum with fidelity to the required time of use.

Another potential disadvantage is that supplemental ITS use does not always produce the intended educational gains either because anticipated learning goals were not met, or because the ITS content assignment does not match the content tested in end-of-year tests (Calhoun, 2011). Inability to achieve the desired learning outcomes are demonstrated in various studies (Calhoun, 2011; Dynarski et al., 2007; Pane et al., 2010; Zacamy, Miller, & Cabalo, 2008). But when teachers implement supplemental ITSs for multiple years, student learning gains increase after the first year (Campuzano, Dynarski, Agodini, & Rall, 2009), which indicates that there is a learning curve for teachers and that short-term implementation may not provide substantial learning outcomes.

In Calhoun's (2011) study of an ITS intervention with ninth-grade students, teachers implemented supplemental ITS instruction by increasing daily mathematics exposure through a required lab. Teachers assigned the students, the majority of whom were performing below grade level, content through the ITS which aligned with fifth-grade curriculum standards. Student performance on the ninth-grade end of level test was disappointingly low and the program was terminated after the first year (J. Calhoun, personal communication, June 14, 2016). This study again highlights the propensity of

teachers to use ITS as a form of remediation.

Supplemental instruction afterschool. Another way for teachers to increase student exposure to mathematics is through afterschool intervention. Afterschool intervention can be advantageous because teachers can supply students with increased exposure to mathematics. In studies by Craig et al. (2013), and Huang et al. (2016), implementation of after school ITS programs for sixth graders were compared with teacher-led instruction in a traditional I do – we do – you do format. Teachers saw comparable performance with two notable differences. First, fewer teachers were needed to conduct the ITS instruction. Second, there was more variability among student outcomes in teacher-led instruction, with respect to gender and race, than in ITS instruction. Thus, teachers may implement ITSs to promote student learning while minimizing gender and ethnic bias (Huang et al., 2016).

Supplemental homework instruction. The third way that teachers implement ITSs as supplemental instruction is through homework assistance. Homework assistance may include programs accompanying the textbook, book assignments submitted through ASSESSment.

Several online programs exist which accompany textbooks and are intended to act as tutors. Pearson's MathXL, is an ITS which allows teachers to generate homework assignments to compliment classwork. The ITS homework is specific to the day's lessons, and allows students to receive immediate feedback and hints, or similar questions to those to which they may desire additional practice. ASSISTments is a program created through government grants which turns textbook assignments into ITS assignments.

Through ASSISTments, students complete textbook-based homework assignments then submit the assignments through a web portal that gives immediate feedback.

Burch and Kuo (2010), as well as Singh et al. (2011), conducted studies of on-line homework and concluded that the feedback feature was essential to student success. Singh et al. also compared the effects of student feedback from the instructor and student feedback from the computer. While student feedback from the computer was shown to promote statistically positive results when compared to ITS homework without feedback, an ANOVA demonstrated similar and significant results for teacher feedback given in a timely manner.

Even though the results from the Burch and Kuo (2010) and Singh et al. (2011) studies may not demonstrate an advantage in learning for students, teachers may find the system advantageous for pedagogical considerations. Student completion of homework on-line necessarily reduces the amount of paperwork for teachers (Stillson & Nag, 2009). It may also ensure that student and teacher interactions focus on more serious conceptual misunderstandings instead of small miscalculations.

Summary

Teachers' conceptions of mathematics consist of both knowledge and beliefs (Steele & Widman, 1997). While previous research revealed a relationship between teachers' conceptions of mathematics and their use of some technologies (Lee, 2007; Tondeur et al., 2016; Wachira et al., 2008), there is a deficit in research on teachers' conceptions of mathematics and their use of ITSs. Understanding how teachers'

conceptions of mathematics relate to their ITS use requires that we know how teachers use ITSs. Previous ITS research, however, has focused on student outcomes in experimental settings rather than teacher practices with ITSs.

To further an understanding of teacher’ use of ITSs, this literature review explored ITS research by focusing on the implementation practices of the researchers or teachers involved in the studies. One of the major findings is that teachers can use ITSs to promote learning despite large student-to-teacher ratios. Teachers can also use ITSs to assist students in learning and practicing mathematics outside the mathematics classroom.

CHAPTER III

METHODS

This study investigated the relationship between teachers' conceptions of mathematics and their use of ITS. An exploratory convergent mixed methods design was used to collect qualitative and quantitative data simultaneously, analyze it separately, and then merge the two data sources (Creswell & Plano Clark, 2018).

Research Questions

The overarching research question in this study was: What is the relationship between teachers' conceptions of mathematics and their use of ITSs for mathematics instruction? Table 3 contains an overview of the chapter information for the following research questions.

Questions Answered Using Quantitative Data

1. What is the relationship between teachers' conceptions of mathematics and their use or non-use of ITSs? This question addresses whether a teacher chooses to use ITSs or not.

2. What is the relationship between teachers' conceptions of mathematics and their use of non-ITS math-focused technologies?

3. Among those teachers who use ITSs, what is the relationship between their conceptions of mathematics and how they use ITSs?

Questions Answered Using Qualitative Data

1. Why do teachers use or not use ITSs?

2. How do teachers use different technologies to teach mathematics?

Table 3

Research Question, Instrumentation, and Data Analysis Information

Research questions	Instrument/data source	Data analysis
1. What is the relationship between teachers' conceptions of mathematics and their use or non-use of ITSs?	Five dimensions of the Conceptions of Mathematics Inventory (Grouws et al., 1996) ITS survey question is a yes/no response to ITS use. Non-ITS users answer a yes/no question on previous ITS use.	One 2x5 Mixed Design ANOVA Descriptive Statistics
2. What is the relationship between teachers' conceptions of mathematics and their use of non-ITS math-focused technologies?	Five dimensions of the Conceptions of Mathematics Inventory (Grouws et al., 1996) ITS survey question gathers information about non-ITS math-focused technologies.	Three Separate 2x5 Mixed Design ANOVAs
3. Among those teachers who use ITSs, what is the relationship between their conceptions of mathematics and how they use ITSs?	Five dimensions of the Conceptions of Mathematics Inventory (Grouws et al., 1996) ITS survey question gathers information about ITS use.	Four Separate 2x5 Mixed Design ANOVAs
4. Why do teachers use or not use ITSs?	ITS survey questions gather open-response information about non-ITS use. ITS Survey questions gather open-response information on ITS use.	Qualitative responses coded using open-coding as outlined by Creswell (2003, 2013), with the use of memos, initial codes, axial codes, and integration.
5. How do teachers use different technologies to teach mathematics?	ITS survey questions gather open-response information on non-ITS math-focused technologies.	Qualitative responses coded using open-coding as outlined by Creswell (2003, 2013), with the use of memos, initial codes, axial codes, and integration.

Research Design

To better understand teachers' conceptions of mathematics and ITS use, this exploratory study employed a convergent mixed methods research design for the collection of qualitative and quantitative data (Creswell & Plano Clark, 2018). In a convergent mixed methods research design the researcher collects both quantitative and qualitative data during the same phase of the study. This type of design was employed because it brought greater insight to the problem than could have been obtained using only qualitative or quantitative data separately. It was also chosen for this dissertation because collecting qualitative and quantitative data from each participant was important under the time constraints of the study (Creswell & Plano Clark, 2018).

The exploratory convergent mixed methods design contains four steps (see Figure 3). During the first step, the researcher designs the quantitative and qualitative strands, then collects the quantitative and qualitative data. During the second step the research analyzes the quantitative and qualitative and qualitative data separately. In the third and fourth steps the researcher merges and interprets the two sets of results.

Participants and Setting

A total of 164 mathematics teachers from 19 junior high schools and one middle school in three school districts were contacted. Junior high and middle school teachers were selected for the study because of the diversity of their mathematical backgrounds (Schmidt et al., 2007). Junior high school mathematics teachers may have entered the

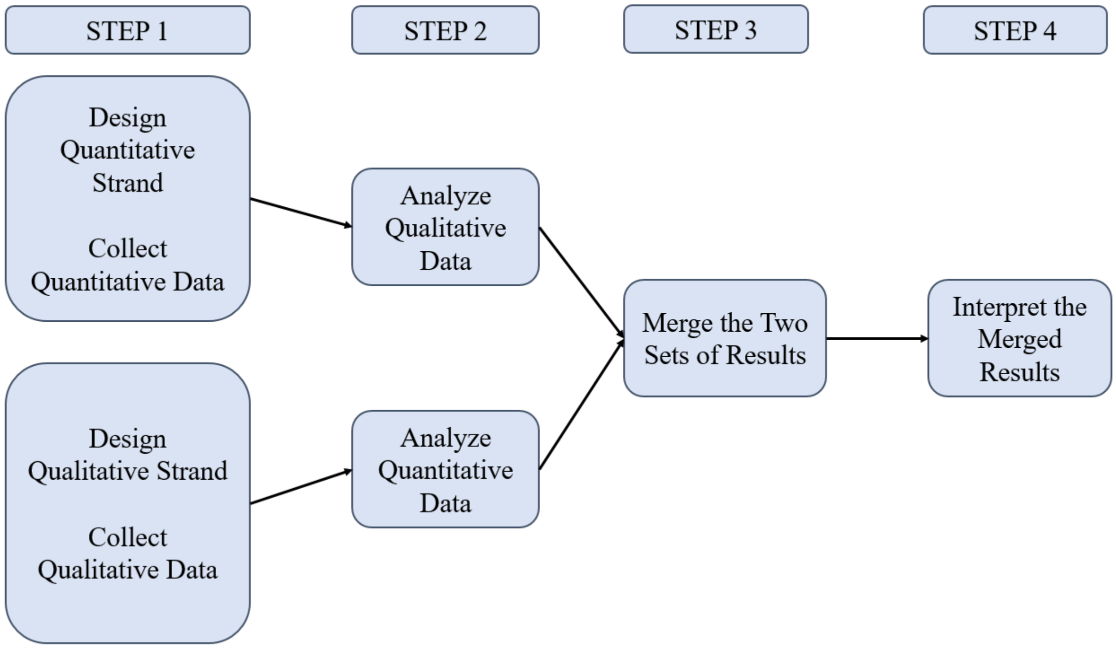

Figure 3. The exploratory convergent mixed methods design.

field after having taught elementary school and attaining a mathematics endorsement. They may also have mathematics degrees. The selection of three school districts, through purposeful sampling, was intended to include teachers from geographically and economically diverse school districts in the western U.S. The largest was an urban school district which served approximately 70,000 students. Approximately 22% of the students in the urban school district received free or reduced lunch. Sixteen of the 20 schools in the study were in the urban district. The second school district served approximately 12,000 students in a metropolitan area where approximately 79% of the students received free or reduced lunch. Each of the three junior high schools in the metropolitan district were Title I schools. The third was a rural school district serving approximately 2,900 students where approximately 20% of students received free or reduced lunch. Geographically, the rural school district, tucked in a mountain valley, was the third

smallest in the state.

Ninety-four of the 168 teachers invited to participate in the study completed the CMI. (One teacher's responses were removed from the survey before analyzing the data. This was because the teacher's answers to the survey questions strongly suggested that the teacher did not read the questions.) This an appropriate response rate for a survey (Baruch & Holtom, 2008). This sample size is consistent with the observation by Creswell and Plano Clark (2018) who observe that a richer blending of qualitative and quantitative data occurs with a sample size of approximately 20-30 individuals in a convergent mixed method design despite the loss of statistical power.

The participants ranged in age from 23 (recently graduated from college) to 65 (near retirement age). While both male and female teachers participated in the study, demographic patterns in the teacher population indicate that most of the teachers were Caucasian (Wood, 2015).

Instrument

The primary instrument used in this study was a survey that included teacher technology use questions and questions from the CMI. The choice of a survey instrument to collect data was appropriate for the following reasons. First, this was a small study with limited resources. Collecting information through a survey allowed distribution to a large number of teachers for a relatively nominal cost (Coastal Services Center, 2007). Second, this type of data collection is a common practice for dissertation research which collects data at a single time from a geographically large region (Punch, 2003). Third,

survey research allows for the collection of data while reducing the bias of a face-to-face interview. Finally, the use of a survey allowed for the collection of both quantitative and qualitative data from the same individuals. This facilitated corroboration and direct comparison of the two types of data (Creswell & Plano Clark, 2018)

The survey had two sections (see Appendix A). The first section of the survey elicited information about teachers' use of ITSs. The second section of the survey measured teachers' conceptions of mathematics using questions from the CMI. The section on teacher use of ITSs was placed first in the data-gathering process because it contained open response questions. It was expected that the teachers would respond more thoughtfully to the open-response questions at the beginning of the survey rather than the end due to fatigue. In addition to the two main sections of the survey, it also contained a link to a second survey for collecting participant information to disseminate incentives. The two survey components for the main survey are described in the next section.

First Section of the Survey: Teacher Use of Intelligent Tutoring Systems

The first section of the survey contained questions eliciting information on teachers' use of ITSs (see Appendix A). The data collected and analyzed quantitatively were used to answer questions 1-3. The data collected and analyzed qualitatively were used to answer questions 4-5.

Survey questions for the first section of the survey were written to reflect the reasons that teachers could choose to use ITSs or other mathematics-focused technology. This section of the survey also elicited information on why teachers could choose not to

use ITSs. The contents of the questions were informed by the literature review and while piloting the survey instrument.

For example, one question asks, “Do you normally assign student use of ITS for any of the following reasons?” Optional responses were: (a) learning new concepts, (b) learning new procedures, (c) practicing procedures, and (d) filling in gaps in student knowledge. These categories of responses were chosen because teachers use graphing calculator technology to enhance conceptual understanding, as well as to perform routine calculations (Brown et al., 2007; Doerr & Zangor, 2000; Lee, 2007). It was likely that teachers would have similar reasons for implementing the ITS. Teachers also use ITS technology for remediation (Calhoun, 2011; Craig et al., 2013; Huang et al., 2016). The term *filling gaps* was used instead of *remediation* to describe using ITSs for deficits in knowledge. This was because remediation often refers to courses for students who are functioning below grade-level. An example of a gap in knowledge for a seventh-grade student might be an understanding of fractions exclusively as part to whole relationships. A gap in knowledge for an algebra student could be the lack of understanding of x and y coordinates in a unit on graphing lines.

Another question gathered information about mathematics-focused non-ITS technology. Teachers were asked if they used graphing calculators, dynamic geometry software, Desmos, or “other” technology. Graphing calculators and dynamic geometry software are well established tools offering teachers the opportunity to teach constructively with technology (Baki, Kosa, & Guven, 2011; Bhagat & Chang, 2015; Brown et al., 2007; Dewey, Singletary, & Kinzel, 2009; Doerr & Zangor, 2000). Because

Desmos is relatively new, research on it is limited. However, practitioner researchers are beginning to publish articles on its potential as a tool for constructivist teaching practices in the mathematics classroom (Bourassa, 2017; King, 2017; Stohlmann, 2017).

Survey questions were written to minimize researcher bias. The questions were written using "straight forward" words (Fink, 2003). Questions were then presented to non-ITS using teachers to determine if the intended meaning of the questions matched the teachers' understood meaning. Questions were also structured to use the present time. Using wording such as "normal activities" increased the likelihood that teachers would recall their most prominent teaching practices (Saris & Gallhofer, 2014).

Second Section of the Survey: Teachers' Conceptions

The second section of the survey included five of the seven dimensions from the Grouws et al. (1996) Teachers' CMI. Each dimension contained eight questions, for a total of 40 questions. Responses to the Teachers' CMI were used in the quantitative analysis.

The five dimensions included from the CMI measured teachers' conceptions of: (1) the composition of mathematics, (2) the structure of mathematics knowledge, (3) doing mathematics, (4) validating ideas in mathematics, and (5) learning mathematics. Each dimension measured teachers' conceptions of mathematics as positioned on a spectrum between two poles (see Figure 4) using a 5-point Likert scale. The original version of the CMI was created using a 5-point Likert scale. Over time the CMI was used with a 6-point scale to encourage individuals to indicate a preference towards one

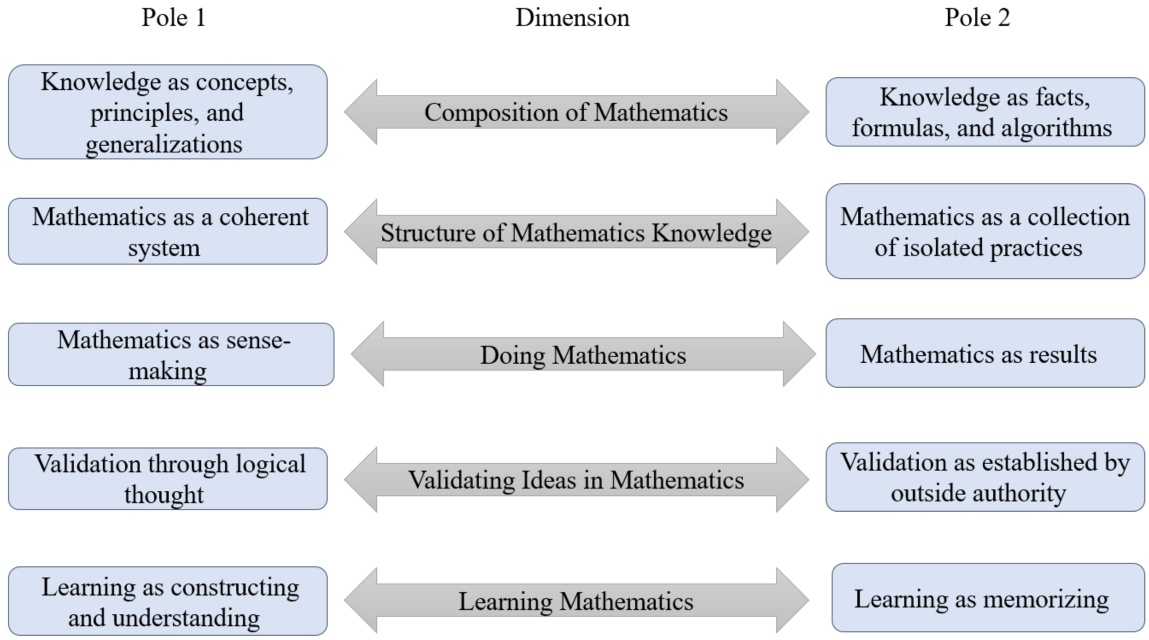

Figure 4. Dimensions of conceptions of mathematics inventory.

dimension or the other (C. L. Howald, personal communication, December 7, 2018). This study utilized the original 5-point scale. The poles considered in the composition of mathematics dimension are mathematics as concepts, principles, and generalization versus knowledge as facts, formulas, and algorithms. The poles in the structure of math knowledge are mathematics as a coherent system versus mathematics as a collection of isolated practices. The poles considered in the doing mathematics dimension are mathematics as sense-making versus mathematics as results. The poles for the validating ideas in mathematic dimension are validation through logical thought versus validation through outside authority. The poles of the learning mathematics dimension are learning as constructing and understanding versus learning as memorizing.

