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PERFORMANCE AND DESIGN OF LABYRINTH SPILLWAY

by

Nosratollah Amanian

A thesis submitted in partial fulfillment  
of the requirements for the degree

of

MASTER OF SCIENCE

in

Civil and Environmental Engineering

Approved:

Major Professor \_\_\_\_\_

Committee Member \_\_\_\_\_

Committee Member \_\_\_\_\_

Dean of Graduate Studies \_\_\_\_\_

UTAH STATE UNIVERSITY  
Logan, Utah

1987



## ACKNOWLEDGEMENTS

I would like to express my appreciation to Dr. J. Paul Tullis, my major advisor, for his advise, patience, and encouragement throughout the completion of this thesis. I would also like to express my appreciation to my other committee members, Dr. Roland W. Jeppson and Dr. Gordon H. Flammer, for their review and helpful suggestions on this study.

Finally, I wish to express my gratitude to my entire family for their financial support and encouragement provided throughout my education.

Nosratollah Amanian

## TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS . . . . .	ii
LIST OF TABLES . . . . .	v
LIST OF FIGURES . . . . .	vii
LIST OF SYMBOLS . . . . .	ix
ABSTRACT . . . . .	x
Chapter	
I. INTRODUCTION . . . . .	1
Spillway Types . . . . .	2
II. REVIEW OF LITERATURE . . . . .	3
The Labyrinth Spillway . . . . .	3
Performance of a Labyrinth Weirs . . . . .	3
Parameters Affecting the Performance of a Labyrinth Weirs . . . . .	7
Dimensional Analysis . . . . .	8
Significance of Independent Ratios in Equation 3 . . . . .	9
III. PRIOR STUDIES ON A LABYRINTH SPILLWAY . . . . .	16
Hydraulic Model Studies of Labyrinth Weirs . . . . .	18
Existing Labyrinth Weirs . . . . .	23
IV. EXPERIMENTAL APPARATUS AND PROCEDURES . . . . .	26
Experimental Facilities . . . . .	26
Scope of the Investigation . . . . .	27
Models Construction . . . . .	30
Testing Procedure . . . . .	31
V. DATA PRESENTATION AND DISCUSSION . . . . .	34
Discharge Coefficients . . . . .	35
Linear Weir . . . . .	36
Analysis of Weir Sectional Crest Shapes . . . . .	39
Analysis of Normal Labyrinth Weirs . . . . .	44
General Discussion of the Behavior of Labyrinth Weirs . . . . .	45
Development of Design Curves . . . . .	48
Comparison of Normal Labyrinth Weirs With Inclined Labyrinth Weirs . . . . .	52

VI. DESIGN AND APPLICATION OF LABYRINTH SPILLWAY . . . . .	59
General Applications of Labyrinth Weirs . . . . .	59
Design Considerations of Labyrinth Weirs . . . . .	59
Different Cases of Labyrinth Spillway Design . . . . .	60
Steps in the Design of Labyrinth Weirs . . . . .	61
Design Example . . . . .	63
VII. SUMMARY AND CONCLUSION . . . . .	67
REFERENCES . . . . .	70
APPENDIX . . . . .	72

## LIST OF TABLES

Table	Page
1. Comparison of Discharge Coefficients $C$ and $c$ for Sharp Crested Weirs with Full Ventilation of the Nappe from This Study and Previous Research . . . . .	43
2. Comparison of the Actual Crest Length of Various Labyrinth Spillways with the Calculated Length Using the Current Study . . . . .	66
3. A Discharge Coefficient of a Rounded Crested Normal Linear Weir . . . . .	73
4. A Discharge Coefficient of a Rounded Crested Normal Linear Weir . . . . .	74
5. A Discharge Coefficient of a Rounded Crested Normal Linear Weir . . . . .	75
6. A Discharge Coefficient of a Rounded Crested Normal Linear Weir . . . . .	76
7. A Discharge Coefficient of a Rounded Crested Normal Linear Weir . . . . .	77
8. A Discharge Coefficient of a Sharp Crested Normal Linear Weir . . . . .	78
9. A Discharge Coefficient of a Sharp Crested Normal Linear Weir . . . . .	79
10. A Discharge Coefficient of a Quarter Rounded Crested Normal Linear Weir . . . . .	80
11. A Discharge Coefficient of a Quarter Rounded Crested Normal Linear Weir . . . . .	81
12. A Discharge Coefficient of a Flat Top Crested Normal Linear Weir . . . . .	82
13. A Discharge Coefficient of a Flat Top Crested Normal Linear Weir . . . . .	83
14. A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	84
15. A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	85
16. A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	86

17.	A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	87
18.	A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	88
19.	A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	89
20.	A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	90
21.	A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	91
22.	A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	92
23.	A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir . . . . .	93
24.	A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir . . . . .	94
25.	A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir . . . . .	95
26.	A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir . . . . .	96
27.	A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir . . . . .	97

## LIST OF FIGURES

Figure	Page
1. Schematic Diagram of Labyrinth Weir (Source: Taylor 1968[11]) . . . . .	4
2. Plan View of Labyrinth Weirs Installed at an Angle in the Channel of Approach (Inclined Labyrinth Weirs) . . . . .	4
3. Diagram Used to Describe the Theoretical Analysis of Labyrinth Weir Flow (Source: Taylor 1968[11]) . . . . .	6
4. Design Chart for Triangular Plan Form Weirs (Source: Hay and Taylor 1970[4]) . . . . .	19
5. Design Chart for Trapezoidal Plan Form Weirs (Source: Hay and Taylor 1970[4]) . . . . .	19
6. Design Chart for Trapezoidal Plan Form Weirs (Source: Darvas 1971[3]) . . . . .	20
7. Discharge Coefficient for Sharp Crested Labyrinth Weirs (Source: Lux 1984[10]) . . . . .	22
8. Design Curves for Quarter Round Labyrinth Weirs (Source: Lux 1984[10]) . . . . .	22
9. Design Curves with Sharp Crested Labyrinth Weirs (Source: Hinchliff and Houston 1984[5]) . . . . .	24
10. Design Curves with Quarter Round Upstream Face, Trapezoidal Form Weir (Source: Hinchliff and Houston 1984[5]) . . . . .	24
11. Shapes of the Crest Section . . . . .	28
12. Effect of Approach Velocity on the Discharge Coefficient of Sharp Crested Weirs . . . . .	37
13. Variation of Discharge Coefficient for Three Types of Ventilation on a Rounded Crested Normal Linear Weir for $P = 0.5$ feet . . . . .	38
14. Effect of Weir Thickness on the Discharge Coefficient of Rounded Crested Weirs . . . . .	40
15. Discharge Coefficients of Normal Linear Weirs with Four Different Crest Shapes. The Nappes are Fully Vented . . . . .	42
16. Flow Condition of Normal Labyrinth Weirs . . . . .	46

17.	Flow Condition of Normal Labyrinth Weirs with Large Angle of Sidewalls . . . . .	47
18.	Design Curves for a Triangular Normal Labyrinth Weirs with Rounded Crest Shape . . . . .	50
19.	Effect of Angle of Sidewalls, $\alpha$ , and H/P on the Performance of Normal Labyrinth Weirs . . . . .	51
20.	Design Chart 1: Triangular Normal Labyrinth Weirs with Rounded Crest Shape. $H_T/P$ from 0.15 to 0.5, and $10 < \alpha < 90$ Degrees . . . . .	53
21.	Design Chart 2: Triangular Normal Labyrinth Weirs with Rounded Crest Shape. $H_T/P$ from 0.5 to 1.0, and $10 < \alpha < 90$ Degrees . . . . .	54
22.	Flow Condition of Inclined Labyrinth Weirs, $\beta = 45$ Degrees, and $\alpha = 24.5$ Degrees . . . . .	56
23.	Effect of Angle of Approach on a Discharge Coefficient of the Unit Length of Inclined Labyrinth Weirs, $\alpha = 24.5$ Degrees . . . . .	57
24.	Effect of Approach Angles on the Performance of Inclined Labyrinth Weirs, for $\alpha = 24.5$ Degrees . . . . .	58

## LIST OF SYMBOLS

a	Half length of labyrinth upstream apex
a'	Half length of labyrinth downstream apex
b	Length of labyrinth sidewall
C	Discharge coefficient (velocity head, $V^2/2g$ , is included)
c	Discharge coefficient (velocity head, $V^2/2g$ , is not included)
$C_L$	Coefficient of discharge
$C_{QR}$	Discharge coefficient of normal linear weir with quarter rounded crest shape
$C_R$	Discharge coefficient of normal linear weir with rounded crest shape
$C_W$	Coefficient of discharge
d	Channel bed elevation differences
g	Acceleration of gravity
H	Upstream piezometric head of approach channel
$H_d$	Downstream head over the crest
$H_T$	Total operating head
$H_o$	Total head
K	Constant value
l	Length of one labyrinth cycle
L	Total length of labyrinth weirs
n	Number of weir cycles in plan
P	Height of weir at the upstream side
Q	Flow discharge
$Q_L$	Discharge over labyrinth weir
$Q_N$	Discharge over normal linear weir occupying the same channel width and with the same total head conditions as the labyrinth weir
RE	Reynolds number ( $VL/\nu$ )
t	Weir thickness
V	Velocity of flow in the channel of approach
w	Width of one cycle of labyrinth spillway
W	Total width of labyrinth spillway
WE	Weber number ( $V\sqrt{L}/\sqrt{\sigma/\rho}$ )
$\alpha$	Angle of sidewall
$\beta$	Angle of labyrinth weir from the normal in the channel of approach
$\gamma$	Specific weight of water
$\mu$	Viscosity
$\rho$	Density
$\sigma$	Surface tension
$\nu$	Kinematic viscosity



## ABSTRACT

## Performance and Design of Labyrinth Spillway

by

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Utah State University, 1987

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Department: Civil and Environmental Engineering

In recent years, the general interest in the safety of dams and reservoirs has grown appreciably. This research describes the application and design of labyrinth weirs to improve the performance of existing reservoirs and also to be used for the construction of new dams. The parameters that can affect the performance of labyrinth weirs are discussed. Tests were conducted on normal linear weirs with four different crest shapes to determine the rate of change of discharge coefficient when each type of weir is used. It is found that weirs with rounded-crest shape can pass more flow than other types of weirs when they are subjected to the same total operating head.

Two design charts are developed for labyrinth weirs with rounded-crest shape with  $H_T/P$  ratio ranges from 0.15 to 0.5 and 0.5 to 1.0. The areas of application of labyrinth weirs, design procedure, different cases of design, and a design example are given. In addition, the performance of labyrinth weirs when the structure placed in an angle to the approaching flow (inclined labyrinth weirs) was tested.

(109 pages)

## CHAPTER I

### INTRODUCTION

Existing dams and reservoirs should be reanalyzed to ensure that they can meet the current standards of dam safety. Advances in the fields of hydrology, meteorology, and also the availability of longer hydrologic records indicate that many old reservoirs which they considered safe in the original design may need to be modified. If design analysis shows that the existing spillway may not adequately pass the probable maximum flood (PMF), resulting in overtopping and possible dam failure, modification of dam structure is required. Many dams need to be enlarged in order to meet increasing downstream water demands, to provide additional head for the hydroelectric power plant, and to give additional flood control capacity to the reservoirs.

Raising the spillway crest and installing a gate over the spillway are two methods to enlarge a reservoir capacity. However, these two methods of modifications may cause some instabilities to the structure. In many cases, an economical method of modification is to raise the existing structure, providing additional storage capacity, instead of increasing the spillway or outlet discharge capacities.

For the existing reservoirs in which the flood control is its major function, and the maximum level of water surface is limited, the best method of modification of the reservoir is to construct a labyrinth weir spillway. This structure may have a raised crest elevation that provides additional storage capacity. In addition to raised crest, it has a longer crest length which provides greater discharge capacity for a fixed spillway crest.

### Spillway Types

Spillways are often classified as controlled or uncontrolled, depending on whether they are gated or ungated. The types of spillways are usually referred to as ogee (overflow), free overfall (straight drop), open channel (trough or chute), side channel, tunnel, conduit, drop inlet (shaft or morning glory), culvert, siphon, and baffle apron drop (13). Also, an overflow can be sharp crested, ogee shaped, broad crested, labyrinth crest, or of different cross sections.

By using a gated spillway, it is possible to increase the discharge per unit length of the crest, thus narrowing the crest length. The automatic gates are more reliable for remote locations and for large reservoirs in which the rise of the water surface is very slow when flooding occurs.

Although the installation of gates have some advantages, it's disadvantages must be considered for the particular site. The advantages of the uncontrolled crest is the elimination of personal attendance at the site to regulate, maintain, and repair the devices. In other words, the uncontrolled crest is absolutely automatic, except for the cases when the flow carries large quantities of floating debris. One of the disadvantages of a gated spillway is that if the gates are opened accidentally, then a significant loss of water and also possible damage to the downstream side of the reservoir may occur.

## CHAPTER II

### REVIEW OF LITERATURE

#### The Labyrinth Spillway

A labyrinth spillway is a normal linear weir which has been folded in the plan in order to increase the effective length of the spillway crest within a fixed spillway width. The labyrinth spillway has been called by many different names, such as, duckbill, folded, accordion, corrugated, and bathtub spillways (10). The plan form of the labyrinth weirs may vary from triangular through trapezoidal to rectangular.

Since the labyrinth spillway is able to discharge more flow than the linear weir for the same head and fixed spillway width, it is a good alternative for modifications of existing reservoirs. Even for construction of new spillways, although the site conditions may allow the building of normal linear weirs, the selection of labyrinth weirs may be found more economical. One disadvantage of a labyrinth spillway might be that it is prone to damage by ice and large floating debris.

#### Performance of a Labyrinth Weirs

Since the flow pattern over a labyrinth weirs is three dimensional and very complicated, the mathematical description of flow cannot be very accurate. Applying the theory of energy and continuity equations in subcritical flow, the water surface level would drop where the flow meets a contraction or rising of channel bottom. Considering one cycle of a labyrinth weirs (Figures 1 and 2), there is a sudden contraction in the upstream channel where flow enters

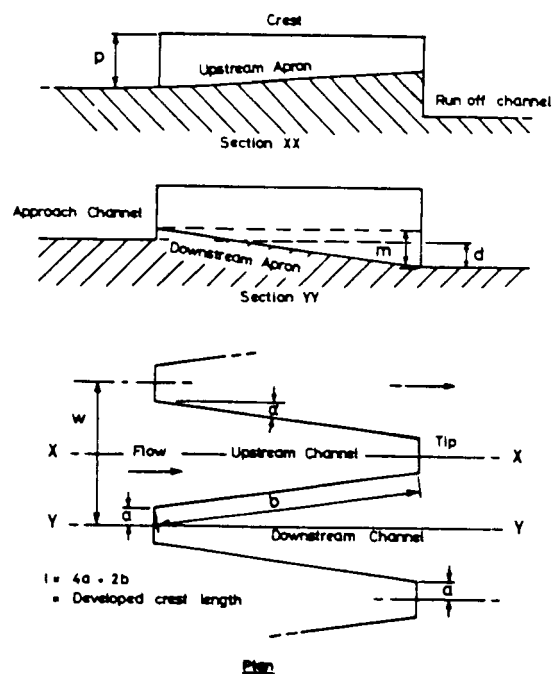


FIG. 1.—Schematic Diagram of Labyrinth Weir (Source: Taylor 1968[11])

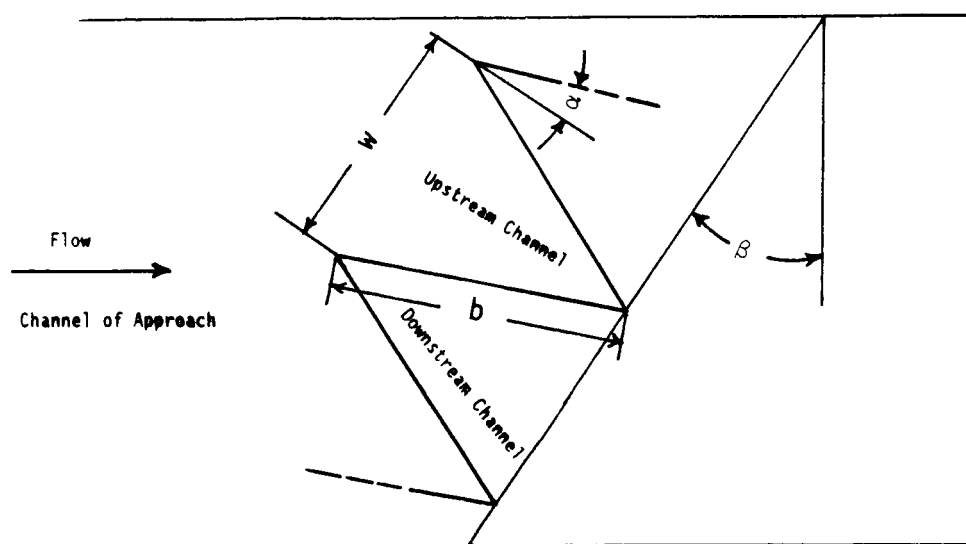


FIG. 2.—Plan View of Labyrinth Weirs Installed at an Angle in the Channel of Approach (Inclined Labyrinth Weirs)

from the approach channel. Consequently, the water level drops. At the same time, the outflow along the weir of the upstream channel acts as an expansion, which tends to raise the level of the water surface. The combination of these two types of operations results in a slow rising of the water surface profile in the channel of approach (4). The same description is true for a downstream channel which has the expansion shape in plan view. The surface profiles of upstream and downstream channels are shown in Figure 3. By assuming the normal weir discharge relation along the labyrinth weirs, the discharge will be less than ideal, because the head along the weir of upstream channel is below the operating head in the approach channel.

The ratio of labyrinth weirs flow to normal linear flow does not increase at the same ratio as the length increment. As the head increases on a labyrinth weirs, its efficiency decreases, and it's performance gradually approaches to the normal linear type of weir which has the same crest length. This is due to two important factors; nappe interference, and the interaction between the upstream and the downstream channel.

The nappe interference occurs when two discharging sides get close to each other. If the angles between the walls of labyrinth weirs are too acute, the performance of higher flows is poor. If the labyrinth angles are very obtuse, the advantage of increasing the crest length is reduced.

Interaction occurs if the water surface anywhere along the downstream channel is above the crest of the weir. If this occurs, the weir operates in a submerged condition, which results in less flow discharge and weak performances.

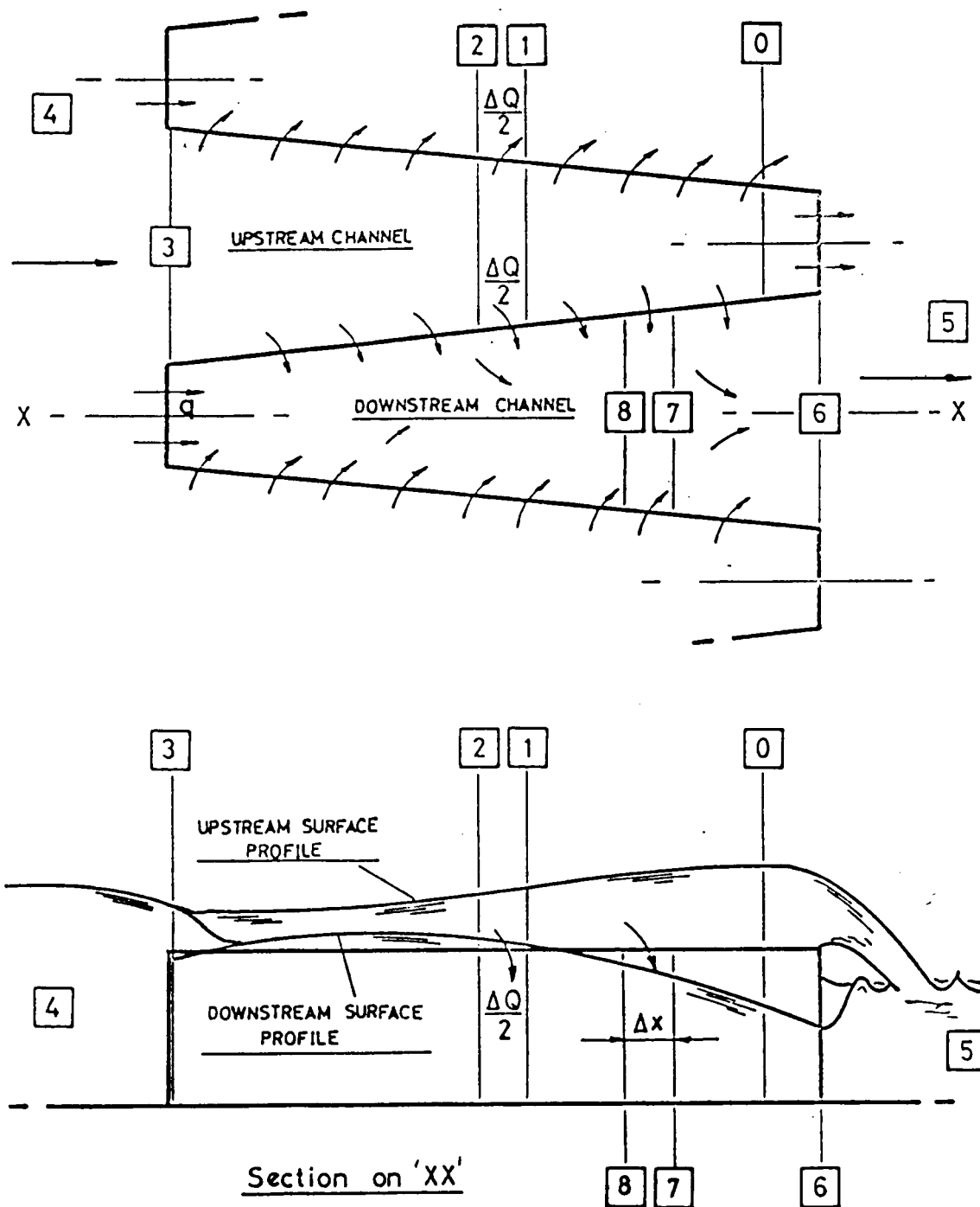


FIG. 3.—Diagram Used to Describe the Theoretical Analysis of Labyrinth Weir Flow (Source: Taylor 1968[11])

### Parameters Affecting the Performance of a Labyrinth Weirs

The discharge of a labyrinth weir is defined by parameters of a normal linear weir, as well as parameters defining the plan form. Based on their significance, these parameters can be divided into two parts: primary and secondary importance.

