A Study of the Flow of Water Over Triangular Weirs and the Determination of Coefficients of Discharge for Small Heads

J. Milton Barrett

Utah State University

Follow this and additional works at: https://digitalcommons.usu.edu/etd

Part of the Civil and Environmental Engineering Commons

Recommended Citation
Utah Agricultural College,
Logan, Utah,
May 26, 1924.

Professor George D. Clyde

Acting Head of the Department of Irrigation
School of Agricultural Engineering
Utah Agricultural College.

Dear Sir:

In accordance with the rules and regulations of the Department of Irrigation and Drainage of the Utah Agricultural College, for the Degree of Bachelor of Science in Irrigation Engineering, I hereby submit a Thesis entitled: A Study of the Flow of Water over Triangular Weirs and the Determination of Coefficients of Discharge for Small Heads.

Respectfully Submitted,

Candidate for Degree of Bachelor Science in Irrigation Engineering.

By

J. Milton Barrett.

THESIS

Submitted in partial satisfaction of the requirements for the Degree of:

BACHELOR OF SCIENCE.

in

Irrigation Engineering

in the

SCHOOL OF AGRICULTURAL ENGINEERING

of

The Utah Agricultural College.

Approved

.................
CONTENTS.

Introduction .................................................. Page 1

CHAP. I

A Review of the Experimental Work Which has been done on Weirs. ........................................ 2

CHAP. II

Methods used in the Determination of Coefficients in the Formula Given. .............................. 7

Summary and Conclusions. ...................................... 13

References. ....................................................... 14
INTRODUCTION

The development of artesian and pumped wells as a source of irrigation water has created the need for a measuring device which will be accurate for small discharges. The thin edged weir has been accepted as one of the most accurate and desirable measuring devices for this work. Three types of weirs are now commonly used: the Rectangular, the Cipolletti, and the Triangular Notch weir. Of these types the triangular notch is probably best suited for small discharges (under 3 second feet). The various formulae for discharge over 0.2 Triangular weirs have been accurately determined for heads over two tenths of a foot. It is the purpose of this investigation to determine the coefficients for use in the general formula for discharge resulting from heads under 0.3 foot.
Most of the experimental work on weirs has been carried out with rectangular weirs. A brief summary of some of the more important experiments will be given here.

**Francis Experiments on Sharp Crested Weirs With Free Overfall.**

Elaborate investigations in the flow of water over weirs were made by James B. Francis at Lowell, Massachusetts during the period from 1847 to 1852. These were published in the Lowell Hydraulic Experiments.

From preliminary experiments on the effect of lateral contractions, Francis developed the general formula of discharge

\[ Q = 6(L - 1NH)H^k \]

where \( N \) number of end contractions and \( H \) the head.

To establish the value of \( C \), Francis made a series of 68 experiments, measuring the volume of flow in a large basin.

The weir used in these experiments was set at the end of an old lock chamber about 22.5 feet long having a plank bottom and masonry side walls.

The head was observed by two hook gages set in each side of the channel about six feet back from the weir crest.

The volume of flow was measured by the rise of water in a rectangular wooden basin into which the flow was directed by means of head gates when the desired rate was established. The basin had a capacity of about 12,000 cubic feet and was nearly filled during each experiment. A slight amount of leakage, and deflection of the sides occurred, but a correction was made for this in each case.
The duration of each experiment was measured on a marine chronometer. The beginning and end of each experiment was announced by an electric bell controlled automatically by the opening of the gates of the measuring tank. The duration of the individual experiments was from 182 to 822 seconds.

The extreme error in the observations used in the determination of the constant $C$ probably did not exceed one half of one per cent.

Francis fixed on 3.33 as the mean value of $C$ making the general formula: $Q = 3.33(L - 1NH)^{1/2}$ for discharge over a rectangular weir with contractions and no velocity of approach.

**Fteley & Stearn's Experiment on Sharp-Crested Weirs With Free Overfall.**

In 1877-79 investigations in the flow of water over weirs were carried out by Alphonse Fteley and Frederic P. Stearns. These experiments were made at the entrance of the Sudbury Aqueduct of the Boston Waterworks. These experiments not only confirmed the work of Francis but also contributed new data concerning velocity of approach and flow of water over weirs resulting from low heads.

The general formula of Fteley and Stearns determined from examination of the results of their experiments is:

$Q = 3.31LH^{1.007}L.$

The present practice largely follows the formula of Fteley and Stearns for heads from .07 to .50 foot, beyond this the Francis formula is used.
Investigations of Hamilton Smith Jr.

H. Smith reviewed and remomputed the work of Lesbros, Poncelet and Lebros, Francis, and Fteley and Stearns; together with 12 experiments of his own on a weir 2.6 feet long with end contractions. He developed the general formula:

\[ Q = c_2/3(2g)^{1/2}LH^{1/2} \]

In which \( c \) is a coefficient on the length of crest, the head, the number of end contractions.

