Optimization of Aileron Spanwise Size and Shape to Minimize Induced Drag in Roll with Correlating Adverse Yaw

Joshua R. Brincklow
Utah State University

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OPTIMIZATION OF AILERON SPANWISE SIZE AND SHAPE TO MINIMIZE
INDUCED DRAG IN ROLL WITH CORRELATING ADVERSE YAW

by

Joshua R. Brincklow

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Mechanical Engineering

Approved:

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Major Professor Committee Member

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Committee Member Vice Provost for Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah

2020
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ABSTRACT

Optimization of Aileron Spanwise Size and Shape to Minimize Induced Drag in Roll with Correlating Adverse Yaw

by

Joshua R. Brincklow, Master of Science
Utah State University, 2020

Major Professor: Dr. Douglas Hunsaker, Ph.D.
Department: Mechanical and Aerospace Engineering

Most modern aircraft employ discrete ailerons for roll control. The induced drag, rolling moment, and yawing moment for an aircraft is dictated in part by the location and spanwise size of the ailerons. To quantify these forces and moments and relate them to aileron design, a potential-flow lifting-line theory is used. This work explores a large design space composed of linearly tapered wing planforms and aileron geometries. Lifting-line theory shows that the optimum aileron location for minimizing induced drag always extends to the wing tip. This aileron design is not influenced by lift and rolling moment requirements. Changes to optimum aileron designs and their impacts to induced drag and yawing moment are considered and provide context for benefits in future morphing aircraft. Results are provided to give insight into aileron placement in the early design process. In most cases, optimum discrete ailerons produce 5–20% more induced drag than a morphing wing at the same rolling moment.

(46 pages)
PUBLIC ABSTRACT

Optimization of Aileron Spanwise Size and Shape to Minimize Induced Drag in Roll with Correlating Adverse Yaw

Joshua R. Brincklow

Most modern aircraft make use of modifying the main wing in flight to begin a roll. In many cases, this is done with a discrete control surface known as an aileron. The lift, drag, and moments for the wing are affected in part by the location and size of the ailerons along the length of the wing. The lift, drag, and moments can be found using a lifting-line theory that considers the circulation in airflow from many small sections of the wing. To minimize the drag due to lift on the wing, the ailerons must be optimized for the best location and size. In every case, the optimum aileron size extends to the wing tip. Results are provided in plots that can be used during the early design process to select optimum aileron size and location, as well as find the corresponding moments and drag due to lift. Compared to morphing wings, or wings that can change their shape for a new lift distribution along the wing, optimum discrete ailerons produce 5–20% more drag due to lift at the same rolling moment.
To my wife and parents, who stand with me during these uncertain times.
ACKNOWLEDGMENTS

This work was funded by the U.S. Office of Naval Research Sea-Based Aviation program (Grant No. N00014-18-1-2502) with Brian Holm-Hansen as the program officer.

Completion of this work would not be possible without my advisor, as he provided the guidance necessary to see the bigger picture and sidestep the pitfalls so common in research.

Joshua R. Brincklow
CONTENTS

ABSTRACT ........................................................................................................................................ iii

PUBLIC ABSTRACT ..................................................................................................................... iv

ACKNOWLEDGMENTS ..................................................................................................................... vi

LIST OF FIGURES ........................................................................................................................... viii

NOMENCLATURE ........................................................................................................................... ix

1 INTRODUCTION .............................................................................................................................. 1
  1.1 Background .................................................................................................................................. 1
  1.2 Lifting-line Theory ..................................................................................................................... 2
  1.3 Optimum Twist Distributions ..................................................................................................... 9

2 LIFTING-LINE ANALYSIS OF ROLL INITIATION ............................................................................. 12
  2.1 Background .................................................................................................................................. 12
  2.2 Derivation of Novel Terms for Aileron Effects ........................................................................... 12

3 APPLICATION OF THE NUMERICAL LIFTING-LINE METHOD ...................................................... 17
  3.1 Comparison of the Classical and Numerical Lifting-line Methods ............................................. 17
  3.2 Case Setup .................................................................................................................................. 20
  3.3 Grid Convergence and Optimization ......................................................................................... 21

4 EMPIRICAL RELATIONS FOR DESIGN BASED ON RESULTS ....................................................... 27
  4.1 Processing the Results ............................................................................................................... 27
  4.2 Results ....................................................................................................................................... 27
    4.2.1 Optimal Aileron Root ........................................................................................................... 27
    4.2.2 Values of $\kappa_{D\ell}$ ............................................................................................................ 28
    4.2.3 Values of $\kappa_n$ ................................................................................................................ 29
  4.3 Application Example ................................................................................................................... 30

5 CONCLUSION ................................................................................................................................... 32