The parameters of primary importance describe the geometry of design and operating head. These parameters are the following:

$H$  = upstream piezometric head of approach channel

$P$  = height of weir at the upstream side

$l$  = effective length of one labyrinth cycle

$a$  = half length of upstream apex

$a'$  = half length of downstream apex

$b$  = length of labyrinth side wall

$w$  = width of one cycle of labyrinth spillway

$\alpha$  = half of the angle between the adjacent walls (angle of side walls)

$\beta$  = angle of labyrinth spillway from the normal in the channel of approach

$g$  = acceleration due to gravity

The parameters of secondary importance arising from constructional details, such as the shape of the crest section, thickness of the weir wall,  $t$ , and the presence of aprons on both the upstream and downstream sides of the weir or from flow conditions over the weir which ranges from subatmospheric pressure and aerated to submerging flow condition. Other parameters of secondary importance can be the channel-bed elevation differences,  $d$ , downstream head,  $H_d$ , and the number of cycles in plan,  $n$ . The fluid properties such as



specific weight,  $\gamma$ , the density,  $\rho$ , the viscosity,  $\mu$ , and the surface tension,  $\sigma$ , are also parameters which affect the performance of the weir.

### Dimensional Analysis

Since the three dimensional flow over the labyrinth weirs has not yet been subjected to exact mathematical description, the most direct solution of the discharge function involves a combination of dimensional analysis and experimentations. Using all independent variables, the discharge function is:

$$Q = F_1 (\text{crest shape, presence of apron, } n, \alpha, \beta, l, a, w, P, d, t, H, H_d, \gamma, g, \mu, \sigma) \quad (1)$$

length of the side wall,  $b$ , and the fluid density,  $P$ , were not included in Equation 1, as they are not independent variables. From Equation 1, nondimensional ratios which describe the discharge function developed as:

$$Q/LH\sqrt{gH} = F_2 (\text{crest shape, presence of aprons, } n, l/w, a/w, w/P, t/P, d/P, H/P, H_d/P, \alpha, \beta, RE, WE) \quad (2)$$

In the above equation,  $g = \gamma/\rho$ , the acceleration due to gravity. The last two variables in Equation 2 represent Reynolds and Weber numbers, respectively. By assuming the dependent ratio in Equation 2 as a coefficient of discharge and including the acceleration due to gravity in this coefficient, the discharge coefficient is defined as:

$$C = Q/LH^{3/2} = F_3 (\text{crest shape, presence of apron, } n, l/w, a/w, w/P, t/P, d/P, H/P, H_d/P, \alpha, \beta, RE, WE) \quad (3)$$

Since the discharge coefficient in Equation 3 cannot be evaluated by analytical procedures, the relative influence of each of the independent ratios must be evaluated by experiment.

### Significance of Independent Ratios in Equation 3

Crest section. A wide range of crest sections can be used for a labyrinth weirs. The performance curves of a particular weir is not applicable to other crest sections having a different crest coefficient. However, as the head over the labyrinth weirs increases, the discharge tends to be independent of the weir crest coefficient, and the use of complex, expensive crest sections is unnecessary from the hydraulic point of view (4).

Aprons. The presence of aprons at the upstream and downstream sides of the weir make gradual variation of height at both sides of the weir. Aprons are considered optional to the structures in which they are formed by filling up the upstream and downstream channels of the weir and are used to strengthen the structure. Since the crest height,  $P$ , varies along the length of the crest, this parameter must be redefined. Due to the loss of the performance of a labyrinth weirs, as  $H/P$  ratio increases, which results from decreasing of  $P$ , the maximum crest height; the smallest permissible aprons should be used. The results of Hay and Taylor (4) indicate that aprons in both upstream and downstream channels are detrimental to labyrinth weirs performance.

Number of weir cycles in plan,  $n$ . By providing or neglecting the influence of the channel side walls enclosing the weir, the number of cycles does not make any significant changes on the performance of a labyrinth weirs. Taylor (11) tested many models with different numbers of cycles in the plan and made a conclusion that the performance of a labyrinth weirs is independent of  $n$ , and the results obtain of the model tests are applicable to a weir constructed of any

number of cycles in the plan.

Length magnification factor,  $l/w$ . Due to the main objective of a labyrinth weirs, which is to increase the total length of a spillway crest, the length magnification ratio is an important factor in defining the effectiveness of such a weir. As this ratio increases, the flow magnification,  $Q_L/Q_N$ , will increase too.

Significance of apex ratio,  $a/w$ . The parameter  $a$  exists whenever the labyrinth spillway is in trapezoidal or rectangular plan forms. Although the upstream and downstream apexes reduce the efficiency of a labyrinth spillway, they are helpful for field construction purposes. Previous studies have shown that a triangular shape in the plan ( $a/w = 0$ ) is the most efficient type of labyrinth weirs (11).

Vertical aspect ratio,  $w/P$ . This parameter shows the ratio of each cycle width to the height of the crest. If the performance of a labyrinth weirs is related to the operating head to crest height ratio, then the variation of  $w/P$  effectively varies the size of the weir cycles in the plan relative to the head, i.e., when  $w/P$  ratio is small, the size of the weir cycles in the plan can become small in comparison with the head. As  $w/P$  approaches to zero, the size of the weir cycles become so small that the weir shape in the plan is not seen by the flow, and the performance will approach that of a normal linear weir. Increasing the  $w/P$  ratio increases the total width of the spillway.

The loss of performance of a labyrinth weirs with small vertical aspect ratio is due to the nappe interference. The nappe interference refers to the interference of flow caused by the impingement of the nappes issuing from opposite side walls of the weir, and it occurs

when the distance between two facing side walls of the weir is small compared with the head on the weir. Taylor (11) recommends that due to the close proximity of the side walls at the upstream tips of triangular plan-form weirs, in order to eliminate the effect of nappe interference, the value of  $w/P$  should not be less than 2.5. For the case of the trapezoidal plan-form weirs, this value is equal to 2.0.

Difference between the elevations of the channel beds upstream and downstream of the weir,  $d$ . Parameter  $d$  determines the degree of interference on the discharge, which results from choking of the offtake channels. In order to reduce the interference to the flow, the bed elevation drop,  $d$ , should be selected sufficiently large. Taylor (11) suggests the value of  $d$  should be equal to or greater than the maximum operating head in order to completely eliminate the effect of interference on the performance.

Significance of head to crest height ratio,  $H/P$ . In Equation 3, a convenient method of including to some extent the influence of approach velocity, as well as the jet contraction, is to relate  $c$  to  $H/P$ . When  $H/P$  is small, the area of approach channel is relatively large compared with the area of the jet at the weir, and the effect of velocity of approach is almost negligible. However, as  $H/P$  increases, the velocity of approach becomes larger in relation to the velocity of the nappe. The contraction of the nappe from the top is caused by the conversion of potential energy to kinetic energy; whereas, the contraction of the nappe at the bottom is caused primarily by the vertical components of the velocity near the upstream face of the weir (9). When  $H/P$  is small, the nappe contraction is relatively large, but as  $H/P$  increases, the influence of vertical components decrease

and there is less contraction.

In a labyrinth weirs, for very small values of  $H/P$ , the performance is almost ideal. Thus, as  $H/P$  approaches zero, the flow magnification,  $Q_L/Q_N$ , approaches the crest length magnification of the weir. The loss of performance as  $H/P$  increases is primarily due to downstream and nappe interference.

Submergence,  $H_d/P$ . Drowning greatly affects the performance of a normal linear weir. The submergence is defined as the conditions of operation when the level of downstream water surface exceeds the height of the crest (4). The degree of submergence is determined by the parameter  $H_d/P$  in which  $H_d$  is a downstream head measured relative to the weir crest. The results of Taylor (11) show that there is no effect on performance when  $H_d/P \leq 0$ . However, he has not recommended the use of a labyrinth weirs in situations where they would normally operate under heavily drowned flow conditions.

Angle of side walls,  $\alpha$ . The parameter  $\alpha$  is of primary importance in determining the performance of the labyrinth weirs, particularly for high length magnifications,  $l/w$ . For a given length magnification, the value of side wall angles can be varied from 0 for a rectangular plan form type of weir to the maximum value ( $\alpha_{\text{maximum}}$ ) associated with a triangular plan form.

An increase in the value of  $\alpha$  for a given length magnification reduces the length of the weir apexes, so the degree of flow contraction at the upstream channels decreases. This results in a smaller drop of water level at entry to the upstream channels and more horizontal water surface profile from the approach channel to the weir (11). Therefore, in order to obtain the maximum performance, a

triangular plan form weirs should be used wherever possible. However, the influence of nappe interference on the performance of a triangular plan-form weirs is greater than on trapezoidal weirs. Taylor (11) has recommended the use of a triangular plan form weirs for  $w/P$  ratios greater than 3, and the trapezoidal weirs with the side wall angle,  $\alpha$ , equal to .75 of the maximum value ( $\alpha = .75 \alpha_{\max.}$ ) when  $w/P$  ratios is less than 3.

Angle of approach flow to the labyrinth spillway,  $\beta$ . At some site conditions, the maximum allowable head might be so limited that using a labyrinth spillway perpendicular to the flowline is not efficient. By placing a labyrinth weirs in the approach channel at an angle to the approaching water, the total overflow length increases even more, which results in more water discharge per unit width of the spillway. The use of this type of labyrinth weirs (inclined labyrinth weirs) decreases the total cost of the structure and the associated downstream channels. The value of angle  $\beta$  may vary from zero for the normal labyrinth weirs to any desired value. However, as angle beta increases, the nappe interference and drowning reduce the effect of the weir.

Significance of fluid characteristics RE and WE. Very little is known about the separate influence of viscosity, represented by the Reynolds number RE, and the surface tension, represented by the Weber number WE, in Equation 3.

The Reynolds number RE is a measure of the relative influence of viscosity, which is related to the energy loss as well as to the velocity variation near the boundaries. The Reynolds number is usually expressed as:

$$RE = VL/\nu \quad (4)$$

or

$$RE = VH/\nu \quad (5)$$

where

$\nu$  is the kinematic viscosity of the flowing fluid.

$L$  and  $H$  are the significant length.

For large weirs, the most significant length is the head.

However, for small and narrow weirs, both head and width of the weir are independently significant (8). Since the velocity in the vicinity of the weir crest is proportional to the square root of the head,  $\sqrt{H}$  can be substituted for  $V$  in equations 4 and 5. Thus for constant values of kinematic viscosity, the total influence of viscosity can be expressed in terms of  $H$  and  $L$ , or  $H$  alone. Since the terms  $L$  and  $H$  are included in Equation 3, the Reynold number  $RE$  need not appear in that equation for  $C$  to reflect the effect of viscosity.

The Weber number  $WE$  is a measure of the relative influence of surface tension, which can be written for horizontal weirs with either the head or the width of the weir as the significant length parameter as follows:

$$WE = V \sqrt{L} / \sqrt{\sigma/\rho} \quad (6)$$

or

$$WE = V \sqrt{H} / \sqrt{\sigma/\rho} \quad (7)$$

In Equations 6 and 7, surface tension,  $\sigma$ , and density,  $\rho$ , are usually considered constant for a given fluid. As in the case of the Reynolds number, the velocity  $V$  is proportional to the square root of the head,  $\sqrt{H}$ . Thus, the influence of surface tension on the weir can also be expressed by the magnitude of  $H$  and  $L$ , or just  $H$  in the

equation for  $C$ .

Because the Reynolds number is inversely proportional to the viscosity, the relative influence of viscosity decreases as  $RE$  increases. Similarly, because the Weber number,  $WE$ , is inversely proportional to the surface tension, the relative influence of surface tension decreases as  $WE$  increases. Since  $RE$  and  $WE$  are expressed in the terms of  $H$  and  $L$ , the relative influence of the combined fluid properties diminishes as either  $H$  or  $L$  becomes larger. Thus, for water at constant temperature and for large heads on large weirs, the influence of viscosity and surface tension is negligible. However, they become significant for the results obtained on small models.



## CHAPTER III

## PRIOR STUDIES ON A LABYRINTH SPILLWAY

Excluding site specific hydraulic models, few direct investigations have been made to study the performance of a labyrinth weirs. One of the most extensive studies was conducted by Taylor (11) in 1968. Prior to Taylor's investigation, three other studies were made by Kořak and Švab, Tison and Fransen, and Gentilini. These investigations are cited by Taylor (11) and will be discussed in the following.

In 1961, Kořak and Švab conducted a series of tests on labyrinth weir models which were trapezoidal in plan. The crests were flat with the edges chamfered on both sides. The design ranges were as follows:

length magnification:  $1.23 \leq l/w \leq 4.35$

vertical aspect ratio:  $1.15 \leq w/P \leq 4.61$

side wall angle:  $5.7^\circ \leq \alpha \leq 20.6^\circ$

head to crest height ratio:  $.05 \leq H/P \leq .25$

Kořak and Švab concluded that the discharge capacity of a labyrinth weirs is much greater than that obtained from a corresponding normal linear weir operating under the same head. They also stated that due to the complexity of a labyrinth weirs, their discharge capacity could only be obtained experimentally. The main criticism to Kořak and Švab's work is that they covered very small ranges of designs and performance of a labyrinth weirs.

Tison and Fransen conducted a series of tests to determine the performance of a labyrinth weirs with the trapezoidal shape in plan.

Unlike Kořak and Švab's work, the head was large in comparison with the size of the weir cycle in plan; and, therefore, the performance of the weir was very dependent on the magnitude of  $w/P$  ratio. Thus, as the head increased, the performance of the weirs approached that of a normal linear weir occupying the same width of channel.

In 1941, Gentilini extended the idea of the oblique (straight) weir into a number of oblique sections, resulting in triangular plan form weirs. The magnitude of side wall angles ( $\alpha$ ) were 60, 45, and 30 degrees. The models were sharp crested, and the height of crest was large in comparison with the width of the channel. Using different combinations of crest height and the number of cycles in plan, a series of tests were conducted with various values of  $w/P$  ratio. Due to the small values of  $w/P$  ratios used in the case of Gentilini's tests, the results were very dependent on the vertical aspect ratio.

One of the most extensive investigations for the behavior of a labyrinth weirs was performed by Taylor in 1968 (11). Taylor recommended 17 steps to take for the design of a labyrinth weirs. He presented his results by plotting the flow magnification,  $Q_L/Q_N$ , against the head to weir crest height ratio,  $H/P$ , where  $Q_L$  is the discharge at head  $H$  over the actual labyrinth weirs of length  $L$ , and  $Q_N$  is the discharge over the hypothetical normal linear sharp crested weir occupying the same channel width as the labyrinth weirs,  $W$ , and with the same values of  $H$  and  $P$  corresponding to the value of  $Q_L$ .

In 1970, Hay and Taylor (4) also published a paper on the performance and design of labyrinth weirs, which provided both procedure and criteria for estimating the discharge over a labyrinth

weirs of triangular or rectangular forms in plan. The work of Hay and Taylor is almost the only source for determining the discharge over a labyrinth spillway. They presented two design charts (Figures 4 and 5), one for triangular weirs and one for trapezoidal weirs. These figures show curves of  $Q_L/Q_N$  plotted against  $H/P$  with length magnifications of 3, 4, 5, 6, 7, and 8 and  $H/P$  range from 0 to .5. Note that as head  $H$  approaches to zero, the flow magnification,  $Q_L/Q_N$ , approaches the length magnification ratio,  $l/w$ , which corresponds to the ideal flow condition.

#### Hydraulic Model Studies of Labyrinth Weirs

A number of hydraulic models have been conducted for a design of a labyrinth spillway. The following are summaries of the more important investigations.

Darvas. Darvas (3) used the experimental results of the model studies of Woronora and Avon weirs in Australia and developed a family of curves for design of labyrinth weirs (Figure 6). He showed that by applying the usual weir performance term, incorporating a discharge coefficient, a simpler design procedure than Taylor's could be achieved. The symbols that Darvas used were:

$$C_L = Q_L/LH^{3/2}$$

and

$$C_{Ws} = Q_L/WH^{3/2}$$

where

$C_L$  and  $C_W$  are the coefficients of discharge

$Q_L$  is the discharge over labyrinth weirs

$L$  is the total developed weir length

$W$  is total width of weir.

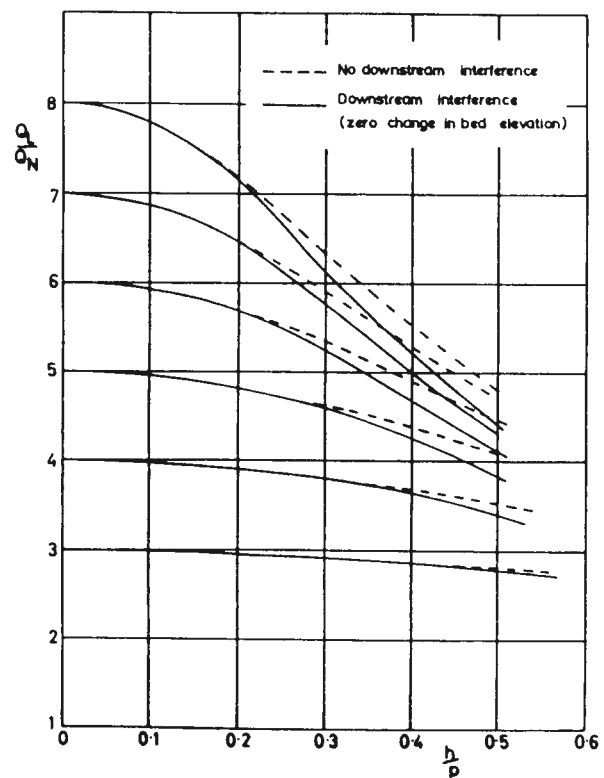


FIG. 4.—Design Chart for Triangular Plan Form Weirs (Source: Hay and Taylor 1970[4])

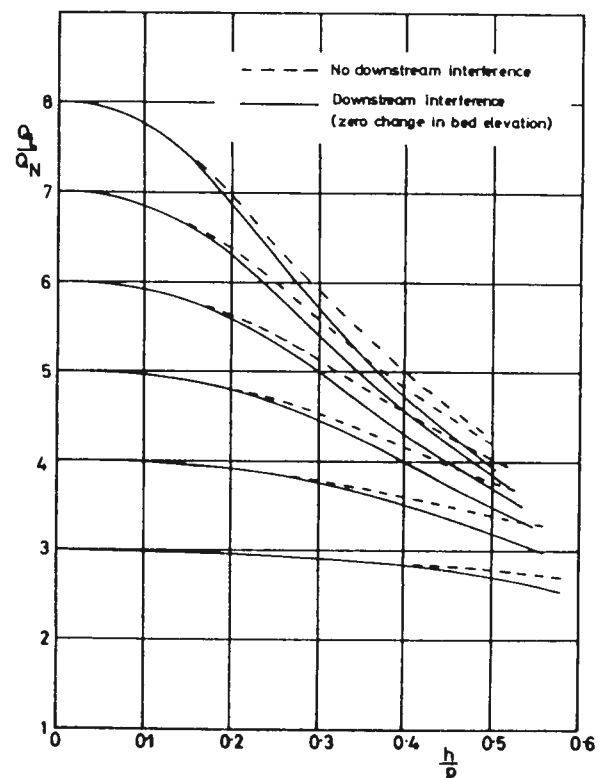


FIG. 5.—Design Chart for Trapezoidal Plan Form Weirs (Source: Hay and Taylor 1970[4])

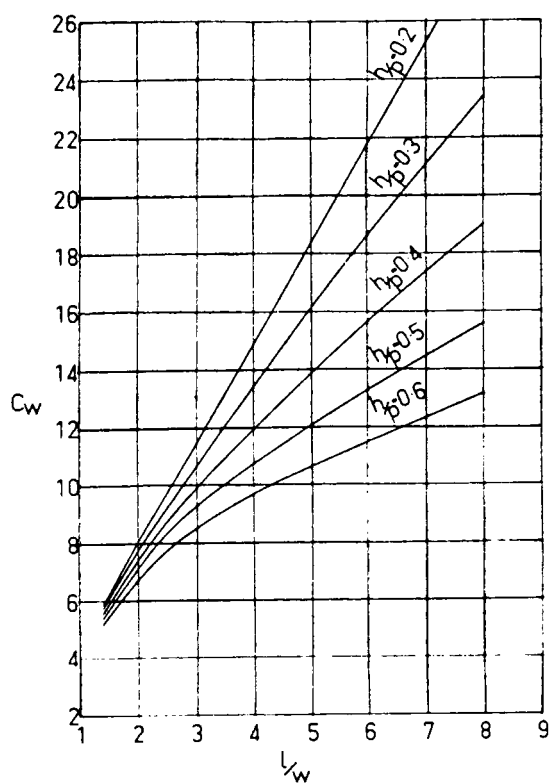


FIG. 6.—Design Chart for Trapezoidal Plan Form Weirs (Source: Darvas 1971[3])

Lux. Lux (10) developed the hydraulic performance of labyrinth weirs from the data obtained from flume studies and site specific models. He used the combination of dimensional analysis and experimentation to develop an equation for the discharge of the labyrinth weirs. Lux also defined the discharge coefficient for triangular and trapezoidal plan forms with the sharp-crested and quarter-round labyrinth weirs (Figure 7 & 8). The equation that Lux developed for the discharge function was:

$$Q = C_w \frac{w/P}{(w/P + K)} w H_o \sqrt{g H_o} \quad (8)$$

where

Q is the discharge of one cycle

H<sub>o</sub> is the total head

K is a constant

For triangular and trapezoidal forms in plan, K has a value of .18 and .10, respectively. The total discharge of a labyrinth weirs can be determined by multiplying Q by the number of cycles, n.

U.S. Bureau of Reclamation (USBR): Ute Dam Labyrinth Spillway.