Bazin Experiments On Sharp-Crested Weirs With Free Overfall.

The most notable contributions to the science of weir measurement are those of Henry Bazin beginning in 1866. Bazin's experiments were carried out on sharp-crested weirs with end contractions. The volume of discharge was measured in a concrete basin.

Bazin adopted the common formula: \( Q = mLh(2gh)^{1/2} \) where \( m \) is the coefficient of discharge and includes the effect of velocity of approach and all variable factors except the head.
WORK DONE ON TRIANGULAR NOTCH WEIR.

The general formula for the discharge of water over a triangular notch weir may be developed as follows:

\[ Q = C \sqrt{2gL(\frac{h-y}{h})} \]

Where:
- \( Q \) is the discharge.
- \( C \) is a constant.
- \( L \) is the width of the weir at water surface.
- \( l \) is the width of the weir at distance \( y \) from water surface.
- \( h \) is the depth of water above weir crest.
- \( y \) is the distance from water surface to any point.
- \( dy \) is the width of elementary strip distance \( y \) from water surface.

For any orifice the discharge \( Q = AV \) where \( A \) is the area of the Orifice and \( V \) is the velocity due the head on its center of gravity. \( V = \sqrt{2gy} \)

Then \( dq \) discharge through the elementary area \( ldy \)

\[ dq = C \frac{dy}{\sqrt{2gy}} \]

From similar triangles \( \frac{l}{L} = \frac{h-y}{h} \) or \( \frac{L}{h} = \frac{l}{h-y} \)

Substituting in equation \( -I- \)

\[ dq = C \frac{L(h-y)}{h} \frac{dy}{y} \]

Then integrating between the limits of \( y = 0 \) and \( y = h \) we get

\[ Q = C \sqrt{2g} \frac{L}{h} \frac{h^2}{15} \]

For 90 degree angle notch \( L = 2h \) substituting this in equation \( -II- \)

\[ Q = C \sqrt{2g} \frac{h^4}{15} \] or \( Q = Ch^\frac{4}{5} \).
Very little data is obtainable on the flow of water over triangular notch weirs. The few investigations carried out have been with 90 degree triangular notches. The following recommendations for coefficients for use in the general formula \( Q = bH^q \), have been given.

V.M. Cone in reporting the work done at the Fort Collins Laboratory by the Experiment Station of the U.S. Department of Agriculture and the Colorado Agricultural Experiment Station gives the formula as \( Q = 2.487H^{3.207} \) for heads from 0.2 to 1.35 feet.

Thompson of Great Britain as the result of his experiments with heads from 0.2 to 0.8 feet gives the formula \( Q = 2.53H^{2.6} \).

Barr's experiments give \( Q = 2.48H^{2.4} \).

University of Michigan experiments give \( Q = 2.52H^{2.6} \).
CHAP. II

METHODS USED IN DETERMINING COEFFICIENTS USED IN THE FORMULA GIVEN.

The investigations reported in this paper were carried out by the writer with the assistance of Professor G. D. Clyde Mr. Christensen, and Mr. Thurber. All of the experimental work was done in the Hydraulic Laboratory of the Utah Agricultural College. In these experiments the discharge over a 90 degree triangular notch weir was determined for low heads, between 0,1 and 0.3 foot.

Because of the small discharge used in these experiments the most accurate method for the measurement of the volume of water discharged was decided to be that of taking the weight of the water discharged in a given length of time, and dividing it by the weight of water taken as 62.5 pounds per cubic foot.

The arrangement of apparatus as used in these experiments is shown in plate I. The cross section of the channel of approach being so great as compared with the cross section of the weir the velocity of approach so nearly approached zero as to be neglected entirely. The weir is cut from a bulkhead of $\frac{1}{8}$" steel plate. Although the edges of the weir were not as sharp as desirable fairly good contraction was obtained except under the very low heads when a little clinging of the water to the crest was observed.

The removable spout was used to convey the water from the weir to the tank while the experiment was in progress, at other times it was removed allowing the water to waste. This
PLAN

SECTION

Sketch Showing Arrangement of Apparatus During Experiment
gave close control over the amount of water entering the tank in the time taken.

The water passing over the weir was caught as described in a circular metal tank. This tank was placed on a standard platform scale used to obtain the weight of the water discharged over the weir.

The time of each experiment was taken with a standard stop watch. The time of run for the individual experiments ranging from about 10 to 180 seconds.

The head on the weir was measured with the hook gage in the still well as shown. Owing to the impossibility of obtaining a perfectly steady flow it was found necessary to read the head at the beginning and the end of each run. As the variation in the two readings was small, not more than 0.05 inches the mean of the two readings could be taken as the constant head with very slight error.

