Practical Optical Survey Strategies for Near Geostationary Orbital Debris

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PRACTICAL OPTICAL SURVEY STRATEGIES FOR NEAR GEOSTATIONARY ORBITAL DEBRIS

by

Akhter Mahmud Nafi

A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Aerospace Engineering

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2020
ABSTRACT

Practical Optical Survey Strategies for Near Geostationary Orbital Debris

by

Akhter Mahmud Nafi, Doctor of Philosophy

Utah State University, 2020

Major Professor: David Geller, Ph.D.
Department: Mechanical and Aerospace Engineering

It is important to have the capability to detect geosynchronous orbit (GEO) objects and update the GEO catalog on a regular basis to ensure the safety of high-value assets in this congested orbit regime. Thus, it is also important to develop and evaluate ground-based optical survey designs and to improve initial orbit determination (IOD) methods to help achieve these objectives. The GEO resident space object (RSO) environment and the observation properties of GEO RSO orbits from an Earth based observatory are studied thoroughly. Based on this information, efficient and practical surveys will be designed and evaluated.
PUBLIC ABSTRACT

Practical Optical Survey Strategies for Near Geostationary Orbital Debris

Akhter Mahmud Nafi

Uncontrolled space objects, more commonly known as space debris, consist of dead satellites, satellite deployment packages, and lost elements of these systems such as insulation blankets. These uncontrolled objects represent hazards to our nation’s most valuable space assets which include communication satellites, weather satellites, Earth monitoring systems, and military assets. To help mitigate this problem, this research proposes using the known astrodynamics of the near geosynchronous orbit (GEO) along with the known concentration of the uncontrolled GEO objects and observation constraints to design ground-based optical surveys that will detect uncatalogued debris. Furthermore, a scoring metric is developed to evaluate the information gain from each survey.
To my parents, who have brought me into this beautiful world and devoted themselves to put me in today’s platform.
ACKNOWLEDGMENTS

From a small town of tropical Bangladesh to the harsh winter of Logan, an extraordinary journey ends with this thesis, but this thesis would not exist without the help of some brilliant and kind individuals. I would like to take this opportunity to acknowledge some of them.

I am deeply grateful to my major professor Dr. David Geller for his constant support and guidance. His knowledge guided me to solve many complex astrodynamics and mathematical problems in my thesis. Not only have I learned problem-solving skills for research from him, but also I have learned about the value of working hard and staying humble. I consider myself very fortunate to work with him.

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Akhter Mahmud Nafi

April, 2020
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<td>AZ-EL</td>
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<td>EO</td>
<td>Electro Optical</td>
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<td>ECEF</td>
<td>Earth Centered Earth Fixed Coordinate System</td>
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<td>ESA</td>
<td>European Space Agency</td>
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<td>GEO</td>
<td>Near-geostationary orbits</td>
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<td>IDAC</td>
<td>Inter Agency Space Debris Coordination Committee</td>
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<td>IOD</td>
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CHAPTER 1

Introduction

Space debris poses an increasing risk in manned space mission and operational satellites. Orbital debris significantly skews the catalog population accounting for over 93% of the current 20,000 cataloged objects [1]. The majority of debris between 1 cm to 10 cm in diameter, are large enough to cause catastrophic damage upon collision - are currently not being tracked and maintained in a catalog. Thus, better space situational awareness (SSA) is required to avoid collisions between orbiting satellites and debris, provide safe reentries, detect on-orbit explosions, and assist missions at launch. Radar is primarily used for tracking debris in Low Earth Orbit (LEO) and ground and space based Electro Optical sensors are used for tracking debris in Medium Earth Orbit (MEO) and Geosynchronous Orbit (GEO). The US Space Surveillance Network (SSN) maintains a catalog of earth-orbiting objects. The catalog nominally includes objects in LEO greater than 10 cm in diameter and larger than 1 m in GEO. Space debris are increasing in GEO and until now, two break-ups have been reported. The 1978 break-up of an EKRAN 2 satellite, SSN 10365, was identified in 1992, and in the same year a Titan 3C Transtage, SSN 3432, break-up produced at least twenty observable pieces [2]. These uncontrolled objects represent hazards to the most valuable space assets in GEO, which include communication satellites, weather satellites, Earth monitoring systems, and military assets. To address the problem this research proposes to develop better survey strategies based on detailed GEO satellite environment analysis.
1.1 Research Statement

The goal of the research is to develop survey techniques to improve the state information of known GEO satellites and to detect previously unknown GEO objects. To achieve these goals, new practical and efficient survey strategies will be designed. These surveys will be simulated and evaluated using newly developed performance metrics.

1.2 Dissertation Overview

The remainder of this document consists of 3 chapters. Chapter 2 covers the associated literature and previous work relevant to the topics presented. Due to the unique nature of the dissertation, Chapter 3 covers the brief description of the USU-STAR and its operation. Chapter 4 states the scope of this research and the objectives to be accomplished. Chapter 5 discuss the current GEO environment. Survey designs for GEO objects are discussed in Chapter 6, and only uncontrolled GEO object survey designs are discussed in Chapter 7. Chapter 8 covers the newly developed information performance metric to evaluate survey designs.
CHAPTER 2
Literature Survey and Related Work

2.1 GEO Satellite Environment

Near-geostationary orbits have a radius approximately equal to 42,164 km, a period equal to the Earth's rotational period, and near zero inclination ($< 20^\circ$). True geostationary orbits with zero inclination appear to be fixed with respect to a ground-based observer. Communication satellites, weather satellites, surveillance and military satellites are often placed in geostationary orbits because of this unique characteristic. Near-geostationary orbits exhibit a well-known figure-8 pattern to ground-based observers. For the purposes of this research, GEO will refer to near-stationary orbits with $< 20^\circ$ inclination.

While geostationary satellites are controlled to maintain near zero inclination, the inclination of uncontrolled GEO satellites changes predictably from $0^\circ$, to approximately $15^\circ$, and then back to $0^\circ$. The period of the oscillation is approximately 54 years [3]. During the first 27 years, an uncontrolled object with an initial inclination of $0^\circ$ will gradually increase in inclination until its inclination has peaked at approximately $15^\circ$. During the next 27 years, its inclination will gradually decrease until it has returned to its original $0^\circ$ inclination, and the cycle will begin again. The main factor causing the unique patterned behavior of the geosynchronous satellites is the perturbation forces from the sun, the moon, and the oblateness of the Earth. These luni-solar and J2 geopotential perturbations produce a torque on the orbit plane of a satellite resulting from the net out-of-plane force component acting on the orbiting object. This torque results in the correlated periodic progression of the inclination and ascending node of each geosynchronous object. The result is satellite orbital evolution similar to gyroscopic motion with secular precession of the orbital angular momentum vector about the ecliptic pole. The effects are observed as an approximately 54-year periodic correlated variation in the inclination and the ascending node [4].
Interesting concentrations of satellites movement can be observed in both the geocentric and the topocentric Right Ascension-Declination (RA-DEC) frame. These crowded region varies over a long time interval. For example, in 2003, these regions were so concentrated in the RA-DEC frame, they were treated as a point, called the ‘pinch point’ [5].

The Resident Space Object (RSO) population in the GEO ring is classified with a taxonomy used by the European Space Agency’s DISCOS database (Database and Information System Characteris- ing Objects in Space) [6] [7]. For GEO RSOs, seven orbit categories are used to classify the type of orbits traversed by these objects.

- C1: Objects under along-track and-cross track control
- C2: Objects under along-track control only
- D: Objects in a drift orbit (Above or below GEO)
- L1: Objects in a libration orbit around the Eastern stable point (longitude 75 degree East)
- L2: Objects in a libration orbit around the Western stable point (longitude 105 degree West)
- L3: Objects in a libration orbit around both stable points
- I: Objects in a highly inclined orbit

All of these GEO objects have an eccentricity smaller than 0.2 and a semi-major axis between 39664 and 45314 km. All inclinations are lower than 70 degrees. Fig.2.1 illustrates the GEO cataloged population.
2.2 Geostationary Survey Design

Better survey designs for the GEO regime will serve two purposes. The first is to gather new information about current GEO debris; this information is crucial to understand the future evolution and removal process of these objects. The second purpose is to improve the available orbit information of known or cataloged objects in GEO. The NASA, Inter Agency Space Debris Coordination Committee (IADC), and ESA initiated programs for optical observation of space debris. These organizations and programs implemented different search strategies based on various objectives. Most of the survey designs tried to observe at 180° solar phase angle or anti-solar point, and avoid pointing at the Moon as well as the galactic plane. Galactic plane consist of many bright stars, especially near the galactic center, and can cause problems when trying to find the faintest objects. More bright star streaks in the field of view may hide the object of interest. By staying at least 10° from the galactic plane the number of stars per square degree can be decreased by several thousand \cite{2}. Schildknecht et al. \cite{8} chose a survey strategy to optimize the coverage for certain bands of...
orbital inclination. The survey tried to observe near Earth’s shadow cone to optimize the illumination condition of objects and avoided pointing towards Milky Way. To increase the probability of re-observing the objects, the same sequence of fields were imaged each night at the same local times.

Another ESA telescope search strategy was to optimize the search for faint objects at high inclinations. The search fields were placed at high declinations in order to reduce the apparent motion of the objects. As a result all objects with inclinations less than the absolute value of telescope pointing declination were not tracked. The follow-up observations were planned and executed between one and two hours after the first detection. In most cases authors tried to recover the object during the following night [9].

NASA’s 1.7° field of view CCD debris telescope was used for survey design to observe a strip of GEO belt which is eight degrees tall, centered at geostationary region. The telescope was pointed to a specific right ascension and declination and parked during each 20 second long exposure [10].

In another paper Schildknecht et. al. [11] described the ESA 2002 GEO survey and compared the results with data from other ESA campaigns. The Search fields were defined by a series of observational constraints: good lighting condition, avoiding dense stellar background, field of view at high elevation; maximizing the angular distance from the moon, and searching near the region of the expected catalogue population. A survey was also designed to detect Geostationary Transfer Objects (GTO). Several clusters of small objects of similar dynamical characteristics had been found. The authors mentioned that the only reasonable explanation for the origin of these clusters were explosions.

The IADC search fields for the spring 2003 GEO campaign were designed to follow a specific declination and anti-solar point of the right ascension, and search fields were changed each night. The survey strategies were simulated before telescope implementation. All potential orbital inclinations and right ascensions of the ascending node, passing through a given FOV were calculated and then cataloged objects were plotted against them. Only overlapped cataloged objects were observable from the observatory [2].
Flohrer et al. [12] discussed different performance related issues for GEO surveys such as, the performance of the optical sensors, the performance of the survey strategy, the performance of the tasking strategy and the performance of the correlation of observations with the catalog. A GEO survey was proposed similar to the IADC survey strategy, observing a declination stripe continuously. The stripe height was 34° in declination and the width in right ascension was equal to the FOV diameter. The stripe was divided into equally sized fields for a particular declination range and 4 observatories were considered to cover all the fields.

For both GEO and GTO searches, Schildknecht et al. [13] designed a survey technique to repeatedly observe the same field in the sky. For the GEO searches, the telescope was tracking with 15”/sec in right ascension and for the GTO surveys, either with 7.5”/sec or 19.5”/sec for the range of expected apparent motion of GTO objects at apogee.

Flury et al. [14] designed surveys to focus on objects in GEO and GTO. The GEO objects were tracked during the exposures to optimize the signal-to-noise ratio (SNR) for objects. The detection technique was based on an algorithm comparing several consecutive frames of the same field in the sky. Background stars were fixed and those are identified on a series of 10 to 30 frames. The remaining parts of the frames were searched for any objects. The telescope was moved after each exposure so that the same area of the sky was passing the field of view at the next exposure. With this method, the telescope slowly scanned the sky from east to west while it was following the stars.

Frueh et al. [15] investigated observation strategies for EO sensors. Surveys were presented by treating the sensor tasking and object coverage as an optimization problem.

Sharma et al. [5] selected the most congested locations in the GEO belt, which he named ‘pinch points’ to design a survey strategy for a space based sensor. The highest density regions, or pinch points, were centered at 0° declination and at approximately 65° and 245° in right ascension. Pinch point operations required the pinch point search region to be continuously observed over the twenty-four hours but in practice Space Based Visible sensor (SBV) operations were limited to eight hours per day.
Seitzer et al. [16] developed GEO survey strategies for the Michigan Orbital Debris Survey Telescope (MODEST) to look for space debris. Each night a field of constant right ascension and declination was chosen that was close to the anti-solar point, but just outside the cone of Earth shadow. This field was tracked at the sidereal rate for as long as possible. The field was selected to be as close to the anti-solar point as possible so that the solar phase angle was at a minimum, and GEO objects are at their maximum apparent brightness. In the 5.2 minutes that it takes a GEO object to move across the full 1.3° field, eight 5s exposures were taken.

In recent work Seitzer et al. [17] observed the GEO debris population at sizes smaller than 10 cm with the 6.5m Magellan telescope ‘Walter Baade’ at the Las Campanas Observatory in Chile. To ensure good lighting conditions observations were taken as close to Earth shadow as possible without being in eclipse. The telescope tracking rates were set to zero, which would be correct for a station-kept object at GEO.

Olmedo et al. [18] describes survey strategies for three telescopes located at south of Spain. Each telescope pointed to a declination strip located at different right ascension angles in such way that the distance between these strips (15°) corresponded to the crossing time of a true GEO object from the first to the second telescope. One hour after the observation of a true GEO object from the first telescope, it would be re-observed from the second telescope, and two hours after the observation from the first telescope the object would be re-observed from the third telescope. In this way, typically, three triplets separated by one and two hours would be available for computing the IOD (in the case of observing a true GEO object).
2.3 Initial Orbit Determination

Initial Orbit Determination (IOD) is an important step towards accurately determining the states of RSOs and maintaining their catalog. Determining an orbit only from angular data or observations (angles only IOD) continues to be an important area of research, as they are being used for ground-based and space-based orbit determination of satellites and space debris, particularly through the use of cameras. Gauss and Laplace developed the very first IOD methods nearly two centuries ago [19]. Recent iterative based IOD methods are developed by Escobal [20] and Gooding [21] known as Double r-iteration and Gooding algorithm respectively.

Gauss initially developed the method to determine the position of the object without determining the velocity. However, the method can be easily extended to find the velocity as well, using the extension method such as Gibbs, Herrick-Gibbs, or a solution to Lambert’s Problem [22]. The method works best for interplanetary studies but is not very accurate for satellite orbit determination [19]. Long et al. [23] suggests that the orbital arc between observation be less than 60° for successful implementation of the method and the method works remarkably well when data is separated by 10° or less. The success of the Gauss’ method also depends on the method used to determine the Lagrange coefficients [22].

In the Double r-iteration method, the mean anomaly and mean motion are computed based on the initial guess of the satellite’s ranges at first two observation times. Then the method attempts to minimize residuals, defined as the difference between the real time interval and estimated time interval. A Newton-Raphson iteration process using the numerical partial derivatives is employed to converge upon the radius magnitudes. The advantage of the Double r-iteration method is that it is designed to solve problems where there might be very large intervals between observations.

The Gooding algorithm was proposed by Gooding in 1993 [21] for a minimum number of 3 observations. This method requires initial guesses of the range at the time of the first and third observations as well as a guess for the direction of the orbit, retrograde or prograde. If a poor guess is made, the method will generally not converge. The method begins with
two guesses of the scalar range from the observation site to the satellite position at the first and third observation times. With these guesses, the first and third radius vectors are then calculated. With these two vectors and the time between the first and third measurements, a Lambert solver is then used to compute the orbit. Using this orbit, a Keplerian propagator estimates the radius vector at the second observation time. In the ideal case of perfect measurements and if the initial guesses were correct, then the unit position vector at 2nd observation time will coincide with the second measurement. In most cases, the unit position vector at second observation time will not coincide with the second measurement due to incorrect guesses and measurement errors. In this case, a shooting method is used to minimize the error.

2.4 Linear Covariance

Linear Covariance Analysis (LinCov) has been applied to general estimation theory problems, as well as in the design and analysis of orbit determination algorithms, inertial navigation systems, and attitude determination systems. LinCov allows the final navigation error covariance to be determined with a single simulation run. The key difference between the extended Kalman filter and the LinCov is that the nonlinear functions and measurements are linearized about the nominal reference trajectory rather than the filter estimated state. This method eliminates the need to propagate or update the system state, while tracking only the error covariance about the nominal trajectory. For most space applications the statistical information are equivalent to those obtained from a similar Monte Carlo analysis. A rigorous linear covariance analysis and error budget method is provided by Maybeck [24] and Christensen and Geller [25]. For angles only measurements, LinCov analysis has been applied to evaluate the efficacy of using optical angles-only measurements of the moons of Jupiter to determine a spacecraft’s position and velocity during a Jupiter approach [26]. In this research, linear information analysis will be used as a performance metric. It will provide how much state information can be obtained from the survey.
CHAPTER 3

USU-STAR

Utah State University Space Situational Awareness Telescope for Astrodynamics Research (USU-STAR) is located in Garden City, UT. The author of this work aided in the development and optimization of a telescope built for the sole purpose of observing and collecting data on satellites and space debris. The main goal of USU-STAR is two fold: to provide a means to empirically validate astrodynamical theory developed for space surveillance, and to feed insight gained from the validation back to the theory. The telescope is located at approximately 41.9333°N, 111.4206°W, and it is 1.981 km above mean sea level. USU-STAR achieved first light on October 21, 2016 at 10:56pm.

3.1 Hardware

One of the aims of this research is to optimize the Electro Optical sensor to view the dimmest possible Resident Space Object (RSO). Signal to Noise Ratio (SNR) and the limiting magnitude of particular setup is required to determine the dimmest detectable RSO. A radiometric model is necessary to compute the SNR and limiting magnitude.