The U.S. Bureau of Reclamation in 1982 made the labyrinth spillway model of the Ute dam in order to study the modification of the existing structure to increase the reservoir to its desired capacity (6). The hydraulic model studies were initiated to verify and extrapolate the existing design curves of Hay and Taylor (4) for application to the Ute dam labyrinth spillway. The originally designed 10-cycle model testing of this labyrinth spillway, which is based on Hay and Taylor design curves, showed the design discharge could not be passed by the spillway within the maximum reservoir elevation. Further studies showed that the design curves developed

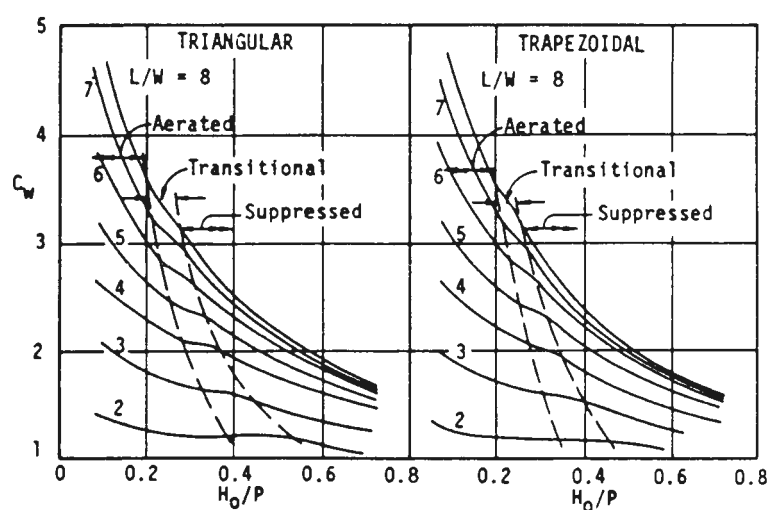


FIG. 7.—Discharge Coefficient for Sharp Crested Labyrinth Weirs (Source: Lux 1984[10])

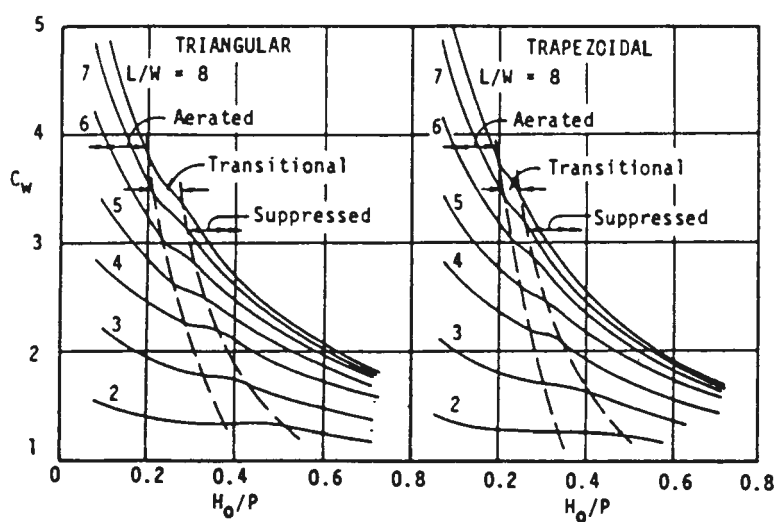


FIG. 8.—Design Curves for Quarter Round Labyrinth Weirs (Source: Lux 1984[10])

by the Bureau were significantly different from the one presented by Hay and Taylor. The differences were particularly noticeable at large H/P values. The Bureau found that the discrepancy between its results and Hay and Taylor's results is partly due to differences in head definitions. Figures 9 and 10 show the revised design curves of the U.S. Bureau of Reclamation for this study in which the head used is the total head; the measured head plus the velocity head,  $V^2/2g$ .

Hyrum Dam Auxiliary Labyrinth spillway. The auxiliary labyrinth spillway for Hyrum Dam was designed from the procedure developed by Hay and Taylor (4) and the modified design curves developed by the USBR from the model study data of Ute Dam. The Hyrum labyrinth was a 2-cycle spillway with two apexes upstream extending 19 feet into the reservoir. The curved sidewalls adjacent to the spillway provided excellent flow distribution with the minimum head loss. The model tests showed that good entrance conditions had more effect on spillway efficiency than spillway orientation (7).

#### Existing Labyrinth Weirs

In addition to the labyrinth spillways mentioned in the hydraulic model studies, there are a number of labyrinth weirs which have been constructed as a direct result of individual model tests.

In 1950, a labyrinth weirs was constructed for the Central Electricity Authority at Skelton Grange Power Station in Great Britain (11). This weir with 20 cycles in plan was designed for the maximum head of three inches. Due to the small design head over the weir, the operation was almost ideal. A normal linear weir of the same head of three inches would need to be five times the present structure.



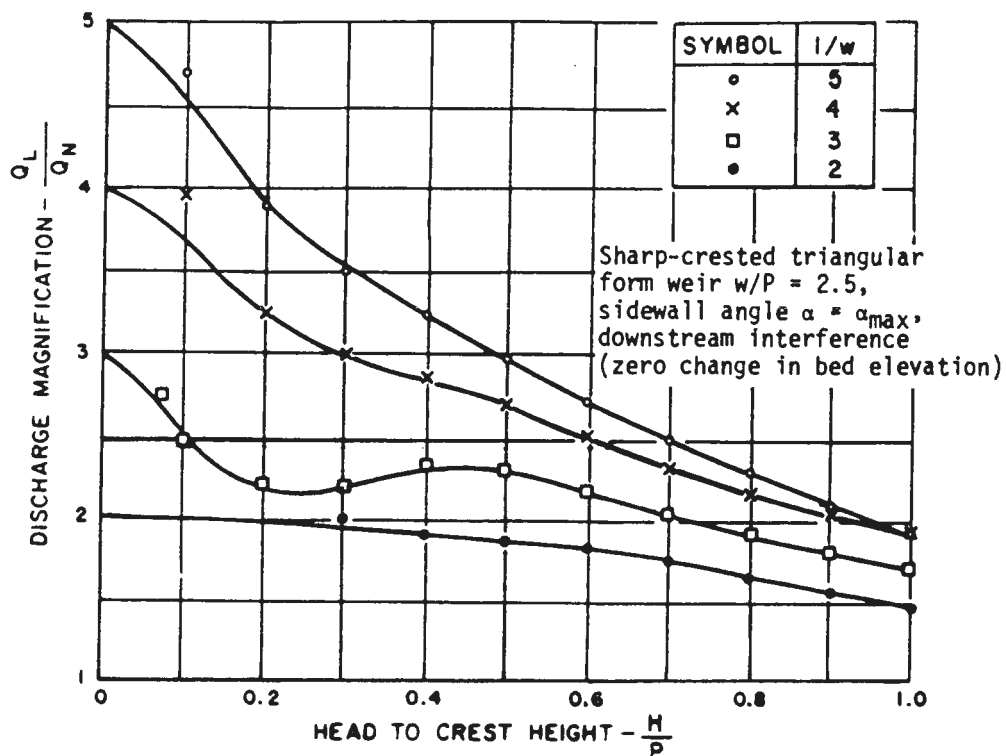


FIG. 9.—Design Curves with Sharp Crested Labyrinth Weirs (Source: Hinchliff and Houston 1984[5])

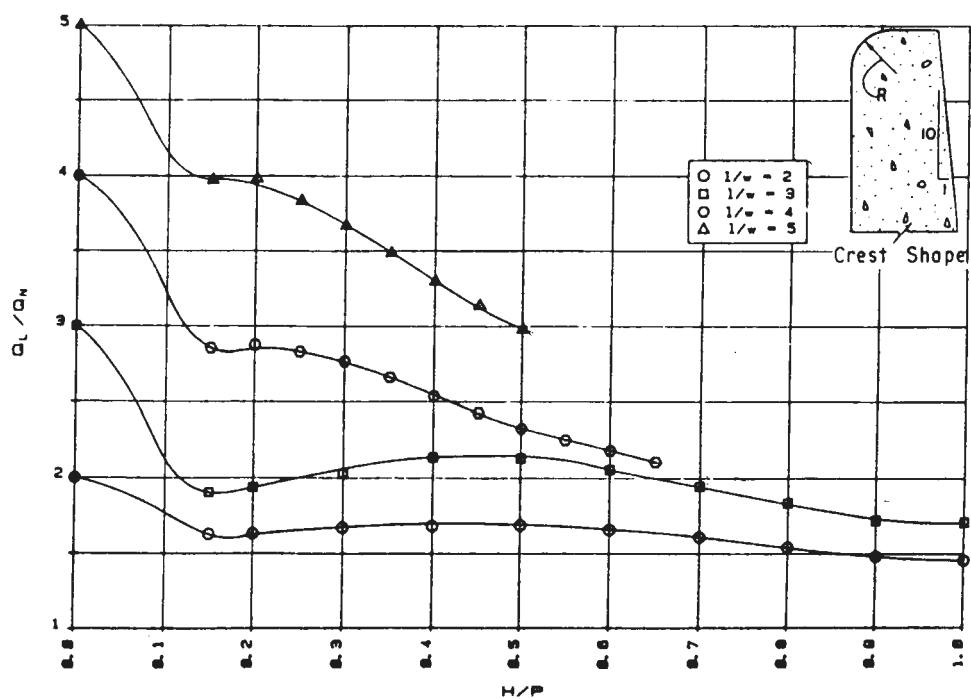


FIG. 10.—Design Curves with Quarter Round Upstream Face, Trapezoidal Form Weir (Source: Hinchliff and Houston 1984[5])

The overflow structure at Beni Bahdel Dam in Algeria is a labyrinth weirs which consists of 20 rectangular sloping lips, referred to as "duckbills" (12). The "duckbills" are built normal to the dam and each projects 30 meters into the reservoir with its width equal to 2 meters. The total crest length of 1200 meters is compressed in a width of 80 m ( $l/w = 15$ ). The structure is capable to discharge  $1000 \text{ m}^3/\text{s}$  at the design head of .5 m. A normal linear weir of the same head of .5 m would discharge about  $95 \text{ m}^3/\text{sec}$ , so the flow magnification is 10.5.

The spillway at Barrage d'Odivelas in Algeria is a shaft spillway in which the principles of labyrinth weirs have been applied to its structure. In this case, eight shaped duckbills, each one extended 15 meters out from a vertical discharge shaft, are used.

The Woronora and Avon Dams in Sydney, Australia, are two large labyrinth weirs serving as reservoir overflow control structures (3).

The original Avon spillway was curved in plan with the crest length of approximately 146 m, and the discharge capacity of  $770 \text{ m}^3/\text{s}$ . The modified Avon's labyrinth weirs is 260 m in length with a discharge capacity of  $1790 \text{ m}^3/\text{sec}$  for a head of 2-8 m (3, 1), thus the flow magnification is 2.4.

Other existing labyrinth spillways are at Quincy Dam, Aurora, Colorado; Mercer Dam, Dallas, Oregon; Bartletts Ferry Dam, Columbus, Georgia (5); Navet Pumped-Storage, Trinidad, Colorado; and Boardman Spillway, Boardman, Oregon (2).

## CHAPTER IV

### EXPERIMENTAL APPARATUS AND PROCEDURES

The experimental work done for this thesis was completed at the Utah Water Research Laboratory in Logan, Utah. The laboratory receives its water supply by gravity flow from a small reservoir on the Logan River and returns by gravity flow to the river. The following is a description of the apparatus and procedures used in this study.

#### Experimental Facilities

An existing laboratory flume 23 feet long, 3 feet wide, and 2 feet deep was chosen. The flume was constructed with plastic glass sidewalls which allowed visual flow observation. A head box with a perforated steel plate aided in entering a uniform and calm water to the flume. A gate at the downstream end of the flume permitted control of the flow depth. A gear jacks is used to control the slope of the flume.

The water was supplied to the flume via a 36-inch diameter mainline. A hand-operated valve, as well as an electrically operated valve, was used to regulate the flow to the flume. The discharge from the flume was entered in channels leading to weighing tanks or volumetric tanks for measurement. Flows up to 29 cfs within .25 percent accuracy can be measured with weight tanks and volumetric tanks. A traversing point gage was mounted on rails running the length of the flume. The point gage measures to an accuracy of .001 ft. A wooden mat, floating upstream of the weir, was used to provide a smooth water surface.

### Scope of the Investigation

The work conducted for this study was divided into three stages:

1. Determining the discharge coefficient of normal linear weir with four different crest shapes in which the results are used for the next two stages.
2. An investigation to determine the significance of the parameters of primary importance on discharge coefficient for the labyrinth spillway installed normal to the flow direction.
3. An investigation to discover the effect of the angle of approaching water to the labyrinth weirs on the discharge coefficient (inclined labyrinth spillway).

In the case of the normal linear weirs, the parameters  $H_T$ , the total head, and  $P$ , the crest height, are used to determine the discharge coefficient. The range of head to crest height ratio,  $H_T/P$ , were between 0 and 1.0. The maximum allowable crest height was limited to eight inches by the dimensions of the test flume. The lower limit of the crest height was set at four inches to minimize the effect of the surface tension. The shapes of the crest were rounded, quarter-rounded, top flat, and sharp-crested weirs (Figure 11). There was no change in floor elevation between the upstream and downstream sides of the weir.

For a labyrinth weirs, the parameters which they considered to be of primary and secondary importance have already been described in Chapter II. Using the parameters of primary importance, the conditions studied by other researchers, and the limitations imposed by the experimental facilities, the scope of the investigation was

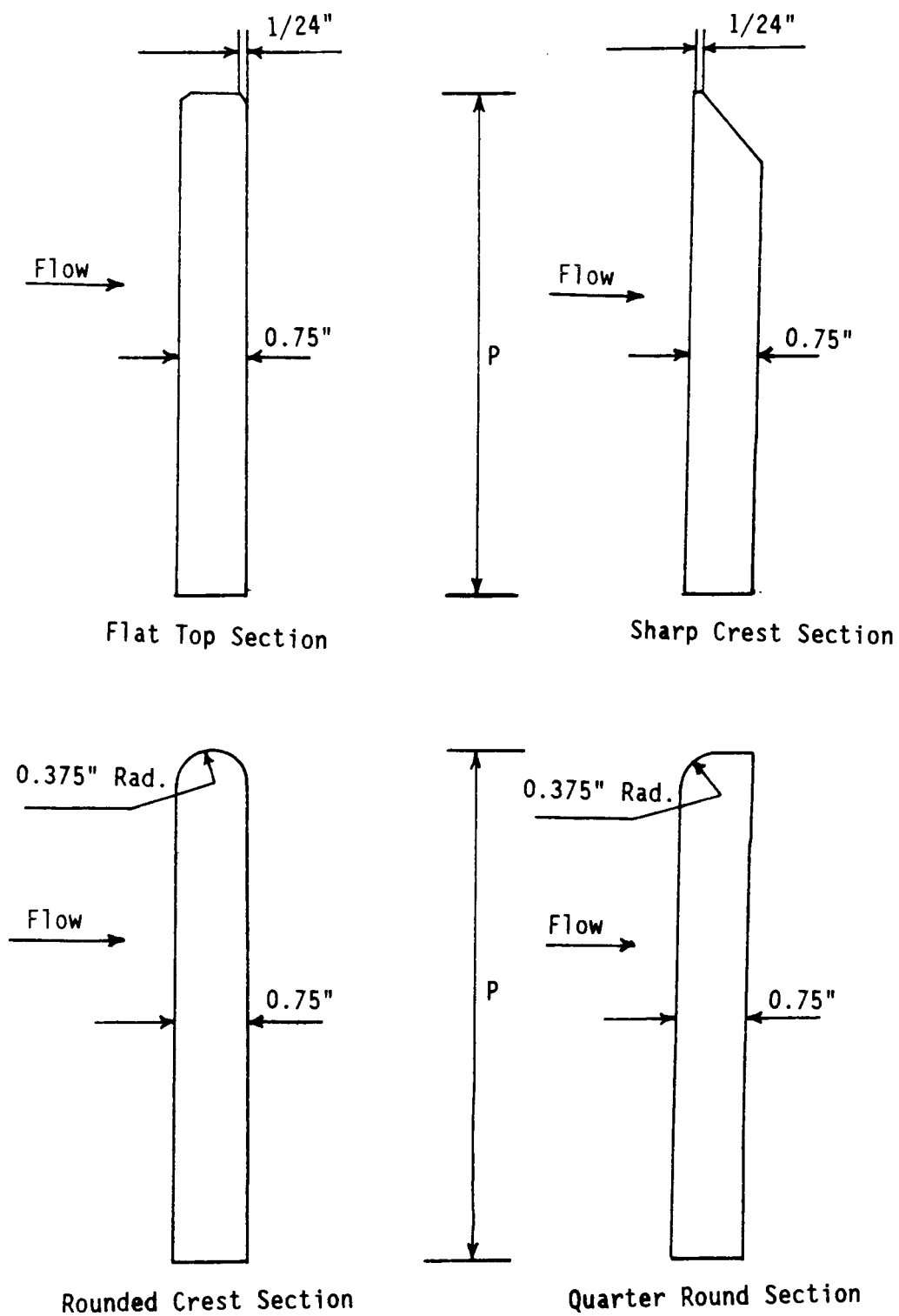


FIG. 11.—Shapes of the Crest Section

determined.

The labyrinth weirs in this study were confined to symmetrical weirs having triangular plan forms, rounded-crest shapes, and the same upstream and downstream apron height. The thickness and the height of the weirs were limited to the constant values of .75 inches and 6 inches, respectively. The number of cycles used in this study was limited either to an integer or an integer plus one-half cycle. Since the number of cycles,  $n$ , had no effect on the labyrinth weir performance, the number was kept to a minimum value in order to create the largest model possible. The number of cycles for normal and inclined labyrinth weirs in these model studies were one and one-half and two cycles, respectively. The performance of a labyrinth weirs was determined for a range of total head to crest height ratios,  $H_T/P$ , between 0 and 1. The length magnification factor,  $l/w$ , and the angle of sidewalls,  $\alpha$ , were considered of primary importance; and the full ranges of these variables needed to be included in the investigation. However, for a triangular plan form, the parameters  $l/w$  and  $\alpha$  describe the same geometry of a labyrinth weirs. Thus, only the sidewall angle,  $\alpha$ , is presented in this model study. The angle of sidewalls were 10.50, 16, 21, 32.12, 49.04, and 90 degrees. The value of 90 degree describes the geometry of a normal linear weir. Since the plan forms of the labyrinth weirs for this study were triangular, the values of apex ratio,  $A/w$ , were equal to 0.

The limiting conditions and the ranges of variables covered in this study for both normal labyrinth weirs and labyrinth weirs placed at an angle in the channel of approach follow:

1.  $0 < H/P < 1.0$
2.  $P = 6"$
3.  $3.89 < w/P < 4.04$       Normal labyrinth weirs  
 $w/P \approx 2.36$                   Inclined labyrinth weirs
4.  $n = 1.5$                       Normal labyrinth weirs
5.  $n = 2$                         Inclined labyrinth weirs
6. plan form - triangular
7. crest shape - rounded crest
8.  $\alpha = 10.50, 16, 21, 32.12, 49.04, \text{ and } 90 \text{ degrees}$
9.  $\beta = 0, 30, \text{ and } 45 \text{ degrees}$

#### Models Construction

All the weir models were constructed from wood. Through the whole experiments, the thickness of the weirs were .75 inches. Adhesive silicone was used to support the weirs in the flume and to prevent leakage of water at the bottom and sides. The weirs were leveled to ensure the same crest height along the length of the weir. The whole model structure was checked to be level both horizontally and vertically.

In the case of inclined labyrinth weirs, the width of the channel upstream of the weirs was reduced in order to prevent the interference of the nappe with the right sidewall of the flume and the submergence downstream of the weirs. A plywood floor, which extended eight feet upstream from the reduction channel, was used as a transition to the channel to minimize losses of head and to prevent disturbance in flow. Two auxiliary walls were constructed at both downstream sides of the labyrinth weirs in order to obtain the simulation of the angle of the sidewalls.

### Testing Procedure

The existing flume was leveled and maintained in this position throughout the experimental work. With the model under test secured in the channel, the bottom of the channel and height of the crest level was measured by the point gage to obtain the height of the weir,  $P$ , with an accuracy of .001 feet.

For each model, the measurement of depth was taken from low flows to high flows. The range of flows was selected to cover  $H/P$  ratio from .1 to approximately 1.0. The increment of  $H/P$  ratio was often equal to .15.

The head of water over the crest of a model was taken as the difference in elevation between the crest and the water surface upstream from the weir. Using this method of measurement in case of any small error in depth measurement, it only affects the velocity head on the weir and would, therefore, represent very minor second order effect in the calculation of weir discharge coefficient. To avoid an error on the head measurement caused from the effect of surface tension and water surface drawdown, the reading of water surface level upstream of the weir was made at a point with a distance of four times the maximum head on the crest.

Three kinds of head measurements based on the types of ventilation were taken on each weir:

a. Ventilated tests:

These tests were conducted by holding a pipe beneath the nappe of the weir to provide a full ventilation of the lower nappe surface. Thus, both the upper and lower nappe surfaces were subjected to full atmospheric pressure. In this case, due to lack of negative pressure



under the nappe, an accurate measurement of water surface level was obtained. The aeration of the nappe was more necessary at lower heads.

b. Half-ventilated tests (cavity):

As the head over the weir increased and with no pipe held in place under the lower surface of the nappe, the air pockets caused an instability in the flow condition in which the nappe alternates between half air entrainment and solid water flow. Difficulty was experienced in accurate reading of head by the point gage due to instability of the flow in this condition.

c. Non-venting tests:

The non-venting tests were conducted in two ways:

1. At low heads, the gate at the downstream end of the channel was temporarily adjusted to maintain the downstream depth greater than the crest height. Then, by fully opening the downstream gate, the air beneath the nappe was completely removed. At this point, the subatmospheric pressure under the nappe caused the nappe to cling to the downstream face of the weir.
2. As the head over the weir increased, the nappe thickness and the downstream depth increased too, making the aeration of the nappe difficult. The ripples on the water surface at very high flow rates made the head measurement difficult. In this case, the reading was made by experience and personal judgement.

Table 3 through Table 27 in the Appendix contains the data and the corresponding calculations.

The measurement of flow rates were conducted with both weight tanks and volumetric tanks. For low flows up to 3 cfs, the weight tanks were used, but for flows exceeding approximately 3 cfs, the measurement of flow rate with volumetric tanks were more accurate. For each test, the flow in the channels was allowed to become stable, and the flow rate readings by the weight tanks or volumetric tanks were repeated to maintain steady readings.

