Model-Based Properties of Earth's Protective Shield: Relating UT-Based Dependencies of the Open/Closed Boundary, Cutoff Latitude, and L-Shell Parameter with Polar Cap Absorption Events

David A. Smith
Utah State University

Follow this and additional works at: https://digitalcommons.usu.edu/etd

Part of the Physics Commons

Recommended Citation

This Dissertation is brought to you for free and open access by the Graduate Studies at DigitalCommons@USU. It has been accepted for inclusion in All Graduate Theses and Dissertations by an authorized administrator of DigitalCommons@USU. For more information, please contact digitalcommons@usu.edu.
MODEL-BASED PROPERTIES OF EARTH'S PROTECTIVE SHIELD: RELATING UT-BASED DEPENDENCIES OF THE OPEN/CLOSED BOUNDARY, CUTOFF LATITUDE, AND L-SHELL PARAMETER WITH POLAR CAP ABSORPTION EVENTS

by

David A. Smith

A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Physics

Approved:

Jan J. Sojka, Ph.D.  
Major Professor

David Peak, Ph.D.  
Committee Member

Bela Fejer, Ph.D.  
Committee Member

Michael Taylor, Ph.D.  
Committee Member

Jonathan Phillips, Ph.D.  
Committee Member

Richard S. Inouye, Ph.D.  
Vice Provost for Graduate Studies

UTAH STATE UNIVERSITY  
Logan, UT

2020
ABSTRACT

Model-based Properties of Earth's Protective Shield:
Relating UT-based Dependencies of the Open/Closed Boundary, Cutoff Latitude, and L-shell Parameter with Polar Cap Absorption Events

by

David A. Smith, Doctor of Philosophy
Utah State University, 2020

Major Professor: Dr. Jan J. Sojka
Department: Physics

Though the technology is decades old, high frequency radio continues to be an important method of communication. Due to the nature of high frequency radio propagation it is possible to transmit and receive signals over great distances. This has proven especially useful to the military, aviation, utility companies, etc.

High frequency radio is not without limitations. Sky waves are subject to varying ionospheric conditions, including radio wave absorption. Under certain conditions radio wave energy can be completely absorbed in the ionosphere. One specific absorption condition occurs in Earth's Polar Regions. So-called polar cap absorption events have been found to follow a well-defined pattern.

The polar cap region is defined by the open/closed boundary of the geomagnetic field. Polar cap absorption events are primarily caused by high energy protons of solar origin between about 1-20 MeV.
There remain critical unanswered questions regarding polar cap absorption events: What is their dependence on solar wind parameters? What is their impact on radio communications? Are there variability sensitivities related to the open/closed boundary and other parameters impacting these events? One goal of this research was to answer some of these questions.

Using the Tsyganenko model of the geomagnetic field, several studies were completed, examining model-based properties of several key characteristics of the geomagnetic field such as the open/closed boundary, the cutoff latitude, the L-shell parameter, and the penumbra. Another goal of this research was to determine possible areas of improvement regarding current methods of forecasting polar cap absorption events.

(222 pages)
PUBLIC ABSTRACT

Model-based Properties of Earth's Protective Shield:
Relating UT-based Dependencies of the Open/Closed Boundary, Cutoff Latitude, and L-shell Parameter with Polar Cap Absorption Events

by

David A. Smith

High frequency radio continues to be an important communications medium. For example, commercial airlines use high frequency radios as their primary communications mode during transpolar crossings. It has been estimated that over 7000 transpolar flights occur each year. Unfortunately, during geomagnetic storms high frequency communications can become unreliable, especially near Earth's Polar Regions.

Space weather forecasters are burdened with the responsibility of predicting how radio signals might be affected during geomagnetic storms and passing that important information on to commercial airlines, allowing them to adjust flight plans accordingly. Such adjustments can be costly, but are necessary to ensure safety of flight crews and passengers.

Currently, the state-of-the-art prediction tool is an empirical model that provides a qualitative analysis of current conditions. The goal of this dissertation was to investigate several important parameters that govern polar cap absorption events in hopes of improving the existing state-of-the-art.
All things bright and beautiful,
All creatures great and small,
All things wise and wonderful,
The Lord God made them all.

- Cecil Francis Alexander

The glory of God is intelligence, or,
in other words, light and truth.

- Joseph Smith, Jr.
For Shannon
You are the love of my life!
ACKNOWLEDGMENTS

As with any remarkable undertaking, an individual can go only so far on their own efforts. Family, friends, peers, and mentors provide support and encouragement, establish boundaries, and provide feedback along the way. Such has been my experience. To this great group of friends and family I offer my heartfelt thanks. Your confidence in my ability to see it through kept me going. This list includes my parents, my siblings, my children and their spouses, grandchildren (yes, grandchildren), and a host of extended family and friends. Thanks to each of you for your faith in me.

Similarly, I am indebted to the USU Physics Department faculty and staff. Karalee Ransom in particular was a veritable reservoir of information, encouragement, and guidance. Thanks Karalee.

Melanie Oldroyd, Darcie Bessinger, Sharon Pappas, and Vanessa Chambers all assisted in one way or another with manuscript preparation, editing, payroll, travel reimbursements, conference registrations, etc, etc, etc. Thank you!

Dr. Jan Sojka was gracious enough to serve as my PhD advisor and the chair of my Supervisory Committee. Not only is Dr. Sojka a brilliant physicist, he is a kind man. I will always be indebted to him for teaching me the ways of the scientist. I'm grateful for his insightful comments and counsel regarding my academic choices, our research, and my future. I'm grateful for the tactful manner in which he offered feedback and correction. Usually his corrections were accompanied by a wry smile and a wink, so the blow wasn't too horrible. Thanks Jan.
I was fortunate indeed to have 4 incredible individuals willing to serve on my Supervisory Committee: Drs. David Peak, Mike Taylor, Bela Fejer, and Jonathan Phillips. Each of them brought an insightful perspective as well as a wealth of knowledge and experience to my Committee. Over the past 5 years I've had numerous opportunities to discuss my research and my future with each of them. Fortunately for me, each was very willing to offer encouragement and support throughout my tenure as a graduate student. I'm grateful to each of them for that support. Thanks David, Mike, and Jonathan.

Dr. Fejer also had the distinction of being the instructor for two of my classes. One, heliophysics, was pretty fun. The other, mathematical methods of physics, wasn't so fun. Dr. Fejer was keenly aware of my struggles with the course. He was also very aware that I was working hard at it. He frequently stopped by my office to reassure and encourage me that I would make it. I will always be grateful for his support, encouragement and guidance. Thanks Bela.

Finally, I offer a few words of thanks to my sweetheart, Shannon. Shannon's love, support, encouragement, and hard work made this all possible. Period. Thanks Shannon.

I love you sweetheart.

- David Alan Smith
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>PUBLIC ABSTRACT</td>
<td>v</td>
</tr>
<tr>
<td>EPIGRAPH</td>
<td>vi</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>vii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiii</td>
</tr>
</tbody>
</table>

## CHAPTER

### I. INTRODUCTION AND BACKGROUND ................................................................. 1

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>The Solar Wind</td>
<td>4</td>
</tr>
<tr>
<td>The Geomagnetic Field</td>
<td>7</td>
</tr>
<tr>
<td>The Ionosphere</td>
<td>10</td>
</tr>
<tr>
<td>Importance of High Frequency Radio Communications</td>
<td>12</td>
</tr>
<tr>
<td>Radio Wave Propagation</td>
<td>15</td>
</tr>
<tr>
<td>High Frequency Radio Wave Absorption</td>
<td>19</td>
</tr>
<tr>
<td>Polar Cap Absorption Events</td>
<td>29</td>
</tr>
</tbody>
</table>

### II. MAGNETOSPHERIC PENETRATION .................................................................. 34

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>34</td>
</tr>
<tr>
<td>Energy and Rigidity</td>
<td>35</td>
</tr>
<tr>
<td>The L-shell Parameter</td>
<td>39</td>
</tr>
<tr>
<td>The Tsyganenko Model</td>
<td>40</td>
</tr>
<tr>
<td>The Open/Closed Boundary</td>
<td>44</td>
</tr>
<tr>
<td>The Cutoff Latitude</td>
<td>49</td>
</tr>
</tbody>
</table>

### III. THE L-SHELL PARAMETER ................................................................. 59

<table>
<thead>
<tr>
<th>Subsection</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>59</td>
</tr>
<tr>
<td>UT-Variability</td>
<td>63</td>
</tr>
</tbody>
</table>

### IV. MODEL-BASED PROPERTIES OF THE OPEN/CLOSED BOUNDARY ........................... 71
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Regions of the Ionosphere</td>
</tr>
<tr>
<td>4.1</td>
<td>T96 Geomagnetic Storm Parameters</td>
</tr>
<tr>
<td>5.1</td>
<td>Energy Levels Used in this Study</td>
</tr>
<tr>
<td>E.1</td>
<td>Typical S-unit Values</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>5</td>
</tr>
<tr>
<td>1.2</td>
<td>8</td>
</tr>
<tr>
<td>1.3</td>
<td>9</td>
</tr>
<tr>
<td>1.4</td>
<td>17</td>
</tr>
<tr>
<td>1.5</td>
<td>27</td>
</tr>
<tr>
<td>1.6</td>
<td>30</td>
</tr>
<tr>
<td>2.1</td>
<td>36</td>
</tr>
<tr>
<td>2.2</td>
<td>38</td>
</tr>
<tr>
<td>2.3</td>
<td>46</td>
</tr>
<tr>
<td>2.4</td>
<td>51</td>
</tr>
<tr>
<td>2.5</td>
<td>55</td>
</tr>
<tr>
<td>2.6</td>
<td>58</td>
</tr>
<tr>
<td>3.1</td>
<td>61</td>
</tr>
<tr>
<td>3.2</td>
<td>62</td>
</tr>
<tr>
<td>3.3</td>
<td>64</td>
</tr>
<tr>
<td>3.4</td>
<td>65</td>
</tr>
<tr>
<td>3.5</td>
<td>66</td>
</tr>
<tr>
<td>3.6</td>
<td>67</td>
</tr>
<tr>
<td>3.7</td>
<td>69</td>
</tr>
<tr>
<td>3.8</td>
<td>70</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES (Cont.)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>OCB Noon meridian cross section in GSM coordinates</td>
<td>75</td>
</tr>
<tr>
<td>4.2</td>
<td>Dayside OCB for three levels of geomagnetic activity</td>
<td>76</td>
</tr>
<tr>
<td>4.3</td>
<td>Universal time dependence of the OCB</td>
<td>78</td>
</tr>
<tr>
<td>4.4</td>
<td>Dayside polar projection for 4 specific UT hours</td>
<td>81</td>
</tr>
<tr>
<td>4.5</td>
<td>Polar projection of the dayside OCB minimum/maximum values</td>
<td>82</td>
</tr>
<tr>
<td>4.6</td>
<td>OPRT auroral energy flux</td>
<td>83</td>
</tr>
<tr>
<td>4.7</td>
<td>OCB comparison with BATS-R-US/RCM</td>
<td>85</td>
</tr>
<tr>
<td>4.8</td>
<td>Northern hemisphere Mercator projection in geographic coordinates</td>
<td>87</td>
</tr>
<tr>
<td>4.9</td>
<td>OCB at noon LT as function of geomagnetic conditions</td>
<td>89</td>
</tr>
<tr>
<td>5.1</td>
<td>A series of 5 escape shells</td>
<td>98</td>
</tr>
<tr>
<td>5.2</td>
<td>UT variation of the cutoff latitude</td>
<td>99</td>
</tr>
<tr>
<td>5.3</td>
<td>The T96 escape surface</td>
<td>101</td>
</tr>
<tr>
<td>5.4</td>
<td>Difference between proton cutoff latitude and OCB</td>
<td>104</td>
</tr>
<tr>
<td>5.5</td>
<td>Geomagnetic activity dependence of the proton cutoff geomagnetic latitude</td>
<td>105</td>
</tr>
<tr>
<td>5.6</td>
<td>Local time dependence of the proton cutoff geomagnetic latitude</td>
<td>107</td>
</tr>
<tr>
<td>5.7</td>
<td>Proton cutoff latitudes for 1.0 and 100 $MeV$ protons for quiet conditions</td>
<td>110</td>
</tr>
<tr>
<td>5.8</td>
<td>Proton cutoff latitude for 1.0 and 100 $MeV$ protons for moderate conditions</td>
<td>114</td>
</tr>
<tr>
<td>5.9</td>
<td>Proton cutoff latitude for 1.0 and 100 $MeV$ protons for severe conditions</td>
<td>115</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>5.10</td>
<td>Standard deviation for fixed-sun coordinate system</td>
<td>116</td>
</tr>
<tr>
<td>5.11</td>
<td>Standard deviation for fixed-earth coordinate system</td>
<td>117</td>
</tr>
<tr>
<td>5.12</td>
<td>Daily variation of proton cutoff energies along 260 deg E meridian</td>
<td>119</td>
</tr>
<tr>
<td>5.13</td>
<td>T96-generated daily variation of proton cutoff energies, 255 deg</td>
<td>120</td>
</tr>
<tr>
<td>5.14</td>
<td>T96-generated daily variation of proton cutoff energies, 75 deg</td>
<td>121</td>
</tr>
<tr>
<td>5.15</td>
<td>Geomagnetic cutoff latitude for various proton energies</td>
<td>122</td>
</tr>
<tr>
<td>5.16</td>
<td>Proton cutoff energy vs. <em>invariant latitude</em></td>
<td>124</td>
</tr>
<tr>
<td>5.17</td>
<td>T96-generated plot for proton cutoff energy</td>
<td>125</td>
</tr>
<tr>
<td>5.18</td>
<td>Comparison of average proton cutoff <em>invariant latitude</em></td>
<td>126</td>
</tr>
<tr>
<td>5.19</td>
<td>Standard deviation of T96 data used to generate Figure 5.18</td>
<td>127</td>
</tr>
<tr>
<td>5.20</td>
<td>Three-hour averaged cutoff rigidity contours</td>
<td>128</td>
</tr>
<tr>
<td>5.21</td>
<td>T96-generated averaged cutoff rigidity contours</td>
<td>129</td>
</tr>
<tr>
<td>5.22</td>
<td>Orbit-averaged cutoff invariant latitude as a function of time</td>
<td>131</td>
</tr>
<tr>
<td>5.23</td>
<td>Polar plot of individual cutoff crossings</td>
<td>133</td>
</tr>
<tr>
<td>5.24</td>
<td>Cutoff latitude for 31.6 and 56.2 <em>MeV</em> protons superimposed over Figure 5.23</td>
<td>134</td>
</tr>
<tr>
<td>5.25</td>
<td>Constant UT vs. constant LT for varying proton energies</td>
<td>136</td>
</tr>
<tr>
<td>5.26</td>
<td>100 <em>MeV</em> proton cutoff geomagnetic latitude presented in LT vs. UT format</td>
<td>138</td>
</tr>
<tr>
<td>5.27</td>
<td>1.0 <em>MeV</em> proton cutoff geomagnetic latitude presented in a LT vs. UT format</td>
<td>140</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES (Cont.)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Calculated cutoff rigidity for specific geographic locations</td>
<td>144</td>
</tr>
<tr>
<td>6.2</td>
<td>T96-generated penumbral width in degrees</td>
<td>145</td>
</tr>
<tr>
<td>6.3</td>
<td>Average penumbral width in degrees for 9 energy levels</td>
<td>148</td>
</tr>
<tr>
<td>6.4</td>
<td>UT average of penumbral width in degrees</td>
<td>150</td>
</tr>
<tr>
<td>6.5</td>
<td>Penumbral width in degrees shown in the LT vs. UT format</td>
<td>151</td>
</tr>
<tr>
<td>7.1</td>
<td>Illustration showing arbitrary flux for 1.0, 10, and 100 MeV protons</td>
<td>159</td>
</tr>
<tr>
<td>D.1</td>
<td>Illustration showing that absorption depends on distance</td>
<td>183</td>
</tr>
<tr>
<td>E.1</td>
<td>Comparison of absorption equations 1.19 and 1.21 from Chapter 1</td>
<td>186</td>
</tr>
<tr>
<td>E.2</td>
<td>Plasma frequency profile</td>
<td>187</td>
</tr>
<tr>
<td>E.3</td>
<td>High frequency radio wave absorption by frequency per region</td>
<td>188</td>
</tr>
<tr>
<td>E.4</td>
<td>Frequency/altitude dependence of the index of refraction</td>
<td>189</td>
</tr>
<tr>
<td>E.5</td>
<td>Comparison of approximated received signal strength</td>
<td>192</td>
</tr>
<tr>
<td>F.1</td>
<td>Polar plots of the relative position of the north magnetic pole, Greenwich meridian and sun in sun-earth coordinate system</td>
<td>195</td>
</tr>
<tr>
<td>F.2</td>
<td>Format to determine if a parameter has a favored ordering</td>
<td>197</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION AND BACKGROUND

Introduction

Though the technology is several decades old high frequency (HF) radio continues to be an important method of communication. This is due in part to the relative low cost of deploying HF communications systems when compared to the cost of maintaining a spaced-based system. In addition, due to the nature of HF radio propagation is it possible to send signals over great distances when cell phone, satellite, or line-of-sight communication modes are unavailable or unreliable. This has proven especially useful to the military, aviation, utility companies, etc.

HF radio is not without its limitations. Since sky waves propagate through the ionosphere, they are subject to varying ionospheric conditions. One important phenomenon that is prone to occur as conditions vary in the ionosphere is HF radio wave absorption. Under certain conditions HF radio wave energy can be absorbed in the ionosphere, causing a degradation of the signal to the point it can be completely lost (Davies, 1965; Lied, 1967).

One specific absorption condition that occurs generally in earth's polar cap regions is fittingly called polar cap absorption (PCA). PCA events have been found to follow a fairly well-defined pattern. The first well-documented PCA event occurred on 23 February 1956. Since that time much has been explored and learned regarding PCA events and their effect on HF radio wave propagation. It is well understood that the polar cap region is defined by the open/closed boundary (OCB) of the geomagnetic field and
that PCA events are primarily caused by high energy protons of solar origin between about 1-200 MeV (Sauer & Wilkinson, 2008), though it will be shown that 1-20 MeV protons are the most likely candidates (Patterson et al., 2001; Lied, 1967).

There remain critical unanswered questions regarding PCA events and their dependence on SW parameters, their impact on HF communications, and variability sensitivities of the OCB and other parameters impacting PCA events. The intent of this research is to answer some of these questions.

The structure of Chapter 1 will proceed as follows: basic background material regarding the solar wind and interplanetary magnetic field will be provided since it is via these systems that energetic protons arrive at earth. Next will be a discussion regarding earth's protective shield, the geomagnetic field. As the name implies, the geomagnetic field protects earth from the destructive effects of solar radiation. Under certain conditions, however, solar radiation in the form of energetic particles is able to reach earth. The following section regarding HF Radio wave absorption makes up the bulk of Chapter 1 since it was the topic of the Candidacy Exam which was part of the initial research of this dissertation. The region of earth's atmosphere known as the ionosphere will be introduced, as well as a description of its function relative to radio wave propagation. The final section will discuss polar cap absorption events.

The overarching narrative of this dissertation is to discuss one important parameter (the McIlwain L-Parameter) and three important characteristics of the geomagnetic field: the open/closed boundary, the energy cutoff latitude, and the penumbra region of the energy cutoff latitude. This will facilitate a better understanding
of certain time-dependent variabilities that may affect PCA events. Chapter 3 is devoted to the McIlwain L-Shell parameter; Chapter 4 includes a comprehensive discussion of the open/closed boundary of the geomagnetic field; Chapter 5 examines the energy cutoff latitude, and finally Chapter 6 investigates the so-called penumbral region of the energy cutoff latitude. Chapter 7 reviews the important findings of this dissertation work.
The solar wind (SW) is the primary medium responsible for communicating activity on the sun into the solar system, especially to the vicinity of earth (Hunsucker & Hargreaves, 2003). It is a fully-ionized plasma that carries the sun's magnetic field outward from the corona. Once a few solar radii away from the sun, this magnetic field is called the interplanetary magnetic field (IMF). The outflow of the SW is primarily driven by expansion of the solar corona and flows radially from the sun. Due to the weakness of the IMF and the rotation of the sun, the IMF spirals away from the sun (Lied, 1967). This has been called the garden-hose or sprinkler effect (Hunsucker & Hargreaves, 2003). Imagine shooting a continuous jet of water out radially while standing motionless. The water jet moves away, outwardly, in a straight line. However, if you begin to spin the water jet appears to spiral out away from you and behind you, though each individual drop of water moves away from you radially. Figure 1.1 demonstrates this effect, showing the sun at the center and the smaller earth at the outer edge in the solar equatorial plane. The arrowed lines moving away from the sun radially represent the solar wind flow while the black lines spiraling away from the sun represent the IMF. Since the SW plasma is an excellent electric conductor and since the IMF is a fairly weak magnetic field, the IMF is "frozen in" to the SW and dragged by the SW out into interplanetary space. At earth the IMF comes in at an angle of roughly 45 degrees (Hunsucker & Hargreaves, 2003).
Figure 1.1: Illustration of the radial flow of the solar wind. Shown also is the spiraling effect of the interplanetary magnetic field. The sun is at the center, with earth on the outer circle.

The SW is composed of solar matter, mostly protons and electrons, though there are other positive ion species present in small numbers. Protons make up about 95% of the positive particles in the SW. However, since the number of electrons is about equal to the number of positive ions the SW is electrically neutral. Some consequences of the SW seen from earth are the northern and southern lights (aurora borealis and aurora australis) and the radially anti-sunward direction of comet tails. The SW is also very important in solar-terrestrial interaction, especially in the behavior of the high-latitude ionosphere (Hunsucker & Hargreaves, 2003). This interaction depends in great measure
on the IMF. The SW is highly variable both spatially and temporally. As the SW flows radially outward from the sun, speeds can range from 200 up to 800 km/s or more as measured at earth. Furthermore, the typical density is about $7 \times 10^6$ ions per cubic meter and a typical temperature is about $4 \times 10^4$ K.

The SW is an important transport vehicle of solar momentum and energy to the near-earth environment. Momentum and energy are transferred mainly via energetic particles from the sun, mostly protons. This continuous stream of solar plasma can at times be significantly perturbed. Coronal mass ejections (CME), for example, inject great quantities of energetic matter as well as electromagnetic energy into space via the SW. When this energy reaches earth geomagnetic storms can develop which may wreak havoc with earth-bound and space-based communications systems. This can be caused by energetic particles of solar origin making their way through earth's protective shield, the geomagnetic field.
The Geomagnetic Field

Due to the so-called geodynamo effect, earth is surrounded by a magnetic field: the geomagnetic field (Fowler, 2005). The geomagnetic field links the IMF with the upper atmosphere and ionosphere and affects the trajectory of energetic particles emitted from the sun. It also acts as a "great accumulator" of solar wind energy (Tsyganenko & Andreeva, 2019). If earth were located far away from any other influences this magnetic field would behave similar to a simple bar magnetic as shown in Figure 1.2, with field lines leaving via the South Pole, reentering via the North Pole. Furthermore, this hypothetical dipole field would be uniform and would extend to infinity. But earth is not in an isolated vacuum, it is embedded within the SW.

Due to the pressure exerted by the SW, the geomagnetic field is constrained to a region called the magnetosphere (Lied, 1967). Furthermore, within the magnetosphere geomagnetic field lines are deformed such that field lines on the sunward side are compressed, field lines on the anti-sunward side are greatly elongated, and field lines along the side can be folded back (Hunsucker & Hargreaves, 2003). A useful image is that of a pebble in a small stream.

Water flows around the pebble in much the same way as the SW flows around the magnetosphere. In front of the pebble waves can build up creating a turbulent region, or bow shock. Behind the pebble a region of calm water, an eddy, forms.
Figure 1.2: Illustration of simplified bar magnet. If the earth were isolated in space far from any other influences, the geomagnetic field would behave similar to a simple bar magnet with a dipolar field.

Depending on the speed and viscosity of the flow, the up-river turbulence and down-river eddies can vary in size. In similar manner, the topology of the magnetosphere changes in response to changes in the SW speed and density as well as the orientation of the IMF.

Figure 1.3 provides an illustration of the geomagnetic field and illustrates several important regions and boundaries. First, notice that although the magnetosphere has an irregular shape when compared to a dipole (compressed on the dayside, elongated into the magnetotail on the nightside), for the most part field lines are closed. This means that each field line begins and ends at some point on earth. Generally one end is connected to the northern hemisphere, the other to the southern hemisphere. These beginning/ending regions are frequently called foot points. However, there exist two important regions,
one in the northern another in the southern polar hemispheres, where field lines have only one foot point connected to earth. This region is called the cusp, the polar cusp, or polar cap region (Newell et al., 2006). Poleward of this cusp region field lines are considered open since only one foot point connects to earth and the other connects directly to the magnetopause. The magnetopause is the area of the magnetosphere where the geomagnetic field and the solar wind interact (Hunsucker & Hargreaves, 2003). Hence, an area exists near the cusp and poleward where solar energetic protons have direct access to earth's upper atmosphere (Lied, 1967). It is in this high-latitude region where solar energetic protons are able to access the ionosphere.

**Figure 1.3**: Simplified rendering of the geomagnetic field. The sun is to the left. Hence the solar wind flow is from left to right. Several important regions are the polar cusp, magnetopause, and the magnetotail. (https://image.gsfc.nasa.gov/poetry/ask/a10019.html)
The Ionosphere

The ionosphere is defined as a region of Earth's atmosphere that has a high enough concentration of free electrons such that it affects radio waves. Because of the very light mass of the free electrons, they are easily moved by the oscillating electric field of the passing radio wave (Lied, 1967). The distribution of these free electrons is not uniform. Rather, it is height dependent.

Earth's neutral atmosphere is divided into regions based on temperature profile. The ionosphere, however, is subdivided based on electron density as shown in Table 1.1. In both cases the critical parameter can be defined as a function of height. The neutral atmosphere consists mostly of molecular nitrogen and oxygen. However, due to photodissociation, significant amounts of atomic nitrogen and oxygen also exist (Nicolet & Aikin, 1960; Hunsucker & Hargreaves, 2003; Schunk & Nagy, 2009).

Table 1.1: Regions of the Ionosphere

<table>
<thead>
<tr>
<th>Altitude Range</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 – 90 km</td>
<td>D-region</td>
</tr>
<tr>
<td>90 – 150 km</td>
<td>E-region</td>
</tr>
<tr>
<td>150 – 500 km</td>
<td>F-region</td>
</tr>
</tbody>
</table>

A key characteristic of the ionosphere is the time scale of the production and loss of free electrons. In the upper regions of the ionosphere this time scale is generally relatively long, lasting many minutes to several hours. Hence, the free electrons are able to absorb energy from the transmitted radio wave and reradiate that energy with little loss. At lower altitudes, however, this time scale may be very short, frequently measured
in seconds or minutes. This leads to a phenomena called HF radio wave absorption. Evidently, the absorption of radio wave energy is dependent on the product of the density of free electrons and the rate at which these free electrons collide with atmospheric neutral particles. Hence, the electron density is a key characteristic in determining the viability of a specific HF frequency for long distance communications and is height-dependent (Sauer & Wilkinson, 2008).
Importance of High Frequency Radio Communications

The high frequency (HF) radio spectrum continues to be a useful communications medium. Air-to-ground aviation communication, military and governmental communication systems, amateur (ham) radio, short-wave broadcasting, and maritime services represent a few users of the HF spectrum. This is due, in part, to the ability of HF signals to propagate for long distance via the ionosphere. A difficulty, then, would be the changing characteristics of the ionosphere over relatively short time scales. In addition, certain conditions can result in total or partial disruption of radio wave propagation via the ionosphere rendering users unable to communicate via regular HF channels. How do these "certain conditions" depend on SW parameters? And what impact do these changing conditions have on HF communications?