The five conceptions included in the first section of the survey were selected based on their relevance to junior high school mathematics instruction. The two

dimensions from the Conceptions of Mathematics survey not included were the conceptions of: (1) the status of mathematics, and (2) and the usefulness of mathematics. They were not included because their connection to middle-grades mathematics are not as strong as the other five dimensions. For example, the status of mathematical knowledge considers mathematics as a dynamic field versus mathematics as a static entity. The mathematical topics addressed in middle grades are relatively static. The conception of the usefulness of mathematics measures mathematics as a useful endeavor versus mathematics as a school subject with little value.

Data Collection

University Institutional Review Board (IRB) approval was granted in late August 2018 (see Appendix C). Email permission from the rural school district's superintendent to conduct the survey was received in late August (see Appendix D). IRB approval from the two larger school districts was granted in September (see Appendix E). Participant recruitment and dissemination of the survey occurred in October.

The researcher followed the Tailored Method Design (Dillman, 2010) for distribution of the survey. Before distributing the survey, the researcher acquired email addresses via school web pages. In the two smaller districts, the researcher sent an initial email to teachers at individual schools informing them of the intent to distribute an email survey. This offered an opportunity to check the validity of the email information and gather information about mathematics teachers who were employed at the schools, but not listed on the school web pages. In the larger school district, web pages were

incomplete during this phase of the research. However, the larger district had a mailing list available to the district math specialist.

In mid-October, the researcher sent an email invitation (see Appendix B) to participate in the survey to all participants. In the inner-city schools and the rural school, the researcher sent email invitations directly to the teachers. In the urban school district, the mathematics specialist distributed the emails. Teachers received two additional reminders to complete the survey within the 3 weeks that the survey was active. The second email was sent during the second week and a third email was sent 2 days prior to the closing of the survey.

During the time that the survey was active, the urban school district's web pages were updated, and email information was made available for the 148 mathematics teachers therein. It was in the second email (the first reminder) that the researcher sent emails, by school, to all the teachers whose names appeared on the web pages. They received a second invitation to participate in the survey and a request to rectify any mistakes to the researcher's mailing list.

Two days prior to the closing of the survey, the researcher sent a personalized email to each of the 168 teachers in all three school districts along with the original email invitation. They were informed that the survey was open for two more days.

Participants were permitted to complete the survey at a time and in a place of their own choosing. No identifying information was shared during the survey completion. To maintain confidentiality, participants were asked to complete two surveys. The first survey collected data on teacher conceptions and ITS use. Upon completion of the first

survey, teachers received a link to complete a second survey requesting a preferred email address to receive a $10 Amazon gift card. The researcher distributed activation codes for gift cards to the participants four days after the survey was closed.

Data Analysis

Following the convergent design analysis prescribed by Creswell and Plano Clark (2018), the researcher analyzed the qualitative and quantitative data separately through standard quantitative and qualitative procedures. This section details the analysis, merging, and interpreting of the quantitative and qualitative data.

Quantitative Analysis

The second section of the survey, which contained items from the CMI, were reported on a five-point Likert scale. Within each of the dimensions, four of the eight questions were written such that an answer of “strongly agree” indicated one pole, and four questions are written such that an answer of “strongly disagree” indicated the same pole. For example, the dimension describing the composition of mathematical knowledge (knowledge as concepts, principles, and generalizations versus knowledge as facts, formulas, and algorithms) contained the following two items:

1. There is always a rule to follow when solving a mathematical problem.
2. While formulas are important in mathematics, the ideas they represent are more useful.

An answer of “strongly agree” on the first question indicated that a teacher might have a conception of mathematical knowledge as a collection of facts, formulas, and

algorithms, whereas an answer of “strongly agree” on the second item indicated that a teacher might have a conception of mathematical knowledge consisting of concepts, principles, and generalizations. Thus, responses to the first four items in a dimension were scaled in ascending order where the number 1 corresponded to “strongly disagree” and the number 5 corresponded to “strongly disagree.” Answers to items 5 through 8 in each dimension were scaled in the opposing order such that the number 1 corresponded to “strongly agree” and the number 5 corresponded to “strongly disagree.” Participants’ responses to the questions within one dimension were averaged in Excel and before transferring them to SPSS for analysis. The averages were used in the analysis of the three quantitative questions.

Question 1. The researcher employed a 2x5 mixed design ANOVA to answer the question: “What is the relationship between teachers’ conceptions of mathematics and their use or non-use of ITSs?” The two-level between-subject factor denoted the response (yes or no) to the question: “Do you use an intelligent tutoring system?” The five-level within-subjects factor denoted the specific dimension. The dependent variable was each dimension’s average regarding sets of eight five-point Likert items from the CMI (Grouws, 1996). Teachers who answered “no” to the question “Do you use an intelligent tutoring system?” received a yes/no follow up question asking if they had ever used an ITS to teach mathematics. The researcher calculated and reported the percent of respondents who answered yes or no.

Question 2. The researcher employed three separate 2x5 mixed design ANOVA models to answer the question: “What is the relationship between teachers’ conceptions

of mathematics and their use of non-ITS math-focused technologies?" The two-level between-subject factor denoted the response (yes or no) to each option of the question: "Which of the following types of technology do you normally use for mathematics instruction?" The researcher treated the three responses as three separate yes/no questions and modeled them independently. Similar to question 1, the five-level within-subjects factor denoted the specific dimension and the dependent variables were each dimension's average regarding sets of eight 5-point Likert items from the CMI (Grouws, 1996).

Question 3. The researcher employed four separate 2x5 mixed design ANOVA models to answer the question: "Among those teachers who use ITSs, what is the relationship between their conceptions of mathematics and how they use ITSs?" This analysis differed from the prior two, in that it was restricted to the sub-sample of participants who answered "yes" to indicate that they were currently using ITSs in their classroom, but otherwise followed the same format as question 2. The two-level between-subject factor denoted the response (yes or no) to each option of the question: "Do you normally assign student use of ITSs for any of the following reasons?" The four responses were treated as four separate yes/no questions and modeled independently. Similar to question 1, the five-level within-subjects factor denoted the specific dimension and the dependent variables were each dimension's average regarding sets of eight five-point Likert items from the CMI (Grouws, 1996). The researcher created tables and graphs of the quantitative data to indicate the results with significant interactions.

All analyses were conducted in SPSS 23.0 (IBM Corp., 2015). Significance was assessed with alpha = .05 for assumption analyses and alpha = .05/8 = .00325 via a

Bonferroni correction for multiple comparisons regarding the 8 independent mixed design ANOVA analyses.

Qualitative Analysis

The qualitative analysis employed a *constant comparative* method (Creswell, 2013). The constant comparative method, originally used with grounded theory, involved comparing one piece of data with all others to determine similarity, differences, and relationships (Creswell & Plano Clark, 2018). The relationship that this research sought to explain was between teachers' conceptions of mathematics and their ITS use.

The responses to the open-ended survey questions were examined in Excel after the survey was closed and all responses were collected. All responses were first *open coded* (Creswell, 2013). Open coding was the "interpretive process in which data is broken down analytically" and it "stimulates generative and comparative questions to guide the researcher upon return to the field" (Corbin & Strauss, 1990, p. 12). All responses were read multiple times while making *memos* to get a general sense of the data (Creswell, 2013; Creswell & Plano Clark, 2018). Memos were short phrases, key concepts, or general ideas (Creswell, 2013) that were used to create *codes.* Codes were themes manifest in the data (Creswell & Plano Clark, 2018). Initially, responses were read and memos were made to questions in sequential order. For example, responses to question one were read before responses to question two, etc. As codes were refined, questions with the same or similar codes were analyzed concurrently.

At various times throughout the coding process, an undergraduate research assistant with experience in mathematics education research participated as a second

coder. After the researcher coded all the responses, the research assistant independently coded 20% or more of the data for each set of responses. The research assistant used either the codes provided or created her own. All discrepancies between the two codes were discussed and amended. Those discussions assisted in the revision of existing codes or the creation of new codes, whereupon the researcher again coded the data. After recoding the data, the research assistant separately coded a different 20% of the data. This process continued until there was at least an 80% intercoder agreement on selected passages (Miles & Huberman, 1994).

The coding process just described led to the use of codes describing different types of differentiation described in Chapter IV. It seemed apparent that the types of ITS uses that teachers were describing could be considered a form of differentiation. After the researcher and research assistant could not come to an 80% agreement on the codes for selected teacher responses, the researcher found a definition of differentiation by Tomlinson (2005) that described three components of differentiation. *Content* differentiation describes what a student learns. *Process* differentiation describes how students learn. *Product* differentiation describes how learning is demonstrated. The researcher recoded the data using those definitions of differentiation as categories. Responses which described differentiation, but were not easily categorized into content, process or product, were given a broader code of *differentiation* (see Table 7 in Chapter IV). After the research assistant recoded another 20% of all responses for ITS use, the threshold of 80% inter-coder agreement (Miles & Huberman, 1994) was easily surpassed.

Throughout this process a constant comparative approach was used (Glaser,

1965). As codes were created and refined, they were compared with different teachers' responses to the same question as well as the same teacher's responses to different questions. This approach helped to assure that the codes accurately described the responses being coded.

The inductive approach to obtaining codes and intercoder reliability is verifiable through an audit trail (Creswell & Plano Clark, 2018) of multiple Excel files. As the researcher refined the codes and the research assistant recoded the data, dated excel files were saved throughout the process to demonstrate the coding progress.

The created codes were the basis for the *axial coding*. Axial coding consisted of finding relationships among the chosen codes to ultimately write a narrative (Creswell, 2013). During axial coding phase, the researcher created subcategories and drew connections between the participants' responses and the identified categories (Creswell & Plano Clark, 2018). This coding procedure was used for each open-ended survey response. After coding each of the open-response questions separately, the researcher compared the results to the separate questions to determine any overarching themes. The researcher created tables with the major codes generated from the open coding process, were presented in tables along with examples of the coded data.

Mixed Methods Analysis

After analyzing and organizing the quantitative and qualitative data separately, the researcher merged the two results and interpreted them together to answer the overarching research question: "What is the relationship between teachers' conceptions of mathematics and their use of ITSs for mathematics instruction?" *Inferences* in mixed

methods studies are conclusions drawn from the separate analyses (Creswell & Plano Clark, 2018). The inferences drawn by combining the qualitative and quantitative analyses are known as *meta-inferences* (Creswell & Plano Clark, 2018). This included the identification of results from the quantitative and qualitative questions that *converged* and *diverged*. Data converges when the quantitative and qualitative results support one other, and diverge when they do not (Creswell, 2003). The results section synthesizes this convergent and divergent data in a narrative to describe teachers' conceptions of mathematics and their use of ITSs for instruction.

Limitations

There were two major limitations to this study. First, the length of the survey might have been a limitation to the quality of data collected. Teachers answered between 45 and 55 questions, which may have caused fatigue and altered their responses. However, while piloting the survey, participants indicated that they completed the survey in approximately 15 minutes. Second, while this research may offer insight into ITS use by secondary mathematics teachers in general, caution must be exercised before applying these findings outside of middle school or junior high school mathematics classrooms.

Validity

The mixed methods approach to this study required that validity was ensured through both quantitative and qualitative aspects of data collection and analysis. Validity was established with the quantitative data by using an established instrument in a method similar to previous use. The CMI has been used for an NSF-funded (OERL, 2018) study

as well as two doctoral dissertations (Howald, 1998; Lee, 2007). The method that this approach employed was analogous to that used by Lee in her doctoral dissertation exploring teachers' conceptions of mathematics and their use of graphing calculators. Ensuring validity for the qualitative data analysis was done through the use of multiple coders (Creswell, 2013) with at least an 80% inter-coder agreement on mutually coded passages (Miles & Huberman, 1994).

Summary

This exploratory study employed a convergent mixed methods research design collecting qualitative and quantitative data to answer the question: "What is the relationship between teachers' conceptions of mathematics and their use of Intelligent Tutoring Systems for mathematics instruction?" The researcher collected quantitative data using five dimensions of the Grouws et al. (1996) CMI and through survey questions on teachers' use of ITSs. Quantitative data was analyzed using separate 2x5 mixed design ANOVA models. The researcher collected qualitative data through survey questions eliciting information about how and why teachers use ITS or non-ITS math-focused technologies. Qualitative and quantitative data were first analyzed separately then merged and analyzed collectively.

CHAPTER IV

RESULTS

The purpose of this research study was to examine the relationship between teachers' conceptions of mathematics and their use of ITSs for mathematics instruction. Ninety-three junior high school and middle school teachers from three school districts responded to questions on a two-part survey. The first part of the survey gathered information on teachers' ITS use and non-use as well as the use of other mathematics-focused technologies. The second part of the survey included 40 Likert questions from the CMI.

The results in this chapter are organized to answer each of the five research questions. After addressing each research question separately, the quantitative and qualitative results are merged and presented together to address the overarching research question.

Quantitative Questions

Question 1. Teachers' Conceptions of Mathematics and Intelligent Tutoring Systems Use

A 2x5 mixed design ANOVA was employed to answer the question: "What is the relationship between teachers' conceptions of mathematics and their use or non-use of ITSs?" The two-level between-subject factor denotes the response (yes or no) to the question: "Do you use an intelligent tutoring system?" The five-level within-subjects factor denotes the specific dimension. The dependent variable is each dimension's

average regarding sets of eight 5-point Likert items composing the CMI (Grouws, 1996).

Of the 93 participants, 71 indicated that they used ITSs for mathematics instruction and 20 indicated that they did not. There were no outliers, as assessed by examination of studentized residuals for values greater than ±3 across each dimension. Conception scores were normally distributed for users and non-users of ITSs except on the dimension of structure as assessed by Shapiro-Wilk's test ($p < .05$). For the dimension of structure, the non-ITS users' scores were normally distributed, but the ITS users' scores were not. The 71 scores for the structure dimension were bimodal (see Figure G1 in Appendix G). Homogeneity of variances and covariances were established through by Levene's test of homogeneity ($p > .05$), Box's M test ($p = .217$). Because Mauchly's test indicated that the assumption of sphericity was violated for the two-way interaction, $\chi^2(9) = 110.728$, $p < .001$, the degrees of freedom were adjusted using the Greenhouse-Geisser correction (Cohen, 2013). Similar corrections were made on all subsequent mixed ANOVAs where sphericity was also violated.

The interaction between conception and ITS use was not statistically significant, $F(2.261, 205.710) = 2.420$, $\varepsilon = 0.565$, $p < .084$. This result shows that there was no significant difference between conception scores for teachers who use ITSs and teachers who do not.

Question 2. Teachers' Conceptions of Mathematics and Mathematics-Focused Technology

Three separate 2x5 mixed design ANOVA models were employed to answer the question: "What is the relationship between teachers' conceptions of mathematics and

their use of other math-focused technologies?" The two-level between-subject factor denotes the response (yes or no) to each option for the question: "Which of the following types of technology do you normally use for mathematics instruction?" The categories of responses were (a) graphing calculator, (b) dynamic geometry software (such as GeoGebra or Geometer's Sketchpad), and (c) Desmos. The three responses were treated as three separate yes/no questions and modeled independently. Similar to question 1, the five-level within-subjects factor denoted the specific dimension and the dependent variable was each dimension's average for sets of eight 5-point Likert items composing the CMI (Grouws, 1996). In the three results presented below, there were no outliers, as assessed by examination of studentized residuals for values greater than ±3.

Teachers' conceptions of mathematics and graphing calculator use. Of the 93 responses, 52 indicated that they used graphing calculators for mathematics instruction while 41 indicated that they did not. The Shaprio-Wilk's test revealed that all the conception scores for calculator use and nonuse were normally distributed, except the dimension of structure for graphing calculator users ($p < .05$). The 52 structure scores were skewed slightly right (see Figure G2 in Appendix G). Lavene's test ($p > .05$) and Box's M ($p = .239$) indicated homogeneity of variance and covariance respectively. The assumption of sphericity was violated for the two-way interaction as demonstrated by Mauchly's test $\chi^2(9) = 105.723$, $p < .001$.

The interaction between conception and calculator usage was statistically significant, $F(2.308, 210.018) = 4.703$, $\varepsilon = 0.577$, $p < .001$, $\eta_p 2 = .049$. This result indicates that that there was a statistically significant difference in conception scores

between teachers who use graphing calculators for instruction and those who do not. Sidak's correction for multiple comparisons was applied to post-hoc analysis. For the dimension of learning, there was no significant difference in conception scores between teachers who used and teachers who did not use calculators for instruction, $p = .215$. There were significant differences in conception scores for the other four dimensions. Whereas teachers who used calculators scored higher on the dimension of doing than teachers who did not use calculators, $p = .039$, for the other three conceptions, non-calculator users' scores were statistically significantly higher (see Table 4).