## CHAPTER V

## DATA PRESENTATION AND DISCUSSION

Since the study of labyrinth weirs requires the application of some concepts of the normal linear weirs, it was decided to first determine the discharge coefficients of normal linear weirs with four different crest shapes. The results of these normal linear weirs are useful for the application of labyrinth weirs in at least two different ways:

1. To compare the flow of labyrinth weirs,  $Q_L$ , with the corresponding normal linear weir flow,  $Q_N$ . The corresponding normal linear weir means a straight weir normal to the flow which occupies the same channel width and with the same total head condition as the labyrinth weir. The comparison can be shown by plotting the flow magnification,  $Q_L/Q_N$ , versus head to crest height ratio,  $H/P$ , for a number of length magnification factors,  $l/w$ .
2. Since this study on labyrinth weirs was only limited to the weirs with rounded crest shape, the ratio of the discharge coefficient of any particular normal linear weir in the crest shape with the rounded crest weir having the same  $H_T/P$  ratio can be used to determine the discharge coefficients of labyrinth weirs with any three other crest shapes which have been tested. This can be achieved by multiplying the ratio of the discharge coefficient of that section to that of a rounded crest shape by the discharge coefficient of rounded crested labyrinth weirs at the same value of  $H_T/P$  ratios.

This chapter will deal with the presentation of results of three different groups of experimental tests. The results of these studies

are useful to design a normal linear weir with four different crest shapes, to design normal labyrinth weirs, and also to compare the performance of inclined labyrinth weirs with normal labyrinth weirs. The support data used in these discussions are found in the appendixes.

### Discharge Coefficients

The usual form of the discharge equation over a spillway crest is:

$$Q = CLH^{3/2} \quad (9)$$

where

$Q$  = discharge in cubic feet per second

$C$  = discharge coefficient

$L$  = length of the weir crest in feet

$H$  = measured head on the weir crest in feet +0.003 (ft)

The value of 0.003 feet which is added to the measured head represents as an adjustment value to compensate for the fluid property effects related to head. These fluid properties are mainly viscosity, and surface tension which their effects are significant where the head over the weir is small.

In the tables of appendix, there are two types of the letter  $c$ : the upper case and the lower case. The former one represented the discharge coefficient in which the velocity head,  $V^2/2g$ , is included, and the latter one is not. It should be noted that in low weirs or when the head over the weir is large compared with the height of the weir, the effect of approaching velocity on the discharge coefficient cannot be ignored. The parameters of  $H_T/P$  and  $H/P$  correspond to  $C$  and  $c$ , respectively. The velocity of approach is computed from the

formula:

$$V = Q/A$$

where

V = velocity of approach in feet per second

Q = flow discharge in cubic feet per second

A = cross-sectional area of the approach channel in square feet

For low flows with small heads on the weir, the effect of approach velocity is negligible; whereas, for moderate flow rates, neglecting the effect of velocity of approach produces different values. Figure 12 shows that as the values of  $H_T/P$  and  $H/P$  increase, the difference between two types of discharge coefficients,  $C$  and  $c$  become larger.

#### Linear Weir

Tables 3 through 13 in the appendix present the discharge coefficients and the relating calculations for four different crest shapes of normal linear weirs. For each type of weir, there are at least two sets of tests corresponding to two different crest heights,  $P$ . For each test, three types of measurements based on the ventilation of the air beneath the nappe are recorded. Due to the existence of low pressure beneath the nappe when it was not aerated, the values of discharge coefficients for nonventing conditions are always larger than other types of aeration for the same flow. However, in order to prevent any kind of undesirable effects, such as damage to the weir construction caused from the negative pressure under the nappe, the nappe should be aerated when possible.

Figure 13 indicates the variation of discharge coefficients for three types of ventilation on a round crest linear weir. This figure

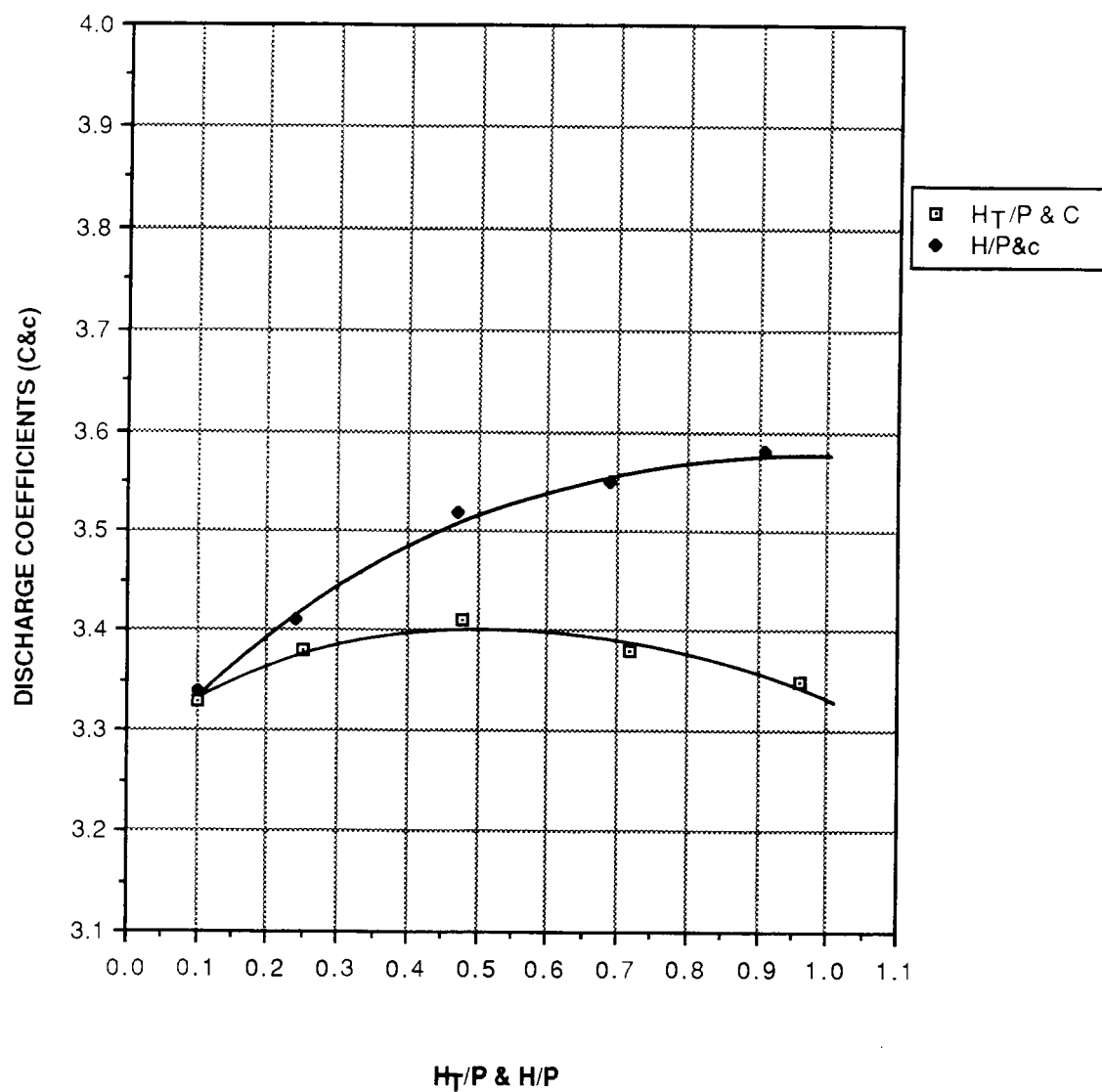


FIG. 12.—Effect of Approach Velocity on the Discharge Coefficient of Sharp Crested Weirs

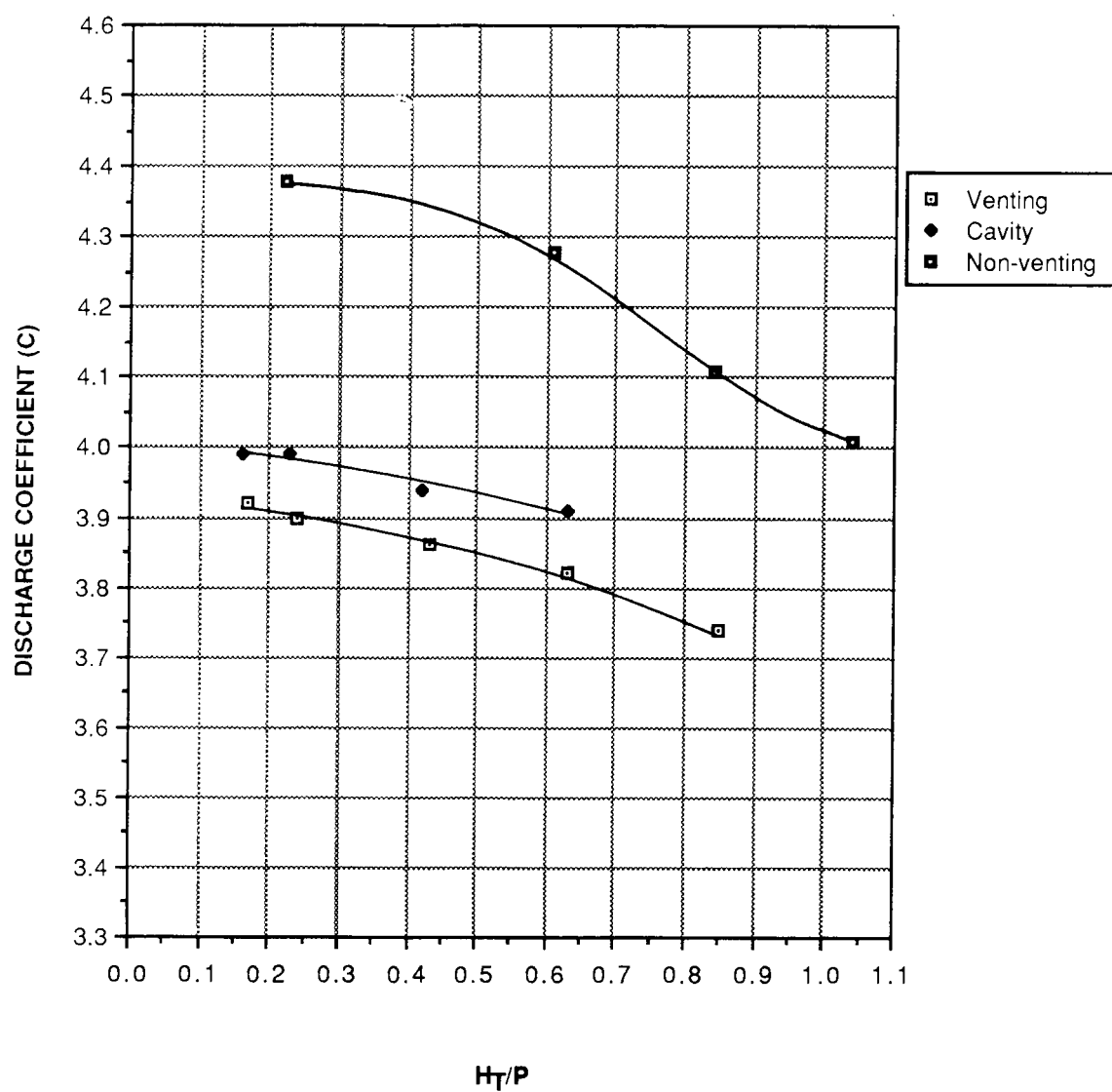


FIG. 13.—Variation of Discharge Coefficient for Three Types of Ventilation on a Rounded Crested Normal Linear Weir for  $P = 0.5$  feet

shows a substantial difference of discharge coefficients at low values of  $H_T/P$ . However, this variation reduces as the head to crest height ratio increases.

All the design curves developed in this study (showing the discharge coefficients) are based on full ventilation of the nappe, except for the case when the upstream head over the weirs was large enough which made aeration of the nappe difficult. In this case, the curves are extended with the data of cavity or nonventing conditions.

Since the discharge coefficients are obtained from the weirs of different crest heights, a comparison of the results should be made to determine whether the discharge coefficients of a particular weir is applicable for another weir with different crest heights. Figure 14 shows that changes of discharge coefficients for different weirs with different crest heights is not significant.

#### Analysis of Weir Sectional Crest Shapes

The type of crest shapes has a major effect on the determination of the discharge coefficient of a weir. Since the flow pattern of one weir differs from another of a different crest shape, the performance and results of a particular crest shape are not applicable to other crest shapes.

Numerous equations have been developed for finding the discharge coefficient of sharp-crested weirs. These equations are derived approximately from the results of the model studies. In this report, weirs with four different crest shapes are tested. The results are shown by plotting a discharge coefficient against the total head to crest height ratio,  $H_T/P$ .



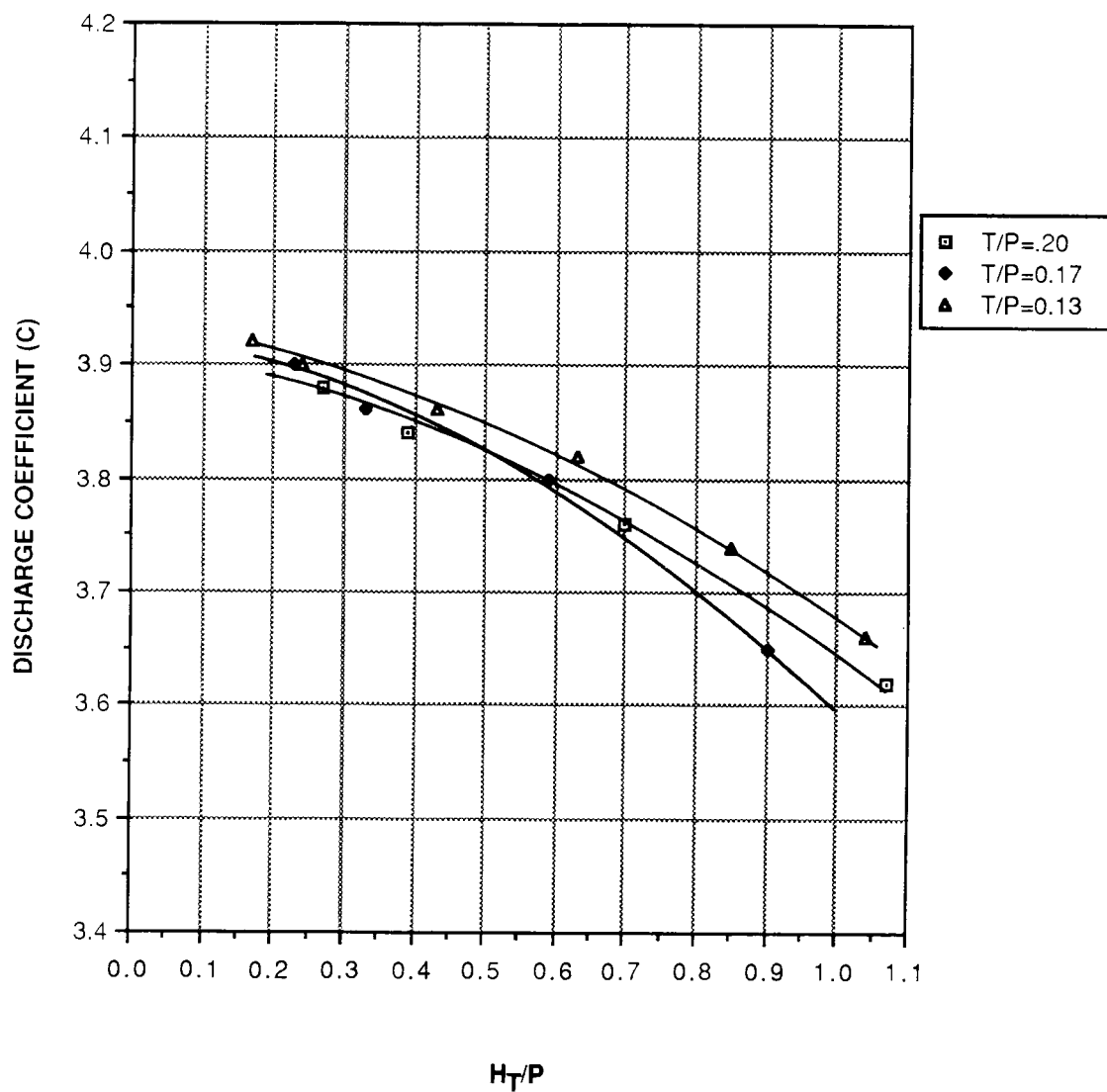


FIG. 14.—Effect of Weir Thickness on the Discharge Coefficient of Rounded Crested Weirs

Figure 15 shows that among the four types of weirs which are tested, weirs with rounded crest shapes have larger discharge coefficients. The discharge coefficients of rounded crest and quarter rounded crest weirs are very close to each other. In a sharp-crested weir, the contraction of the nappe from the bottom of the crest is larger than other types of weirs. The higher nappe contraction from the top causes the upstream head over the weir to increase and, therefore, the discharge coefficient would decrease. The discharge coefficient of top flat crest at small values of  $H_T/P$  ratio are quite far from sharp-crested weirs. However, as the value of  $H_T/P$  increases, the discharge coefficient of these two types of weirs become similar. Therefore, weirs with top flat crest behave as sharp-crested weir as the head over the weir increases.

The discharge coefficients of all types of weirs, except the sharp-crested weir, are decreasing as the values of head to crest height ratio,  $H_T/P$ , increase. In the case of sharp-crested weirs, the discharge coefficients increase up to  $H_T/P$  value of approximately 0.5; then, it decreases as  $H_T/P$  becomes larger. It is evident from Figure 15 that the differences among the weir discharge coefficients of different crest shapes are most apparent at small  $H_T/P$  ratios, and as the values of  $H_T/P$  increase, these variations become smaller.

In order to check the validity of the presentation of the results, the discharge coefficients obtained from the sharp-crested weir are compared with the previous research. Table 1 gives a comparison of these results. The relating equations are listed in Reference 9.

Table 1 indicates that the values of the discharge coefficients

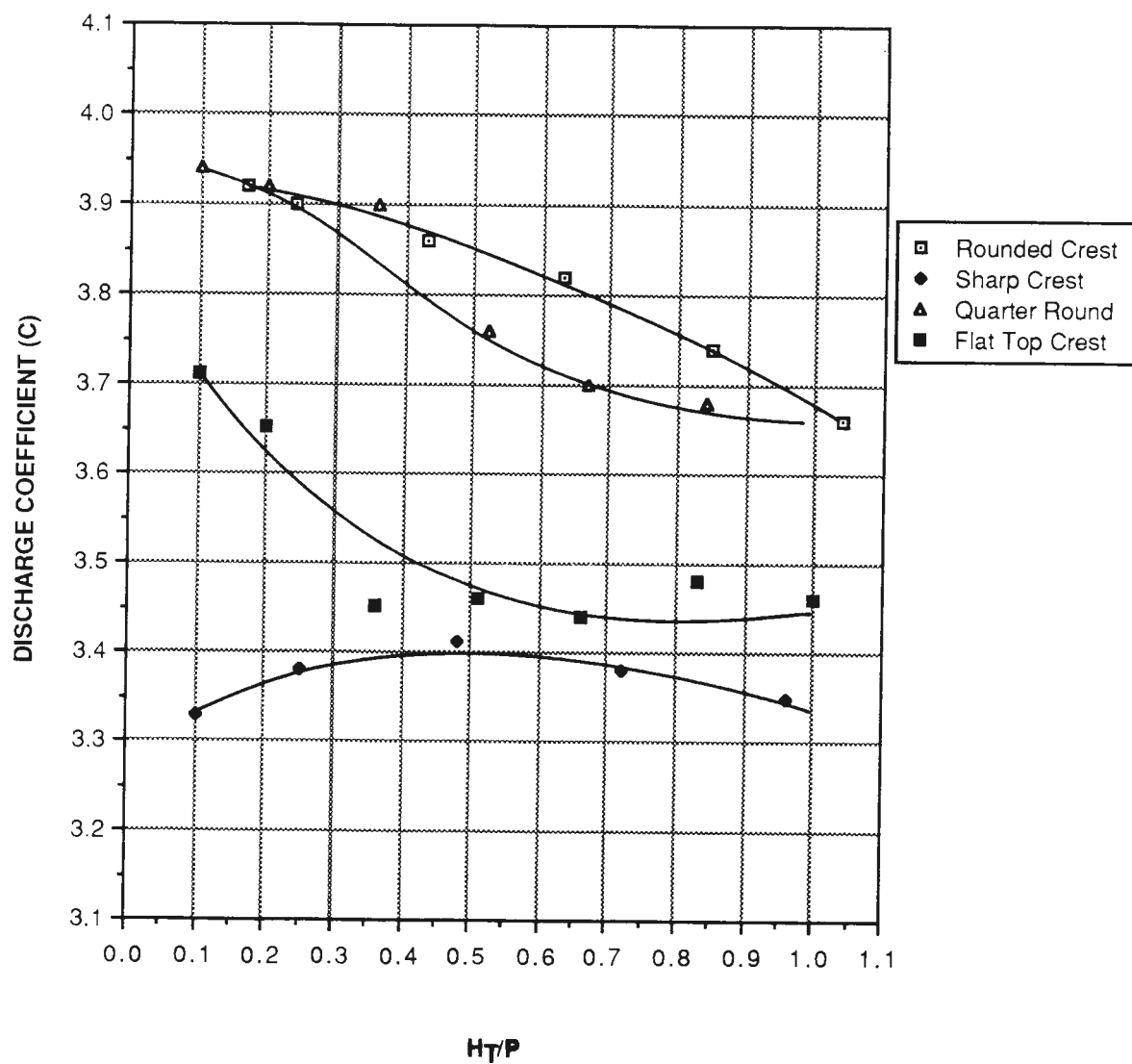


FIG. 15.—Discharge Coefficients of Normal Linear Weirs with Four Different Crest Shapes. The Nappes are Fully Vented

TABLE 1.—Comparison of Discharge Coefficients  $C$  and  $c$  for Sharp Crested Weirs with Full Ventilation of the Nappe from This Study and Previous Research.