The ionosphere, and hence the number of free electrons, depends heavily on the sun. Therefore, variations are expected throughout the day, depending in large measure on the Local Time (LT) of the area in question. This LT variation is also called a diurnal or daily variation. Hence, the local time of day (LT) is an important consideration in the production and loss of free electrons. Furthermore, any other disturbance that increases or decreases the free electron density on short or long timescales will alter the ionosphere. Solar flares and CME's are two such disturbances.

A primary goal of this dissertation research is to improve overall understanding of PCA events. The following section touches on the mechanisms of HF radio wave absorption in the ionosphere and has two primary focuses: a review of HF radio wave propagation and absorption in the ionosphere an introduction to PCA events.
The first goal is accomplished by reviewing two primary sources: Davies (1965) and Lied (1967). Though these references may seem somewhat dated, they provide a solid basis for this review. As an example, Hunsucker & Hargreaves (2003) cover radio wave propagation through the ionosphere. In their discussion regarding HF absorption they use the same equations as Davies (1965) and Lied (1967). Furthermore, many of their listed references are from the same time period. Lied (1967) provides a qualitative description of absorption, focusing initially on a geometric optics approach to lay the groundwork for an understanding of HF radio wave propagation and absorption. Davies (1965) provides a mathematical approach, providing tie-in with wave theory, the quasi-longitudinal (QL) approach, and Maxwell's equations.

The geometric approach is based on the index of refraction of weakly ionized plasmas. The mathematical approach is based on the absorption coefficient as derived from electrodynamics. Both of these approaches have at their heart a reliance on the altitude-dependence of the electron density. As will be shown, the index of refraction and the absorption coefficient depend on electron number density.

This section also relies heavily on Bain and Harrison (1972) and their D-region model as a starting point for understanding the application of absorption and to provide a suitable D-region model. A useful FORTRAN code was developed to calculate the different parameters and determine absorption under varying situations. In addition, Rawer (1952) provided solid underpinnings to assist in understanding field strength at a distant location and providing application to the theoretical basis; e.g. what signal
strength might a ham radio operator expect at the receiving end of an HF signal after passing through the ionosphere?
Radio Wave Propagation

Sky Waves

Radio waves generally propagate in two ways: ground waves and sky waves. ARRL (1991) provides an excellent definition for ground waves as any wave that stays close to the Earth, reaching the receiving point without leaving Earth's lower atmosphere. Ground waves generally provide line-of-sight communication. Sky waves, on the other hand, usually leave Earth's lower atmosphere. Sky waves allow radio signals to travel great distances, enabling world-wide communication via the ionosphere. As described previously, the ionosphere is that area of Earth's atmosphere that lies between about 50 – 1000 km in altitude. What makes the ionosphere extraordinary is the relatively high number of free electrons that exist within this weakly-ionized region.

Geometric Optic Approach

One approach to understanding qualitatively how radio waves propagate through the ionosphere is geometric optics (Lied, 1967). Snell's Law, also known as the law of refraction, is shown to be,

\[ \mu_1 \sin(\theta_1) = \mu_2 \sin(\theta_2) \]  

Here \( \mu_1 \) is the index of refraction of the initial medium, \( \mu_2 \) is the index of refraction of the new medium, \( \theta_1 \) is the angle of incidence, and \( \theta_2 \) is the refracted angle. In introductory physics courses the initial focus of Snell's Law is on macroscopic systems and solids where the index of refraction of the various media is always greater than unity. It can be shown, however, that the index of refraction of a plasma can be less than unity.
Since the ionosphere is a weakly-ionized plasma, it is assumed that the index of refraction of the ionosphere can be less than unity. The importance of this assumption will be made evident.

As illustrated in Figure 1.4, it can be shown that as monochromatic light passes from a medium with a lower index of refraction (initial medium) into a medium with a higher index of refraction (new medium) such that \( \mu_1 < \mu_2 \), a light ray with an initial angle of incidence will be refracted, or bent, toward the normal (red line normal to surface) causing a small refracted angle. In the opposite case \( \mu_1 > \mu_2 \) it is expected that the ray to be refracted, or bent, away from the normal. This would certainly be the case if the light ray was moving from air \((\mu \sim 1.0)\) into the ionospheric plasma \((\mu < 1)\).

The index of refraction can be defined mathematically in several ways. One is in terms of the phase velocity of a wave through the medium versus the speed of light in a vacuum,

\[
\mu = \frac{c}{v_p} \quad (1.2)
\]

Another definition of the index of refraction is given in terms of the physical properties of the medium,

\[
\mu = \sqrt{\varepsilon_r} \quad (1.3)
\]

where \( \varepsilon_r \) is defined as the dielectric constant of the medium. Furthermore, the dielectric
constant of a plasma can be defined in terms of the plasma frequency (Jackson, 1999; Griffiths, 2013)

$$
\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}
$$

where the angular frequency can be defined in terms of the radio wave frequency, \( \omega = 2\pi f \) and the plasma frequency as,

$$
\omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m_e}
$$

Here \( n_e \) is defined as the electron number density, \( e \) as the elementary charge, \( \varepsilon_0 \) as the permittivity of free space, and \( m_e \) as the electron mass. Recasting the index of refraction in terms of the plasma frequency,
Then putting equation 1.7 in terms of the radio wave frequency it is evident that,

\[ \mu = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \]  

\[ \Rightarrow \mu = \sqrt{1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2}} \]  

Then putting equation 1.7 in terms of the radio wave frequency it is evident that,

\[ \mu = \sqrt{1 - \frac{n_e e^2}{4\pi^2 \varepsilon_0 m \omega^2}} \]  

showing that the index of refraction in a plasma depends on electron density and inversely on the square of the radio wave frequency. Hence, as the electron density increases, the index of refraction decreases, as expected. Envision the ionosphere as a region of stratified layers. As altitude increases, so does the electron density. At each boundary between these stratified layers one would expect the ray to be refracted more and more away from the normal (toward the horizontal). Eventually the ray path becomes horizontal and the ray will bend over and return to earth by a symmetrical path (Lied, 1967). This is the crux of the geometrical argument. A medium wherein the index of refraction is frequency-dependent is considered a dispersive media (Jackson, 1999; Lied, 1967).
High Frequency Radio Wave Absorption

Absorption Equation

Davies (1965) provides an excellent starting point for an investigation of high frequency radio wave absorption with a general equation for total system loss in decibels (dB),

$$L_s = L_{ta} + L_{tp} - G_t + L_p + L_{rp} - G_r + L_{ra}$$  \hspace{1cm} (1.9)

where $L_s$ is the total system loss, $L_{ta}$ the ohmic loss in the transmitting antenna system, $L_{tp}$ any polarization mismatch of the transmitting antenna, $G_t$ is the transmitting antenna gain, $L_p$ the path loss, $L_{rp}$ is the polarization mismatch of receiving antenna, $G_r$ the gain of the receiving antenna, and $L_{ra}$ the ohmic loss in the receiving antenna system. Of the several components contributing to system loss, the one relevant to this discussion is path loss ($L_p$). Davies (1965) defines the total path loss in $dB$ as the sum of three path loss contributors,

$$L_p = L_d + L_a + L_f$$  \hspace{1cm} (1.10)

where $L_d$ is the path loss due to distance, $L_a$ is path loss due to absorption in the ionosphere, and $L_f$ is path loss due to focusing and/or defocusing. Of the three, path loss due to absorption in the ionosphere is germane to this discussion. Generally the absorption term can be described by,

$$L_a = 10 \log \left( \frac{p_r}{p_u} \right) = 20 \log (\rho)$$  \hspace{1cm} (1.11)

Here $p_r$ is defined as the actual power received, $p_u$ as the power that would have been received in the absence of absorption, and $\rho$ as the effective amplitude reflection.
coefficient. Davies (1965) defines this term qualitatively as the ratio of the square of the amplitude of a wave which is reflected once in the ionosphere to the amplitude which would have been received in the absence of dissipative attenuation. Davies (1965) shows this to be,

\[ \frac{l}{l_0'} = e^{-\int \kappa \, ds} \]  \hspace{1cm} (1.12)

Here, \( l \) is the amplitude of a wave which is reflected once in the ionosphere, \( l_0' \) is the amplitude which would have been received in the absence of dissipative attenuation, and \( \kappa \) is the absorption coefficient. Recasting this equation in terms of the effective amplitude reflection coefficient shows that,

\[ \rho = e^{-\int \kappa \, ds} \]  \hspace{1cm} (1.13)

\[ \Rightarrow \ln(\rho) = -\int \kappa \, ds \]  \hspace{1cm} (1.14)

From the rules of logarithms,

\[ \log(x) = \frac{\ln(x)}{\ln(10)} \]  \hspace{1cm} (1.15)

showing that,

\[ L_a = -8.69 \int \kappa \, ds \]  \hspace{1cm} (1.16)

The units for the absorption coefficient as defined by Davies (1965) are nepers per meter. As there are about 8.69 decibels per neper it is apparent that the absorption equation is now in units of \( dB \) per meter. The integral will return units of distance so we expect the above equation to yield absorption in \( dB \). The absorption coefficient can be defined mathematically in several ways, the particular equation depending generally on the type of absorption affecting the radio wave as well as the relation between the ionospheric plasma frequency and the collision frequency.
Types of Absorption

Lied (1967) and Davies (1965) describe two basic types of absorption: deviative absorption and non-deviative absorption.

Deviative absorption

Deviative absorption occurs when the frequency of the incoming radio wave is near the plasma frequency of the local ionosphere \((f \sim f_p)\). In this case the refractive index will be small and the wave will propagate slowly through the region. This can be understood by recalling that the group velocity of a wave propagating through the ionosphere is,

\[
v_g = c\mu\quad(1.17)
\]

Hence, the ray path of the radio wave may be appreciably deviated. Furthermore, when the index of refraction becomes zero, the group velocity becomes zero and the wave is reflected (Lied, 1967). A review of phase velocity and group velocity is discussed in Appendix A. The reflection point becomes an important consideration at the lower end of the HF spectrum.

Non-deviative absorption

Non-deviative absorption occurs when the frequency of the incoming radio wave is greater than the plasma frequency of the local ionosphere \((f > f_p)\). In this case the refractive index will be near (but less than) unity and the wave will move through the region at about the speed of light in vacuum. Hence, the ray path of the radio wave is not
appreciably deviated. It will be shown that within the D-region, non-deviative absorption dominates.

**Absorption Coefficient**

Benson (1964) and Davies (1965) define the absorption coefficient for high frequency absorption using the quasi-longitudinal approximation,

\[
\kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{1}{\mu} \right) \left( \frac{n_e \nu}{\omega^2 + \nu^2} \right)
\]  

(1.18)

in units of nepers per meter. A detailed derivation is found in Appendix B. Equation 1.18 introduces \( \nu \) as collision frequency. It will be shown that within the D-region \( \mu \sim 1 \). Therefore, it follows that for HF signals within the D-region,

\[
L_a = -8.69 \left( \frac{e^2}{2\varepsilon_0 mc} \right) \int \frac{n_e \nu}{\omega^2 + \nu^2} \, ds
\]  

(1.19)

**Electron Density**

Electron density \( (n_e) \) is important because the electron interactions allow radio waves to propagate through the ionosphere (Saur & Wilkinson, 2008). Specifically, it is the free electrons which, due to their light mass, can be moved in resonance with the oscillating electric field of the radio wave and thus control the refracting properties of the ionized medium (Lied, 1967).

Besides free electrons there are ions and neutral species in the atmosphere. It is possible, in fact quite likely, that as free electrons move about they will collide with ions and/or neutrals. A key factor in radio wave propagation through the ionosphere is the time between these collisions. Each time an electron collides with an ion or neutral
species it gives up some of the energy it absorbed from the incoming radio wave. Since neutral species are much more numerous in the lower regions of the ionosphere there is a much, much shorter time period between collisions at lower altitudes. Hence, electrons give up their energy much more quickly. Furthermore, electrons may recombine with certain ions, reducing the number of free electrons. Free electrons are produced as ionization takes place and free electrons are lost as recombination takes place. Important sources of ionization energy come from sunlight (solar radiation, well understood), auroral particles (mostly electrons, fairly well understood), and energetic particles of solar origin (mostly protons, least understood and the topic of this research). Free electrons gain energy as they are moved by incoming radio waves but they lose energy as they collide with larger ions and neutrals. It is this continuous production and loss of free electrons and re-radiation of radio wave energy that creates such havoc in the D-region.

A particularly important neutral species in the D-region is nitric oxide (NO), though NO is considered a minor species in the atmosphere overall. Within the D-region NO is ionized by extreme ultraviolet solar radiation (Nicolet & Aikin, 1960; Liu, et al., 2004). Specifically,

\[ NO + h\nu \rightarrow NO^+ + e^- \quad (1.20) \]

Here \( h \) is Planck's constant \( \sim 4.14 \times 10^{-15} \text{eV} \cdot \text{s} \) and \( \nu \) is the photon frequency (about \( 2.2 \times 10^{15} \text{Hz} \) for Lyman-\( \alpha \) during non-storm periods). Evidently, during normal conditions, a majority of free electrons in the D-region are produced via Lyman-\( \alpha \) solar radiation (Nicolet & Aikin, 1960). During solar flares hard X-rays can significantly increase ionizations rates. Likewise, during coronal mass ejections energetic protons of
solar origin can significantly increase ionization rates (Liu et al., 2004; Handzo et al., 2014).

As mentioned previously, electrons are lost via recombination. One specific type of fast recombination is dissociative recombination,

\[ NO^+ + e^- \rightarrow N + O \] (1.21)

Here, a nitric oxide cation combines with an electron to produce nitrogen and oxygen atoms. Thus, electrons and ions are continually being produced and lost. Under normal conditions the number density of electrons at a particular altitude remains fairly consistent. However, there are significant diurnal and seasonal changes. The electron density is also sensitive to the 11-year solar cycle.

Also, energetic particles, mostly protons, emitted by the sun with energies ranging from 1-100 MeV have been shown to have sufficient energy to ionize neutral atmospheric constituents, enhancing the production of free electrons. When a significant number of solar energetic protons (SEP) are measured, as typically occurs during a CME, it is referred to as an SEP event. In fact, SEP's are a major contributor to enhanced ionization, especially poleward of the cusp region. In Chapter 6 SEP events and the energy ranges most likely to enhance free electron production within the D-region of the ionosphere are discussed.

One difficulty that arises in trying to forecast HF radio wave propagation via polar paths is the dynamic nature of the ionosphere, especially in the high-latitude region above about 60 degrees. This is due in part to the ease with which SEP's are able to penetrate earth's protective shield in this region. Electromagnetic effects due to solar
flares can be extreme, but are generally short-lived, lasting several minutes to a few hours. Effects due to SEP events can last for many days. Since SEP's rely on the topology of the geomagnetic field, an important consideration in forecasting their effect on propagation would be the changing structure of the geomagnetic field.

Hence, understanding free electron production and loss processes within the D-region are a critical factor in understanding and predicting radio wave absorption.

**Collision Frequency**

As the free electrons collide with neutral particles they lose energy. As they lose energy the intensity of the radio wave is reduced. This is the crux of high frequency absorption in the D-region. From about $50 - 100 \ km$ the electron density range is about $10^6 - 10^{11} \ m^{-3}$. Within that same altitude range the total density of all neutrals is on the order of $10^{18} - 10^{22} \ m^{-3}$. Earlier the assumption was made that the number of ions is about equal to the number of electrons, so the ion density is on the order of $10^6 - 10^{11} \ m^{-3}$ as well. The most abundant neutrals in the D-region are $N_2$ and $O_2$ (Hunsucker & Hargreaves, 2003) but as discussed earlier, $NO$ is an important neutral since Lyman-$\alpha$ photons are able to penetrate down to this altitude, making $NO^+$ a critical ion (Zolesi & Cander, 2014). When considering the total collision frequency is it important to account for electron-ion and electron-neutral collisions? The *total electron* collision frequency can be given as,

$$\nu_e = \nu_{el} + \nu_{en}$$

(1.22)
Here $\nu_e$, $\nu_{el}$, and $\nu_{en}$ are the total electron, electron-ion, and electron-neutral collision frequencies. Kelley (1989) shows that the electron-ion collision frequency is given by,

$$\nu_{el} = [34 + 4.18 \ln(T_e^3 / n_e)]n_e T_e^{-3/2} \text{ s}^{-1} \quad (1.23)$$

Here $T_e$ is the electron temperature. Evidently, under most conditions at D-region altitudes the electron temperature is about equal to the neutral and ion temps (personal email discussion with Dr. Vince Eccles). Hence, the electron-neutral collision frequency is given by,

$$\nu_{en} = (5.4 \times 10^{-10})n_n \sqrt{T_e} \text{ s}^{-1} \quad (1.24)$$

Here $n_n$ is the neutral number density. An example may be helpful: at an altitude of 100 km,

$$n_e \sim 1.0 \times 10^5 \text{ cm}^{-3}$$
$$n_n \sim 6.0 \times 10^{12} \text{ cm}^{-3}$$
$$T_e \sim 208 \text{ K}$$

Using a rough estimate of the electron collision frequencies,

$$\nu_{el} = [34 + 4.18 \ln(T_e^3 / n_e)]n_e T_e^{-3/2}$$

$$\Rightarrow \nu_{el} \sim [34 + 4.18 \ln(208^3 / 1.0 \times 10^5)](1.0 \times 10^5)(208)^{-3/2}$$

$$\Rightarrow \nu_{el} \sim 1.8 \times 10^3 \text{ s}^{-1} \quad (1.25)$$

$$\nu_{en} = (5.4 \times 10^{-10})n_n \sqrt{T_e} \text{ s}^{-1}$$

$$\Rightarrow \nu_{en} = (5.4 \times 10^{-10})(6 \times 10^{12})\sqrt{208}$$

$$\Rightarrow \nu_{en} \sim 4.7 \times 10^4 \text{ s}^{-1} \quad (1.26)$$
The example shows that $\nu_{en} \gg \nu_{ei}$. Hence, to good approximation it is reasonable to neglect the electron-ion collisions. Notice from Figure 1.5 that the electron collision rate decreases with increasing altitude. Using only electron-neutral collisions one can recast equation 1.19, substituting for collision frequency,

$$L_a = \frac{-1.26 \times 10^{-15}}{\sin(a) f^2} \int_{h_0}^{h_f} n_e(h) n_n(h) \sqrt{T_e(h)} \, dh$$  \hspace{1cm} (1.27)$$

Equation 1.27 allows a determination of the absorption of high frequency radio waves propagating through the D-region as a function of height where $h_0 \sim 50 \text{ km}$ and $h_f \sim 90 \text{ km}$.

**Figure 1.5:** Simplified illustration of electron density and collision frequency. Density ($N$) and frequency ($\nu$) are shown as functions of altitude. Under normal conditions the product ($N \cdot \nu$) is maximum in the D-region. Image from Lied (1967), under the NATO fair use policy, https://www.nato.int/cps/en/natohq/79511.htm
An extremely important consideration is the product $[n_e \cdot v]$ from equation 1.19. Notice from Figure 1.5 that the product $[n_e \cdot v]$ reaches a maximum at the upper boundary of the D-region. Since absorption is proportional to the product $[n_e \cdot v_{en}]$ under quiescent conditions we expect a maximum in absorption to occur around 85 – 95 km. A key assertion made by Nicolet & Aikin (1960) was that the D-region could in essence be "pushed down" due to solar flare emissions. In similar manner, it is expected that SEP events, independent of solar flares, could have a similar effect on the D-region, especially in high-latitude regions. Chapter 5 includes a discussion of the cusp region and how it is influenced by changing geomagnetic conditions due, in part, to SEP events. Furthermore, Chapter 6 presents information regarding the "push-down" effect discussed by Nicolet & Aikin (1960). An interesting consideration is that SEP events, though generally linked to flares and CME’s, can occur independently. And, as will be discussed later, SEP events have been shown to last for days. Hence, the effects of SEP events on the D-region can be of significant duration.
Polar Cap Absorption Events

In the section regarding HF radio wave absorption it was shown that most absorption is non-deviative in nature and takes place within the D-region of the ionosphere. It was also shown that absorption within the D-region is proportional to the product of the electron number density and the collision frequency. Under quiescent conditions most ionization in the D-region is the result of Lyman-\(\alpha\) radiation emitted from the sun. Therefore, during quiescent conditions HF radio wave absorption in the D-region is primarily a daytime effect.

SEPs have direct access to earth's polar regions via open field lines of the geomagnetic field in the cusp regions. SEPs of varying energies are able to penetrate to different depths of earth's atmosphere. This penetration depth, also known as the optical depth, increases as proton energy increases. Figure 1.6, based on data from Lied (1967), illustrates the penetration depth of SEPs with energies ranging from 1-100 MeV. In Figure 1.6 the \textit{x-axis} is the log scale of proton energy, the \textit{y-axis} is altitude in km. The shaded area represents the penetration depth of 1-20 MeV protons within the D-region. Notice that 100 MeV protons typically penetrate beyond (below) the D-region. Patterson et al., (2001) notes that most absorption occurs between the altitudes of 40-70 km and is most strongly affected by SEPs with energy \(\sim 20\) MeV, which show a penetration depth of about 62 km. Hence, SEPs with energies of 1-20 MeV not only penetrate to the D-region, they penetrate to a specific area most responsible for increased ionization and, therefore, absorption (Patterson et al., 2001; Lied, 1967).
Figure 1.6: Proton penetration depth as function of proton energy. Protons with energy within the 1-20 MeV regime penetrate to altitudes of about 90 down to roughly 62 km. This figure is based on data obtained from Lied (1967).

Thus, increased ionization in the D-region can be brought about by an increase in solar radiation within the required wavelength and/or by an increase in SEPs. Increased ionization via electromagnetic radiation generally occurs on the dayside of earth since the electromagnetic radiation moves at the speed of light and typically arrives within 8-10 minutes of the emission. SEP emissions on the other hand, can take several hours to
reach earth due to their sub-light speed and their helical path along the field lines of the IMF. Plus, since their primary access point is via the polar cusp region, the increased D-region ionization due to SEPs frequently takes place in earth's polar regions. Hence, events due to SEPs causing increased ionization, and therefore absorption, in earth's polar regions are called polar cap absorption events (PCA).

The first well-documented PCA event occurred on 23 February 1956 (Hunsucker & Hargreaves, 2003; Patterson et al., 2001; Lied, 1967; Bailey, 1964). Since that time much has been learned and explored regarding PCA events and their effect on high frequency (HF) radio wave propagation in earth's polar regions. It is well understood that PCA events are primarily caused by high-energy protons of solar origin (Kouznetsov et al., 2014) and that the polar cap region is defined by the open/closed boundary (OCB) of the geomagnetic field (Wild et al., 2004). It has been shown that solar energetic protons (SEP) ranging in energy from 1-200 MeV have easy access to earth's ionosphere near the geomagnetic poles and are responsible for the majority of ionospheric absorption (Sauer & Wilkinson, 2008). Further studies have postulated that the energy range most-responsible for absorption lie within the 1-20 MeV range, (Kouznetsov et al., 2014; Patterson et al., 2001; Lied, 1967).

Qualitatively it appears that PCA events follow this general process: SEPs are emitted from the sun following a solar flare, or more likely a CME. The SEPs, being positively charged, follow the field lines of the IMF toward earth. Once near earth, the SEPs interact with the geomagnetic field, where some are able to enter the cusp region
via open field lines. However, as will be discussed later, SEPs of higher energy can enter earth's upper atmosphere along closed field lines near the open/closed boundary.

PCA events tend to have similar properties. Hunsucker & Hargreaves (2003) describe several PCA event properties. They observe that PCA events occur on average about 6 times per year. However, there can be well over 10 during an active year and none during quiet times. Furthermore, they found that once a PCA event begins it will tend to last for about 1-4 days on average, though some have lasted more than a week. Apparently the initial increase in absorption begins very near earth's magnetic pole then over a period of several hours the increased absorption spreads equatorward covering the polar cap region with higher energy SEPs reaching further equatorward. Unlike the cutoff boundary for protons above about $100 \text{ MeV}$, the cutoff boundary for protons in the 1-20 $\text{MeV}$ range is poorly understood and is a key focus of this research.

Understanding the nature of PCA events allows those that rely on HF communications to prepare for and adapt to communications disruptions due to PCA events. Understanding the mechanisms that cause PCA events allows space weather forecasters to assist those that rely on HF communication to have advanced warning of PCA events and forewarning of potential consequences.

Chapter 2 describes magnetospheric penetration and several important associated terms and concepts, including numerical modeling techniques that allow tracking SEP's from the SW through the magnetosphere into the D-region of the ionosphere. Chapter 3 is devoted to the McIlwain $L$-Parameter and its importance in the study of proton trajectories through the geomagnetic field. Chapter 4 presents interesting findings
regarding the OCB and a departure from a pure LT (diurnal) dependence. Chapter 5 reports on the transport of $1-100\ MeV$ protons through the magnetosphere, leading to an approximation of the cutoff latitude and a presentation of new insights regarding the variability of the cutoff latitude. Chapter 6 discusses the penumbral region and its importance to the energy cutoff latitude. Finally, Chapter 7 includes a summary of this work and potential applications.
CHAPTER II

MAGNETOSPHERIC PENETRATION

Introduction

As discussed in Chapter 1, PCA events are primarily caused by SEPs penetrating the magnetosphere, gaining access to the D-region of the ionosphere in earth's polar regions. Chapter 2 will cover the important parameters of energy and rigidity, how they are related, their significance to PCA events, and how they are used to show that the computer program is functioning properly. A brief introduction will be given regarding the McIlwain L-Parameter and the related invariant latitude (McIlwain, 1961). In addition, the computer model that was used for this research, the 1996 version of the Tsyganenko Model of the Geomagnetic Field will be introduced. Finally, the open/closed boundary, cutoff latitude, and penumbra region will be introduced.
Energy and Rigidity

Generally, SEPs are characterized by their energy, typically in electron volts (eV) or mega electron volts (MeV). An electron volt is the amount of kinetic energy gained by a charged particle as it is accelerated through a potential field of 1 volt and is equal to about $1.6 \times 10^{-19}$ joules. As was discussed in Chapter 1, proton energies responsible for PCA events typically have energies of 1-100 MeV, with the 1-20 MeV range being most responsible. Using the basic equation for kinetic energy it becomes evident that protons within this energy range (1-100 MeV) have speeds of roughly $1.4 \times 10^{7} - 1.4 \times 10^{8}$ m/s. An initial concern is whether account should be made for relativistic effects.

Relativistic effects are manifest as the speed of light is approached. Recalling that the speed of light is about $3.0 \times 10^{8}$ m/s it can be shown that the above velocities, as a percent of the speed of light are about $0.05c - 0.5c$, where $c$ is the speed of light in vacuum. Recall that relativistic effects are related to the Lorentz factor,

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$  \hspace{1cm} (2.1)

In the above, gamma ($\gamma$) is the Lorentz factor, $v$ the particle velocity and $c$ the speed of light.