Table 4

Estimated Marginal Means for Teachers' Conceptions and Calculator Use with Post Hoc Interaction Tests

	Calculator							
	No		Yes		Difference			
Dimension	*M*	*SE*	*M*	*SE*	*M*	*SE*	Sig.†	*ES* Cohen's d
Composition	2.308	0.075	2.060	0.067	0.248	0.101	0.016*	0.485
Structure	1.771	0.056	1.543	0.049	0.228	0.074	0.003**	0.446
Doing	3.892	0.064	4.161	0.057	0.179	0.086	0.039*	0.351
Validating	2.027	0.071	1.813	0.063	0.215	0.094	0.025*	0.421
Learning	1.985	0.067	1.873	0.060	0.112	0.090	0.215	-

† *p* values use the Sidak adjustment for multiple comparisons
* Interaction is significant at the 0.05 level.
** Interaction is significant at the 0.01 level.

The higher average composition score for an answer of no indicated that teachers who conceived that mathematics was about concepts, principles, and generalizations were more likely to use graphing calculators for instruction than teachers who conceived that

mathematics was about facts, formulas and algorithms. The higher average structure score for an answer of no indicated that teachers who conceived that mathematics was a coherent system were more likely to use graphing calculators than teachers who conceived that mathematics was a collection of isolated practices. The higher average doing score for an answer of yes means that teachers who conceived of mathematics as a results-centered practice were more likely to use a graphing calculator than those who conceived of mathematics as sense-making practice. The higher average validating score of no revealed that teachers who conceived that mathematical validation should be established through logical thought were more likely to use graphing calculators than teachers who conceived that validation should come through outside authority (see Figure 5).

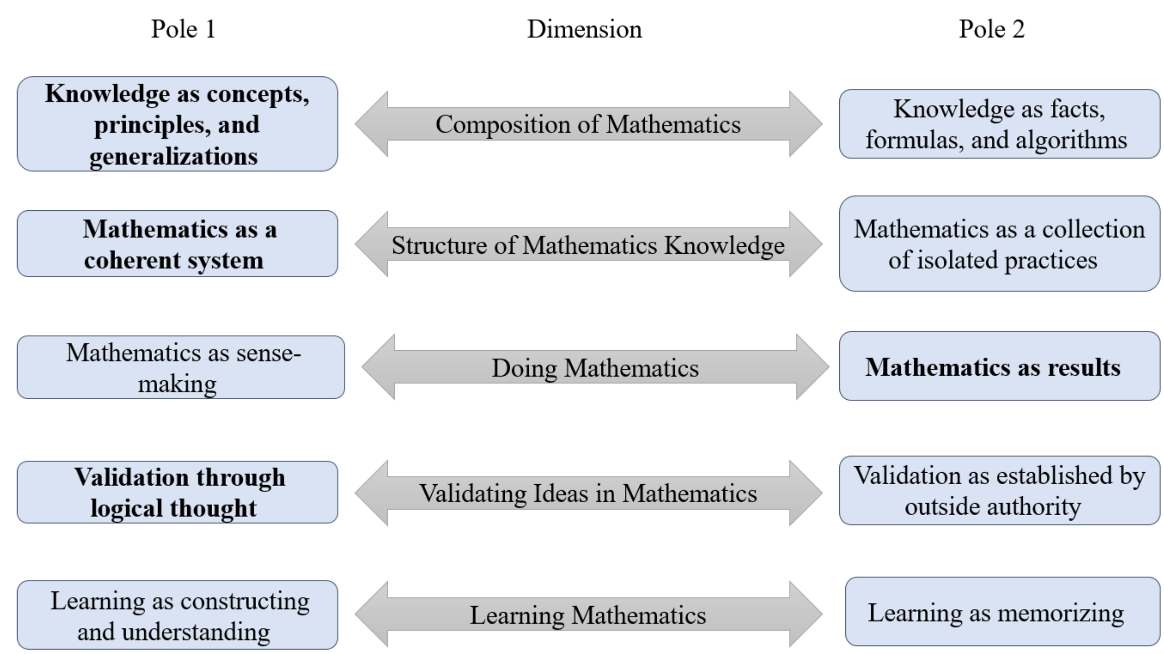

Figure 5. Interpretation of the Sidak adjustment and post-hoc analysis test on graphing calculator use. The bolded lettering indicates the conceptions of teachers who favored graphing calculators for instruction.

Teachers' conceptions of mathematics and Desmos use. The users and non-users of Desmos were more evenly distributed, with 48 indicating that they do and 45 indicating that they do not use Desmos for instruction. In the structure of mathematics, the non-users of Desmos had conception scores which were normally distributed, but the Desmos users' scores were not, as assessed by Shapiro-Wilk's test ($p < .05$). DGS users' conception scores were skewed right (see Figure G3 in Appendix G). Homogeneity of variances and covariances were established with Levene's test ($p > .05$) and Box's test ($p = .253$). Mauchly's test of sphericity indicated that the assumption of sphericity was violated for the two-way interaction, $\chi^2(9) = 110.728$, $p < .001$.

The interaction between conception and Desmos use was statistically significant, $F(2.317, 210.834) = 5.132$, $\varepsilon = 0.579$, $p < .001$, partial $\eta_p 2 = .053$. Sidak's correction for multiple comparisons was applied to post-hoc analysis. While mean differences for teachers who used Desmos were not significantly different for the dimension of doing (p = .050) or learning ($p = 0.167$), they were significantly different for the other three dimensions (see Table 5). For the dimensions of composition, structure, and validating, teachers who used Desmos scored lower on the dimension than those who did not use Desmos.

These findings indicate that teachers who conceived of mathematics as concepts, principles, and generalizations were more likely to use Desmos than those who conceived of mathematics as facts, formulas, and algorithms. Teachers who conceived of mathematics as a coherent system were more likely to use Desmos than those who conceived of mathematics as a collection of isolated practices. Teachers who conceived

that validation should come through logical thought were more likely to use Desmos than those who thought that validation should come through outside authority (see Figure 6).

Table 5

Estimated Marginal Means for Teachers' Conceptions and Desmos Use with Post Hoc Interaction Tests

	Desmos							
	No		Yes		Difference			
Dimension	*M*	*SE*	*M*	*SE*	*M*	*SE*	Sig.†	*ES* Cohen's d
Composition	2.325	0.071	2.023	0.069	0.302	0.098	0.003**	0.592
Structure	1.758	0.053	1.536	0.051	0.222	0.074	0.004**	0.436
Doing	3.994	0.061	4.164	0.059	0.170	0.085	0.050	-
Validating	2.022	0.067	1.799	0.065	0.223	0.094	0.019*	0.437
Learning	1.986	0.064	1.862	0.062	0.124	0.089	0.167	-

† *p* values use the Sidak adjustment for multiple comparisons
* Interaction is significant at the 0.05 level.
** Interaction is significant at the 0.01 level.

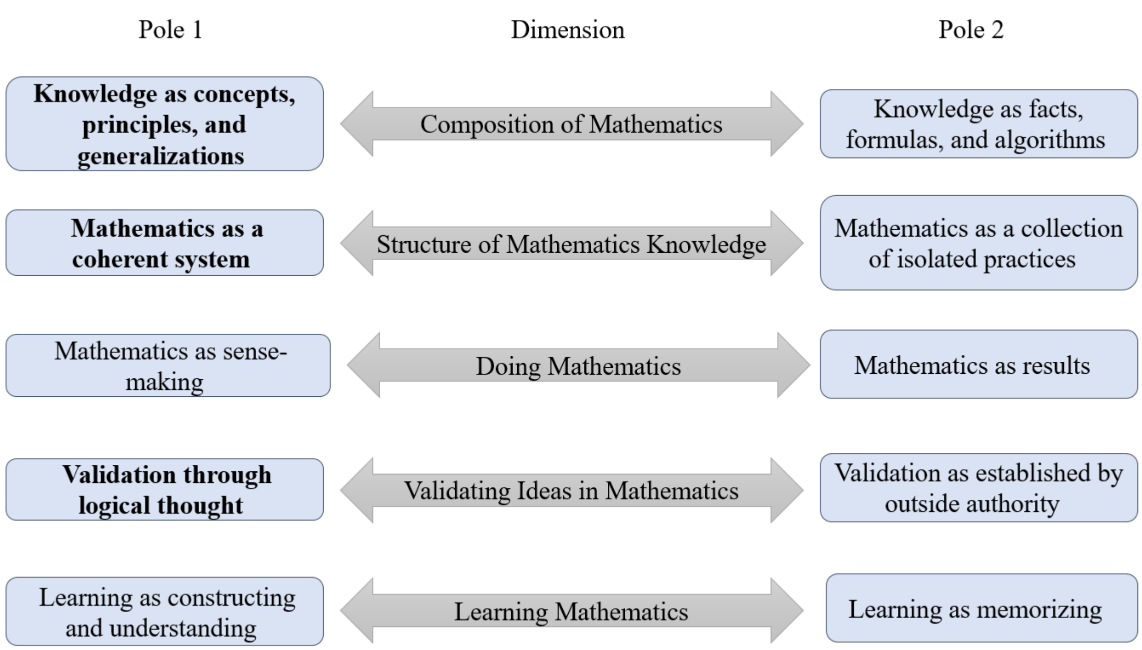

Figure 6. Interpretation of the Sidak adjustment and post-hoc analysis test on Desmos use. The bolded lettering indicates the conceptions of teachers who favored Desmos for instruction.

Teachers' conceptions of mathematics and dynamic geometry software (DGS) use. Of the 93 responses, 19 teachers indicated that they used DGS for classroom instruction while 74 indicated that they did not. Again, DGS users and nonusers' conception scores were normally distributed in all the dimensions except for structure. In this dimension, the non-users' scores had a multi-modal distribution (see Figure G4 in Appendix G). Homogeniety of variances and covariances were established with Levene's test ($p > .05$) and Box's M test ($p = .239$). Mauchly's test indicated that the assumption of sphericity was violated for the two-way interaction, $\chi^2(9) = 110.243$, $p < .001$.

The interaction between conception and DGS use was statistically significant $F(2.271, 206.686) = 3.337$, $\varepsilon = 0.568$, $p < .001$ $\eta_p 2 = .035$. This result shows that there was a significant difference in conception scores between teachers who used DGS for instruction and teachers who did not. Sidak's correction for multiple comparisons revealed a significant difference in means on the dimension of validating, $p = .018$. Teachers who used DGS for instruction scored lower on the validating dimension than teachers who did not. There were no significant differences in mean scores for the other four dimensions (see Table 6). The higher average validating score of no indicates that teachers who had a conception that mathematical validation should be established through logical thought were more likely to use DGS than teachers with a conception that mathematical validation should come through outside authority (see Figure 7).

Question 3. Teachers' Conceptions of Mathematics and Purpose of ITS Use

Four separate 2x5 mixed design ANOVA models were employed to answer the

Table 6

Estimated Marginal Means for Teachers' Conceptions and Dynamic Geometry Software Use with Post Hoc Interaction Tests

	Dynamic geometry software							
	No		Yes		Difference			
Dimension	*M*	*SE*	*M*	*SE*	*M*	*SE*	Sig.†	*ES* Cohen's d
Composition	2.206	0.057	2.026	0.113	0.180	0.127	0.160	-
Structure	1.679	0.043	1.507	0.084	0.172	0.094	0.071	-
Doing	4.041	0.048	4.243	0.094	0.203	0.106	0.058	-
Validating	1.965	0.052	1.684	0.103	0.280	0.116	0.018*	0.539
Learning	1.932	0.050	1.882	0.099	0.051	0.112	0.650	-

† *p* values use the Sidak adjustment for multiple comparisons.
* Interaction is significant at the 0.05 level.

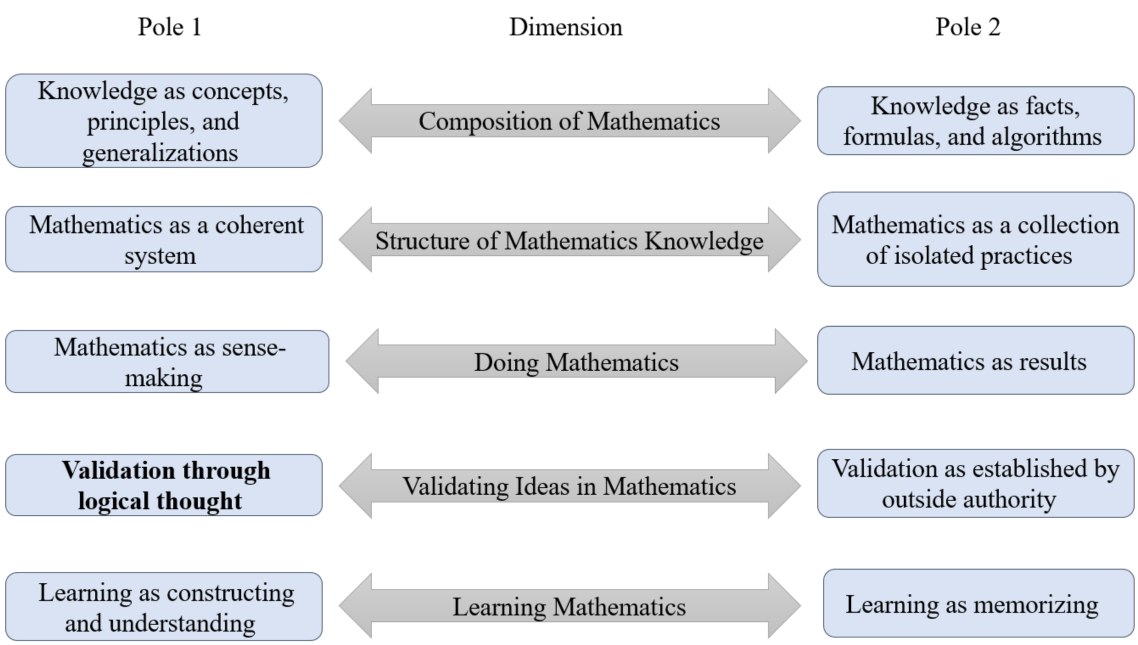

Figure 7. Interpretation of the Sidak adjustment and post-hoc analysis test on Dynamic Geometry Software use. The bolded lettering indicates the conceptions of teachers who favored DGS for instruction.

following question: “Among those teachers who use ITSs, what is the relationship between their conceptions of mathematics and how they use ITSs?” This analysis differed from the prior two, in that it was restricted to the subsample of participants who answered “yes” to currently using ITSs in their classroom, but otherwise will follow the same format as question 2. The two-level between-subject factor denoted the response (yes or no) to each option of the question: “Do you normally assign student use of ITSs for any of the following reasons?” The four responses were treated as four separate yes/no questions and modeled independently. Similar to question 1, the five-level within-subjects factor denoted the specific dimension and the dependent variable was each dimension’s average regarding sets of eight 5-point Likert items composing the CMI (Grouws, 1996). Similar to question 2 results, there were no outliers for any of the analyses below, as assessed by examination of studentized residuals for values greater than ±3. Where violations of normality were present, the mixed ANOVA calculation was still used based on the central limit theorem.

Teachers’ conceptions of mathematics and ITS use to fill gaps in student knowledge. Of the 71 ITS users who responded to this survey, 66 indicated that they use ITSs to fill gaps in student knowledge while 11 indicated that they did not. The small number of individuals who did not use ITSs to fill gaps make testing the assumptions for a mixed ANOVA problematic. Tests of normality were suspect. The Shapiro-Wilk’s test ($p < .05$) demonstrated violations of normality on the dimension of structure for teachers who used ITSs to fill gaps as well as for those who did not (see Figures G5 and G6 in Appendix G). There was homogeneity of variances, as assessed by Levene’s test ($p >$

.05), but Box's M test could not be computed by SPSS. As with the previous analyses, Mauchly's test of sphericity showed a violation of sphericity for the two-way interaction, $\chi 2(9) = 94.913$, $p < .001$.

There was no statistically significant interaction between conceptions and teachers' use of ITSs to fill gaps in knowledge, $F(2.122, 146.441) = .189$, $\varepsilon = 0.531$, $p = .840$. These results indicate that there was no statistical difference in conception scores for teachers who used ITSs to fill gaps and teachers who did not.

Teachers' conceptions of mathematics and ITS use to practice procedures. Of the 71 ITS users who responded to this survey, 60 indicated that they use ITSs for students to practice procedures while 11 indicated that they did not. As with other tests, normality was not present on the dimension of structure. Scores for teachers who used ITSs to teach procedures were multimodal (see Figure G7 in Appendix G). There was homogeneity of variances and covariances as determined by Levene's test ($p > .05$), and Box's M test ($p = .921$). Because Mauchly's test of sphericity indicated that the assumption of sphericity was violated for the two-way interaction, $\chi 2(9) = 92.802$, $p < .001$.

There was no statistically significant interaction between conceptions and teachers' use of ITSs for practicing procedures, $F(2.145, 147.986) = .837$, $\varepsilon = 0.536$, $p = .442$. This result shows that teachers who use ITSs to practice procedures do not have statistically different conception scores than teachers who do not.

Teachers' conceptions of mathematics and ITS use to learn new procedures. Of the 71 responses, 16 indicated that they employ ITSs for students to learn new

procedures while 55 indicated that they did not. The Shapiro-Wilk's test revealed conception scores were normally distributed except on the dimension of learning ($p < .05$). The conception scores for learning for teachers who did not use ITSs to teach procedures were not normal (see Figure G8 in Appendix G). There was homogeneity of variances, as assessed by Levene's test of homogeneity of variance ($p > .05$) for four of the conceptions. However, the conception of "doing mathematics" failed the test of homogeneity, $F(1,69)=6.291$, $p = .014$. There was homogeneity of covariances, as assessed by Box's test of equality of covariance matrices ($p = .041$). Despite the homogeneity of variance for the conception of doing, the mixed ANOVA was still employed. Mauchly's test of sphericity indicated that the assumption of sphericity was violated for the two-way interaction, $\chi2(9) = 94.640$, $p < .001$.