REFERENCES ....................................................................................................................................... 37
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Change in local section angle of attack due to pure rolling rate</td>
</tr>
<tr>
<td>2</td>
<td>Induced drag planform penalty factor for untwisted linearly tapered wings</td>
</tr>
<tr>
<td>3</td>
<td>Rectangular planforms with varying methods of spanwise node placement with a lifting-line along the quarter-chord</td>
</tr>
<tr>
<td>4</td>
<td>Induced drag increment error between two lifting-line methods as a function of nodes per semispan</td>
</tr>
<tr>
<td>5</td>
<td>Grid-convergence analysis of induced drag coefficient at several prescribed rolling moment coefficients</td>
</tr>
<tr>
<td>6</td>
<td>Aircraft properties as a function of grid density</td>
</tr>
<tr>
<td>7</td>
<td>Contour of induced drag coefficient at a rolling-moment coefficient of 0.04</td>
</tr>
<tr>
<td>8</td>
<td>Contour of induced drag coefficient at a rolling-moment coefficient of 0.1</td>
</tr>
<tr>
<td>9</td>
<td>Contour plot of deflection angle (in degrees)</td>
</tr>
<tr>
<td>10</td>
<td>Contour plot of yawing moment coefficient</td>
</tr>
<tr>
<td>11</td>
<td>Aileron root positions based on aspect ratio and taper ratio to achieve minimum induced drag</td>
</tr>
<tr>
<td>12</td>
<td>Values of $\kappa_{D\ell}$ using optimal aileron design as a function of taper ratio for aspect ratios ranging from 4 to 20</td>
</tr>
<tr>
<td>13</td>
<td>Values of $\kappa_{n}$ using optimal aileron design as a function of taper ratio for aspect ratios ranging from 4 to 20</td>
</tr>
</tbody>
</table>
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_j$</td>
<td>Fourier coefficients in the lifting-line solution</td>
</tr>
<tr>
<td>$a_j$</td>
<td>decomposed Fourier coefficients related to planform</td>
</tr>
<tr>
<td>$b$</td>
<td>semispan of the wing</td>
</tr>
<tr>
<td>$b_j$</td>
<td>decomposed Fourier coefficients related to symmetric twist</td>
</tr>
<tr>
<td>$C_{D_i}$</td>
<td>induced drag coefficient</td>
</tr>
<tr>
<td>$C_{D_0}$</td>
<td>simplified induced drag coefficient</td>
</tr>
<tr>
<td>$\tilde{C}_{L,\alpha}$</td>
<td>section-lift slope</td>
</tr>
<tr>
<td>$C_{\ell}$</td>
<td>rolling-moment coefficient</td>
</tr>
<tr>
<td>$C_n$</td>
<td>yawing-moment coefficient</td>
</tr>
<tr>
<td>$c$</td>
<td>local section chord length</td>
</tr>
<tr>
<td>$c_j$</td>
<td>decomposed Fourier coefficients related to aileron deflection</td>
</tr>
<tr>
<td>$d_j$</td>
<td>decomposed Fourier coefficients related to rolling rate</td>
</tr>
<tr>
<td>$\bar{L}$</td>
<td>local section lift</td>
</tr>
<tr>
<td>$N$</td>
<td>number of terms retained in a truncated infinite series</td>
</tr>
<tr>
<td>$p$</td>
<td>angular rolling rate, positive right wing down</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>dimensionless angular rolling rate</td>
</tr>
<tr>
<td>$R_A$</td>
<td>wing aspect ratio</td>
</tr>
<tr>
<td>$R_T$</td>
<td>wing taper ratio</td>
</tr>
<tr>
<td>$V_\infty$</td>
<td>freestream velocity magnitude</td>
</tr>
<tr>
<td>$z$</td>
<td>spanwise coordinate from mid-span, positive left</td>
</tr>
<tr>
<td>$z_{\delta_r}$</td>
<td>spanwise position of the aileron closest to the wing root</td>
</tr>
</tbody>
</table>
$z_{\delta t} = \text{spanwise position of the aileron closest to the wing tip}$

$\alpha = \text{local geometric angle of attack relative to the freestream}$

$\alpha_{L0} = \text{local zero-lift angle of attack}$

$\Gamma = \text{local section circulation}$

$\Delta C_{Di} = \text{incremental change in induced-drag coefficient}$

$\delta_a = \text{aileron deflection angle in radians}$

$\delta_r = \text{semispan position of the aileron closest to the wing root}$

$\delta_t = \text{semispan position of the aileron closest to the wing tip}$

$\varepsilon_f = \text{local airfoil-section flap effectiveness}$

$\varepsilon_{\Omega} = \text{twist effectiveness}$

$\theta = \text{change of variables for the spanwise coordinate}$

$\kappa_D = \text{planform penalty factor in induced drag calculations}$

$\kappa_{DL} = \text{lift factor in induced drag calculations}$

$\kappa_{D\ell} = \text{rolling-moment factor in induced drag calculations}$

$\kappa_{D\Omega} = \text{twist factor in induced drag calculations}$

$\kappa_n = \text{yawing-moment factor in yawing-moment calculations}$

$\rho = \text{air density}$

$\chi = \text{spanwise antisymmetric twist distribution function}$

$\Omega = \text{negative of the twist value at the location of max magnitude twist}$

$\omega = \text{spanwise symmetric twist distribution function}$
CHAPTER 1
INTRODUCTION

1.1 Background

Discrete control surfaces are often used on a main wing for roll control and are often referred to as ailerons. Aircraft performance, structure, and system configuration, which vary with each airframe design, often determines the size and placement of the ailerons [1–4]. In recent years, extensive studies have been made into morphing aircraft that can deflect the wing trailing-edge continuously. For example, NASA has studied morphing wing concepts such as the Variable-Camber Continuous Trailing Edge (VCCTE) [5]. Flexsys is working on a continuous trailing-edge flap [6] for a Gulfstream aircraft. The Utah State University Aerolab in partnership with AFRL has developed and flight-tested a variable-camber continuous wing (VCCW) [7–11]. Often the main benefit of morphing-wing technology over ailerons is minimizing drag for a range of flight conditions [5,12–14]. Aileron deflection produces increased induced drag and radar observability, and decreased roll-yaw coupling control compared to morphing wings at a given rolling moment [15,16].

The location and spanwise size of the ailerons partially determine the magnitude of the yawing moment and drag. An optimal aileron geometry is desired to compare against morphing wings, as non-optimal solutions for discrete control surfaces compared against optimal continuous trailing-edge surfaces will lead to incorrect conclusions. Optimal aileron geometries are also desired for insight into aileron placement during the early stages of aircraft design. A potential-flow vortex lattice analysis for optimal aileron placement was performed by Feifel [17] for elliptic
planforms with a rolling moment requirement to minimize induced drag. This work broadens the scope by considering linearly tapered wings with ailerons using potential-flow lifting-line theory and gradient optimization. A relationship between aileron placement, induced drag, rolling moment, and yawing moment is provided as part of Prandtl’s classical lifting-line theory [18] and gives further insight into the conclusions made from the numerical results.

1.2 Lifting-line Theory

The section-lift distribution and induced drag on a finite wing is expressed in a Fourier sine series in Prandtl’s classical lifting-line (LL) theory [18,19]. The classical LL solution for the circulation distribution can be expressed as

$$\Gamma(\theta) = 2bV_\infty \sum_{j=1}^{N} A_j \sin(j\theta)$$

(1)

where $b$ represents the semispan of the wing, $V_\infty$ represents the freestream velocity, and $\theta$ represents a change of variables in the spanwise direction,

$$\theta \equiv \cos^{-1}(-2z/b)$$

(2)

Combining Eq. (1) with the Kutta-Joukowski law [20,21] gives

$$\bar{L}(\theta) = 2\rho V_\infty^2 b \sum_{j=1}^{N} A_j \sin(j\theta)$$

(3)