Figure 2.1 shows the relation between the non-relativistic energy (dashed black line), and the relativistic energy (solid black line). The solid red line below shows the percent difference between the non-relativistic energy and relativistic energy. The horizontal axis is the scaled proton velocity in units of $1.0 \times 10^{8}$ m/s and the vertical axis is proton energy in MeV. We see that within the 1-100 MeV range the percent
difference reaches a maximum of about 10% for 100 MeV protons. As stated previously, the 1-20 MeV protons are most responsible for PCA events. It is apparent from the Figure that 20 MeV protons show very little relativistic effects. Furthermore, Smart et al. (1969) indicate that protons within this range are non-relativistic. Hence, it is appropriate to continue without relativistic effects.

![Comparison of Relativistic vs. Non-Relativistic Velocities](image)

**Figure 2.1**: Relativistic corrections. Relativistic corrections are an important concern when studying energetic protons. This figure illustrates that for the energy regime under investigation (1-20 MeV) relativistic corrections are not needed. The solid and dashed black curves represent non-relativistic and relativistic effects respectively. The red curve represents the percent difference between the two.

When studying particle trajectories through a magnetic field it is also common to use rigidity. Rigidity is a measure of a charged particle's ability to maintain a straight path through a magnetic field. Specifically, rigidity is defined by Smart & Shea (1985) as, "A measure of (a particle's) resistance to a magnetic force that deflects the particle from a straight-line trajectory." Furthermore, Hunsucker & Hargreaves (2003) state that
one advantage of rigidity is that all particles with the same rigidity will follow the same path through a given magnetic field.

Mathematically, rigidity \((P)\) is described as,

\[
P = \frac{p c}{q}
\]

Here, \(p\) is the particle's momentum, \(c\) the speed of light, and \(q\) the charge of the particle.

Hence, we can recast rigidity in various forms,

\[
P = \frac{(m v) c}{q}
\]

It was shown previously that \(1-100\) MeV protons are non-relativistic; hence we use the non-relativistic momentum. Examining units we find that,

\[
P = kg \left( \frac{m}{s} \right) \left( \frac{m}{s^2 c} \right) \left( \frac{1}{c} \right) = \frac{kg m^2}{s^2 c} = \frac{f}{c} = \frac{c v}{c} = V
\]

Hence, the unit of rigidity is the volt. It is common to use megavolts and gigavolts in many applications.

Particle velocities can be defined in terms of kinetic energy, preferably in electron volts. Therefore,

\[
E = \frac{1}{2} m v^2
\]

\[
\Rightarrow v = \sqrt{\frac{2E}{m}}
\]

\[
\Rightarrow P = \frac{m}{q} \sqrt{\frac{2E}{m} c}
\]

\[
\Rightarrow P = \frac{\sqrt{2m^2 c^2 E}}{mq^2}
\]

\[
= \frac{\sqrt{2(mc^2)E}}{q}
\]
In the above, $mc^2$ is the proton's rest mass. In electron volts the proton's rest mass is roughly 938.3 $MeV/c^2$. Thus, stating the proton energy in $MeV$ and defining $m_r$ as the rest mass with units of $MeV/c^2$ then,

$$P = \frac{\sqrt{2m_rE}}{q}$$  \hspace{1cm} 2.10

When using electron volts as the unit of choice, $q$ then represents the number of charge units (Smart & Shea, 1985). If we're assuming a single proton then,

$$P = \sqrt{2m_rE}$$  \hspace{1cm} 2.11

In this form, rigidity is given in megavolts per particle. Conversion from rigidity to energy is straightforward via,

$$E = \frac{p^2}{2m_r}$$  \hspace{1cm} 2.12

As shown in Figure 2.2, protons with energy between 1-100 $MeV$ have rigidity of about 0.05-0.45 $GV$.

**Figure 2.2**: Rigidity as a function of proton energy.
The L-shell Parameter

The L-parameter \((L)\) was first introduced by McIlwain (1961) in an attempt to more-accurately organize and represent magnetically trapped particles. The coordinate system suggested was based on the magnitude of the magnetic field and adiabatic invariants. \(L\) was described by McIlwain (1961) as a parameter that "...retains most of the desirable properties of (the adiabatic invariant) and that has the additional property of organizing measurements along lines of force." \(L\) and the related *invariant latitude* will be discussed in greater detail in Chapter 3.
The Tsyganenko Model

An important consideration when studying SEP trajectories through the magnetosphere is to have a functioning, realistic model of the geomagnetic field. One such model is the 1996 version of the Tsyganenko model of the geomagnetic field (T96). T96 is a semi-empirical best-fit model for Earth’s magnetic shield based on years of satellite observations (Tsyganenko, 1995; Tsyganenko & Andreeva, 2018). The basic approach behind a model such as T96 is to break the total magnetic field into several partial fields, each based on a particular current system, then vectorially add them together (Tsyganenko & Andreeva, 2018). In addition, the field is parameterized by several key inputs, namely the solar wind vector components, the solar wind ram pressure, and the y and z components of the IMF. Within the T96 model, these current systems are the magnetopause, cross-tail, and ring tail currents. According to the Community Coordinated Modeling Center, "...the T96 model contains an explicitly defined realistic magnetopause, large-scale Region 1 and 2 Birkeland current systems, and IMF penetration across the boundary," (Tsyganenko Geomagnetic Field Model and GEOPACK libraries, 2019).

The total geomagnetic field is then modeled using internal and external components. The internal component can be either a dipole approximation or the full-scale International Geomagnetic Reference Field (IGRF) model. The external component is the actual T96 subroutine. The total field is then the sum of the internal and external components. A key consideration is that the internal source, in this dissertation the
IGRF, is rigidly coupled to the rotating Earth. Hence, the topology of the internal field with respect to the Sun varies with UT and day of year (Tsyganenko & Andreeva, 2019).

T96 was a critical component of this dissertation. The code, along with the included GEOPACK subroutines, provided geomagnetic field components, traced magnetic field lines, and tracked the magnetopause boundary. Each of these elements was required to develop SEP trajectories and model the open/closed boundary. Using T96 as the foundation, FORTRAN computer subroutines were developed to perform many of these functions.

**Coordinate Systems**

T96 uses a variety of coordinate systems to describe the geomagnetic field. A few of them are the GSM, GSW, and GSE systems. Following is a description of each system as well as some clarification as to where and how each may be used.

**Geocentric Solar Magnetospheric (GSM)**

A GSM coordinate system has its $x$-$axis$ pointing positive from Earth toward the Sun. The $z$-$axis$ is described very well by (Hones et al., 1986) as "...the projection of the earth's dipole axis in a plane perpendicular to the earth-sun line..." The $y$-$axis$ then completes the right-handed coordinate system. Again quoting (Hones et al., 1986),"This coordinate system reflects the fact that the magnetotail is more strongly influenced by the solar wind flow (assumed radial from the sun) and by the direction of the dipole."
Tsyganenko uses GSM coordinates for T96 to determine corrections to the geomagnetic dipole (external field).

**Geocentric Solar Wind (GSW)**

Since the solar wind does not arrive radially (as assumed in GSM coordinates) the GSW coordinate system was devised. The GSW system has its $x$-axis aligned anti-parallel to the velocity vector of the solar wind and is positive toward the Sun. The $z$-axis and $y$-axis are as in the GSM coordinate system. Tsyganenko uses GSW coordinates for the IGRF_GSW_08 subroutine to determine the geomagnetic dipole (Inner field).

**Geocentric Solar Ecliptic (GSE)**

A GSE coordinates system has its $x$-axis towards the Sun and its $z$-axis perpendicular to the plane of the Earth's orbit around the Sun (positive North). This system is fixed with respect to the Earth-Sun line. As in the previous systems, the $y$-axis completes a right-handed coordinate system. Tsyganenko uses GSE coordinates for the solar wind input in the above referenced subroutines. T96 includes subroutines to convert between GSW and GSE coordinates as well as other useful routines that allow transformations to/from regular Cartesian and spherical coordinates.

**Inputs**

The vector components of the solar wind are used exclusively to set the solar wind vector used to determine the $x$-axis of the GSW system. They do not affect
geomagnetic structure. An important simplification is that GSM and GSW coordinates become identical when the solar wind velocity components are directed radially along the $x$-axis such that $v_{swx} = -400 \text{ km/s}$ and the other components are set equal to zero (Tsyganenko & Andreeva, 2019). This assumption is used throughout this dissertation. The structure of the field is determined by inputs of solar wind ram pressure, $D_{st}$ index, and the $B_y$ and $B_z$ components of the IMF.

GSM and GSW coordinate systems require the definition of the tilt angle of the geodipole and the related inner field. One way this is accomplished is by using a purely dipole approximation of the field and explicitly defining the tilt angle. Another method is to use the IGRF approximation of the inner field along with the RECALC subroutine. Using input parameters of year, day of year, hour, minute, and the 3 components of the solar wind velocity, RECALC determines the unique tilt angle as well as the necessary components of the rotation matrices needed for transformations between GSM, GSW, and GSE coordinate systems.

**Methodology**

T96 filled two primary roles. The first was to provide geomagnetic field components to move SEPs through the field to test for allowed vs. forbidden trajectories. The second was to evaluate field lines to determine the boundary where field lines transitioned from being closed to open. This also facilitated a straightforward approximation for $L$ as demonstrated by Pilchowski et al., (2010).
The Open/Closed Boundary

The open-closed boundary (OCB) defines the region, moving pole-ward, where geomagnetic field lines transition from being closed to open. Closed field lines have both foot points at or near Earth's internal magnetic field in opposing hemispheres. Open field lines have one foot point at Earth while the other maps to the IMF and the SW (Newell et al., 2006; Kabin et al., 2004; Wild et al., 2004). Charged particles are able to follow these open field lines into Earth's upper atmosphere.

The OCB defines the polar cap boundary (PCB). Being able to identify and track the OCB allows study of several of the most important dynamic process in Earth's geomagnetic system (Wang et al., 2016). Variations in the OCB and related changes in the size of the PCB have been linked to the net rate of magnetic reconnection on both the dayside and nightside (Mende et al., 2016; Wild et al., 2004; Chisham et al., 2004; Kabin et al., 2004). The OCB is critical to other topics in space physics including planetary magnetospheres and magnetosphere-ionosphere (M-I) coupling (Dixon et al., 2015). The OCB could also provide important insights into the equatorward limits of precipitation of energetic particles into the D-region of the ionosphere, causing polar cap absorption (PCA) events (Smart et al., 2000).

The OCB is often estimated using in situ measurements made by spacecraft transiting polar regions or by ground-based optical imagers to develop proxies for mapping the OCB (Dixon et al., 2015; Chisham et al., 2004). In addition, physics-based computational models can be used to estimate the location of the OCB (Rae et al., 2010; Wang et al., 2016; Wild et al., 2004; Newell et al., 2009). A difficulty with this approach
is two-fold. First, spacecraft observations are limited by their single-point sampling at the OCB. Second, ground-based imagers typically have a limited field of view. A significant barrier to a global determination of the OCB lies in the necessity to combine diverse data sets and a lack of true global coverage of observational data (Rae et al., 2010; Chisham et al., 2004).

Figure 2.3 provides an example of this transitional region. The solid black line represents a closed field line with a foot point in the northern hemisphere at 72 degrees geographic latitude. This field line is considered closed because it has foot points in the northern and southern hemispheres and is a closed loop. The dotted and dashed lines, however, represent open field lines at 73 degrees and 74 degrees geographic latitude respectively. These lines are considered open because they have only one foot point in the northern hemisphere, the other foot point mapping to the IMF. Hence, moving poleward, a boundary in encountered (the OCB) where field lines transition from closed to open.

A specific region of the dayside OCB is called the cusp, polar cusp, or separatrix. The cusp is located near the noon meridian plane (Tsyganenko & Russell, 1999) and is an area where geomagnetic field lines provide direct connection between the ionosphere and the SW via open magnetic field lines (Hunsucker & Hargreaves, 2003). An important study regarding variability of the cusp was performed by Russell (2000), showing that the geomagnetic latitude of the cusp depends on several factors including dynamic pressure of the SW, IMF orientation, and dipole tilt angle. Coxon et al. (2016) put the cusp at 75

Figure 2.3: Illustration of open vs. closed magnetic field lines. Field lines are shown in GSM coordinates. The sun is to the right. Closed field lines have both foot points at the surface of the earth, open field lines have one foot point at earth, the other maps to the solar wind.

The location of the OCB as a reference latitude is utilized by a wide range of scientists and space weather forecasters, and indirectly by amateur (Ham) radio operators.
This OCB usage is to regard its latitude as an approximation of the equatorward edge of a polar region that is being impacted by energetic particles, causing PCA events. This edge is known to move equatorward as the intensity of the PCA event increases. However this boundary at local noon, for example, would be expected to have the same geomagnetic latitude to all observers regardless of their longitude.

Since the T96 model has a good track record of reasonably predicting the geomagnetic field (Mende et al., 2016; Wild et al., 2004), the initial goal was to use the T96 model to study trajectories of protons with energies of 1-100 MeV through the geomagnetic field, attempting to develop a better understanding of the proton energy cutoff latitude and how the cutoff latitude, solar energetic proton (SEP) events, and PCA events were related (important works in this field include Taylor, 1967; Sauer, 1963; Sauer & Wilkinson, 2008; Smart et al., 1969; Smart et al., 2000; Smart & Shea 2001). However, model runs of proton trajectories require substantial computational resources, especially for low-energy protons (~1 MeV) that experience a great deal of geomagnetic bending and require a significant number of steps to determine if the trajectory is allowed or forbidden (Smart et al., 2000). The hypothesis driving this particular study is that the OCB could serve as a proxy for the energy cutoff latitude of low-energy protons.

Available with the T96 model is a suite of subroutines commonly called GeoPack 2008 (Tsyganenko & Andreeva, 2019). One of these subroutines allows the user to trace geomagnetic field lines. T96 inputs include the date (year, day, hour), SW velocity vector components, SW ram pressure, Dst index, and the y and z components of the IMF($B_y, B_z$). Using a strictly radial SW flow allows Geocentric Solar-Wind (GSW) and
Geocentric Solar-Magnetospheric (GSM) coordinates to align (Tsyganenko, 1995). This is a useful assumption, especially when investigating proton trajectories, but has no significant bearing on tracing geomagnetic field lines.

Chapter 4 includes a detailed discussion regarding the variability of the OCB and proposes several new findings.
The Cutoff Latitude

Poleward of the OCB, geomagnetic field lines are open, allowing easy SEP access. Equator-ward of the OCB geomagnetic field lines are closed. This does not mean, however, that protons are unable to penetrate equator-ward of the OCB (Nesse Tyssøy & Stadsnes, 2015; Leske et al., 2001). An important boundary when considering geomagnetic field topology and PCA events is commonly referred to as the cutoff latitude, energy cutoff latitude, etc. It is defined as the lowest latitude to which a solar proton of a given energy can penetrate into the upper atmosphere (Nesse Tyssøy & Stadsnes, 2015; Smart et al., 2006; Smart & Shea, 2001; Shea et al., 1965).

One method in common use (Rigidity Method) seeks to define the cutoff rigidity (or energy) for a specific geographic latitude/longitude pair (Smart et al., 1999; Smart et al., 2006; Smart et al., 2000; Sauer & Wilkinson, 2008). Once a location of interest is selected, a broad range of rigidities (proton energies) is examined to determine the cutoff rigidity (proton energy) for that specific location. Since a specific geographic location relates to the foot point of a specific magnetic field line, this method determines the cutoff rigidity for a specific geomagnetic field line. Hence, the previously-discussed McIlwain $L$-parameter can be used to describe the relationship between the rigidity and the invariant latitude (McIlwain, 1961; Sauer & Wilkinson, 2008).

The method used for this dissertation work (Latitude Method) takes a different approach, looking instead for the cutoff latitude associated with a specific proton energy. To compare methods, the Rigidity Method starts with a specific latitude and tests a range of rigidities; the Latitude Method starts with a specific energy and runs through lower and
lower geographic latitudes in search of the lowest latitude that provided access to the
selected altitude for the chosen proton energy. In both methods the initial velocity vector
is directed radially outward from the geographic location under consideration. Hence, the
Rigidity Method typically uses the term "cutoff rigidity" or "cutoff energy," whereas the
Latitude Method uses the term "cutoff latitude" or "energy cutoff latitude."

Figure 2.4 provides an example of the cutoff latitude. Figure 2.4 is a polar
projection with the polar axis representing magnetic latitude and the azimuthal axis
representing Local Time. The solid black line is the maximum equatorward extent of 1.0
MeV protons based on the given conditions. The dashed black line represents the
maximum equatorward extent of 100 MeV protons for the same conditions. An important
observation is that the 100 MeV protons reach lower cutoff latitudes than 1 MeV protons.
This will be discussed further in Chapter 5.

Throughout the years a tremendous amount of attention has been paid to the
cutoff latitude and its various properties. Vallarata (1935) noted that the total intensity of
cosmic radiation measured in different locations along the same geomagnetic latitude but
at different longitudes would vary considerably. They attributed this "longitude effect" to
an asymmetry of the geomagnetic field. Similar mention of the longitude effect and the
asymmetry of the geomagnetic field is made in Smart & Shea (1985, 2003) and others.

There exists an extensive body of research involving the cutoff latitude (Smart et
al., 2006, 2000, 1999). One common approach in finding the cutoff latitude is to use a
model of the geomagnetic field and trace proton trajectories through the field.
Figure 2.4: Simplified rendition of a polar projection of the cutoff latitude. 1.0 and 100 MeV protons are shown at 0000 UT. The radial axis represents geomagnetic latitude; the azimuthal axis represents local time, with noon LT at the top. The solid black line represents the cutoff latitude (maximum equatorward extent) of 1.0 MeV protons, the dashed black line shows the cutoff latitude for 100 MeV protons.

An important consideration is that, unlike the OCB, the cutoff boundary may not be at one specific latitude. Smart et al., (2000) use three distinct cutoff regions to describe the cutoff latitude: an upper cutoff above which all trajectories are allowed, a lower cutoff below which all trajectories are forbidden, and an effective cutoff which represents an average of the upper and lower cutoff latitudes. The intermediate region between the upper and lower cutoff is called the penumbral region (Smart et al., 2006, 2000; Smart & Shea 2001) and is introduced more fully in Chapter 2.10. Chapter 6 is devoted to analysis of this important region.
When using observational data, Nesse Tyssøy & Stadsnes (2015) following Leske et al. (2001) define the cutoff location to be that *invariant latitude* at which the count rate is half of its mean value above 70 degrees. Section 2.10 includes a discussion regarding the width of the penumbral region, while Chapter 6 includes further discussion of this important region.

To accomplish the proper motion of a proton through the geomagnetic field, the straightforward approach of solving the Lorentz force equation was used. The Lorentz force equation is,

\[ \mathbf{F} = m \mathbf{a} = q \mathbf{E} + q \mathbf{v} \times \mathbf{B} \quad 2.13 \]

Where \( \mathbf{F} \) is the force vector, \( \mathbf{a} \) is acceleration, \( \mathbf{E} \) the electric field, \( \mathbf{B} \) the magnetic field, \( \mathbf{v} \) the velocity vector, \( q \) the proton charge, and \( m \) the proton mass. A nominal magnitude for the electric field near earth is about 10 mV/m. Assuming that the velocity and magnetic field are orthogonal, and assuming a nominal geomagnetic field magnitude of 25 \( \mu T \) we see that for protons of energy \( 1-100 \text{ MeV} \) \( (qv \times B) \gg qE \). Hence, for this study the electric field is neglected. Therefore,

\[ m \mathbf{a} = q \mathbf{v} \times \mathbf{B} \quad 2.14 \]

\[ \Rightarrow \frac{d\mathbf{v}}{dt} = \frac{q}{m} \mathbf{v} \times \mathbf{B} \quad 2.15 \]

\[ \Rightarrow d\mathbf{v} = \left( \frac{q}{m} \mathbf{v} \times \mathbf{B} \right) dt \quad 2.16 \]

Given a set of initial conditions, the test proton is moved through the model geomagnetic field. There are many methods for numerically solving ordinary differential equations (ODE) such as Equation 2.16. Frequently used methods for solving Equation 2.16 include Bulirsch-Stoer (Smart et al., 2006, 1999) and 4th-order Runge-Kutta (Smart
A primary goal of such numerical methods is to provide a reliable numerical solution with minimal computational effort. Hence, they are especially useful when computer resources are limited.

The resources of the Center for High Performance Computing (CHPC) at the University of Utah were made available for this dissertation research. Hence, computer resources were not limited. It was decided to use the basic, straightforward method of solving the ODE for the change in velocity \((dv)\) at each time step. Since Equation 2.16 is in vector form, it is necessary to decompose it into its three Cartesian variations,

\[

dv_x = \left(\frac{q}{m}\right) (v_y B_z - v_z B_y) dt \\

dv_y = \left(\frac{q}{m}\right) (v_x B_z - v_z B_x) dt \\

dv_z = \left(\frac{q}{m}\right) (v_x B_y - v_y B_x) dt
\]

(2.17a) (2.17b) (2.17c)

Given a solution for the vector components of \((dv)\), the new velocity components were then computed and the proton moved to the new position. The magnetic field vector components were then determined for the new position, allowing a new solution for the change in velocity, etc.

A challenge arises in that SEPs may have very high energies and may be moving at significant fractions of the speed of light. Therefore, if the time step \((dt)\) is too large the resolution may be too coarse to correctly move the particle through the field. This is true regardless of the method employed to numerically solve Equation 2.16. Hence, an important consideration when modeling energized particle trajectories is the relationship between the number of steps \((N)\), and \(dt\).
For a given time period (a few seconds) as $dt$ gets smaller, $N$ must get larger. The concern is two-fold. First, though a small $dt$ would certainly be beneficial, $N$ can become very large. This results in model runs becoming very time consuming. Second, as $dt$ gets very small the round-off error in calculations can become a considerable fraction of $dt$.

A good reality check to validate results can be the fact that within a magnetic field where changes take place over a slow time period, energy will be conserved. Therefore, the more "conserved" energy is for a given time period, the more physically correct will be the particle motion through the field.

To test this hypothesis the following test was performed: a proton was placed at a point 6 $R_E$ from Earth along the $x$-axis, placing the proton within the magnetosphere. The velocity was set such that $v_x = v_y = v_z = 8.0 \times 10^6$ m/s. This yields a speed of roughly $1.39 \times 10^7$ m/s and an energy of about 1 MeV. The initial number of steps was set to 10,000 ($N = 10000$) and the initial time step was set to 1 ms ($dt = 1.0 \times 10^{-3}$). In this case the product of $N dt = 10$ s. The goal, therefore, was to adjust these two parameters (increasing $N$, decreasing $dt$) for each run such that $t = 10$ s to see how well energy would be conserved as the number of steps increased.

Figure 2.5 tells the story, but requires a little explaining. Notice that the vertical axis in each panel of Figure 2.5 is different. Though the curves in Figure 2.5 appear similar, they are quite different. In Figure 2.5a the vertical axis spans a range of 1-10 MeV. Hence, for the chosen number of steps and the chosen time step, energy increased drastically. In Figure 2.5b the vertical axis has a range of 1-1.3 MeV. Here, over the 10 second interval, energy increased by about 30%. Hence, one could argue it still was not
conserved properly. Figure 2.5c shows a vertical axis of $1.0 - 1.03 \text{ MeV}$ and a related energy increases by about 3% during the 10 second interval.

![Energy Plot](image)

**Figure 2.5:** 4 plots showing conservation of proton energy. Different values of $dt$ and $N$ are illustrated over a 10 second interval. This figure demonstrates one difficulty in numerically solving ordinary differential equations.

Figure 2.5d shows the effect of approaching the time scale where round off error becomes significant, as demonstrated by the noisy curve, thus demonstrating the
importance of selecting an appropriate number of steps and a related time step such that results are physically correct, all the while being efficient with computer resources.

Frequently throughout the research process "sanity checks" were done to be sure the numerical analysis was performing adequately. Ultimately, $N = 2.0 \times 10^6$ was determined to be a reasonable number of steps. A varying time step ($dt = 8.25 \times 10^{-12}/B$) was used to take advantage of the fact that as the magnetic field strength diminishes the force on the proton also diminishes allowing for a larger time step.

Another important consideration is the initial proton location. In the early stages of this research the method was to place SEP's at some initial location outside the magnetosphere and propagate them toward earth, hoping that they would hit earth, or at least get close. A more appropriate method, the so-called reverse engineering process, was found to be much more efficient. The reverse engineering process will be discussed further in Chapter 5.

The goal of the first study was to find certain combinations of SEP energies that would allow the SEP's to approach close enough to Earth that the proton would enter the magnetosphere. This was an important first step because only protons that enter the magnetosphere will be of interest. Proton energies within the range $1.0 \leq E \leq 100.0$ MeV were evaluated, with an initial position of $x = 15.0 \, R_E$ and positions along the $z$-axis $0 \leq z \leq 25 \, R_E$. The intent was to determine if there existed a "sweet spot," or area of easy access, along the $z$-axis where protons with a certain range of energies could enter the geomagnetic field. For this study the first day of summer, at a time when the
geomagnetic axis was pointed anti-sunward were examined. SW and IMF conditions were obtained from ACE via the OMNI website, though the radial flow SW simplification as described in Section 2.6 was used. This study included the following assumptions:

1) Motion only in the $GSW$ coordinate $x$-$z$ plane

2) Proton travels in straight line from the Sun to a point 15 Earth radii distant from Earth. In $GSW$ coordinates this is the point $x = 15.0 \, R_E$. Hence, at that point the proton will have only $v_x \neq 0$

3) Initial positions along the $z$-axis will be $0 \leq z \leq 25 \, R_E$.

4) Ignore any motion along the $y$-axis, but scrub final data such that only protons that have a $y$-axis component within the geomagnetic field will be accepted.

5) Include geomagnetic field components, $(B_x, B_y, B_z)$

6) Neglect any electric field. Hence, $\vec{E} = 0$

7) Evaluate proton energies from $1.0 \leq E \leq 100.0 \, MeV$. This is based on information from (Lied, 1967) in section 3, table 3.3. Apparently this energy range allows proton penetration between about 90-50 km, which would be appropriate for the D-region.

8) The solar wind speed input will be as indicated by Tsyganenko to set the $GSW$ and $GSM$ systems identical.

The following IMF parameters were used:

1) $Pressure$: 2.74 $nP$,

2) $Dst$: 1.0 $nT$,

3) $By$: 2.5 $nT$,
4) $B_z$: -3.6 nT.

Figure 2.6 shows the results from this study. Each diamond on the plot represents SEP's that successfully penetrated the magnetosphere to the specified radial distance from earth. It is evident that a greater number of high-energy SEP's entered the magnetosphere given the initial conditions. Furthermore, the lowest energy SEP that reached the magnetopause was about 40 MeV.

![Protons Reaching Geomagnetic Field @ 5 Earth Radii](image)

**Figure 2.6:** Protons reaching the geomagnetic field. The initial starting point for this study was 5 earth radii from the geomagnetic field. The horizontal axis represents the $z$-axis plane in earth radii; the vertical axis is proton energy.