There was no statistically significant interaction between teachers' conceptions and the use of ITSs for learning new procedures, $F(2.126, 146.691) = .234$, $\varepsilon = 0.531$, $p = .804$. Similar to the previous result, this implies that teachers who use ITSs for their students to practice procedures do not have a significantly different conception score than teachers who do not.

Teachers' conceptions of mathematics and ITS use to learn new concepts. Of the 71 responses, 29 indicated that they used ITSs for students to learn new concepts and 42 indicated that they do not. Normality was present except for ITS users on the dimension of structure ($p < .05$). ITS users for conception use had conception scores which were bimodal (see Figure G9 in Appendix G). There was homogeneity of variances, as assessed by Levene's test ($p > .05$) and homogeneity of covariances, as

assessed by Box's test (p = .274). Mauchly's test of sphericity indicated that the assumption of sphericity was violated for the two-way interaction, $\chi2(9) = 101.141$, $p <$ .001.

There was no statistically significant interaction between teachers' conceptions and the use of ITSs for learning new concepts, $F(2.082, 143.651) = 1.760$, $\varepsilon = 0.520$, $p =$ .174. This indicates that there was no difference in conception scores between ITS users who used ITSs to teach new concepts and those who did not.

Qualitative Questions

This section contains the results for the two qualitative questions. The fourth research question focused on teacher use of ITSs and the fifth research question focused on teacher use of mathematics-specific technology for teaching.

Question 4. Why Teachers Use or Do Not Use ITSs

To address research question 4, teachers responded to the question: "Do you normally assign student use of ITSs for any of the following reasons?" Optional responses were: (a) learning new concepts, (b) learning new procedures, (c) practicing procedures, and (d) filling in gaps in student knowledge. If a teacher responded in the affirmative, she/he was directed to a follow-up question. The follow-up questions were: (1) Explain why you use and intelligent tutoring system to teach new concepts; (2) Explain why you use an intelligent tutoring system to teach new procedures; (3) Explain why you use an intelligent tutoring system to practice procedures; and (4) Explain why

you use an intelligent tutoring system to fill in gaps in student knowledge. The following sections detail the results to the analysis for the follow-up questions.

Why teachers use ITS. The overarching theme describing teacher use of ITSs was differentiation. Tomlinson (2005) describes differentiation as altering an approach to learning to change one (or more) of three curricular elements. The first element, *content,* describes what a student learns. The second element, process, describes how students "go about making sense of ideas and information" (p. 4). The third element, *product,* describes the different ways in which student learning can be demonstrated. Responses related to content and process differentiation surfaced with enough regularity that they are introduced here before proceeding to each of the follow-up questions. Six sample responses demonstrating differentiation are given in Table 7.

Table 7

Examples of Differentiation in Teacher Responses

Type of differentiation	Sample response
Content	"Some students are ready to move on and learn something new before the rest of the class."
	"Based on the initial knowledge check, it pushes my students to learn new things that we have not taught yet."
Process	"Sometimes students don't understand my explanation but seeing it another way and being able to practice it many times helps."
	"Many students enjoy interacting with technology as a way to learn new things."
Non-specific	"Students can learn at their own pace and move forward if they are ready."
	"[It] allows students to work and practice the individualized items they are learning."

The examples from Table 7 of content differentiation demonstrate how teachers used ITSs to modify what individual students learn. In the first response, the teacher indicated that the ITS was used to give individual students access to new content while the rest of the class was working with current content. The second response demonstrates how a teacher allowed the ITS to guide student learning. The process differentiation exemplifies the way a teacher used the ITS to facilitate a different mode of instruction. The first response indicates the teacher was using the ITS to (1) present content in a different way, and (2) give multiple practice opportunities. The second response demonstrates how a teacher valued learning through an ITS because it incorporated technology practices in the classroom.

The nonspecific differentiations are given as examples of differentiation which are not easily categorized into either content or process. In the first example, the teacher indicates that the ITS was used to allow students to progress as a personal pace. The ITS was used by the teacher to facilitate learning at the learner's pace, but it lacks detail about what the student is learning (new material vs. old material) or how the student is learning (through examples, videos, etc.). A similar difficulty is seen in the second example as well. While it is noteworthy that the different characterizations of differentiation were present in teacher responses, for the purposes of this narrative, the term differentiation is used to describe all forms of differentiation unless necessary for clarification.

In addition to differentiation rendered through the ITS itself, some teachers noted that using the program offered opportunities to differentiate through general classroom strategies. Multiple teachers noted that they could use classroom ITS time to separate

students for focused group instruction as exemplified through the response of a teacher who said, “Students can work at their own pace, gives me more time to pull students into small groups while the rest of the class is working on ALEKS.”

ITS use to teach concepts. Approximately half (37 of 71) of the teachers who used ITSs indicated that they used the software to teach new concepts. In addition to differentiation for advanced and remedial learners, a few teachers indicated that they used ITSs for advanced exposure to new topics. *Additional exposure* refers to the use of ITSs because they offer more exposure to procedures or concepts addressed during classroom instruction. *Advanced exposure* is a practice in which teachers used ITSs to introduce students to new topics for the express purpose of facilitating classroom learning when the concept is learned in class.

One method of administering differentiated learning described in these responses was with computer directed learning paths and problem selections as described in the literature review. Some examples of this are shown in the following teacher responses. One teacher justified ITS use “because it allows students to learn at their comfortability and knowledge level.” Another teacher wrote that the ITS was used “to allow students to learn a new concept at their own pace.” These responses indicate that teachers found value in allowing students to work on topics and at a pace personalized through the ITS.

Another sentiment reflecting differentiation shared by a teacher was that the ITS was useful for tracking student learning. The teacher wrote that ITSs were used to “teach new concepts because as a teacher I need to see what skills they have and where they might be struggling. This is a form of a pre-assessment.” This response demonstrates how

the teacher used ITSs computer directed learning paths to assess student learning. The teacher was using information about a students' location on a learning path to ascertain information about the students' state of knowledge. This an example of product differentiation because the teacher was using student information from an ITS to demonstrate learning (Tomlinson, 2005).

Differentiation through computer directed learning paths and problem selection filled a niche for teachers who with students who were ready to learn new material. Some teachers saw the use of ITSs as a tool to teach concepts to students who would otherwise be held back by the pace of a class. One teacher expressed it by stating: "Sometimes students are ready to move on to a new topic before the whole class is ready. I use the software to help those kids have somewhere to go rather than being bored during class." Another teacher wrote that "I use ITSs to teach new concepts to help extend the learning of my higher-level students."

While ITSs were used for teaching concepts to accelerated students, they were also used for teaching concepts to students who were not accelerated. For example, one teacher wrote that "Students who have missed past concepts can learn them with ITS." This is similar to an idea expressed by a resource mathematics teacher who wrote "Aleks helps me to offer some students more assistance on topics they have not yet mastered…"

One theme of ITS use unique to teachers using it for teaching concepts was advanced exposure. Note how one teacher articulates the use of advanced exposure to prepare her students for a classroom lesson: "I find that if the students have already been exposed to the info when I teach it they understand it better and can ask better

clarification questions." Another teacher used ITSs to teach concepts "so students will have a notion of the concept when we teach it in class." Implicit in these descriptions of ITS use is the notion that students are not learning the topics using ITSs alone. This is a form of supplemental instruction discussed in the literature review. However, unlike supplementary instruction discussed in other questions and in the literature review, these are examples of supplemental classroom instruction in advance of the classroom lesson. The teachers saw value in classroom instruction, but they also saw value in the use of ITSs to augment conceptual understanding.

ITS use to teach procedures. Teachers who used ITSs to teach procedures account for the smallest sample of ITS users. Only 16 of the 71 ITS users in this survey fit into this category. Consequently, there was not a lot of commonality among the responses. A minor theme, which is unique to this research question, is that of differentiation through learning procedures other than those taught in the class or through a textbook. This sentiment was expressed by one teacher who wrote that she used ITSs to "show kids ways that are different than the way I do it." Another teacher wrote that ITS was used to teach procedures "to let students know there are multiple ways to get answers."

ITS use to practice procedures. Second to filling in gaps, most teachers used ITSs for the purpose of practicing procedures (60 of 71). The large difference between the number of teachers who used ITSs for practicing procedures and those who used ITSs for teaching concepts may indicate that teachers felt ITS instruction was procedural in nature. There were four prominent themes in this pattern of practice. Teachers used ITSs

to practice procedures because they (1) valued the differentiation through computer directed learning paths and problem selection, (2) valued the instant feedback feature, (3) wanted to provide additional exposure to topics learned in class, and (4) wanted to conserve resources.

Like teacher use of ITSs for teaching concepts, teachers saw value in its ability to offer differentiation through computer-directed learning paths and problem selection. Teachers expressed this sentiment through their approval of ITSs to offer mathematical practice at ones' own pace and with focused practice on only needed topics. For example, one teacher wrote that ITSs were used to "give students more practice on procedures they need only a little more help on—to increase fluency." Another teacher wrote that, "It generates multiple problems until a student is able to do it correctly multiple times in a row. If one student only needs three problems that is all they get but another student can get multiple problems to help them." A third teacher iterated that, "Each of my students need practice in different areas. My program allows me to differentiate for the needs of my students."

In addition to offering individual learning paths, teachers really appreciated the instant feedback capabilities in ITSs. Consider this response written by a teacher: "I think it is a good resource for students to practice and get immediate feedback if they are doing it correctly or not. It is more immediate than homework, lessons, etc." This teacher recognized that feedback from a computer was going to be quicker than anything she could offer. Another teacher wrote: "This is a growth mindset for them. When mistakes are made, they can see their mistake and make corrections on the next problem." This

teacher recognized the potential for mathematical growth when students can receive quick feedback and continue in their practice.

Some teachers saw value in ITSs for offering additional exposure to topics previously addressed in class. For example, one teacher wrote that, “The assignments are short enough that for some students it is simply not enough practice.” Another teacher wrote that, “Sometimes the students just need the practice with the material that I have taught them.” These responses hint that the textbooks used in class were insufficient to offer the quantity of practice that students needed to master new material.

The final theme addressed in the use of ITSs for procedural practice was the conservation of resources. *Conservation of resources* refers to the use of ITSs for conservation of classroom or teacher resources such as time, paper, and instructional material. One teacher explained the issue by stating: “There isn’t enough time to give practice in class.” This sentiment was echoed by another teacher who wrote: “There isn’t enough time in class [to] provide sufficient practice and teach new content.” Their method of classroom instruction did not provide ample time to adequately address the mathematics instructional strand of procedural fluency (National Research Council, 2001) so they relied on ITS use to address it. Another teacher stated that it “limits paperwork.” By noting the importance of limiting paperwork, the teacher may have been referring to the time needed to accomplish the paperwork, or the extra paper needed to accomplish the same amount of work on paper. In either case, classroom resources were conserved.

ITS use to fill gaps. This research question elicited the most responses from

teachers (66 of 71 teachers). In addition to the overarching theme of differentiation teachers tended to use ITS as a tool to conserve resources.

Teachers saw the ability of ITSs to provide differentiation as invaluable – especially as they pertained to gaps in student knowledge. One teacher wrote that “it targets specific gaps…instead of having the whole class practice a task they don’t all need to practice.” Another wrote that “I use intelligent tutoring to fill in gaps because each student has different individual needs.” The value of ITSs for detecting and meeting individual needs is even more evident when one considers the perspective shared by a teacher who wrote that “it allows students to go back and relearn concepts that they did not get to in previous years.”

In addition to the usefulness of ITS to detect and address learning needs, some teachers also expressed appreciation for the differentiation it could offer through alternate methods of instruction used by ITSs. For example, a MATHia user wrote that, “MATHia makes students explain the step to step process in solving applied problems.” Another teacher expressed appreciation for the gap-filling process of ITSs by stating that “the students have an option to click ‘I don’t understand this’ and it will walk them through step by step how to do the problem before they move on. This helps in filling in gaps.” A teacher using an ITS with embedded video tutorials wrote: “It gives them explanations and sometimes videos showing them how to do the problems. It is a nice way to catch up on things they have forgotten.”

The theme of conservation of resources was the most prominent for teachers using ITSs to fill gaps. Teachers seemed to appreciate that ITS use allowed them to meet the

diverse learning needs of students. One teacher shared this sentiment by stating that "there is not time in class to recover previous years' concepts and all the gaps in knowledge." Another teacher shared: "ALEKS gives students practice at the student's individual level. I can't replicate that with paper and pencil across 30-36 kids in a classroom." The response from the first teacher indicates that time was the major constraint to addressing individual learning needs while the second teacher's response indicates that creating individual practice sheets for students' diverse needs was something that would otherwise be impossible without the use of an ITS. The ability of ITSs to fill gaps while keeping pace with current curricular needs was expressed in the response of a teacher who wrote that: "Students come to us with all different gaps. Some of them small and some of them large. It would be nearly impossible to fill in all gaps and continue with learning in a years' time. The ITSs are a great way to fill in gaps that students have without utilizing much in-class time."

Having discussed teacher uses of ITSs for teaching concepts, teaching procedures, practicing procedures, and filling gaps, the next section the next section will detail responses by teachers who did not use ITSs.

Why teachers do not use ITSs. Of the 93 survey participants, 22 indicated that they did not use ITSs. Their reasons for not using them reflected three of the instructional buffers in the theoretical framework: lack of access to technology, lack of time, and a disposition unfavorable to the use of ITSs. Except for teacher disposition, reasons for teacher non-use did not appear to reflect teachers' conceptions.

One reason teachers did not use ITSs was because they lacked knowledge about

the program. Of the five teachers who reported a lack of knowledge about ITSs, only one expounded on that response. She wrote, “I have just re-entered teaching after being a stay-at-home mom for eight years (I taught full-time for 8 years before my first child was born). I am not sure what an intelligent tutoring system is.”

Teachers who lacked the technology reported that funding was a major obstacle to implementation. One teacher wrote that computers shared with too many teachers in the school rendered them inaccessible for regular use. Another teacher listed funding for the licenses as difficult to achieve but wrote: “In the past when I have used them, I have found that most of my students were making genuine gains using the software.” Only one teacher indicated that the internet speed was insufficient to run the program.

Two teachers reported not using ITSs because they lacked time. One wrote, “There isn’t enough time in the regular class to use the system when I only have 80 minutes with them every other day.” A second teacher indicated that time spent understanding program usage was the impediment. “I used Carnegie and found that it took forever and the theorems had to be word exact. Too much time [was] wasted figuring out wording and no learning was happening.”

Teachers with unfavorable dispositions towards ITSs indicated that their own personal instruction would be more valuable to their students than the ITS. For example, one teacher wrote: “I have used it as remediation, not as the primary teaching tool. I believe discussion is a better way to teach and learn math.” Another teacher responded: “I feel like the time students spend working on an intelligent tutoring system is not as effective as time they could spend with me targeting their misconceptions.” This

sentiment was expressed by other teachers who stated: “I don’t believe it would do as good a job as I can,” and “I haven’t found any that help students understand mathematics at the depth that I would like.” What makes these responses noteworthy is that they are in direct contrast with those offered by teachers who use ITSs to fill gaps. Teachers who used ITSs to fill gaps did so, in part, because they lacked time to address individual students’ needs on their own.

Question 5. How Teachers use Other Mathematics-Specific Technology to Teach Mathematics?

Each of the 93 participants in the study answered questions on their use of other mathematics-specific technologies. Each participant was asked: “Do you normally use any of the following mathematics-specific technologies for instruction?” Optional responses were (a) graphing calculator, (b) DGS, (c) Desmos, or (d) other. Participants received a follow-up question to elicit further information for each affirmative answer. The follow-up question prompted the teachers to “describe in detail a typical lesson where you used the __________. How did you use the technology?” This section contains the results from these follow-up questions.

Three major themes emerged in this analysis. Calculators, DGS and Desmos were used for calculations, visualizations, and explorations (see Table 8). *Calculations* referred to the use of technology for computational purposes. This is analogous to Doerr and Zangor’s (2000) description of graphing calculator use as a computational tool to evaluate numerical expressions, to round, or to estimate. Doerr and Zangor also noted how students used graphing calculators as a visualizing tool. *Visualizations*, in this study,

Table 8

Prominent Codes for Non-Intelligent Tutoring Systems Technology

Type of differentiation	Sample response
Calculation	The use of technology for computational purposes.
Exploration	Technology use that actively promotes conceptual understanding. Examples of this include the use of sliders, changing variables, or manipulating physical aspects of a construction.
Visualization	Using visual displays to “determine the nature of an underlying structure of a function (or object), to link the visual representation to the physical phenomena, and to solve equations” (Doerr & Zangor, 2000, pp. 155-156)

referred to three practices observed by Doerr and Zangor to “determine the nature of an underlying structure of a function, to link the visual representation to the physical phenomena, and to solve equations” (pp. 154-155). Though Doerr and Zangor utilize this description explicitly for graphing calculator use, this description lends itself well to DGS and Desmos practices as well. *Explorations* described technology use that actively promoted conceptual understanding by having students interact with the technology through use of sliders, changing variables, or manipulating physical aspects of a construction to elicit information about their effects.

Teachers’ use of graphing calculators. All three of the themes described above were present in teachers’ use of graphing calculators. Additionally, more teachers (53 out of 93) reported using graphing calculators than reported using dynamic geometry software or Desmos. Graphing calculator technologies may be considered the most versatile and accessible technologies available to teachers. However, the types of responses elicited from teachers indicated that graphing calculators were used almost exclusively in the upper grades, most likely due to their availability (Dewey et al., 2009).