$H_T/P$	$H/P$	Current Study		U.S.B.R	Kinds-water	Basin	Schoder & Turner
		$C$	$c$	$c$	$c$	$c$	$c$
0.1	0.1	3.33	3.34	3.26	3.26	3.29	3.26
0.25	0.24	3.38	3.41	3.33	3.32	3.36	3.32
0.48	0.47	3.41	3.52	3.43	3.41	3.50	3.42
0.72	0.69	3.40	3.55	3.52	3.50	3.56	3.52
0.96	0.91	3.35	3.58	3.62	3.58	3.65	3.62

(C and c) are in general agreement with previous research. There is a variation of approximately 3% in values of c from this study with past research. At small  $H_T/P$  ratios, the values of C are nearly identical with others. However, at high  $H_T/P$  ratios, the values of C are considerably lower than others. These variations might be due to the effect of velocity head,  $V^2/2g$ , which is included in this study to calculate the discharge coefficient C. In general, small differences in discharge coefficients of the current study and previous research can be justified from a number of approximations, assumptions, ranges of different parameters, and the experimental conditions of each individual test.

#### Analysis of Normal Labyrinth Weirs

Discharge coefficient. Tables 14 through 23 in the appendix show the discharge coefficients of normal labyrinth weirs for different values of the angle of sidewalls,  $\alpha$ , with  $H_T/P$  ranges from approximately 0.1 to 1.0. To find a discharge coefficient, equation (9), which is used for normal linear weirs, is applied. Since the heads are measured a distance from the upstream apexes, the effects of surface tension and viscosity are neglected. Therefore, the value of 0.003 feet which was added to the measured head for normal linear weirs is not included in these calculations.

Due to the special characteristics of the operation of labyrinth weirs, most of the head measurements were made for nonventing conditions. For the angle of sidewalls,  $\alpha$  (up to 21 degrees), the nappes were automatically aerated along the length of the weirs originated from the upstream apexes. However, as the angle of  $\alpha$  increases, the nappes should ventilate manually. Since it was not

possible to maintain ventilated conditions when the angle of sidewalls were large, the discharge coefficients which are calculated for the range of angle of sidewalls above 25 degrees are relatively high.

The discharge coefficients are shown by two types of the letter  $c$  as they are used in the case of normal linear weirs. The discharge coefficients  $C$  and  $c$  are multiplied by the ratio of total length to total width of labyrinth weirs to represent the discharge coefficient of a unit width of weir.

#### General Discussion of the Behavior of Labyrinth Weirs

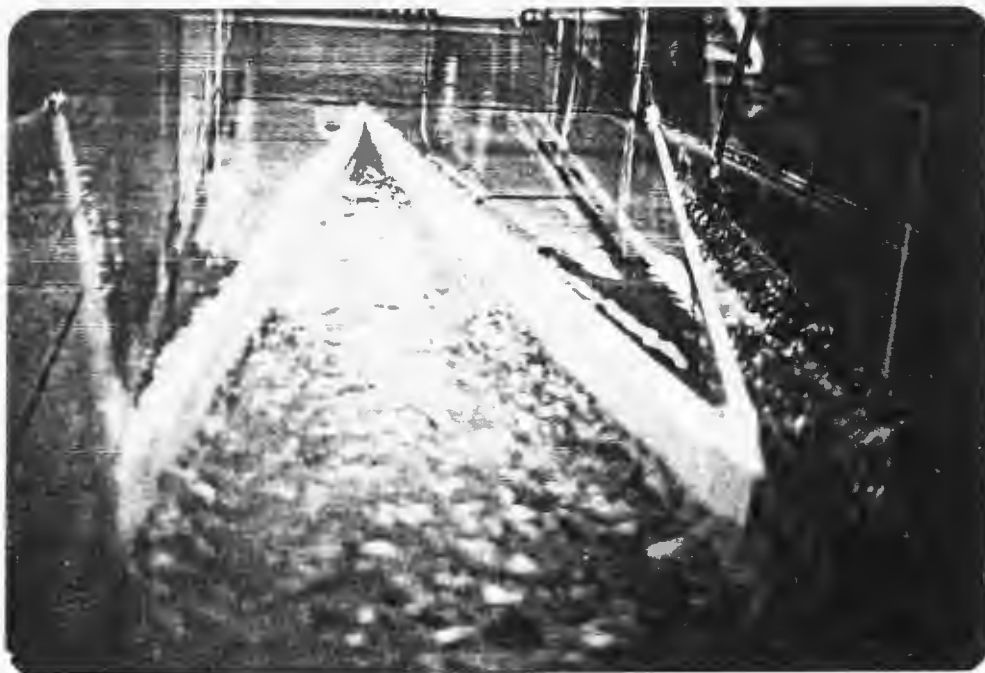
The discharge over a labyrinth spillway is a function of many parameters which have been described in detail in Chapter II. Total operating head, upstream and downstream heights of the weir, weir crest shape, labyrinth plan forms, and the angle of sidewalls are parameters which significantly affect the performance of labyrinth weirs. Figures 16 and 17 present the flow conditions of labyrinth weirs with different angles of sidewalls,  $\alpha$ .

Since the head over a labyrinth weirs varies from section to section, in order to obtain an optimum performance, the attempt should be made to maintain a uniform head along the length of the weirs and also minimize the entry losses to gain the maximum operating head.

In the case of triangular labyrinth weirs, the contraction at the entry section and the energy losses are minimum. In addition, the rate of contraction of upstream channels and the outflow are equal, which causes the water surface profile to remain horizontal; whereas in a rectangular or trapezoidal weir, the upstream tips at the entry section cause both the head loss and contraction of the channel. This



a. Small angle of sidewalls, and facing downstream

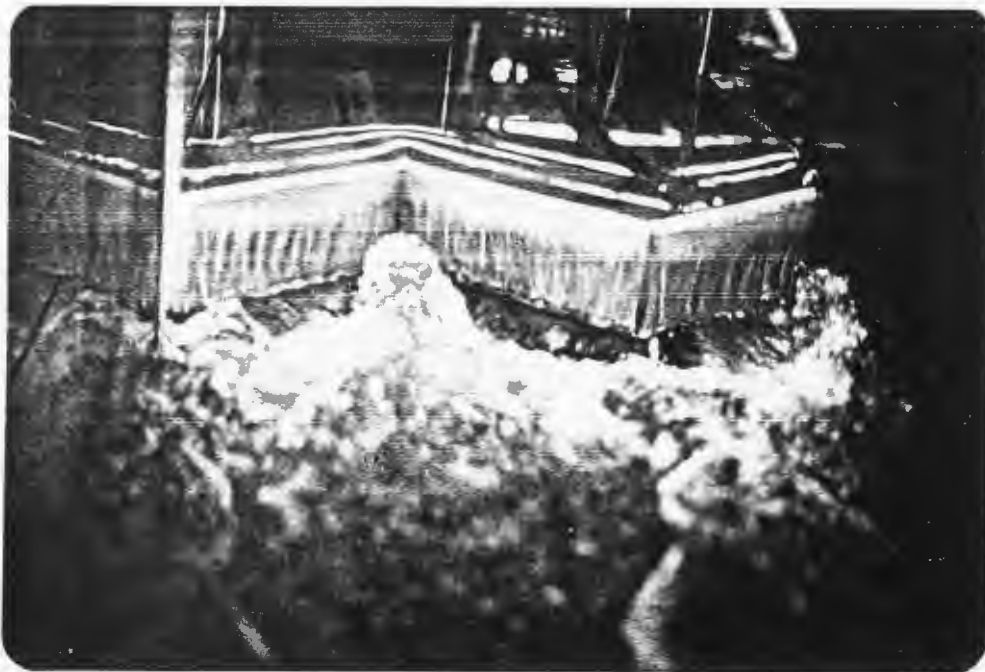


b. Small angle of sidewalls, and facing upstream

FIG. 16.—Flow Condition of Normal Labyrinth Weirs



a. Manually aerated



b. Non-aerated

FIG. 17.—Flow Condition of Normal Labyrinth Weirs with Large Angle of Sidewalls



results in a reduction of water surface profile at the beginning of upstream channels of a labyrinth weir. The water surface level gradually rises along the length of the channel as the outflow over the weirs expands the flow. Thus, in order to obtain the best performance, the triangular labyrinth weir is recommended.

For a very small head over the labyrinth weirs, the performance is almost ideal. However, because of interference of the nappes, the performance declines with the increase in the value of  $H/P$ . The nappe interference has a greater influence on triangular weir designs than those of rectangular or trapezoidal weirs. For larger values of  $H/P$ , increasing the total length of the structure by reducing the angle of sidewalls does not always result in a higher performance of labyrinth weirs.

Since the parameter  $w/P$  describes the degree of interference of the nappes, in the case of triangular labyrinth weirs, this parameter should be selected greater than 2.5 to prevent an excessive flow impingement of adjacent weirs. However, increasing the value of  $w/P$  increases weir area and, therefore, the costs of the structure.

The drowning condition is another factor which reduces the performance of labyrinth weirs. The flow depth downstream of the weirs should be maintained below the weir crest to prevent drowning. In most of the designs, the upstream and downstream bed elevations are equal. However, in case of drowning, lowering the level of the downstream channel bed would help to nearly solve the problem.

#### Development of Design Curves

Most of the designs of labyrinth weirs have been based on the generalized design curves of Hay and Taylor (Ref. 4). Since these

published curves were inaccurate for the design at high heads, and also the ranges of  $H/P$  ratio were not adequate, development of new curves with a possibility of presenting a new procedure for design was necessary.

In their studies, Hay and Taylor did not include the effect of velocity head which significantly affects the performance of labyrinth weirs when the operating head is large. This study is confined to the triangular labyrinth weirs. The design curves are developed in two methods. In both methods, the effect of approaching velocity is included in the calculations.

The first method presented is similar to the one which was developed by Hay and Taylor. Figure 18 shows the design curves with flow magnification ratio,  $Q_L/Q_N$ , ranges from 1.27 to 5.53, and  $H_T/P$  ratio ranges from 0.1 to 1. These curves can be used to design a labyrinth spillway following the design procedures which were recommended by Hay and Taylor.

The second method, which is presenting a new design curve and procedure, is also developed. In this method, a designer is able to design a labyrinth spillway directly by applying equation (9) using an appropriate discharge coefficient,  $C$ , obtained from newly developed design curves.

The results obtained for normal labyrinth weirs are presented in Figures 19, 20, and 21. Figure 19 shows that for a particular angle of sidewalls,  $\alpha$ , the discharge coefficient,  $C$ , decreases as the ratio of total head to crest height,  $H_T/P$ , increases. In addition, as the angle of sidewalls,  $\alpha$ , becomes larger, the discharge coefficient in a unit length increases. This is because of less nappe interference

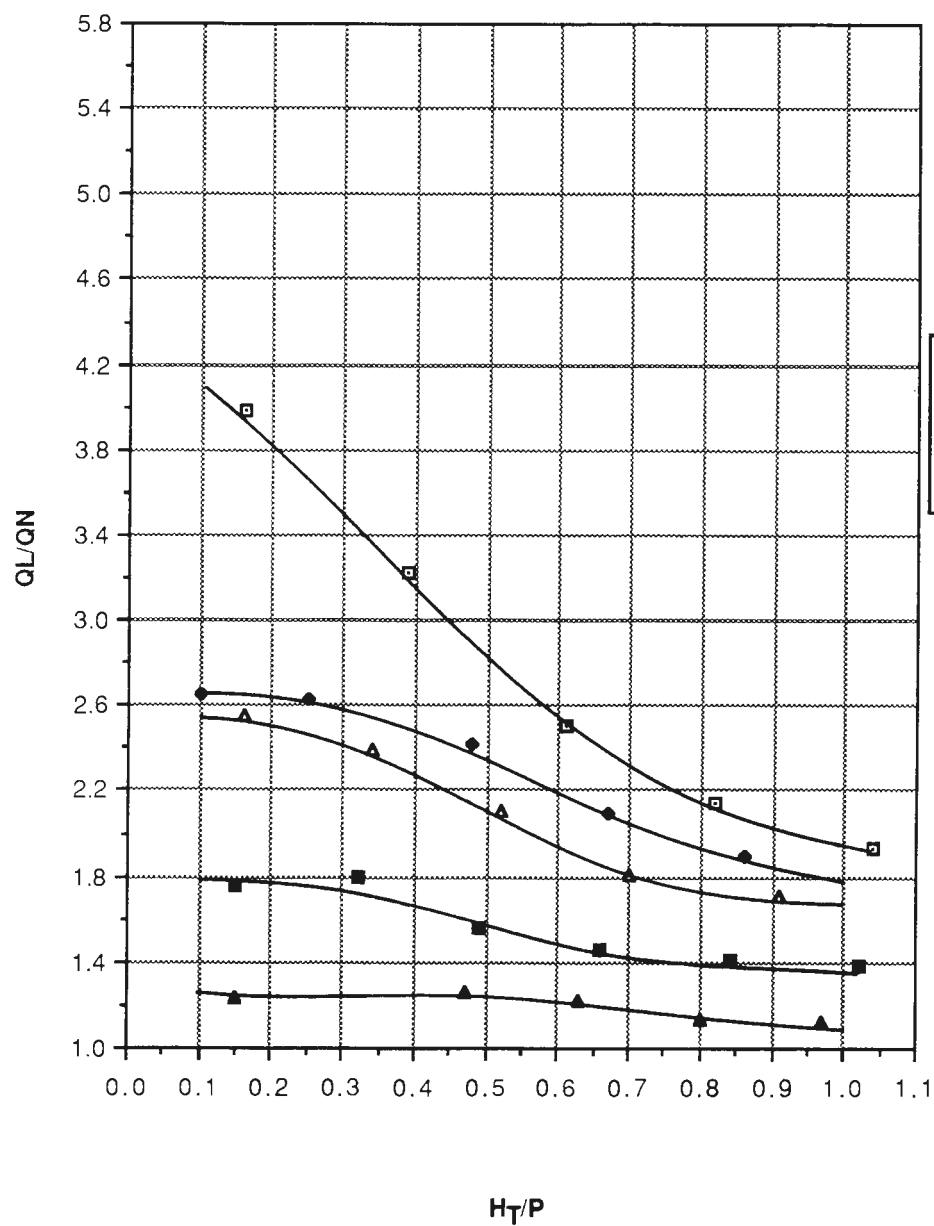


FIG. 18.—Design Curves for a Triangular Normal Labyrinth Weirs with Rounded Crest Shape

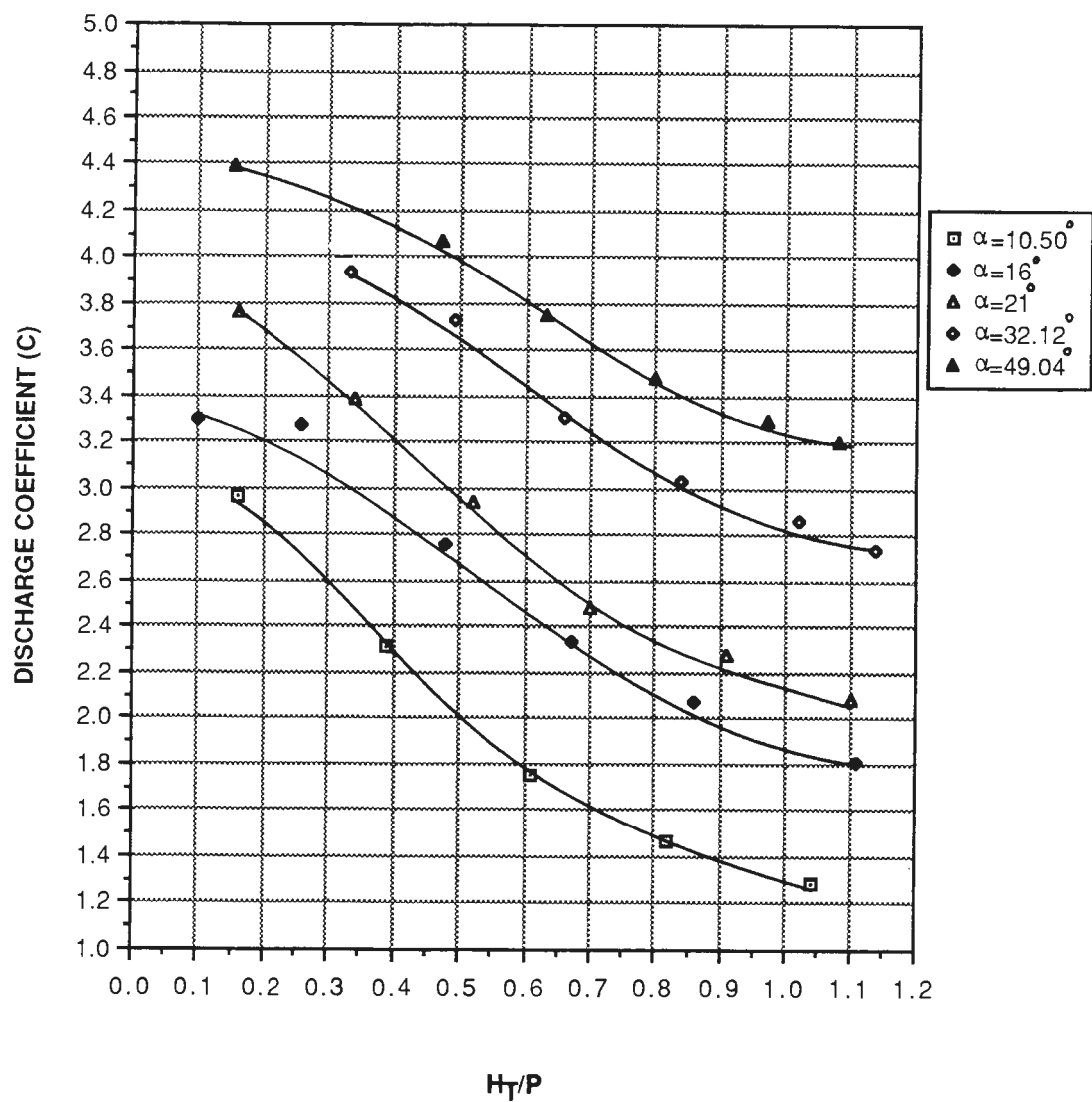


FIG. 19.—Effect of Angle of Sidewalls,  $\alpha$ , and  $H_T/P$  on the Performance of Normal Labyrinth Weirs

when  $\alpha$  is large.

Figures 20 and 21 illustrate the discharge coefficient,  $C$ , against angle of sidewalls,  $\alpha$ , for a number of  $H_T/P$  ratios. The angle of sidewalls are ranging from 10.5 to 90 degrees. The variations of total head to crest height,  $H_T/P$ , are from 0.15 to 0.5, and from 0.6 to 1.0 in Figures 20 and 21, respectively.

Figure 20 indicates that the discharge coefficients for  $H_T/P$  ratios of 0.1, 0.2, 0.3, 0.4, and 0.5 (with the angle of sidewalls above 25 degrees) are larger than the discharge coefficient of normal linear weirs ( $\alpha = 90$  degrees), operating under the same head condition. There are at least two major factors which can possibly cause higher discharge coefficients in a unit length of a labyrinth spillway than normal linear weirs when the  $H_T/P$  ratio is small and  $\alpha$  is large:

1. The discharge coefficients in Figure 20 for  $\alpha$  equal to 90 degrees are based on fully ventilated conditions; whereas, for a labyrinth weir with the angle of sidewalls greater than 25 degrees, the ventilation did not occur either automatically nor manually.

2. In labyrinth weirs, for a large angle of sidewalls, the degree of nappe contraction from the top seems to be higher than normal linear weirs.