The developed radiometric model consist of two different types of parameters, design parameters and constant or assumed parameters. The things that we have control over are design parameters, and in this problem they are: aperture diameter (D), f-number (N) and pixel size (p) of the CCD. Constant or assumed parameters are inherent to the sensor or are assumed to reduce the dimensionality of the problem like sky brightness ($q_{p,sky}$), CCD quantum efficiency (QE), CCD dark current ($q_{p,dark}$), atmospheric transmittance ($\tau_{atm}$), range of RSO and angular velocity of RSO ($\omega$).

The limiting magnitude of an electro optical system is computed using the following equation from Coder-Holzinger [27]
\[
m_v = -2.5 \log_{10} \left( \frac{\text{SNR}_{\text{alg}} \sqrt{m_i \omega ND (q_{p,\text{sky}} + q_{p,\text{dark}})}}{\phi_0 \tau_{\text{atm}} \tau_{\text{opt}} (\pi D^2/4) Q E \sqrt{p}} \right)^{1/2}
\]  

(3.1)

Fig. 3.1: Limiting magnitude vs aperture diameter

Every single point in Fig. 3.1 is a potential electro optical sensor but not all the systems are physically possible. Those ones that are physically possible, are bounded by the black curve that is called Pareto Frontier. The cyan region of the figure contains all the realistic set up for an electro optical system. The red star in Fig. 3.1 represents the USU-STAR system which has 0.25 m aperture diameter, 9 micrometer pixel size and f-number 5 which has a Limiting Magnitude 17.

The USU-STAR system consists of optics, a -20° cooled CCD camera, mount and structure. Details are provided in the list below:

- Optics: A 0.25 m aperture diameter, AG Optical Systems. Field of view of the system is 1.28°.
- Camera: A 4096 × 4096 pixel CCD camera, IFOV is 9 μm. The manufacturer is Finger Lakes Imaging and the model is ProLine PL16801 4K4K CCD (KAF-16801).
• Mount: Astrosysteme Austria DDM-85 Basic German equatorial mount

• Structure: The telescope is installed inside a Astro Haven Enterprises 7 ft dome

3.2 Software

Four major software packages are used to control the hardware of the USU-STAR system. Their brief functions are stated below:

• Autoslew is used for operating the mount. The functions Autoslew executes are adjusting the motor parameters, optimizing weight balance, synchronizing telescope position, creating a ‘Pointing file’ for normal operation and defining ‘Park positions’. The pointing file is used to increase the pointing accuracy of the telescope and the park position is used to park the mount in a certain park position after the end of a session.

• Maxim DL is specifically designed for astronomical imaging and other low-light level applications. This software is used to control the exposure time, capture different frames for calibration, read the raw FITS file, and monitor the operating temperature of the camera.

• The AGO thermal system is designed to maintain the temperature of the primary and secondary mirrors to above ambient air temperature by automatically turning on and off the heating and cooling elements of the thermal control system according to a set of parameters set by the user.

• Gemini provides both telescope focusing and camera rotation in a single, robust package. The Gemini package includes standard ASCOM driver software for use with any ASCOM compliant software client, flash upgradable firmware, and an easy-to-use interface program.
3.3 Scheduler

Scheduling is done using a MATLAB script. This script takes a TLE catalog as an input. Based on the survey design, the user selects the exposure time, total time per track, total time allowed for slewing and number of frames per track. Then it provides a Java script to schedule a week’s worth of measurements so as to first maximize coverage, then brightness.

3.4 Operations

Weather plays a crucial role in telescope operation. Precipitation, temperature and wind speed are checked for the entire night before starting the operation. It is always made sure that the telescope is in proper parking position before opening the dome since the dome is smaller than the total height of the telescope and the pier combined. Before running the scheduler script, calibration is required. An image of part of sky is captured and then compared with an star catalog to get the RA-DEC of the center of the frame. Then the scheduler script is executed. Finally after the observations are completed, the telescope is returned to its parking position, and after a remote visual check to ensure the telescope is in the parking position, the dome is closed.
4.1 Scope

The proposed research concentrates on determining accurate states of known GEO satellites and uncontrolled GEO debris objects that will be eventually maintained in catalog. This research will use the information from current TLE catalog to design surveys, and it is expected that unknown debris objects will be detected in the process as they will mimic the dynamics of known satellites. In order to reach the goal, it is imperative to optimize the Electro Optical (EO) sensor design and operation, develop a full understanding of the GEO RSO environment, and develop efficient survey strategies. Additionally performance metrics will be utilized to evaluate the designed surveys.

Understanding the behavior of the existing GEO RSOs can help develop better methods to detect debris object. It is also in the scope of the dissertation to understand and visualize the motion of GEO RSOs in different coordinate frames, e.g. the geocentric RA-DEC frame, the topocentric RA-DEC frame, the topocentric horizontal frame, and the ECEF frame. Surveys will be designed for a single ground based observatory. The data association problem will not be discussed in this research. The USU-STAR validation of those surveys depends on the availability of the system and the weather.
4.2 Objectives

The goal of this research is to develop techniques to improve the state information of known GEO satellites and to detect previously unknown GEO objects. To achieve these goals, the following objectives have been identified:

1. Design and install the Electro Optical sensor for tracking Earth orbiting satellites from a ground based observatory specifically for detecting orbital debris.

2. Develop a clear understanding of the GEO satellite environment, the long term behavior of GEO satellites, and the observational properties of GEO RSO orbits.

3. Design practical and efficient surveys for GEO objects based on their observational properties.

4. Develop metrics to evaluate survey designs based on visibility, coverage and accuracy.

5. Develop simulation to evaluate survey designs.

6. Validate simulation using data from the USU-STAR.

Unfortunately objective 6 was not achieved due to the time constraint.
CHAPTER 5
GEO Environment

Near-geostationary orbits have a radius approximately equal to 42,164 km, a period equal to the Earth’s rotational period, and near zero inclination ($< 20^\circ$). True geostationary orbits with zero inclination appear to be fixed with respect to a ground-based observer. Communication satellites, weather satellites, surveillance and military satellites are often placed in a geostationary orbit because of this unique characteristic. For the purposes of this paper, GEO will refer to near-stationary orbits with $< 20^\circ$ inclination. Uncontrolled space objects, more commonly known as space debris, consist of dead satellites, satellite deployment packages, and lost elements of these systems such as insulation blankets. These uncontrolled objects represent hazards to the valuable satellites placed in GEO. To detect GEO RSOs and update the GEO catalog, it is necessary to analyze and understand the GEO environment and the long-term behavior of the objects in GEO orbit. In this paper, publicly available RSO data from space-track.org (11 November 2017, and 17 July 2018) is used to analyze the GEO orbit region. There are 1273 GEO objects in the catalog, and Figure 5.1 shows the histogram of inclination of all the GEO satellites in the catalog.

While geostationary satellites are controlled to maintain near zero inclination, the inclination of uncontrolled GEO satellites changes predictably from $0^\circ$, to approximately $15^\circ$, and then back to $0^\circ$. This oscillation is produced by the combined effects of solar and lunar perturbations as well as the Earth’s oblateness (J2 term), and the period of the oscillation is approximately 54 years [3]. During the first 27 years, an uncontrolled object with an initial inclination of $0^\circ$ will gradually increase in inclination until its inclination has peaked at approximately $15^\circ$. During the next 27 years, its inclination will gradually decrease until it has returned to its original $0^\circ$ inclination, and the cycle will begin again.

Figures 5.2 and 5.3 shows the correlation of the inclination and right ascension of the ascending node of uncontrolled GEO RSOs due to lunisolar and J2 geopotential perturba-
Inclination of GEO satellites

Inclination vs right ascension of the ascending node of the geosynchronous satellites. Figure 5.2 shows catalog data from 11 November 2017, and Figure 5.3 shows the periodic changes in inclination and RAAN of a single uncontrolled GEO satellite propagated forward from zero initial inclination for 53 years. In the simulation, the RAAN of the object quickly jumps to 100° and then starts to follow the period pattern illustrated in Figure 5.3. The right ascension of the ascending node starts to decrease and eventually after 26.5 years goes to 0° when the inclination is maximum, 15°. For the next 26.5 years, the RAAN starts to increase in opposite direction while the inclination decreases and returns to zero. This information can be helpful in designing efficient ground-based optical surveys for GEO satellites. For example, the majority of uncontrolled objects in Figure 5.2
have RAAN between $-40^\circ$ and $+90^\circ$. This fact contributes to the pinch point phenomena discussed later in this paper.

Fig. 5.3: Inclination vs right ascension of the ascending node of a single geosynchronous satellites propagated for 50 years. Blue represents the first 26.5 year and red represent next 26.5 year of the satellite propagation

(a) Geocentric right ascension and declination frame  (b) Topocentric right ascension and declination frame, latitude $41^\circ$ N, longitude $110^\circ$ W

Fig. 5.4: 24 hour motion of GEO objects in the geocentric and topocentric inertial frame

5.1 GEO Object Distribution in RA-DEC and AZ-EL Space

Figure 5.4 shows the motion of GEO objects over a 24 hour period in both geocentric
and topocentric inertial frames. The motion is from left to right. In the geocentric frame, uncontrolled objects oscillate above and below $0^\circ$ declination ($20^\circ$ max), while the controlled geostationary satellites have near zero declination and little oscillation. While crossing the $0^\circ$ declination, many uncontrolled GEO objects pass through a small window in right ascension. This phenomenon occurs due to the fact that, the majority of the uncontrolled satellite are concentrated in RAAN space.

In the topocentric frame associated with USU-STAR (latitude $46^\circ$ N, longitude $111^\circ$ W), the uncontrolled object oscillate above and below $-6^\circ$ declination, from $-25^\circ$ to $+15^\circ$, while the controlled geostationary satellites have approximately $-6^\circ$ declination and little oscillation.

The 24-hour motion of the same GEO satellites in the local azimuth-elevation frame associated with USU-STAR is shown in Figure 5.5. In this frame controlled geostationary satellites appear to be fixed, while uncontrolled objects follow a standard figure-8 pattern.

![Figure 5.5: 24 hour GEO satellite motion in azimuth and elevation frame](image)

5.2 Phase Angle

Phase angle defined in this paper is the angle between the unit line of sight vector and the unit sun vector. $\hat{\rho}$ is the unit line of sight vector and $\hat{s}$ is the sun direction vector and
Fig. 5.6: Phase angle illustration

Fig. 5.7: Contour plot of phase angle as a function of RSO geocentric RA-DEC, for 2 different days in the year at mid night, for latitude 41° N, longitude 110°W

\( \phi \) is the phase angle showed in Figure 5.6. 180° phase angle is the best for the observation as satellites will appear brightest in the image. Figure 5.7 shows the contour plot of phase angle in GEO in 2 different days in the year at midnight.

5.3 Galactic Plane

The galactic plane consists of many bright stars, especially near the galactic center, and can cause problems when trying to find faint objects and even bright objects. Many bright star streaks in the field of view may hide the object of interest. Figures 5.8(a) and
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(a) Galactic plane (blue), and galactic center (red) in geocentric RA-DEC coordinate

(b) Galactic plane change over 24 hr (11 Nov, 2017) viewed from latitude 41° N, longitude 110° W, motion is right to left

Fig. 5.8: 24 hour motion of galactic plane in geocentric inertial frame and in azimuth and elevation frame

5.8(b) show the galactic plane in RA-DEC frame and azimuth-elevation frame respectively where the red circle indicates the galactic center. A well-designed survey will avoid the galactic plane by at least 10° [2].

(a) Geocentric right ascension and declination frame

(b) Geocentric right ascension and declination frame

Fig. 5.9: Satellite concentration in the geosynchronous belt over a 24 hour period, 2017
5.4 Pinch point Phenomena

Interesting concentrations of satellites movement can be observed in both geocentric and topocentric right ascension-declination (RA-DEC) frame. These crowded regions vary over a long time interval. For example, in 2003, these regions were so concentrated in the RA-DEC frame, they were treated as a point, called the “Pinch Point” [5]. However, due to long-term behavior of GEO objects and the addition of new GEO satellites the pinch point has expanded into pinch point “regions”. This is shown in Figure 5.9 where the GEO satellites concentrations are shown for 2017. Highly concentrated dark red regions are the area of interest for ground-based survey design. In the geocentric frame the zero declination region is more concentrated than any other declination, and in the topocentric frame $-6^\circ$ declination is more concentrated. Some dark-red areas are more crowded than others due to the passing of uncontrolled satellites through the region over 24 hour period, i.e., the most concentrated pinch point regions are produced as uncontrolled objects travel through their ascending nodes (Region 1) and their descending nodes (Region 2). Figure 5.9 gives a visualization of the number of satellites passing through RA-DEC space in 24 hour period. It can be seen from the figure that there are some concentrated regions in the RA-DEC space. Most of the controlled satellites follow the geocentric declination close to $0^\circ$ and topocentric declination $-6^\circ$. Regions near $70^\circ$ and $-250^\circ$ geocentric right ascension are densely populated relative to rest of the right ascension space.
CHAPTER 6
Survey Design

Seven different surveys will be designed and evaluated in this chapter. The first, a Pinch Point Survey design, is presented in Section 6.1. The main idea of this design is to first determine the number of RSOs that pass through each point in RA-DEC space over a 24 hour period. Then the survey is designed to point at the RA-DEC location with the highest concentration of RSOs. This survey design is operationally the simplest since very little telescope slewing is required. The next 5 surveys, presented in Sections 6.2-6.6, are each intended to be continuous improvements of the Pinch Point survey design. Significant telescope slewing maybe required in these cases. In section 6.7, the last survey design of this chapter, the Pinch Point concept is abandoned and replaced with a design to ensure that each observable RSO is detected at least three times, as this is necessary for Initial Orbit Determination (IOD). A summary of all 7 surveys is presented in Section 6.8.

The observation period for ground-based optical telescope varies widely from season to season. To see the effect of observation time for two different seasons, surveys are designed for both summer (17 July, 2018) and winter (11 November, 2017). The observation time is set from sunset to sunrise with astronomical twilight taken into account. This provided 10 hrs and 50 mins of observation time for the evening of 11 November 2017 and only 5 hrs of observation time for evening of 17 July 2018. For each survey, summer and winter results will be represented by S and W respectively, e.g. Survey 1W, Survey 3S etc. For direct comparison, all surveys are designed for 11 Nov, 2017 and 17 July 2018 using the publicly available GEO RSO data from space-track.org.

System parameters consist of the location of the observatory, telescope and camera specification and observation time interval. Surveys are designed based on the system parameters of USU-STAR telescope installed near Bear Lake Utah. The system parameters for all surveys are shown in Table 6.1.
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<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude of the observatory</td>
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</tr>
<tr>
<td>Longitude of the observatory</td>
<td>-111.42°</td>
</tr>
<tr>
<td>Altitude of the observatory</td>
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</tr>
<tr>
<td>Aperture Diameter</td>
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</tr>
<tr>
<td>Field of View</td>
<td>1.28 × 1.28°</td>
</tr>
<tr>
<td>Size of Frame</td>
<td>4096 × 4096</td>
</tr>
<tr>
<td>Pixel size</td>
<td>9 µm</td>
</tr>
<tr>
<td>Imaging mode</td>
<td>Earth tracking mode</td>
</tr>
<tr>
<td>Time between measurements</td>
<td>20 sec (Survey 1), 30 sec (Survey 2-7)</td>
</tr>
<tr>
<td>Exposure time</td>
<td>5 sec</td>
</tr>
<tr>
<td>Read out time</td>
<td>5 sec</td>
</tr>
<tr>
<td>Time for Slewing</td>
<td>20 sec</td>
</tr>
</tbody>
</table>

Table 6.1: System parameters for survey design

With the exception of the Pinch Point survey, these surveys will not track a single point in RA-DEC space and significant slewing of the telescope may be required. Thus it is important for the survey design to ensure telescope has enough time to reach a given RA-DEC. $\Delta \theta$ is introduced to score the telescope movement of the entire survey, where $\Delta \theta$ is the angle between the telescope pointing direction $l_t$ and the telescope pointing direction at previous time step $l_{t_{i-1}}$ in the local azimuth-elevation frame.

$$\Delta \theta = \cos^{-1}(l_t, l_{t_{i-1}})$$ (6.1)

The average $\Delta \theta$ and the standard deviation of $\Delta \theta$ of every survey will indicate the feasibility of the implementation based on the slewing capability of the telescope. Zero $\Delta \theta$ means the telescope is tracking the same point in local azimuth-elevation frame. Higher $\Delta \theta$ does not guarantee object detection as the telescope may not have enough time to slew to the intended RA-DEC at the desired time.

As will be seen, the Pinch Point survey (Survey 1), requires very little telescope slewing. The time between measurements is thus selected to be only 20 sec.

For Surveys 2-7, the time between every observation is selected as 30 sec and a slew rate of 5°/sec is chosen. All surveys assume exposure times of 5 sec, camera readout of 5
sec and the remaining 20 sec for slewing. So maximum $\Delta \theta$ is considered to be $100^\circ$, since if $\Delta \theta > 100^\circ$, the telescope may not have enough time to reach the commanded RA-DEC.

In addition to $\Delta \theta$, four other key angles related to survey performance will be utilized: 1) the solar phase angle of the center of the FOV ($\phi$) is a measure of visibility of an RSO, the elevation of the center of the FOV ($\lambda$) indicates whether or not RSO is above horizon, the angle between moon and the center of the FOV ($\xi$) indicates the level of moonlight interference, and the angle between the galactic center and the center of the FOV ($\psi$) is an indication of the level of background star concentration. These four angles as well as $\Delta \theta$ will be presented for each survey.

6.1 Survey Design 1 – Pinch Point Tracking (PPT)

The main idea behind this new survey strategy is to track the two Pinch Points discussed at Chapter 5 and shown in Figure 6.1. By following the highest concentration regions, more satellites and possible debris can be detected. Pinch Point 1 is located at $0^\circ\ DEC$ and $70^\circ\ RA$ in the geocentric frame. Pinch Point 2 is located at $0^\circ\ DEC$ and $250^\circ\ RA$ in the geocentric frame.