The Fourier coefficients in Eqs. (1) and (3) are related to the distributions of the chord-length and aerodynamic angle-of-attack. Prandtl’s LL equation can be used for any
wing planform and twist distribution to determine the spanwise section-lift distribution. To obtain the Fourier coefficients $A_j$ in Eqs. (1) and (3), the LL equation must be satisfied at $N$ locations along the wing. This results in a linear system that can be solved to yield the Fourier coefficients

$$
\sum_{j=1}^{N} A_j \left[ \frac{4b}{\bar{c}_l} c(\theta) + \frac{j}{\sin(\theta)} \right] \sin(j\theta) = \alpha(\theta) - \alpha_{L0}(\theta)
$$

(4)

where $\alpha(\theta)$ and $\alpha_{L0}(\theta)$ are functions of spanwise location and represent the geometric and aerodynamic angle of attack respectively. These can be used once the Fourier coefficients have been obtained to solve for the integrated forces and moments on the wing. With rigid-body roll effects included, the resultant lift, induced drag, rolling moment, and yawing moment coefficients are

$$
C_L = \pi R_A A_1
$$

(5)

$$
C_{Di} = \pi R_A \sum_{j=1}^{N} j A_j^2 - \frac{\pi R_A \bar{p}}{2} A_2
$$

(6)

$$
C_\ell = -\frac{\pi R_A}{4} A_2
$$

(7)

$$
C_n = \frac{\pi R_A}{4} \sum_{j=2}^{N} (2j - 1) A_{j-1} A_j - \frac{\pi R_A \bar{p}}{8} (A_1 + A_3)
$$

(8)

where $\bar{p}$ is the nondimensional roll rate about the stability axis, and is defined as

$$
\bar{p} \equiv pb/2V_\infty
$$

(9)
The stability axis is the axis parallel to the freestream and intersecting the center of gravity as shown in Fig. 1.

![Diagram showing change in local section angle of attack due to pure rolling rate.](image)

**Fig. 1** Change in local section angle of attack due to pure rolling rate.

Equations (4)–(8) have the disadvantage of requiring recalculation for each change in operating condition, including angle of attack, control-surface deflection, and rolling rate. A more useful form of the LL solution has been presented by Phillips and Snyder [22] and allows for operating conditions to be solved for independently [23]. A similar approach is applied here with the definition

\[
A_j = a_j (\alpha - \alpha_{L0})_{\text{root}} - b_j \Omega + c_j \delta_a \varepsilon_f + d_j \bar{p}
\]

(10)

where \(a_j\), \(b_j\), \(c_j\), \(d_j\) are decomposed Fourier coefficients representing planform, twist, aileron deflection, and roll rate, respectively. Here twist is defined to be spanwise symmetric with scaling \(-\Omega\), and the roll control mechanism to be symmetric in magnitude and opposite in sign, termed antisymmetric, with a magnitude of \(\delta_a\). The symbol \(\varepsilon_f\) is the aileron section flap effectiveness, which in this work is assumed to be constant across the
span of the flap. The decomposed Fourier coefficients can be found by using the relations

\[
\sum_{j=1}^{N} a_j \left[ \frac{4b}{\bar{C}_{L,a}c(\theta)} + \frac{j}{\sin(\theta)} \right] \sin(j\theta) = 1
\]  

(11)

\[
\sum_{j=1}^{N} b_j \left[ \frac{4b}{\bar{C}_{L,a}c(\theta)} + \frac{j}{\sin(\theta)} \right] \sin(j\theta) = \omega(\theta)
\]  

(12)

\[
\sum_{j=1}^{N} c_j \left[ \frac{4b}{\bar{C}_{L,a}c(\theta)} + \frac{j}{\sin(\theta)} \right] \sin(j\theta) = \chi(\theta)
\]  

(13)

\[
\sum_{j=1}^{N} d_j \left[ \frac{4b}{\bar{C}_{L,a}c(\theta)} - \frac{j}{\sin(\theta)} \right] \sin(j\theta) = \cos(\theta)
\]  

(14)

where \(\omega(\theta)\) is a symmetric twist distribution function, and \(\chi(\theta)\) is a spanwise antisymmetric twist distribution function, which can be represented as an indicator function

\[
\chi(z) = \begin{cases} 
0, & z < -z_{\delta_t} \\
1, & -z_{\delta_t} \leq z \leq -z_{\delta_r} \\
0, & -z_{\delta_r} < z < z_{\delta_r} \\
-1, & z_{\delta_r} \leq z \leq z_{\delta_t} \\
0, & z > z_{\delta_t}
\end{cases}
\]  

(15)

where \(z_{\delta_r}\) is the spanwise position of the aileron closest to the wing root and \(z_{\delta_t}\) is the spanwise position of the aileron closest to the wing tip. The aileron root and tip are here defined as the spanwise edge position of the aileron closest to the wing root and tip, respectively.

The normalized twist distribution functions \(\omega(\theta)\) and \(\chi(\theta)\) are multiplied by the corresponding scalings \(-\Omega\) and \(\delta_a \epsilon_f\) to give the resultant total twist distribution in the
wing. The total symmetric twist is $-\Omega \omega(\theta)$, and the total antisymmetric twist is $\delta_a \varepsilon_f \chi(\theta)$.

For a given wing planform, symmetric twist distribution function, and antisymmetric control deflection distribution, Eqs. (11)–(14) can be solved for the decomposed Fourier coefficients. These coefficients along with angle-of-attack, symmetric twist scaling, control deflection scaling, section flap effectiveness, and rolling rate can then be used in Eq. (10) to compute the Fourier coefficients in Eqs. (5)–(8).