#### Comparison of Normal Labyrinth Weirs with Inclined Labyrinth Weirs

A few tests were conducted to determine the rate of increase in the total efficiency of a structure if a labyrinth weir is constructed at an angle to the approaching water in the channel of approach rather than to be perpendicular to the flow line. Tables 24 through 27 in

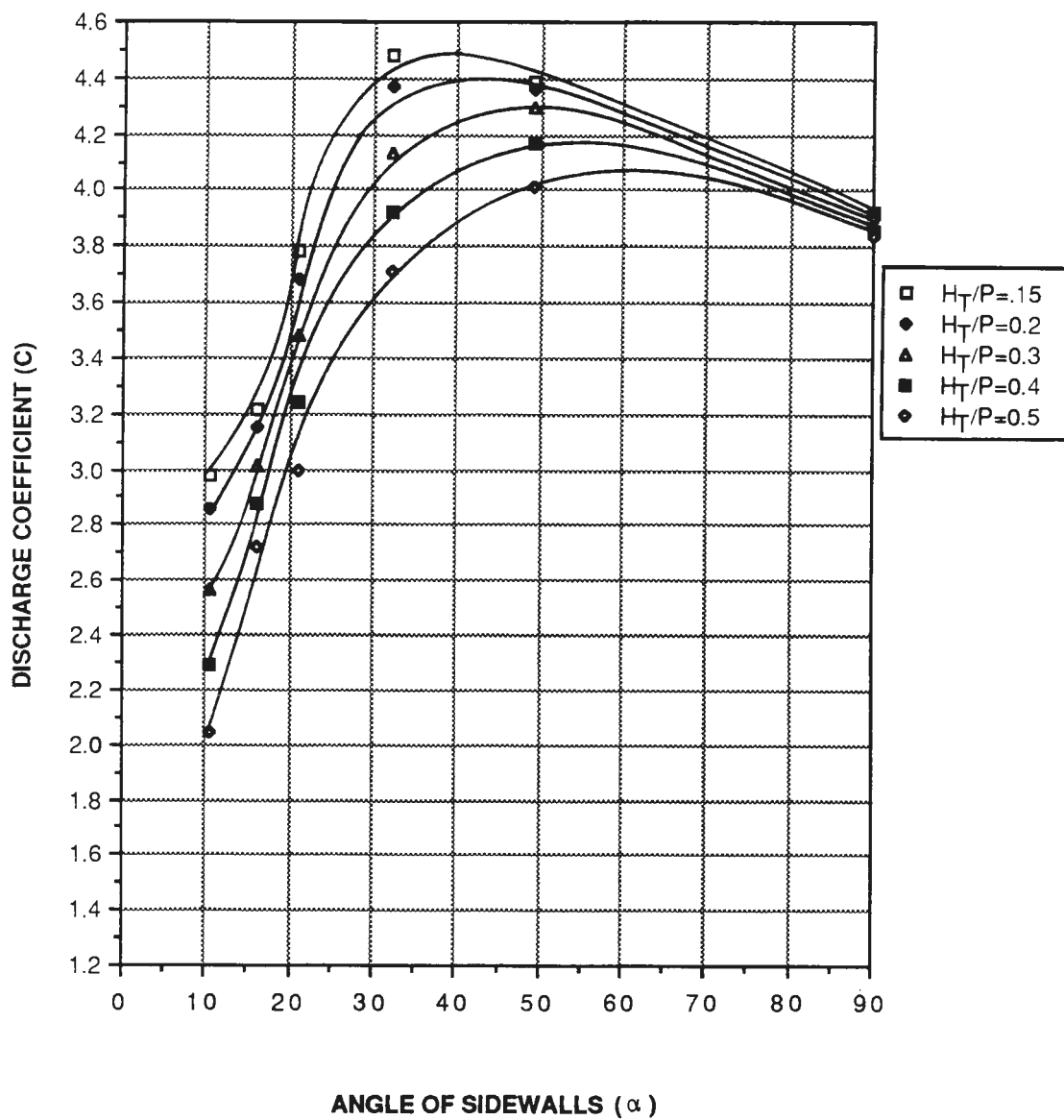


FIG. 20.—Design Chart 1: Triangular Normal Labyrinth Weirs with Rounded Crest Shape.  $H_T/P$  from 0.15 to 0.5, and  $10 < \alpha < 90$  Degrees

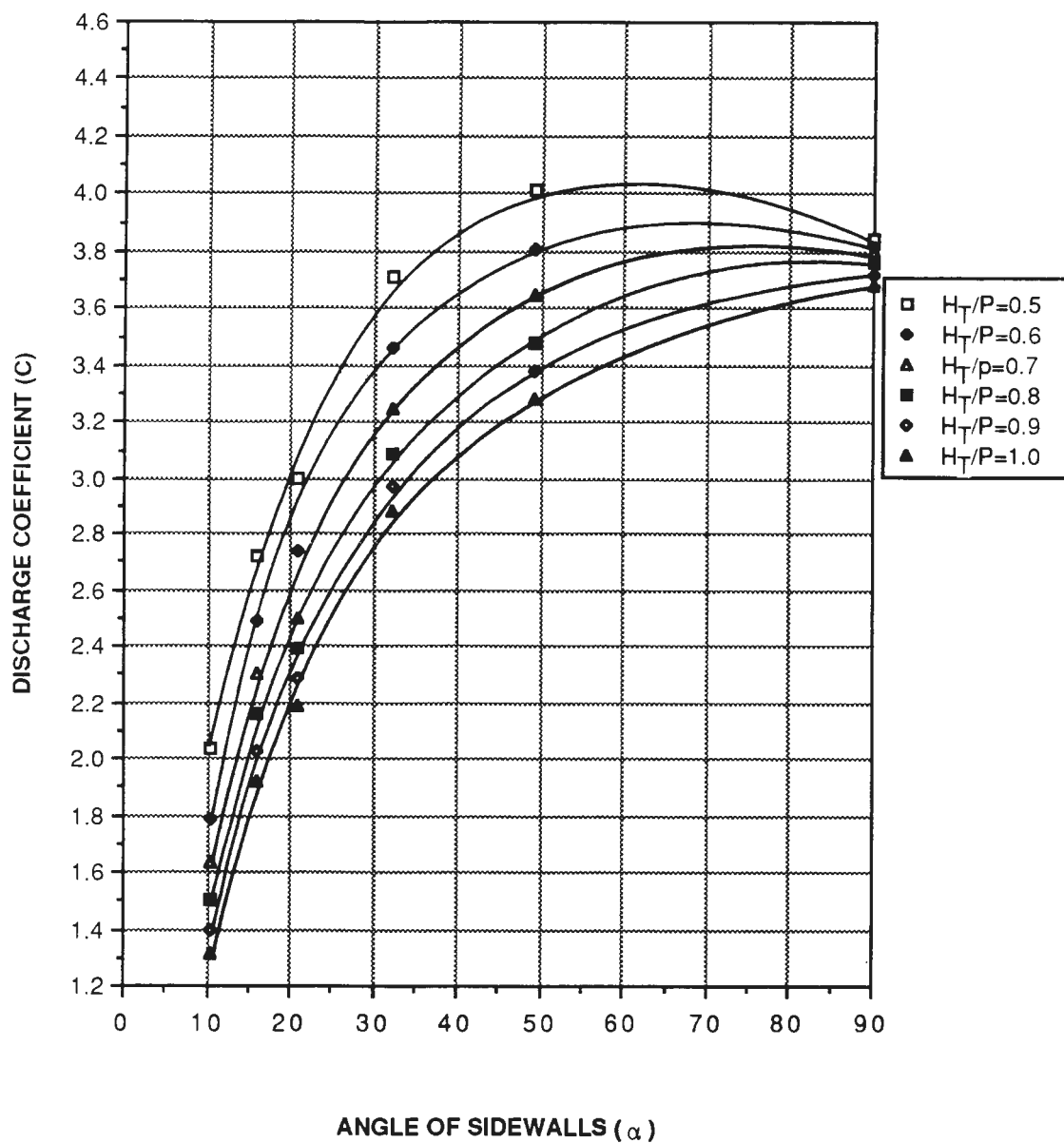


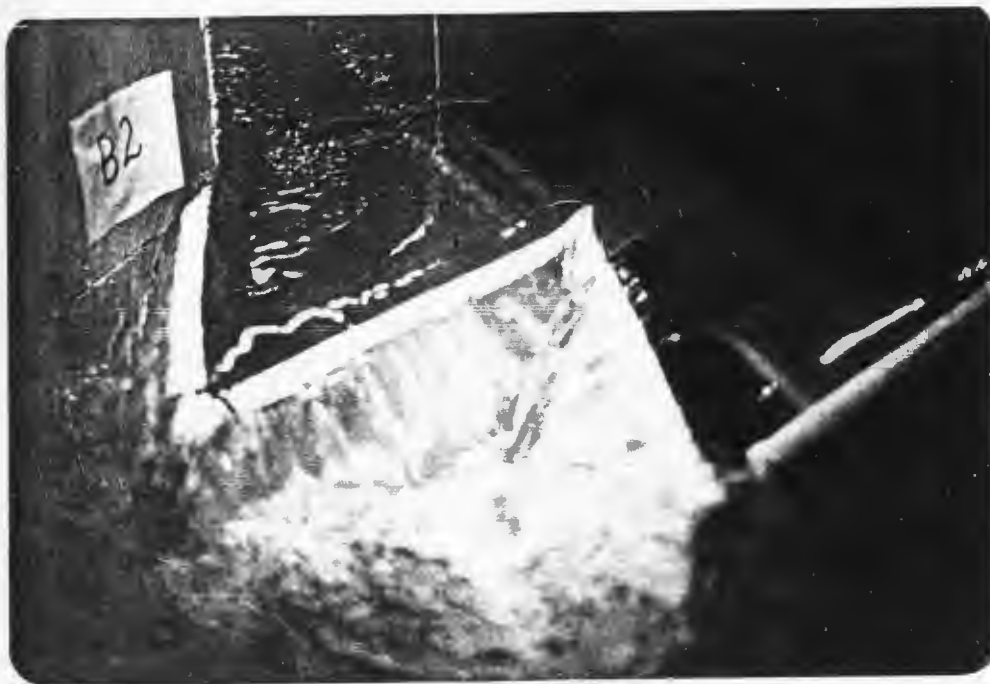
FIG. 21.—Design Chart 2: Triangular Normal Labyrinth Weirs with Rounded Crest Shape.  $H_T/P$  from 0.5 to 1.0, and  $10 < \alpha < 90$  Degrees

the appendix show the data and the relating calculations for two different values of angle of  $\beta$ , while the value of angle of sidewalls,  $\alpha$ , was held constant. The discharge coefficient and the flow magnification,  $Q_L/Q_N$ , are calculated similar to normal labyrinth weirs. Figure 22 shows the flow conditions of inclined labyrinth weirs for two values of angle of  $\beta$ .

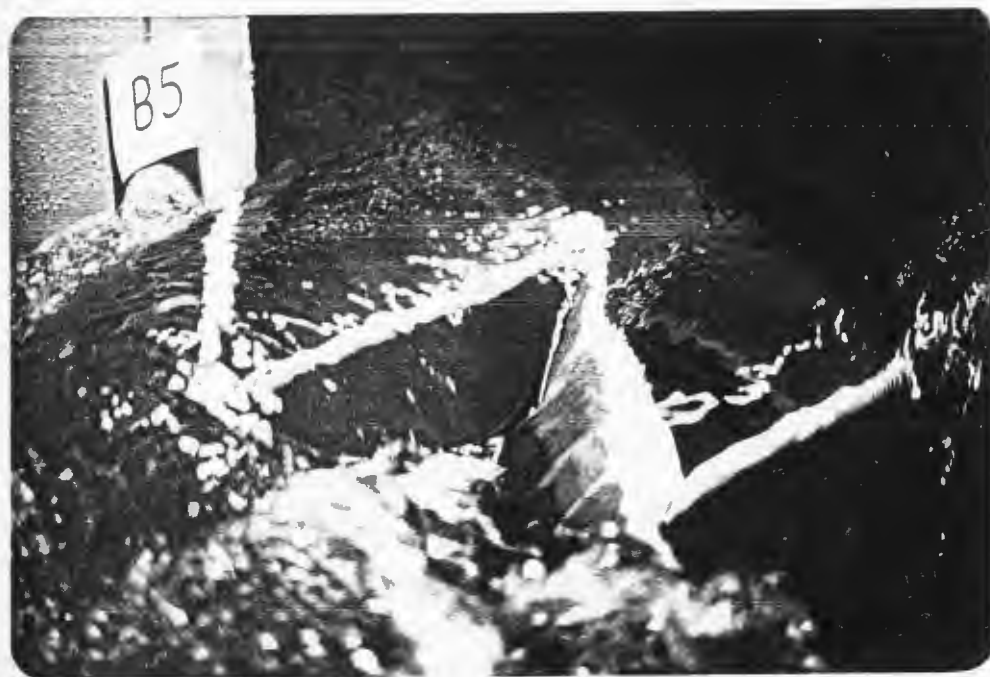
Figure 23 allows comparison of the performance of normal labyrinth weirs with inclined labyrinth weirs. This figure shows that the discharge coefficient in the unit length of inclined labyrinth weirs is less than normal labyrinth weirs. In addition, the discharge coefficient in a unit length of the weir decreases as the angle of  $\beta$  becomes larger. These are due to the higher nappe impingement of adjacent sidewalls.

Figure 24 indicates that the flow magnification of inclined labyrinth weirs varies significantly as the angle of  $\beta$  changes. An increase in the total length of the structure due to the construction of labyrinth weirs inclined to the approaching water causes the flow magnification to increase. At high operating heads, only a small gain in the performance is achieved. Thus, in this case the use of inclined labyrinth weirs with excess cost of the structure might not be a good alternative for design.





a. Low flow performance



b. High flow performance

FIG. 22.—Flow Condition of Inclined Labyrinth Weirs,  $\beta = 45$  Degrees, and  $\alpha = 24.5$  Degrees

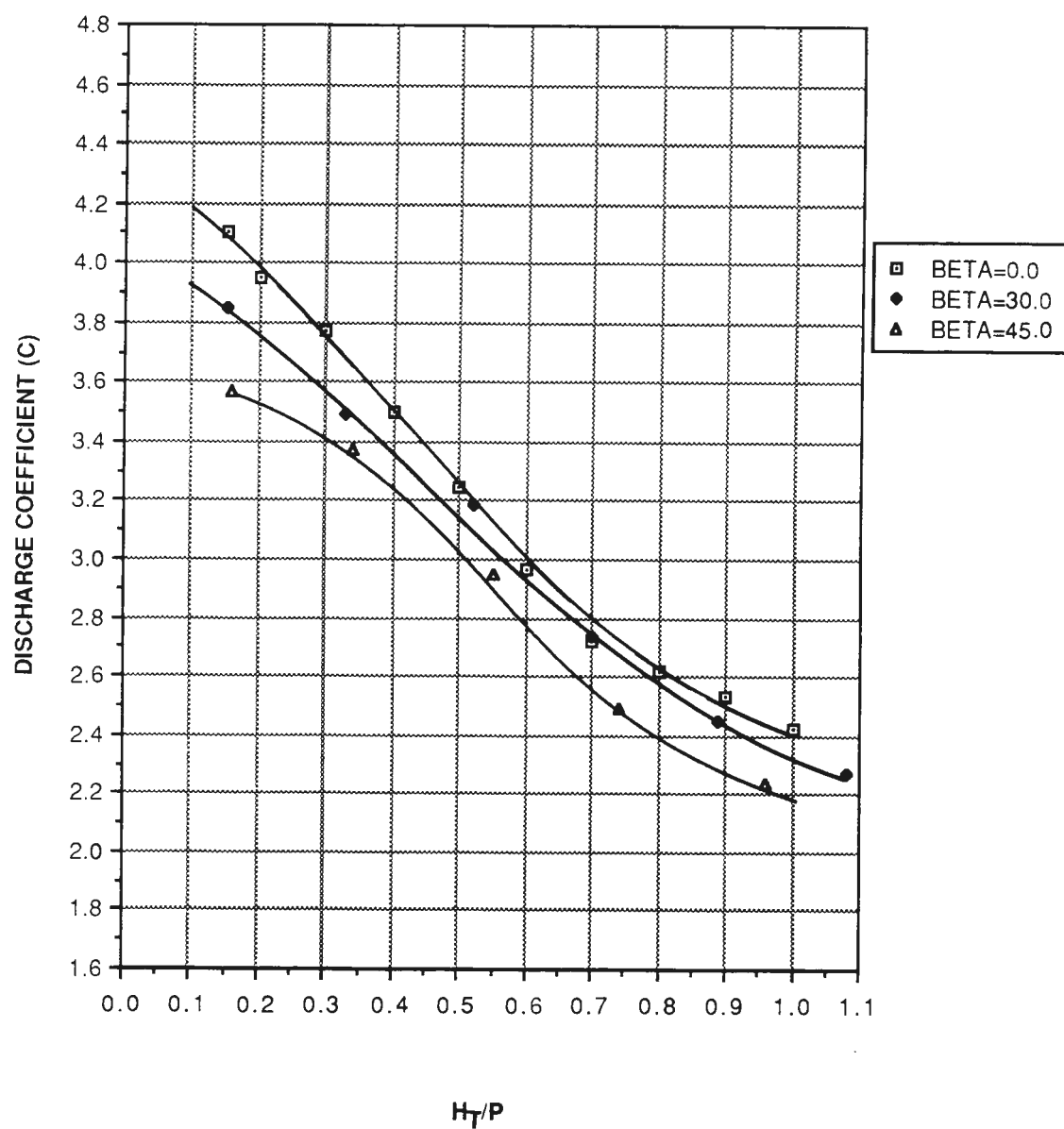


FIG. 23.—Effect of Angle of Approach on a Discharge Coefficient of the Unit Length of Inclined Labyrinth Weirs,  $\alpha = 24.5$  Degrees

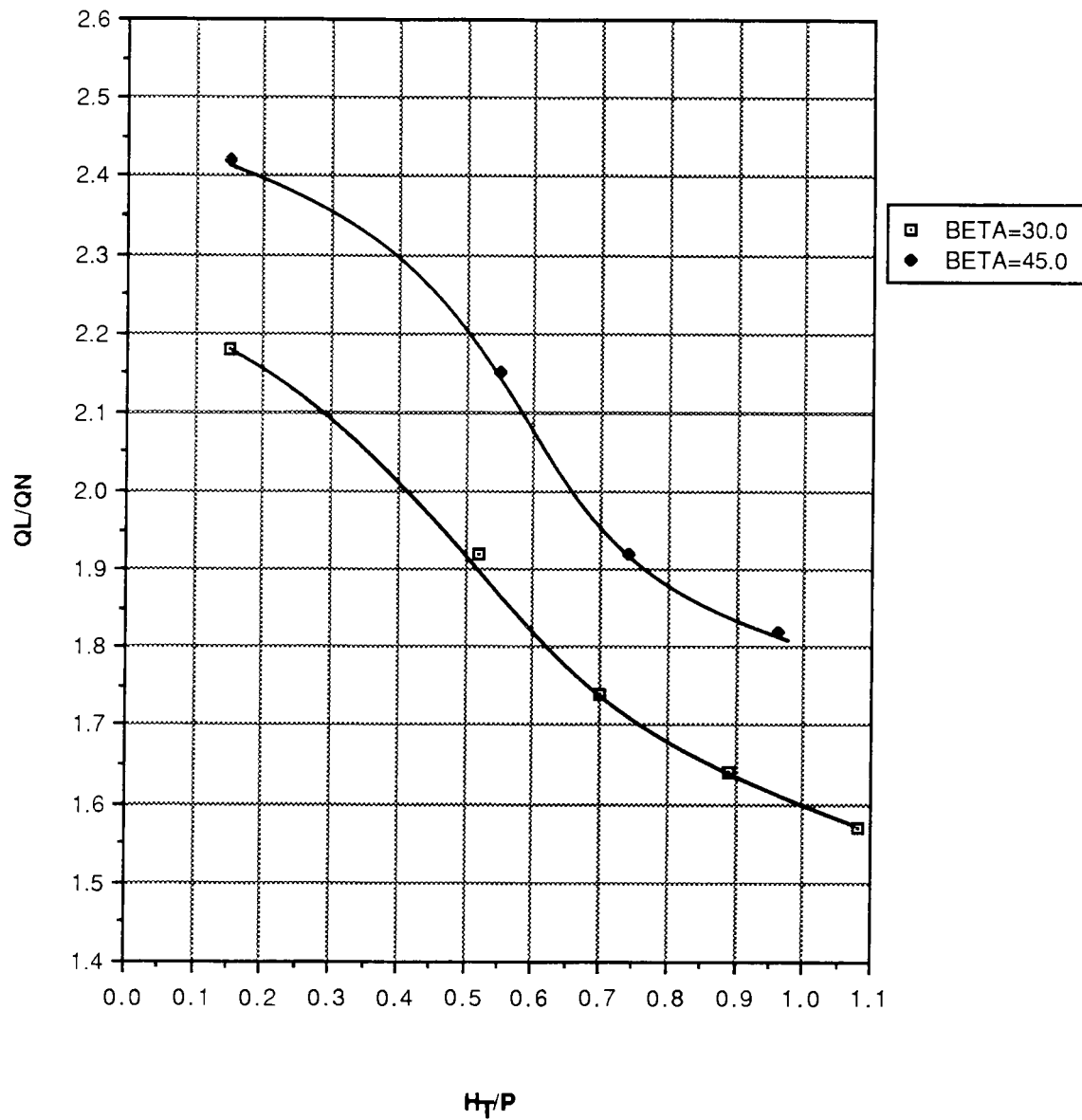


FIG. 24.—Effect of Approach Angles on the Performance of Inclined Labyrinth Weirs, for  $\alpha = 24.5$  Degrees

## CHAPTER VI

## DESIGN AND APPLICATION OF LABYRINTH SPILLWAY

General Applications of Labyrinth Weirs

Although the labyrinth spillways have not been built in a large number of designs, they have potential applications and advantages to use in particular site conditions. The following are major areas of application of labyrinth weirs:

1. To modify an existing reservoir in order to improve the dam safety when the discharge capacity of the spillway is not adequate.
2. To increase the storage of those types of reservoirs whose functions are to control the floods, providing an adequate head for hydroelectric power plants, irrigation, and so on. In this case, the construction of a labyrinth spillway is an alternative to installing a gate.
3. When the maximum water surface level of the reservoir and its adjacent upstream rivers are limited. In this case, the capability of a large flow discharge of labyrinth weirs is very significant if flooding occurs.
4. When the site condition is too narrow so that the normal linear spillway does not provide enough discharge capacity.
5. Since the labyrinth spillway can be compressed into a narrow width to discharge the same flow rate with an equal head as does the normal linear weir with longer length, its construction is considered to be more economical.

Design Considerations of Labyrinth Weirs

Since the performance of a labyrinth spillway depends on

different parameters in order to obtain optimum performance, the approach condition, spillway placements and orientations, and the parameters of primary importance should be selected properly.

The flow distribution in the channel of approach and upstream channels should be uniform. A wavy water surface reduces the performance of labyrinth weirs. The most efficient type of spillway entrance is to make a curved approach adjacent to each end cycle of the labyrinth spillway.

The spillway placement is more important than the orientation. In order to reduce the head loss in the channel of approach and at the upstream apexes, the spillway should be installed as far upstream in the reservoir as possible. The spillway orientations do not significantly affect the performance of labyrinth weirs. However, in the case where the apexes of the end cycles are located upstream, the water surface along the end spillway sidewalls are rough, which causes the reduction in head and discharge.

For low heads, nappe oscillation and noise will occur. These phenomena are due to the alternation of atmospheric and subatmospheric pressure under the nappe. To prevent any damage to the spillway due to the existence of negative pressure under the nappe, the best method is to install splitter piers along the spillway sidewalls. The placement of these piers are generally a couple of feet from the downstream apexes. In this case, the reduction of length of labyrinth spillway due to the width of splitter piers should be considered in the design. These piers might be submerged during high flows.

#### Different Cases of Labyrinth Spillway Design

There are at least four possible cases of labyrinth spillway

design relating either to the site condition constraints or economical considerations. What follows are the cases which the designer should consider when applying the design procedures of the current study:

1. The maximum allowable head is limited, but there is no restriction on the width of the spillway. In this case, it is recommended that larger values of angle of sidewalls,  $\alpha$ , be used in order to get the higher performance resulting from less nappe interference.

2. There is no restriction on the maximum level of water surface, but the site condition does not allow a linear spillway with appropriate length. In this situation, it may not be economical to raise the height of the spillway. In addition, it will reduce the factor of safety of the reservoir. The use of a labyrinth spillway with a small angle of sidewalls,  $\alpha$ , can be a good alternative.

3. Neither the maximum water surface level nor the width of the spillway are limited. In this case, the construction of labyrinth weirs can reduce the total cost of the structure. An optimization study must be made to determine the most suitable angle of sidewalls.

4. The fourth case is when the maximum allowable head and the width of the spillway are both limited. In this condition, the use of a labyrinth spillway is the best alternative for design.