Calculation uses described by teachers included calculating lines of best fit, solutions to systems of equations, correlation coefficients, square roots, and powers of numbers. These calculations range from routine (square roots and powers of numbers) to complicated (regression lines and correlation coefficients.) Teachers of advanced ninth graders reported using the technology to compute sines and cosines.

The usefulness of the graphing capability for visualizations was notable in teachers' responses. Teachers instructed students to graph systems of equations to visualize and find solutions. They instructed students on the creation of scatter plots and box-and-whisker plots. Most teachers responded that they used graphing calculators to perform linear regressions as exemplified in the following response:

> I taught my Secondary Math II Honors students how to find the place(s) where two graphs (two lines, two parabolas, or a line and a parabola) intersect by hand and then taught them how to find the place(s) of intersection on their graphing calculators.

Though the teacher was using the graphing capabilities of the graphing calculator for a more complex topic, the teacher did not describe using the calculator for exploratory purposes.

Some responses by teachers indicated that they used the graphical capabilities of the calculators to promote student exploration. For example, this response reflects a typical exploration lesson with a graphing calculator as described by a teacher: "When working with exponential functions, my students graph functions to discover the effects on y = a(b)^x of different values for a and b. They also discover that b cannot equal 0 or 1 and what happens if b is a negative number." No teacher, however, described using preloaded images in Casio or TI calculators to model equations. That is in contrast to the

way teachers describe using Desmos later in this section.

Teacher use of Desmos. Second to graphing calculator use, 48 of 93 teachers used Desmos for visualization, calculation, and exploration. Unlike calculator use, however, Desmos was employed by teachers from all grade-bands.

The teachers who employed Desmos for visualization seemed to use the graphing calculator feature almost exclusively. Like the graphing calculator uses described previously, teachers used Desmos to graph equations and scatter plots. In addition, teachers described using Desmos to graph circles, inequalities, and lines in standard form. While graphing calculators also offer the capability to create circles and inequalities, teachers only reported graphing them with Desmos. The following response describes one reason why Desmos may have been used for a wider variety of inputs:

> When solving systems of equations in standard form, Desmos makes it easy and simple for students to graph and visually see what is going on. The different colors that Desmos provides, as well as the ability to put equations right into Desmos in standard form instead of converting to slope-intercept form make this tool extremely handy and student-friendly.

As indicated by the teacher, Desmos easily graphs relations in a multiplicity of forms with a colorful output. These features are not found together in all graphing calculators.

Teachers who used Desmos for exploration reported using the classroom activities as well as the graphing calculator. For example, one teacher responded: “I often use Desmos when I want students to explore parts of an equation. I have used this when I want students to discover what makes an exponential function increase or decrease.”

Desmos use differed from other technology use by teachers in the interactive

activities. For example, the Desmos classroom activities are pre-made and designed to engage students in exploring mathematical topics. One teacher responded, “I really like the marble slides... It provides a good structure for them to explore.” The marble slide activity is one in which students are asked to adjust variables and domains to facilitate a cluster of marbles to roll into and delete a series of stars on the screen. Another teacher reported the following:

> I had the students play around with different situations in which they had to graph the course that the Ferris wheel made over time. They then were able to adjust the speed and direction of the Ferris wheel and re-graph.

In the Ferris wheel activity, students manipulate sliders to adjust the radius and speed of a turning wheel. Students, in turn, relate those adjustments to the height of a person above ground over time. These were examples of interactive activities not currently found on graphing calculators.

The teachers who employed Desmos for calculation purposes appeared to use the on-line calculator instead of a handheld calculator. For example, one teacher responded, “I’ve showed the students that it has a good calculator to use on it.” Another teacher wrote, “Desmos is used as a link on Canvas to their online calculator since most students don’t have a calculator at home.” While these comments only represented a minutia of the total number of responses, they are included here to demonstrate the variability in the types of uses for Desmos reported by teachers.

Teachers use of DGS. DGS technology was the least reported use of technology by teachers in this survey with only 19 of 93 responders indicated that they used it in their classrooms. Teachers’ uses of DGS reflected visualization and exploration.

The theme of visualization was present when teachers described using DGSs. In some instances, teachers were creating visuals for the class or students were creating the visuals. For example, one teacher wrote: “When I want to manipulate geometry, I project something like GeoGebra so that the students can see the actual transformation or completion of the problem.” In this example, the teacher was using DGS to create a dynamic construction to assist in teaching a concept. This was in contrast with the use of DGS described by another teacher who used it to generate problem sets: “I have used GeoGebra to create pictures of shapes and create new problems for my students when the book doesn’t give enough examples or KUTA doesn’t have the type of problem I need.” In this example, the teacher used DGS, but not for its dynamic capabilities.

DGS has the potential for creating exploratory lessons. For example, one teacher wrote: “I don’t get to use this often, but I like to use GeoGebra as a way of having students explore diagrams to discover relationships rather than be told them.” Another teacher responded, “I used GeoGebra to help students see visually the triangle sum theorem. I had them experiment with different lengths of the sides of the triangle to see why a+b has to be greater than c.” These responses show teachers encouraging students to use DGS to create activities meant to engage and explore, rather than merely demonstrate a concept.

This completes the presentation of the qualitative results. This section demonstrated how teachers used ITSs for differentiation purposes and graphing calculators, DGS, and Desmos for calculation, visualization, and exploration.

Mixed-Methods Analysis

This mixed-methods analysis focuses on a pattern of technology use manifest through the qualitative and quantitative results. This pattern was defined in three parts: (1) Teachers' use of ITSs for procedural practice and gap-filling activities, (2) Teachers used Desmos to promote engaging and exploratory learning experiences, and (3) Teachers reserved graphing calculator use for routine, but complex, calculations or visualizations.

Intelligent Tutoring Systems use for Procedural Practice and Gap-Filling

To better understand the widespread use of ITSs for procedural practice and gap-filling, it is instructive to return to the overarching question: "What is the relationship between teachers' conceptions of mathematics and their use of ITSs for mathematics instruction?" The lack of significant interaction for teachers' use of ITSs is noteworthy for two reasons. First, it indicates that teachers' conceptions of mathematics were not determining factors in their use or non-use of ITSs. Second, it indicates that teachers' conceptions of mathematics were not determining factors in how teachers used ITSs.

A majority (71 of 93) of participants indicated that they used ITSs. Many also indicated that they used ITSs to practice procedures (60 of 71), and fill in gaps (66 of 71) (see Table 9). Only 29 teachers reported using ITSs to teach concepts. Though the magnitude of the difference in teachers' responses indicated a potential interaction, the interaction effect was not statistically significant. In addition, a variety of teachers were using ITSs, and they were ITSs for reasons that were not statistically different.

Table 9

Reasons for Teacher Use of Intelligent Tutoring Systems by Percent

Purpose	Percent
Fill-gaps	93
Practice procedures	85
Teach concepts	41
Teach procedures	23

The most prominent reasons for using ITSs was to promote procedural fluency and fill knowledge gaps. Having students interact with an ITS to improve procedural fluency was appealing for the teachers because it offered students immediate feedback that the teachers could not offer otherwise. ITSs also gave teachers the ability to customize problem sets for additional practice. For teachers, ITSs offered a way to focus on the precise procedures that they wanted to students to practice.

In addition to ITS use for procedural practice, it also offered teachers a convenient way to address gaps in students' knowledge without dramatically altering the pace of instruction on current material. Teacher responses implied that filling gaps might not have happened at all if it were not for ITSs. Teachers' comments indicated that, were it not for ITSs, gap-filling and teaching current content would be mutually exclusive activities.

Desmos use for Exploratory Lessons and Calculators for Calculations

There was a contrast in teachers' reported use of graphing calculators and Desmos. Graphing calculators were used primarily for calculation purposes, while Desmos was used for calculation and exploration. To better understand teachers use of

both technologies, it is instructive to revisit the interaction effects for conceptions and graphing calculator or Desmos use.

Unlike teachers' ITS use, there was a significant interaction between teachers' conceptions of mathematics and graphing calculator use. However, there was a notable significant interaction between teachers' conceptions and calculator use for the dimension of doing. This indicated that teachers with a conception of mathematics as a results-centered practice were more likely to use graphing calculators than teachers with a conception of mathematics as sense-making. Teachers who reported using the graphing calculators for exploratory purposes only accounted for 15% of the total responses. Most teachers preferred to use graphing calculators for calculation or visualization purposes. Overall, these results show that teachers in this study viewed calculators as devices for obtaining results.

Unlike calculator use, there was not a significant interaction on the dimension of doing for Desmos use. This result implies that teachers were not more likely to use Desmos based on a conception of mathematics as a results-centered or a sense-making practice. Despite this result, over half (58%) of the teachers who used Desmos wrote about using it in an exploratory manner, while only 19% indicated that they used it specifically for calculation purposes. What makes this finding noteworthy is that, in many cases, teachers who used graphing calculators for calculation purposes were using Desmos for exploratory purposes.

The teachers' choice of Desmos for exploratory activities is noteworthy. Despite efforts by calculator manufacturers, such as Casio and TI, to incorporate features into

their devices, such as conical graphing capabilities, geometry apps, and background images for modeling functions, teachers in this study did not indicate that they were using them. The teachers were using Desmos instead. Graphing calculators can be difficult to use and can require a considerable amount of time investment before students are proficient with them (Berry, Graham, Honey, & Headlam, 2007; Ruthven et al., 2009).

A Pattern of Practice

In this section, the patterns of ITS, Desmos, and graphing calculator use described above are illustrated through a summary table of teacher examples (see Table 10). The pattern shows that teachers assigned ITSs to support procedural or gap filling processes, employed Desmos for exploratory work, and used graphing calculators for calculation or visualization purposes. This pattern of technology use was found among individual teachers' responses throughout their surveys.

As Table 10 shows, one teacher who employed ITSs, graphing calculators, and Desmos for three unique practices wrote: "My students come from different backgrounds and have different gaps. With my program I can help multiple students fill in gaps at the same time." The same teacher described a calculation-based practice of graphing calculator use as follows:

> In order to find a linear regression equation, students need to use a graphing calculator. First, they must populate lists with statistical data. Then they have to calculate the a and b values, and finally they need to use significant digits to create an equation.

In this example, the teacher made graphing calculators available to facilitate an otherwise lengthy and difficult calculation. The teacher's example of Desmos use, though brief,

Table 10

Individual Teacher's Descriptions of Intelligent Tutoring Systems, Graphing Calculator, and Desmos Use

ITS use for practicing procedures	ITS use for filling gaps	Graphing calculator use	Desmos use
When they practice with intelligent tutoring the student can receive instant feedback.	There is not time in class to recover previous years' concepts and all the gaps in knowledge.	The concept taught is box and whisker plots. After collecting data students enter the data into graphing calculators then are taught how to create a box and whisker plot.	In a Desmos app already created students learn about least squares regression. They move a line of best fit around and try to make squares that are attached to the line as small as possible.
It generates multiple problems until a student is able to do it correctly multiple times in a row. If one student only needs 3 problems that is all they get but another student can get multiple problems to help them.	It targets specific gaps as well as allows me to target specific gaps for selected students instead of having the whole class practice a task they don't all need to practice.	Relating roots of a quadratic equation to the x-intercepts for the quadratic function. I had students solve the equation by hand and then had them graph the equation to locate the x-intercepts.	I used the classroom activity on Desmos dealing with domain and range for functions. This allowed the students to have a dynamic visual for what they were writing with the domain and range.
Sometimes students don't understand my explanation but seeing it another way and being able to practice it many times helps.	I don't always catch what kids are missing- ALEKS is supposed to do that.	… I am currently doing scatter plots, so I will do a linear regression with my honors students.	To introduce scatterplots, we did a celebrity guessing game on Desmos. It creates a scatter plot for them and then takes them through the different describing words for the graphs.

indicates that it was employed for the purpose of deepening the students' understanding of domain and range through an exploratory classroom activity. She wrote: "In order to supplement instruction on Domain and Range I used a Desmos activity that allowed the students to explore domain and range on a graph." In this teacher's view, the three pieces of technology had three distinct purposes.

While the pattern of exploratory use of Desmos and calculation use of graphing

calculators is prominent, it is not descriptive of all responses. As noted in the qualitative section, there were some teachers who used graphing calculators for exploratory purposes. A few teachers described using graphing calculators for exploratory purposes and Desmos for non-exploratory purposes. For example, one teacher described a graphing calculator lesson in the following way: “Each student had a graphing calculator. We were exploring the shapes of graphs and learning how to input a function into the graphing calculator.” The teacher then described her Desmos lesson by writing “I used Desmos projected on the screen to show students the steps for putting in a function and viewing the graph.” In this example, the teacher used graphing calculators to aid in student exploration but used Desmos to project a graph on the board for visual purposes.

Other teachers described graphing calculator use and Desmos use in exploratory terms. One teacher wrote that she used graphing calculators for “exploring what happens as you change the slope or the y-intercept independent of each other.” She described her Desmos use in similar terms when she wrote that she used it for “exploring scatterplots, slopes, [and] functional relationships.” No explanation was given for why one tool was used instead of another.

While the presence of these alternative uses of technology are used to demonstrate divergent responses (Creswell & Plano Clark, 2018), a multiplicity of teachers who used ITSs, graphing calculators, and Desmos, used them consistent with the pattern described above. This concludes the presentation of the mixed methods results.

Summary

The results of the quantitative analysis showed no significant interaction effects between teachers' conceptions and ITS use or nonuse. Additionally, there was no significant interaction effect between teachers' conceptions and purpose of ITS use. There were, however, significant interaction effects between teachers' conceptions and non-ITS technology use. There were significant interactions between teachers' conceptions and calculator use on the dimensions of composition, structure, doing, and validating. For DGS use, there was a significant interaction between teachers' conceptions on the dimension of validating. For Desmos use, there were significant interaction effects on the dimensions of composition, structure, and validating.

The qualitative analysis showed that teacher use of ITSs and other technologies was influenced by differentiation. Teachers used ITSs to provide access to different content as well as access to different forms of instruction. Lack of knowledge about ITSs, lack of resources, and unfavorable disposition towards ITSs were reasons that teachers did not use ITSs for instruction. Teachers who used other technologies (i.e., graphing calculators, DGSs, and Desmos) used them for three purposes. They used them (1) to perform calculations, (2) to assist in exploration activities, and (3) for visualization purposes. While the theme of visualization was present across all three technologies, the theme of calculation was most pronounced for graphing calculator use, and exploration was most pronounced for DGS use.

In the mixed-methods analysis, the pattern that emerged was that teachers employed ITSs for procedural practice and gap-filling activities, Desmos was used to

promote exploratory learning experiences, and graphing calculators were used for routine calculations and visualizations.

CHAPTER V

DISCUSSION

The purpose of this mixed-methods study was to research the relationship between junior high school mathematics teachers' conceptions of mathematics and their use of ITSs for mathematics instruction. Teachers' conceptions influence general technology use (Kim et al., 2013) as well as their mathematics-specific technology use (Lee & McDougall, 2010; Wachira et al., 2008). However, no studies have addressed teachers' conceptions of mathematics and their use of ITSs.

The overarching question addressed in this study was: "What is the relationship between teachers' conceptions of mathematics and their use of ITSs for mathematics instruction?" Both quantitative and qualitative questions were used to address the overarching question. The questions answered using quantitative data were:

1. What is the relationship between teachers' conceptions of mathematics and their use or non-use of ITSs?

2. What is the relationship between teachers' conceptions of mathematics and their use of non-ITS math-focused technologies?

3. Among those teachers who use ITSs, what is the relationship between their conceptions of mathematics and how they use ITSs?

The questions answered using qualitative data were:

1. Why do teachers use or not use ITSs?

2. How do teachers use different technologies to teach mathematics?

The convergent mixed methods design employed in this dissertation used a survey to collect both quantitative and qualitative data simultaneously (Creswell & Plano Clark,

2018). Quantitative questions were analyzed using eight separate 2x5 mixed design ANOVAs. The qualitative data was analyzed using a constant comparative method (Creswell & Plano Clark, 2018). After these analyses, the data were merged and interpreted together.

Three noteworthy findings from this study are: (1) Teachers used ITSs independent of conceptions; (2) Teachers used ITSs primarily for differentiation that focused on procedures and filling gaps; (3) A subset of ITS-using teachers demonstrated a pattern of technology use which incorporated graphing calculators and Desmos to address a variety of mathematical practices.

Teachers' Intelligent Tutoring System Use and Conceptions

There was no significant interaction effect between teachers' conceptions of mathematics and their use or nonuse of ITSs. For the 76% of teachers who used ITSs, there were also no significant interaction effects between their conceptions of mathematics and use of ITSs for learning new concepts, learning new procedures, practicing procedures, or filling gaps in knowledge. Stated more generally, teachers in this study with a variety of conceptions used ITSs. This result does not support previous findings that teachers' conceptions were linked to technology practices (Kim et al., 2013; Lee, 2007; Wachira et al., 2008). This may be because of the unique structure of ITSs noted in the literature review.

Twenty-four percent of the teachers in the survey did not use ITSs. Three prominent reasons offered for not using ITSs are also components of the theoretical

framework for this dissertation. Indeed, the close match to the instructional buffers in the theoretical framework (see Figure 2) is the reason for reporting these findings. First, some teachers had a general disposition that did not favor ITS use. This finding supports previous findings that teachers tend to not use technology which does not match their beliefs or instructional practices (Ertmer & Ottenbreit-Leftwich, 2010; Zhao et al., 2002). Second, some teachers lacked resources for implementation such as quality internet access or computer access. Indeed, computer ownership and internet access do vary across the U.S. (Rainie & Cohn, 2014) and this survey did include one rural school district which may have similar technology needs. Third, some teachers lacked knowledge of the products. Teachers who cited a lack of knowledge about the ITSs lacked what Koehler and Mishra (2009) refer to as TPACK. They did not use the technology because they did not know about it and did not know what services ITSs could offer their students. One will note that only one of the reasons, the general disposition, could be tied to teachers' conceptions. Thus, teachers' nonuse of ITSs was associated with lack of resources or lack of knowledge of the product.