This method proved to be a tedious process. This problem was solved once a better understanding of the reverse engineer process was developed. Using the reverse engineering technique greatly improved research efficiency. Chapter 5 includes a discussion regarding the cutoff latitude and describes findings relative to the variability of the cutoff latitude.
CHAPTER III

THE L-SHELL PARAMETER

Introduction

The L-Shell Parameter ($L$) was first introduced by McIlwain (1961) in an attempt to more accurately organize and represent particles trapped in the then-recently-discovered Van Allen radiation belts (Van Allen et al., 1959). The ($B_m, L$) coordinate system presented by McIlwain (1961) uniquely defines a location by the intensity of the magnetic field at the mirror point ($B_m$) and the $L$-parameter. $L$ was described by McIlwain (1961) as a parameter that "...retains most of the desirable properties of (the adiabatic invariant) and that has the additional property of organizing measurements along lines of force."

In principle, $L$ represents the equatorial radius of a field line (McIlwain, 1961), and is only well-defined for an aligned dipole configuration (Pilchowski et al., 2010). Pilchowski et al. (2010) demonstrate 4 methods in which $L$ can be approximated:

- $B_{min}, K = 0, R_{max}, z = 0$ and are defined as follows: $B_{min}$ approximates $L$ as the distance from Earth's center to the point of minimum field strength along a field line;
- $K = 0$ approximates $L$ as the distance from Earth's center to the point where the field line crosses the equatorial plane of the tilted dipole field; $R_{max}$ approximates $L$ as the maximal radial distance of a field line from Earth's center;
- $z = 0$ approximates $L$ by the passage of a field line through the equatorial plane. Pilchowski et al. (2010) reasoned that $R_{max}$ was the most useful because it contains "...geometrical information about the
respective field line." In this dissertation work, the $R_{\text{max}}$ approximation of $L$ is used throughout.

Figure 3.1 allows a qualitative look at what $L$ represents. The contour lines (1.5, 2, 2.5, etc) represent lines of constant $L$. Hence, along each contour line we expect to find a foot point of a geomagnetic field line that approximately crosses the geomagnetic equator at the distance specified given in Earth radii. An $L$ value of 1.5 indicates a field line with a maximum radial distance of 1.5 Earth radii, serving as an approximation of an equatorial radius 1.5 Earth radii; an $L$ value of 5.0 represents an equatorial radius of 5.0 Earth radii, and so on. This allows one to systematically group geomagnetic field lines by their associated $L$-value and retain important physical properties of the field line without encountering the difficulties associated with actually calculating the adiabatic invariants (Pilchowski et al., 2010; McIlwain, 1961).

Associated with $L$ is the invariant latitude. McIlwain (1961) describes the relationship between $L$ and the invariant latitude ($\lambda$) by,

$$R = L \cos^2(\lambda)$$

where $R$ is the radial distance to a foot point of a magnetic field line. Thus, a foot point near the surface of the earth would have $R=1.0$, and for the D-region (about 100 km) $R=1.015$. Solving for the invariant latitude we find that,

$$\lambda = \cos^{-1}\left(\frac{R}{\sqrt{L}}\right)$$
**Figure 3.1:** Contour plot of L-shell values. The x-axis represents geographic longitude; the y-axis represents geographic latitude. Lines of constant $L$ represent foot points of geomagnetic field lines with the same $L$ value.

Hence, for each $L$ there is an associated *invariant latitude*. A geographical point, described by a geographical latitude/longitude pair, can also be described by $L$ and an *invariant latitude*. For example, Millstone Observatory is located at 42.617 N by 289.483 E. It is well known that Millstone Hill lies at $L=2.6$ which equates to an *invariant latitude* of about 51.8 degrees. Some geophysical phenomena that occur in the ionosphere or magnetosphere are well-defined by their unique $L$. For example, the Van Allen Radiation Belts occur between about $L=1.5-2.5$ (inner belt) and $L=4-6$ (outer belt). Figure 3.2 gives an example of $L$ as it relates to an *invariant latitude* of 54.7 degrees.

$L$ is important to this dissertation work because it is frequently used to estimate and describe cutoff energies for energetic particles (Pilchowski et al., 2010; Smart et al., 1999; Shea et al., 1987). Furthermore, the *invariant latitude* (and therefore $L$) is used to estimate HF radio wave absorption. For example, Sauer & Wilkinson (2008) discuss a method using the *invariant latitude* and related cutoff energy to determine energy...
thresholds used to estimate HF absorption in the D-region during PCA events (Akmaev et al., 2010; Space Weather Prediction Center, 2019).

Figure 3.2: Illustration of determination of the invariant latitude. It is assumed that the L-shell parameter is known. In this case, $R$ represent the foot point of the closed field line at the surface of the earth ($R=1$).

\[ R = L \cos^2(\lambda) \Rightarrow \lambda = \cos^{-1}\left(\frac{R}{L}\right) \Rightarrow \lambda = \cos^{-1}\left(\frac{1}{3}\right) = 54.7^\circ \]
UT-Variability

The assumption, of course, is that the invariant latitude is, in fact, invariant. This isn't necessarily the case. Using T96 as the model magnetic field, Figure 3.3 examines $L$ as a function of geographical longitude ($x$-axis) and UT (individual curves). Each panel of Figure 3.3 plots $L$ for all 24 hours UT across all longitudes at a specific geographic latitude. Hence, each panel contains 24 curves, one curve for each hour UT. Notice in Figure 3.3a (30° geographic latitude) that all 24 curves superpose onto one line. This is an indication that at that geographic latitude $L$ is constant along each meridian for all UT. In other words, $L$ has a value of 1.2 and an associated invariant latitude along the 30 E meridian for all UT, and a value of 1.5 and an associated invariant latitude along the 240 E meridian, etc. This is confirmed by reexamining Figure 3.1. Notice that at the Lat/Lon point (30N, 240E) the value is 1.5, and at the point (30N, 30E) the interpolated value is near 1.2, consistent with the IGRF approximation of an offset-tilted dipole magnetic field.

Figure 3.3c plots the 24 UT curves at 50° N latitude. Again, all 24 UT curves superpose onto one line. However, there's a bump near the 270 E meridian. This "bump" is evident as the dip near 270 degrees longitude in Figure 3.1. A reexamination of Figure 3.1 shows that moving laterally across the 50 N line several contour lines are crossed ranging from 2.5 to about 4, with the highest $L$ value occurring at about the 270 E meridian. An important feature evident in Figure 3.3 is the lack of variance as a function of UT. All 24 UT curves are exactly the same. Thus, a conclusion is that for mid-
latitudes, $L$ and the associated *invariant latitude* are constant along a line of constant longitude and do not demonstrate a UT-dependent variation, as expected.

![Figure 3.3](image)

**Figure 3.3:** L-shell parameter as function of geographic longitude (Low latitude). The L-shell parameter is examined at specific geographic latitudes (30, 40, and 50 deg). Each panel contains 24 curves, one for each UT. Hence, all 24 lines are superposed onto one line, indicating that for these latitudes, the L-shell parameter does not show a UT dependence.

Figure 3.4 tells a different story. In Figure 3.4a (60 N), from 0 to about 210 degrees longitude the UT curve appears to superpose as in Figure 3.3. However, there is a significant variance in $L$ as a function of UT, especially around the 270 E meridian. The variance is even greater in Figure 3.4b (70 N). Hence, it appears that above a certain geographical latitude $L$ and the related *invariant latitude* experience a UT-dependent variation, especially near 270 E longitude. It would appear, then, that to adequately
describe a location above mid-latitudes by $L$ or the invariant latitude, one must know the UT hour.

\[ \text{Figure 3.4: L-shell parameter as function of geographic longitude (high-latitude region). The L-shell parameter is examined at 60 and 70 degrees geographic latitude. A UT dependence is evident at these higher latitudes.} \]

It might be helpful to examine this UT effect at two specific locations. As was mentioned previously, Millstone Hill is located at (42.6 N, 289.5 E). Figure 3.5 shows $L$ at that location as a function of UT ($x$-axis) and season (separate curves), with each panel representing different geomagnetic storm conditions (quiet, moderate, severe). Each curve represents $L$ during the first day of spring, summer, fall, and winter. It is apparent that $L$ and the related invariant latitude are constant across all UT and under all seasonal
conditions at Millstone Hill. This confirms the notion that at mid-latitudes \( L \) (and the related *invariant latitude*) is indeed constant and describes very well that physical location. There is, however, a slight increase in \( L \) due to geomagnetic storm conditions. This will be investigated later.

**Figure 3.5**: \( L \)-shell parameter for a specific geographic location. The \( L \)-shell parameter is shown as a function of UT and season, shown for 3 variations of geomagnetic activity. For this location (Millstone Hill) the \( L \)-shell parameter is constant, as described in the scientific literature.

Figure 3.6 moves to the Lat/Lon point of (55.0 N, 289.5 E), roughly 13 degrees north of Millstone Hill. Figure 3.6a shows a constant \( L \), but the value has moved to about \( L=5.5 \). Figures 3.6b and 3.6c, however, show significant variations in \( L \) in the UT frame.
as well as the geomagnetic storm frame. Hence, it appears that $L$ and the related
*invariant latitude* can be UT-dependent at higher latitudes. A high-latitude storm
dependence is also observed. This is an important consideration given that the OCB is
usually above 60 N and that the proton energies most responsible for PCA events (1-20
MeV) typically have cutoff latitudes above 50 N.

**Figure 3.6:** L-shell parameter at a specific geographic location. Here the L-shell
parameter is shown 15 degrees north of Millstone Hill. As in Figure 3.5, $L$ is shown as a
function of UT and season over 3 variations of geomagnetic activity. $L$ is quite variable
seasonally as driven by geomagnetic conditions.
One important use of $L$ is to describe the cutoff latitude of energetic protons. For example, (Smart et al., 1999) performed a thorough study of cutoff rigidities for a variety of geomagnetic storm conditions, establishing a world-grid structure of cutoff rigidities at a penetration altitude of 450 km. They calculated $L$ for each grid location, thus linking the cutoff rigidity, $L$, and the associated invariant latitude. Sauer & Wilkinson (2008) use the world grid structure from Smart et al. (1999) as a means for including geomagnetic effects in their radio wave absorption model. Specifically, they use $L$ as a means for adjusting their absorption equations to account for geomagnetic cutoff energies that may be above either of the standard energy thresholds of their absorption equations.

Figure 3.7 (Figure 1 from Sauer & Wilkinson, 2008) illustrates the cutoff energy of energetic protons at an altitude of 450 km as reported by Smart et al. (1999). From Figure 3.7 it is evident that the cutoff latitude for 100 MeV protons under quiet conditions is about 63 degrees invariant latitude. This results in a value of about $L=5$. An important consideration is that under all but the most extreme geomagnetic conditions, the cutoff latitude is well above 60 degrees invariant latitude for protons with energy 1-20 MeV, the key energies responsible for PCA events.
Figure 3.7: Invariant latitude as function of proton cutoff energy. Sauer & Wilkinson (2008)

Figure 3.8a shows modeled values for $L$ for 100 MeV protons for 4 separate UT hours (5, 11, 17, 23 UT). It is apparent that for those 4 hours UT, $L$ has a value of about $L=5$, in good agreement with Sauer and Wilkinson (2008). Figures 3.7b and 3.7c show $L$ for 17.8 and 1.00 MeV protons. In these cases $L$ varies between about 6-10 and 10-20 earth radii for 17.8 and 1.00 MeV protons respectively. Hence, a statement placing the cutoff energy for 100 MeV protons at $L=5$ is a true statement; placing the cutoff energy for 1.00 MeV protons at a specific $L$ would require the UT and longitude be known. Thus is demonstrated a UT-dependent variation in $L$ and the associated invariant latitude.
Figure 3.8: L-shell parameter as function of geographic longitude. $L$ is shown for 5, 11, 17, and 23 UT and 100, 17.8, and 1.0 MeV. For the chosen times, 100 MeV protons have a constant L-shell value, as expected. Protons with lower energy show a variable L-shell parameter.

It has been shown that the energy range most responsible for PCA events falls within the 1-20 MeV range. Hence, it seems apparent that when evaluating $L$ (or the associated invariant latitude) with the intent of estimating HF absorption, the UT-dependence is an important consideration.
CHAPTER IV

MODEL-BASED PROPERTIES OF THE OPEN/CLOSED BOUNDARY

Introduction

As discussed in Chapter 2, the open-closed boundary (OCB) defines the region where geomagnetic field lines transition from being closed to open.

A UT dependence of the OCB has been postulated in previous works. For example, Sojka et al. (1981) identified a UT variation in high-latitude ionospheric convection arising from the superposition of an M-I convection electric field with a terrestrial co-rotation electric field, while Coxon et al. (2016) quantified a UT-type of dependence in the location of the dayside field aligned currents, showing that over many years and seasons the integrated strength of the dayside currents depend upon the solar zenith angle of the ionosphere where the current closure ionospheric conductivities are located. These conductivities are dependent on solar XUV radiation, and hence are modulated by changing solar zenith angle. They referred to this phenomenon as a diurnal effect.

Strictly defining open vs. closed field lines on the night side of Earth can be arbitrary (Kabin et al., 2004). This is due, in part, to the great distances to which geomagnetic field lines may be stretched in the magnetotail. Field lines that may eventually map to the SW (open) do so within time and distance scales that greatly exceeds what is happening in the near-Earth geomagnetic environment. Wang et al. (2016) discuss the trade off between reducing computational time vs. an appropriate
amount of data. Hence, for this study the focus is strictly on the dayside of the northern hemisphere, defined in this work as 0600-1800, centered at about noon local time (LT).

To negate any seasonal variation due to the tilt of Earth's rotational axis toward or away from the Sun, the investigation was limited to the vernal equinox which in 2010 occurs on day 79.

Results of this study are compared with the Ovation Prime Real-Time (OPRT) auroral precipitation model (OVATION Prime Real-Time, 2018) described in Newell et al. (2009). OPRT and the resulting plots of auroral energy flux and total hemispheric power (HP) provide a means to compare the OCB determined by T96 with that of an empirical model of auroral electron precipitation, a frequently-used proxy for determining the OCB (Coxon et al., 2016; Newell et al., 2009; Wild et al., 2004). Results of this study are also compared with those of Rae et al. (2010), including an interval (5 June 1998) where the OCB could be determined using a combination of instruments during a sharp northward to southward IMF turning.

The OCB is evaluated as a function of several different parameters. Section 2 of this chapter examines the OCB at a specific UT as a function of LT. Section 3 examines the OCB at a specific LT as a function of UT. Section 4 compares the UT-dependent OCB derived from the T96 model with OPRT and with results from Rae et al. (2010). Section 5 discusses a Space Weather (SWx) impact of the UT-dependent OCB, and Section 6 offers a discussion regarding the apparent UT dependence associated with the OCB as well as final conclusions.
Locating the Open/Closed Boundary

An important aspect of this research is the impact of geomagnetic storm conditions on the OCB. The T96 user’s guide (Tsyganenko & Andreeva, 2019) states that T96 uses a linear dependence of the amplitudes of the field sources on the SW pressure, $D_{st}$, and IMF-related parameters. Therefore, best results are expected near the most probable value of the input parameters. This corresponds to the region within the specific parameter space with the highest density of spacecraft measurements. Hence, caution should be exercised in modeling situations with extremely high or low values for these parameters. Thus, extrapolating T96 too far beyond the range of reliable approximation may lead to unrealistic results. Table 4.1 shows the input values used in this study for quiet, moderate, and severe geomagnetic storm conditions. Note that the values for the $D_{st}$ index and for $B_z$ slightly exceed the envelope of T96 capabilities but do not fall in the range of "extreme" values as indicated by Tsyganenko & Andreeva (2019).

<table>
<thead>
<tr>
<th>Input</th>
<th>Range</th>
<th>Quiet</th>
<th>Moderate</th>
<th>Severe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{st}$ ($nT$)</td>
<td>$-100 &lt; D_{st} &lt; 20$</td>
<td>1.0</td>
<td>-50</td>
<td>-150</td>
</tr>
<tr>
<td>Pressure ($nPa$)</td>
<td>$0.5 &lt; P &lt; 10$</td>
<td>1.0</td>
<td>4.0</td>
<td>8.0</td>
</tr>
<tr>
<td>$By$ ($nT$)</td>
<td>$-10 &lt; By &lt; 10$</td>
<td>1.0</td>
<td>4.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$Bz$ ($nT$)</td>
<td>$-10 &lt; Bz &lt; 10$</td>
<td>1.0</td>
<td>-5.0</td>
<td>-15.0</td>
</tr>
</tbody>
</table>

To determine the OCB, geomagnetic field lines were plotted starting at 50 degrees geographic latitude so there was relative certainty that the initial point would yield a
closed field line. The program then moved pole-ward in 1-degree increments until an open field line was reached. In this study a closed field line is defined as one which has both foot points located at Earth's surface; open field lines have one foot point at Earth, the other mapping into the SW. This determination was made by first checking the radial distance \((R)\) of the current trace point from Earth. If the final trace point had a value of \(R\) that was about equal to the initial value of \(R\) at the initial foot point then the field line was considered closed. Also, a parameter was used from one of the subroutines for tracing the boundary between the geomagnetic field and the SW, the magnetopause. If the field line pierced the magnetopause it was assumed to be an open field line. For the study and figures associated with Section 5.2 the OCB was studied at noon LT at 0500 UT.

Figure 4.1 shows one closed and two open field lines in each of three geomagnetic storm conditions at noon LT. Figure 4.1a shows that the closed field line (72.0 deg) extends to about \(L=12\), where \(L\) is a variation of the well-known McIlwain parameter (McIlwain, 1961) introduced by Pilchowski et al. (2010) as "the maximum distance from the centre of the Earth, which a point along a magnetic field line can attain." The McIlwain parameter is further discussed in Chapters 2.3 and 6.2. Note that the first open field line (73.0 deg) maps to the SW. Hence, Figure 4.1a shows that along that particular meridian at noon LT (105.0 E) the OCB exists between 72.0 and 73.0 degrees geomagnetic latitude for the given conditions, where geomagnetic latitude is defined by T96, based on coordinate system transformations from spherical geographic into spherical geomagnetic coordinates. Figures 4.1b and 4.1c show the OCB under moderate and severe conditions respectively. Apparently, as storm conditions increase
the OCB moves equator-ward and the $L$ value of the last closed field line decreases along both axes, as expected. For moderate conditions the OCB lies between 66.0 and 67.0 degrees, and for severe conditions between 60.0 and 61.0 degrees geomagnetic latitude.

Figure 4.1: OCB Noon meridian cross section in GSM coordinates. The OCB is shown during quiet, moderate, and severe geomagnetic conditions. A solid line in the above panels represents a closed magnetic field line; dashed and dotted lines represent open field lines. In all three panels the horizontal axis represents the $x$-plane, the vertical axis represents the $z$-plane.

Figure 4.2 is an example of dayside OCB locations at 0500 UT once all necessary geographic longitude planes at 15-degree intervals have been evaluated. Figure 4.2 shows the predicted location of the northern hemisphere OCB during the vernal equinox under quiet conditions at 0500 UT (solid line). Note that at noon LT the OCB is at roughly 72.5 deg geomagnetic latitude. During the 0600-1800 LT regime the OCB ranges from a minimum of 72.0 (1000 LT) to a maximum of about 78.0 (0600 LT) degrees geomagnetic latitude. These geomagnetic parameter values are within those stated previously in this section. The impact of storm conditions is to move the OCB
equatorward due to factors that apparently are largest around noon LT. Between quiet and severe conditions the equatorward shift is about 12 degrees at noon LT and 3.0 to 4.0 degrees at 0600 and 1800 LT.

Figure 4.2: Dayside OCB for three levels of geomagnetic activity. Panel 4.2a shows the dayside geomagnetic latitude of the OCB for three levels of geomagnetic activity at 0500 UT. Panel 4.2b shows the same boundaries in a polar projection also at 0500 UT. In both panels the solid line shows quiet, the dashed line moderate, and the dotted line severe geomagnetic conditions.
UT-Dependence of the Open/Closed Boundary

An important feature of the T96 model is the self-defined UT control based on the International Geomagnetic Reference Field (IGRF). In a 24-hour period, as the geographic Earth rotates in the Sun-Earth geometry, the Earth's magnetic field will have different orientations relative to the Sun and to the SW. Several coordinate systems exist aimed at describing this geometry. Two such systems are the GSW and GSM coordinate systems. Both are used by the T96 model.

How does this periodic behavior of the geomagnetic field affect the OCB? Section 4.2 discussed how the T96 model was used to locate the OCB. This section discusses what the T96 model showed regarding the UT-dependence of the geomagnetic field and the OCB. It is less ambiguous to determine open vs. closed geomagnetic field lines on Earth's dayside. Hence, this study evaluated 3 local times, all within the dayside, plotting the OCB as a function of UT. To plot the OCB over a 24-hour period the same procedure as described in Section 2 was used along lines of geographic longitude at 15-degree intervals, beginning with the Prime Meridian. This facilitated the OCB to be mapped in terms of UT as well as geographic LT. Figure 4.3 shows these results. Several important features are apparent.

Each panel of Figure 4.3 has UT as the \textit{x-axis} and geomagnetic latitude as the \textit{y-axis}. Each panel of Figure 4.3 shows the OCB under quiet, moderate, and severe geomagnetic storm conditions. All three panels of Figure 4.3 show a periodic nature to the OCB. This is most evident in Figure 4.3b at noon LT, though the cyclic behavior is
manifest in all three panels. In all cases the OCB shifts equator-ward as geomagnetic storm conditions intensify, as expected.

**Figure 4.3:** Universal time dependence of the OCB. Here the OCB is shown at 3 levels of geomagnetic activity at 0800, 1200, and 1600 LT. In panel 4.3b the 78-deg magnetic latitude approximation of the OCB at noon LT (cusp region) is shown as a heavy red line.

Figure 4.3b illustrates that under quiet conditions the OCB min/max difference is about 10 degrees, for moderate conditions about 12 degrees, and for severe conditions
about 13 degrees geomagnetic latitude. Furthermore, comparing the OCB maximum occurring at about 1500 UT in Figure 4.3b demonstrates that the OCB maximum due to geomagnetic condition changes by about 5 degrees going from quiet to moderate and about 8 degrees going from moderate to severe.

The UT effect has a significant impact on OCB topology. Though the cyclic behavior at 0800 and 1600 LT is not as well defined as that at noon LT, the cyclic nature is readily observable. Also, it is noted that the peak in each plot has a definite shift towards a later UT as activity level increases.

In Figure 4.3b (Noon LT) an additional dashed line is drawn at 78 degrees geomagnetic latitude. This provides a reference for a commonly adopted location of the dayside noon cusp-OCB. In the introduction it was discussed that this OCB location ranged from 75 to 78 degrees geomagnetic latitude (Coxon et al., 2016; Hunsucker & Hargreaves, 2003; Russell, 2000). These values all lie within the UT range of the quiet activity noon OCB.
The Open/Closed Boundary and Dayside Auroral Oval

One approach commonly used for estimating the OCB is to use data gathered via ground-based measurements and/or via spacecraft as they pass through Earth's polar regions, using the data to estimate auroral electron precipitation from which an estimate of the OCB can be made. To test the OCB determination of the T96 model, results of this study were compared with two separate studies: 1) the OPRT model of auroral precipitation (OVATION Prime Real-Time, 2018) as described by Newell et al. (2009); 2) The Space Weather Modeling Framework (SWMF) and associated observational data as reported by Rae et al. (2010).

OVATION Prime Real-Time

Figure 4.2 shows what a typical plot of the OCB taken at a single UT might look like. Since a goal of this study was to compare T96 OCB estimates with OPRT estimates care was used to be sure inputs matched those used to calculate OPRT images. The initial study examined quiet conditions during the vernal equinox. Once a better understanding of the UT effect on the OCB was achieved, it made sense to plot a full 24-hour UT estimate of the OCB. Several of these are shown in Figure 4.4 (0500, 1100, 1700, and 2300 UT).

Several key features are apparent in Figure 4.4. Notice that each hour UT has an independent configuration or shape of the OCB. At noon LT for each hour UT there is a range of about 10 degrees geomagnetic latitude. Under quiet conditions it the OCB has a
maximum equator-ward extent of about 70.0 degrees geomagnetic latitude. Furthermore, as storm conditions increase there is an equatorward expansion of the OCB, as expected.

**Figure 4.4**: Dayside polar projection for 4 specific UT hours. The dayside OCB is shown under quiet (a), moderate (b) and severe (c) conditions.

Figure 4.5 shows the minimum/maximum envelope for quiet, moderate, and severe conditions. The minimum represents the lowest geomagnetic latitude reached by the predicted OCB; the maximum value is the highest latitude. The min/max values are determined for each UT hour for the dayside sector only. Under quiet conditions the min/max values are 72 and 83 degrees, for moderate conditions 65 and 81 degrees, and for severe conditions 57 and 80 degrees geomagnetic latitude.
Figure 4.5: Polar projection of the dayside OCB minimum/maximum values. Illustrated are quiet (a), moderate (b), and severe (c) conditions. The polar axis is geomagnetic latitude, the azimuthal axis is local time. The dashed line represents the minimum OCB, the solid line represents the maximum OCB. The shaded area represents the region in which the min/max occurs.

Figure 4.6 is taken directly from the OPRT data set (OVATION Prime Real-Time, 2018), based on Newell et al. (2009). It shows the auroral electron energy flux forecast based on the same inputs used by this study to generate Figures 4.2 and 4.4. Section 1 discussed that the cusp is generally regarded as being at geomagnetic latitude of 75-78 degrees. Also, the OCB can be approximated as a circle (Coxon et al., 2016). This approximation is represented by the red arc in Figure 4.6.

The auroral electron energy flux is based on observed conditions and does not include an implicit UT dependence. Hence, for equivalent geophysical conditions all OPRT plots should look the same, regardless of UT in this coordinate system. As discussed in Section 4.3, apparently the OCB is dynamic due to UT effect, showing a large geomagnetic latitude variance. Therefore, a more realistic comparison with the OPRT maps would include the min/max envelope as shown in Figure 4.5. This is shown as the superimposed yellow arcs in Figure 4.6. Clearly the 75-78 degree circular approximation of the cusp lies within the UT range of the OCB derived from the T96.
**Figure 4.6**: OPRT auroral energy flux. Here we see the energy flux in a magnetic latitude/magnetic local time polar projection for conditions optimized for the quiet conditions defined in this study. The heavy red circle represents the OCB located over the 75-78 degree range. The solid yellow semicircles represent the OCB min/max values during conditions similar to those used to obtain the OPRT prediction (OVATION Prime Real-Time, 2018).

These UT-OCB’s tend to lie at the poleward edge of the dayside oval and extend by as much as 10 degrees latitude into the polar region. Given the lack of a direct UT component in the OPRT the comparison is at best qualitative. However, the degree of consistency between OPRT and T96 is encouraging.
The Space Weather Modeling Framework

The Space Weather Modeling Framework (SWMF) is a suite of computational models used for modeling physical processes from the Sun to the Earth and consists of several models including BATS-R-US, ionospheric electrodynamics (IE), and the Rice Convection Model (RCM). Rae et al. (2010) tested two configurations of SWMF against a large observational dataset during a sharp northward-to-southward IMF turning observed on 5 June 1998. Figures 2 and 3 from Rae et al. (2010) are especially useful since these figures show the modeled OCB based on the two configurations of SWMF along with the observed OCB from a collection of data from several sources (Rae et al., 2010).