For a wing with symmetric planform and twist, the even terms of the $a_j$ and $b_j$ coefficients are zero. For any wing with an antisymmetric control surface distribution, the odd terms of the $c_j$ coefficients are zero. The odd terms of the $d_j$ coefficients are also zero, since the aerodynamic angle-of-attack changes antisymmetrically with rigid-body roll about the stability axis. Equation (10) can then be expressed as

$$A_j = \begin{cases} 
    a_j(\alpha - \alpha_{L0})_{\text{root}} - b_j \Omega & \text{j odd} \\
    c_j \delta_a \varepsilon_f + d_j \bar{p} & \text{j even}
\end{cases}$$

(16)

Using Eq. (16) in Eqs. (5)–(8) gives

$$C_L = \pi R_A [a_1(\alpha - \alpha_{L0})_{\text{root}} - b_1 \Omega]$$

(17)

$$C_{Dl} = \pi R_A \sum_{j=1}^{N} j[a_j(\alpha - \alpha_{L0}) - b_j \Omega]^2 + \pi R_A \sum_{j=2}^{N} j(c_j \delta_a \varepsilon_f + d_j \bar{p})^2$$

$$- \frac{\pi R_A \bar{p}}{2} (c_2 \delta_a \varepsilon_f + d_2 \bar{p})$$

(18)

$$C_\ell = -\frac{\pi R_A}{4} (c_2 \delta_a \varepsilon_f + d_2 \bar{p})$$

(19)
\[ C_n = -\frac{\pi R_A \bar{p}}{8} (a_1 (\alpha - \alpha_{L0})_{\text{root}} - b_1 \Omega + a_3 (\alpha - \alpha_{L0})_{\text{root}} - b_3 \Omega) \]

\[ + \frac{\pi R_A}{4} \left( \sum_{j=2}^{N} (2j - 1) (a_{j-1} (\alpha - \alpha_{L0})_{\text{root}} - b_{j-1} \Omega) (c_j \delta \epsilon_f + d_j \bar{p}) \right)_{\text{even}} \]

\[ + \left( \sum_{j=3}^{N} (2j - 1) (c_{j-1} \delta \epsilon_f + d_{j-1} \bar{p}) (a_j (\alpha - \alpha_{L0})_{\text{root}} - b_j \Omega) \right)_{\text{odd}} \]

Recognizing the last term in Eq. (18) is the same as Eq. (19), Eq. (18) can be expressed as

\[ C_{D_i} = \pi R_A \sum_{j=1}^{N} j [a_j (\alpha - \alpha_{L0}) - b_j \Omega]^2 + \pi R_A \sum_{j=2}^{N} j (c_j \delta \epsilon_f + d_j \bar{p})^2 \]

\[ - 2 \bar{p} C_\ell \]

In the absence of aileron deflection and rolling rate, the induced drag simplifies to

\[ C_{D0} = \pi R_A \sum_{j=1}^{N} j [a_j (\alpha - \alpha_{L0}) - b_j \Omega]^2 \]

(22)

and the induced drag can be rearranged in the form [24]

\[ C_{D0} = \frac{C_L^2 (1 + \kappa_D) - \kappa_{DL} C_L C_{L,\alpha} \Omega + \kappa_{DL} (C_{L,\alpha} \Omega)^2}{\pi R_A} \]

(23)

where

\[ C_L = C_{L,\alpha} [(\alpha - \alpha_{L0})_{\text{root}} - \epsilon \Omega \Omega] \]

(24)
\[ C_{L,\alpha} = \pi R_A a_1 = \frac{\tilde{C}_{L,\alpha}}{[1 + \tilde{C}_{L,\alpha}/(\pi R_A)](1 + \kappa_L)} \]  

(25)

\[ \kappa_L = \frac{1 - (1 + \pi R_A/\tilde{C}_{L,\alpha})a_1}{(1 + \pi R_A/\tilde{C}_{L,\alpha})a_1} \]  

(26)

\[ \varepsilon_\Omega \equiv \frac{b_1}{a_1} \]  

(27)

\[ \kappa_D \equiv \sum_{j=2}^{N} \frac{a_j^2}{a_1^2} \]  

(28)

\[ \kappa_{DL} \equiv 2 \frac{b_1}{a_1} \sum_{j=2}^{N} j \frac{a_j}{a_1} \left( \frac{b_j}{b_1} - \frac{a_j}{a_1} \right) \]  

(29)

\[ \kappa_{DL\Omega} \equiv \left( \frac{b_1}{a_1} \right)^2 \sum_{j=2}^{N} j \left( \frac{b_j}{b_1} - \frac{a_j}{a_1} \right)^2 \]  

(30)

Here \( a_n \) depends on planform as shown in Eq. (11) and \( b_n \) depends on wing twist as shown in Eq. (12), and therefore \( \kappa_L, \varepsilon_\Omega, \kappa_D, \kappa_{DL} \), and \( \kappa_{DL\Omega} \) depend on planform and twist, with examples shown by Phillips et. al. [25]. From these relationships, valuable conclusions can be drawn about optimum taper ratio and symmetric twist design. For example, in the absence of twist, Eq. (23) simplifies to

\[ \C_{D0} = \frac{C_L^2}{\pi R_A} (1 + \kappa_D) \]  

(31)

The term \( \kappa_D \) represents the wing planform penalty factor in induced drag relative to an untwisted elliptic wing. The wing planform penalty factor can be computed for any planform from Eqs. (28) and (31). Glauert [26] was the first to visualize \( \kappa_D \), and more
recently, Phillips [27] produced a similar figure. Work by Phillips et. al [28] cleared up misconceptions based on Glauert’s limited results. In Fig. 2, a visualization for $\kappa_D$ as a function of taper ratio and aspect ratio is given, similar to Phillips [27]. The common rule-of-thumb that induced drag is minimized with a taper ratio of 0.4 comes from these types of computations [26,27].

![Fig. 2 Induced drag planform penalty factor for untwisted linearly tapered wings.](image)

These types of design-space explorations are useful for providing intuition in the early stages of aircraft design. Here, a similar approach is used to examine the influence of aileron placement on induced drag. In order to evaluate the effect of discrete ailerons on induced drag and find the correlated yawing moment, it is helpful to understand two optimal twist distributions.

1.3 Optimum Twist Distributions

Phillips, et. al [14] and Phillips and Hunsaker [29] showed a normalized spanwise
twist distribution function that minimizes induced drag for any symmetric wing planform in steady level flight. This can be written as

$$\omega(\theta) = 1 - \frac{\sin(\theta)}{c(\theta)/c_{root}}$$  \hspace{1cm} (32)

with the required symmetric twist scaling based on the lift coefficient

$$\Omega = \frac{4bC_L}{\pi R_A \tilde{C}_{L,\alpha} c_{root}}$$  \hspace{1cm} (33)

And the required angle of attack is

$$(\alpha - \alpha_{L0})_{root} = \frac{C_L}{\pi R_A} \left( \frac{4b}{\tilde{C}_{L,\alpha} c_{root}} + 1 \right)$$  \hspace{1cm} (34)

Any wing employing this twist distribution function at the angle of attack given in Eq. (34) will produce an elliptic lift distribution and result in an induced drag of

$$C_{Dl} = \frac{C_L^2}{\pi R_A}$$  \hspace{1cm} (35)