Differentiation and Filling Gaps

In addition to largescale ITS use by teachers regardless of their mathematical conceptions, evidence of another pattern of broad ITS use emerged in this study. Ninety-three percent of ITS-using teachers used them to fill gaps, and 85% used them to practice procedures.

Use of ITS for gap filling and procedural practice presented a pragmatic approach

to educational challenges associated with larger class sizes and typical time constraints associated with teaching responsibilities. Through ITSs, teachers could offer more focused instruction on relevant topics without adding tasks to their already busy schedules. They valued ITSs as a differentiation tool to address learning needs that they otherwise would not address altogether or address as effectively. Certainly, there is merit to this practice. Knowledge assessments by humans are not necessarily better than knowledge checks by ITSs (VanLehn, 2011). Not only are ITSs programmed to effectively assess student knowledge and learning (Shute & Psotka, 1996), they are also not subject to implicit human biases (Huang et al., 2016).

While the use of ITSs for practicing procedures supports the finding that technology use by teachers favors practicing basic skills (Prieto-Rodriguez, 2016), it is not clear whether or not teachers viewed gap-filling as a procedural endeavor. What is clear is the overarching trend suggested by these findings: Approximately three fourths of the teachers use ITSs, independent of their mathematical conceptions, for the express purpose of addressing procedural needs and gaps in knowledge. This information suggests that teachers considers ITSs as classroom assistants rather than substitute teachers. Teachers are not using them to replace their instruction, but rather to augment it.

It is not clear if this ITS implementation to promote procedural fluency is in alignment with the programmers' intentions. It is possible that teachers view ITSs as best suited for procedural fluency while ITS programmers view them as tools for promoting conceptual understanding. In other words, this use may demonstrate a gap between intended and implemented curriculum.

Intelligent Tutoring System, Graphing Calculators, and Desmos Integration

Were the use of ITS technology to focus solely on the prior findings, the results from this research might paint a rather dull picture of mathematics education. Technology needs to be used to enhance conceptual understanding and give students a chance to engage with mathematics that could not be accomplished with paper and pencil alone (NCTM, 2000, 2014). Looking at a subset of teachers in this study, one can see a more holistic approach to technology implementation for mathematics instruction. As described previously, teachers employed ITSs to facilitate procedural practice and filling gaps. A subset of ITS-using teachers also employed graphing calculators to assist with routine calculations and visualization, and Desmos to facilitate exploratory activities. This is a noteworthy pattern of technology because it demonstrates how teachers include and exclude these technologies based on each technology's propensity to assist students in different aspects of mathematics learning (Koehler & Mishra, 2009) and solve issues related to professional practices (Ertmer & Ottenbreit-Leftwich, 2010).

The use of graphing calculators for routine calculations and visualization is not an uncommon practice. As noted in the literature review, secondary mathematics teachers tend to use calculators as computational tools or instruments to improve the accuracy and appearance of student work (Brown et al., 2007; Ruthven et al., 2009; Simmt, 1997). However, it was also noted that teachers used graphing calculators for exploratory and sense-making activities (Doerr & Zangor, 2000; Lee & McDougall, 2010). What makes the findings of this study noteworthy is not the lack of exploratory practices with

calculators, but rather the shifting of those practices from a hand-held graphing calculator to Desmos. This might be because using a handheld graphing calculator requires a considerable amount of classroom time investment for students to be able to use them effectively (Doerr & Zangor, 2000; Lee, 2007), whereas the Desmos graphing calculator is much easier to navigate. This could also hint at the reason why most of the teachers in this study did not use DGSs. Geogebra, a prominent free DGS requires the use of typed commands and various sub-menus to navigate it effectively.

While the teachers in this subset used ITSs and graphing calculators to address computational or procedural needs, their primary use of Desmos was to provide exploratory lessons. These exploratory activities included both the Desmos classroom activities as well as the graphing calculator application. This is a hopeful finding. It indicates that there are teachers who are who value technology for its ability to engage students in exploratory practices, not just for routine calculations or to supplement classroom instruction. As noted previously, the teachers described in this section represent a distinct subset of ITS users in this study. They may, however, represent an overall class of teacher whose practices are desirable of emulation.

Implications

One implication from this study is the need to incorporate best practices or guiding principles for ITS use into preservice teacher programs and professional development. This, in turn, implies the need to create a set of best practices or guiding principles. If teachers are going to continue using ITSs, as this study suggests, then their

use thereof should be thoughtful and not indiscriminate. It is possible that teachers in this study did not know why they used ITSs, but were only invited to reflect upon their use when participating in the survey.

Prominent teacher use of ITSs leads to a second, and more important, implication: Teachers are not using available technology to promote mathematical practices that promote technological investigations as called upon by the NCTM (2014). Whereas 76% of teachers indicated that they used ITSs, only 52% indicated that they used Desmos and 20% indicated that they used DGSs. Based on these numbers, it is clear that the majority of student exposure to mathematics focused technology was not intended to promote investigation. As noted in this study, a major reason for using Desmos was to offer exploratory opportunities to students. Other programs, such as DGSs of computer apps, also afford opportunities to explore mathematical topics at a conceptual level (NCTM, 2000). Teachers need to increase their use of these types of programs.

The observation that teachers are favoring ITS use over other technologies, in conjunction with the observation that teachers need to use technology to promote exploratory activities should act as a clarion call to ITS designers to incorporate more exploratory apps and activities into their ITS design. ITSs should do more than address math knowledge in a routine manner. ITSs should engage students in engaging and sense-making activities.

The results from this study also suggest that educational leaders should continue to make technology available for mathematics classrooms. Teachers need access to computers because of the versatility they offer in accessing a variety of programs and

apps.

Finally, teachers need to be transparent with stakeholders about their intended ITS use. It would be easy for stakeholders to assume that teachers are using ITSs as a substitution for mathematical instruction. This type of misunderstanding could feed public misunderstanding of the type of work that mathematics teachers do on a daily basis.

Suggestions for Future Research

This was an exploratory study. Therefore, the observations emanating therefrom need further research to fully understand the underlying practices. For example, while most of the ITS users employed the programs for procedural practice and gap filling, nearly half of the ITS users indicated that they used them to teach concepts. Were teachers who used ITSs to promote procedural fluency using different ITSs than teachers who used them to teach concepts? Are teachers using ITSs for the purposes that the designers intended?

To fully address these questions, a study which compared specific ITSs to teacher use would need to be conducted. The study would also need to identify the designers' intent in creating the ITSs.

Conclusion

The use of ITSs by most teachers in this study for procedural practice and filling gaps in knowledge indicates that these programs are supplying a much-needed service to

teachers. The pattern of teachers using multiple technologies in this study also indicates that they were not handing over instruction to computers. Teachers were thoughtfully selecting technologies to address specific learning needs. These findings suggest that teachers need regular and frequent access to computers, ITSs, and handheld calculators.

The results of this mixed-methods study showed that teachers' conceptions of mathematics (as measured by five dimensions of the CMI) were not related to their use of ITSs (Grouws et al., 1996). A large majority of teachers in the study used ITSs to provide procedural practice and gap-filling opportunities for their students because of the differentiation opportunities it provided. Not only did the ITSs provide instant feedback and targeted instruction, it also gave teachers the ability to easily provide additional practice within the classroom time constraints.

This study also revealed that a subset of teachers employed ITSs, graphing calculators, and Desmos to address specific and unique learning needs of their students. Approximately half of the teachers in this study indicated that they used graphing calculators primarily for calculation and visualization purposes. Teachers indicated that they used Desmos for exploratory and visualization purposes. This indicates that teachers were infusing their classroom instruction with multiple technologies in varied and purposeful manners.

This research adds to a growing body of ITS research by demonstrating that teachers in this sample population used ITSs independent of their mathematical conceptions. Their use was pragmatic because it was intended to fill gaps and offer procedural practice that might not have otherwise been administered. Other research

indicates that the type of ITS use demonstrated in this study can be effective for increasing student mathematical growth (Burch & Kuo, 2010; Erümit & Vagifoglu Nabiyev, 2015; Ma et al., 2014; Steenbergen-Hu & Cooper, 2013). While ITS use can be effective at increasing mathematical knowledge, teachers whose technology use in the mathematics classroom consists exclusively of ITS use are not fully engaging students in the types of exploratory activities that modern technologies can offer.

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APPENDICES

Appendix A

Survey Instrument

Survey Instrument

The survey included questions about teachers' conceptions of mathematics and their use of technology to teach mathematics. The Qualtrics survey mixed the questions from the conceptions of mathematics survey. In the appendix, however, the questions are presented by section.

First Question

As per IRB requirement, the first question of the survey is an informed consent to participate. All other questions are survey-specific.

Survey Introduction

This survey contains two parts and should take you approximately 45 minutes to complete. There are no right or wrong answers. The first section contains five to 15 multiple choice and open-ended questions eliciting information on your use of technology. The second section contains 40 Likert scale questions eliciting information on your beliefs and knowledge of mathematics.

Teacher Use of ITS

This portion of the survey contains questions about Intelligent Tutoring Systems and other math-specific technologies. Intelligent Tutoring Systems are web-based computer programs such as ALEKS, Carnegie's MATHia, iReady, and Imagine Math (TTM) which provide opportunities for students to learn at their own pace.

Please answer the following few questions about your use (or non-use) of intelligent tutoring systems and other math-specific technologies in your teaching

practices.

Research questions 1 and 4: Use or non-use of ITS and why

2. Do you use an intelligent tutoring system (ALEKS, Carnegie's MATHia, iReady, or Imagine Math (TTM)) to teach mathematics?

    a. Yes
    b. No

3. (If no on 2) Have you ever tried using an intelligent tutoring system to teach mathematics?

    a. Yes
    b. No

4. (If no on 2) Explain why you do not use an intelligent tutoring system to teach mathematics. (Skip to question 10)

Research questions 3 and 4: how and why teachers use ITS

5. Do you normally assign student use of intelligent tutoring systems (ALEKS, Carnegie's MATHia, iReady, and Imagine Math (TTM)) for any of the following reasons? (Check all that apply.)

    a. Learning new concepts
    b. Learning new procedures
    c. Practicing procedures
    d. Filling in gaps in student knowledge
    e. At the request of my school or district administration

6. (If yes on 5a) Explain why you use and intelligent tutoring system to teach new concepts.

7. (If yes on 5b) Explain why you use an intelligent tutoring system to teach new procedures.

8. (If yes on 5c) Explain why you use an intelligent tutoring system to practice procedures.

9. (If yes on 5d) Explain why you use an intelligent tutoring system to fill in gaps in student knowledge.
   (Go to question 10)

Research questions 2 and 5: use of non-ITS math-focused technologies

10. Do you normally use any of the following mathematics-specific technologies for instruction?

    a. Graphing Calculator
    b. Dynamic Geometry Software (such as GeoGebra or Geometer's Sketchpad)
    c. Desmos
    d. Other (Open Response)

11. (If yes on 10a) Describe in detail a typical lesson where you used the graphing calculator. How did you use the technology?

12. (If yes on 10b) Describe in detail a typical lesson where you used the dynamic geometry software. How did you use the technology?

13. (If yes on 10c) Describe in detail a typical lesson where you used Desmos. How did you use the technology?

14. (If yes on 10d). What is the "other" technology which you normally use for mathematics instruction?

15. (If yes on 10d). Describe in detail a typical lesson where you used the "other" technology indicated in question 14.

Conceptions of Mathematics Inventory Sorted

Read each question carefully and mark your answer (strongly agree, agree, neutral, disagree, or strongly disagree). Do not spend too much time on any one item.

Composition of mathematical knowledge. These questions measure a conception of mathematics as knowledge as concepts, principles, and generalizations versus knowledge as facts, formulas, and algorithms.

1. There is always a rule to follow when solving a mathematical problem.

6. Mathematicians work with symbols rather than ideas.

11. Learning computational skills, like addition and multiplication, is more important than learning to solve problems.

16. The field of mathematics is for the most part made up of procedures and facts.

21. While formulas are important in mathematics, the ideas they represent are more useful.

26. Computation and formulas are only a small part of mathematics.

31. In mathematics there are many problems that can't be solved by following a given set of steps.

36. Mathematical knowledge consists mainly of ideas and concepts and the connections among them.

Structure of mathematical knowledge. These questions measure the conceptions that mathematics is a coherent system versus mathematics as a system of isolated practices.

2. Diagrams and graphs have little to do with other things in mathematics like operations and equations.

7. Mathematics consists of many unrelated topics.

12. Finding solutions to one type of mathematics problem cannot help you solve other types of problems.

17. There is little in common between the different mathematical topics you have studied, like measurements and fractions.

22. Often a single mathematical concept will explain the basis of a variety of formulas.

27. Mathematics is mostly thinking about relationships among things such as numbers, points, and lines.

32. Concepts learned in one mathematics class can help you understand material in the next mathematics class.

37. Most mathematical ideas are related to one another.

Doing mathematics. This section measures a conception that mathematics is about sensemaking versus mathematics is about results.

3. Knowing why an answer is correct in mathematics is as important as getting a correct answer.

8. When working mathematics problems, it is important that what you are doing makes sense to you.

13. Understanding the statements a person makes is an important part of mathematics.

18. When a problem doesn't make sense, you can usually solve it by using some different but related mathematics you already know.

23. One can be quite successful at doing mathematics without understanding it.

28. If you cannot solve a mathematics problem quickly, then spending more time on it won't help.

33. Being able to use formulas well is enough to understand the mathematical concept behind the formulas.

38. If you knew every possible formula, then you could easily solve any mathematical problem.

Validating ideas in mathematics. These questions measure a conception that mathematics may be validated through logical thoughts versus validation through outside authority.

4. When two students don't agree on an answer in mathematics, they need to ask the teacher or check the book to see who is correct.

9. You know something is true in mathematics when it is in a book or an instructor tells you.

14. You can only find out that an answer to a mathematics problem is wrong when it is different from the book's answer or when the teacher tells you.

19. In mathematics, the instructor has the answer and it is the students' job to figure it out.

24. Justifying the statements a person makes is an important part of mathematics.

29. It is important that you can convince yourself of the truth of a mathematical statement.

34. When two classmates don’t agree on an answer, they can usually think through the problem together until they have a reason for what is correct.

39. When one’s method of solving a mathematics problem is different from the instructor’s method, both methods can be correct.

Learning mathematics. These questions measure a conception of learning as constructing and understanding versus learning as memorizing intact knowledge.

5. Learning to do mathematics problems is mostly a matter of memorizing the steps to follow.

10. Learning mathematics involves memorizing information presented to you.

15. Asking questions in mathematics class means you didn’t listen to the instructor well enough.

20. You can only learn mathematics when someone shows you how to work a problem.

25. Memorizing formulas and steps is not that helpful for learning how to solve mathematics problems.

30. When learning mathematics, it is helpful to analyze your mistakes.

35. When you learn mathematics, it is helpful to compare new ideas to mathematics you already know.

40. Learning mathematics involves more thinking than remembering information.

Appendix B

Survey Recruitment Email

Dear Junior High/Middle School Mathematics Teacher,

You are being invited to take part in a research project examining the relationship between teachers' conceptions of mathematics and intelligent tutoring system (ITS) use (or non-use). ITSs are web-based computer programs such as ALEKS, Carnegie's MATHia, iReady, and Imagine Math (TTM) which provide opportunities for students to learn at their own pace.

The data are being gathered using an anonymous on-line survey. The survey should take no more than 40 minutes to complete. To thank you for your time, you will receive a $15 Amazon gift card.

The survey begins with (up to) 15 questions about your use (or non-use) of ITSs and your use of mathematics-specific classroom technology. The technology questions are followed by 40 Likert-type questions to determine your thoughts on the (a) composition of mathematical knowledge, (b) structure of mathematical knowledge, (c) doing mathematics, (d) validating ideas in mathematics, and (e) learning mathematics. To preserve anonymity, the last item on the survey is a link to another survey which gathers information necessary for you to receive the Amazon gift card.

The link to the survey is: https://usu.co1.qualtrics.com/jfe/form/SV_cCJP6xnUdzE9d9X

If you have any questions, please contact Andrew Glaze at andrewrglaze@gmail.com or 385-350-3633; Dr. Patricia Moyer-Packenham at patricia.moyer-packenham@usu.edu or 435-797-2597; or Dr. Max Longhurst at max.longhurst@usu.edu.

Appendix C

University IRB Approval

Institutional Review Board

USU Assurance: FWA#00003308

UtahState University
OFFICE OF RESEARCH AND GRADUATE STUDIES

Exemption #2

Certificate of Exemption

FROM:

Melanie Domenech Rodriguez, IRB Chair

Nicole Vouvalis, IRB Administrator

To:	Patricia Moyer-Packenham, Max Longhurst, Andrew Glaze
Date:	August 27, 2018
Protocol #:	8491
Title:	Teachers' Conceptions Of Mathematics And Intelligent Tutoring System Use: A Mixed Methods Approach

The Institutional Review Board has determined that the above-referenced study is exempt from review under federal guidelines 45 CFR Part 46.101(b) category #2:

> Research involving the use of educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures or observation of public behavior, unless: (a) information obtained is recorded in such a manner that human subjects can be identified, directly or through the identifiers linked to the subjects: and (b) any disclosure of human subjects' responses outside the research could reasonably place the subjects at risk of criminal or civil liability or be damaging to the subjects' financial standing, employability, or reputation.

This exemption is valid for three years from the date of this correspondence, after which the study will be closed. If tł research will extend beyond three years, it is your responsibility as the Principal Investigator to notify the IRB before the study's expiration date and submit a new application to continue the research. Research activities that continue beyond the expiration date without new certification of exempt status will be in violation of those federal guidelines which permit the exempt status.