However, Figure 6.2 shows that the Pinch Points may not be above the horizon during the night from the perspective of the specific observatory, or may have a very small time period before it sets below the horizon, or may have poor phase angles. In the first case it is impossible to conduct any survey. For the second case, the total observation time is not utilized efficiently and in the third case, objects may not be visible. The elevation of the Pinch Points also depends on the season of the year.

6.1.1 Survey 1W (Winter, 11 Nov 2017)

Observation time was set from sunset to sunrise with astronomical twilight taken into account for the evening of 11 November 2017. This provided 10 hours and 50 minutes of observation time. Considering $15^\circ$ as minimum elevation criteria, Figure 6.2 shows that the Pinch Point 2 is not visible from the observatory. Thus Survey 1W focused entirely
A summary of Survey 1W is shown in Figure 6.3-6.5. Figure 6.3 shows the pointing direction of telescope in geocentric RA-DEC frame and topocentric azimuth-elevation frame. In the latter case, azimuth is measured counterclockwise from east. Figure 6.3(b) and 6.3(e) shows that Survey 1W scans the sky from east to south and then to west.

Figure 6.4 presents the performance of Survey 1 on 4 key factors. To reduce the problems associated with high star concentration and light saturation, a well-designed survey
should avoid the galactic plane by at least 10° and the moon by 15°. Figure 6.4(a) shows that Pinch Point 1 is sufficiently away from the moon, and Figure 6.4(b) shows that Pinch Point 1 is sufficiently away from the galactic center. Figure 6.4(c) shows that Pinch Point 1 has good phase angle greater than 90° throughout the entire night. Although no algorithm was used to force the favorable observation environment in this survey for this date. Different dates and times may show unfavorable observation environment. The mean slew angle, \( \mu_{\Delta \theta} = 0.077° \), and the mean plus 2\( \sigma \) slew angle, \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 0.084° \). Figure 6.4(d) shows that the Survey 1S has very small telescope movements and telescope slewing time are well within the bounds.

For this survey design, a total of 139 unique cataloged RSOs are captured in the field of view. A total of 2078 images of the 139 unique RSOs are taken throughout the entire survey. Figure 6.5(a) illustrates that most of the GEO RSOs are captured approximately 15-16 times. 365 images do not have any RSO as the Pinch Point does not guarantee RSO detection at each time step. The highest number of RSOs captured in a single image was 8 as showed in Figure 6.5(c). Figure 6.5(d) shows the time history of accumulated RSOs. The first Point was not above the horizon for the first 3 hrs and 20 mins.
Fig. 6.3: Telescope pointing, Survey 1W (Winter, 11 Nov 2017)
Fig. 6.4: Constraints Survey 1W (Winter, 11 Nov 2017)
6.1.2 Survey 1S (Summer, 17 July 2018)

For the evening of 17 July 2018, there are 5 hours of observation time and Pinch Point 1 was not visible from the observatory. Pinch Point 2 was visible and tracked for 3 hrs 45 mins. Similar to Survey 1W, images are taken every 20 sec with 5 second exposure. A summary of Survey 1W is shown in Figure 6.6-6.8.

Figure 6.6(c) shows that the survey scans from east to south and then to west. The elevation of the center of the FOV is above 15° for 3 hrs 45 mins. Figure 6.7 shows that the Pinch Point 2 has good phase angle and is also sufficiently away from the moon and the galactic center. The mean slew angle, $\mu_{\Delta\theta} = 0.089^\circ$, and the mean plus $2\sigma$ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 0.0981^\circ$. Figure 6.7(d) shows that the Survey 1S has very small telescope
movements and telescope slewing time are well within the bounds.

Because of the shorter observation time in summer, only 43 unique cataloged RSOs are captured. Most of the unique satellites are detected 15 times as shown in Figure 6.8(a). The 43 unique RSOs are observed 618 times. The highest number of RSOs captured in a single image was 6 as shown in Figure 6.8(c). Similar to Survey 1W, 347 images do not have any RSOs as the Pinch Point does not guarantee RSO detection at each time step.
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 6.6: Telescope pointing, Survey 1S (Summer, 17 July 2018)
Fig. 6.7: Constraints Survey 1S (Summer, 17 July 2018)
6.2 Survey Design 2 – Modified Pinch Point Tracking (MPPT)

In Survey 1, the Pinch Point may not be visible from the observatory throughout the night, the phase angle may not be favorable, or the moon or galactic center may be very close to the field of view. Survey 2 is designed to overcome these challenges by providing flexibility to move away from the Pinch Point when the above unfavorable conditions exist.
For Survey 2 (and subsequent survey designs), the following constraints are enforced:

\[
\begin{align*}
\text{solar phase angle} & \quad \phi(\alpha_{\text{FOV}}, \delta_{\text{FOV}}, t) > 90^\circ \quad (6.2) \\
\text{local elevation} & \quad \lambda(\alpha_{\text{FOV}}, \delta_{\text{FOV}}, t) > 15^\circ \quad (6.3) \\
\text{lunar angular displacement} & \quad \xi(\alpha_{\text{FOV}}, \delta_{\text{FOV}}, t) > 20^\circ \quad (6.4) \\
\text{galactic center angular displacement} & \quad \psi(\alpha_{\text{FOV}}, \delta_{\text{FOV}}, t) > 10^\circ \quad (6.5)
\end{align*}
\]

Next, a grid in RA-DEC space is created and populated by the number of catalogued GEO RSOs, \( n \), where \( n \) is the number of catalogued RSOs in each cell throughout the entire observation period. Each cell of the grid has the same size as the FOV of the ground based telescope. \( \phi, \lambda, \xi, \) and \( \psi \) are computed at each time observation time, \( t_i \), for each cell in the grid. Then, each cell is given a weight, \( w_i \), equal to 100 if the constraint is satisfied or zero if not. Finally, a performance index \( J_i \) is computed and assigned to each cell in the grid at each observation time, \( t_i \).

\[
J_i = n \times w_{\phi,i} \times w_{\lambda,i} \times w_{\xi,i} \times w_{\psi,i} \quad (6.6)
\]

The survey is designed by selecting the cell that has the highest performance index at each time step, i.e., the cell with the highest value of \( J_i \). The survey will always image the RA-DEC with the largest number of RSOs subject to the constraints in Eqs 6.2-6.5. The result of this approach for surveys designed are shown in Figure 6.9-6.14.

### 6.2.1 Survey 2W (Winter, 11 Nov 2017)

Figure 6.9(a) shows that the Survey 2W has multiple points in RA space, but the DEC is always at 0°. Unlike Survey 1W, Survey 2W was forced to meet all the user defined constraints, and the catalog propagation was for only 10 hrs and 50 mins (entire observation time) instead of 24 hours.

At the beginning for Survey 2W, the cell that contained the largest number of GEO
objects is below the horizon, so, as designed, the survey looks for the next best cell that satisfies all the criteria stated above. Figure 6.9(c) illustrates that the survey starts at 10° RA and switches to higher RA and eventually settles for 65°. Figure 6.9(f) also shows that the survey tracked several other cells before the cell with the maximum number of objects rises above 15° elevation. This is an improvement of Survey 1W as it was initially below the horizon for initial 3 hrs 20 mins as shown in Figure 6.3(b). As soon as the cell with the maximum number of objects satisfies all of the key angle constraints, it is tracked by Survey 2W in the same manner as the Pinch Point in Survey 1W. Figure 6.9(e) shows that Survey 2W scans the sky from east to near south while waiting for the Pinch Point to show up and then again scans the sky from east to south and then to west through the remaining observation time.

Figure 6.10 presents the performance of Survey 2W on 4 key factors. It shows that throughout the entire observation period $\phi$ is greater than 138°, $\xi$ is greater than 70°, $\psi$ is greater than 100°, which ensures good observation condition throughout the entire survey. The $\Delta\theta$ time history plot in Figure 6.10(d) shows that the telescope slew well bellow the 100° constraint throughout the entire observation time. The mean slew angle, $\mu_{\Delta\theta} = 0.179^\circ$, and the mean plus 2σ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 1.076^\circ$ ensures that Survey 2W can be accomplished without excessive slewing.

Figure 6.11(d) shows that a total of 156 unique RSOs are detected and 153 of them are detected at least 3 times (at least 3 observations are required for angles only initial orbit determination). 330 image frames did not contain any satellites and a maximum 8 satellites are detected in a single image frame.
Fig. 6.9: Telescope pointing, Survey 2W (Winter, 11 Nov 2017)
Fig. 6.10: Constraints Survey 2W (Winter, 11 Nov 2017)
6.2.2 Survey 2S (Summer, 17 July 2018)

Using the same approach, Survey 2S was run for the evening of 17 July 2018, where the observation time was only 5 hours. Interestingly, Survey 2S has only one point in RA-DEC space as shown in Figure 6.12(a) similar to the Pinch Point survey. Unlike the Survey 1S survey, Survey 2S was forced to meet all the user defined constraints, and the catalog propagation was for only 5 hours (entire observation time) instead of 24 hours. Figure 6.12(f) shows that the survey scans from east to south and then towards the west and the elevation of the center of the field of view is always above 15°. Figure 6.13 shows that throughout the entire observation period $\phi$ is greater than $160^\circ$, $\xi$ is greater than $124^\circ$, and $\psi$ is greater than $35^\circ$, which ensures good observation conditions throughout the entire
survey. The mean slew angle and mean plus 2σ slew angle are $\mu_{\Delta \theta} = 0.138^\circ$ and $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 0.15^\circ$, respectively, and show that telescope slewing times are well within the bounds.

Figure 6.14(c) shows that the maximum number of RSOs in an image frame is 6, and Figure 6.14(a) shows that most of the detected RSOs are tracked 10 or 11 times, which is similar to Survey 1S. A total of 114 unique RSOs are detected and 113 of them are detected at least three times as shown in Figure 6.14(d). The 114 unique catalogued RSOs are imaged 1122 times.
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 6.12: Telescope pointing, Survey 2S (Summer, 17 July 2018)
Fig. 6.13: Constraints Survey 2S (Summer, 17 July 2018)
(a) Frequency of the GEO satellite observations
(b) GEO satellites detection time history
(c) Number of satellites in each image
(d) Accumulation of RSO time history

Fig. 6.14: RSO data, Survey 2S (Summer, 17 July 2018)

6.3 Survey Design 3 – Maximize Object Observations (MOO)

For Survey 1 and 2, a single grid in RA-DEC space was created where each cell representing a potential telescope pointing direction was populated with the number of objects passing through that cell over an entire observation period utilizing a GEO catalogue. For Survey 2, each cell was also weighted according the constraints that had been satisfied at each time $t_i$.

For Survey 3, a grid is created at each possible observation time $t_i$, and the cells are populated by the number of objects that occupy that cell at time $t_i$. The cells are then weighted by the performance index $J_i$ in Eq 7.6. In effect, this creates a grid in RA-DEC space at each time $t_i$. If multiple cells of the grid satisfy all the survey requirements, then
the performance index only depends on the number of RSOs stored in the cell. If multiple
cells have equal performance indices, the algorithm will select the cell that is closest to the
cell associated with the previous observation. The results of this approach are shown in
Figure 6.15-6.20.

Unfortunately, one characteristic of GEO objects is that if a large number of RSOs with
favorable observing conditions are captured in the FOV, they may be tracked repeatedly
throughout the entire observation period. This phenomena is due to the inherent nature
of the GEO satellite population. The performance index will be always higher for the cell
containing that group of satellites. Thus a cell with a large number of GEO RSOs will be
tracked repeatedly because it’s performance index will always be the highest so long as the
constraints are met.

6.3.1 Survey 3W (Winter, 11 Nov 2017)

The result for Survey 3W are shown in Figure 6.15-6.17. Figure 6.15(a) shows that
Survey 3W stays at 0° DEC, but scans through -45° RA to 135° RA. Figure 6.15(c) illus-
trates some linear tracking features in RA space. This indicates that the survey is tracking
only a few groups of satellite through the entire observation period. Figure 6.15(e) shows
that the survey is generally slewing back and forth from east to south, and later part of the
survey is slewing back and forth from south to southwest. The elevation was never below
15° as shown in Figure 6.15(b).

Figure 6.16 shows that Survey 3W satisfies all the key angle constraints. \( \phi \) is greater
than 90°, \( \xi \) is greater than 20°, and \( \psi \) is greater than 60° throughout the entire survey.
The mean slew angle, \( \mu_{\Delta \theta} = 10.757° \), and the mean plus 2\( \sigma \) slew angle, \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 48.24° \)
shows that there will be more slewing activity than previous surveys, but the slewing is
within the maximum capability of the telescope.

Figure 6.17(b) shows the GEO satellites detection time history and Figure 6.17(d)
shows the accumulated RSO data over time. Both figures confirm the repeating charac-
teristics of the survey. An abundance of horizontal lines in the accumulated RSOs curve
clearly indicates that some RSOs are tracked repeatedly, and in some cases the RSOs are
tracked more than 900 times as shown in Figure 6.17(a). Figure 6.17(c) shows that the maximum number of RSO in a single image was 8. Survey 3W has at least 4 RSOs in a single image compared to 330 images with zero satellites in Survey 2W. One of the goals of the newly designed survey was to maximize the number of observed objects, which is surely visible in Figure 6.17(b).

However, a total of only 104 unique RSOs are detected, 84 of them at least 3 times. As compared to Survey 2W where 156 unique RSOs are detected, 153 of them at least 3 times. So, in this sense Survey 3W failed to increase the number of unique RSOs.

However, the 104 unique catalogued RSOs are imaged 7724 times, while the 156 unique RSOs in Survey 2W are imaged only 2093 times. In this sense, Survey 3W did what it was supposed to do.
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 6.15: Telescope pointing, Survey 3W (Winter, 11 Nov, 2017)
Fig. 6.16: Constraints Survey 3W (Winter, 11 Nov, 2017)
6.3.2 Survey 3S (Summer, 17 July 2018)

Using the same approach, Survey 3S was run for the evening of 17 July 2018, where observation time was only 5 hours. The result for Survey 3S are shown in Figures 6.18-6.20. Survey 3S shows similar characteristics as Survey 3W. Repetition in RSO tracking is still visible in Figure 6.18(c). In Survey 3S, as in Survey 3W, only a few groups of RSOs are tracked repeatedly. The survey is fixed in 0° DEC but moves around in RA space. Figure 6.18(e) shows that the survey is generally slewing back and forth from east to south, and later part of the survey is slewing back and forth from south to southwest. The elevation is never below 15° as shown in Figure 6.20(d).

Figure 6.19 shows that, throughout the entire observation period \( \phi \) is greater than 90°,
\( \xi \) is greater than 60°, and \( \psi \) is greater than 20° which ensures good observation condition throughout the entire survey. The mean slew angle, \( \mu_{\Delta \theta} = 12.83^\circ \) and the mean plus 2\( \sigma \) slew angle, \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 53.64^\circ \) show that telescope slewing well within the bound.

Long streaks in Figure 6.20(b) and the higher number in Figure 6.20(a) also confirms that only few catalogued RSOs are tracked repeatedly. Figure 6.20(c) shows that the maximum number of RSO in image frame is 8.

However, a total of only 63 unique RSOs are detected, 52 of them at least 3 times. As compared to Survey 2S where 114 unique RSOs are detected, 113 of them at least 3 times. So, in this sense Survey 3S failed to increase the number of unique RSOs. However, the 63 unique catalogued RSOs are imaged 3417 times, while the 114 unique RSOs in Survey 2W are imaged only 1122 times. In this sense, Survey 3S did what it was supposed to do.
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame  

(c) RA time history  
(d) DEC time history  

(e) Azimuth time history  
(f) Elevation time history  

Fig. 6.18: Telescope pointing, Survey 3S (Summer, 17 July 2018)
Fig. 6.19: Constraints Survey 3S (Summer, 17 July 2018)
6.4 Survey Design 4 – Random Ra-Dec Selection (RRDS)

In Survey 3, some of the cells in the RA-DEC space are tracked repeatedly because they nearly always had the highest performance index. Survey 4 is designed to overcome this issue. In Survey 4, the grid cells at each time $t_i$ will be populated in the same manner as in Survey 3, and each cell at each time will be scored using Eq 7.6. However, instead of selecting the cell with the highest performance index (as in Survey 3), Survey 4 will randomly select from all the cells with performance index $J_i$ greater than zero in the grid. Performance index $J_i$ greater than zero ensures at least 1 satellites in the FOV at that time step. Because of the random selection, this survey method will not track the same group of satellites repeatedly. The results of this approach are shown in Figure 6.21-6.26.
6.4.1 Survey 4W (Winter, 11 Nov 2017)

The result for Survey 4W are shown in Figure 6.21-6.23. Figure 6.21(a) shows that Survey 4W scans through -50° RA to 135° RA and -15° DEC to 15° DEC for the entire visible GEO belt area in RA-DEC space. Figure 6.21(b) and Figure 6.21(e) show that the survey scans generally from east then to south, and to west parts of the sky but is highly variable. The elevation is always above 15° as shown in Figure 6.21(f).

Figure 6.22 shows that throughout the entire observation period φ is greater than 90°, ξ is greater than 20°, and ψ is greater than 40° which ensures good observation condition throughout the entire survey. The survey design ensured at least 1 RSO in each image. A maximum of 7 RSOs can be detected in a single image. It can be seen that the randomness of this survey requires significant slewing between observation. As expected, \( \mu_{\Delta \theta} = 37.04° \) and the mean plus 2σ slew angle, \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 87.31.64° \) is much higher than previous surveys. The elevation and the \( \Delta \theta \) in Figure 6.22(d) shows that the angular movement of the telescope is large. In some cases \( \Delta \theta \) may be too large to complete the slew.