#### Steps in the Design of Labyrinth Weirs

Although this study was conducted on the behavior of labyrinth weirs with the triangular plan forms, the application of the results shows that the design curves can be used for trapezoidal labyrinth weirs. The following are the steps which should be taken to design either triangular or trapezoidal labyrinth weirs:

1. Determine the total width of the approach channel,  $W$ , and height of the weir crest,  $P$ , from the site conditions.
2. From the hydrological studies, calculate the maximum flow discharge to pass from the spillway.
3. Specify the maximum allowable head for the reservoir.
4. Select the desired type of crest shapes. The rounded crest shape has a higher discharge coefficient than other crest shapes.
5. Choose triangular or trapezoidal plan forms. In the case of trapezoidal plan forms, the length of the upstream and downstream apexes should be selected as small as possible. These apexes are being used for constructional reasons.
6. From the information of previous steps, calculate the ratio of total head to crest height,  $H_T/P$ .
7. Select any value of angle of sidewalls,  $\alpha$ .
8. Determine the design discharge coefficient,  $C$ , using Figures 20 or 21. These figures are developed for rounded crest shape.
9. In case of the use of other types of weir crest shapes, rather than rounded crest shapes, determine the ratio of discharge coefficient to the rounded crest shapes by using Figure 15. Multiply this ratio by  $C$ , which is obtained in Step 9, and use it as a design discharge coefficient in Step 10.
10. Determine the total length of labyrinth spillway using equations  $Q = CLH_T^{3/2}$ . The value of  $C$  are those calculated from Steps 8 or 9.
11. Assume a reasonable value for the number of spillway cycles, considering that the vertical aspect ratio,  $w/P$ , should not be less than 2.5 and 2 for triangular and trapezoidal labyrinth weirs, respectively.

12. The length of a cycle can be determined from the previous steps. By the multiplication of this value and the assumed number of cycles in Step 11, the total length of a labyrinth weirs is determined.

13. Compare the result of Step 12 with the length of labyrinth weirs obtained in Step 10. If this value was greater than the value of Step 10, repeat the calculations of Steps 7, 8, 10, and 12 with a larger value of angle of sidewalls,  $\alpha$ , until these two values become approximately equal. If a smaller value is obtained of total length of labyrinth weirs in Step 12 than Step 10, try the above calculations with a smaller value of  $\alpha$ . The assumed number of cycles will remain constant for the whole process.

#### Design Example

The following sample problem represents the use of the design criteria suggested in this paper:

Step 1. Width of the approach channel = 448 feet  
Height of the weir crest = 10 feet

Step 2. Maximum discharge = 50,000 cfs

Step 3. Maximum allowable head = 7.10 feet

Step 4. Shape of the weir crest = quarter round (1/4 arc)

Step 5. Type of the plan form = trapezoidal  
Length of upstream apex =  $2a = 2$  feet  
Length of downstream apex =  $2a' = 2$  feet

Step 6. Calculate  $H_T/P$   
Approach velocity =  $\frac{Q}{(P + H)W} = \frac{50000}{(10 + 7.10)448} = 6.53$  fps

$$H_T = H + V^2/2g = 7.10 + \frac{(6.53)^2}{2 \times 32.2} = 7.76 \text{ feet}$$

$$H_T/P = 0.78$$

Step 7. Assume  $\alpha = 20$  degrees



- Step 8. From Figure 21, the design discharge coefficient for the rounded crest shape = 2.36
- Step 9. Calculate the ratio of the discharge coefficient of quarter rounded crest shape,  $C_{QR}$ , to rounded crest shape,  $C_R$ , from Figure 15 for  $H_T/P$  equal to 0.78:

$$\frac{C_{QR}}{C_R} = \frac{3.68}{3.76}$$

$$C = \frac{3.68}{3.76} * 2.36 = 2.31$$

- Step 10. The total length of a labyrinth weirs is determined as follows:

$$Q = CLH_T^{3/2}$$

$$50000 = 2.31 * L * (7.76)^{3/2}$$

$$L = 1001 \text{ feet}$$

- Step 11. Assume the number of cycles,  $n = 10$

$$w = \frac{448}{10} = 44.8 \text{ feet}$$

$$\text{and } w/P = \frac{44.8}{10} = 4.48$$

- Step 12. Calculate the length of a cycle:

$$l = 2b + 2a + 2a'$$

$$b = \frac{(44.8 - 4) \div 2}{\sin \alpha} = \frac{20.4}{\sin 20} = 59.65 \text{ feet}$$

$$l = 2 * 59.65 + 2 + 2 = 123.3 \text{ feet}$$

$$L = l * n = 123.3 * 10 = 1233 \text{ feet}$$

- Step 13. Since the length of labyrinth weirs obtained in Step 12 is greater than the one in Step 10, repeat the calculations from Step 7, with the greater assumed value for  $\alpha$ .

If  $\alpha = 25$  degrees

$$\text{then } C = 2.66 * \frac{3.68}{3.76} = 2.6$$

$$L = 888 \text{ feet}$$

$$n = 10 \text{ cycles}$$

$$b = 48.27 \text{ feet}$$

$$l = 100.5 \text{ feet}$$

$$L = l * n = 1005 \text{ feet}$$

since  $L = l * n = 1005 > 888$  feet, try for another larger value of  $\alpha$ .

finally if  $\alpha = 35.9$  degrees

$$\text{then } C = 3.21 * \frac{3.68}{3.76} = 736 \text{ feet}$$

$$L = 736 \text{ feet}$$

$$n = 10 \text{ cycles}$$

$$b = 34.79 \text{ feet}$$

$$l = 73.6 \text{ feet}$$

$$L = 73.6 * 10 = 736 \text{ feet}$$

The same labyrinth weirs can be designed for  $\alpha = 34.65$  degrees,  $n = 15$  cycles,  $b = 22.75$  feet, and the total length of the weirs equal to 743 feet.

In order to check the degree of accuracy of the newly developed curves and procedures of the current study, the parameters and the major dimensions of some existing labyrinth spillways are used to determine the total length of these structures. The comparison of the calculated total length of these labyrinth spillways with their actual lengths show that the result of the current study can be applied to design a labyrinth spillway with a high degree of accuracy. The total discharge with the maximum head, the major dimensions, and also the percent difference of the total length of some existing labyrinth weirs are shown in Table 2. The average discrepancy of  $\pm 5\%$  can be seen between the results.

TABLE 2.—Comparison of the Actual Crest Length of Various Labyrinth Spillways with the Calculated Length Using the Current Study

Name	Design discharge, ft <sup>3</sup> /s	Total Width, ft	Maximum head, ft	Weir Height, ft	Crest Shape	Plan Form	Angle of sidewalls, $\alpha$ , degrees	Crest length, ft	Calculated crest length, ft	% Difference
Ute Dam	550,000	840	19.00	30.0	1/4 arc	Trapez.	11.64	3,360	3151	-6.3
Woronora Dam	36,000	484	4.46	7.25	1/4 arc	Trian.	25.4	1,127	1151	+2.1
Avon Dam	50,000	448	7.10	10.00	1/4 arc	Trapez.	27.5	868	832	-4.1
Bartletts Ferry Dam	240,000	1,230	6.00	11.25	1/4 arc	Trapez.	14.48	4,729	5153	+9
Navet Pumped Storage	17,000	180	5.0	10.00	1/2 arc	Trian.	23.58	450	426	-5.3
Hyrum Dam	9,050	60	5.50	12.00	1/4 arc	Trapez.	8.9	300	305	1.7
Boardman Spillway	13,660	120	5.80	9.06	1/2 arc	Trian.	19.53	350	327	-6.5

## CHAPTER VII

## SUMMARY AND CONCLUSION

The purpose of this study was to develop a new procedure for the design of a labyrinth spillway. The new procedure was intended to be easier to apply compared with previous methods, to cover a wider range of  $H/P$  ratio, to maintain a higher degree of accuracy, and to be applicable for weirs with crest shapes other than sharp crested.

A dimensional analysis was made, and the significance of the parameters of primary and secondary importance were described. The parameters which were used in this study for labyrinth weirs were those that have been called of primary importance by Taylor (11). Since different crest shapes can be used for weirs, four types of crest (sharp crest, rounded crest, quarter rounded crest, and top flat crest) are tested for normal linear weirs. The effect of approach velocity in the discharge coefficient is included by using the total head instead of measured head in the calculations. For each test, data for three types of ventilation (fully vented, cavity, and nonvented) were recorded. Two types of labyrinth weirs, normal and inclined, are introduced. The former was installed normal to the flow direction; whereas, the latter one was at an angle to the approaching water. The shape of the crest for both types of labyrinth weirs was rounded crest, and the plan form was triangular. The range of angle of sidewalls,  $\alpha$ , was from 10 to 90 degrees. The angles of labyrinth weirs to the approaching flow,  $\beta$ , were 0, 30, and 45 degrees.

A design curve for a normal linear weir with four different crest shapes (Figure 15) was developed. This curve is the discharge

coefficient,  $C$ , against the total head to crest height ratio,  $H_T/P$ , with  $H_T/P$  ranges from 0.1 to 1.1. For the normal labyrinth weirs, two design charts (Figures 20 and 21) with  $H_T/P$  ratio ranges from 0.15 to 0.5 and 0.5 to 1.0 are developed. Figures 15, 20, and 21 can be used by the designer to design a labyrinth weirs with four different crest shapes. The design procedure and the steps which must be followed with a design example have been presented.

The following are conclusions and recommendations from the current study:

1. The ability of labyrinth weirs to pass relatively large flows at low heads is a good alternative both hydraulically and economically to modify an existing reservoir.
2. The discharge coefficient of weirs with rounded crest shape is higher than quarter rounded, top flat, and sharp crested weirs. Therefore, to obtain larger flow discharge, the rounded crest weirs are recommended.
3. The comparison of the discharge coefficient of sharp-crested weirs from this study shows a close agreement with the results of previous research.
4. The performance of labyrinth weirs decreases as the head increases. This is due to the interference of the nappes and the submergence.
5. The comparison of the current developed procedure in calculating the total length of some existing labyrinth weirs shows a discrepancy of 5% with their actual crest length.
6. The installation of labyrinth weirs with an angle to the approaching water can increase the capacity of the spillway significantly.

7. More research can be done to check the performance of labyrinth weirs with plan forms other than triangular, trapezoidal, or rectangular, but plan forms such as parabolic or elliptic.

8. The effect of angle of approach water to the labyrinth weirs has not been fully described in this study. This effect can be checked for the angle of sidewalls,  $\alpha$ , ranges from 0 to 90 degrees with the angle of approaching water ranges from 0 to some reasonable values.

9. An optimization model can be conducted to determine the optimal dimensions of labyrinth weirs to minimize the cost and satisfy the hydraulic performance.

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## APPENDIX

TABLE 3.— A Discharge Coefficient of a Rounded Crested Normal Linear Weir  
 $P=3.35$  FT.     $L=3.0$  FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2g$ (FT)	$H+V^2/2g$ (FT)	$H_T/P$	C	c
1A	Fully-venting	0.102	0.30	0.398	0.30	0.001	0.103	0.31	3.82	3.90
	Non-venting	0.096	0.29	0.398	0.31	0.001	0.097	0.29	4.17	4.26
2A	Fully-venting	0.139	0.41	0.631	0.44	0.003	0.142	0.42	3.81	3.93
	Non-venting	0.132	0.39	0.631	0.45	0.003	0.135	0.40	4.10	4.24
3A	Fully-venting	0.209	0.62	1.166	0.71	0.008	0.217	0.65	3.77	3.98
	Non-venting	0.203	0.61	1.166	0.72	0.008	0.211	0.63	3.92	4.16
4A	Fully-venting	0.271	0.81	1.728	0.95	0.014	0.285	0.85	3.73	4.02
	Non-venting	0.267	0.80	1.728	0.96	0.014	0.281	0.84	3.80	4.11
5A	Non-venting	0.336	1.00	2.400	1.19	0.022	0.358	1.07	3.69	4.05
6A	Non-venting	0.416	1.24	3.260	1.45	0.033	0.449	1.34	3.58	4.01
7A	Non-venting	0.474	1.41	3.950	1.63	0.041	0.515	1.54	3.53	4.00
8A	Non-venting	0.566	1.69	5.100	1.89	0.055	0.621	1.85	3.45	3.96
9A	Non-venting	0.659	1.97	6.420	2.15	0.072	0.731	2.18	3.40	3.97

TABLE 4.— A Discharge Coefficient of a Rounded Crested Normal Linear Weir  
P=.668 FT. L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
1B	Fully-venting	0.100	0.15	0.398	0.17	.000	0.100	0.15	3.99	4.01
	Non-venting	0.093	0.14	0.398	0.17	.000	0.093	0.14	4.43	4.46
2B	Fully-venting	0.135	0.20	0.631	0.26	0.001	0.136	0.20	4.06	4.11
	Non-venting	0.129	0.19	0.631	0.26	0.001	0.130	0.19	4.33	4.39
3B	Fully-venting	0.210	0.31	1.166	0.44	0.003	0.213	0.32	3.87	3.95
	Non-venting	0.194	0.29	1.166	0.45	0.003	0.197	0.30	4.34	4.45
4B	Fully-venting	0.272	0.41	1.728	0.61	0.006	0.278	0.42	3.87	3.99
	Non-venting	0.257	0.38	1.728	0.62	0.006	0.263	0.39	4.20	4.34
5B	Fully-venting	0.341	0.51	2.400	0.79	0.010	0.351	0.53	3.80	3.97
	Non-venting	0.329	0.49	2.400	0.80	0.010	0.339	0.51	4.00	4.18
6B	Fully-venting	0.416	0.62	3.260	1.00	0.016	0.432	0.65	3.79	4.01
	Non-venting	0.410	0.61	3.260	1.01	0.016	0.426	0.64	3.87	4.09
7B	Non-venting	0.466	0.70	3.950	1.16	0.021	0.487	0.73	3.84	4.10
8B	Non-venting	0.553	0.83	5.100	1.39	0.030	0.583	0.87	3.79	4.10
9B	Non-venting	0.639	0.96	6.420	1.64	0.042	0.681	1.02	3.79	4.16

TABLE 5.— A Discharge Coefficient of a Rounded Crested Normal Linear Weir

P=.5 FT.      L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
10C	Fully-venting Cavity	0.083	0.17	0.299	0.17	.000	0.083	0.17	3.92	3.95
		0.082	0.16	0.299	0.17	.000	0.082	0.16	3.99	4.02
11C	Fully-venting Cavity	0.118	0.24	0.499	0.27	0.001	0.119	0.24	3.90	3.95
		0.116	0.23	0.499	0.27	0.001	0.117	0.23	3.99	4.05
	Non-venting	0.110	0.22	0.499	0.27	0.001	0.111	0.22	4.31	4.38
12C	Fully-venting Cavity	0.208	0.42	1.160	0.55	0.005	0.213	0.43	3.86	3.99
		0.205	0.41	1.160	0.55	0.005	0.210	0.42	3.94	4.08
	Non-venting	0.192	0.38	1.160	0.56	0.005	0.197	0.39	4.33	4.49
13C	Fully-venting Cavity	0.306	0.61	2.080	0.86	0.011	0.317	0.63	3.82	4.04
		0.301	0.60	2.080	0.87	0.012	0.313	0.63	3.91	4.14
	Non-venting	0.294	0.59	2.080	0.87	0.012	0.306	0.61	4.04	4.28
14C	Fully-venting	0.406	0.81	3.167	1.17	0.021	0.427	0.85	3.74	4.04
	Non-venting	0.401	0.80	3.167	1.17	0.021	0.422	0.84	3.81	4.11
15C	Non-venting	0.490	0.98	4.163	1.40	0.031	0.521	1.04	3.66	4.01

TABLE 6.—A Discharge Coefficient of a Rounded Crested Normal Linear Weir

P=.363 FT. L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
100	Fully-venting	0.083	0.23	0.299	0.22	0.001	0.084	0.23	3.90	3.95
	Cavity	0.082	0.23	0.299	0.22	0.001	0.083	0.23	3.97	4.02
	Non-venting	0.081	0.22	0.299	0.22	0.001	0.082	0.23	4.04	4.09
110	Fully-venting	0.118	0.33	0.499	0.35	0.002	0.120	0.33	3.86	3.95
	Cavity	0.116	0.32	0.499	0.35	0.002	0.118	0.32	3.96	4.05
	Non-venting	0.110	0.30	0.499	0.35	0.002	0.112	0.31	4.27	4.38
120	Fully-venting	0.208	0.57	1.160	0.68	0.007	0.215	0.59	3.80	3.99
	Cavity	0.200	0.55	1.160	0.69	0.007	0.207	0.57	4.01	4.23
	Non-venting	0.193	0.53	1.160	0.70	0.008	0.201	0.55	4.21	4.46
130	Fully-venting	0.311	0.86	2.080	1.03	0.016	0.327	0.90	3.65	3.94
	Cavity	0.303	0.83	2.080	1.04	0.017	0.320	0.88	3.78	4.10
	Non-venting	0.297	0.82	2.080	1.05	0.017	0.314	0.87	3.88	4.22
140	Fully-venting	0.404	1.11	3.167	1.38	0.029	0.433	1.19	3.66	4.07
	Cavity	0.402	1.11	3.167	1.38	0.030	0.432	1.19	3.69	4.10
	Non-venting	0.400	1.10	3.167	1.38	0.030	0.430	1.18	3.71	4.13
150	Fully-venting									
	Cavity									
	Non-venting	0.486	1.34	4.163	1.63	0.041	0.527	1.45	3.59	4.06

TABLE 7.—A Discharge Coefficient of a Rounded Crested Normal Linear Weir

P=.309 FT. L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
10E	Fully-venting	0.083	0.27	0.299	0.25	0.001	0.084	0.27	3.88	3.95
	Cavity	0.082	0.27	0.299	0.25	0.001	0.083	0.27	3.95	4.02
	Non-venting	0.081	0.26	0.299	0.26	0.001	0.082	0.27	4.02	4.09
11E	Fully-venting	0.118	0.38	0.499	0.39	0.002	0.120	0.39	3.84	3.95
	Cavity	0.116	0.38	0.499	0.39	0.002	0.118	0.38	3.93	4.05
	Non-venting	0.110	0.36	0.499	0.40	0.002	0.112	0.36	4.24	4.38
12E	Fully-venting	0.208	0.67	1.160	0.75	0.009	0.217	0.70	3.76	3.99
	Cavity	0.205	0.66	1.160	0.75	0.009	0.214	0.69	3.83	4.08
	Non-venting	0.195	0.63	1.160	0.77	0.009	0.204	0.66	4.10	4.39
13E	Fully-venting	0.310	1.00	2.080	1.12	0.019	0.329	1.07	3.62	3.96
	Cavity	0.306	0.99	2.080	1.13	0.020	0.326	1.05	3.68	4.04
	Non-venting	0.297	0.96	2.080	1.14	0.020	0.317	1.03	3.82	4.22
14E	Fully-venting	0.405	1.31	3.167	1.48	0.034	0.439	1.42	3.59	4.05
	Non-venting	0.402	1.30	3.167	1.48	0.034	0.436	1.41	3.63	4.10
15E	Non-venting	0.486	1.57	4.163	1.75	0.047	0.533	1.73	3.53	4.06

TABLE 8.—A Discharge Coefficient of a Sharp Crested Normal Linear Weir

P=.674 FT. L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
16A	Fully-venting	0.069	0.10	0.193	0.09	.000	0.069	0.10	3.33	3.34
	Cavity	0.069	0.10	0.193	0.09	.000	0.069	0.10	3.33	3.34
	Non-venting	0.065	0.10	0.193	0.09	.000	0.065	0.10	3.63	3.64
17A	Fully-venting	0.165	0.24	0.705	0.28	0.001	0.166	0.25	3.38	3.41
	Cavity	0.163	0.24	0.705	0.28	0.001	0.164	0.24	3.44	3.47
	Non-venting	0.157	0.23	0.705	0.28	0.001	0.158	0.23	3.63	3.67
18A	Fully-venting	0.314	0.47	1.883	0.64	0.006	0.320	0.48	3.41	3.52
	Cavity	0.311	0.46	1.883	0.64	0.006	0.317	0.47	3.46	3.57
	Non-venting	0.303	0.45	1.883	0.64	0.006	0.309	0.46	3.59	3.71
19A	Non-venting	0.467	0.69	3.431	1.00	0.016	0.483	0.72	3.38	3.55

TABLE 9.—A Discharge Coefficient of a Sharp Crested Normal Linear Weir

P=.339 FT.