Similarly, a LL analysis by Hunsaker et. al [30] produced an antisymmetric twist distribution function that minimizes induced drag for any symmetric wing planform and prescribed rolling moment with zero rolling rate. This can be written as

$$\chi(\theta) = \left[ 1 + \frac{2b \sin(\theta)}{\tilde{C}_{L,\alpha} c(\theta)} \right] \cos(\theta)$$  \hspace{1cm} (36)

This twist distribution function provides the optimum continuous twist along the wingspan to produce a given rolling moment and minimize induced drag in the absence of rolling rate. For any wing geometry employing the optimal antisymmetric twist distribution
function from Hunsaker et. al [30] in Eq. (36), the corresponding induced drag increase relative to steady level flight conditions is

\[(\Delta C_{Di})_\ell = 32 \frac{C_\ell^2}{\pi R_A} \]  

(37)

Equation (37) gives the minimum increase in drag for any rolling moment. Most aileron designs produce more induced drag than this. Using the antisymmetric twist distribution function given by Eq. (36), the resulting yawing moment at roll initiation is given by

\[C_n = -\frac{3C_L C_\ell}{\pi R_A} - 5C_\ell (a_3 (\alpha - \alpha_{L0})_{root} - b_3 \Omega) \]  

(38)

With this LL formulation, the effect of discrete ailerons on induced drag can now be considered.
2.1 Background

Classical LL theory uses a Fourier sine series to represent the lift distribution along a wing as shown in Eq. (3). Including more Fourier coefficients increases the number of frequencies considered in the analysis as well as the rank of the linear system of equations that must be solved. An aileron deflection represents a step change in twist distribution along the wing, and therefore introduces many frequencies into the lift distribution. Because solutions to this system of equations were obtained by hand in the early days of aeronautics, they were limited in the number of Fourier coefficients that could be included. Hence, it was difficult to use this method to accurately evaluate the effect of ailerons on induced drag before the advent of the computer. Today, however, it is quite simple to solve large systems of equations with little effort, so many more Fourier coefficients can be included.

2.2 Derivation of Novel Terms for Aileron Effects

The change in induced drag based only on aileron deflection and rolling rate can be obtained by subtracting Eq. (22) from Eq. (21). This gives

\[ (\Delta C_{Dl})_{\delta \rho} \equiv C_{Dl} - C_{D0} = \pi R_A \sum_{j=2}^{N} j \left( c_j \delta_a \epsilon_{f_j} + d_j \rho \right)^2 - 2 \rho C_{\ell} \]  (39)

During roll initiation, the rolling rate is zero, while the aileron deflection and rolling
moment are nonzero. In this case, the rolling moment, yawing moment, and change in induced drag can be found from Eqs. (19), (20), and (39) respectively by using $\bar{p} = 0$, which gives

$$C_\ell = -\frac{\pi R_A}{4} c_2 \delta_a \epsilon_f$$  \hfill (40)

$$C_n = \frac{\pi R_A \delta_a \epsilon_f}{4} \left( \sum_{j=2}^{N} (2j-1)(a_{j-1}(\alpha - \alpha_{L0})_{\text{root}} - b_{j-1}\Omega)(c_j) \right)_{\text{even}}$$

$$+ \left( \sum_{j=3}^{N} (2j-1)(c_{j-1})(a_j(\alpha - \alpha_{L0})_{\text{root}} - b_j\Omega) \right)_{\text{odd}}$$ \hfill (41)

$$(\Delta C_{Di})_\delta = \pi R_A \delta_a^2 \frac{\epsilon_f^2}{2} \sum_{j=2}^{N} j c_j^2$$ \hfill (42)

Using Eq. (40) in Eq. (42) to eliminate the aileron deflection magnitude $\delta_a$ gives

$$(\Delta C_{Di})_\delta = \frac{32 C_\ell^2 (1 + \kappa_{D\ell})}{\pi R_A}$$ \hfill (43)

where

$$\kappa_{D\ell} = \frac{1}{2} \sum_{j=4}^{N} j \left( \frac{c_j}{c_2} \right)^2$$ \hfill (44)
From Eq. (43), it is shown that the increase in induced drag is a function of the aspect ratio, rolling moment, and \( \kappa_{D\ell} \). The decomposed Fourier coefficients \( c_j \) depend on planform as shown in Eq. (13), as well as the spanwise aileron edge positions as shown in Eq. (15). Hence, the value for \( \kappa_{D\ell} \) is a function of planform, aileron position, and aileron spanwise length. Note however that this analysis predicts that the increase in induced drag given in Eq. (43) is independent of section flap effectiveness \( \epsilon_f \), lift, and symmetric twist. It is also interesting to note that for this case, the increase in induced drag is directly proportional to the square of the rolling moment, much in the same way that the induced drag in the absence of twist is proportional to the square of the lift coefficient, as shown in Eq. (31).

The induced drag of an untwisted wing of any planform with ailerons can be given as

\[
C_{Di} = \frac{C_L^2(1 + \kappa_D) + 32C_{\ell}^2(1 + \kappa_{D\ell})}{\pi R_A} \tag{45}
\]

If the symmetric twist distribution function given in Eqs. (32) and (33) is used, \( \kappa_D \) is zero. If the optimal antisymmetric twist distribution function from Hunsaker et. al [30] given in Eq. (36) is used, \( \kappa_{D\ell} \) is zero. If both twist distribution functions are used simultaneously, the minimum induced drag for a given lift and rolling moment is [14]

\[
C_{Di} = \frac{C_L^2 + 32C_{\ell}^2}{\pi R_A} \tag{46}
\]

As a first step to understand aileron design, only wings without twist will be considered, making \( \Omega \) zero. This is similar to the approach Glauart [26] and Phillips [27] employed, since they neglected twist in their initial studies on the effects of wing planform on induced drag. Applying these simplifications to Eqs. (17) and (41) gives
\[ C_L = \pi R_A [a_1 (\alpha - \alpha_{L0})_{\text{root}}] \quad (47) \]

\[ C_n = \frac{\pi R_A \delta_a \varepsilon_f}{4} \left( \sum_{j=2}^{N} (2j - 1) \left( a_{j-1} (\alpha - \alpha_{L0})_{\text{root}} \right) c_j \right)_{j \text{ even}} \]

\[ + \left( \sum_{j=3}^{N} (2j - 1) \left( c_{j-1} (\alpha - \alpha_{L0})_{\text{root}} \right) a_j \right)_{j \text{ odd}} \quad (48) \]