Since the input data used in Rae et al. (2010) was correctly adjusted to account for the lag time between observation by the WIND instrument and interaction with the geomagnetic field (about an hour) it was determined that the following T96-required conditions existed at 1247 UT on 5 June 1998, prior to the $B_z$ reversal:

$$B_y \sim -4.8 \, nT, B_z \sim 3.45 \, nT, D_{st} \sim -3.5 \, nT, P_{SW} \sim 5.65 \, nPa$$

Figure 4.7a shows the T96 dayside OCB results superimposed over Figure 3a from Rae et al. (2010). In Figure 4.7a the solid red line represents the T96 dayside OCB at 1347 UT (data suitably lagged) based on the above conditions while the dashed orange line and dashed/dotted black line show the modeled OCB and the observed OCB from Rae et al. (2010) respectively. Throughout the dayside sector, the T96 projection of the OCB is generally within about 2-3 degrees of their results.
**Figure 4.7:** OCB comparison with BATS-R-US/RCM. The T96 OCB (solid red line) is superimposed over Figure 3a and 3b from Rae et al. (2010). 4.7a shows the OCB prior to the $B_z$ reversal at 1307 UT. 4.7b shows the OCB at 1547 UT, after the reversal. In both panels the dotted orange line shows the BATS-R-US/RCM OCB; the dotted/dashed black line shows the observed OCB as described in Rae et al., 2010.

It was then determined that the following conditions existed at 1407 UT, after the $B_z$ reversal:

$$B_y \sim -8.3 \, nT, B_z \sim -6.4 \, nT, D_{st} \sim 2.0 \, nT, P_{SW} \sim 2.0 \, nPa.$$  

Figure 4.7b shows the T96 dayside OCB results at 1507 UT superimposed over Figure 3d from Rae et al. (2010). In Figure 4.7b the T96 projection of the OCB is generally within about 3-5 degrees of their results. In the 1200-1400 LT sector the T96 OCB is within 2-3 degrees, and in the 0600-0800 sector within 1-2 degrees of their projections. Furthermore, T96 appropriately captured the essence of the equatorward expansion of the OCB during the $B_z$ reversal. Hence, we conclude that T96 provides a reasonable prediction of the OCB.
The Open/Closed Boundary in Geographical Coordinates

As previously discussed, the location of the OCB is utilized by a wide range of scientists and space weather forecasters, and indirectly by amateur radio operators. Typically, investigators regard the OCB latitude as an approximation of the equatorward edge of a polar region that is being impacted by energetic particles, causing PCA events. This edge is known to move equatorward as the intensity of the PCA event increases, as has been shown. The expectation, however, is that at local noon this boundary would have the same geomagnetic latitude to all observers regardless of their longitude. This study suggests differently.

Figure 4.8 is an unconventional use of the northern hemisphere Mercator geographic representation. Each longitude is at a different UT such that each meridian is at solar noon LT. The commonly used 78-deg magnetic latitude location of the noon OCB under quiet conditions is represented by the dotted line in Figure 4.8, showing the expected geographic variability in latitude. The location of the OCB under quiet conditions with the associated UT variability is shown as a solid line. These two quiet time OCB locations differ by as much as 10 degrees in latitude; from 14 degrees to 280 degrees East longitude the UT dependent OCB lies equatorward of the fixed OCB. The approximate location of the geomagnetic dipole is shown as a black dot. The most equatorward latitude for either OCB curve does not lie on the magnetic pole longitude. The fixed 78° OCB lies about 15° eastward while the UT OCB is westwards by about 15°.
Figure 4.8: Northern hemisphere Mercator projection in geographic coordinates. In this projection the OCB is shown at noon LT for each longitude. The solid red line represents quiet conditions; the dashed and dotted red lines show moderate and severe conditions respectively. The dotted black line represents the OCB at 78 degrees magnetic latitude. The black dot approximates the location of the geomagnetic North Pole.

In Figure 4.8 two addition curves are drawn to represent the UT noon OCB for moderate (dashed line) and severe (most equatorward curve) geomagnetic conditions. Note that the difference between each of the three UT dependent OCB curves is of the same order as the difference between the two quiet OCB curves. Hence the basic UT effect being identified in this study is commensurate with the main SWx impact on this boundary. A further note is that this study focused primarily on one seasonal condition, the vernal equinox. The following section includes a further discussion regarding this point.
Discussion

M-I coupling processes lead to many standard ionospheric phenomena such as the auroral region (Hardy et al., 1985), high-latitude electric field convection (Weimer, 1995), the region of Birkeland current closures (Ijima & Poterma, 1982), and cross-polar cap potential (Reiff et al., 1981). These models are empirical models that use simple or very complex dependences on solar, solar wind, magnetospheric, and atmospheric conditions. These dependencies are based on indices and parameters representative of SWx dynamics. *But none of these utilize an explicit UT dependence*, although all the indices do have a UT reference as time series parameters.

This study has demonstrated, using a coupled representation of the solar-wind, the magnetosphere, and the geomagnetic field, specifically T96, that the dayside OCB has a strong UT dependence (Smith & Sojka, 2019). The dayside cusp region has a strong association with the OCB since its ionospheric magnetic fields map through the dayside magnetosphere into a region of magnetic reconnection and hence the SW. The cusp, therefore, is morphologically a region that straddles closed, (equatorward) field lines and open (poleward) field lines.

Conventionally the latitude dynamics of the cusp would be associated with SWx status, i.e., during disturbed periods the cusp would have an equatorward motion. But it would be the same magnetic latitude dependence for all geographic longitude meridians. *Our study contradicts this, Figure 4.3b and 4.4 provide a glimpse of the T96 UT dependence of the noon OCB*. Figure 4.9 extends this noon OCB UT dependence as a function of SWx activity; the SWx indices listed in Table 1 have been three points
through which a smoothly varying curve was created for each index. Across this range of SWx conditions the UT dependence of the Noon OCB has a consistent variability of 10 degrees to 15 degrees in magnetic latitude.

Figure 4.9: OCB at noon LT as function of geomagnetic conditions.

Generating observational evidence for this suggestion is particularly challenging. A ground station is by definition located at one longitude and hence has a unique UT when it passes under the cusp region. Hence, several ground-based sites would be
needed to have long-term data sets of the cusp such that statistically the SWx versus UT can be separated. Satellites perhaps have a better likelihood to have data streams capable of identifying a cusp UT dependence.

This initial study focused on the vernal equinox. There is a strong probability that other seasons have a somewhat different UT dependence based on seasonal dependence of the geographic and hence magnetic poles in the solar/SW frame. Perhaps an additional modeling study using advanced MHD models of the magnetosphere can provide independent quantification of this UT effect.

A rather interesting consequence of this finding is that when the previously mentioned statistical models were generated they were missing a key parameter, the UT effect. Figure 5.4 graphically shows how variable the location of a polar cap auroral boundary would be based on the location of the OCB. Historically, the locations of boundaries such as the auroral polar boundary or the equatorward precipitation boundary have never been tied to a specific SWx parameter, or combination of parameters. Rather scatter plots, with large scatter, are associated with equatorward motion of a boundary against an increasing SWx index. As stated previously, Coxon et al. (2016) quantified a UT-type dependence in the location of the dayside field aligned currents that was not evident in Iijima & Potemra (1982). Coxon et al. (2016) also showed that over many years and seasons the integrated strength of the dayside currents in either the northern or southern hemispheres depended upon the solar zenith angle of the ionosphere where the current closure ionospheric conductivities are located. These conductivities are dependent on solar XUV radiation, and hence are modulated by changing solar zenith
angle. Coxon et al. (2016) referred to this phenomenon as a diurnal effect, which in this study has been labeled a UT effect to differentiate it from a diurnal, local time, effect.

The evidence presented by Coxon et al. (2016) extends over seasons for both the northern and southern hemispheres. Inferences from this study imply that all M-I phenomena have a UT modulation that is of a similar magnitude as the SWx drivers. Sojka et al. (1981) identified a UT variation in high-latitude ionospheric convection arising from the superposition of an M-I convection electric field with a terrestrial co-rotation electric field.
CHAPTER V

MODEL-BASED PROPERTIES OF THE CUTOFF LATITUDE

Introduction

The cutoff latitude was briefly introduced in Chapter 2. It has been shown that the cutoff latitude (or cutoff rigidity) varies on a diurnal basis. Terms describing the diurnal variations of the cutoff latitude include daily variation, local time effect, latitude effect, and longitude effect to name a few. For example, Smart et al. (1969), in describing their use of a particular geomagnetic field model, note that the model contains a local time dependency, allowing them to study the daily variation of the cutoff latitude. Similarly, Smart et al. (2000) describe a daily variation in the high-latitude energy cutoff as a natural consequence of coupling of the internal and external magnetic fields as the earth rotates within a magnetosphere oriented in a SW coordinate system. Taylor (1967) likewise observes that the cutoff latitude is dependent on local time and experiences diurnal variations. Smart et al. (2003) discuss a longitude effect which they relate to local time. They discuss how the longitude effect relates to local solar time and is a result of "complex interactions" with the geomagnetic field.

Sojka et al. (1981) found that physical characteristics of several ionospheric features, including the polar ionization hole and the main ionospheric trough, exhibit what they called a UT dependence. They observed that incoherent scatter radar facilities at different geographic longitudes measured different diurnal patterns even though the data covered similar magnetic latitudes at similar local times. They were able to show that
ionospheric features previously thought to be dependent only on the local time of day showed a UT dependence. This UT dependence, they claim, is due to the offset of the geomagnetic and geographic poles.

The cutoff latitude is dependent on many factors. Since geomagnetic disturbance levels are important, solar wind conditions must be accounted for. Proton energy is a critical factor in the location of the cutoff latitude, though it has been shown by Fiori and Danskin (2016) that not all energies are equally effective in creating absorption. Finally, since the cutoff latitude is equator-ward of the OCB, and as it was shown in Chapter 4 that the OCB experiences a UT-dependent variation, there could be a related UT-dependence inherent in the cutoff latitude, especially in protons with energy between about 1-20 MeV.

But why is this important? Although the technology is many decades old, HF radio continues to offer a reliable and efficient means of communication. As an example, commercial airlines continue to use HF radio for polar routes that pass above about 80 degrees latitude (Fiori & Danskin 2016; Sauer & Wilkinson 2008). As previously discussed in Chapter 1, HF radio waves can experience significant attenuation due to absorption in the D-region, especially when propagating over the poles. Hence, it is critical that HF radio operators have some reliable means available to monitor and predict HF propagation conditions in polar regions.

Fiori & Danskin (2016) describes their study regarding the sensitivity of a well-known PCA prediction tool, the D-Region Absorption Prediction Tool (D-RAP) (See Space Weather Prediction Center, 2019; Sauer & Wilkinson, 2008; Akmaev et al., 2010
for a detailed discussion of D-RAP). Fiori & Danskin (2016) state that at best, D-RAP is most useful as a "qualitative indicator" and that the model frequently "misrepresents absorption values." In their study Fiori & Danskin (2016) describe a "latitudinal dependence" which had not been previously reported and they recommend its inclusion in D-RAP.

Those most interested in HF propagation conditions, especially in polar regions, rely heavily on D-RAP since it represents the state-of-the-art in PCA prediction. Therefore, it is imperative that additional studies take place to test other sensitivities which may help improve D-RAP’s prediction capabilities. Section 2 briefly discusses the well-known L-shell parameter; Section 3 sets forth the method used to determine the cutoff latitude; Sections 4, 5, and 6 discuss the various dependencies of the cutoff latitude; Section 7 reviews several important studies regarding solar proton trajectories and the cutoff latitude, allowing a comparison with previous work; Section 8 brings the preceding sections together in a comprehensive discussion of the new results obtained via this study.
The L-shell Parameter

As discussed in Chapter 3, $L$ was first introduced by McIlwain (1961) in an attempt to more-accurately organize and represent magnetically trapped particles. Furthermore, an important conclusion was developed in Chapter 3 regarding the variability of $L$ at geographic latitudes above about 55 degrees. Since $L$ is used as a parameter to describe the cutoff latitude (or cutoff energy) in terms of the related invariant latitude, it is important to account for that variability.
Locating the Cutoff Latitude

Defining the cutoff latitude is a challenge. In Chapter 4 defining the OCB on the dayside along a field line was straightforward. Using the field line tracing code and depending upon the resolution chosen, there was some location where, moving equatorward, a definite transition took place from open to closed field lines. The cutoff latitude, on the other hand, requires examining proton trajectories subject to various conditions that would dictate whether a particular trajectory was allowed or forbidden.

For example, the initial proton velocity components have an effect on the proton trajectory. Following Smart et al. (2000), all velocities were directed radially from earth's center through the initial location. Other important considerations which will be discussed in detail later include trajectories that pass through the magnetopause, trajectories that propagate through the magnetotail, and trajectories that pass through the solid earth.

As discussed in Chapter 2.4, IGRF was used to model the inner geomagnetic field while the 1996 version of the Tsyganenko model (T96) was used to approximate the outer components of the geomagnetic field. 2010 was arbitrarily chosen as the year, day 79 was chosen due it being the vernal equinox. The equinox was chosen to minimize any variation due to seasonal offset.

To accomplish the proper motion of a proton through the geomagnetic field, the straightforward approach of solving the Lorentz force equation was used. The Lorentz force equation is,

\[ \mathbf{F} = m \mathbf{a} = q \mathbf{E} + q \mathbf{v} \times \mathbf{B} \]  

5.1
Here, $F$ is the force vector, $a$ is acceleration, $E$ the electric field, $B$ the magnetic field, $v$ the velocity vector, $q$ the proton charge, and $m$ the proton mass. Chapter 2 contains a detailed review of the procedure used to approximate the motion of a proton through the geomagnetic field as well as some difficulties encountered.

An important consideration is defining allowed vs. forbidden trajectories. Smart et al. (2000) describe this difficulty in great detail. One example they cite is Boberg et al. (1995) in which any trajectory that begins at a low earth altitude and reaches the altitude of a geosynchronous orbit, about 6.6 Earth radii ($R_E$), is considered valid. One consideration here is that in nearly all situations, the magnetopause is further than 6.6 $R_E$ distant. An allowed trajectory would indicate that the proton started outside the geomagnetic field and would pierce the magnetopause. Hence, this method could show false positives.

To test the sensitivity of this "escape shell," a test was devised to determine how proton trajectories responded to various cutoff shells. Figure 6.1 shows the setup for this initial study. Escape shells were established at 6.6, 8, 10, 12, and 14 $R_E$. Any trajectory that pierced the shell during the code run was determined to be an allowed trajectory. Figure 5.1 illustrates that the escape shells for 6.6, 8, and 10 $R_E$ lie fully within the magnetopause while the shell at 12 $R_E$ lies at the magnetopause on the dayside. Only the shell at 14 $R_E$ has a portion that lies outside the magnetopause. In all cases the escape shell lies within the magnetotail on the nightside.
Figure 5.1: A series of 5 escape shells. Each is represented by circles in the GSM non-midnight meridian plane. The heavy dashed line represents the T96 quiet condition magnetopause.

The results of the study for 1, 10, 56.2, and 100 MeV are shown in Figure 5.2. One striking observation is the varying amplitude of the cutoff latitude, especially at lower energies. In Figure 5.2a, corresponding to the 6.6 Earth radii shell, it is observed that for all 4 energy levels there is little modulation in the cutoff latitude as a function of local time (LT). The solid black line representing the cutoff latitude for a 1.00 \textit{MeV} proton has an amplitude of about 1-2 degrees. As the escape shell radius is increased there is an increase in the amplitude of the modulation of the cutoff latitude as a function of LT for the 1.00 and 10.0 \textit{MeV} protons. Figure 6.2e shows an amplitude of about 9
degrees for the 1.00 $MeV$ proton. The variation of the higher energy protons (56.2 and 100 $MeV$) remains fairly consistent in Figures 5.2a-5.2e. Figure 5.2f shows the results using the methodology that will be described momentarily.

**Figure 5.2**: UT variation of the cutoff latitude. Each panel shows 4 proton energies. Furthermore, each panel corresponds to a recalculation of the cutoff latitude using an increased radius based on the 5 escape shells shown in Figure 5.1, the exception being Figure 5.2f which replaces the spherical escape shell with the magnetopause-based escape surface described in the text.

One concern was how to account for proton trajectories in the magnetotail. Smart et al. (2000) state that there is no de facto geomagnetic field model. As such, results will be model-dependent and should be interpreted accordingly. Smart et al. (2000) found that trajectories of lower energy protons experience a great deal of geomagnetic bending,
and that trajectories in the magnetotail may take many time steps to resolve. This research saw similar results.

A practical solution to the time management problem is to set two parameters that define allowed trajectories. First, if the proton pierces the magnetopause at any time during the code run it is considered an allowed trajectory. Second, if the proton reaches some predetermined radial distance away from Earth it is also considered an allowed trajectory. Using this approach, Smart et al. (2000, 2006) use the radial distance of 6.6 $R_E$ in the magnetotail.

It became apparent that protons could reach a radial distance greater than 6.6 $R_E$ and remain in trapped trajectories. Furthermore, since the distance from the Sun-Earth line (the $x$-axis in Figure 5.1) to the magnetopause reaches a distance of about 30 $R_E$ in the magnetotail it became apparent that a large area existed beyond 6.6 $R_E$ in the magnetotail that could contain forbidden trajectories. Hence, following Smart et al. (1999) it was determined that 25 $R_E$ provided a more realistic radial distance within the magnetotail to act as the dividing line between allowed and forbidden trajectories in the magnetotail.

Another consideration involves trajectories that at some point intersect or pass through the solid Earth. Smart et al. (2000, 2006) considered trajectories that intersected the solid Earth as forbidden. Using this method they found that above a certain upper latitude all trajectories were valid, and below a certain lower latitude all trajectories were forbidden. Within the band of the upper and lower latitude there existed a gradient of allowed trajectories. They called this region the penumbra, and it was an important part
of their research. As discussed in Chapter 2, the penumbral region did not constitute a significant variation in the cutoff latitude for the energy levels investigated. Hence, the cutoff latitude used in this research would correspond with the upper cutoff described by Smart et al. (2000).

Figure 5.3 shows the parameters used in this study. A trajectory was considered allowed if at any time during the code run a) it pierced the magnetopause (solid black line in Figure 5.3), or b) it reached a radial distance of 25 $R_E$ (solid red curve in Figure 5.3). A trajectory was considered forbidden if neither of those conditions was met. Note that the magnetopause is a 3-D structure determined by the Tsyganenko model.

![Figure 5.3: The T96 escape surface. The escape surface is based on the T96 magnetopause and a 25 Earth radius sphere and is shown in the noon/night meridian plane of the GSM coordinate system.](image)

Figure 5.2f illustrates how results from the methodology used for this study differ from the previously discussed methods. It is important to note that the method ultimately
used in the study closely resembles results using $14 \, R_E$ except for slight variations. Hence, it is appropriate to include the magnetopause as an indicator for allowed vs. forbidden trajectories. Extending the condition for the magnetotail out to $25 \, R_E$ was appropriate as well. One noteworthy observation from Figure 5.2 is that the 100 $MeV$ proton seems to maintain the same cutoff latitude regardless of the method chosen. This is an important observation and will be discussed in detail later in this chapter.
UT-Dependence of the Cutoff Latitude

Chapter 4 demonstrated an important UT variability in the OCB. Based on that initial work, a central hypothesis of this study is that a similar UT variability should be evident in the cutoff latitude. There exists a range of latitudes below the OCB that may provide allowed trajectories even though the field lines are closed (Nesse Tyssøy & Stadsnes, 2015; Leske et al., 2001).

A preliminary objective was to compare the OCB results from Chapter 4 with cutoff latitude data previously compiled using the T96 model. A straightforward approach was to determine the difference, or offset (OCB-Cutoff Latitude), between the OCB and the cutoff latitude for each data point for each energy level. Each energy level contains 576 data points, 24 hours UT, each with 24 hours LT. The average of all the offsets for each energy level were calculated and plotted in Figure 5.4.

In Figure 5.4 the horizontal axis is the log scale of proton energy, the vertical axis is the average offset in degrees of geomagnetic latitude. Figure 5.4 demonstrates that as proton energy increases the offset increases as well, indicating that as energy increases, the cutoff latitude is pushed further equatorward from the OCB.

Chapter 4 included a useful plot of the OCB at noon LT (the polar cusp) for each UT (see Figure 4.7). Figure 4.7 showed the UT effect specifically in the cusp region. Figure 5.5 shows a similar plot including the cutoff latitude for 3 energy levels (1, 10, 100 MeV).
Figure 5.4: Difference between proton cutoff latitude and OCB. The difference between a proton's cutoff latitude and the corresponding OCB during quiet conditions, averaged over 24 hours UT and 24 hours LT (576 points). The horizontal axis is proton energy from 1-100 MeV.

Figure 5.5a shows the OCB (cusp region) as well as the cutoff latitude at noon LT as a function of UT for quiet conditions. Figure 5.5b and 5.5c show the same cusp region and energy levels for moderate and severe storm conditions. These conditions are discussed in Chapter 4.

One striking feature of Figure 5.5a is the similar variations observed in the OCB and the cutoff latitude for each energy level. Also, as storm conditions increase (quiet to moderate to severe) the trend is for the OCB and the cutoff latitude to move equatorward. This is as expected (Nesse Tyssøy & Stadsnes, 2015; Sauer & Wilkinson, 2008). In Figure 5.5a the cutoff latitude peak for 1.00 MeV protons is at 80 degrees, in Figure 5.5b at 75 degrees, and in Figure 5.5c at 70 degrees geomagnetic latitude. Similar results are evident for the 10.0 and 100 MeV protons.
Figure 5.5: Geomagnetic activity dependence of the proton cutoff geomagnetic latitude. The cutoff latitude is shown at noon LT as a function of UT for 1.0, 10, and 100 MeV along with the corresponding OCB (solid black line). Quiet, moderate, and severe conditions are shown in Figures 5.5a, 5.5b, and 5.5c respectively.

Another interesting feature in Figure 5.5 is that as storm conditions increase, the OCB, 1.00 and 10.0 MeV protons move closer together. In Figure 5.5a the 1.0 and 10.0 MeV peaks are about 5 degrees apart. In Figure 5.5b the two peaks are about 2-3 degree apart, and in Figure 5.5c the separation appears to be less than 1 degree.

Furthermore, the peak-to-peak modulation of each energy level remains fairly consistent. For quiet conditions the peak-to-peak variance is about 10 degrees for all
three energies; for moderate conditions the peak-to-peak modulation is about 10 for the 1.00 MeV protons, and closer to 11 for the 10 and 100 MeV protons. Under severe conditions the peak-to-peak is roughly 11 degrees for all three energy levels.

Figure 5.6 illustrates the cutoff latitude along several arbitrary meridians. Figures 5.6a-5.6d show the cutoff latitude for three energies along the 15, 105, 195, and 285 degree meridians as a function of LT. These longitudes were chosen due to the specific alignment that occurs between the geographic and geomagnetic poles and the sun. Figure 5.7 further investigates the significance of longitude. In all 4 Panels of Figure 5.6 1 MeV protons show a large modulation. The modulation is not as pronounced for 10 MeV protons. There appears to be little to no modulation for 100 MeV protons. It is also interesting to note that in each panel of Figure 5.6 there is a significant dip in the cutoff latitude around 0100 LT. This may be due in part to the day/night asymmetry discussed by Nesse Tyssøy & Stadsnes (2015).

Also noteworthy is the behavior of the 100 MeV cutoff latitude curves. The curves show little variance in all 4 panels. However, Figures 5.6a, 5.6c and 5.6d show the cutoff latitude at roughly the same location, while Figure 6.6b shows the cutoff latitude shifted equatorward. Taking the average of the cutoff latitude for each meridian over all LT shows that Figures 5.6a, 5.6c and 5.6d have averages of 63.3, 62.5 and 62.3 degrees respectively.

Figure 5.6b shows an average of 57 degrees. Hence, there is an apparent modulation of the cutoff latitude that seems to be energy-dependent and exhibits some sort of longitudinal dependence.
Figure 5.6: Local time dependence of the proton cutoff geomagnetic latitude. The cutoff latitude for three proton energies (1.0, 10, 100 MeV) at four fixed longitudes (15, 105, 195, and 285 degrees geographic longitude) are shown in Figures 5.6a, 5.6b, 5.6c, and 5.6d respectively.
Energy-Dependence of the Cutoff Latitude

By its very nature the cutoff latitude is energy-dependent. All other inputs being equal, the expectation is that as proton energy increases the cutoff latitude will move equatorward. But is this energy dependence somehow coupled to the modulations discussed in the previous section? To examine any potential correlation between these effects, 9 energy levels (MeV) were selected using a logarithmic distribution:

\[ E = 10^n \]

\[ 0 \leq n \leq 2 \]

Table 5.1 relates the value of \( n \) to the energy and rigidity values used in this study.

**Table 5.1:** Energy Levels Used in this Study

<table>
<thead>
<tr>
<th>( n )</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>0.25</td>
<td>1.78</td>
</tr>
<tr>
<td>0.50</td>
<td>3.16</td>
</tr>
<tr>
<td>0.75</td>
<td>5.62</td>
</tr>
<tr>
<td>1.00</td>
<td>10.0</td>
</tr>
<tr>
<td>1.25</td>
<td>17.8</td>
</tr>
<tr>
<td>1.50</td>
<td>31.6</td>
</tr>
<tr>
<td>1.75</td>
<td>56.2</td>
</tr>
<tr>
<td>2.00</td>
<td>100</td>
</tr>
</tbody>
</table>

As shown in Chapter 4, due to the UT effect, the OCB has a unique boundary or structure for a 24 hour LT period for each hour of UT. Hence, a useful representation to show the variability of the OCB and the cutoff latitude is to plot the boundary structure
for all 24 hours UT on a single polar plot centered on the north geomagnetic pole. This provides a useful view of how each hourly boundary builds a structure that allows an investigation of relevant questions.

To enhance the capability to examine the cutoff latitude under varying conditions, two sets of coordinates systems were used: a fixed-sun system and a fixed-earth system. Both systems use polar coordinates and have the same radial axis, magnetic latitude. However, they have different azimuthal axes, each showing a different and important effect on the cutoff latitude.

**Fixed-sun System**

Figures 5.7a (1 MeV) and 5.7b (100 MeV) use a fixed-sun coordinate system. This system uses local time (LT) as the azimuthal axis with noon LT at the top, midnight LT at the bottom. Hence, the position of the sun relative to earth is fixed. Each boundary represents the cutoff latitude for a specific hour in UT over 24 hours of LT. Hence, for each UT boundary, the earth moves to a different orientation. At noon LT the 180 deg meridian aligns with 1200 LT, at 0500 UT the 105 deg meridian aligns with 1200 LT, etc. Using the fixed-sun system allows the observation of changes in the cutoff latitude as earth rotates, reflecting changes in the orientation of the geomagnetic field. Hence, the fixed-sun coordinate system allows observation of changes in the cutoff latitude due to changes in the alignment of the geomagnetic field.
**Figure 5.7:** Proton cutoff latitudes for 1.0 and 100 $MeV$ protons for quiet conditions. Proton cutoff latitudes are shown for 24-hours UT in 2 polar projections. Figures 5.7a and 5.7b use a fixed-sun system with azimuth angle being solar local time. Figures 5.7c and 5.7d use a fixed-earth system with geographic longitude being the azimuthal angle.