Figure 6.23(b) shows that the Survey 4 covers the satellite id vs observation time space more uniformly. The maximum frequency of any RSO is 11. That shows that the designed survey is no longer tracking particular groups of RSOs as desired. Figure 6.23(d) shows that a total of 398 unique RSOs are detected which is much higher than previous surveys and 306 of them are detected at least three times. The 398 unique catalogued RSOs are imaged 1755 times.
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 6.21: Telescope pointing, Survey 4W (Winter, 11 Nov 2017)
Fig. 6.22: Constraints Survey 4W (Winter, 11 Nov 2017)
6.4.2 Survey 4S (Summer, 17 July 2018)

Using the same approach, Survey 4S was run for the evening of 17 July 2018, where observation time was only 5 hours. Survey 4S shows similar characteristics as Survey 4W, and scans through the entire visible GEO belt. Similar to Survey 4S, Figure 6.24(b) and Figure 6.24(e) shows that the survey scans the sky from east to south in a random manner and the elevation is always above 15°. As a result the mean slew angle, $\mu_{\Delta \theta}$ will be much higher than Survey 2S and 3S. The mean slew angle, $\mu_{\Delta \theta} = 38.65^\circ$ and the mean plus 2σ slew angle, $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 91.08^\circ$ which is on the high side.

Figure 6.25 shows that throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, and $\psi$ is greater than 10° which ensures good observation conditions.
Figure 6.25(d) shows that, in some cases the telescope may not be able to detect the desired RSO because $\Delta \theta$ is higher than 100°.

Figure 6.26(b) shows the RSO detection time history and it is clearly visible that more RSOs are tracked uniformly rather than repeated detection of group of RSOs. The maximum frequency of any satellite is only 7. At least one RSO is detected in each image frame and a maximum 7 RSOs can be found in a single image. Figure 6.26(d) shows that a total of 306 unique RSOs are detected and 146 of them are detected at least 3 times. The 306 unique catalogued RSOs are imaged 837 times.
(a) Survey in geocentric RA-DEC frame  (b) Survey in topocentric azimuth and elevation frame

(c) RA time history  (d) DEC time history

(e) Azimuth time history  (f) Elevation time history

Fig. 6.24: Telescope pointing, Survey 4S (Summer, 17 July 2018)
Fig. 6.25: Constraints Survey 4S (Summer, 17 July 2018)
6.5 Survey Design 5 – Weighted Object Observation (WOO)

Survey 4 eliminated the Survey 3 problem of repeatedly tracking the same objects, but the random selection of RA-DEC location resulted in very large $\Delta \theta$ from one observation to the next. Further, Survey 4 did not do well in the area of detecting unique RSO at least 3 times. Specifically, Survey 4S detected 306 unique RSOs but only 146 of them are tracked at least three times.

Survey 5 represents a second attempt to improve Survey 3 by implementing a new constraint such that whenever any cell contains a RSO that has been already tracked more than 4 times, performance index $J_i$ is down-weighted by a factor of 20. This will force the survey to go to another location after the down-weighting occurs rather than repeatedly.
tracking the same group of satellites (or same RA-DEC). Dividing by 20 ensures the telescope will again track the group or individual RSO that has already been tracked 4 times only if there is no other individual RSOs in the whole grid other than the group of RSOs. The down-weight method ensures that even one previously undetected RSO in a different cell will have greater performance index than the cell having a group of RSOs tracked at least 4 times. The results of this approach are shown in Figure 6.27-6.32.

6.5.1 Survey 5W (Winter, 11 Nov 2017)

The result for Survey 5W are shown in Figure 6.27-6.29. Figure 6.27(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 6.27(e) and Figure 6.27(b) show that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey, and the elevation is always above 15°.

Figure 6.28 shows that, throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, and $\psi$ is greater than 10° which ensures good observation conditions throughout the entire survey. The mean slew angle, $\mu_{\Delta \theta} = 26.71$° and the mean plus 2$\sigma$ slew angle, $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 89.126$° shows that telescope slewing is within the bound.

Figure 6.29(a) illustrates that most of the RSOs are tracked less than 4 times and only relatively few of them are tracked repeatedly. Figure 6.29(b) also confirms the previous statement. It can be seen from the Figure 6.29(b) and relatively flat line after 8 hrs of observation as shown in Figure 6.29(d) that before 8 hours of observation time, the RSOs are tracked uniformly but after 8 hours of observation some RSOs are tracked repeatedly similar to Survey 3. Remember that the performance factor $n$ was down weighted if any RSO was tracked 4 times. After enough observations almost all the unique RSOs are tracked at least 4 times and then the survey only tracks groups of RSOs having higher performance factor $n$ even after down-weighting.

In Figure 6.29(c) a maximum 7 RSOs can be in a single image and each image contains at least 1 RSO. Figure 6.29(d) shows that a total of 412 unique RSOs are detected and 411 of them are detected at least 3 times which is much higher than previous surveys. The 412 unique catalogued RSOs are imaged 3984 times.
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame  

(c) RA time history  
(d) DEC time history  

(e) Azimuth time history  
(f) Elevation time history  

Fig. 6.27: Telescope pointing, Survey 5W (Winter, 11 Nov 2017)
Fig. 6.28: Constraints Survey 5W (Winter, 11 Nov 2017)
6.5.2 Survey 5S (Summer, 17 July 2018)

Using this same approach, Survey 5S was designed for the evening of 17 July 2018, where the observation time was only 5 hours. The results for Survey 5S are shown in Figure 6.30-6.32. Figure 6.30(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 6.30(e) and Figure 6.30(f) shows that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey.

Figure 6.31 shows that throughout the entire observation period $\phi$ is greater than $90^\circ$, $\xi$ is greater than $20^\circ$, and $\psi$ is greater than $10^\circ$ which ensures good observation conditions throughout the entire survey. Further, the mean slew angle, $\mu_{\Delta\theta} = 29.52^\circ$ and the mean plus $2\sigma$ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 89.81^\circ$ show that telescope slewing is on the high side but
within the sensor limit.

Figure 6.32(a) shows that the most RSOs are tracked more than 3 times. Figure 6.32(b) shows that RSO detection are scattered in in GEO satellite time history plot. This means that groups of RSOs are not tracked repeatedly. Figure 6.32(c) shows that a maximum 7 RSOs can be detected in a single image and here every image has at least 2 RSOs. Figure 6.32(d) shows that a total of 299 unique RSOs are detected and 264 of them are detected at least 3 times which is much higher than Survey 2S and Survey 3S and Survey 4S. The 299 unique catalogued RSOs are imaged 1778 times.
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 6.30: Telescope pointing, Survey 5S (Summer, 17 July 2018)
Fig. 6.31: Constraints Survey 5S (Summer, 17 July 2018)
(a) Frequency of the GEO satellite observations
(b) GEO satellites detection time history
(c) Number of satellites in each image
(d) Accumulation of RSO time history

Fig. 6.32: RSO data, Survey 5S (Summer, 17 July 2018)

Unfortunately, this survey results in many GEO object begin tracked in 4 consecutive observations frames where the time between observations is only 20 sec. Because of this, the actual state information collected will be low, i.e, better state information can be obtained when the time between observations is larger.

6.6 Survey Design 6 – Random Observation Time Selection (ROTS)

Survey 6 is identical to Survey 5 except instead of designing the survey observations by choosing the cell with the highest performance index in chronological order (and getting 4 back-to-back observations of the same object), the survey observations are selected randomly in time until a full night of observations are scheduled. Groups of RSOs that
are observed four times are still down-weighed by a factor of 20 after each observation, but
the observations will be more spaced out over the night in a random manner resulting in
higher state information content. Thus, in Survey 6 a group of RSOs will not be tracked
repeatedly as Survey 3, and $\Delta \theta$ will not be as large as in Survey 4, individual RSO detection
will be more spaced out in time than Survey 5 as down-weighting and random observation
time selection is occurring simultaneously. The results of this approach are shown in Figure
6.33-6.38.

6.6.1 Survey 6W (Winter, 11 Nov 2017)

The result for Survey 6W are shown in Figure 6.33-6.35. Figure 6.33(a) shows that the
survey scans through the visible GEO belt in RA-DEC space. Figure 6.33(e) and Figure
6.33(f) shows that the azimuth and the elevation angles of the telescope pointing direction
are scattered throughout the survey. Figure 6.33(f) illustrates that the elevation is always
above 15°.

Figure 6.34 presents the performance of survey 6 on 4 key factors. It shows that
throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, and $\psi$
is greater than 40°. The mean slew angle, $\mu_{\Delta \theta}$ was 39.21° and the system will be able to get
most of the objects in the center of the FOV as the mean plus $2\sigma$ slew angle, $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta}$
is 98.42° ($<100$°).

Figure 6.35(b) shows how the GEO RSOs detections are distributed as a function
of observation time. Compared to Figure 6.29(b) the dots in the plot look more evenly
distributed. Figure 6.35(c) shows that the maximum number of satellite in the image frame
was 6. Some RSOs are tracked more than 100 times but most of the RSOs are detected
between 4 to 6 times. Figure 6.35(d) shows that the he total number of unique RSOs
detected was 416. The number of RSOs detected more than 3 times are 413. The 416
unique catalogued RSOs are imaged 4842 times.
(a) Survey in geocentric RA-DEC frame

(b) Survey in topocentric azimuth and elevation frame

(c) RA time history

(d) DEC time history

(e) Azimuth time history

(f) Elevation time history

Fig. 6.33: Telescope pointing, Survey 6W (Winter, 11 Nov 2017)
Fig. 6.34: Constraints Survey 6W (Winter, 11 Nov 2017)
6.6.2 Survey 6S (Summer, 17 July 2018)

Using the same approach, Survey 6S was designed for the evening of 17 July 2018, where observation time was only 5 hours. The result for Survey 6S are shown in Figure 6.36-6.38. Figure 6.36(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 6.36(e) and Figure 6.36(f) shows that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey. Figure 6.36(f) illustrates that the elevation is always above 15°.

Figure 6.37 shows that throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, $\psi$ is greater than 10°, and elevation greater than 15° which ensures good observation condition throughout the entire survey. The mean slew angle, $\mu_{\Delta\theta} =$
33.53° and the mean plus $2\sigma$ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 81.87°$ shows that telescope slewing is in the higher side but within the sensor limit.

Figure 6.38(a) shows that most RSOs are tracked more than 3 times. Figure 6.38(b) shows that RSO detections are scattered in in GEO satellite time history plot which means no particular groups of RSOs are not being tracked repeatedly. Figure 6.38(c) shows that a maximum 7 RSOs can be detected in a single image, and here every image will have at least 2 RSOs. Figure 6.38(d) shows that a total of 319 unique RSOs are detected and 272 of them are detected at least 3 times which is much higher than Survey 2S and Survey 3S and slightly higher than 5S. The 319 unique catalogued RSOs are imaged 1888 times.
Fig. 6.36: Telescope pointing, Survey 6S (Summer, 17 July 2018)
Fig. 6.37: Constraints Survey 6S (Summer, 17 July 2018)
6.7 Survey Design 7– Guaranteed Initial Orbit Determination (GIOD)

For the initial orbit determination (IOD) of an RSO, at least 3 sets of angle measurements are required. For a GEO RSO, the time difference between each measurement should be reasonably large due to the geometry of the orbit. Survey 7 ensures 3 observations (3 sets of angle measurements) with a fixed time interval between measurements for each RSO. Initially an $S \times K$ matrix $Q$ is created, where $S$ is the number of satellites and $K$ is the total number of observation times. If an RSO satisfies all the observation constraints stated earlier at any observation time, a value of 1 is stored at that element of the matrix, if not 0 is stored.
For Survey 7, not all RSOs are selected for observation. Only those that can be detected at least 3 times at a 20 minute interval are selected for the survey. That means an RSO has to satisfy all the observation constraints for at least 60 minutes in the entire night to be eligible for the survey.

Next, satellites are prioritized based on their availability for observation in the entire observation time. RSOs with lower availability get higher priority. Some RSOs have a 1 in every column of the matrix $Q$, and that indicates the RSO is visible with favorable observation condition for the entire night and the priority of those RSOs will be less compared to the other RSOs having fewer ones in the matrix $Q$. High priority satellites will be assigned time slots first to collect 3 images 20 min apart, then the time slots will be allocated to lower priority satellites. The results of this approach are shown in Figure 6.39-6.44.

6.7.1 Survey 7W (Winter, 11 Nov 2017)

The result for Survey 7W are shown in Figure 6.39-6.41. Figure 6.39(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 6.39(e) and Figure 6.39(f) shows that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey. Figure 6.39(f) illustrates that the elevation is always above $15^\circ$.

Figure 6.40 clearly indicates that the survey has favorable viewing condition throughout the entire night. The mean slew angle $\mu_{\Delta\theta} = 26.921^\circ$ and the mean plus 2$\sigma$ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 87.74^\circ$, which is below the maximum capacity of the telescope slewing. The $\Delta\theta$ plot indicates that in some cases the slew of the telescope will be very large (> 100$^\circ$). In those cases the telescope may not be able to point to the desired direction at that specific time.

Figure 6.41(c) shows that a maximum 7 RSOs can be detected in a single image. Figure 6.41(d) shows that the number of unique RSOs detected was 409, and among them 408 of the RSOs was detected at least 3 times. The 409 unique catalogued RSOs are imaged 2196 times...
(a) Survey in geocentric RA-DEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 6.39: Telescope pointing, Survey 7W (Winter, 11 Nov 2017)
Fig. 6.40: Constraints Survey 7W (Winter, 11 Nov 2017)
6.7.2 Survey 7S (Summer, 17 July 2018)

Using the same approach, Survey 7S was designed for the evening of 17 July 2018, where observation time was only 5 hours. The result for Survey 7S are shown in Figure 6.42-6.44. Figure 6.42(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 6.42(e) and Figure 6.42(f) shows that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey. Figure 6.42(f) illustrates that the elevation is always above 15°. Figure 6.43 clearly indicates that the survey has favorable viewing condition throughout the entire night. \( \mu_{\Delta \theta} = 34.38^\circ \) and \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 88.12^\circ \), which is below the maximum capacity of the telescope slewing.

Figure 6.44(d) shows that Survey 7S detects 222 unique RSOs and 206 of them are...
detected at least 3 times. Maximum 6 satellites can be detected in a single image frame as shown in Figure 6.44(c) and highest number of detection for single satellite is 11 as shown in Figure 6.44(a). The 222 unique catalogued RSOs are imaged 766 times.
Fig. 6.42: Telescope pointing, Survey 7S (Summer, 17 July 2018)
Fig. 6.43: Constraints Survey 7S (Summer, 17 July 2018)
6.8 Survey Summary

A total of 7 survey designs for GEO RSO detection with their key performance metrics are discussed in detail in this chapter. Survey 1 does not utilize the entire observation time due to the unavailability of the Pinch Point nor does it account for unfavorable observation conditions. Survey 2 attempts to remedy the problems of Survey 1, as it satisfies the observation constraints, attempts to maximize the observation time, and only evaluates RSO concentration over the observation period rather than 24 hrs. An increase in the number of unique object detection reflects the improvement of Survey 2.

In Survey 2, the Pinch Points are the high RSO concentrated points, in RA-DEC frame, when GEO RSOs are propagated for the entire observation time. Survey 3 is designed to
investigate the Pinch Point concept at each observation time instead of entire observation time. Unfortunately due to the inherent nature of GEO RSOs, Survey 3 repeatedly tracks particular groups of RSOs. For Survey 3 the unique number of RSO detection is less than Survey 2.

Survey 4 provides a solution for the issue of repeated detection in Survey 3, and results in an increase in number of unique object detected. However random selection of the cell increases the $\Delta\theta$ and makes this survey operationally challenging.

Survey 5 provides an alternative approach to solving the issues of Survey 3 by appropriately down-weighting cells that are imaged more than 4 times. This survey produces good results with lower $\Delta\theta$. One shortcoming of the Survey 5 however, is that the RSO observations are closely spaced and may not provide higher state information content.

In Survey 6, observations of a RSO are spaced out over the night in a random manner to improve the state information content. The result is that the unique number of object detection is greater than Survey 3-5 and $\Delta\theta$ is reasonably low.

Survey 7 is different than the rest of the surveys stated above. Three observations are spaced out 20 min for each RSO which guarantees it’s initial orbit determination. The number of unique objects detected is similar to Surveys 4-6 in. A summary of all results is presented in Tables 6.2 and 6.3 and Figure 6.45.
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<th># of Unique RSOs Observed &gt; 2 times</th>
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<td>Survey 5W (WOO)</td>
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Table 6.2: Survey summary (Winter, 11 Nov 2017)

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<thead>
<tr>
<th>Survey ID</th>
<th># of RSO Observations</th>
<th># of Unique RSOs Observed</th>
<th># of Unique RSOs Observed &gt; 2 times</th>
<th>Mean $\Delta \theta$ $\bar{\mu}_{\Delta \theta}$(Deg)</th>
<th>$\bar{\mu}<em>{\Delta \theta} + 2 \sigma</em>{\Delta \theta}$(Deg)</th>
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<tr>
<td>Survey 1S (PPT)</td>
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Table 6.3: Survey summary (Summer, 17 July 2018)
Fig. 6.45: Summary
<table>
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<th># of RSO Observations</th>
<th># of Unique RSOs Observed</th>
<th># of Unique RSOs Observed &gt; 2 times</th>
<th>Mean $\Delta \theta$ $\mu_{\Delta \theta}$ (Deg)</th>
<th>$\mu_{\Delta \theta} + 2 \sigma_{\Delta \theta}$ (Deg)</th>
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Table 6.4: Survey summary for AMOS (Winter, 11 Nov 2017)

6.9 Hawaii Data

All the surveys were generated for AMOS observatory, Hawaii. A summary of the AMOS survey data is shown in Table 6.4. Survey 1 shows less number of unique object detection compared to USU-STAR. Pinch point is only available for 4 hour 28 min from the AMOS observatory while the pinch point was available 7 hour 30 min from USU-STAR. However, Survey 2-7 shows similarity in all metrics.
CHAPTER 7
Uncontrolled Object Survey Design

The survey designs approaches of the previous chapter will be applied to a catalogue of uncontrolled GEO objects, in this research uncontrolled GEO objects are defined as near GEO objects with inclinations \(i\) greater than \(2^\circ\) and semi-major axes \(a\) between 40000 km and 44000 km. For the survey designs the uncontrolled/debris orbital set \(S\) defined as

\[
S = \{a \in [40000 \text{ km}, 44000 \text{ km}] \cap i > 2^\circ\}
\] (7.1)

The catalogued uncontrolled/debris RSOs in set \(S\) are then given a unique identification (id) based on their order in the TLE catalogue.