As part of the IRB's quality assurance procedures, this research may be randomly selected for continuing review durir the three year period of exemption. If so, you will receive a request for completion of a Protocol Status Report during the month of the anniversary date of this certification.

In all cases, it is your responsibility to notify the IRB prior to making any changes to the study by submitting an Amendment/Modification request. This will document whether or not the study still meets the requirements for exempt status under federal regulations.

Upon receipt of this memo, you may begin your research. If you have questions, please call the IRB office at (435) 797-1821 or email to irb@usu.edu.

The IRB wishes you success with your research.

4460 Old Main Hill Logan, UT 84322-4460 PH: (435) 797-1821 Fax: (435) 797-3769 WEB: irb.usu.edu EMAIL: irb@usu.edu

Appendix D

Rural District Email Approval

From: Andrew Glaze [mailto:aglaze@dsdmail.net]
Sent: Tuesday, August 28, 2018 1:19 PM
To: Doug Jacobs
Subject: Research Question

Dear Mr. Jacobs,

My name is Andrew Glaze. I am a doctoral student at Utah State University and a mathematics teacher in Davis School District. This summer while I was taking an administrative course with Dr. Richard Nye from the Ogden School District, I asked him to suggest a couple rural school districts where I could invite junior high mathematics teachers to participate in a survey. He suggested that I talk to you.

I am conducting a study to investigate a relationship between teachers' conceptions of mathematics and their use of intelligent tutoring systems such as iReady, ALEKS, or Thank Through Math. I just received university IRB approval and am ready to move forward to receiving school district approval. I anticipate that I will have approval from both Ogden and Davis school districts in the next couple of weeks.

Would it be possible for me to request approval to conduct a survey in your school district also? I would be glad to call you to talk about the survey, what it involves, and the incentive that I am offering educators for participation.

Thank you.

Andrew Glaze

From: Doug Jacobs <djacobs@morgansd.org>
Sent: Wednesday, August 29, 2018 7:24 AM
To: Andrew Glaze <aglaze@dsdmail.net>
Subject: RE: Research Question

Hi Andrew. Yes, we would be happy to participate. Good Luck. Doug Jacobs

Appendix E

School Districts’ IRB Approvals

Davis School District
Established 1911
Reid P. Newey, Superintendent
Learning First!

September 21, 2018

Andrew Glaze
aglaze@dsdmail.net

Dear Mr. Glaze,

The application you submitted has been reviewed by the Superintendent's Executive Staff. The research project entitled "Teachers' Conceptions of Mathematics and Intelligent Tutoring System Use: A Mixed Methods Approach" is approved for the 2018-19 school year.

As a researcher you are responsible for all aspects of the study. District resources may not be used to conduct the study. All costs associated with the study are paid by the researcher. Please request data, as needed, from the Davis District Assessment Department by submitting a data request (https://www.davis.k12.ut.us/departments/assessment) under "Tools" or calling (801) 402-5305.

Approval at the district level allows each site to then determine whether to participate in your proposed research study or project. District approval, therefore, is not a guarantee that you will be able to conduct the study at the locations with the employees or the students you wish to include in the study or project.

Please remember that any anticipated changes to the study and approved procedures must be submitted to this office prior to implementation. It is our understanding that you will protect the anonymity of individuals involved in the research.

We hope your research proves insightful and fulfilling.

Sincerely

Ms. Amy Siegel
Assessment Supervisor

45 East State Street • PO Box 588 • Farmington, Utah 84025 • (801) 402-5261

AdvancED Accredited | NWAC

OGDEN SCHOOL DISTRICT
Empowering Excellence Through Education
Educational Research

September 21, 2018

Dear Mr. Glaze:

We have received your proposal to conduct the research study entitled "Teachers' Conceptions of Mathematics and Intelligent Tutoring System Use: A Mixed Methods Approach." We understand all participant subjects will grant consent forms as presented in your IRB application and your request to conduct research with OSD. We further understand that you have received IRB exemption dated August 27, 2018.

The Ogden School District team has reviewed your request and completed confidentiality agreement. We have can now conclude that this research is in line with both our district mission and with appropriate IRB approved research practices for working with human subjects. With the aforementioned understandings we can agree to this voluntary research study within our school district.

This letter is to serve as recognition of our approval for you to proceed with your research as described. Note that Ogden School District requires documentation that you obtained informed consent and any data collected and presented publicly must not allow for the identification of individual participants. Your point of contact with our school district will be Mrs. Janean Montgomery. Mrs. Montgomery will be your resource and your first contact if any questions or concerns arise related to the research in our school district.

Any concerns about the research or the protocols employed will be handled jointly by Ogden School District and the researcher to determine appropriate response and any needed next steps, deferring to the researcher and the IRB committee for responsibility to fulfill the IRB approved processes. Further clarification and coordination will be obtained through your assigned point of contact, Mrs. Montgomery, montgomeryj@odensd.org.

We look forward to receiving a copy of your findings and reviewing these with you at the conclusion of your study. Thank you for your efforts to further our profession.

Earnestly,

Sarah Roberts
Ogden School District
Executive Director
Instructional Leadership
801 737-7335, robertss@ogdensd.org

Appendix F

University Candidacy Approval

RESEARCH and GRADUATE STUDIES
UtahStateUniversity

Date: 9/20/2018

Application for Candidacy for a Doctoral Degree

Use this form to apply for candidacy in a doctoral degree program. Submit the form after your dissertation proposal has been approved, after all regulatory approvals are in place, and after you have passed your comprehensive examination. (Directions for completing form are on page 4.)

Student's Name: Andrew R. Glaze   A#: A01768462

Student's Email: andrewrglaze@gmail.com   Signature

Col-Dept-Prog-Deg: Education - TEAL - Education - PhD - Curriculum and Instruction

I currently hold (check one): ☐ a Bachelor's degree ☒ both Bachelor's and Master's degrees

My dissertation proposal was approved on (date): 8/27/2018

Dissertation proposal title: TEACHERS' CONCEPTIONS OF MATHEMATICS AND INTELLIGENT TUTORING SYSTEM USE: A MIXED METHODS APPROACH

☐ No regulatory approvals are required for this dissertation research

Regulatory Approvals (check if required)

			Approval or Completion Date
☐ Animal Subjects	IACUC #:		
☒ Human Subjects	IRB protocol #:	irb-8491	8/27/2018
☐ Radioactive Materials	Authorization #:		
☐ Biohazards	Authorization #:		
☐ Recombinant DNA/Synthetic Biology	IBC #:		
☐ EHS Lab Safety Training			
☐ Responsible Conduct of Research (USU 6900 should appear on student's transcript)			

Notice: All students who began a doctoral degree program after July 2013 are required to take the Responsible Conduct of Research (RCR) training. Any doctoral student who was supported on a grant from the National Science Foundation after 2010 is required to take the RCR training.

Type abstract in the area provided below.

Abstract requirements:

In 300-350 words summarize your project, its objectives, the methods that will be employed (how and where you will obtain your data), and any expected results.

This exploratory study investigates teachers' conceptions of mathematics and their use (or non-use) of Intelligent Tutoring Systems (ITSs). The overarching question addressed by this study is "What is the relationship between teachers' conceptions of mathematics and their use of Intelligent Tutoring Systems (ITS) for mathematics instruction?" A conception of mathematics is a combination of knowledge and beliefs about mathematics. An ITS is a program that: 1) performs tutoring functions; 2) constructs a cognitive model of a student's psychological state, or locates the psychological state in a previously defined model; and, 3) uses information from item number two to adjust an element from item number one (Ma, Adesope, Nesbit, & Qing, 2014).

This survey study employs a convergent mixed methods research design for the collection of qualitative and quantitative data (Creswell & Plano Clark, 2018). Approximately 150 mathematics teachers from 19 junior high schools and one middle school in three school districts will be invited to participate in the study. The first section of the survey contains quantitative and qualitative questions on teachers' use of ITSs. The ITS questions were designed for this study and piloted ten times. The second section of the survey includes five of the eight dimensions from the Grouws, Howald, and Colangelo (1996) Teachers' Conceptions of Mathematics Inventory. The five included dimensions are teachers' conceptions of: 1) the composition of mathematics, 2) the structure of mathematics knowledge, 3) doing mathematics, 4) validating ideas in mathematics, and 5) learning mathematics.

The quantitative analysis will employ a 2 x 5 mixed design ANOVA to investigate the relationship between teachers' conceptions of mathematics and their use (or non-use) of ITSs. Qualitative responses will be coded using the open-coding procedure outlined by Creswell (2003, 2013). Other studies have shown a relationship between teachers' conceptions of mathematics and their use of graphing calculators to teach mathematics (Doerr & Zangor, 2000; Lee, 2007). Results from this study may suggest that teachers with an instrumental view of mathematics employ ITSs to practice procedures while teachers with a relational view do not use ITSs or use ITSs to promote a conceptual understanding of mathematics.

References:

Creswell, J. W. (2003). Research design: Qualitative, quantitative, and mixed methods approaches (2nd ed.). Thousand Oaks, CA: SAGE Publications Inc.

Creswell, J. W. (2013). Qualitative inquiry & research design; Choosing among five approaches (3rd ed.). Thousand Oaks, CA: SAGE Publications Inc.

Creswell, J. W., & Plano Clark, V. L. (2018). Designing and conducting mixed methods research (3rd ed.). Thousand Oaks, CA: SAGE Publications Inc.

Doerr, H. M., & Zangor, R. (2000). Creating Meaning for and with the graphing calculator. Educational Studies in Mathematics, 41(2), 143–163.

Grouws, D. A., Howald, C. L., & Colangelo, N. (1996). Student conceptions of mathematics; A comparison of mathematically talented students and typical high school algebra students. Presented at the Annual Meeting of the American Educational Research Association, New York.

Lee, J. A. (2007). Using graphing calculators in the mathematics classroom: Teachers' conceptions and their teaching practices (Unpublished doctoral dissertation). University of Toronto, Toronto, Ontario, Canada. Retrieved from https://search-proquest-com.dist.lib.usu.edu/pqdtglobal/docview/304745440/C2F9A3728A5D42A3PQ/1?accountid=14761

Comprehensive Examination: list each section of the exam and the date it was passed.

Section	Date	Examiner
Written Examination	June 2017	Dr. Moyer-Packenham

By signing this form, the Advisor/Major Professor and all committee members acknowledge that the student has met the residency requirement (see page 3), they have approved the student's dissertation research proposal, and they believe that the student is ready to conduct independent dissertation research.

Advisor/Major Professor	Email address	Signature
Dr. Patricia Moyer-Packenham	patricia.moyer-packenham@usu.edu	
Committee Members		
Dr. Max Longhurst	max.longhurst@usu.edu	
Dr. Beth MacDonald	beth.macdonald@usu.edu	
Dr. Kady Schneiter	kady.schneiter@usu.edu	
Dr. Susan Turner	susan.turner@usu.edu	
Department Head		
Kit Mohr	kathleen.mohr@usu.edu	

School of Graduate Studies: Richard Inouye richard.inouye@usu.edu

Signature Order

1) Student, Major Professor
2) Other Committee Members
3) Department Head
4) IRB, IACUC (if needed)
5) School of Graduate Studies

DocuSign CCs (other individuals who will receive a copy of the signed form)

	Name	Email
GPC:	Krista Terrell	krista.terrell@usu.edu
SGS:	Kurt Adison	kurt.adison@usu.edu
Other:		
Other:		

Residency Requirement (from the Catalog)

The purpose of the residency requirement is to ensure that the doctoral student experience includes at least one period of concentrated attention to study, research, and interaction with faculty. This period of immersion in the culture of the student's department is an important part of preparation for future work in academic communities. The residency requirement for doctoral studies (PhD, EdD, and professional doctoral degrees) consists of the following:

Credits for residency. Of the credits on the approved Program of Study, either 50% or 33 semester credits, whichever is fewer, must be from USU. The balance of credits may be from USU or from other institutions, subject to transfer credit limits and the approval of the student's supervisory committee. This aspect of the residency requirement is verified by the School of Graduate Studies as part of the check of the Program of Study.

Participation in the academic community. Meeting the residency requirement also means that doctoral students must take part in the academic community of their program. Participation could include collaborative scholarship with faculty or peers, working as a research assistant or graduate instructor, attending professional meetings, being involved with student or professional organizations, and participating in colloquia, orientation programs, etc. This participation may or may not coincide with the period of concentrated study. Departments have the responsibility to determine appropriate ways for their doctoral students to participate in the academic life of their field and to provide opportunities for this participation. This aspect of the residency requirement is certified by the graduate supervisory committee and noted as acceptable by signatures on the application for candidacy form.

Each degree program may set more intense requirements for residency. Students should review college, departmental, and program requirements.

Doctoral Students: If you have not already done so, you should discuss with your advisor, and your committee, your rights to copyright and data associated with your research and your plans for authorship of any publications that result from your research. These issues are discussed in a memo from Dean McLellan, and there are two forms (Copyright Form, Authorship) that you will be required to submit before your degree will be awarded. The memo and the forms are available on the SGS web site (www.rgs.usu.edu/graduateschool/forms/), under 'Dissertation / Thesis / Plan B Forms.' If you have questions about the USU copyright policy, contact the Merrill-Cazier Library.

Appendix G

Histograms for Non-Normal Data Distribution

The histograms represent the teachers' conception scores with non-normal distribution referenced in the 2x 5 mixed ANOVA analysis.

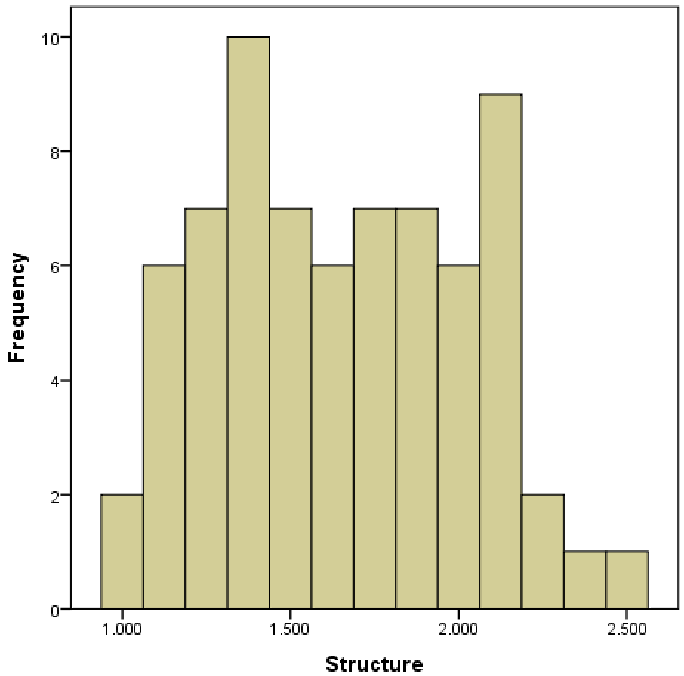

Figure G1. Histogram for structure in question 1 for Intelligent Tutoring Systems use.

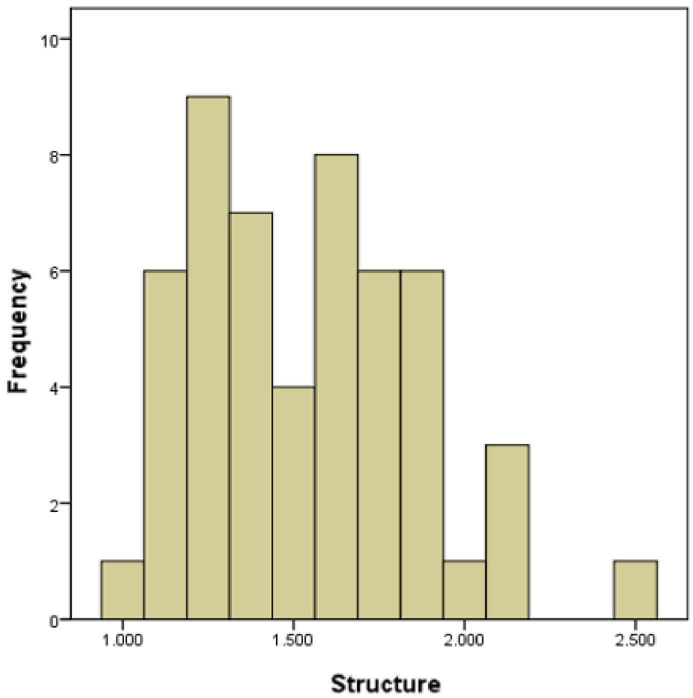

Figure G2. Histogram for structure in question 2 for graphing calculator users.

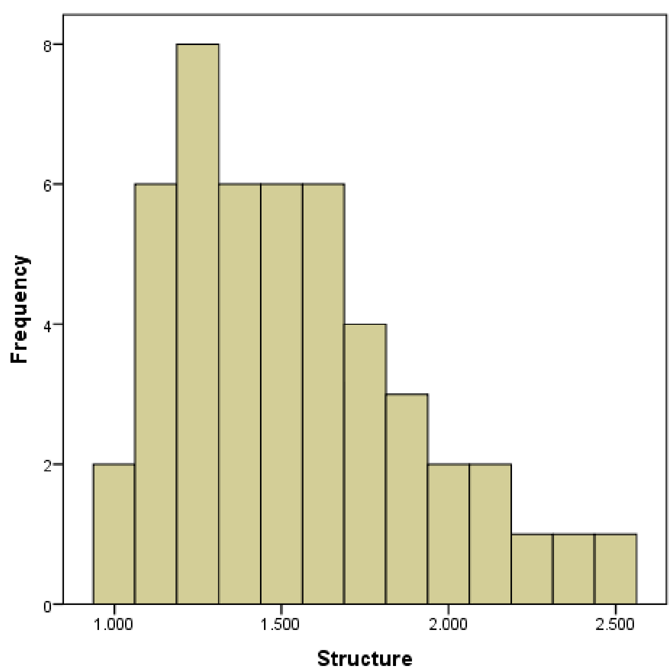

Figure G3. Histogram for structure in question 2 for Desmos users.