**Fixed-earth System**

Figures 5.7c (1 $MeV$) and 5.7d (100 $MeV$) use a fixed-earth coordinate system. Here geographic longitude is the azimuthal axis. In this case the earth is “fixed” with 180 degrees at the top, 0 degrees at the bottom. LT is defined by the position of the sun in this fixed-earth system. Hence, at 00 UT it is noon LT along the 180 deg meridian, local midnight along the 0 deg meridian. At 0500 UT it is noon LT along the 105 deg meridian, midnight LT along the 285 deg meridian. Since this is a fixed-earth system, the
changing location of the sun around the azimuthal axis allows the observation of changes in the cutoff latitude due to solar wind parameters (Nesse Tyssøy & Stadsnes, 2015). Hence, the fixed-earth system allows observation of changes in the cutoff latitude due to sun/earth alignment.

Figure 5.7a and 5.7b show a full 24 hour UT plot of the cutoff latitude boundary using the fixed-sun system under quiet geomagnetic conditions. The radial axis represents the magnetic latitude while the azimuthal axis is LT. Hence, the expectation is to observe effects due to geomagnetic field alignment. Each plot contains 24 individual boundaries superimposed upon one another to create a boundary structure. Figure 5.7a shows 24 separate cutoff latitude boundaries for 1.00 MeV protons and Figure 5.7b shows the same for 100 MeV protons. Several differences are quickly identifiable.

Note that the radius for the structure in Figure 5.7a (1 MeV) is smaller than the radius in Figure 5.7b (100 MeV). Also note that the latitudinal extent of the structure in Figure 5.7a is higher than that of Figure 5.7b (70 vs. 55 deg). Also, the structure in Figure 5.7b is more symmetrical than is that in Figure 5.7a. Also, the thickness of the structure along any given meridian of LT is roughly the same in Figures 5.7a and 5.7b. One interpretation of the thickness of the boundary structure along any given line of constant LT or constant longitude could be an indication of the variability of the cutoff latitude along that line for the specified conditions.

Figures 5.7c and 5.7d show the same 24 hour structure using the fixed-earth system. Notice that the radius of the structure in Figure 5.7c (1 MeV) is smaller than that of Figure 5.7d (100 MeV). Unlike Figure 5.7a and 5.7b, there is a significant difference
in the thickness of the structures in Figures 5.7c and 5.7d. However, the maximum equatorward extent is similar. The difference in structure thickness is a characteristic that warrants additional discussion. Hence, whatever effect is evident in Figures 5.7a and 5.7b is not energy dependent while the effect shown in Figures 5.7c and 5.7d is energy dependent.
Solar Wind Dependence of the Cutoff Latitude

Another important consideration is the effect increasing geomagnetic storm conditions may have on the cutoff latitude. It was shown previously that the alignment of the geomagnetic field as well as proton energy can affect the location of the cutoff latitude. Furthermore, Figure 5.5 demonstrated that geomagnetic conditions, which rely on SW parameters, are a determining factor in the location of the cutoff latitude. To test the effect of changing geomagnetic conditions on the cutoff latitude, parameters for moderate and severe conditions as defined in Chapter 4 were used. Figures 5.8 and 5.9 use an identical format as Figure 5.7 and describe moderate and severe geomagnetic conditions respectively. As in Figure 5.7 a fixed-earth (panels a and b) and a fixed-sun (panels c and d) coordinate system are used.

In all cases, as geomagnetic conditions increase an overall expansion of the boundary structure in Figure 5.8 and 5.9 relative to Figure 5.7 is observed. Furthermore, notice that the relative thickness of each structure remains consistent. For example, Figures 5.7a, 5.8a, and 5.9a show a consistent thickness, though the structure boundaries appears to be pushed equatorward with increasing geomagnetic conditions. The interesting case is the 100 MeV proton in the fixed-earth system (Figures 5.7d, 5.8d, and 5.9d). Here, a fairly thin boundary structure, indicating little variation in the cutoff latitude is observed. As before, note that the structure is pushed equatorward as storm conditions increase, as expected.
Figure 5.8: Proton cutoff latitude for 1.0 and 100 $MeV$ protons for moderate conditions. Proton cutoff latitudes are shown using the same format as Figure 5.7.
Figure 5.9: Proton cutoff latitude for 1.0 and 100 MeV protons for severe conditions. Proton cutoff latitudes are shown using the same format as Figure 5.7.

Figure 5.10 examines the standard deviation of the average variance for each local time (e.g. Figure 5.7a, 5.7b) for each energy level and each storm condition in the fixed-sun system. It is apparent that the variation demonstrated in the fixed-sun system is fairly consistent for all energy levels examined. Evidently, in the fixed-sun system a geographical location is insufficient to define the cutoff latitude. It is also necessary to define LT at that location.
Figure 5.10: Standard deviation for fixed-sun coordinate system. The fixed-sun system is used to generate Figures 5.7, 5.8, and 5.9.

Figure 5.11 examines the standard deviation of the average variance along each longitude (e.g. 5.7c, 5.7d) for each energy level and each storm condition in the fixed-earth system. It is evident that 100 $MeV$ protons exhibit little variation, showing a standard deviation of about 0.5. 1 $MeV$ protons show a standard deviation of nearly 3.0. It is apparent that in the fixed-earth frame 100 $MeV$ protons demonstrate a specific cutoff latitude for a specific location, consistent with the scientific literature. Lower-energy protons ($\sim$1-20 $MeV$), however, show a varying cutoff latitude for a specific location.
Figure 5.11: Standard deviation for fixed-earth coordinate system. The fixed-earth system is used to generate Figures 5.7, 5.8, and 5.9.
Comparisons with Previous Work

A primary aim of this study was to take the results discussed in Chapter 4 and apply them to the study of PCA events, specifically the proton energy cutoff latitude. Building upon previous studies, the ultimate goal was to address the following unanswered questions: a) is there a previously unaccounted for UT dependence in the cutoff latitude; b) if so, is the sensitivity of the dependency sufficient that it should be included in PCA models.

An important consideration of any new study is its agreement with previous results. Though many studies have been done regarding cutoff latitude over the years, this study relies heavily on the works of Smart, Shea, and their collaborators to validate its results. There also exists a robust body of current work in this field, allowing a comparison of results of this study with an extensive group of theoretical and observation-based studies. It was found that in all applicable cases, results of this study matched very well with previous results.

Smart et al., 1969

An attempt was made to replicate results from Smart et al. (1969). Figure 5.12 from this dissertation work is Figure 3 from Smart et al. (1969) showing the daily variation of the cutoff latitude as a function of local time and proton energy along the 260 degree E meridian. Since the data for this dissertation was intended to look specifically at longitudes associated with UT, data did not exist for the 260 degree meridian. However, data did exist for the 255 degree E meridian.
Figure 5.12: Daily variation of proton cutoff energies along 260 deg E meridian. The dashed lines indicate extrapolated values. \( \lambda \) indicates the geographic latitude along the 260 deg meridian and \( \Lambda \) denotes the invariant latitude (Smart et al., 1969).

Figure 5.13 is an attempt to reproduce Figure 3 (Figure 5.10) from Smart et al. (1969). Geomagnetic cutoff latitudes from about 74 degrees (low energy) to 62 degrees (high energy) assuming quiet geomagnetic conditions were calculated that correlate very well with Smart et al. (1969). Also apparent in Figure 5.12 and 5.13 is an asymmetry near noon local time as described in the literature.
Figure 5.13: T96-generated daily variation of proton cutoff energies, 255 deg.

Figure 5.14 shows that geomagnetic cutoff latitudes along the 75E meridian range from 72 degrees (low energy) to 58 degree (high energy). An important observation is that the geomagnetic cutoff latitudes are different at specific local times. For example, at 0900 LT along the 255 degree meridian the cutoff latitude for 3.16 MeV protons is roughly 72 degrees; along the 75 degree meridian the 3.16 MeV cutoff latitude is roughly 69 degrees.
Figure 5.14: T96-generated daily variation of proton cutoff energies, 75 deg.

Figure 5.15 shows a different view of the same data. In Figure 5.15 the geomagnetic cutoff latitude is a function of local time for each energy level along the 105 degree meridian. The 100 MeV curve is fairly flat, whereas the cutoff latitude for the lower energy protons varies significantly. Apparently protons with higher energy (above about 30 MeV) seem to have a well-defined cutoff latitude. Protons with lower energy, however, seem to have a variable cutoff latitude that depends on the time of day as well as the orientation of the geomagnetic field as defined by the orientation of the geomagnetic and geographic poles.
This seems to contradict portions of the data from Shea et al. (1965), especially the cutoff energies for many latitude/longitude pairs. For example, Shea et al. (1965) show the cutoff rigidity at College, USA (64.85, 212.16) as 0.54 GV (roughly 150 MeV proton). Based on the results discussed previously, this dissertation work would agree with that result. Shea et al. (1965) give the cutoff rigidity at Alert, Canada (82.5, 297.6) as 0.05 GV (roughly 1 MeV proton). The results of this dissertation work would indicate that to define the cutoff latitude for Alert, Canada, it would be necessary to know both LT and UT. Hence, it would appear that an important consideration is missing, especially for low-energy protons. Since protons with energy ~1-20 MeV are responsible for the
majority of HF absorption during PCA events, this new finding could have important significance.

**Smart et al., 2006**

Figure 1 in Smart et al. (2006) is very well known and has been used in various forms in several publications (Sauer & Wilkinson, 2008; Smart et al., 2000, 1999). Figure 5.16 is taken from Sauer & Wilkinson (2008) and is attributed to Smart et al. (1999). Figure 5.16 represents an interpolation of the Smart et al. (1999) world grid data of cutoff rigidities in terms of the previously discussed McIlwain L-shell parameter for the inner geomagnetic field allowing the cutoff rigidity to be plotted in terms of the cutoff energy and invariant cutoff latitude (Smart et al., 2000). Figure 5.16 plots the proton cutoff latitude as a function of proton energy (1-1000 MeV) and as a function of $K_p$ index. The shaded region represents the applicable energy range covered by this dissertation work (10-100 MeV).
Figure 5.16: Proton cutoff energy vs. *invariant latitude*. Both are shown as functions of geomagnetic activity. Shaded region represents 10-100 MeV energy range (Sauer & Wilkinson, 2008).

Figure 5.17 shows the result from this dissertation work in the same format as Figure 5.16. Figure 5.17 shows the average of 576 data points for each energy level (24 hours UT times 24 hours LT). This is the same method used by Smart et al. (2006, 2000, 1999) to generate Figure 5.16. Again, the shaded area of Figure 5.17 represents the energy regime common to Figure 5.16, 5.17, and 5.18.
Figure 5.17: T96-generated plot for proton cutoff energy. This format is similar to Sauer & Wilkinson, (2008). Shaded area represents 10-100 MeV energy range, vertical axis represents magnetic latitude.

Figure 5.18 shows a comparison between the data from this dissertation work (dashed line) and the data from Smart et al. (1999) as quoted by Sauer & Wilkinson (2008) (solid line). The data are in good agreement.
**Figure 5.18**: Comparison of average proton cutoff *invariant latitude*. Shown are T96 proton energies between 10 and 100 MeV with the interpolated Smart et al. (2006) simulations at three geomagnetic activity levels.

Figure 5.19 shows the standard deviation of the average variance of the simulated data averaged over 576 data points for each energy level. It appears that in all three cases (quiet, moderate, severe) the difference between the data from this dissertation work and the Smart et al. (1999) data is well within the standard deviation.
Figure 5.19: Standard deviation of T96 data used to generate Figure 5.18.

Also, Figure 5.19 illustrates that as proton energy increases, the standard deviation decreases. The larger sigma value for low-energy protons could be interpreted as additional noise due to penumbral transparency, for example. It is the hypothesis of this study that this is an indication of an inherent variation in the cutoff latitude that has yet to be fully explored, especially for protons with energy less than about 20 MeV.

Figure 5.20 is used with permission from Smart et al. (2006) and shows the cutoff latitude in geographic coordinates as a function of geographic longitude. Figure 5.21 uses the data from this dissertation to create similar contours. An important consideration is that Smart et al. (2006) uses $K_p$ index as the geostorm indicator, whereas this
dissertation work uses the $D_{st}$ index. Furthermore, the minimum value used by Smart et al. (2006) is $K_p = 2.0$ which represents a slightly enhanced geostorm condition. Hence, the expectation is that Figure 5.20 shows contours slightly equatorward compared to Figure 5.21.

**Figure 5.20:** Three-hour averaged cutoff rigidity contours. Shown are contours for quiet conditions plotted in geographic coordinates (Smart et al., 2006) used with permission from AGU. https://publications.agu.org/author-resource-center/usage-permissions/
Figure 5.21: T96-generated averaged cutoff rigidity contours. Panel 5.21a shows a three-hour average around 0000UT; Panel 5.21b shows a three-hour average around 1200 UT; Figure 5.21c shows the average over all UT.

An important aspect of this study was an attempt to quantify a hypothesized variation inherent to the cutoff latitude that in previous studies such as Smart et al., (2006) could have been considered noise.

Leske et al. 2001

Leske et al. (2001) provides an opportunity to compare observed results with modeled results of this study. They used SAMPEX data (see Baker et al., 1993) to
determine the cutoff latitude during 6 significant SEP events. In each case the data are tracked over 8 days. In their study they defined the cutoff latitude to be the invariant latitude where the count rate is half of the average count rate above 70 degrees invariant latitude. Figure 5.22 (Figure 3 from Leske et al., 2001) shows the orbit-averaged cutoff latitude as a function of time during the 6 SEP events. It also plots the $D_{st}$ index during those events. The shaded areas of Figure 5.22 show the $D_{st}$ range used for this dissertation work for moderate and severe geomagnetic storm conditions. Figure 5.23 (Figure 7 from Leske et al., 2001) shows a polar plot of individual cutoff crossings, specifically for the SAMPEX MAST Z2 rates (Baker et al., 1993) which include He ions with an energy of 8-15 MeV/nucleon, which compares with 30-60 MeV protons (Leske et al., 2001).
Figure 5.22: Orbit-averaged cutoff invariant latitude as a function of time. T96 is compared to the geomagnetic activity index $D_{st}$ from Leske et al. (2001). The shaded box was added to show the 50-100 $D_{st}$ range which is used to model moderate to severe conditions for this dissertation work. Data from these plots was interpolated as described in the text.

Figure 5.23 shows that for the Northern Hemisphere the average cutoff latitude (black circle) is about 64 degrees invariant latitude. This average cutoff latitude is based
on observed conditions over a variety of $D_{ste}$ indices. Hence, for comparison purposes a weighted average based on modeled data from this dissertation work allowed the determination of cutoff latitudes under similar conditions. By observation of Figure 5.22 it was determined that of the 48 total days 39 experienced quiet conditions, 7 were moderate, and 2 severe. Hence, the weighted average over quiet, moderate, and severe conditions for 31.6 and 56.2 MeV protons was determined. This study yielded an average cutoff latitude of 63.2 degrees geomagnetic latitude which compares very favorably with Leske et al. (2001). Figure 5.24 shows the cutoff latitude for 31.6 and 56.2 MeV superposed over Figure 5.23. Agreement is excellent.
Figure 5.23: Polar plot of individual cutoff crossings. Shown are crossings for ~8-15 MeV/nucleon He during the time periods indicated, in invariant latitude vs. magnetic local time. Black circle represents average location of cutoff latitude (Leske et al., 2001).
Figure 5.24: Cutoff latitude for 31.6 and 56.2 MeV protons superimposed over Figure 5.23. 31.6 MeV protons are shown as solid red circle, 56.2 MeV protons are shown as dashed red circles.

Hence, confidence is high that the new results obtained are based on reasonable assumptions.
Discussion

In presenting the cutoff latitude simulation data the variables being considered were proton energy, UT, LT, and three levels of geomagnetic activity. As discussed previously, the equinox was used for the input day parameter as was a radially-directed solar wind flow. It was observed that the trend was generally consistent with expected results in that the important boundaries moved equatorward with increasing geomagnetic activity. This is also the case with increasing proton energy. The figures demonstrating these trends do not represent well defined cutoff latitudes; rather, they illustrate bands of significant UT/LT dependence. An exception to this might be Figures 5.7d, 5.8d, and 5.9d in which relatively well-defined cutoff latitudes are present at 100 MeV.

This discussion is an attempt to expand on the UT/LT interpretation of the cutoff latitude. Appendix F provides our definition and procedure for identifying a UT and LT dependence. Appendix F also relates these two time parameters via the terrestrial longitude, introducing a third parameter. In the end, an objective was to determine if the cutoff latitude has a preferred organizational parameter. If so, how could that parameter be best defined?

The three panels of Figure 5.25 show the geomagnetic cutoff latitude for 1 and 100 MeV protons for specific UT and LT times. Figures 5.25a and 5.25b show that the boundary varies over 24 hours when either LT or UT are fixed, introducing longitudinal variability. Panel 5.25c fixes the longitude and shows how the boundary varies over 24 hours. Figure 5.25a plots the cutoff latitude of a 1.0 MeV proton. The solid black line shows the cutoff latitude as a function of LT (UT fixed at 18.00) and the dashed black
line shows the cutoff latitude as a function of UT (LT fixed at 04.00). In both cases there is a significant variation in the cutoff latitude. Figure 5.25b shows the cutoff latitude for a 100 MeV proton for the same constant UT (solid line) and LT (dashed line). Though the variation is not as great, there is still an obvious variation regardless of LT or UT being held constant. Hence, there is some effect causing a variation in the cutoff latitude that is independent of proton energy, but probably depends on specific geometry of the Earth and solar wind that is specified by combinations of \{LT, UT, Longitude\}. 

**Figure 5.25:** Constant UT vs. constant LT for varying proton energies. Figure 5.25a shows the 1.0 MeV cutoff dependence on both UT and LT where a unique LT and UT have been selected for each line. Figure 5.25b repeats this for 100 MeV protons. Figure 5.25c shows the 1.0 and 100 MeV UT dependence for a fixed longitude of 15 degrees E.
Figure 5.25c assumes a constant longitude, in this case 15 degrees geographic longitude. The solid black line represents the cutoff latitude of the 100 MeV protons, while the dashed line is the cutoff latitude for the 1 MeV protons. Notice that the 100 MeV protons are basically flat lined along 64 +/- 0.5 degree geomagnetic latitude while the 1 MeV proton has a significant variation 73 +/- 3 degrees. The horizontal axis of Figure 5.25c is UT. Hence, each point on the plot represents the cutoff latitude along the constant longitude at a specific UT. As stated previously, UT, LT and longitude are linearly related. In this case, \( LT = UT + 10 \) (hr) given the constant longitude of 15 degrees. Unlike Figures 5.25a and 5.25b were there was little difference in energy dependence of the cutoff latitudes, Figure 5.25c shows a cutoff latitude effect that is energy-dependent. Certainly one could argue that the higher energy proton would be less susceptible to change due to its higher rigidity. But when one compares Figure 5.25c with Figures 5.25a and 5.25b it is evident that both 1 MeV and 100 MeV protons are subject to a very similar variability.

In Appendix F Figure F.2 provides a format that can provide information on how the cutoff boundary might be organized in the \{LT, UT, Longitude\} format. Figure 5.26 shows 100 MeV protons while Figure 5.27 shows the 1 MeV cutoff latitudes plotted in a LT versus UT format. The geomagnetic cutoff latitudes are color coded as given by the color key on the right of each panel. In this LT versus UT format a diagonal line parallel to the 0:0 (LT:UT) to 24:24 (LT:UT) corresponds to a line of constant longitude (see Appendix F).
Figure 5.26: 100 MeV proton cutoff geomagnetic latitude presented in LT vs. UT format. The color key ranges from 56 to 68 degrees of geomagnetic latitude.

In Figure 5.26 the highest 100 MeV cutoff latitude is at 66 degrees and is identified by the dark orange/red color band. This band occurs predominantly along the diagonal line $LT = UT + 2$. From Figure F2 we see this equates to a constant longitude of 30 degrees. Likewise, the lowest cutoff boundary is at 55 degrees and is shown in deep blue, occurring at $LT = UT - 6$. From Figure F2 we find this constant longitude to
be 270 degrees. Therefore, Figure 5.26 clearly shows that the 100 \( MeV \) proton is well organized by a longitudinal constancy that is independent of LT and UT. It is important to notice, however, that along lines of constant UT and LT (vertical/horizontal) the 100 \( MeV \) proton shows significant variance as illustrated by the varying color bands. The cutoff boundary latitude does have a strong longitude modulation between geomagnetic latitudes of 55 and 66 degrees. This is quite different from the expectation that the 100 \( MeV \) cutoff boundary might be associated with a specific inner geomagnetic field L-shell. A specific L-shell of the inner magnetosphere dipole field would have the same geomagnetic latitude at all longitudes.
Figure 5.27: 1.0 MeV proton cutoff geomagnetic latitude presented in a LT vs. UT format. The color key ranges from 63.5 to 82 degrees of geomagnetic latitude.

In Figure 5.27 the diagonal symmetry is much less evident. Certainly there are elements of longitudinal constancy, but only in portions of Figure 5.27. For example, an upward diagonal band is evident near the center of Figure 5.27. However, this band does not exhibit the linear relationship as illustrated in Figure 5.26. Hence, the 1 MeV proton is not specifically organized by a longitude dependence that is independent of LT or UT.
The open/closed boundary is susceptible to the UT effect as discussed in Chapter 4. Since the OCB marks the transition region from open to closed field lines, any proton regardless of energy can enter the polar cap region which, by definition, is at or above the OCB. It was demonstrated that the offset between the cutoff latitude and the OCB is about 4 degrees for 1 MeV protons and roughly 15 degrees for 100 MeV protons. Hence, it could be argued that within a reasonable margin of error, 1 MeV protons require open field lines to propagate into the upper atmosphere, while 100 MeV protons can penetrate equatorward well beyond the open-closed magnetic field topology.

Figure 5.4 showed that on average, the difference between the 1 and 100 MeV cutoff latitudes is 10 degrees. One potential consequence of this could be that the proton spectrum equatorward of the OCB progressively becomes harder since the lower energy protons are unable to penetrate to these lower latitudes. This, in turn, could lead to different ionization altitude profiles in the atmosphere. This is a significant consideration because it is the lower energy protons (1-20 MeV) that are most responsible for increased ionization and thus HF radio wave absorption in the D-region of the ionosphere.

Much of the scientific literature regarding cutoff latitudes and rigidities concentrates on higher-energy protons that reach lower in the atmosphere and have a direct human impact. For example, Smart et al. (2006) discuss two important points. The first is that protons with energy greater than 10 MeV can penetrate a space suit; the second is that "...the most important energies are those that can penetrate spacecraft shielding and continue into the human body and affect the blood forming organs."
Hence, much of the research in this important field is devoted to proton energies much
greater than the focus of this dissertation work, higher than 400 \(MeV\)!

This dissertation work has shown, in agreement with the scientific literature, that
protons with energy above about 30 \(MeV\) are not subjected to a UT variability. This
dissertation work has also shown that protons in the \(\sim 1-20 MeV\) range do experience a
UT variability. This variability is an important consideration when modeling PCA event
effects on HF absorption.
CHAPTER VI

THE PENUMBRA

Introduction

What exactly is a penumbra? There are two important, well-defined regions of a shadow, such as during an eclipse. The first is called the umbra, from the Latin for "shadow." The umbra is the region that receives no light from the source. It is in complete shadow. The second region is called the penumbra, including the prefix pen- from the Latin "paene" meaning "nearly, or almost." Hence, the penumbra is that region that receives some portion of light from the source and is considered to be in partial shadow.

The units used to measure the width of the penumbra depend upon the method used. In the Rigidity Method, the width of the penumbra is measured in rigidity, typically in GV. Shea et al. (1965) determined the width of the penumbra for many geographic locations. Figure 6.1 is a table from Shea et al. (1965) wherein they show that the penumbra width was about 0.05 GV near Apatity, USSR (67.55 N, 33.33 E). This equates to a penumbral width of roughly 1.33 MeV. Using the Latitude Method, the width of the penumbra is measured in degrees. Figure 6.2 shows that the penumbra for a 10 MeV proton is on average about 2.0° wide. Hence, it is important to know the method being used. For the remainder of this section the penumbra width is based on the Latitude Method. Hence the width is measured in degrees.
Figure 6.1: Calculated cutoff rigidity for specific geographic locations. Original data is from Shea et al. (1965). Shaded area added to highlight the penumbral region; location used for this study has been boxed in.

<table>
<thead>
<tr>
<th>Place</th>
<th>Geographic Latitude</th>
<th>Geographic Longitude</th>
<th>Penumbral Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahmedabad, India</td>
<td>23.01</td>
<td>72.61</td>
<td>15.94</td>
</tr>
<tr>
<td>Alert, Canada</td>
<td>82.50</td>
<td>297.67</td>
<td>&lt;0.05</td>
</tr>
<tr>
<td>Alma Ata, USSR</td>
<td>43.20</td>
<td>76.94</td>
<td>6.77</td>
</tr>
<tr>
<td>Apatity, USSR</td>
<td>67.55</td>
<td>33.33</td>
<td>0.68</td>
</tr>
<tr>
<td>Bariloche, Argentina</td>
<td>−41.15</td>
<td>288.98</td>
<td>10.36</td>
</tr>
<tr>
<td>Belgrano, Antarctica</td>
<td>−77.97</td>
<td>321.20</td>
<td>0.80</td>
</tr>
<tr>
<td>Bergen, Norway</td>
<td>60.40</td>
<td>5.32</td>
<td>1.18</td>
</tr>
<tr>
<td>Berkeley, United States</td>
<td>37.86</td>
<td>237.70</td>
<td>4.88</td>
</tr>
<tr>
<td>Brisbane, Australia</td>
<td>−27.50</td>
<td>153.01</td>
<td>7.75</td>
</tr>
<tr>
<td>Budapest, Hungary</td>
<td>47.50</td>
<td>18.90</td>
<td>4.79</td>
</tr>
<tr>
<td>Buenos Aires, Argentina</td>
<td>−34.58</td>
<td>301.50</td>
<td>10.91</td>
</tr>
<tr>
<td>Calgary, Canada</td>
<td>51.08</td>
<td>245.91</td>
<td>1.17</td>
</tr>
<tr>
<td>Cambridge, United States</td>
<td>41.38</td>
<td>286.88</td>
<td>1.79</td>
</tr>
<tr>
<td>Cape Schmidt, USSR</td>
<td>68.87</td>
<td>180.31</td>
<td>0.64</td>
</tr>
<tr>
<td>Charlestown, Rhode Island</td>
<td>−14.21</td>
<td>961.25</td>
<td>13.10</td>
</tr>
</tbody>
</table>

*P_e (Q&W)* and †P_e (Calc.)*
Figure 6.2: T96-generated penumbral width in degrees. Penumbral region is shown as a function of proton energy. The horizontal axis is the log scale of proton energy; the vertical axis is the average width.

How does a penumbral region relate to finding the cutoff latitude discussed in Chapters 2 and 6? Apparently there is no exact cutoff latitude. Rather, there is some upper cutoff latitude above which all trajectories are allowed and a lower cutoff latitude below which all trajectories are forbidden. The same could be said regarding rigidities and the Rigidity Method, i.e. there is an upper rigidity above which all trajectories are allowed and a lower rigidity below which all trajectories are forbidden. The region between the upper and lower limits would be considered the penumbra, or penumbral region. In this research the upper cutoff latitude was used. The upper cutoff latitude is
defined as the last allowed trajectory before the first forbidden trajectory. Similarly, the lower cutoff latitude is defined as the last allowed trajectory. Another useful definition is the effective cutoff latitude which is defined as an average of the upper and lower cutoff latitudes (Smart et al., 2000).