The observation period for ground-based optical telescope varies widely from season to season. To see the effect of observation time for two different seasons, surveys are designed for both summer (17 July, 2018) and winter (11 November, 2017). The observation time is set from sunset to sunrise with astronomical twilight taken into account. This provided 10 hrs and 50 mins of observation time for the evening of 11 November 2017 and only 5 hrs of observation time for evening of 17 July 2018. For each survey, summer and winter results will be represented by S and W respectively, e.g. Survey 1W, Survey 3S etc. Publicly available RSO data from space-track.org is used to propagate uncontrolled GEO RSOs for both dates.

The first survey, a Pinch Point Survey design, is presented in Section 7.1. The main idea of this design is to first determine the number of uncontrolled GEO RSOs that pass through each point in RA-DEC space over entire evening of observations. Then the survey is designed to point at the RA-DEC location with the highest concentration of RSOs. This survey design is operationally the simplest since very little telescope slewing is required. The next 5 surveys, presented in Sections 7.2-7.6, are each intended to be continuous improvements of
the Pinch Point survey design. Significant telescope slewing may be required in these cases. In section 7.7, the last survey design of this chapter, the previous surveys are abandoned and replaced with a design to ensure that each observable RSO is detected at least three times, as this is necessary for Initial Orbit Determination (IOD). A summary of all 7 surveys is presented in Section 7.8.

For Surveys 2-7, the time between every observation is selected as 30 sec and a slew rate of 5°/sec is chosen. Similar to chapter 6, all surveys assume camera exposure times of 5 sec, camera readout of 5 sec and the remaining 20 sec for slewing. Thus, the maximum $\Delta \theta$ is considered to be 100°, since if $\Delta \theta > 100 ^\circ$, the telescope may not have enough time to reach the commanded RA-DEC.

7.1 Survey Design 1 – Pinch Point Tracking (PPT)

The main idea behind this survey strategy is to track the Pinch Points discussed in Chapter 5 and the satellite concentration of uncontrolled GEO objects shown in Figure 7.1. By following the highest concentration regions, it is hypothesized that more uncatalogued uncontrolled satellites and possibly uncatalogued debris can be detected. For only uncontrolled GEO RSOs, there are several high concentration regions or, Pinch Points, instead of two as discussed in previous chapter. Among the several Pinch Points, -4.5° DEC, 14.5° RA and -14.7° DEC, 253° RA are the two Pinch Points selected for this research.

However, Figure 7.2 shows that the Pinch Points may not be above the horizon during the night from the perspective of the specific observatory, or may have a very small time period before it sets below the horizon, or may have poor solar phase angles. In the first case it is impossible to conduct any survey. For the second case, the total observation time is not utilized efficiently and in the third case, objects may not be visible. The elevation of the Pinch Points also depends on the season of the year.
Fig. 7.1: Uncontrolled satellite concentration in the geosynchronous belt over entire observation hr period, 2017

(a) Pinch Point elevation 11 November, 2017
(b) Pinch Point elevation 17 July, 2018

Fig. 7.2: Pinch Point elevation
### 7.1.1 Survey 1W (Winter, 11 Nov 2017)

Considering 15° as minimum elevation criteria, Figure 7.2(a) shows that Pinch Point 2 is not visible from the observatory. Thus Survey 1W focused entirely on Pinch Point 1. Pinch Point 1 was above 15° elevation for 7 hrs. The survey images are taken every 20 sec with 5 sec exposure.

A summary of Survey 1W results are shown in Figure 7.3-7.5. Figure 7.3 shows the pointing direction of telescope in geocentric RA-DEC frame and in topocentric azimuth-elevation frame. In the latter case, azimuth is measured counterclockwise from east. Figure 7.3(b) and 7.3(e) shows that Survey 1W scans the sky from east to south and then to west.

Figure 7.4 presents the performance of Survey 1 on 4 key factors. To reduce the problems associated with high star concentration and light saturation, a well-designed survey should avoid the galactic plane by at least 10° and the moon by 15°. Figure 7.4(a) shows that Pinch Point 1 is sufficiently away from the moon, and Figure 7.4(b) shows that Pinch Point 1 is sufficiently away from the galactic center. Figure 7.4(c) shows that Pinch Point 1 has a good solar phase angle greater than 90° throughout the entire night. Although no algorithm was used to force the favorable observation environment in this survey for this date different dates and times may show unfavorable observation environment. The mean slew angle, $\mu_{\Delta\theta} = 0.0737^\circ$, and the mean plus 2σ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 0.0803^\circ$, are small. Figure 7.4(d) shows that Survey 1W has very small telescope movements and telescope slewing time are well within the bounds.

For this survey design, a total of 64 unique cataloged RSOs are captured in the field of view and all of them are detected at least 3 times. A total of 917 images of the 64 unique RSOs are taken throughout the entire survey. Figure 7.5(a) illustrates that most of the GEO RSOs are captured approximately 15-16 times. 622 images do not have any RSO as the Pinch Point does not guarantee RSO detection at each time step. The highest number of RSOs captured in a single image is 4 as showed in Figure 7.5(c). Figure 7.5(d) shows the time history of accumulated RSOs. It should be noted that the Pinch Point is not above the horizon for the last 3 hrs 50 mins of the evening.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 7.3: Telescope pointing, Survey 1W (Winter, 11 Nov 2017)
Fig. 7.4: Constraints Survey 1W (Winter, 11 Nov 2017)
7.1.2 Survey 1S (Summer, 17 July 2018)

For the evening of 17 July 2018, there were 5 hrs of observation time and Pinch Point 1 was visible for a very short period of time from the observatory. Pinch Point 2 was visible and tracked for 2 hrs 30 mins. Similar to Survey 1W, images were taken every 20 sec with 5 second exposure. A summary of Survey 1W is shown in Figure 7.6-7.8.

Figure 7.6(e) shows that the survey scans from south to west. The elevation of the center of the FOV is above 15° for 2 hrs 30 mins. Figure 7.7 shows that Pinch Point 2 has good solar phase angle and is also sufficiently away from the moon and the galactic center. The mean slew angle, \( \mu_{\Delta \theta} = 0.084° \), and the mean plus 2\( \sigma \) slew angle, \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 0.0918° \) are small. Figure 7.7(d) shows that the Survey 1S has very small telescope movements and
telescope slewing time are well within the bounds.

Because of the shorter observation time in summer, only 21 unique cataloged objects were captured and all of them were detected at least 3 times. Most of the unique satellites were detected 15 times as shown in Figure 7.8(a). The 21 unique RSOs were observed 297 times. The highest number of RSOs captured in a single image was 4 as shown in Figure 7.8(c). Similar to Survey 1W, 261 images do not have any RSOs as the Pinch Point does not guarantee RSO detection at each time step.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in topocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 7.6: Telescope pointing, Survey 1S (Summer, 17 July 2018)
Fig. 7.7: Constraints Survey 1S (Summer, 17 July 2018)
Fig. 7.8: RSO data, Survey 1S (Summer, 17 July 2018)

7.2 Survey Design 2 – Modified Pinch Point Tracking (MPPT)

In Survey 1, the Pinch Point may not be visible from the observatory throughout the
night, the phase angle may not be favorable, or the moon or galactic center may be very
close to the field of view. Survey 2 is designed to overcome these challenges by providing
flexibility to move away from the Pinch Point when the above unfavorable conditions exist.
For Survey 2 (and subsequent survey designs), the following constraints are enforced:

\[
\begin{align*}
\text{solar phase angle} & \quad \phi(\alpha_{FOV}, \delta_{FOV}, t) > 90^\circ \quad (7.2) \\
\text{local elevation} & \quad \lambda(\alpha_{FOV}, \delta_{FOV}, t) > 15^\circ \quad (7.3) \\
\text{lunar angular displacement} & \quad \xi(\alpha_{FOV}, \delta_{FOV}, t) > 20^\circ \quad (7.4) \\
\text{galactic center angular displacement} & \quad \psi(\alpha_{FOV}, \delta_{FOV}, t) > 10^\circ \quad (7.5)
\end{align*}
\]

Where \( \alpha_{FOV} \) and \( \delta_{FOV} \) represents the center of the FOV in the RA-DEC frame.

Next, a grid in RA-DEC space is created and populated by the number \( n \) of uncontrolled catalogued RSOs that pass through each cell throughout the entire observation period. Each cell of the grid has the same size as the FOV of the ground based telescope. \( \phi, \lambda, \xi, \) and \( \psi \) are computed at each time observation time, \( t_i \), for each cell in the grid. Then, each cell is assigned 4 weights, \( w_{\phi,i}, w_{\lambda,i}, w_{\xi,i}, \) and \( w_{\psi,i} \), all are equal to 100 if the constraint is satisfied or zero if not. Finally, a performance index \( J_i \) is computed and assigned to each cell in the grid at each observation time, \( t_i \).

\[
J_i = n \times w_{\phi,i} \times w_{\lambda,i} \times w_{\xi,i} \times w_{\psi,i} \quad (7.6)
\]

The survey is designed by selecting the cell that has the highest performance index at each time step, i.e., the cell with the highest value of \( J_i \). The survey will always image the RA-DEC with the largest number of RSOs subject to the constraints in Eqs 7.2-7.5. The result of this approach are shown in Figure 7.9-7.14.

7.2.1 Survey 2W (Winter, 11 Nov 2017)

Unlike Survey 1W, Survey 2W was forced to meet all the user defined constraints. Images are taken at 30 sec time interval through the entire observation time. Figure 7.9(c) and Figure 7.9(d) show that other than for a brief period at the beginning, the survey is tracking a single point in RA-DEC space, and as soon as the point does not have favorable observation conditions (e.g. the cell that contained the largest number of GEO objects is
below the horizon), the survey looks for the next best cell that satisfies all the criteria stated above. Thus, the survey switches to higher RA-DEC and eventually settles for 70° RA and 14.08° DEC.

Figure 7.9(c) shows that Survey 2W scans the sky from east to west for more than 6 hrs of observation time and then south to west through the remaining observation time.

Figure 7.10 presents the performance of Survey 2W on 4 key factors. It shows that throughout the entire observation period $\phi$ is greater than 130°, $\xi$ is greater than 70°, and $\psi$ is greater than 90°, which ensures good observation conditions throughout the entire survey. The $\Delta \theta$ time history plot in Figure 7.10(d) shows that the telescope slewing requirement is well below the 100° constraint throughout the entire observation time. The mean slew angle, $\mu_{\Delta \theta} = 0.19^\circ$, and the mean plus 2σ slew angle, $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 3.075^\circ$ are small and ensures that Survey 2W can be accomplished without excessive slewing.

Figure 7.11(d) shows that a total of 83 unique objects are detected and 80 of them are detected at least 3 times (at least 3 observations are required for angles only initial orbit determination). The 83 unique catalogued RSOs are imaged 1742 times. However, 708 image frames did not contain any satellites and a maximum 4 satellites are detected in a single image frame.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in geocentric azimuth and elevation frame  

c) RA time history  
(d) DEC time history  
(e) Azimuth time history  
(f) Elevation time history  

Fig. 7.9: Telescope pointing, Survey 2W (Winter, 11 Nov 2017)
(a) $\xi$ time history

(b) $\psi$ time history

(c) $\phi$ time history

(d) $\Delta \theta$ time history

Fig. 7.10: Constraints Survey 2W (Winter, 11 Nov 2017)
7.2.2 Survey 2S (Summer, 17 July 2018)

Using the same approach, Survey 2S was run for the evening of 17 July 2018, where the observation time was only 5 hrs. Figure 7.12(a) shows the pointing direction of telescope in RA-DEC frame and it is clear that the survey is tracking several Pinch Points. Figure 7.12(b) and Figure 7.12(e) show that the survey scans generally from south to west and then eastern parts of the sky. Unlike the Survey 1S survey, Survey 2S was forced to meet all the user defined constraints. Figure 7.13 shows that throughout the entire observation period $\phi$ is greater than $90^\circ$, $\xi$ is greater than $60^\circ$, and $\psi$ is greater than $10^\circ$, which ensures good observation conditions throughout the entire survey. The mean slew angle and mean plus $2\sigma$ slew angle are $\mu_{\Delta\theta} = 0.364^\circ$ and $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 6.0715^\circ$, respectively, and show that
telescope slewing times are well within the bounds.

Figure 7.14(c) shows that the maximum number of RSOs in an image frame is 4, and Figure 7.14(a) shows that most of the detected RSOs are tracked 10 or 11 times. A total of 37 unique objects were detected and 34 of them were detected at least three times as shown in Figure 7.14(d). The 37 unique catalogued RSOs were imaged 429 times.
(a) Survey in geocentric RA-EDEC frame

(b) Survey in geocentric azimuth and elevation frame

(c) RA time history

(d) DEC time history

(e) Azimuth time history

(f) Elevation time history

Fig. 7.12: Telescope pointing, Survey 2S (Summer, 17 July 2018)
(a) $\xi$ time history  
(b) $\psi$ time history  
(c) $\phi$ time history  
(d) $\Delta \theta$ time history

Fig. 7.13: Constraints Survey 2S (Summer, 17 July 2018)
7.3 Survey Design 3 – Maximize Object Observations (MOO)

For Survey 1 and 2, a single grid in RA-DEC space was created where each cell representing a potential telescope pointing direction and was populated with the number of objects passing through that cell over an entire observation period utilizing a GEO catalogue of uncontrolled objects. For Survey 2, each cell was also weighted according the constraints that had been satisfied at each time $t_i$.

For Survey 3, a grid is created at each possible observation time $t_i$, and the cells are populated by the number of objects that occupy that cell at time $t_i$. The cells are then weighted by the performance index $J_i$ in Eq 7.6. In effect, this creates a grid in RA-DEC space at each time $t_i$. If multiple cells of the grid satisfy all the survey requirements, then
the performance index only depends on the number of RSOs stored in the cell. If multiple cells have equal performance indices, the algorithm will select the cell that is closest to the cell associated with the previous observation. The results of this approach are shown in Figure 7.15-7.20.

Unfortunately, one characteristic of GEO objects is that if a large number of RSOs with favorable observing conditions are captured in the FOV, they may be tracked repeatedly throughout the entire observation period. This phenomena is due to the inherent nature of the GEO satellite population. The performance index will be always higher for the cell containing that group of satellites. Thus a cell with a large number of GEO RSOs will be tracked repeatedly because it’s performance index will always be the highest so long as the constraints are met.

7.3.1 Survey 3W (Winter, 11 Nov 2017)

The results for Survey 3W are shown in Figure 7.15-7.17. Figure 7.15(a) shows that Survey 3W scans through the entire visible GEO belt area in RA-DEC space. Figure 7.15(c) illustrates some linear tracking features in RA space. This indicates that the survey is tracking on the order of 20 groups of satellite through the entire observation period. Figure 7.15(e) shows that the survey is generally slewing back and forth from east to south at the beginning of the survey and back and forth from south to west at the end of the survey. The elevation is never below 15° as shown in Figure 7.15(b).

Figure 7.16 shows that Survey 3W satisfies all the key angle constraints. $\phi$ is greater than 90°, $\xi$ is greater than 20°, and $\psi$ is greater than 40° throughout the entire survey. The mean slew angle, $\mu_{\Delta \theta} = 22.91^\circ$, and the mean plus 2σ slew angle, $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 77.426^\circ$, shows that there will be more slewing activity than previous surveys, but the slewing is within the maximum capability of the telescope.

Figure 7.17(b) shows the GEO satellites detection time history and Figure 7.17(d) shows the accumulated RSO data over time. Both figures confirm the repeating characteristics of the survey. An abundance of horizontal streaks in Figure 7.17(b) clearly indicates that some RSOs are tracked repeatedly, and in some cases the RSOs are tracked more than
150 times as shown in Figure 7.17(a). Figure 7.17(c) shows that the maximum number of RSO in a single image was 5. Survey 3W has at least 2 RSOs in every image compared to 708 images with zero satellites in Survey 2W. One of the goals of the newly designed survey was to maximize the number of object observations, which is surely visible in Figure 7.17(b).

Figure 7.17(d) shows that a total of only 206 unique RSOs are detected, 158 of them at least 3 times, as compared to Survey 2W where 83 unique RSOs were detected, 80 of them at least 3 times. The 206 unique catalogued RSOs are imaged 3833 times, while the 83 unique RSOs in Survey 2W are imaged only 801 times. In this sense, Survey 3W did what it was supposed to do.
Fig. 7.15: Telescope pointing, Survey 3W (Winter, 11 Nov, 2017)
Fig. 7.16: Constraints Survey 3W (Winter, 11 Nov, 2017)
7.3.2 Survey 3S (Summer, 17 July 2018)

Using the same approach, Survey 3S was run for the evening of 17 July 2018, where observation time was only 5 hrs. The results for Survey 3S are shown in Figures 7.18-7.20. Survey 3S shows similar characteristics as Survey 3W. Repeated RSO tracking is again evident in Figure 7.18(c) where approximately 15 groups of RSOs are tracked repeatedly. Figure 7.18(a) shows that the telescope will slew over the visible GEO belt in RA-DEC frame. Figure 7.18(e) and Figure 7.18(f) show that the azimuth and the elevation values are scattered throughout the survey. The elevation is never below 15° as shown in Figure 7.20(d).

Figure 7.19 shows that, throughout the entire observation period φ is greater than 90°,
\( \xi \) is greater than 20\(^\circ\), and \( \psi \) is greater than 10\(^\circ\) which ensures good observation conditions throughout the entire survey. The mean slew angle, \( \mu_{\Delta \theta} = 23.69^\circ \), and the mean plus 2\( \sigma \) slew angle, \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 79.25^\circ \) show that telescope slewing is well within the bounds.