The data collected in these experiments is presented in Table I and in graphic form on Plate II.

In plotting the data obtained, for convenience in calculating the coefficients, the logarithms of the values were plotted in place of the values themselves, or rather the values were plotted direct on logarithmically ruled paper, which gives the same result. The values when thus plotted fell approximately on a straight line. This should be so as can be seen from an examination of the logarithmic form of the general equation \( Q = CH^n \) which is the same as \( \log Q = n \log H + \log C \), which is essentially the same as the slope intercept equation of a straight line: \( y = mx + b \). In which \( m \) = the slope of the line
and \( b \) = the \( x \) intercept. The straight line obtained is shown in Plate II. The slope of the line was obtained from the coordinates of a point, being measured with the natural scale. The slope being the tangent of the angle the line makes with the abscissae. In this case the slope equals \( \frac{8.7}{3.8} \approx 2.289 \).

Due to error in projecting the line up to intersect the ordinate where the head equaled unity the constant \( C \) was obtained direct by solving the formula \( Q = CH^{2.27} \) for \( C \). From several calculations the average value of \( C = 2.76 \) was obtained.

From these investigations the general formula of \( Q = 2.76H^{2.27} \) was obtained for flow of water over 90 degree triangular notch weirs with full contractions under heads from 0.1 to 0.3 foot.
<table>
<thead>
<tr>
<th>HEAD IN INCHES</th>
<th>TIME in SECONDS</th>
<th>WEIGHT in POUNDS</th>
<th>QUANTITY in CU. FT.</th>
<th>Q in GALLONS</th>
<th>HEAD in FEET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Begin</td>
<td>End</td>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.10</td>
<td>3.10</td>
<td>3.10</td>
<td>10.8</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>3.06</td>
<td>3.06</td>
<td>3.06</td>
<td>11.4</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>3.02</td>
<td>3.04</td>
<td>3.04</td>
<td>11.2</td>
<td>80</td>
<td>1.882</td>
</tr>
<tr>
<td>3.01</td>
<td>3.01</td>
<td>3.01</td>
<td>11.8</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>2.20</td>
<td>2.14</td>
<td>2.17</td>
<td>24.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>2.04</td>
<td>2.02</td>
<td>2.03</td>
<td>25.6</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>2.01</td>
<td>1.98</td>
<td>1.995</td>
<td>28.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.94</td>
<td>1.89</td>
<td>1.915</td>
<td>32.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.90</td>
<td>1.81</td>
<td>1.805</td>
<td>35.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.79</td>
<td>1.74</td>
<td>1.765</td>
<td>38.8</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.73</td>
<td>1.70</td>
<td>1.715</td>
<td>42.8</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.70</td>
<td>1.69</td>
<td>1.695</td>
<td>42.8</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.66</td>
<td>1.64</td>
<td>1.65</td>
<td>45.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.63</td>
<td>1.60</td>
<td>1.615</td>
<td>47.4</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.50</td>
<td>1.59</td>
<td>1.595</td>
<td>49.3</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.58</td>
<td>1.58</td>
<td>1.58</td>
<td>50.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.53</td>
<td>1.49</td>
<td>1.51</td>
<td>56.2</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.47</td>
<td>1.40</td>
<td>1.435</td>
<td>62.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.37</td>
<td>1.30</td>
<td>1.335</td>
<td>71.2</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>86.0</td>
<td>80</td>
<td>1.282</td>
</tr>
<tr>
<td>1.20</td>
<td>1.15</td>
<td>1.15</td>
<td>103.0</td>
<td>80</td>
<td>1.282</td>
</tr>
</tbody>
</table>
Diagram for Determining Coefficients

In Formula $Q = CH^n$

Log $Q = \log C + n \log H$

$n = \frac{67}{3} = 2.289$

$C = 2.76$

$H =$ Head in Feet

$Q =$ Discharge in CFS
SUMMARY AND CONCLUSIONS.

From the above discussion the following conclusions can be drawn:

1- The discharge over 90 degree triangular notch weirs for heads from 0.1 to 0.5 foot do agree with the theoretical values obtained from the formulas given for the discharge of water over such weirs for larger heads.

2- The work done in these experiments justifies the presentation of the formula \( Q = 2.76H^{2.287} \) for use in calculating the flow of water over 90 degree triangular notch weirs for heads under 0.3 foot.

3- For practical use in the measurement of water a weir of this type, but having a thinner edged crest should be used. With this weir a smaller value of \( C \) in the above equation should be used, this value to be obtained by further experimental work.
REFERENCES

Merriman Mansfield A Treatise on Hydraulics.
Hughes & Safford Hydraulics
Lowell Hydraulic Experiments
King & Wisler Hydraulics.