Since this dissertation research focused primarily on the upper cutoff, it was important to determine if the penumbra region was sufficiently wide as to introduce significant uncertainty in the cutoff latitude results. Using only the upper cutoff would not be realistic, for example, if the penumbra had a width of 15 degrees.

Another interesting property of the penumbra is its transparency. Smart, et al., (2006), using the Rigidity Method, define the transparency as a quantity that describes the particle flux transmitted through the penumbral region. A transparency approaching unity means most of the flux in the penumbra is allowed through the geomagnetic field, while a transparency approaching zero means that very little flux is allowed.
Finding the Penumbra

For this study of the penumbra, the cutoff latitude data for quiet conditions was used. The program was modified to begin one degree above the previously determined cutoff latitude (upper bound) and evaluated latitudes moving equatorward in 0.20 degree increments for a total of 61 steps, covering 12 degrees of latitude. The program then evaluated each set of data to determine the upper, lower, and effective cutoff latitudes as well as the width in degrees of the penumbra for each energy level. Figure 6.2 shows the average width of the penumbra as a function of proton energy. At the extremes (1, 100 MeV) the width of the penumbra is less than 1 degree. The maximum average width of about 1.80 degrees occurs between 5.62 and 10 MeV. Note that each point in Figure 6.2 represents an average of 576 data points, 24 LT hours over 24 UT hours.

Figure 6.3 shows the average width of the penumbra for the 9 energy levels used (1.00, 1.78, 3.16, 5.62, 10.0, 17.8, 31.6, 56.2, and 100 MeV) as functions of LT and UT. The LT average takes the penumbra width for each of the 24 UT hours for a specific LT and calculates the average. The UT average takes the penumbra width for each of the 24 LT hours for a specific UT and calculates the average.
Figure 6.3: Average penumbral width in degrees for 9 energy levels. These are the 9 energy levels used for this study. The solid black line represents the LT average and the solid gray line represents the UT average as described in the text.

The grey line in each plot is the UT average; the black line is the LT average. One noticeable characteristic is the relatively flat line for the UT average throughout the 9 panels. In each of the first 5 (3.3a-3.3e) the UT average line trends up slightly as energy increases to about 10.0 MeV, then trends downward as energy increases to 100 MeV. Furthermore, as energy increases the variability of the UT average decreases (the line gets flatter). At the extreme, the 100 MeV UT average is flat.
In Figure 6.3a (1 MeV) the LT average (black line) has a distinct maximum of 3.0 degrees at about 19.00 LT and a smaller peak of about 1.0 degree near 03.00 LT. A minimum trough is evident from about 04.00 - 14.00 LT. Comparing the LT curve of Figure 6.9a with the other low-energy figures shows a pattern developing.

First, the peak seems to become less defined, broader and not as sharp. Second, the peak shifts to the left (earlier LT). In Figure 6.3e (10 MeV) the minima and maxima have reversed positions. In Figure 6.3a there is a trough during daylight hours and peaks during nighttime, while Figure 6.3e shows a maxima table or plateau during daylight hours and minima occurring during nighttime hours. Beginning with Figure 6.3f through 6.3i the two lines converge until Figure 6.3i shows nearly identical lines for the LT and UT averages.

Figure 6.4 shows only the UT average for all 9 energy levels, superimposed into one plot. What stands out here is that the UT average varies only slightly within each energy level. Furthermore, the overall variation for the UT average is small, generally less than a degree. Apparently the penumbra experiences little if any UT effect across the energy regime examined; the preceding figures show this clearly. There is a slight energy effect related to the UT average of the penumbra, on the order of only a fraction of a degree between bands, overall a degree or two.
Figure 6.4: UT average of penumbral width in degrees. Shown are the 9 energy levels evaluated in this study.

An LT effect, however, is more noticeable. It seems that the LT effect is not uniform across the study's energy spectrum. It is evident that the 1.0 MeV penumbra has a day/night variation with minima occurring during daylight hours. The 10.0 MeV penumbra shows a similar variation but with minima occurring during nighttime hours. The 100 MeV penumbra seems to be immune to LT effects as well as UT effects.

Figure 6.5 shows a 3-dimensional analysis of the penumbra width as a function of UT (horizontal axis) and LT (vertical axis). Figure 6.5a (1.0 MeV) shows peak values (greatest penumbra width) during nighttime hours after sunset and another slight peak in the pre-dawn hours. Also evident is the shift toward daytime maxima in Figure 6.5c (10.0 MeV), as energy increases. Figure 6.5d (100 MeV) contains fewer maxima,
indicating that the penumbra width becomes fairly consistent across the energy spectrum investigated.

**Figure 6.5:** Penumbral width in degrees shown in the LT vs. UT format. Shown are 1.00 (a), 5.62 (b), 10.0 (c), and 100 (d) MeV protons. Here the horizontal axis is UT, the vertical axis is LT, and the color scale runs from 0-9 degrees geomagnetic latitude.

These UT and LT behaviors are interesting and warrant further investigation, but are not critical to this research. What is important is that the penumbra region is on the order of 1-3 degrees in width and does not represent a significant variability in the cutoff latitude, especially for the 1-20 MeV protons, which are most responsible for PCA events.
Physical Interpretation

Recall that in this dissertation, as in other studies, the initial velocity of the proton is "vertical," meaning it moves out radially from Earth's surface. This could indicate that at different latitudes, although the vertical speed is the same, the speed parallel to the magnetic field line (and therefore the pitch angle) is different. Hence, an important variable, the pitch angle, is not considered. It is very likely that varying the pitch angle as well as the azimuthal angle of the initial velocity vector could have an impact on the width of the penumbral region as well as proton trajectories and will be considered in further studies.
CHAPTER VII

CONCLUSIONS

High Frequency Radio Wave Absorption

An important concept from Chapter 1 is the penetration depth of energetic protons. It was shown that protons with energy \( \sim 1 - 20 \text{ MeV} \) have easy access to the D-region, with 20 MeV protons most responsible for increased ionization within the D-region, leading to increased absorption. Hence, the conclusions from the remaining chapters will focus specifically on energetic protons between 1-20 MeV.

It was also shown that near the bottom of the D-region the plasma frequency is much less than an HF radio wave frequency. This would indicate that within the very low frequency (VLF) range the D-region should form the upper boundary of a waveguide. This was the motivation for Bain and Harrison (1972), and is still an active research field. Researchers are attempting to use waveguide theory to determine D-region characteristics such as electron density from VLF signals propagating along the D-region waveguide (Marshall, 2017).

There are several specific questions to address. For example, Handzo et al. (2014) demonstrated a relationship between solar flares and an increase in electron density above 100 km. Nicolet & Aikin (1960) postulated that as hard X-rays penetrate deeper into the D-region one should expect "a general lowering of the layer." This certainly seems reasonable but how does one obtain D-region data at that low altitude? Perhaps VLF studies will provide an answer. Davies (1965) mentioned a solar flare from
11 July 1961 that increased absorption "...by a factor of 10." What change in electron density would need to occur for that significant of a change in absorption to take place? Hopefully incoherent scatter radar will provide some clues.

Furthermore, how do storm-related energetic precipitation events depend on solar wind parameters? What impact do these events have on HF communications? With answers to those questions it may be possible to assist the various stakeholders that rely on HF radio communications (Odenwald, 2010).
The L-shell Parameter

Two important conclusions have been made regarding the L-shell parameters as well as the *invariant latitude*. First, for protons above about 30 MeV, this dissertation work was able to validate the current scientific literature in that the L-shell parameter and *invariant latitude* are fixed. Using either the rigidity cutoff method or the latitude cutoff method (Chapter 2, Cutoff Latitude), the L-shell parameter and associated *invariant latitude* are well-defined and do not exhibit a UT-dependent modulation.

Second, for protons below about 20 MeV, this dissertation work showed a UT-dependent variation in the L-shell parameter. One consequence of this variation is that when describing the cutoff rigidity or cutoff latitude for protons with energy less than about 20 MeV the *invariant latitude* alone does not provide a suitable description of the location due to the UT-dependent variation. To accurately describe the L-shell parameter and *invariant latitude* it is necessary to define the UT.

This makes sense when one considers that protons above about 30 MeV have lower cutoff latitudes, meaning their L-shell parameter will be related to the inner geomagnetic field. For T96, this inner field is modeled by the IGRF as explained in Chapter 2.4, the Tsyganenko Model. The inner field is not affected by solar wind conditions and, therefore, exhibits a dipole-like topology. Hence, geomagnetic latitude, *invariant latitude*, and the L-shell parameter are not dependent upon UT. Protons with energy below about 20 MeV will be governed by the external field which is highly variable due to solar wind conditions, does not exhibit a dipole-like topology, and is affected by the orientation of the geomagnetic/geographic poles (the UT-effect).
Several important findings have been made regarding the open/closed boundary. First, the geomagnetic latitude of the OCB depends on UT, even if all other space weather drivers are held constant. Second, since the cusp region is closely associated with the noon LT OCB, it experiences a UT-dependent modulation as well. This variation was shown to be greater than about 10 degrees in geomagnetic latitude. This variation suggests significant consequences. For example, if the OCB is a viable proxy for the cutoff latitude for low-energy protons, a 10 degree variation (over 1100 km!) could significantly alter the safest flight path during a transpolar route. Finally, since this work only considered a single day of the year (the vernal equinox) it is highly probable that seasonal variations exist as well.

Research regarding the open/closed boundary was motivated by an interest in knowing how energetic particles such as protons can penetrate equatorward of the OCB during PCA events, specifically searching for a workable proxy for the energy cutoff latitude of low-energy protons. Future research on this topic is to quantify the seasonal dependence of the OCB UT effect. Possible additional research will search for measurement data sets that can produce evidence of this UT effect.
The Cutoff Latitude

The primary focus of this work was to investigate the energy cutoff latitude and its relationship to polar cap absorption events. The importance of HF communication paths via polar regions has been discussed, demonstrating that the ability to describe current conditions as well as forecast future propagation conditions is a critical safety issue. This work was able to demonstrate several key findings regarding the energy cutoff latitude that have important consequences to modeling HF propagations conditions, especially during PCA events. First, it has been shown that the geomagnetic cutoff latitude depends on LT and UT, even if all other space weather drivers are held constant. Second, it was shown that the UT effect is independent of proton energy. An important consideration is that PCA events are primarily driven by protons with energies between about 1-20 MeV. This same energy regime is not well-defined by the L-shell parameter as was discussed previously.

This research was motivated by a desire to improve the sensitivity of current PCA predicting models. It has been shown that an important sensitivity consideration is the UT effect, especially for solar energetic protons below about 30 MeV. Future research could include a seasonal modification to the UT and LT effects.
The Penumbra

The penumbral region was not initially considered an important component of this dissertation work. However, it became apparent that an important consideration in both study and application was the width of the penumbra and the associated penumbral transparency. Frankly, the deeper this work delved into the penumbra, more questions arose than were answered. Still, there were several key findings. First, for lower-energy protons there was a significant difference between the LT average and the UT average. Evidently LT is an important consideration for protons with energy below about 20 MeV. Protons with energy above about 30 MeV seemed to experience minimal variation due to LT or UT.

Figure 7.1 provides an illustration of the significance of the penumbra region in terms of proton energy intensity (flux). The \textit{x-axis} represents geomagnetic latitude; the \textit{y-axis} represents flux. \textit{Figure 7.1 is for illustration purposes only and does not represent specific data nor is it drawn to scale.} Figure 7.1 shows the proton energy flux of three levels of proton energy: 1.00, 10.0, and 100 MeV. Illustration components that refer to 100 MeV protons are shown in red, 10 MeV protons in green, 1 MeV protons in purple. The OCB region is shown in blue. Using the illustrated parameters, the OCB occurs at about 87 degrees, but is highly variable depending on UT; out to about 85 degrees geomagnetic latitude it is apparent that all illustrated proton energies are present within that specific region; from 85 down to about 75 degrees geomagnetic latitude the 10 and 100 MeV protons are still showing constant flux; from 75 down to 65 only the 100 MeV flux is constant. Since the 100 MeV protons have a deeper penetration depth (Chapter
1.8, PCA Events, Figure 1.6) they do not contribute significantly to increased ionization in the D-region. Hence, from Figure 7.1 it appears that increased ionization would occur down to about 65 degrees geomagnetic latitude. However, from about 75 down to 65 degrees the amount of ionization would drop off as the 10 MeV flux decreases toward zero. Important considerations are: the 100 MeV cutoff latitude and penumbra region experience no UT variation; the 10 MeV cutoff latitude and penumbra experience some UT variation; the 1 MeV cutoff latitude, the associated penumbra region, and the OCB all experience significant UT variation.

![Illustration 7.1](image)

**Figure 7.1:** Illustration showing arbitrary flux for 1.0, 10, and 100 MeV protons. This figure illustrates a suggested penumbral region. Shaded regions represent areas of variability due to UT or other dependencies.
Since PCA events are linked to protons in the 1-20 $MeV$ range, LT and UT modulations must be considered. Furthermore, it was found that the average width of the penumbral region is greatest at about 10 $MeV$. This seems counter-intuitive and should be investigated further.
Final Comments

The driving force behind this dissertation research was a desire to improve existing forecasting techniques regarding PCA events and their effect on HF radio wave propagation. The hope was to provide motivation for a physics-based model of HF absorption due to PCA events, using information learned through this work. Currently, the gold-standard of HF absorption prediction during PCA events is the D-region Absorption Prediction tool (D-RAP) as discussed in Chapter 5.1. Studies in this work of the open/closed boundary, L-shell parameter, penumbral region, and the energy cutoff latitude have provided an important parameter that could help improve D-RAP forecasting capabilities: a UT-dependent variation that is not currently accounted for in existing D-region models. D-RAP uses the L-shell parameter and invariant latitude to account for geomagnetic cutoff effects. However, this dissertation work has shown that using the L-shell parameter or the associated invariant latitude may not provide the most accurate way to account for UT-effects since at the lower energies most responsible for increased ionization in the D-region the L-shell parameter and invariant latitude depend on UT.

Several additional investigations are suggested that have the potential to improve forecasting of PCA effects. These could include: apply observed spectral fluxes to this relative energy study; examine how the spectrum changes as a function of latitude, moving equatorward from the OCB; study of HF radio wave absorption changes as a function of latitude, moving equatorward from the OCB; investigation of HF radio communications, including propagation and absorption, through the penumbral regions.
It is the conclusion of this dissertation work that D-RAP, and PCA modeling in general, could be improved by explicitly including UT and LT dependencies in their efforts.
REFERENCES


APPENDICES
Appendix A

Derivation of the Index of Refraction

It has been shown that the phase velocity of a radio wave propagating through the ionosphere is given as,

\[ v_p = \frac{c}{n} \]  \hspace{1cm} A.1

Here \( n \) is defined as the index of refraction. Since the ionosphere is defined as a dispersive medium, we expect the index of refraction to be a function of the frequency of the radio wave.

We also expect the group velocity to be defined in terms of some index of refraction relative to the group velocity,

\[ v_g = \frac{c}{n'} \]  \hspace{1cm} A.2

with \( n' \) being the index of refraction of the ionosphere relative to the group velocity.

Recasting the above in terms of the indices of refraction we find that,

\[ n = \frac{c}{v_p} \quad \text{and} \quad n' = \frac{c}{v_g} \]

Recalling wave theory from electrodynamics, we again recast the equations for phase and group velocities in terms of wave properties,

\[ v_p = \frac{\omega}{k} \quad \text{and} \quad v_g = \frac{d\omega}{dk} \]  \hspace{1cm} A.3, A.4
Here $\omega$ is defined as the angular frequency of the wave and $k$ as the wave number.

Inserting the equation for group velocity into the equation for index of refraction we find that,

$$ n' = \frac{c}{v_g} $$

$$ \implies n' = c \frac{dk}{d\omega} \quad \text{(A.5)} $$

But we can define the angular frequency of the radio wave in terms of the frequency,

$$ \omega = 2\pi f $$

$$ \implies n' = c \frac{dk}{2\pi df} \quad \text{(A.6)} $$

Equating the two phase velocity equations gives us an equation for $k$,

$$ v_p = \frac{c}{n} \text{ and } v_p = \frac{\omega}{k} = \frac{2\pi f}{k} $$

$$ \implies k = \frac{2\pi f}{c} n(f) \quad \text{(A.7)} $$

And again, we recall that the index of refraction, $n$, is a function of frequency. Then taking the derivative of $k$ with respect to $f$,

$$ \frac{dk}{df} = \frac{2\pi n}{c} + \frac{2\pi f}{c} \frac{dn}{df} \quad \text{(A.8)} $$

we can solve for the index of refraction relative to the group velocity. Recall that,

$$ n' = \frac{c}{2\pi} \frac{dk}{df} $$

$$ \implies n' = \frac{c}{2\pi} \left[ \frac{2\pi n}{c} + \frac{2\pi f}{c} \frac{dn}{df} \right] $$

Cancelling terms we find that,
Furthermore, it was shown previously that for a weakly-ionized gas the index of refraction is defined by,

\[ n^\prime = [n(f) + f \frac{dn}{df}] \]  

Here, \( f_p \) and \( f \) are the plasma frequency and radio wave frequency respectively.

Inserting the above into the equation for \( n^\prime \) we find that,

\[ n^\prime = \frac{1}{\sqrt{1 - \frac{f_p^2}{f^2}}} = \frac{1}{n} \]  

Now we define the group velocity in terms of the index of refraction relative to the phase velocity,

\[ v_p = \frac{c}{n} \text{ and } v_g = \frac{c}{n} \]

\[ \Rightarrow v_g = \frac{c}{\left(\frac{1}{n}\right)} = cn \]

Hence, the equations for phase velocity and group velocity become,

\[ v_p = \frac{c}{n} \text{ and } v_g = cn \]
Appendix B

Derivation of the Absorption Coefficient

Davies (1965) defines the absorption coefficient generally as,

$$\kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{1}{\mu} \right) \left( \frac{n_c v}{\omega^2 + v^2} \right)$$

But where does this absorption coefficient have its origins? We turn our attention to basic electrodynamics and Maxwell’s equations. Recall that in free space,

$$\nabla \cdot \mathbf{E} = 0 \quad \nabla \cdot \mathbf{B} = 0 \quad \text{B.1, B.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad \text{B.3, B.4}$$

These represent a set of first-order, coupled, partial-differential equations. These can be decoupled by applying the curl,

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\Rightarrow \nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad \text{B.5}$$

and the same can be shown for the magnetic field. Using separation of variables we can solve the above wave equation showing that,

$$\mathbf{E} = E_0 \cos(kx_1 - \omega t) \quad \text{B.6}$$

and $x_1$ is the direction of propagation. In exponential terms the preceding becomes,

$$\mathbf{E} = E_0 e^{i(kx_1 - \omega t)} \quad \text{B.7}$$

In a dispersive media the electric field can be defined by a complex wave equation,

$$\nabla^2 \mathbf{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \text{B.8}$$

which allows complex solutions of the form,
and we note that \( \vec{k} \) is the complex wave number and can be defined as,

\[
\vec{k} = \sqrt{\varepsilon \mu_0} \omega
\]

We can recast the complex wave number in terms of its real and imaginary components,

\[
\vec{k} = k + i \frac{\kappa}{2}
\]

In the above, \( \kappa \) is defined as the absorption coefficient. If we define the parameter \( \alpha = \frac{\kappa}{2} \) and recall that the intensity of the wave falls off as \( e^{-2\alpha x} \), then,

\[
\Rightarrow \vec{E} = \vec{E}_0 e^{-\kappa x} e^{i(kx_1-\omega t)}
\]

Davies (1965) defines the absorption coefficient as "...a measure of the decay of amplitude per unit distance,"

\[
\kappa = \chi \frac{\omega}{c}
\]

\[
\Rightarrow E = E_0 e^{\left(\chi \frac{\omega}{c} \right)x_1} e^{i(kx_1-\omega t)}
\]

and \( \chi \) is defined as the imaginary part of the complex index of refraction. Thus we see that the absorption coefficient has its origins in electrodynamics and is derived from Maxwell's equations.

Davies (1965) then makes the connection between the complex index of refraction

\[
n^2 = (\mu - i\chi)
\]

where \( \mu \) is the real component and \( \chi \) is the imaginary component, and the absorption coefficient

\[
\kappa = \frac{\omega}{c} \chi
\]
by combining them to show that,

$$\kappa = \frac{e^2}{2\epsilon_0 mc} \left( \frac{1}{\mu} \right) \left( \frac{n_{cv}}{\omega^2 + \nu^2} \right)$$
Appendix C

Data Used to Determine Collision Frequency

The absorption results are based on 4 critical parameters: electron density, neutral density, electron temp, and collision frequency. However, collision frequency is a function of neutral density and electron temperature.

Electron Density

The electron density profile is based on work done by Bain & Harrison (1972). The altitude resolution is 1 km and covers the altitude range 50 < h < 100 km. Hence, it was important to have the same resolution for the other critical parameters. Bain & Harrison (1972) base their model on solar maximum at local noon.

Neutral Density

First we assume that the neutral density is constant throughout the D-region. The data set for neutral density comes from Kelley (1989). For this model we used Table B1, near sunspot maximum. This compares favorably with Bain & Harrison (1972) since their model is based on solar maximum as well. The problem is that Kelley (1989) use an altitude resolution of 10 km within the D-region. Therefore, we plotted their data points and plotted a best-fit exponential function that approximates Kelley (1989) fairly well,

\[ n_n \approx 5.0 \times 10^{20} \exp[-0.182h] \]  

C.1
Electron Temperature

Again, we used data from Kelley (1989), interpolating their results to get a 1 km resolution for altitude,

$$ T_e = 0.0012 h^3 - 0.147 h^2 - 0.2221 h + 517.6 $$  \[C.2\]

Collision Frequency

Here we turned again to Kelley (1989), using their equation for the electron-neutral collision frequency,

$$ \nu_{en} = [5.4 \times 10^{10}] n_n \sqrt{T_e} $$  \[C.3\]

The data set follows:

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Neutral ( \text{cm}^{-3} )</th>
<th>Electron ( m^{-3} )</th>
<th>Temp (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0</td>
<td>5.64E+16</td>
<td>1.41E+06</td>
<td>289</td>
</tr>
<tr>
<td>51.0</td>
<td>4.70E+16</td>
<td>2.82E+06</td>
<td>283</td>
</tr>
<tr>
<td>52.0</td>
<td>3.92E+16</td>
<td>5.62E+06</td>
<td>277</td>
</tr>
<tr>
<td>53.0</td>
<td>3.27E+16</td>
<td>1.07E+07</td>
<td>272</td>
</tr>
<tr>
<td>54.0</td>
<td>2.73E+16</td>
<td>1.91E+07</td>
<td>266</td>
</tr>
<tr>
<td>55.0</td>
<td>2.27E+16</td>
<td>3.09E+07</td>
<td>260</td>
</tr>
<tr>
<td>56.0</td>
<td>1.89E+16</td>
<td>4.68E+07</td>
<td>255</td>
</tr>
<tr>
<td>57.0</td>
<td>1.58E+16</td>
<td>6.46E+07</td>
<td>250</td>
</tr>
<tr>
<td>58.0</td>
<td>1.32E+16</td>
<td>7.94E+07</td>
<td>244</td>
</tr>
<tr>
<td>59.0</td>
<td>1.10E+16</td>
<td>8.91E+07</td>
<td>239</td>
</tr>
<tr>
<td>60.0</td>
<td>9.16E+15</td>
<td>8.32E+07</td>
<td>234</td>
</tr>
<tr>
<td>61.0</td>
<td>7.63E+15</td>
<td>7.41E+07</td>
<td>229</td>
</tr>
<tr>
<td>62.0</td>
<td>6.36E+15</td>
<td>6.31E+07</td>
<td>225</td>
</tr>
<tr>
<td>63.0</td>
<td>5.31E+15</td>
<td>5.13E+07</td>
<td>220</td>
</tr>
<tr>
<td>64.0</td>
<td>4.42E+15</td>
<td>4.37E+07</td>
<td>216</td>
</tr>
<tr>
<td>65.0</td>
<td>3.69E+15</td>
<td>4.90E+07</td>
<td>212</td>
</tr>
<tr>
<td>66.0</td>
<td>3.08E+15</td>
<td>8.71E+07</td>
<td>208</td>
</tr>
<tr>
<td>67.0</td>
<td>2.56E+15</td>
<td>1.91E+08</td>
<td>204</td>
</tr>
<tr>
<td>68.0</td>
<td>2.14E+15</td>
<td>2.63E+08</td>
<td>200</td>
</tr>
<tr>
<td>69.0</td>
<td>1.78E+15</td>
<td>3.55E+08</td>
<td>197</td>
</tr>
<tr>
<td>70.0</td>
<td>1.49E+15</td>
<td>4.57E+08</td>
<td>193</td>
</tr>
<tr>
<td>71.0</td>
<td>1.24E+15</td>
<td>5.50E+08</td>
<td>190</td>
</tr>
<tr>
<td>72.0</td>
<td>1.03E+15</td>
<td>6.46E+08</td>
<td>187</td>
</tr>
<tr>
<td>73.0</td>
<td>8.62E+14</td>
<td>7.59E+08</td>
<td>185</td>
</tr>
<tr>
<td>74.0</td>
<td>7.18E+14</td>
<td>8.91E+08</td>
<td>182</td>
</tr>
</tbody>
</table>
For the plot comparing absorption by region and frequency we also used data from a homework assignment to get electron densities up to 400 km.

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Electron Density (m⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.0</td>
<td>1.12e11</td>
</tr>
<tr>
<td>120.0</td>
<td>1.29e11</td>
</tr>
<tr>
<td>140.0</td>
<td>1.96e11</td>
</tr>
<tr>
<td>160.0</td>
<td>3.02e11</td>
</tr>
<tr>
<td>180.0</td>
<td>4.76e11</td>
</tr>
<tr>
<td>200.0</td>
<td>7.12e11</td>
</tr>
<tr>
<td>220.0</td>
<td>9.88e11</td>
</tr>
<tr>
<td>240.0</td>
<td>1.27e12</td>
</tr>
<tr>
<td>260.0</td>
<td>1.50e12</td>
</tr>
<tr>
<td>280.0</td>
<td>1.63e12</td>
</tr>
<tr>
<td>300.0</td>
<td>1.63e12</td>
</tr>
<tr>
<td>320.0</td>
<td>1.56e12</td>
</tr>
<tr>
<td>340.0</td>
<td>1.43e12</td>
</tr>
<tr>
<td>360.0</td>
<td>1.29e12</td>
</tr>
<tr>
<td>380.0</td>
<td>1.14e12</td>
</tr>
<tr>
<td>400.0</td>
<td>1.00e12</td>
</tr>
</tbody>
</table>
Appendix D

Case Studies

Davies (1965) provides several conditions, deriving absorption coefficients for each. Appendix D will examine three. In each case the assumption is non-deviative absorption. In addition, each version of the absorption coefficient will be given in terms of radians per second as well as cycles per second.

Case 1

In case 1, the radio wave frequency is much greater than the collision frequency ($\omega \gg \nu$) and any geomagnetic effects are neglected (low altitude, mid-latitude).