Long streaks in Figure 7.20(b) and the higher number in Figure 7.20(a) also confirms that groups of Uncontrolled catalogued RSOs are tracked repeatedly. Figure 7.20(c) shows that the maximum number of RSO in image frame is 4.

A total of only 126 unique RSOs are detected, 95 of them at least 3 times, as compared to Survey 2S where 37 unique RSOs were detected, 34 of them at least 3 times. So, in this sense Survey 3S increased the number of unique RSOs. The 126 unique uncontrolled catalogued RSOs are imaged 1742 times, while the 37 unique RSOs in Survey 2S are imaged only 429 times. In this sense, Survey 3S did what it was supposed to do.
(a) Survey in geocentric RA-EDEC frame  (b) Survey in geocentric azimuth and elevation frame

(c) RA time history  (d) DEC time history

(e) Azimuth time history  (f) Elevation time history

Fig. 7.18: Telescope pointing, Survey 3S (Summer, 17 July 2018)
Fig. 7.19: Constraints Survey 3S (Summer, 17 July 2018)
7.4 Survey Design 4 – Random RA-DEC Selection (RRDS)

In Survey 3, some of the cells in the RA-DEC space were tracked repeatedly because they nearly always had the highest performance index. Survey 4 is designed to overcome this issue. In Survey 4, the grid cells at each time $t_i$ will be populated in the same manner as in Survey 3, and each cell at each time will be scored using Eq 7.6. However, instead of selecting the cell with the highest performance index (as in Survey 3), Survey 4 will randomly select from all the cells with performance index $J_i$ greater than zero in the grid. Performance index $J_i$ greater than zero ensures at least 1 satellites in the FOV at that time step. Because of the random selection, this survey method will not track the same group of satellites repeatedly. The results of this approach are shown in Figure 7.21-7.26.

Fig. 7.20: RSO data, Survey 3S (Summer, 17 July 2018)
7.4.1 Survey 4W (Winter, 11 Nov 2017)

The results for Survey 4W are shown in Figure 7.21-7.23. Figure 7.21(a) shows that Survey 4W scans through the entire visible GEO belt area in RA-DEC space. Figure 7.21(b) and Figure 7.21(e) show that the survey scans generally from east then to south, and to west parts of the sky but is highly variable. The elevation is always above 15° as shown in Figure 7.21(f).

Figure 7.22 shows that throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, and $\psi$ is greater than 40° which ensures good observation conditions throughout the entire survey. The survey design ensured at least 1 RSO in each image. A maximum of 3 RSOs can be detected in a single image. It can be seen that the randomness of this survey requires significant slewing between observation. As expected, $\mu_\Delta = 36.863^\circ$ and the mean plus $2\sigma$ slew angle, $\mu_\Delta + 2\sigma_\Delta = 89.0172^\circ$ is much higher than previous surveys. The elevation and the $\Delta \theta$ in Figure 7.22(d) shows that the angular movement of the telescope is large. In some cases $\Delta \theta$ may be too large to complete the slew.

Figure 7.23(b) shows that the Survey 4 covers the 800 uncontrolled objects more uniformly over time. The maximum observation frequency of any RSO is 18. That shows that the designed survey is no longer tracking particular groups of RSOs as desired. Figure 7.23(d) shows that a total of 282 unique objects are detected which is much higher than previous surveys and 234 of them are detected at least three times. The 282 unique catalogued RSOs are imaged 1467 times.
(a) Survey in geocentric RA-EDEC frame             (b) Survey in geocentric azimuth and elevation frame

(c) RA time history                                 (d) DEC time history

(e) Azimuth time history                            (f) Elevation time history

Fig. 7.21: Telescope pointing, Survey 4W (Winter, 11 Nov 2017)
Fig. 7.22: Constraints Survey 4W (Winter, 11 Nov 2017)
7.4.2 Survey 4S (Summer, 17 July 2018)

Using the same approach, Survey 4S was run for the evening of 17 July 2018, where observation time was only 5 hrs. Survey 4S shows similar characteristics as Survey 4W, and scans through the entire visible GEO belt. Similar to Survey 4S, Figure 7.24(e) and Figure 7.24(f) shows that the survey scans the sky from east to south in a random manner and the elevation is always above 15°. As a result the mean slew angle, $\mu_{\Delta\theta}$ is much higher than Survey 2S and 3S. The mean slew angle, $\mu_{\Delta\theta} = 37.98^\circ$ and the mean plus 2$\sigma$ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 90.36^\circ$ are on the high side.

Figure 7.25 shows that throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, and $\psi$ is greater than 10° which ensures good observation conditions.
Figure 7.25(d) shows that, in some cases the telescope may not be able to detect the desired RSO because $\Delta \theta$ is higher than 100°.

Figure 7.26(b) shows the RSO detection time history and it is clearly visible that more RSOs are tracked uniformly rather than repeated detection of group of RSOs. The maximum frequency of any satellite is only 8. At least one RSO is detected in each image frame and a maximum 3 RSOs can be found in a single image. Figure 7.26(d) shows that a total of 215 unique RSOs are detected and 125 of them are detected at least 3 times. The 215 unique catalogued RSOs are imaged 668 times.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in geocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 7.24: Telescope pointing, Survey 4S (Summer, 17 July 2018)
Fig. 7.25: Constraints Survey 4S (Summer, 17 July 2018)
7.5 Survey Design 5 – Weighted Object Observation (WOO)

Survey 4 eliminated the Survey 3 problem of repeatedly tracking the same groups of objects, but the random selection of RA-DEC locations resulted in very large $\Delta \theta$ from one observation to the next. Further, Survey 4 did not do well in the area of detecting unique RSO at least 3 times. Specifically, although Survey 4S detected 215 unique objects, only 125 of them, less than $\sim 60\%$ are tracked at least three times.

Survey 5 represents a second attempt to improve Survey 3 by implementing a new constraint such that whenever any cell contains a RSO that has been already tracked more than 4 times, performance index $J_i$ is down-weighted by a factor of 20. This will force the survey to go to another location after the down-weighting occurs rather than repeatedly...
tracking the same groups of satellites (or same RA-DEC). Dividing by 20 ensures the telescope will again track the group or individual RSO that has already been tracked 4 times only if there is no other individual RSOs in the whole grid other than the group of RSOs. The down-weight method ensures that even one previously undetected RSO in a different cell will have greater performance index than the cell having a group of RSOs tracked at least 4 times. The results of this approach are shown in Figure 7.27-7.32.

7.5.1 Survey 5W (Winter, 11 Nov 2017)

The results for Survey 5W are shown in Figure 7.27-7.29. Figure 7.27(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 7.27(e) and Figure 7.27(b) show that the azimuth and the elevation angles of the telescope pointing directions are scattered throughout the survey, and the elevation is always above 15°.

Figure 7.28 shows that throughout the entire observation period φ is greater than 90°, ξ is greater than 20°, and ψ is greater than 40° which ensures good observation conditions throughout the entire survey. The mean slew angle, μΔθ = 26.55° and the mean plus 2σ slew angle, μΔθ + 2σΔθ = 88.24° show that telescope slewing is within the bounds.

Figure 7.29(a) illustrates that most of the RSOs are tracked less than 10 times and only relatively few of them are tracked repeatedly. Figure 7.29(b) also confirms the previous statement. It can be seen from the Figure 7.29(b) and the relatively flat line after 6 hrs of observation in Figure 7.29(d) that the RSOs are tracked rather uniformly up to 6 hrs, but somewhat repeatedly afterwards as in Survey 3. Remember that the performance factor n is down weighted if any RSO was tracked 4 times. After enough observations almost all the unique RSOs are tracked at least 4 times and then the survey only tracks groups of RSOs having higher performance factor n.

In Figure 7.29(c) a maximum 5 RSOs can be in a single image and each image contains at least 1 RSO. Figure 7.29(d) shows that a total of 294 unique RSOs are detected and 293 of them are detected at least 3 times which is much higher than previous surveys. The 294 unique uncontrolled catalogued RSOs are imaged 2908 times.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in geocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 7.27: Telescope pointing, Survey 5W (Winter, 11 Nov 2017)
Fig. 7.28: Constraints Survey 5W (Winter, 11 Nov 2017)

(a) $\xi$ time history  
(b) $\psi$ time history  
(c) $\phi$ time history  
(d) $\Delta \theta$ time history
7.5.2 Survey 5S (Summer, 17 July 2018)

Using this same approach, Survey 5S was designed for the evening of 17 July 2018, where the observation time was only 5 hrs. The results for Survey 5S are shown in Figure 7.30-7.32. Figure 7.30(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 7.30(e) and Figure 7.30(f) shows that the azimuth and the elevation angles of the telescope pointing directions are scattered throughout the survey.

Figure 7.31 shows that throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, and $\psi$ is greater than 10° which ensures good observation conditions throughout the entire survey. Further, the mean slew angle, $\mu_{\Delta \theta} = 31.98^\circ$, and the mean plus 2$\sigma$ slew angle, $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 92^\circ$, show that telescope slewing is on the high side but
within the sensor limit.

Figure 7.32(a) shows that most RSOs are tracked 4 or more times. Figure 7.32(b) shows that RSO detections are uniformly scattered in time. This means that groups of RSOs are not tracked repeatedly. Figure 7.32(c) shows that a maximum 4 RSOs can be detected in a single image and here every image has at least 2 RSOs. Figure 7.32(d) shows that a total of 233 unique RSOs are detected and 219 of them are detected at least 3 times which is much higher than Survey 2S, Survey 3S, and Survey 4S. The 233 unique uncontrolled catalogued RSOs are imaged 1199 times.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in geocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 7.30: Telescope pointing, Survey 5S (Summer, 17 July 2018)
Fig. 7.31: Constraints Survey 5S (Summer, 17 July 2018)
Unfortunately, this survey results in many GEO object begin tracked in 4 consecutive observations frames where the time between observations is only 20 sec. Because of this, the actual state information collected will be low, i.e, better state information can be obtained when the time between observations is larger.

7.6 **Survey Design 6 – Random Observation Time Selection (ROTS)**

Survey 6 is identical to Survey 5 except instead of designing the survey observations by choosing the cell with the highest performance index in chronological order (and getting 4 back-to-back observations of the same object), the survey observations are selected randomly in time until a full night of observations are scheduled. Groups of RSOs that
are observed four times are still down-weighed by a factor of 20 after each observation, but
the observations will be more spaced out over the night in a random manner resulting in
higher state information content. Thus, in Survey 6 a group of RSOs will not be tracked
repeatedly as Survey 3, $\Delta \theta$ will not be as large as in Survey 4, and individual RSO detection
will be more spaced out in time than Survey 5 as down-weighting and random observation
time selection is occurring simultaneously. The results of this approach are shown in Figure
7.33-7.38.

7.6.1 Survey 6W (Winter, 11 Nov 2017)

The result for Survey 6W are shown in Figure 7.33-7.35. Figure 7.33(a) shows that
the survey scans through the visible GEO belt in RA-DEC space. Figure 7.33(e) and
Figure 7.33(f) shows that the azimuth and the elevation angles of the pointing directions
are scattered throughout the survey. Figure 7.33(f) illustrates that the elevation is always
above 15°.

Figure 7.34 presents the performance of survey 6 on 4 key factors. It shows that
throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, and
$\psi$ is greater than 40°. The mean slew angle, $\mu_{\Delta \theta}$ was 35.84° and the system will be able to
slew to most of the objects in the required 20 sec as the mean plus 2$\sigma$ slew angle, $\mu_{\Delta \theta} + 2\sigma_{\Delta \theta}$
is 93.97° ($< 100$°).

Figure 7.35(b) shows how the GEO RSOs detections are distributed as a function
of observation time. Compared to Figure 7.29(b) the detections plot look more evenly
distributed. Figure 7.35(c) shows that the maximum number of satellite in the image frame
is 4. Some RSOs are tracked more than 50 times but most of the RSOs are detected between
4 to 10 times. Figure 7.35(d) shows that the he total number of unique RSOs detected is
295. The number of RSOs detected more than 3 times are 294. The 295 unique uncontrolled
catalogued RSOs are imaged 3219 times.
Fig. 7.33: Telescope pointing, Survey 6W (Winter, 11 Nov 2017)
Fig. 7.34: Constraints Survey 6W (Winter, 11 Nov 2017)

(a) $\xi$ time history
(b) $\psi$ time history
(c) $\phi$ time history
(d) $\Delta\theta$ time history
7.6.2 Survey 6S (Summer, 17 July 2018)

Using the same approach, Survey 6S was designed for the evening of 17 July 2018, where observation time was only 5 hrs. The result for Survey 6S are shown in Figure 7.36-7.38. Figure 7.36(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 7.36(e) and Figure 7.36(f) show that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey. Figure 7.36(f) illustrates that the elevation is always above 15°.

Figure 7.37 shows that throughout the entire observation period $\phi$ is greater than 90°, $\xi$ is greater than 20°, $\psi$ is greater than 10°, and elevation above 15° which ensures good observation condition throughout the entire survey. The mean slew angle, $\mu_{\Delta\theta} = 38.52^\circ$
and the mean plus 2σ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 91.8^\circ$ indicates that telescope slewing is in the higher side but within the sensor limit.

Figure 7.38(a) shows that most RSOs are tracked more than 3 times. Figure 7.38(b) shows that RSO detections are scattered uniformly in time which means no particular groups of RSOs are not being tracked repeatedly. Figure 7.38(c) shows that a maximum 4 RSOs can be detected in a single image, and here every image will have at least 1 RSO. Figure 7.38(d) shows that a total of 239 unique RSOs are detected and 230 of them are detected at least 3 times which is much higher than Survey 2S, Survey 3S, and slightly higher than 5S. The 239 unique catalogued RSOs are imaged 1235 times.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in geocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 7.36:  Telescope pointing, Survey 6S (Summer, 17 July 2018)
Fig. 7.37: Constraints Survey 6S (Summer, 17 July 2018)
(a) Frequency of the GEO satellite observations  
(b) GEO satellites detection time history

(c) Number of satellites in each image  
(d) Accumulation of RSO time history

Fig. 7.38: RSO data, Survey 6S (Summer, 17 July 2018)

7.7 Survey Design 7– Guaranteed Initial Orbit Determination (GIOD)

Initial orbit determination (IOD) of an RSO requires at least 3 sets of angle measurements are required. For a GEO RSO, the time difference between each measurement should be reasonably large due to the geometry of the orbit. Survey 7 ensures 3 observations (3 sets of angle measurements) with a specified time interval between measurements for each RSO. Initially an $S \times K$ matrix $Q$ is created, where $S$ is the number of satellites and $K$ is the total number of observation times. If an RSO satisfies all the observation constraints (eq. 7.1 - eq. 7.4) at any observation time, a value of 1 is stored at that element of the matrix, if not 0 is stored.
For Survey 7, not all RSOs are selected for observation. Only those that can be detected at least 3 times separated by a 20 min interval are selected for the survey. That means an RSO has to satisfy all the observation constraints for at least 60 mins in the entire night to be eligible for the survey.

Next, satellites are prioritized based on their availability for observation in the entire observation time. RSOs with lower availability get higher priority. Some RSOs have a 1 in every column of the matrix $Q$, and that indicates the RSO is visible with favorable observation condition for the entire night and the priority of those RSOs will be less compared to the other RSOs having fewer ones in the matrix $Q$. High priority satellites will be assigned time slots first to collect 3 images 20 min apart, then the time slots will be allocated to lower priority satellites. The results of this approach are shown in Figure 7.39-7.44.

### 7.7.1 Survey 7W (Winter, 11 Nov 2017)

The result for Survey 7W are shown in Figure 7.39-7.41. Figure 7.39(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 7.39(e) and Figure 7.39(f) shows that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey. Figure 7.39(f) illustrates that the elevation is always above 15°.

Figure 7.40 clearly indicates that the survey has favorable viewing condition throughout the entire night. The mean slew angle $\mu_{\Delta\theta} = 27.21^\circ$ and the mean plus 2$\sigma$ slew angle, $\mu_{\Delta\theta} + 2\sigma_{\Delta\theta} = 85.81^\circ$, is below the maximum slewing capacity of the telescope. The $\Delta\theta$ plot indicates that in some cases the slew requirement of the telescope will be very large (> 100°). In those cases, the telescope may not be able to point to the desired direction at that specific time.

Figure 7.41(c) shows that a maximum 4 RSOs can be detected in a single image. Figure 7.41(d) shows that the number of unique RSOs detected is 284, and among them 284 of the objects is detected at least 3 times. The 284 unique catalogued RSOs are imaged 1053 times.
(a) Survey in geocentric RA-EDEC frame
(b) Survey in geocentric azimuth and elevation frame

(c) RA time history
(d) DEC time history

(e) Azimuth time history
(f) Elevation time history

Fig. 7.39: Telescope pointing, Survey 7W (Winter, 11 Nov 2017)
Fig. 7.40: Constraints Survey 7W (Winter, 11 Nov 2017)

(a) $\xi$ time history

(b) $\psi$ time history

(c) $\phi$ time history

(d) $\Delta \theta$ time history
7.7.2 Survey 7S (Summer, 17 July 2018)

Using the same approach, Survey 7S was designed for the evening of 17 July 2018, where observation time was only 5 hrs. The results for Survey 7S are shown in Figure 7.42-7.44. Figure 7.42(a) shows that the survey scans through the visible GEO belt in RA-DEC space. Figure 7.42(e) and Figure 7.42(f) shows that the azimuth and the elevation angles of the telescope pointing direction are scattered throughout the survey. Figure 7.42(f) illustrates that the elevation is always above 15°. Figure 7.43 clearly indicates that the survey has favorable viewing condition throughout the entire night. \( \mu_{\Delta \theta} = 35.18^\circ \) and \( \mu_{\Delta \theta} + 2\sigma_{\Delta \theta} = 90.54^\circ \), within the maximum slewing capacity of the telescope.