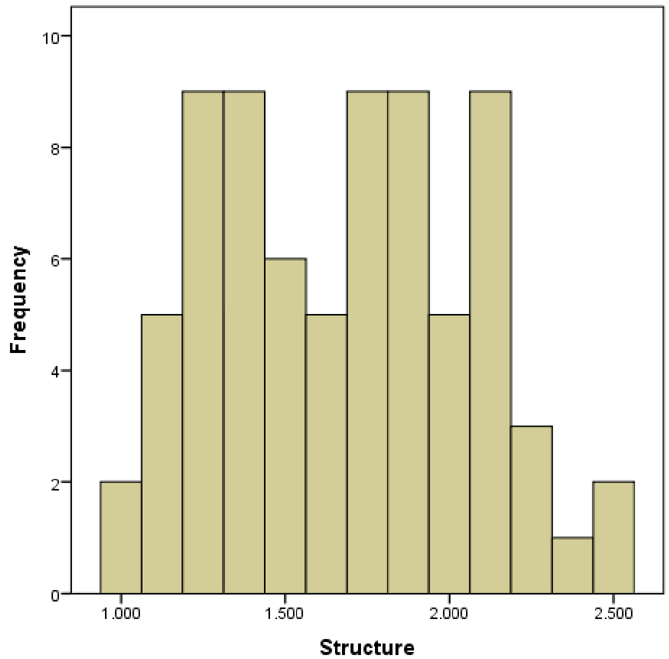

Figure G4. Histogram for structure in question 2 for Dynamic Geometry Software nonusers.

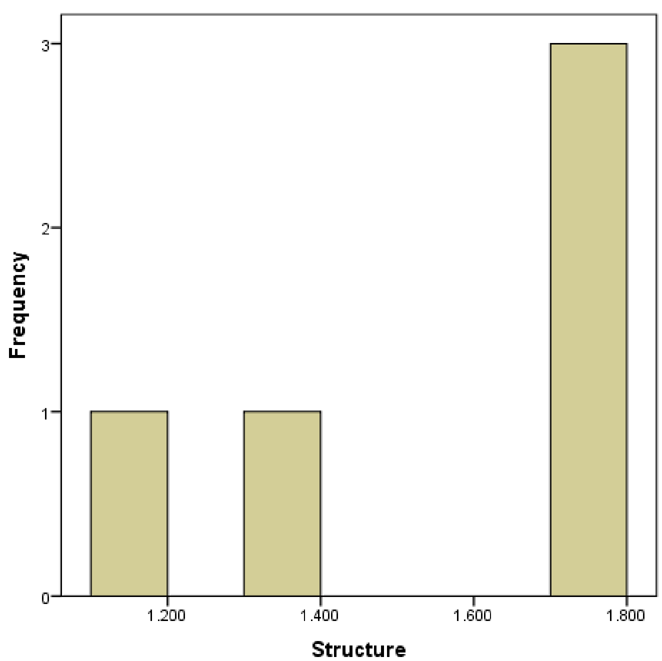

Figure G5. Histogram for structure in question 3 for teachers not using Intelligent Tutoring Systems to fill gaps.

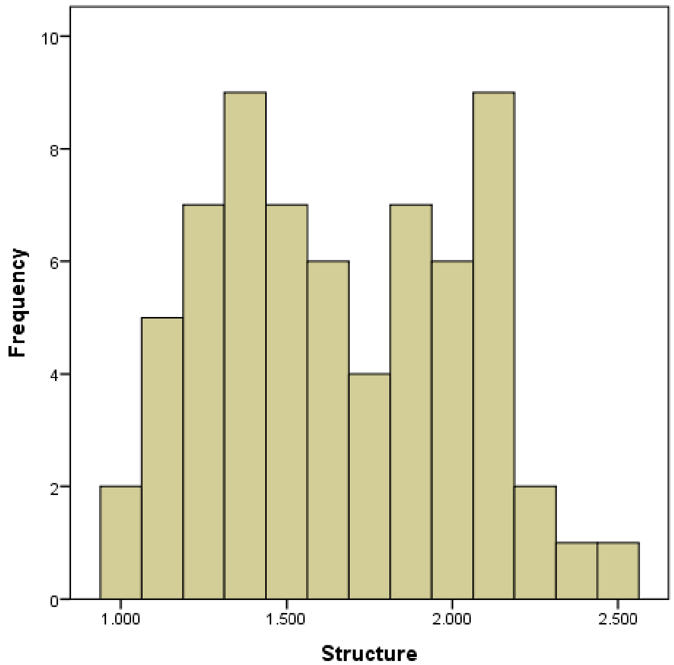

Figure G6. Histogram for structure in question 3 for teachers using Intelligent Tutoring Systems to fill gaps.

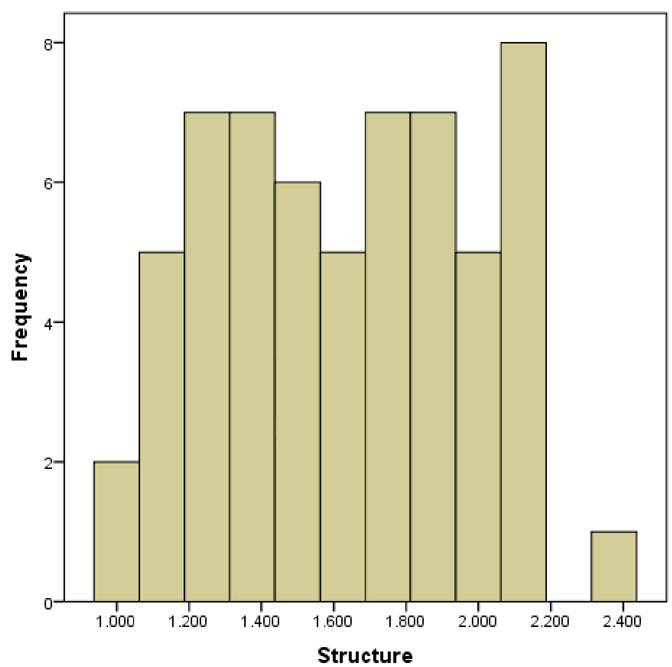

Figure G7. Histogram for structure in question 3 for teachers using Intelligent Tutoring Systems to practice procedures.

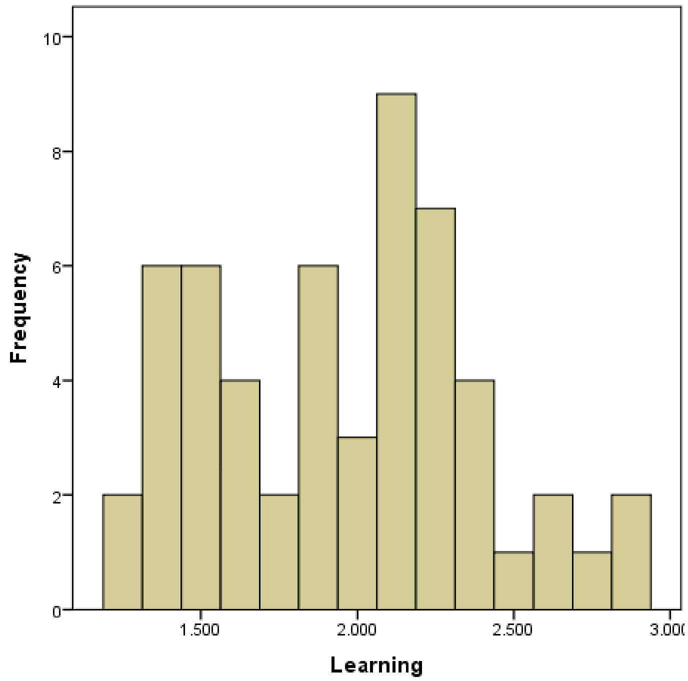

Figure G8. Histogram for learning in question 3 for teachers not using Intelligent Tutoring Systems to teach procedures.

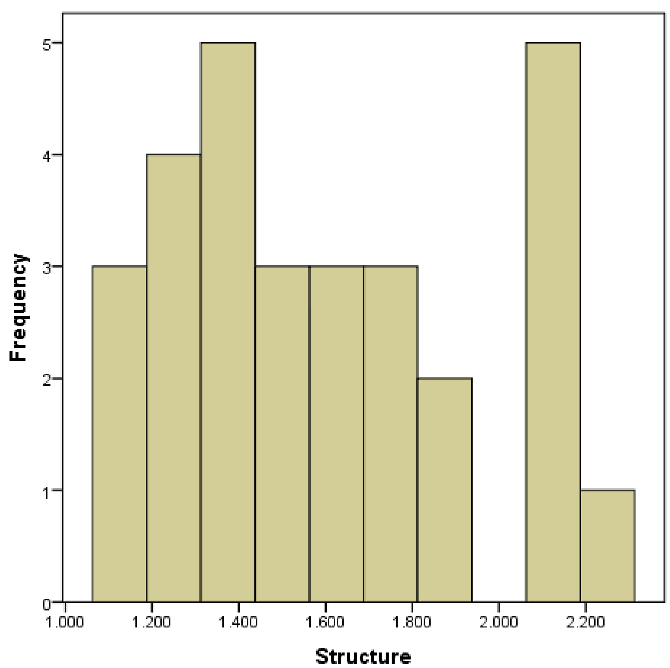

Figure G9. Histogram for structure in question 3 for Intelligent Tutoring Systems use to teach concepts.

VITAE

ANDREW R. GLAZE

Business Address:
Farmington Junior High School
150 S. 200 W.
Farmington, UT 84025
Email:
andrewrglaze@gmail.com

Home Address:
284 S. 850 E.
Kaysville, UT 84037
801-678-9563

EDUCATION

Ph.D. May 2019
Education, Utah State University, Logan
Specialization: Curriculum and Instruction
Emphasis: Mathematics Education and Leadership
Dissertation: Teachers' Conceptions of Mathematics and Intelligent Tutoring System Use

M.A. August 2006
Mathematics Education, Brigham Young University, Provo
Thesis: The Nature and Frequency of Mathematical Discussion During Lesson Study That Implemented the CMI Framework

B.A. August 2004
Mathematics Education, Brigham Young University, Provo
Minor: Physics Education
Utah Level 3 Teaching License

EMPLOYMENT HISTORY

Junior High School Teacher, Grades 7 – 9 (2013 – Present)
Davis School District, Farmington, Utah
Mathematics Department Chair
Courses: 7th Grade Mathematics, 8th Grade Mathematics, Mathematics I

High School Teacher, Grades 10 – 12 (2006 – 2013)
Davis School District, Farmington, Utah
Courses: Common Core Math II, Geometry, Algebra II, Precalculus

Facilitator, Core Academy (2016 – 2017)
Utah State School Board
Responsibilities: Facilitating a professional development promoting increased content knowledge and pedagogical content knowledge for teachers of the 7th grade curriculum.

Facilitator, Principles to Actions (2015)
Utah State Office of Education
Responsibilities: Facilitating a professional development promoting the personal implementation of NCTM's Principles to Actions.

Facilitator, Utah Common Core Academy (2012)
Utah State Office of Education
Responsibilities: Facilitating a professional development on the common core Math II for secondary teachers of mathematics throughout the state

Facilitator, Western Governor's University (2007 – 2008)
Responsibilities: Facilitated learning for undergraduate mathematics courses and mathematics methods courses.

University Mathematics Instructor and TA, BYU Provo (2004 – 2006)

- MTHED 306 Concepts of Mathematics: A course for pre-service elementary school teachers. Concept-oriented exploration of rational numbers and proportional reasoning, probability, and early algebraic reasoning in relation to children's learning was taught.
- MTHED 305 Basic Concepts of Mathematics: A course for pre-service elementary school teachers. Concept-oriented exploration of number, measurement, and informal geometry in relation to children's learning was taught.
- TEAL 4630/6630: Middle School Mathematics Methods is a course covering major topics of middle school mathematics and pedagogical techniques.
- Math 119 Introduction to Calculus: This is a calculus course for students in the Colleges of Biology, Agriculture and Business.
- Math 110 College Algebra: Functions, polynomials, theory of equations, exponentials and logarithmic functions, matrices, determinants, systems of linear equations, permutations, combinations, and binomial theorem were taught.

Lead Tutor, University Tutoring Lab (2002 – 2004)
Brigham Young University, Department of Mathematics, Provo
Responsibilities: Facilitating the training of undergraduate tutors for upper and lower division mathematics courses.

RESEARCH

Research Interests:

- Equity in education
- Mathematics teacher development
- Technology in the mathematics classroom

Research Assistant:

- Research assistant for the Active Learning Lab at Utah State University
- Research assistant for Dr. Blake Peterson
- Research assistant for Dr. Janet Walters

Research Completed:

- Master's Thesis: *The Nature and Frequency of Mathematical Discussion During Lesson Study That Implemented the Comprehensive Mathematics Instruction Framework (2006).*

- *Math Initiative Committee* (2004-2006).
  Faculty Advisor: Dr. Blake E. Peterson
  My Role: Gathering data via video camera, transcribing recorded data, and evaluating written data collected from study participants.

- *Innovative Professional Development in Mathematics: A Three-Year Study of a Cohort of Elementary Teachers* (2003-2004)
  Faculty Advisor: Dr. Janet G. Walter
  My Role: Gathered and analyzed data with Field Notes and video equipment.

PRESENTATIONS

National Conferences

Glaze, A. R. (March 2017). *Invisible Mathematics*, MidSchool Math National Conference, Santa Fe, NM.

Utah Council of Teachers of Mathematics (UCTM)

Glaze, A. R. (November 2016). *Invisible Mathematics.* Annual Conference of the Utah Council of Teachers of Mathematics (UCTM), Salt Lake City, Utah

Glaze, A. R. (November 2014). *Beans to Bullying.* Annual Conference of the Utah Council of Teachers of Mathematics (UCTM), Layton, Utah

Glaze, A. R. (November 2013). *Am I Doing it Right? Implementing Dynamic Geometry Software in Your Classroom.* Annual Conference of the Utah Council of Teachers of Mathematics (UCTM), Salt Lake City, Utah.

Glaze, A. R. (October 2012). *Shifts, Stretches, and Slides, Oh My.* Annual Conference of the Utah Council of Teachers of Mathematics (UCTM), American Fork, Utah.

Glaze, A. R. (April 2011). *An Excel-ent Way to Teach Statistics.* Outreach Conference of the Utah Council of Teachers of Mathematics (UCTM), Cedar City, Utah.

Glaze, A. R. (October 2009). *Professional Learning Community Collaborations.* Annual Conference of the Utah Council of Teachers of Mathematics (UCTM), Orem, Utah.

Davis School District

Glaze, A. R. (August 2017). *Intelligent Uses of Intelligent Tutoring Systems.* Davis School District In-service Training, Kaysville, Utah

Glaze, A. R. (October 2012). *The Five Practices.* Davis School District In-service Training, Farmington, Utah.

Glaze, A. R. (January 2011). *Excel-ent Classroom Strategies with Technology.* Davis School District Technology Conference, Woods Cross, Utah.

STATE AND DISTRICT SERVICE – LEADERSHIP ACTIVITIES

USBE Instructional Materials Review

October 2018

I participated in a committee to review then recommend or reject mathematics instructional materials submitted by text book publishers to the Utah State Board of Education.

Utah RISE Content Bias Review

September 2018

During this three-day committee, I assisted in reviewing testing items to be piloted for 6^{th}, 7^{th}, and 8^{th} grade RISE assessment during the 2018-2019 school year.

Utah State Board of Education Rubric Validation

Summer 2018

I served on a committee to validate rubrics for SAGE testing items from the 2017-2018 school year.

Utah State Board of Education Item Review

Summer 2017

I served on a committee to review SAGE testing items from the 2017 school year.

Utah State Board of Education Item Writer

Spring 2017

I served on a committee to write test questions for the SAGE assessment.

Northern Utah Curriculum Consortium (NUCC) Curriculum Writer

Spring 2013

I served on a committee designated to write curriculum content for the implementation of Math III for the Utah Core Curriculum.

Publicity Specialist for the Utah Council of Teachers of Mathematics

2014-2015

Duties include the distribution of timely and important information to the Utah mathematics community at large.

National Council of Teachers of Mathematics (NCTM) Affiliate Representative

2012-2014

Duties included representing the state of Utah as a delegate for the national conferences, promoting NCTM membership for affiliate members, and helping to recognize outstanding mathematics teachers through affiliate sponsored awards.

Board Member at Large for the Utah Council of Teachers of Mathematics (UCTM)

2009-2011

My primary duty was to prepare the program for the annual conference.

Math Standards Committee

2009

I served on a committee aiding in determining cut scores for the Algebra II CRT level descriptors.

UBSCT Item Writing

2008

I served on a state committee to write items for the Utah Basic Skills Competency Test.

Text to Core Committee Member

2007-2008

I served on a committee that helped teachers align their textbook use with the then current state core.

CRT Item Writing

2007-2008

I wrote and submitted questions for the Algebra II and Geometry Criterion Reference Tests.

PUBLICATIONS

Glaze, A. (2017). Invisible Mathematics. Utah Mathematics Teacher, 10 (Fall/Winter), 16–25.

Nadelson, L. S., Juth, S., Sias, C. M., Gedeborg, S., & Glaze, A. (Under Review). Pioneers in unknown territory: Teacher perceptions and use of non-conventional instructional tools. Journal of Research on Technology in Education.

CONSULTANT WORK

Technology Consultant

July 2017

I was a consultant for the Utah Military Academy in their technology implementation for mathematics classes.