For case 1 Davies (1965) defines the absorption coefficient as,

$$\kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{\omega^2} \right]$$  \hspace{1cm} D.1

$$\sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{4\pi^2 f^2} \right]$$

$$\Rightarrow L_a = -8.69 \left( \frac{e^2}{8\pi^2\varepsilon_0 m_e c} \right) \int \frac{n_e \nu}{f^2} \, ds$$  \hspace{1cm} D.2

Note that for case 1 absorption varies inversely as the square of the wave frequency. It will be shown that within the D-region, case 1 holds when the radio wave frequency is greater than about 10 MHz.
Case 2

Here the radio wave frequency is much less than the collision frequency \((\omega \ll \nu)\) and again geomagnetic effects are neglected. Davies (1965) defines the absorption coefficient in this manner,

\[
\kappa \sim \frac{e^2}{2\epsilon_0 m_e c} \frac{n_e}{\nu}
\]

\[
\Rightarrow L_a = -8.69 \left( \frac{e^2}{2\epsilon_0 m_e c} \right) \int \frac{n_e}{\nu} \, ds \quad \text{D.3}
\]

For case 2 absorption varies inversely with the collision frequency.

It is interesting to note that in case 2 as the collision frequency increases, absorption decreases. Apparently this stems from the confinement of electron motions and the small amount of energy extracted from collisions. Evidently Equation D3 is intended specifically for polar cap absorption events since electrons could be produced at much lower heights (50 to 60 km). Davies (1965) states that at these levels it is necessary to use the above equation to determine \(\kappa\). However, under such conditions absorption may be taking place over a range of heights and the exact determination of \(\kappa\) may be a complicated process. It would be interesting to compare the two preceding versions of the absorption coefficient. That is left for a later study.

Case 3

For case three the assumption is that the radio wave frequency is about equal to the collision frequency \((\omega \sim \nu)\). In this case geomagnetic effects are included.
Here Davies (1965) defines the absorption coefficient as,

\[
\kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{(\omega \pm \omega_H)^2 + \nu^2} \right]
\]

where \(\nu\) is the frequency of radio wave in hertz, \(f\) is the frequency of radio wave in hertz, \(f_H\) is the gyrofrequency of the electron, and \(f \pm f_H\) is the so-called effective frequency. A thorough study of case 3 will lead toward high-latitude regions.

Lied (1967) uses similar versions for each case. For case 1,

\[
\kappa \sim 1.345 \times 10^{-7} \left[ \frac{n_e \nu}{f^2} \right]
\]

and for case 3,

\[
\kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{4\pi^2 (f \pm f_L)^2 + \nu^2} \right]
\]

\[
\sim 5.29 \times 10^{-6} \left[ \frac{n_e \nu}{4\pi^2 (f \pm f_L)^2 + \nu^2} \right]
\]

Lied (1967) notes that case 1 and case 3 are normally satisfied for HF radio waves propagating through the ionosphere. In this section case 1 will be examined within the D-region.

Note also that for case 3 Lied (1967) does not explicitly use gyrofrequency. Rather, an effective frequency (the \(f \pm f_L\) term) is used. \(f_L\) is defined in terms of the gyrofrequency such that \(f_L \equiv f_H \cos(\theta)\) where \(f_H\) is the gyrofrequency and \(\theta\) is the
angle between the direction of propagation of the wave and the geomagnetic field.

Davies (1965) defines this term as the gyro frequency corresponding to the component of the earth's magnetic field in the direction of phase propagation, the quasi-longitudinal approach (Benson, 1964).

**Derivation of absorption equation**

In this section the attempt is made to derive an appropriate equation which will allow calculation of D-region absorption per unit distance. Restating important relationships from Chapter 1,

\[ L_a = -8.69 \int \kappa \, ds \quad D.9 \]

\[ \kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{n_e \nu}{\omega^2 + \nu^2} \right) \quad D.10 \]

Then inserting the equation for the absorption coefficient derived above into Equation D10,

\[ L_a = -8.69 \left( \frac{e^2}{2\varepsilon_0 mc} \right) \int \frac{n_e \nu}{\omega^2 + \nu^2} \, ds \quad D.11 \]

Then for case 1,

\[ L_a = -8.69 \left( \frac{e^2}{8\pi^2 \varepsilon_0 m_e c} \right) \int \frac{n_e \nu}{f^2} \, ds \quad D.12 \]

Combining all the constant terms we find that,

\[ L_a = - \left( \frac{1.17 \times 10^{-6}}{f^2} \right) \int n_e \nu \, ds \quad D.13 \]

where \( n_e \) and \( \nu \) are functions of height and the integral is over the path. Figure D1 shows the basic geometry of a radio wave propagating along a specific path. Within the high frequency regime nearly all absorption takes place within the D-region and is non-
deviative in nature (Davies, 1965). From Figure D1 it is apparent that as the ray propagates along some path it passes through an *absorbing region*. This would be the D-region.

![Figure D1](image)

**Figure D.1:** Illustration showing that absorption depends on distance. Also, absorption depends on the number of times a radio wave passes through the so-called absorbing region.

Since the absorption is non-deviative the assumption is made that the ray propagates along a straight line. Since the ray has some initial elevation angle, $\alpha$, it is useful to show the *right triangle* formed as the ray passes through the D-region and some basic trigonometry to get the path length from the height of the D-region,

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{s} \quad \text{D.14}$$

with $\alpha$ defined as the elevation angle, $h$ as height and $s$ as the path length through the absorbing region. Then solving for the path length,

$$\sin \alpha = \frac{h}{s} \quad \text{D.15}$$
Using the above relationship puts the differential for path length in terms of height,

\[ ds = \frac{dh}{\sin \alpha} \]

\[ La = -\frac{1.17 \times 10^{-6}}{\sin \alpha f^2} \int n_e v dh \]  \hspace{1cm} \text{D.18}

Figure D1 shows that the ray passes through the D-region two times per hop. Then let \( p \) be the number of hops yielding a final version of the absorption equation,

\[ La = -\frac{1.17 \times 10^{-6}(2p)}{\sin \alpha f^2} \int n_e v dh \]  \hspace{1cm} \text{D.19}

\[ La = -\frac{2.34 \times 10^{-6}p}{\sin \alpha f^2} \int n_e v_{en} dh \]  \hspace{1cm} \text{D.20}

where the integral is now over the height of the D-region.
Appendix E

Absorption Data Analysis

Using data from Bain and Harrison (1972) a base-line electron density is established for a model D-region with 1.0 km data points from 50 – 100 km, using data from Kelley (1989) to provide base-line neutral densities and temperatures. Since Kelley's data is at 10 km intervals, a simple Excel sheet was used to generate an appropriate exponential function that fit the data well. Furthermore, data from Kelley (1989) as well as electron density from a homework assignment to create a model ionosphere up to about 400 km. Appendix C provides the particulars.

The data set allowed testing of several assumptions put forth in this section by evaluating the applicability of case 1 in the D-region, the relationship between radio wave frequency and plasma frequency, and the behavior of the index of refraction.

Case 1 Validity

In this section absorption obtained using equations (19) and (21) are compared. Figure E1 shows that limiting approximations were valid above about 10 MHz. Since the HF regime covers 3.0 – 30.0 MHz a more accurate absorption figure within the D-region would be obtained using kappa as in (19). Using equation 21 (case 1 approximation) could lead to an overstatement of absorption by many dB at the lower end
of the HF spectrum. At higher frequencies the difference would at best be a dB or so and could be considered inconsequential. This is discussed further in a following section.

![Graph showing absorption equations](image)

**Figure E.1**: Comparison of absorption equations 1.19 and 1.21 from Chapter 1. Above about 10 MHz the lines begin to converge, indicating that both equation yield a similar result. Below 10 MHz equation 1.21 shows greater absorption for the given conditions.

Figure E2 shows that within the D-region the plasma frequency was always less than the radio wave frequency. Hence, non-deviative absorption dominates within the D-region. The D-region covers roughly \((50.0 \leq h \leq 90.0 \text{ km})\)
Figure E.2: Plasma frequency profile. Horizontal axis represents the plasma frequency in MHz, the vertical axis shows the altitude in km. The shaded area represents the high frequency regime within the D-region. Below about 90 km the plasma frequency is much less than the radio wave frequency.

Absorption Frequency-dependence

Figure E3 demonstrates the frequency-dependence of HF absorption and shows that the bulk of HF radio wave absorption takes place within the D-region. This is primarily due to the exponential decrease in the neutral density with increasing altitude which means a decrease in the collision frequency.
**Figure E.3**: High frequency radio wave absorption by frequency per region. The vertical blue bar represents absorption due to the D-region, the E-region and F-region are represented by the red and black bars respectively.

**Index of Refraction**

Figure E4 shows the frequency dependence as well as altitude dependence of the index of refraction,

\[ \mu = \sqrt{1 - \frac{f_p}{f^2}} \]  \hspace{1cm} (E.1)

The altitude dependence comes from the changing electron density as altitude increases through the D-region (shaded area in Figure E4). For a given frequency, as the plasma frequency increases due to increasing electron density we move further away
from unity, toward zero. Notice that at about 98 km the index of refraction for 3 MHz hits zero. This is significant since \( \mu = 0 \) defines a reflection point for a vertically incident radio wave (Lied, 1967). Another key observation is that up through the transition region between the D-region and E-region (about 85 km) the index of refraction is always near unity. It isn't until an altitude of about 86 km is reached that the index of refraction varies appreciably from unity.

![Figure E.4: Frequency/altitude dependence of the index of refraction.](image)

Generally three scenarios are expected:

\[
f \sim f_p \Rightarrow \mu \sim 0 \Rightarrow \text{Reflection point} \quad \text{E.2}
\]

\[
f > f_p \Rightarrow \mu < 1 \Rightarrow \text{Refraction} \quad \text{E.3}
\]
From Figure E2 within the D-region the radio wave frequency is always greater than the scaled plasma frequency. Hence, the first two cases (E.2, E.3) are likely, with E.3 being most likely. E.3 further bolsters the assertion that non-deviative absorption dominates within the D-region. The third scenario (E.4) would depend more on the properties of the F-region and would lead to a discussion of the maximum useable frequency (MUF).

**Example**

A transmitter drives an isotropic antenna with an input power measured in watts. Assume a perfect system such that there are no losses and the antenna puts out the full power. Computing the field strength at a distance of 1 km from the antenna (Rawer, 1952) yields,

\[ E_0 = 5470\sqrt{P} \quad \text{E.5} \]

\( P \) is the radiated power in watts and the units of \( E_0 \) are microvolts per meter (\( \mu V/m \)). Since it is helpful to work with decibels, Rawer (1952) shows how to convert the field strength to decibels above 1 \( \mu V/m \),

\[ E_0 = 74.8 + 10 \log(P) = \text{dB over } 1 \frac{\mu V}{m} \quad \text{E.6} \]

Assuming a radiated power of 100 watts,

\[ E_0 = 5470\sqrt{100} = 54700 \frac{\mu V}{m} \quad \text{E.7} \]

\[ \Rightarrow E_0 = 74.8 + 10 \log(100) = 94.8 \text{ dB over } 1 \frac{\mu V}{m} \quad \text{E.8} \]

Rawer (1952) shows that the total field at some distant point is given as,
For this example, $L_f$ is neglected and information from Rawer (1952) is used to estimate $L_d$ for single-hop refraction via the F-region. Rawer (1952) shows that assuming a total distance of about 2000 km one would expect path loss due to distance of roughly 63 dB. This report will limit the investigation to radio wave absorption in the D-region at mid-latitudes and will only examine non-deviative absorption. Hence, for the D-region using equation 19 as a starting point and combining constants and assuming a vertical path,

$$E = E_0 - [L_d + L_a + L_f]$$  \hspace{1cm} \text{E.9}

For this example, the table of values found in appendix C is used, then run through a simple FORTAN program that computes the D-region absorption. One hop is assumed so the radio wave will pass through the D-region twice. Referring to Figure E3 the frequency dependence of absorption within the D-region is evident. From Equation E6 the electric field at the receiving end is defined to be,

$$L_a = -9.2 \times 10^{-5} \int \left( \frac{n_e}{4\pi^2 f^2 + v^2} \right) dh$$ \hspace{1cm} \text{E.10}

Figure E5 shows the comparison of the calculated received signal strength within the high frequency spectrum. It is interesting to note that as the upper-end of the HF spectrum is reached very little change in signal strength is manifest.
Figure E.5: Comparison of approximated received signal strength. Figure E.5 is based on absorption equations 1.19 and 1.21.

To put Figure E.5 into perspective it is helpful to understand how the received signal strength is generally reported to the user. A typical high frequency receiver is equipped with an S-meter. In basic terms, an S-meter provides the operator a reading of the received signal strength. A typical S-meter will have markings ranging from S1-S9. Many models include S9 + 10 dB or higher ranges as well. A change of one S-unit represents a change of 6 dB. Furthermore, by convention each S-unit corresponds to some number of decibels above one microvolts per meter (field strength). Table E.1 provides the necessary information.
Comparing Table E.1 with Figure E.5 one can determine approximate S-meter readings for various frequencies. For example, based on the assumptions an S-meter reading of between S8-S9 for the 30 MHz signal would be expected. By contrast, one would expect a reading of less than S2 for the 3 MHz signal. One key ingredient so far ignored here is the noise floor; i.e. the background noise. An S3 signal is very readable if the background noise is at S1, but an S8 signal is of no use if the background noise is well above S9. Hence, many factor come in to play.

**Table E.1:** Typical S-unit Values.

<table>
<thead>
<tr>
<th>S-unit</th>
<th>dB over 1 μV/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>S9+10 dB</td>
<td>44</td>
</tr>
<tr>
<td>S9</td>
<td>34</td>
</tr>
<tr>
<td>S8</td>
<td>28</td>
</tr>
<tr>
<td>S7</td>
<td>22</td>
</tr>
<tr>
<td>S6</td>
<td>16</td>
</tr>
<tr>
<td>S5</td>
<td>10</td>
</tr>
<tr>
<td>S4</td>
<td>4</td>
</tr>
<tr>
<td>S3</td>
<td>-2</td>
</tr>
<tr>
<td>S2</td>
<td>-8</td>
</tr>
<tr>
<td>S1</td>
<td>-14</td>
</tr>
</tbody>
</table>
Appendix F

Longitudinal and UT Effects

Universal Time (UT) is a time standard based on earth's rotation, whereas Local Time (LT) is based on the angular position of the sun at a specific location. Figure F.1 shows a 24-hour UT progression in 6-hour steps. Since each hour represents a rotation of 15 degrees, each 6-hour step represent rotation of 90 degrees. Let the polar axis represent geographical longitude and the radial axis represent geographical latitude. Then let the blue arrow indicate rotational direction, the black circle the latitude of the geomagnetic pole ($80.6^\circ N$), the small black dot the geomagnetic North Pole at longitude ($287.4^\circ E$), the red square is then the Prime Meridian at Greenwich, England, and the large orange dot is the sun.

It becomes apparent that for each hour UT there is a specific geographical configuration of physical locations such as the prime meridian and the geomagnetic North Pole, as well as a specific LT configuration. In each panel of Figure F.1 the sun remains in the same location at the top of each plot. Hence, it is always 12:00 LT (Noon) at the top, midnight at the bottom, and dawn/dusk at the right/left. From Figure 3.1, at 00.00 UT it is Noon LT along the 180 degree meridian; at 18.00 UT it is Noon LT along the 270 degree meridian and so forth.

Since the geomagnetic field is coupled to the geomagnetic poles it is easily inferred that there must be a unique configuration of the geomagnetic field for each hour
of UT i.e. at 0600 UT the geomagnetic field has a different topology than at 1200 UT. This is due in part to the offset of the geomagnetic poles from the geographic poles.

**Figure F.1:** Polar plots of the relative position of the north magnetic pole, Greenwich meridian and sun in sun-earth coordinate system. The North magnetic pole is shown as a black dot, Greenwich is represented by a red square, and the sun as an orange dot. The blue arrow indicates direction of rotation.

It's helpful to understand the relationship between Local Time (LT) and Universal Time (UT) by generating a matrix with UT as the horizontal axis and LT as the vertical axis, then plotting the resulting longitude within that framework as in Figure F.2. Each hour UT will have a specific configuration of 24-hours LT. An important consideration is that along the Prime Meridian LT equals UT i.e. at 0500 UT it is 5:00 AM LT in Greenwich, England. Hence, whenever $LT=UT$ the resulting longitude is always zero.
For example, from Figure F.2 it is apparent that 18:00 LT at 1800 UT has a resulting longitude of 0.0, as does 12:00 LT at 1200 UT. It becomes evident that along each left-to-right upward diagonal there is a line of constant longitude. As an example, 6:00 AM LT at 1200 UT occurs along the 270 degree meridian, as does 7:00 AM LT at 1300 UT and 8:00 AM LT at 1400 UT, etc. It is, therefore, reasonable to assume that if a physical phenomena were plotted in the UT/LT format described in Figure F.2, one could determine if the phenomena exhibited a highly linear relationship as described above. Such an example will be described later.

Another way to interpret this is that along each vertical and horizontal line there is a unique longitude as a function of each LT/UT pair. However, the longitude is constant for each UT/LT pair along the left-right upward diagonal as indicated in Figure F.2. In fact, there is a unique relationship between UT, LT and longitude. Holding longitude constant results in a varying relationship between UT and LT. For example, we notice that for $LT = UT + 1$ when $UT = 2$ then $LT = 3$ such that the point $2, 3$ on the grid yields a longitude of 15 degrees. In similar manner, the point $(3, 4) = 15^\circ$, as does $(10, 11) = 15^\circ$. Hence, the relationship $LT = UT + 1$ yields the constant longitude of 15 degrees.
Figure F.2: Format to determine if a parameter has a favored ordering. This is a proposed format to determine if a parameter has a favored LT, UT, or longitude ordering. The format would require the parameter to be scaled in a LT vs. UT plot such that a longitude dependence is seen as a diagonal line. Figures 5.26 and 5.27 provide examples of this format in use.

Setting $LT = UT + 9$ results in UT/LT pairs $(1, 10), (2, 11)$ and $(11, 20)$ each of which yields a longitude of $135^\circ$. Hence, there is a simple, yet intricate relationship between UT, LT and longitude. Since they're linearly related, holding UT constant is the same as holding LT constant. Furthermore, when holding the longitude constant it doesn't really matter whether one uses LT or UT as the horizontal axis because they are linearly related in a specific way for each longitude. If looking at 15 degrees E longitude
then LT is related to UT by $LT = UT + 1$ and so forth. Hence, one would just shift the figure so many places to the left or right depending on which longitude was being held constant. An important consideration, however, is that in the physical world, UT and LT describe very different things.

Apparently there are at least two individual yet coupled dependencies. The first, following Sojka, et al. (1981) is called in this dissertation work a "UT-effect". It is based primarily on the topology of the geomagnetic field due to the offset of the geomagnetic and geographic poles. The other, which is called an LT-effect, shows a typical day/night variation. The open/closed boundary is susceptible to the UT effect. This effect, relative to the OCB, was discussed in Chapter IV. Remember, the OCB marks the transition region from open to closed field lines. Pretty much any proton regardless of energy can enter the polar cap region which, by definition, is at or above the OCB. As such, the field lines are open. It has been shown in Chapter 5 that the offset between the cutoff latitude and the OCB is about 4 degrees for 1 MeV protons and roughly 15 for 100 MeV protons. Hence, it could be argued that within a reasonable margin of error, 1 MeV protons require open field lines to propagate into the upper atmosphere, while 100 MeV protons can pretty much do what they want i.e. they don’t necessarily need open field lines.

**Local Time Effect**

The local time (LT) effect, aka "diurnal variation," or "daily variation," is based on the orientation of earth relative to the sun and the corresponding effect that alignment has on the geomagnetic field. When discussing LT effects we use terms such as day,
night, dawn, dusk, midnight, noon, etc. Frequently the geomagnetic field is shown in a
day/night orientation showing the elongated magnetotail (night) and the compressed day
side. Hence, when considering geomagnetic field lines and their related L-Shell
parameters (Chapter IV), the expectation is some sort of day/night asymmetry.

**Longitude Effect**

The longitude effect was a bit more difficult to understand. Frequently the
literature discusses LT and longitude effects as if they are the same thing. Certainly LT
and longitude are related. However, there is a specific longitudinal effect that is due to
the longitudinal asymmetry of the geomagnetic field. Recall that the geomagnetic poles
are offset about 11 degrees from the geographic poles. In the northern hemisphere the
geomagnetic north pole is located at about 80N, 287E. Therefore, as the Earth rotates the
asymmetry with respect to geographic longitude will be evident.

If the geomagnetic pole was aligned exactly with the geographic pole we would
expect symmetry about Noon LT (Smart et al., 1969; Smart & Shea, 2003). Since the
poles are not aligned, we see a slight offset as was discussed by Smart et al. (1969). As
in their work, we find an asymmetry about Noon LT as a manifestation of the roughly 11
degree offset between the geographic and geomagnetic North poles.

We agree completely with the extensive body of existing work. We are able to
conclusively show an LT effect based on sun/earth alignment and the topology of the
geomagnetic field due to that alignment. We are also able to demonstrate the
longitudinal effect based on the offset between the geographic and geomagnetic poles.
Universal Time Effect

We believe a third important effect should be considered. We call it the UT effect. Akin to the longitudinal effect, the UT effect is a manifestation of the offset between the geographic and geomagnetic poles. However, like the LT effect, the UT effect demonstrates the changing topology of the geomagnetic field in relation to the sun/earth line. Ignoring other inputs such as the solar wind, if earth didn't rotate, or if the poles were aligned, we would expect the geomagnetic field to exhibit a nice, clean dipolar structure. But the poles are not aligned and earth is rotating. Hence, even if we exclude such inputs as the solar wind, the topology of the geomagnetic field would be dynamic through a 24-hour period. We believe that the UT effect captures this dynamic topology of the geomagnetic field.

Summary

We have been able to reproduce the well-known effects of local time and longitudinal asymmetry as they relate to energy cutoffs. Furthermore, we have introduced an additional variability due to the dynamic nature of the geomagnetic field caused by the offset between the geographic and geomagnetic poles. It appears that the UT effect is most pronounced at the lower energies, especially 1-20 MeV, the range most responsible for PCA events.
Appendix G

Permissions

Publisher

Permission to Use Figure 3 From Smart et al., 2006

10/29/2019 RightsLink Printable License
https://s100.copyright.com/CustomerAdmin/PLF.jsp?ref=88ff2ed0-5938-4dc5-8317-98e95157a15d 1/7

ELSEVIER LICENSE
TERMS AND CONDITIONS

Oct 29, 2019

This Agreement between 547 E 1100 N ("You") and Elsevier ("Elsevier") consists of your license details and the terms and conditions provided by Elsevier and Copyright Clearance Center.

License Number 4690491222277

License Date Oct 15, 2019

Licensed Content Publisher: Elsevier
Licensed Content Publication: Advances in Space Research
Licensed Content Title: A geomagnetic cutoff rigidity interpolation tool: Accuracy verification and application to space weather
Licensed Content Author: D.F. Smart, M.A. Shea, A.J. Tylka, P.R. Boberg
Licensed Content Date: Jan 1, 2006
Licensed Content Volume: 37
Licensed Content Issue: 6
Licensed Content Pages: 12
Start Page 1206
End Page 1217
Type of Use: reuse in a thesis/dissertation
Portion: Figures/tables/illustrations
Number of figures/tables/illustrations: 1
Format both print and electronic
Are you the author of this Elsevier article? No
Will you be translating? No
Original Figure numbers Figure 3
Requestor Location
547 E 1100 N
547 E 1100 N
LOGAN, UT 84341
United States
Attn: 547 E 1100 N
Publisher Tax ID 98-0397604

Total 0.00 USD
CAREER OBJECTIVES

Continued studying of high frequency radio wave absorption as well as polar cap absorption events, the geomagnetic field, energetic particle precipitation, the open/closed boundary, etc.

EDUCATION
PhD, Physics, Utah State University, Logan, UT, (Defended Nov 2019)
  Dissertation:
  Model-based properties of Earth's protective shield:
  Relating UT-based dependencies of the open/closed boundary, cutoff latitude and L-shell parameter.

BS, Physics (Astronomy track), Humboldt State University, Arcata, CA, May 2014

TEACHING EXPERIENCE
Instructor, Physics, Utah State University, Logan, UT, Fall 2015 - Current
  Physics 2210 (online)

Instructor, Physics, Utah State University, Logan, UT, Summer 2016 - Current
  Physics 2210 (face-to-face)

Teaching Assistant, Physics, Utah State University, Logan, UT, Fall 2014 - Current
  Physics 2210
  Physics 2220

Lab Instructor, Physics, Utah State University, Logan, UT, Fall 2014 - Spring 2015
  Physics 2225

CONTINUING EDUCATION
Space Weather Summer School, Boulder, CO, Summer 2016
RESEARCH EXPERIENCE

AFRL Scholar/Researcher, Kirtland AFB, Albuquerque, NM, Summer of 2015
   PI: Dr. Stephen Kahler
   Energetic Solar Protons

Undergraduate Research Assistant, Humboldt State University, Arcata, CA, 2011-2014
   PI: Dr. Ken Owens
   Plasma Physics

Undergraduate Research Assistant, Humboldt State University, Arcata, CA, 2012-2014
   PI: Dr. C.D. Hoyle
   Gravity

Undergraduate Research Assistant, Humboldt State University, Arcata, CA, 2012-2013
   PI: Dr. Ryan Campbell
   Observational Astronomy

PEER REVIEWED PUBLICATIONS


PRESENTATIONS


Smith, D. A. & Sojka, J. J., Topology of earth's protective shield, APS 4-Corners Fall Meeting, Salt Lake City, UT, Oct. 2018
Smith, D. A. & Sojka, J. J., Open/closed boundary and energy cutoff latitudes for varying Dst indices, Space Weather Workshop, Boulder, CO, May 2018


Kahler, S. W. & Smith, D. A., Comparisons of two techniques for magnetically connecting in-situ observations of energetic particles with their solar sources, AFRL Poster Session, Kirtland AFB, June 2015

Smith, D. A., The practicality of a seemingly simple project to learn the secrets of the universe. Or how physics is like making a cake, Humboldt State University, Arcata, CA May 2014


Smith, D. A., & Hoyle, C. D., Tests of gravity below the 50-micron distance scale, National conference on Undergraduate Research (NCUR), La Crosse, WI, Apr. 2013


Owens, K., Smith, D. A., Plasma Research Update, Dept of Mathematics Colloquium, Humboldt State University, Arcata, CA, 2012

Owens, K., Smith, D. A., Bringing a star to Earth, Dept of Mathematics Colloquium, Humboldt State University, Arcata, CA, 2011

Non-Published Reports


SERVICE TO PROFESSION

Invited Reviewer
Journal of Geophysical Research - Space Physics (May, 2017)

AWARDS/SCHOLARSHIPS

Outstanding Graduate Student Presentation, APS 4-Corners Fall Meeting, 2018
Outstanding Graduate Teaching Assistant, Utah State University, 2016
AFRL Research Scholar, Kirtland AFB, Albuquerque, NM, 2015
Gene Adams Endowed Scholarship, Utah State University, Logan, UT, 2015
President's Scholar Support Fund, Humboldt State University, Arcata, CA, 2012, 2013
Montgomery Scholarship, Humboldt State University, Arcata, CA, 2012

v du Prize, Humboldt State University, Arcata, CA, 2011