Figure 7.44(d) shows that Survey 7S detects 201 unique objects and 195 of them are
detected at least 3 times. A maximum of 3 satellites are detected in a single image frame as shown in Figure 7.44(c) and the highest number of detection for single satellite is 9 as shown in Figure 7.44(a). The 201 unique uncontrolled catalogued RSOs are imaged 713 times.
(a) Survey in geocentric RA-EDEC frame  
(b) Survey in geocentric azimuth and elevation frame

(c) RA time history  
(d) DEC time history

(e) Azimuth time history  
(f) Elevation time history

Fig. 7.42: Telescope pointing, Survey 7S (Summer, 17 July 2018)
Fig. 7.43: Constraints Survey 7S (Summer, 17 July 2018)
150

(a) Frequency of the GEO satellite observations

(b) GEO satellites detection time history

(c) Number of satellites in each image

(d) Accumulation of RSO time history

Fig. 7.44: RSO data, Survey 7S (Summer, 17 July 2018)

7.8 Summary

A total of 7 survey designs for GEO RSO detection with their key performance metrics are discussed in detail in this chapter. Survey 1 does not utilize the entire observation time due to the unavailability of the Pinch Point nor does it account for unfavorable observation conditions. Survey 2 attempts to remedy the problems of Survey 1, as it satisfies the observation constraints, attempts to maximize the observation time, and only evaluates RSO concentration over the observation period rather than 24 hrs. An increase in the number of unique object detection reflects the improvement of Survey 2.

In Survey 2, the Pinch Points are the high RSO concentrated points, in RA-DEC frame, when GEO RSOs are propagated for the entire observation time. Survey 3 is designed to
investigate the Pinch Point concept at each observation time instead of entire observation time. Unfortunately, due to the inherent nature of GEO RSOs, Survey 3 repeatedly tracks particular groups of RSOs. For Survey 3 the unique number of RSO detection is less than Survey 2.

Survey 4 provides a solution for the issue of repeated detection in Survey 3, and results in an increase in number of unique object detected. However random selection of the cell increases the $\Delta \theta$ and makes this survey operationally challenging.

Survey 5 (Weighted Object Observations) provides an alternative approach to solving the issues of Survey 3 by appropriately down-weighting cells that are imaged more than 4 times. This survey produces good results with lower $\Delta \theta$. One shortcoming of the Survey 5 however, is that the RSO observations are closely spaced and may not provide higher state information content.

In Survey 6, observations of a RSO are spaced out over the night in a random manner to improve the state information content. The result is that the unique number of object detection is greater than Survey 3-5 and $\Delta \theta$ is reasonably low.

Survey 7 is different from the rest of the surveys stated above. Three observations are spaced out 20 min for each RSO which guarantees it’s initial orbit determination. The number of unique objects detected is similar to Surveys 4-6 in. A summary of all results is presented in Tables 7.1 and 7.2 and Figure 7.45.
Fig. 7.45: Summary
<table>
<thead>
<tr>
<th>Survey ID</th>
<th># of RSO Observations</th>
<th># of Unique RSOs Observed</th>
<th># of Unique RSOs Observed &gt; 2 times</th>
<th>Mean $\Delta\theta$ $\mu_{\Delta\theta}$(Deg)</th>
<th>$\mu_{\Delta\theta} + 2 \sigma_{\Delta\theta}$(Deg)</th>
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<tr>
<td>Survey 1W (PPT)</td>
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<td>64</td>
<td>64</td>
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<td>0.08</td>
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<td>Survey 2W (MPPT)</td>
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<td>80</td>
<td>0.19</td>
<td>3.07</td>
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<td>Survey 3W (MOO)</td>
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<td>206</td>
<td>158</td>
<td>22.91</td>
<td>77.42</td>
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<td>Survey 4W (RRDS)</td>
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<td>282</td>
<td>234</td>
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<td>89.01</td>
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<td>293</td>
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<td>Survey 7W (GIOD)</td>
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<td>284</td>
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<td>85.81</td>
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Table 7.1: Survey summary (Winter, 11 Nov 2017)

<table>
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<tr>
<th>Survey ID</th>
<th># of RSO Observations</th>
<th># of Unique RSOs Observed</th>
<th># of Unique RSOs Observed &gt; 2 times</th>
<th>Mean $\Delta\theta$ $\mu_{\Delta\theta}$(Deg)</th>
<th>$\mu_{\Delta\theta} + 2 \sigma_{\Delta\theta}$(Deg)</th>
</tr>
</thead>
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<tr>
<td>Survey 1S (PPT)</td>
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</tr>
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<td>34</td>
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<td>Survey 3S (MOO)</td>
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<td>95</td>
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<td>233</td>
<td>219</td>
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<tr>
<td>Survey 6S (ROTS)</td>
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<td>239</td>
<td>230</td>
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<td>713</td>
<td>201</td>
<td>195</td>
<td>35.18</td>
<td>90.54</td>
</tr>
</tbody>
</table>

Table 7.2: Survey summary (Summer, 17 July 2018)
It is vital to design efficient surveys to detect and track GEO RSOs, and it is equally essential to analyze the performance of designed surveys. Performance metrics can be used to evaluate surveys before they are implemented to ensure that they will be useful and meet user requirements. The key purpose of a survey is to improve the state information of catalogued objects or determine new state information of uncatalogued objects.

In this chapter, a key performance metric will be developed that will score the state information of the designed surveys. Furthermore, a detailed investigation of how key survey parameters are related to the state information will be conducted. Finally, a closed-form representation of the information performance metrics based on the survey parameters will be developed.

### 8.1 Performance Metric Selection

The a priori state information of an RSO detected in a FOV is often incomplete. In this scenario of infinite initial covariance, the Fisher Information matrix is capable of providing the state information even without full state information. Angle and angle-rate information can be extracted from images of ground-based observatories without any estimation, however it is impossible to get the range and range-rate information from only one or two images. At least three images of specific RSO are required to compute the range and range-rate. Thus, successful RSO surveys can be evaluated based on the range and range rate information extraction since angle and angle-rate information is primarily based on the accuracy of the sensor. Furthermore, range and range-rate information conveys information about both the position and velocity of the RSO and will help the survey designer accordingly.
8.1.1 Problem Dynamics

The state vector of an RSO can be represented by right ascension ($\alpha$), right ascension rate ($\dot{\alpha}$), declination ($\delta$), declination rate ($\dot{\delta}$), range ($\rho$), and range rate ($\dot{\rho}$) in spherical coordinates, $X = [\alpha \ \dot{\alpha} \ \delta \ \dot{\delta} \ \rho \ \dot{\rho}]^T$. The dynamics of the state vector can be expressed as

\begin{align}
\ddot{\alpha} &= 2\dot{\alpha}\dot{\delta}\tan{\delta} - 2\dot{\alpha}\dot{\rho}\rho^{-1} \\
\ddot{\delta} &= \sin{\delta}\cos{\delta}\dot{\alpha}^2 + 2\dot{\delta}\dot{\rho}\rho^{-1} \\
\ddot{\rho} &= \dot{\delta}^2\rho + \frac{1}{2}\dot{\alpha}^2\rho - \mu\rho^{-2} + \frac{1}{2}\dot{\alpha}^2\cos{2\delta}\rho
\end{align}

The dynamics can be expressed by the following nonlinear vector differential equation.

$$\dot{X} = f(X_i)$$

where $X_i$ is the state at time $t_i$. 

Fig. 8.1: Satellite position
8.1.2 Measurement Model

Measurements are the topocentric RA and DEC angle measurements $\alpha_T$ and $\delta_T$ from electro-optical sensor. The nonlinear measurement with observation error $v_i$ can be expressed as in Eq. 8.5, where, $v$ is measurement noise with covariance $R$.

$$y = h(t_i, X) + v_i$$  \hspace{1cm} (8.5)

$$v \sim \mathcal{N}(0, R)$$  \hspace{1cm} (8.6)

The linearized measurement model can be expressed as

$$\delta y_i = H(t_i)\delta x_i + v_i$$  \hspace{1cm} (8.7)

where $H(t_i)$ is defined by

$$H(t_i) = \frac{\partial h(t_i, X)}{\partial X} = \left[ \frac{\partial \alpha_T}{\partial X} \quad \frac{\partial \delta_T}{\partial X} \right]^T$$  \hspace{1cm} (8.8)

and,

$$\frac{\partial \alpha_t}{\partial X} = \frac{\partial \alpha_T}{\partial i_{los}} \frac{\partial i_{los}}{\partial r_s} \frac{\partial r_s}{\partial X}$$  \hspace{1cm} (8.9)

$$\frac{\partial \delta_t}{\partial X} = \frac{\partial \delta_T}{\partial i_{los}} \frac{\partial i_{los}}{\partial r_s} \frac{\partial r_s}{\partial X}$$  \hspace{1cm} (8.10)

A detailed derivation of Eqs. 8.9 and 8.10 are shown in Appendix A.1
8.1.3 Fisher Information Matrix

The Fisher Information matrix in recursive format is given by

$$\Lambda(t_i, t_1) = \Phi^T(t_{i-1}, t_i)\Lambda(t_{i-1}, t_i)\Phi(t_{i-1}, t_i) + H^T(t_i)R^{-1}(t_i)H(t_i)$$  \hspace{1cm} (8.11)

It can be seen that the information in a single measurement at time $t_1$ is $H^T(t_i)R^{-1}(t_i)H(t_i)$, where $\Phi$ is the state transition matrix and $H$ is the linearized measurement model matrix.

The state transition matrix $\phi(t_i, t_{i-1})$ is computed by integrating the differential equation

$$\dot{\phi}(t_i, t_{i-1}) = A(t_i)\phi(t_i, t_{i-1})$$  \hspace{1cm} (8.12)

where,

$$A(t_i) = \frac{\partial f(X_i)}{\partial X} \bigg|_{X_n(t_i)}$$  \hspace{1cm} (8.13)

and where $X_n$ is the reference orbit. A detailed derivation of the elements of $A(t_i)$ is provided in Appendix A.2.

Since the information associated with the RA, DEC, and their associated rates can be obtained directly from the measurements, what remains to be determined is the range information $\sqrt{\Lambda(5, 5)}$ and the range-rate information $\sqrt{\Lambda(6, 6)}$. These two terms will be referred to as the range and range-rate information performance metrics $I_\rho$ and $I_{\dot{\rho}}$ respectively. The units are $\frac{1}{\text{km}}$ and $\frac{\text{sec}}{\text{km}}$, respectively.

8.2 Sensitivity of Performance Metrics to GEO Orbital Elements

From the previous section, it is clear that the Fisher information matrix is a function of the observatory location, the number of measurements, the measurement time interval, and the orbit of the RSO. Thus, if the location of the observatory, the number of measurements, and measurement time interval are held fixed, the Fisher information matrix and the
associated range and range-rate information performance metrics will be a function only of the RSO orbit.

Information performance metrics will be dependent on the true anomaly $\nu$ as there will be no information if the RSO is below the horizon. In this research a new parameter $\epsilon$ will be used instead of true anomaly to analyze the characteristics of performance information metrics. The relative angular separation of an object with respect to the observatory is defined by $\epsilon$

$$\epsilon = \theta - \nu$$

where, $\theta$ is the local sidereal time and $\nu$ is the true anomaly of the RSO. Irrespective to time, $\epsilon$ will provide identical performance metrics scores if other problem parameters and orbital elements remain same.

![Fig. 8.2: RSO geometry with $\theta$ and $\epsilon$](image)

Because of the geometry of the GEO satellites relative to the observatory, $\epsilon$ has a limited range. The range varies based on the latitude of the observatory and is computed below.
Figure 8.2 illustrates a scenario where the observatory is aligned with vernal equinox so local sidereal time is zero. The observatory position vector \( \vec{r}_e \) can be expressed as a function of observatory latitude, \( \vec{r}_e = R_e [\cos \lambda \ 0 \ \sin \lambda]^T \) and the Geostationary RSO position vector can be expressed as a function of \( \epsilon \), \( \vec{r}_s = R_g [\cos \epsilon \ \sin \epsilon \ 0]^T \), where \( R_e \) is the distance of the observatory from the center of the earth and \( R_g \) is the range of the satellite.

From Figure 8.2 it can be shown that

\[
|\vec{r}_g - \vec{r}_e|^2 = R_g^2 + R_e^2 - 2R_gR_e \cos \epsilon \cos \lambda \tag{8.15}
\]

and

\[
\vec{r}_e \cdot (\vec{r}_g - \vec{r}_e) = R_e|\vec{r}_g - \vec{r}_e| \cos \left(\frac{\pi}{2} - \alpha\right) = R_e|\vec{r}_g - \vec{r}_e| \sin \alpha \tag{8.16}
\]

where \( \alpha \) is the elevation of the RSO Using Eqs. 8.15 and 8.16 the following can be derived

\[
\vec{r}_e \cdot \vec{r}_g - \vec{r}_e \cdot \vec{r}_e = R_e|\vec{r}_g - \vec{r}_e| \sin \alpha \tag{8.17}
\]

\[
R_eR_g \cos \epsilon \cos \lambda - R_e^2 = R_e|\vec{r}_g - \vec{r}_e| \sin \alpha \tag{8.18}
\]

\[
(\epsilon \times \vec{r}_e)^2 = (R_e |\vec{r}_g - \vec{r}_e| \sin \alpha)^2 \tag{8.19}
\]

\[
R_e^2R_g^2 \cos^2 \epsilon \cos^2 \lambda - 2R_e^3R_g \cos \epsilon \cos \lambda + R_e^4 = R_e^2(Rg^2 + R_e^2 - 2R_gR_e \cos \epsilon \cos \lambda) \sin^2 \alpha \tag{8.20}
\]

\[
(\epsilon \times \vec{r}_e)^2 = (R_e^2R_g^2 \cos^2 \lambda) \cos^2 \epsilon + (2R_e^3R_g \cos \lambda \sin^2 \alpha) - 2R_e^3R_g \cos \lambda \cos \epsilon \cos \lambda \sin^2 \alpha \cos \alpha + (R_e^4 - R_e^2R_g^2 \sin^2 \alpha - R_e^4 \sin^2 \alpha) = 0 \tag{8.21}
\]

Eq. 8.21 is a quadratic in \( \cos \epsilon \) and the equation can be solved for \( \cos \epsilon \) which will provide \( \epsilon \). The value of \( \epsilon \) is a function of observatory latitude. If \( \alpha \) becomes the minimum elevation angle, Eq. 8.21 will provide maximum and minimum value of \( \epsilon \). Figure 8.3 shows
that the $\epsilon$ is largest for the observatories on the equator and symmetrically decreases in both hemispheres.

Fig. 8.3: $\epsilon$ vs latitude for $\alpha = 15^\circ$

The range of epsilon depends on the position of the observatory. For $\alpha = 15^\circ$, the range is $-66.7^\circ$ to $+66.7^\circ$ for an observatory on the equator and the range of $\epsilon$ decreases as the latitude changes in both hemispheres. The range of $\epsilon$ is approximately $-57^\circ$ to $+57^\circ$ for Logan, Utah.

A RSO reference trajectory is required to compute the information matrix. The purpose of this section is to show that range and range-rate information performance metrics are relatively insensitive to RSO orbital parameters except $\epsilon$, when the RSO is in a near-geostationary orbit, for a given observatory location, measurement number, and measurement interval.

To show this, the information matrix and the associated range and range-rate information performance metrics are computed for a number of near-geostationary orbits. The metrics will be systematically compared to a geostationary orbit. It is assumed that observations can only be made when the local elevation of the RSO is greater than 15 degrees above the horizon.

As the focus of the research is in near geostationary region, information performance
metrics sensitivity is observed on three prominent orbital elements in this region: semi-major axis, inclination and, eccentricity. The main idea is to see how much information change occurs if the RSO trajectory is varied from the reference trajectory. The reference trajectory is set as a geostationary orbit with orbital elements semi-major axis \( a = 42164 \) km, eccentricity \( e = 0 \) and inclination \( i = 0^\circ \).

For the simulation, the observatory location is selected as the location of USU-STAR. The true anomaly is set to ensure the RSO is visible from the observatory. The time between measurements is selected as 30 min, and the number of measurements is selected as 3. The range of orbital elements selected for the sensitivity test is shown in Table 8.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axis, ( a )</td>
<td>( 42164 \pm 400 ) Km</td>
</tr>
<tr>
<td>Inclination, ( i )</td>
<td>1° to 15°</td>
</tr>
<tr>
<td>Eccentricity, ( e )</td>
<td>0 to 0.01</td>
</tr>
</tbody>
</table>

Table 8.1: Parameters considered for survey design sensitivity analysis
Fig. 8.4: Sensitivity analysis of information metrics
It can be seen from Figure 8.4, that there is a maximum 14% change in the range information and a maximum 5% change in the range rate information in semi-major axis and inclination space for near Geostationary orbits. The maximum range information change in semi-major axis and eccentricity space is only 2.5%. The maximum change in range information in eccentricity and inclination space is 12% and the range rate information change is only 2.5%. Similar analysis was conducted with orbital elements Ω and ω, but they have lesser effects on the information performance metrics values.

Thus it is feasible to use a reference trajectory a = 42164 km, e = 0, i = 0°, ω = 0°, and Ω = 0° to evaluate information performance metrics of near Geostationary objects instead of using the actual trajectory. These five orbital elements with their associated value will be referred as the nominal trajectory.

8.3 Characteristics of Information Performance Metrics

For a specific observatory, the information performance metrics $I_\rho$ and $I_\dot{\rho}$ depend on three parameters including the number of measurements (N), time between measurements ($\Delta T$), and the difference between the local sidereal time and true anomaly ($\epsilon$).

Three scenarios will be used to show the characteristics of $I_\rho$ and $I_\dot{\rho}$. First, the effect of $\epsilon$ on information metrics will be investigated. Then, $\epsilon$ and $\Delta T$ will be varied to see their effect on information metrics. Finally, the effect of N will be analyzed with realistic range of $\epsilon$ and $\Delta T$.