Substituting Eq. (47) in Eq. (48) gives:

\[ C_n = \frac{C_L \delta_a \varepsilon_f}{4} \left( \sum_{j=2}^{N} (2j - 1) \left( \frac{a_{j-1}}{a_1} \right) c_j \right)_{j \text{ even}} \]

\[ + \left( \sum_{j=3}^{N} (2j - 1) \left( c_{j-1} \frac{a_j}{a_1} \right) c_{j-1} \right)_{j \text{ odd}} \quad (49) \]

Equation (40) can be rearranged and used in Eq. (49) to eliminate the aileron deflection magnitude \( \delta_a \):

\[ C_n = -\frac{C_L C_{\ell}}{\pi R_A} \left( \sum_{j=2}^{N} (2j - 1) \left( \frac{a_{j-1}}{a_1} \right) c_j \right)_{j \text{ even}} \]

\[ + \left( \sum_{j=3}^{N} (2j - 1) \left( c_{j-1} \frac{a_j}{a_1} \right) c_{j-1} \right)_{j \text{ odd}} \quad (50) \]

This can be rearranged to give
\[ C_n = -\frac{C_L C_\ell \kappa_n}{\pi R_A} \]  

(51)

where

\[ \kappa_n = 3 + \left( \sum_{j=3}^{N} (2j - 1) \left( \frac{c_j - 1}{c_2} \right) \left( \frac{a_j}{a_1} \right) \right)_{j \text{ odd}} \]

\[ + \left( \sum_{j=4}^{N} (2j - 1) \left( \frac{a_{j-1}}{a_1} \right) \left( \frac{c_j}{c_2} \right) \right)_{j \text{ even}} \]  

(52)
CHAPTER 3

APPLICATION OF THE NUMERICAL LIFTING-LINE METHOD

3.1 Comparison of the Classical and Numerical Lifting-line Methods

A current major drawback of using classical LL theory in the evaluation of the effects of ailerons on induced drag is the inability to use grid clustering to achieve second-order convergence. In the classical LL theory, traditionally nodes are clustered along the wing using Eq. (2) with cosine-clustering near the wing tips by evenly spacing the nodes in $\theta$. However, this method of clustering does not consider how the node clustering will fall relative to the placement of the aileron. Figure 3(a) provides a visualization of the traditional cosine-clustering with 80 nodes and symmetrically-placed ailerons. Note that the edge of the aileron may be in a position between two nodes or directly on a node. As the number of nodes used in the calculation increases, the accuracy of the induced drag and yawing moment solutions will vary as the cosine-clustered nodes change position relative to the location of the edge of the aileron. As a comparison, the clustering used in Fig. 3(b) allows greater control over the placement of nodes relative to the aileron position, so that the edge of an aileron falls directly on a node regardless of its span or spanwise location. This clustering can be termed as aileron-sensitive clustering. Aileron-sensitive clustering can be utilized in the classical LL theory, but because an aileron forces a step change in twist along the span, an infinite number of frequencies are introduced, and the finite Fourier series cannot accurately model the step change in twist. Finding mathematical workarounds to use the classical LL method with aileron-sensitive clustering is a future topic of study.

Given the prior work in deriving the LL theory, the current inability to access a working induced drag and yawing moment solution using discrete control surfaces may
seem a great discouragement. However, a numerical LL algorithm published by Phillips and Snyder [22], which is a close numerical analog to the classical LL theory, can effectively use aileron-sensitive clustering and accurately find the aerodynamic effects of ailerons.

Fig. 3 Rectangular planforms with varying methods of spanwise node placement with a lifting-line along the quarter-chord.

The numerical LL method differs from the vortex lattice method that Feifel [17] used because the surface flow boundary condition is not required at the three-quarter-chord [31] along the panel center line. Instead, the algorithm finds the local circulation at each wing section with a relationship between the three-dimensional vortex lifting law [32] and the section airfoil lift. This algorithm depends on a system of lifting surfaces connected by discrete horseshoe vortices, creating a vorticity field [33]. This method can be applied to multiple lifting surfaces with sweep and dihedral and gives accurate solutions for wings with aspect ratios greater than about 4 [22]. This algorithm is applied in MachUp [14,34], an open-source code available on GitHub¹.

As an example, Fig. 4 shows the difference in convergence between MachUp and

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¹ https://github.com/usuaero/MachUp
the classical LL method with traditional cosine-clustering for an induced drag increment caused by aileron deflection as a function of nodes per semispan. Error is calculated as the difference in induced drag increment between a given number of nodes per semispan and a significantly greater number of nodes per semispan, in this case 640 nodes, using the same method. Note that the classical LL method does not consistently decrease in error as the number of nodes is increased. The numerical LL algorithm is therefore a more consistently accurate tool to explore the design space of aileron sizing and placement.

![Figure 4](image-url)

**Fig. 4** Induced drag increment error between two lifting-line methods as a function of nodes per semispan.

A downside of using the numerical lifting-line algorithm is that the results for the decomposed Fourier coefficients cannot be found directly. The numerical lifting-line algorithm provides integrated force and moment solutions for the complete wing, which can be used to estimate $\kappa_{D_\ell}$ and $\kappa_n$. Rearranging Eqs. (31), (43), and (51) gives

$$\kappa_D = \frac{C_{D_0} \pi R_A}{C_L^2} - 1$$  \hspace{1cm} (53)
\begin{equation}
\kappa_{D\ell} = \frac{(\Delta C_{Di})_\delta (\pi R_A)}{32 C^2_{\ell}}
\end{equation}

\begin{equation}
\kappa_n = -\frac{\pi C_n R_A}{C_L C_{\ell}}
\end{equation}

Results for the integrated induced-drag increment and yawing moment were then used in Eqs. (53)–(55) to estimate \( \kappa_D, \kappa_{D\ell}, \) and \( \kappa_n \) for a given planform and aileron geometry.

### 3.2 Case Setup

Each wing semispan is specified in 3 wing sections, with the center section containing the aileron, and each wing section is cosine-clustered with a number of nodes relative to section span, as shown in Fig. 3(b). The control surfaces are modeled with a flap-chord fraction of 1.0 for the entire control surface length. As Feifel [5] notes, the induced drag increment predicted by potential flow algorithms is independent of the aileron flap-chord fraction and depends only on the prescribed rolling moment, explained in detail by Phillips [15] as well as Abbott and Doenhoff [22]. A Newton-Secant method is used to find the aileron deflection that would provide a target rolling moment within machine precision at double-precision computing.