L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
16B	Fully-venting	0.071	0.21	0.193	0.16	.000	0.071	0.21	3.18	3.20
	Cavity	0.070	0.21	0.193	0.16	.000	0.070	0.21	3.24	3.27
	Non-venting	0.069	0.20	0.193	0.16	.000	0.069	0.20	3.31	3.34
17B	Fully-venting	0.167	0.49	0.705	0.46	0.003	0.170	0.50	3.26	3.35
	Cavity	0.165	0.49	0.705	0.47	0.003	0.168	0.50	3.31	3.41
	Non-venting	0.160	0.47	0.705	0.47	0.003	0.163	0.48	3.46	3.57
18B	Non-venting	0.310	0.91	1.883	0.97	0.015	0.325	0.96	3.35	3.58
19B	Non-venting	0.469	1.38	3.431	1.42	0.031	0.500	1.48	3.20	3.53



TABLE 10.—A Discharge Coefficient of a Quarter Rounded Crested Normal Linear Weir

P=.675 FT. L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H/P$	C	c
20A	Fully-venting	0.068	0.10	0.225	0.10	.000	0.068	0.10	3.94	3.96
	Cavity	0.067	0.10	0.225	0.10	.000	0.067	0.10	4.03	4.04
	Non-venting	0.066	0.10	0.225	0.10	.000	0.066	0.10	4.12	4.13
21A	Fully-venting	0.135	0.20	0.610	0.25	0.001	0.136	0.20	3.92	3.97
	Cavity	0.135	0.20	0.610	0.25	0.001	0.136	0.20	3.92	3.97
	Non-venting	0.128	0.19	0.610	0.25	0.001	0.129	0.19	4.24	4.29
22A	Fully-venting	0.236	0.35	1.402	0.51	0.004	0.240	0.36	3.90	4.00
	Cavity	0.229	0.34	1.402	0.52	0.004	0.233	0.35	4.07	4.18
	Non-venting	0.225	0.33	1.402	0.52	0.004	0.229	0.34	4.18	4.29
23A	Fully-venting	0.339	0.50	2.346	0.77	0.009	0.348	0.52	3.76	3.91
	Cavity	0.336	0.50	2.346	0.77	0.009	0.345	0.51	3.80	3.96
	Non-venting	0.325	0.48	2.346	0.78	0.009	0.334	0.50	3.99	4.16
24A	Fully-venting	0.439	0.65	3.443	1.03	0.016	0.455	0.67	3.70	3.91
	Cavity	0.431	0.64	3.443	1.04	0.017	0.448	0.66	3.79	4.01
	Non-venting	0.426	0.63	3.443	1.04	0.017	0.443	0.66	3.85	4.08
25A	Fully-venting	0.540	0.80	4.736	1.30	0.026	0.566	0.84	3.68	3.95
	Non-venting	0.527	0.78	4.736	1.31	0.027	0.554	0.82	3.80	4.09

TABLE 11.—A Discharge Coefficient of a Quarter Rounded Crested Normal Linear Weir

P=.343 FT.    L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
20B	Fully-venting	0.070	0.20	0.225	0.18	0.001	0.071	0.21	3.76	3.79
	Cavity	0.069	0.20	0.225	0.18	0.001	0.070	0.20	3.83	3.87
	Non-venting	0.068	0.20	0.225	0.18	0.001	0.069	0.20	3.91	3.96
21B	Fully-venting	0.137	0.40	0.610	0.42	0.003	0.140	0.41	3.77	3.88
	Cavity	0.137	0.40	0.610	0.42	0.003	0.140	0.41	3.77	3.88
	Non-venting	0.132	0.38	0.610	0.43	0.003	0.135	0.39	3.97	4.10
22B	Fully-venting	0.238	0.69	1.402	0.80	0.010	0.248	0.72	3.72	3.95
	Non-venting	0.226	0.66	1.402	0.82	0.010	0.236	0.69	3.99	4.26
23B	Non-venting	0.326	0.95	2.346	1.17	0.021	0.347	1.01	3.77	4.14
24B	Non-venting	0.432	1.26	3.443	1.48	0.034	0.466	1.36	3.57	4.00

TABLE 12.—A Discharge Coefficient of a Flat Top Crested Normal Linear Weir

P=.673 FT. L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
26A	Fully-venting	0.066	0.10	0.202	0.09	.000	0.066	0.10	3.71	3.72
	Non-venting	0.065	0.10	0.202	0.09	.000	0.065	0.10	3.79	3.81
27A	Fully-venting	0.135	0.20	0.566	0.23	0.001	0.136	0.20	3.65	3.68
	Cavity	0.134	0.20	0.566	0.23	0.001	0.135	0.20	3.69	3.72
	Non-venting	0.129	0.19	0.566	0.24	0.001	0.130	0.19	3.90	3.93
28A	Fully-venting	0.236	0.35	1.233	0.45	0.003	0.239	0.36	3.45	3.52
	Cavity	0.232	0.34	1.233	0.45	0.003	0.235	0.35	3.54	3.61
	Non-venting	0.223	0.33	1.233	0.46	0.003	0.226	0.34	3.74	3.83
29A	Fully-venting	0.337	0.50	2.125	0.70	0.008	0.345	0.51	3.46	3.57
	Cavity	0.334	0.50	2.125	0.70	0.008	0.342	0.51	3.50	3.62
	Non-venting	0.328	0.49	2.125	0.71	0.008	0.336	0.50	3.59	3.72
30A	Fully-venting	0.429	0.64	3.072	0.93	0.013	0.442	0.66	3.44	3.61
	Cavity	0.419	0.62	3.072	0.94	0.014	0.433	0.64	3.56	3.74
	Non-venting	0.414	0.62	3.072	0.94	0.014	0.428	0.64	3.62	3.80
31A	Fully-venting	0.538	0.80	4.415	1.22	0.023	0.561	0.83	3.48	3.70
	Cavity	0.531	0.79	4.415	1.22	0.023	0.554	0.82	3.54	3.77
	Non-venting	0.526	0.78	4.415	1.23	0.023	0.549	0.82	3.58	3.82

TABLE 13.—A Discharge Coefficient of a Flat Top Crested Normal Linear Weir

P=.343

L=3.0 FT.

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	c
26B	Fully-venting	0.069	0.20	0.202	0.16	.000	0.069	0.20	3.46	3.49
	Non-venting	0.068	0.20	0.202	0.16	.000	0.068	0.20	3.54	3.57
27B	Fully-venting	0.138	0.40	0.566	0.39	0.002	0.140	0.41	3.47	3.56
	Cavity	0.135	0.39	0.566	0.39	0.002	0.137	0.40	3.59	3.68
	Non-venting	0.133	0.39	0.566	0.40	0.002	0.135	0.39	3.66	3.76
28B	Fully-venting	0.236	0.69	1.233	0.71	0.008	0.244	0.71	3.35	3.52
	Cavity	0.234	0.68	1.233	0.71	0.008	0.242	0.71	3.39	3.56
	Non-venting	0.225	0.66	1.233	0.72	0.008	0.233	0.68	3.58	3.78
29B	Non-venting	0.327	0.95	2.125	1.06	0.017	0.344	1.00	3.46	3.74
30B	Non-venting	0.420	1.22	3.072	1.34	0.028	0.448	1.31	3.38	3.72

TABLE 14.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.508 FT.      w=2.0 FT.      w/P=3.94      ALPHA=10.50  
W=3.0 FT.      L=16.583 FT.      L/W=5.53

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C.L/W	C	c.L/W	c
31A	Non-venting	0.076	0.15	1.180	0.67	0.007	0.083	0.16	16.44	2.97	18.77	3.40
32A	Non-venting	0.152	0.30	3.320	1.68	0.044	0.196	0.39	12.79	2.31	18.67	3.38
33A	Non-venting	0.229	0.45	5.010	2.27	0.080	0.309	0.61	9.74	1.76	15.24	2.76
34A	Non-venting	0.305	0.60	6.600	2.71	0.114	0.419	0.82	8.12	1.47	13.06	2.36
35A	Non-venting	0.381	0.75	8.260	3.10	0.149	0.530	1.04	7.14	1.29	11.71	2.12
36A	Non-venting	0.457	0.90	9.600	3.32	0.171	0.628	1.24	6.43	1.16	10.36	1.87
37A	Non-venting	0.508	1.00	10.610	3.48	0.188	0.696	1.37	6.09	1.10	9.77	1.77

TABLE 15.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.508 FT.      w=2.0 FT.      w/P=3.94      ALPHA=10.50  
W=3.0 FT.      L=16.583 FT.      L/W=5.53

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	QL (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C*	QN (CFS)	QL/QN
31A	Non-venting	0.076	0.15	1.180	0.67	0.007	0.083	0.16	3.92	0.297	3.98
32A	Non-venting	0.152	0.30	3.320	1.68	0.044	0.196	0.39	3.87	1.028	3.23
33A	Non-venting	0.229	0.45	5.010	2.27	0.080	0.309	0.61	3.82	1.995	2.51
34A	Non-venting	0.305	0.60	6.600	2.71	0.114	0.419	0.82	3.75	3.081	2.14
35A	Non-venting	0.381	0.75	8.260	3.10	0.149	0.530	1.04	3.66	4.272	1.93
36A	Non-venting	0.457	0.90	9.600	3.32	0.171	0.628	1.24	-	-	-
37A	Non-venting	0.508	1.00	10.610	3.48	0.188	0.696	1.37	-	-	-

\* C represents the discharge coefficient of normal linear weir.

TABLE 16.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

$P=.495$  FT.       $w=2.0$  FT.       $w/P=4.04$        $\text{ALPHA}=16$   
 $W=3.0$  FT.       $L=10.344$  FT.       $L/W=3.45$

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C.L/W	C	C.L/W	C
38A	Non-venting	0.050	0.10	0.392	0.24	0.001	0.051	0.10	11.38	3.30	11.69	3.39
39A	Cavity	0.116		1.530	0.83	0.011	0.127	0.26	11.29	3.27	12.91	3.74
40A	Non-venting	0.197	0.40	3.260	1.57	0.038	0.235	0.48	9.52	2.76	12.43	3.60
41A	Non-venting	0.268	0.54	4.620	2.02	0.063	0.331	0.67	8.08	2.34	11.10	3.22
42A	Non-venting	0.339	0.68	5.990	2.39	0.089	0.428	0.86	7.13	2.07	10.12	2.93
43A	Non-venting	0.432	0.87	7.600	2.73	0.116	0.548	1.11	6.25	1.81	8.92	2.59

TABLE 17.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

$P=0.495$  FT.       $w=2.0$  FT.       $w/P=4.04$        $\text{ALPHA}=16$   
 $W=3.0$  FT.       $L=10.344$  FT.       $L/W=3.45$

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	QL (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	QN (CFS)	QL/QN
38A	Non-venting	0.050	0.10	0.392	0.24	0.001	0.051	0.10	3.94	0.15	2.65
39A	Cavity	0.116	0.23	1.410	0.77	0.009	0.125	0.25	3.90	0.54	2.63
40A	Non-venting	0.197	0.40	3.260	1.57	0.038	0.235	0.48	3.86	1.35	2.42
41A	Non-venting	0.268	0.54	4.620	2.02	0.063	0.331	0.67	3.80	2.20	2.10
42A	Non-venting	0.339	0.68	5.990	2.39	0.089	0.428	0.86	3.74	3.17	1.89
43A	Non-venting	0.432	0.87	7.600	2.73	0.116	0.548	1.11	-	-	-



TABLE 18.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.502 FT.      w=2.0 FT.      w/P=3.98      ALPHA=21  
W=3.0 FT.      L=8.44 FT.      L/W=2.81

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2g$ (FT)	$H+V^2/2g$ (FT)	$H_T/P$	C.L/W	C	c.L/W	c
44A	Non-venting	0.076	0.15	0.700	0.40	0.003	0.079	0.16	10.60	3.77	11.14	3.96
45A	Non-venting	0.153	0.30	1.980	1.01	0.016	0.169	0.34	9.52	3.39	11.03	3.92
46A	Non-venting	0.225	0.45	3.290	1.51	0.035	0.260	0.52	8.26	2.94	10.28	3.65
47A	Non-venting	0.301	0.60	4.370	1.81	0.051	0.352	0.70	6.97	2.48	8.82	3.14
48A	Non-venting	0.377	0.75	5.920	2.24	0.078	0.455	0.91	6.42	2.28	8.52	3.03
49A	Non-venting	0.452	0.90	7.210	2.52	0.099	0.551	1.10	5.88	2.09	7.91	2.81
50A	Non-venting	0.502	1.00	8.140	2.70	0.113	0.615	1.23	5.62	2.00	7.63	2.71

TABLE 19.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.502 FT.      w=2.0 FT.      w/P=3.98      ALPHA=21  
W=3.0 FT.      L=8.44 FT.      L/W=2.81

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	QL (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	QN (CFS)	QL/QN
44A	Non-venting	0.076	0.15	0.700	0.40	0.003	0.079	0.16	3.92	0.274	2.56
45A	Non-venting	0.153	0.30	1.980	1.01	0.016	0.169	0.34	3.88	0.829	2.39
46A	Non-venting	0.225	0.45	3.290	1.51	0.035	0.260	0.52	3.84	1.557	2.11
47A	Non-venting	0.301	0.60	4.370	1.81	0.051	0.352	0.70	3.80	2.412	1.81
48A	Non-venting	0.377	0.75	5.920	2.24	0.078	0.455	0.91	3.72	3.462	1.71
49A	Non-venting	0.452	0.90	7.210	2.52	0.099	0.551	1.10	-	-	-
50A	Non-venting	0.502	1.00	8.140	2.70	0.113	0.615	1.23	-	-	-

TABLE 20.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.506 FT.      w=2.0 FT.      w/P=3.95      ALPHA=32.12  
W=3.0 FT.      L=5.38 FT.      L/W=1.79

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C.L/W	C	c.L/W	c
51A	Non-venting	0.076	0.15	0.520	0.30	0.001	0.077	0.15	8.05	4.49	8.27	4.62
52A	Fully-venting Cavity	0.157	0.31	1.420	0.71	0.008	0.165	0.33	7.07	3.94	7.61	4.25
		0.153	0.30	1.420	0.72	0.008	0.161	0.32	7.33	4.09	7.91	4.41
53A	Non-venting	0.228	0.45	2.470	1.12	0.020	0.248	0.49	6.69	3.73	7.56	4.22
54A	Non-venting	0.304	0.60	3.450	1.42	0.031	0.335	0.66	5.92	3.31	6.86	3.83
55A	Non-venting	0.380	0.75	4.500	1.69	0.045	0.425	0.84	5.42	3.03	6.40	3.57
56A	Non-venting	0.455	0.90	5.690	1.97	0.060	0.515	1.02	5.12	2.86	6.18	3.45
57A	Non-venting	0.506	1.00	6.420	2.11	0.069	0.575	1.14	4.90	2.74	5.95	3.32

TABLE 21.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.506 FT.      w=2.0 FT.      w/P=3.95      ALPHA=32.12  
W=3.0 FT.      L=5.38 FT.      L/W=1.79

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	QL (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	QN (CFS)	QL/QN
51A	Non-venting	0.076	0.15	0.520	0.30	0.001	0.077	0.15	4.33	0.296	1.76
52A	Fully-venting Cavity	0.157	0.31	1.420	0.71	0.008	0.165	0.33	-	-	-
		0.153	0.30	1.420	0.72	0.008	0.161	0.32	3.98	0.793	1.79
53A	Non-venting	0.228	0.45	2.470	1.12	0.020	0.248	0.49	4.20	1.580	1.56
54A	Non-venting	0.304	0.60	3.450	1.42	0.031	0.335	0.66	3.99	2.355	1.46
55A	Non-venting	0.380	0.75	4.500	1.69	0.045	0.425	0.84	3.81	3.195	1.41
56A	Non-venting	0.455	0.90	5.690	1.97	0.060	0.515	1.02	3.68	4.122	1.38
57A	Non-venting	0.506	1.00	6.420	2.11	0.069	0.575	1.14	-	-	-

TABLE 22.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.514 FT.      w=2.0 FT.      w/P=3.89      ALPHA=49.04  
W=3.0 FT.      L=3.81 FT.      L/W=1.271

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C.L/W	C	c.L/W	c
58A	Cavity Non-venting	0.078 0.077	0.15 0.15	0.369 0.369	0.21 0.21	0.001 0.001	0.079 0.078	0.15 0.15	5.57 5.68	4.39 4.47	5.65 5.76	4.44 4.53
59A	Fully-venting Cavity Non-venting	0.161 0.154 0.145	0.31 0.30 0.28	1.030 1.030 1.030	0.51 0.51 0.52	0.004 0.004 0.004	0.165 0.158 0.149	0.32 0.31 0.29	5.12 5.46 5.96	4.03 4.30 4.69	5.31 5.68 6.22	4.18 4.47 4.89
60A	Non-venting	0.231	0.45	1.840	0.82	0.011	0.242	0.47	5.17	4.07	5.52	4.35
61A	Non-venting	0.308	0.60	2.670	1.08	0.018	0.326	0.63	4.78	3.76	5.21	4.10
62A	Non-venting	0.386	0.75	3.510	1.30	0.026	0.412	0.80	4.42	3.48	4.88	3.84
63A	Non-venting	0.463	0.90	4.420	1.51	0.035	0.498	0.97	4.19	3.30	4.68	3.68
64A	Non-venting	0.514	1.00	5.070	1.64	0.042	0.556	1.08	4.08	3.21	4.59	3.61

TABLE 23.—A Discharge Coefficient of a Rounded Crested Normal Labyrinth Weir

P=.514 FT.      w=2.0 FT.      w/P=3.89      ALPHA=49.04  
W=3.0 FT.      L=3.81 FT.      L/W=1.271

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	QL (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	QN (CFS)	QL/QN
58A	Cavity	0.078	0.15	0.369	0.21	0.001	0.079	0.15	-	-	-
	Non-venting	0.077	0.15	0.369	0.21	0.001	0.078	0.15	4.33	0.298	1.24
59A	Fully-venting	0.161	0.31	1.030	0.51	0.004	0.165	0.32	-	-	-
	Cavity	0.154	0.30	1.030	0.51	0.004	0.158	0.31	-	-	-
	Non-venting	0.145	0.28	1.030	0.52	0.004	0.149	0.29	-	-	-
60A	Non-venting	0.231	0.45	1.840	0.82	0.011	0.242	0.47	4.02	1.458	1.26
61A	Non-venting	0.308	0.60	2.670	1.08	0.018	0.326	0.63	3.85	2.182	1.22
62A	Non-venting	0.386	0.75	3.510	1.30	0.026	0.412	0.80	3.85	3.091	1.14
63A	Non-venting	0.463	0.90	4.420	1.51	0.035	0.498	0.97	3.71	3.951	1.12
64A	Non-venting	0.514	1.00	5.070	1.64	0.042	0.556	1.08	-	-	-

TABLE 24.—A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir

P=.506 FT.

w=1.208 FT.

w/P=2.39

ALPHA=24.5

BETA=30

W=2.094 FT.

L=5.33 FT.

L/W=2.55

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C.L/W	C	c.L/W	c
65A	Cavity Non-venting	0.076	0.15	0.448	0.37	0.002	0.078	0.15	9.80	3.85	10.21	4.01
		0.075	0.15	0.448	0.37	0.002	0.077	0.15	9.99	3.93	10.42	4.09
66A	Cavity	0.152	0.30	1.241	0.90	0.013	0.165	0.33	8.87	3.49	10.00	3.93
67A	Non-venting	0.228	0.45	2.280	1.48	0.034	0.262	0.52	8.11	3.19	10.00	3.93
68A	Non-venting	0.304	0.60	3.090	1.82	0.052	0.356	0.70	6.96	2.74	8.80	3.46
69A	Non-venting	0.380	0.75	3.937	2.12	0.070	0.450	0.89	6.23	2.45	8.03	3.15
70A	Non-venting	0.455	0.90	4.885	2.43	0.092	0.547	1.08	5.77	2.27	7.60	2.99
71A	Non-venting	0.506	1.00	5.523	2.61	0.105	0.611	1.21	5.52	2.17	7.33	2.88

TABLE 25.—A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir

P=.506 FT.      w=1.208 FT.      w/P=2.39      ALPHA=24.5      BETA=30  
W=2.094 FT.      L=5.33 FT.      L/W=2.55

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	QL (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	QN (CFS)	QL/QN
65A	Cavity	0.076	0.15	0.448	0.37	0.002	0.078	0.15	-	-	-
	Non-venting	0.075	0.15	0.448	0.37	0.002	0.077	0.15	4.33	0.206	2.18
66A	Cavity	0.152	0.30	1.241	0.90	0.013	0.165	0.33	-	-	-
67A	Non-venting	0.228	0.45	2.280	1.48	0.034	0.262	0.52	4.16	1.189	1.92
68A	Non-venting	0.304	0.60	3.090	1.82	0.052	0.356	0.70	3.95	1.776	1.74
69A	Non-venting	0.380	0.75	3.937	2.12	0.070	0.450	0.89	3.77	2.406	1.64
70A	Non-venting	0.455	0.90	4.885	2.43	0.092	0.547	1.08	3.65	3.113	1.57
71A	Non-venting	0.506	1.00	5.523	2.61	0.105	0.611	1.21	-	-	-



TABLE 26.—A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir

P=.512 FT.

w=1.208 FT.

w/P=2.36

ALPHA=24.5

BETA=45

W=1.75 FT.

L=5.33 FT.

L/W=3.048

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	Q (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C.L/W	C	c.L/W	c
72A	Cavity Non-venting	0.077	0.15	0.428	0.42	0.003	0.080	0.16	10.87	3.57	9.57	3.14
		0.076	0.15	0.428	0.42	0.003	0.079	0.15	11.08	3.63	9.76	3.20
73A	Cavity	0.154	0.30	1.294	1.11	0.019	0.173	0.34	10.26	3.37	10.23	3.35
74A	Cavity	0.230		2.339	1.80	0.050	0.280	0.55	9.00	2.95	10.13	3.32
75A	Non-venting	0.307	0.60	3.119	2.18	0.074	0.381	0.74	7.59	2.49	8.76	2.87
76A	Non-venting	0.384	0.75	4.098	2.61	0.106	0.490	0.96	6.83	2.24	8.22	2.70

TABLE 27.—A Discharge Coefficient of a Rounded Crested Inclined Labyrinth Weir

P=.512 FT.      w=1.208 FT.      w/P=2.36      ALPHA=24.5      BETA=45  
W=1.75 FT.      L=5.33 FT.      L/W=3.048

RUN NO.	TYPES OF VENTILATION	H (FT)	H/P	QL (CFS)	V (FPS)	$V^2/2G$ (FT)	$H+V^2/2G$ (FT)	$H_T/P$	C	QN (CFS)	QL/QN
72A	Cavity Non-venting	0.077	0.15	0.428	0.42	0.003	0.080	0.16	-	-	-
		0.076	0.15	0.428	0.42	0.003	0.079	0.15	4.33	0.177	2.42
73A	Cavity	0.154	0.30	1.294	1.11	0.019	0.173	0.34	-	-	-
74A	Cavity	0.230		2.339	1.80	0.050	0.280	0.55	4.12	1.088	2.15
75A	Non-venting	0.307	0.60	3.119	2.18	0.074	0.381	0.74	3.91	1.625	1.92
76A	Non-venting	0.384	0.75	4.098	2.61	0.106	0.490	0.96	3.72	2.254	1.82

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Dear Sir:

I am currently studying as a graduate student at the Utah State University. My major is Hydraulics Engineering, and the subject of my thesis is "A Discharge Coefficient of Labyrinth Weirs". I would like to put the following figures from your publications in my thesis:

1. Journal of the Hydraulic Division  
Vol. 97 NO. HY8. Aug. 1971  
figure 18 page 1250
2. Journal of the Hydraulic Division  
Vol. 96 NO. HY11. Nov. 1970  
figures 12 & 13 page 2351

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Mr. Ana Nian  
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Dear Mr. Nian:

Permission is granted for use in your thesis of the labyrinth spillway figures discussed during your telephone call. Specifically figures 1-16 and 1-17 from the paper "Hydraulic Design and Application of Labyrinth Spillways" published in the January 1984 USCOLD Proceedings and figures 2 and 3 from "Discharge Characteristics of Labyrinth Weirs" published in the August 1984 ASCE Hydraulics Division Water for Resource Development Conference Proceedings. It would be appreciated if you would reference the Bureau of Reclamation for use of these figures.

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Philip H. Burgi  
Chief, Hydraulics Branch