8.3.1 Effect of $\epsilon$ on Information Metric

Figure 8.5 shows the effect of $\epsilon$ and elevation on $I_\rho$ and $I_\dot{\rho}$ from Logan, Utah. Figure 8.5(a) shows that $I_\rho$ is minimum at $\epsilon = 0^\circ$ and grows in both positive and negative directions in the range of $\epsilon$. On the other hand, Figure 8.6(a) shows that $I_\dot{\rho}$ is minimum at +57° and maximum at -57°. Figure 8.5(c) and Figure 8.6(c) show the effect of RSO elevation on $I_\rho$ and $I_\dot{\rho}$, and an interesting fact is that the highest elevation does not have maximum information performance metrics score.
Fig. 8.5: Range information characteristics (N=3 and ΔT = 30 min)
8.3.2 Effect of Measurement Interval on Information Metric

Figure 8.7 illustrates the effect of $\Delta T$ on $I_\rho$ and $I_{\dot{\rho}}$. Information performance metrics, $I_\rho$ and $I_{\dot{\rho}}$ both increase as $\Delta T$ increases. Lower negative values of $\epsilon$ with higher $\Delta T$ produces the best results.
8.3.3 Effect of Number of Measurements on Information Performance Metrics

As expected, $I_\rho$ and $I_\dot{\rho}$ also increase as the number of measurement, $N$, increases. Every surface in Figure 8.8 corresponds to different value of $N$. Three to six measurements of $\Delta T$ ranging from 30 min to 4 hours for all possible $\epsilon$ were simulated. Surfaces of information metrics $I_\rho$ and $I_\dot{\rho}$ were computed for each $N$. For $I_\rho$, Figure 8.8(a) shows that each surface is distinct and the larger $N$ is, the higher the surface. The surfaces have curvature and height variation due to the effect of $\epsilon$ and $\Delta T$.
8.4 Closed-form Representation of the Information Performance Metric for Specific N

$\epsilon$ and $\Delta T$ both have a range of possible values for a specific ground based observatory and a specific N. So it is feasible to create a closed-form representation of surfaces of $I_\rho$ and $I_\dot{\rho}$ based on the range of values of $\epsilon$ and $\Delta T$ for a specific N. A closed-form representation of $I_\rho$ and $I_\dot{\rho}$ surface will help a user to compare survey performance and can be a deciding factor for survey design. Users will have a fast practical way to evaluate a survey before implementation.

The general equation for quadratic surface in $\epsilon$-$\Delta T$ space can be defined as:

$$I_n = A\epsilon_n^2 + B\Delta T_n^2 + C\epsilon_n\Delta T_n + D\epsilon_n + E\Delta T_n + F$$  \hspace{1cm} (8.22)

where $I_n$ represent both range and range-rate information performance metrics and n is the point in the surface. A, B, C, D, E, F are the coefficients.

Knowing the coordinate of n (where n ≤ 6) points in the surface (information performance metrics of n possible pair of $\epsilon$ and $\Delta T$) it is possible to find the coefficients and compute the close form representation of the performance metrics for specific N for GEO objects.
\[
\begin{bmatrix}
\epsilon_1^2 & \Delta T_1^2 & \epsilon_1 \Delta T_1 & \epsilon_1 & \Delta T_1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\epsilon_n^2 & \Delta T_n^2 & \epsilon_n \Delta T_n & \epsilon_n & \Delta T_n & 1 \\
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C \\
D \\
E \\
F \\
\end{bmatrix}
= 
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
\end{bmatrix}
\]

Treating the equation as:

\[Mx = b\] (8.24)

Solving for coefficient \(x\)

\[M^T M \hat{x} = M^T b\] (8.25)

\[\hat{x} = (M^T M)^{-1} M^T b\] (8.26)

Figure 8.9 shows the closed-form surface of information performance metrics for \(N=3\). Green stars represented the \(I_\rho\) and \(I_{\dot{\rho}}\) score for a limited number of \(\epsilon\) and \(\Delta T\) and the surface is generated from the coefficient computed the least square method. The surfaces represent the information metrics score from all the possible value of \(\epsilon\) and \(\Delta T\) for \(N = 3\).
8.5 Survey Example

The concept of information performance metric analysis was implemented for a GEO survey (Survey 7). Every satellite was imaged 3 times and the time between measurements was 20 min. \( I_\rho \) and \( \dot{I}_\rho \) is computed from actual trajectory of the RSO, using nominal trajectory, and from closed form solution.

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Original</th>
<th>Nominal</th>
<th>Closed form solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The average score for ( I_\rho )</td>
<td>1.0998 1/km</td>
<td>1.1989 1/km</td>
<td>1.940 1/km</td>
</tr>
<tr>
<td>The average score for ( \dot{I}_\rho )</td>
<td>2964.5 sec/km</td>
<td>2774.8 sec/km</td>
<td>2810.1 sec/km</td>
</tr>
</tbody>
</table>

Table 8.2: Information performance metrics score
Figure 8.11 shows the score of $I_\rho$ and $I_{\dot{\rho}}$ on the y-axis and satellite id on x-axis.

For the survey, average values for $I_\rho$ and $I_{\dot{\rho}}$ are very similar. Again it shows that, it is possible to use nominal trajectory and closed form solution to get the information performance metrics score.
CHAPTER 9
Summary and Future Work

9.1 Summary

A total of 7 survey designs for all catalogued GEO RSO and only uncontrolled GEO RSO with their key performances are discussed elaborately in this research. For all catalogued GEO RSO survey design, Survey 1 does not use the entire observation time due to the unavailability of the Pinch Point as well as favorable observation environment. Survey 2 is the immediate improvement of Survey 1, as it satisfies the observation constraints and also maximize the observation time. Increase in the number of unique object detection reflects the improvement of Survey 2.

Pinch Point is the high concentrated points in RA-DEC frame when GEO RSOs are propagated for entire observation time period or a sidereal day. Survey 3 is designed to investigate the Pinch Point concept at each observation time step instead of entire observation time. Unfortunately, due to the inherent nature of GEO RSOs, particular groups of RSOs are tracked repeatedly in this Survey. The unique number of RSO detection is less than Survey 2.

Survey 4 provides one solution for the issue of repeated detection in Survey 3. Increase in the number of unique object detection than Survey 1, 2 and 3 reflects the improvement of Survey 4. Random selection of the cell increases the $\Delta \theta$ and also has large difference in number of RSO detected $> 2$ times and the number of Unique RSO detection, specially for shorter observation time (Summer).

Survey 5 provides alternative approach to solve the issues of Survey 3. The difference in number of RSO detected $> 2$ times and the number of Unique RSO detection is not as large as Survey 4. One shortcoming of the Survey 5 is that, the RSO observations are closely spaced that does not provide higher state information content.
Observations of a RSO are more spaced out over the night in a random manner in Survey 6 which results in higher state information content. The unique number of object detection is greater than Survey 3, Survey 4 and Survey 5. Also, the difference in number of RSO detected $> 2$ times and the number of Unique RSO detection is not as large as Survey 4.

Survey 7 is different than rest of the surveys. Three observations are spaced out 20 min for each RSO which guarantees it’s initial orbit determination. The number of unique object detection is similar to Survey 4, 5 and 6 in summer, but less in winter.

For only uncontrolled GEO object survey design, Surveys 1 and 2 will detect fewer unique objects, but the telescope slewing requirement is very small and implementation will not require a sophisticated telescope system. However, Survey 6 detects the most unique objects in both winter and summer, and is the best survey if the requirement is to maximize the number of unique RSO detected. On the other hand, Survey 7 detects fewer RSOs than Survey 6 but has 20 min time intervals between all observations. Both Survey 6 and 7 cover much of GEO belt which increase the probability of detecting uncatalogued objects.

Survey designs were also simulated for AMOS observatory and performance is identical to USU-STAR.

A key performance metric to evaluate the state information was developed. Development of a closed form representation of the information performance metrics based on the survey parameters was another unique contribution of the research.

9.2 Future Work

Surveys are designed for GEO objects and highly eccentric objects (GTO) were not included. An extension of the current survey designs would include these highly eccentric objects. Recent observation shows an increase of debris in the GTO regime, making this a region of active interest. In the future, an additional constraint slew rate can be added to the survey design performance index. Slew rate was computed for each survey design but it was not explicitly enforced as a constraint. Slew rate could be related to exposure time, which is
currently an adjustable parameter in the survey design. The closed-form representation of the information performance metrics was estimated through a least-square fit of a surface modeled by a quadratic polynomial. The accuracy of this fit may be increased with a higher-order polynomial. Finally, future work will include validating the survey results using USU-STAR.

9.2.1 USU-STAR validation

For validation, survey simulation showed in chapter 6 and 7 should agree with the telescope data. The number of RSO detected at each time step can be checked for validation.

Now, Simulation will generate the following data at each time step for the entire observation period: exposure time, the time between measurement, detected satellite ID, and the pointing direction of the satellite as corresponding topocentric RA-DEC. The pointing directions will be feed into a Java script that will automate the telescope slew for the entire observation period and take image at the commanded time step. Then, the Javascript will eventually send to the observatory computer for survey execution.

The telescope will be pre-calibrated before running the Javascript. Then, the Java script will be run in observatory computer that will command the telescope and the camera to execute the survey. After the end of the survey, images will be compared with the simulated data. Theoretically, the number of RSO in each image at each time step should match with the simulated survey design but, because of the cloud, the reflective material and size of the RSO, and larger slew angle some may not be detected in the image. Though, it is expected to get more RSO in the image than simulated data because the search region contains many uncatalogued objects and that is one of the goals of this research.
REFERENCES


APPENDIX A

Partial Derivatives

A.1 Measurements Partials, $H$

\[
\frac{\partial \alpha_T}{\partial i_{los}} = \frac{\partial \tan^{-1}(\frac{i_{Ly}}{i_L})}{\partial i_L} = \frac{1}{1 + (\frac{i_{Ly}}{i_L})^2} \begin{bmatrix} -1 & 0 & 1 \\ i_{Ly} & i^2_{Ly}i_L & 0 \end{bmatrix} \tag{A.1}
\]

\[
\frac{\partial \delta_T}{\partial i_L} = \frac{\partial \sin^{-1}(i_{Lz})}{\partial i_L} = \frac{1}{\sqrt{1 - i^2_{Lz}}} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \tag{A.2}
\]

\[
\frac{\partial i_L}{\partial \vec{r}_a} = \frac{1}{|\vec{r}_a - \vec{r}_0|} \begin{bmatrix} I - i_Li_L^T \end{bmatrix} \tag{A.3}
\]

\[
\frac{\partial r_a}{\partial X} = \begin{bmatrix} -\rho \cos \delta \sin \alpha & 0 & -\rho \sin \delta \cos \alpha & 0 & \cos \delta \cos \alpha & 0 \\ \rho \cos \delta \cos \alpha & 0 & -\rho \sin \delta \sin \alpha & 0 & \cos \delta \sin \alpha & 0 \\ 0 & 0 & -\rho \cos \delta & 0 & \sin \delta & 0 \end{bmatrix} \tag{A.4}
\]

A.2 Dynamics Partials, $A$

\[
A = \frac{\partial f(X)}{\partial X} \tag{A.5}
\]

\[
A = \begin{bmatrix}
\frac{\partial \dot{\alpha}}{\partial \alpha} & \frac{\partial \dot{\alpha}}{\partial \alpha} & \frac{\partial \dot{\alpha}}{\partial \delta} & \frac{\partial \dot{\alpha}}{\partial \delta} & \frac{\partial \dot{\alpha}}{\partial \rho} & \frac{\partial \dot{\alpha}}{\partial \rho} \\
\frac{\partial \ddot{\alpha}}{\partial \alpha} & \frac{\partial \ddot{\alpha}}{\partial \alpha} & \frac{\partial \ddot{\alpha}}{\partial \delta} & \frac{\partial \ddot{\alpha}}{\partial \delta} & \frac{\partial \ddot{\alpha}}{\partial \rho} & \frac{\partial \ddot{\alpha}}{\partial \rho} \\
\frac{\partial \dot{\delta}}{\partial \alpha} & \frac{\partial \dot{\delta}}{\partial \alpha} & \frac{\partial \dot{\delta}}{\partial \delta} & \frac{\partial \dot{\delta}}{\partial \delta} & \frac{\partial \dot{\delta}}{\partial \rho} & \frac{\partial \dot{\delta}}{\partial \rho} \\
\frac{\partial \ddot{\delta}}{\partial \alpha} & \frac{\partial \ddot{\delta}}{\partial \alpha} & \frac{\partial \ddot{\delta}}{\partial \delta} & \frac{\partial \ddot{\delta}}{\partial \delta} & \frac{\partial \ddot{\delta}}{\partial \rho} & \frac{\partial \ddot{\delta}}{\partial \rho} \\
\frac{\partial \dot{\rho}}{\partial \alpha} & \frac{\partial \dot{\rho}}{\partial \alpha} & \frac{\partial \dot{\rho}}{\partial \delta} & \frac{\partial \dot{\rho}}{\partial \delta} & \frac{\partial \dot{\rho}}{\partial \rho} & \frac{\partial \dot{\rho}}{\partial \rho} \\
\frac{\partial \ddot{\rho}}{\partial \alpha} & \frac{\partial \ddot{\rho}}{\partial \alpha} & \frac{\partial \ddot{\rho}}{\partial \delta} & \frac{\partial \ddot{\rho}}{\partial \delta} & \frac{\partial \ddot{\rho}}{\partial \rho} & \frac{\partial \ddot{\rho}}{\partial \rho} \\
\end{bmatrix} \tag{A.6}
\]
\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\
0 & 0 & 0 & 1 & 0 & 0 \\
A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\
0 & 0 & 0 & 0 & 0 & 1 \\
A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66}
\end{bmatrix}
\]

now

\( A_{21} = \frac{\partial \ddot{x}}{\partial \alpha} = 0 \) \hspace{1cm} (A.8)
\( A_{22} = \frac{\partial \ddot{x}}{\partial \dot{\alpha}} = 2\dot{\delta} \tan \delta - 2\dot{\rho} \rho^{-1} \) \hspace{1cm} (A.9)
\( A_{23} = \frac{\partial \ddot{x}}{\partial \delta} = 2\dot{\alpha} \dot{\delta} \sec^2 \delta \) \hspace{1cm} (A.10)
\( A_{24} = \frac{\partial \ddot{x}}{\partial \dot{\delta}} = 2\dot{\alpha} \dot{\delta} \sec \delta \) \hspace{1cm} (A.11)
\( A_{25} = \frac{\partial \ddot{\alpha}}{\partial \rho} = 2\dot{\alpha} \rho \rho^{-2} \) \hspace{1cm} (A.12)
\( A_{26} = \frac{\partial \ddot{\alpha}}{\partial \dot{\rho}} = -2\dot{\alpha} \rho^{-1} \) \hspace{1cm} (A.13)

\( A_{41} = \frac{\partial \dot{\delta}}{\partial \alpha} = 0 \) \hspace{1cm} (A.14)
\( A_{42} = \frac{\partial \dot{\delta}}{\partial \dot{\alpha}} = 2\dot{\alpha} \sin \delta \cos \delta \) \hspace{1cm} (A.15)
\( A_{43} = \frac{\partial \ddot{\delta}}{\partial \delta} = \dot{\alpha}^2(\cos^2 \delta - \sin^2 \delta) \) \hspace{1cm} (A.16)
\( A_{44} = \frac{\partial \ddot{\delta}}{\partial \dot{\delta}} = 2\dot{\delta} \rho^{-1} \) \hspace{1cm} (A.17)
\( A_{45} = \frac{\partial \ddot{\delta}}{\partial \rho} = -2\dot{\delta} \rho \rho^{-2} \) \hspace{1cm} (A.18)
\( A_{46} = \frac{\partial \ddot{\delta}}{\partial \dot{\rho}} = 2\dot{\delta} \rho^{-1} \) \hspace{1cm} (A.19)
\[ A_{61} = \frac{\partial \bar{\rho}}{\partial \alpha} = 0 \quad (A.20) \]

\[ A_{62} = \frac{\partial \bar{\rho}}{\partial \dot{\alpha}} = \dot{\alpha} \rho + \dot{\alpha} \rho \cos 2\delta \quad (A.21) \]

\[ A_{63} = \frac{\partial \bar{\rho}}{\partial \delta} = -\rho \dot{\alpha}^2 \sin 2\delta \quad (A.22) \]

\[ A_{64} = \frac{\partial \bar{\rho}}{\partial \dot{\delta}} = 2\dot{\delta} \rho \quad (A.23) \]

\[ A_{65} = \frac{\partial \bar{\rho}}{\partial \rho} = \dot{\delta}^2 + \frac{1}{2} \dot{\alpha}^2 + \mu \rho^{-3} + \frac{1}{2} \dot{\alpha}^2 \cos 2\delta \quad (A.24) \]

\[ A_{66} = \frac{\partial \bar{\rho}}{\partial \dot{\rho}} = 0 \quad (A.25) \]
CURRICULUM VITAE

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To secure a challenging position in a reputable organization to solve complex astrodynamics, guidance and control system problems with earned knowledge and skills

Education

Utah State University, Logan, Utah

• Doctor of Philosophy in Aerospace Engineering, May 2020
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  – Emphasis: Aircraft navigation
  – GPA: 3.69
Experience

- Graduate Research Assistant, Utah State University, Logan, Utah (August 2015-present)
  - Operated, calibrated and maintained the telescope at USU to obtain images of the satellites and space debris
  - Performed image processing on captured images to determine the positions of the satellites and space debris
  - Designed a sequential estimator to estimate the Fisher information in a space object survey
  - Implemented travelling salesman (linear programming) optimization technique to optimize the tracking of Space debris using a ground-based telescope

- Lecturer, Military Institute of Science and Technology, Bangladesh (January 2014 to June 2015)
  - Theory courses taught: Signals and system, fundamentals of electronics, and digital signal processing
  - Lab courses taught: C programming, communication engineering lab, digital signal processing lab and radar lab
  - Assistant course coordinator of junior undergrad student of aeronautical engineering department
Journal Publication


Conference Publications


- Practical Optical Survey Strategies for GEO from A Single Ground Based Observatory, AM Nafi, D Geller, *AIAA/AAS Astrodynamics Specialist Conference 2018, At Snowbird, Utah*

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