The angle of attack and rolling moment coefficient were adjusted iteratively to arrive at the prescribed lift coefficient and rolling-moment coefficient for a given wing planform and aileron geometry. Convergence criteria of \( 1.0 \times 10^{-12} \) and \( 1.0 \times 10^{-16} \) were used for the lift coefficient and rolling-moment coefficient, respectively. Newton’s method was used for each case as outlined by Phillips [25] with a convergence criterion of \( 1.0 \times 10^{-12} \) to satisfy the Jacobian system of equations for the numerical lifting-line algorithm. For the computations shown here, an airfoil section with a lift slope of \( 2\pi \)
and a zero-lift angle of attack of 0 were used, which corresponds to a thin airfoil with zero camber.

### 3.3 Grid Convergence and Optimization

Multiple grid densities were analyzed to ensure fully grid-converged values from the numerical LL algorithm. Figure 5 shows results for induced drag as a function of grid density for several rolling-moment coefficients. Figure 6 shows the induced drag coefficient, deflection angle, and yawing-moment coefficient predicted by the numerical LL algorithm as a function of grid density given a rolling-moment coefficient of 0.1. The values suggest grid convergence is achieved with 80 nodes over the semispan. A grid density of 100 nodes over the semispan is used in the rest of the analysis to give a preferable balance of accuracy and computational cost.

![Grid Convergence Analysis](image)

**Fig. 5** Grid-convergence analysis of induced drag coefficient at several prescribed rolling-moment coefficients.

The optimum spanwise size and location of ailerons for minimum induced drag is found by using an open-source optimization algorithm, Optix, created and used by Hodson,
et al. [34], available on GitHub². The algorithm makes use of the Broyden [35], Fletcher [36], Goldfarb [37], and Shanno [38] (BFGS) method to iteratively find a minimum. For the aileron root and tip, the optimization algorithm used decimal numbers out to machine precision at double-precision computing. With a grid density of 100 nodes and small changes to the span of the aileron section, the number of nodes assigned to the aileron would change, changing the induced drag and presenting a similar convergence challenge as the classical LL theory. To counter this issue, the optimization algorithm used two loops. The inner loop could exclusively change the aileron geometry, while the outer loop could exclusively change the redistribution of nodes so that the number of nodes assigned to a section of the wing would remain proportional to the span of the section. In other words, an aileron making up 50% of the semispan would contain 50 nodes after redistribution.

Fig. 6 Aircraft properties as a function of grid density.

² https://github.com/usuaero/Optix
Figures 7 and 8 show induced drag contours for a wing with a prescribed rolling-moment coefficient, lift coefficient, and aspect ratio. A diagonal line boundary defines the limiting case of an infinitely small aileron at any location in the semispan. The $(x,y)$ coordinates on this plot represent the beginning spanwise location $(x)$ and the ending spanwise location $(y)$ of the aileron. Contour lines extend to aileron deflections past 25 degrees to show data trends, even though this study only analyzes the inviscid case and does not consider stall characteristics. Aileron deflections more than 25 degrees are typically not practical for aircraft design. A circle at the top of the plots shows the minimum induced drag, with the corresponding value shown in text at the bottom right. In Fig. 7, which shows induced drag contours with a prescribed rolling-moment coefficient of 0.04, lift coefficient of 0.5, and aspect ratio of 8, a low gradient near the minimum allows for movement of the aileron tip between 10 and 40 percent of the wing semispan with less than a 2% increase in induced drag above the minimum. Figure 8 changes the prescribed rolling-moment coefficient to 0.1 while keeping all other parameters the same as Fig. 7, which shows an increased sensitivity of the induced drag to aileron size and position based on prescribed rolling moment. However, movement of the aileron tip between 10 and 40 percent of the wing semispan only increases the induced drag less than 4% above the minimum.
Fig. 7 Contour of induced drag coefficient at a rolling-moment coefficient of 0.04.

Fig. 8 Contour of induced drag coefficient at a rolling-moment coefficient of 0.1.

Figures 9 and 10 show the contour plots for deflection angle and yawing moment, respectively, matching the parameters used in Fig. 8. For the deflection angle, the minimum is found at the point (0,100) in the plot, indicating an aileron extending from the wing root to the wing tip. For the yawing moment, the minimum can be found by making the aileron span small and close to the wing root. These minimums are consistently at the same edges of the domain with each specified rolling moment, lift, aspect ratio, and taper ratio.
Fig. 9 Contour plot of deflection angle (in degrees).

Fig. 10 Contour plot of yawing moment coefficient.

Regardless of rolling moment or lift, the minimum induced drag in Figs. 7 and 8 is found where the aileron tip meets the wing tip. Gradient-based optimization techniques produce difficulties when the optimum is close to a boundary. With these difficulties in mind, the aileron tip was limited to coincide with the wing tip for the following analysis,
or $\delta_t = 1$. In other words, the wing section containing the wing tip is infinitely small, and only the aileron root location could vary to minimize induced drag.
CHAPTER 4

EMPIRICAL RELATIONS FOR DESIGN BASED ON RESULTS

4.1 Processing the Results

From Eq. (43)(37), it is evident that the increase in induced drag due to aileron deflection is dependent on $\kappa_{D\ell}$. From Eq. (51), it is evident that the corresponding yawing moment depends on $\kappa_n$ in the absence of symmetric twist. Equations (13), (44), and (52) show that $\kappa_{D\ell}$ and $\kappa_n$ are functions of the wing planform as well as the aileron geometry because of their dependence on the decomposed Fourier coefficient $c_j$. These equations also show that $\kappa_{D\ell}$ is independent of prescribed rolling-moment and lift. As the classical lifting-line theory presented convergence limitations as shown previously, the numerical lifting-line algorithm combined with a gradient-based optimization algorithm discussed above were used to optimize the aileron geometry to minimize induced drag as well as find the corresponding yawing moment for a range of aileron geometries. By combining the results from the numerical LL method and Eqs. (53)–(55), empirical relations can be obtained for the aileron root $\delta_r$, the rolling-moment factor $\kappa_{D\ell}$, and the yawing-moment factor $\kappa_n$. Cases were run with aspect ratio varied from 4 to 20 in increments of 2 and taper ratio varied from 0.0 to 1.0 in increments of 0.01.

4.2 Results

4.2.1 Optimal Aileron Root

A plot of aileron roots that minimize induced drag for various wing planforms are given in Fig. 11. As aspect ratio and taper ratio increase, the aileron root must move closer to the root of the wing to achieve the minimum induced drag for the wing. Feifel [17]
reported that conventional single-segment ailerons are optimally sized for elliptical wings at 70% semispan. Referencing Fig. 2, the closest tapered wing planform to an elliptical planform is about a wing with a taper ratio of 0.4. The aileron root values at $R_T = 0.4$ agree closely with what Feifel [17] reported.

![Graph showing aileron root positions based on aspect ratio and taper ratio to achieve minimum induced drag.]

**Fig. 11** Aileron root positions based on aspect ratio and taper ratio to achieve minimum induced drag.

4.2.2 Values of $\kappa_{D\ell}$

A graph of $\kappa_{D\ell}$ for optimal aileron design with changes in aspect ratio and taper ratio is given in Fig. 12. Looking at Fig. 12 and Eq. (43), results for $\kappa_{D\ell}$ can be seen as a percent increase in the induced drag increase due to aileron geometry versus a wing using the optimum twist distribution function given in Eq. (33), which would give a $\kappa_{D\ell}$ of 0. Since a variable-continuous trailing-edge wing could be designed to produce the optimum twist distribution function given in Eq. (33), the results in Fig. 12 give insight into the advantages of VCCTE and VCCW technology. As an example, a wing using discrete ailerons with an aspect ratio of 8 and taper ratio of 0.4 would produce 10% more induced
drag than a variable-continuous trailing-edge wing. Note from Fig. 12 that the minimum $\kappa_{De}$ for any aspect ratio is found at a taper ratio of 1.0.

![Graph of $\kappa_{De}$](image)

**Fig. 12** Values of $\kappa_{De}$ using optimal aileron design as a function of taper ratio for aspect ratios ranging from 4 to 20.

4.2.3 Values of $\kappa_n$

A graph of $\kappa_n$ is given in Fig. 13 for multiple aspect ratios and taper ratios. Note at a taper ratio of about 0.32, the value for $\kappa_n$ is the same for all aspect ratios. At lower taper ratios, $\kappa_n$ is generally less, which decreases the adverse yawing moment. Note that in Eq. (51), a $\kappa_n$ less than 0 provides proverse yaw. Regardless of the wing planform, Fig. 12 shows that if zero twist is applied to the wing geometry, a wing with optimally-placed ailerons to minimize induced drag will not produce proverse yaw.
4.3 Application Example

An example of how to design a wing with optimally sized ailerons and find the induced drag and correlated yawing moment is warranted. A designer with a previously chosen aspect and taper ratio, in this example 14 and 0.6 respectively, could look at Fig. 11 and see that setting the aileron root at 26.5% of the wing semispan minimizes the induced drag from the aileron. Figure 12 gives a corresponding $\kappa_{D\ell}$ of about 0.11, while Fig. 2 gives a $\kappa_D$ of about 0.045. These values can be used along with a desired lift and rolling moment in Eq. (45) to give the total induced drag for the wing. With a prescribed lift coefficient of 0.5 and rolling moment coefficient of 0.05, the induced drag coefficient is $7.99 \times 10^{-3}$. Figure 13 gives a $\kappa_n$ value of about 3.42, which can be used along with a desired lift and rolling moment in Eq. (51) to give the yawing moment for the wing, which in this case is $-1.94 \times 10^{-3}$. This methodology offers excellent estimates for initial wing design.

An important note is that symmetric twist (washout) has not been considered in this
analysis, which would change the values for $\kappa_n$. With symmetric twist, the values of $\kappa_n$ could drop below 0 in certain conditions and the wing could then create proverse yaw during roll. This is a topic of future work.
Aileron geometry determines the induced drag produced by an aileron deflection for a prescribed rolling moment. A potential flow lifting-line optimization for aileron spanwise size and position can minimize the induced drag increase from ailerons. Prandtl's classical LL theory is the foundation for this approach and allows for calculation of spanwise section-lift distribution for any wing planform and twist distribution function. An optimum normalized spanwise symmetric twist distribution function minimizes induced drag for a symmetric wing planform in steady level flight, while an antisymmetric twist distribution function minimizes induced drag for any symmetric wing planform and prescribed rolling moment with zero rolling rate. Ailerons will typically produce more induced drag than the optimum twist solution to produce a rolling moment. Results from this theory show how the induced drag and yawing moment are related to planform, aileron design, lift, and rolling moment. For this study, the yawing moment calculations neglected symmetric wing twist (washout) effects.

This optimization made use of a numerical LL algorithm, as the classical lifting-line theory had grid-clustering limitations. Based on the grid resolution and convergence for the induced drag, deflection angle, and yawing moment, 100 nodes per semispan were used in this analysis. A gradient-based optimization technique is used to find the aileron spanwise size and position for minimum induced drag. The aileron tip for minimum induced drag is found to meet the wing tip position, so the optimization was limited to changing the aileron root while the aileron tip was locked to the wing tip. The optimization produced results for aileron root positions for minimum induced drag, as well as...
coefficients for finding the corresponding minimum induced drag and yawing moment.

Coefficients for finding the induced drag include $\kappa_D$ and $\kappa_{D\ell}$, where $\kappa_D$ can be viewed as an increase in drag due to a deviation in wing planform from an elliptic wing, and $\kappa_{D\ell}$ can be viewed as a percent increase in induced drag due to a deviation from the optimal twist distribution function through using aileron design. Coefficients for finding the corresponding yawing moment include $\kappa_n$, where $\kappa_n$ can be viewed as a proportionality constant for adverse yaw. The minimum increase in induced drag from ailerons is found with a taper ratio of 1.0, while the least adverse yaw can be found with a taper ratio of 0. Regardless of the wing planform, if zero twist is applied to the wing geometry, a wing with optimally-placed ailerons to minimize induced drag will not produce proverse yaw.

Optimal aileron root results can be used in initial design work for aileron geometries to minimize induced drag. Although optimal aileron solutions for induced drag in all cases produce adverse yaw, theory suggests that wing symmetric twist (washout) can be used such that aileron deflection would produce proverse yaw. This is a topic of future research.
REFERENCES


