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Social Justice Mathematical Modeling for Teacher Preparation

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SOCIAL JUSTICE MATHEMATICAL MODELING FOR TEACHER PREPARATION

by

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ABSTRACT

Social Justice Mathematical Modeling for Teacher Preparation

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Contrary to popular belief, mathematics education possesses no intrinsic immunity to the sociopolitical labyrinths modern educators are required to navigate. In spite of this fact, teachers receive little training, resulting in a great many math teachers, both new and veteran, who lack the critical social consciousness necessary to provide diverse learners with affordances vital to their success in mathematics and in life. The incorporation of carefully scaffolded mathematical explorations of social justice issues into required mathematics content courses can serve as both models for effective mathematics instruction as well as tools for the inculcation of social consciousness in prospective teachers. Unfortunately, high-quality mathematics for social justice curricular materials are few in number. This paper recounts the development, test enactment, evaluation, and revision of three social justice mathematical modeling projects intended to help address the dearth of existing materials. Student work samples revealed greater awareness of institutional racism in the US criminal justice system, the inequities surrounding tracking in school mathematics, and an increase in positive affect toward mathematical modeling. The results suggest social justice mathematical modeling can be effective for influencing pre-service teacher beliefs.

(314 pages)

PUBLIC ABSTRACT

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Today's math teachers face significant social and political challenges for which they receive little preparation. Mathematics content courses can potentially provide additional preparation in this regard by providing future teachers with experiences to mathematically explore social justice issues. This provides them with opportunities to increase their awareness and sensitivity to social justice issues, develop greater empathy for their future students, and serve as examples for high quality instruction that they can emulate in their future careers. This dissertation recounts the development and revision of three social justice mathematical modeling projects, and shares evidence from student work samples of the ways in which the experience impacted students' lives. The implications of this work for teacher preparation and modeling education are discussed.

For Heather, Dean, and Owen.

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CHAPTER 1

INTRODUCTION

Mathematics teachers and teacher educators face a myriad of challenges and obstacles—many of which are likely to exist in perpetuity. From the difficulties inherent in teaching diverse student populations, to the challenge of accurately assessing student understanding of concepts and their connections with procedures, to the social and political obstacles they face from well-meaning but uninformed administrators, difficult parents, and education policy. Both tragically and inevitably, the difficulties which occupy a position of prominence in the minds of mathematics educators and researchers tend to vary little with time (Banilower et al., 2013).

Even a superficial perusal of noteworthy publications in the history of mathematics education literature leads to a simple conclusion: the problems of yesteryear remain problems today. From the beginning of the "reform" movement to the present, math education researchers have highlighted the challenges of providing students with opportunities to learn and do mathematics in the way that professionals learn and do mathematics, consistently pushing for a transition from an excessive emphasis on mathematical proficiency through repetition of procedural calculations to deep understanding grounded in conceptual knowledge. The means to this end has primarily taken the form of schools adopting, and training their mathematics teachers in the use of, curricular materials better suited to the application of constructivist pedagogies where students discover and create their own meaning in the mathematics classroom. This focus still looms large in the minds of quality mathematics teachers, teacher educators, and researchers, but the scope has since broadened to include a strong emphasis on the importance of providing all students, of every race, ethnicity, gender identity, socioeconomic status, and so forth, access to quality mathematics education (see, for example, National Council of Supervisors of Mathematics and TODOS: Mathematics for All (2015)).

Equity has become a central focus in math education for good reason. To this day, students that do not fit the middle-to-upper class white male mold—with the exception of many East-Asian students due in part to a *different* form of racial discrimination—tend to be held to lower expectations, have fewer choices of mathematics courses due to tracking practices (let alone the option of advanced coursework), and are not taught mathematics relevant to their lived experience or that prepares them for post-secondary education and a career. Further exacerbating these inequities is the penchant among schools to assign inexperienced teachers to the so-called "low" tracks, a practice known colloquially as "teacher tracking" (National Council of Teachers of Mathematics, 2018). The reasons such inequities persist are varied and complex, with causes grounded in a number of issues that extend well-beyond the walls of a school, but nonetheless permeate their grounds, including: income inequality, implicit racial and gender biases, established societal roles, preconceptions of who can "do mathematics," zealously uninformed parents—to say nothing of the political policies that ebb and flow with the tide of public opinion, and yet exert strong forces that shape education in public schools (Burris and Garrity, 2008; Boaler and Dweck, 2016).

The solutions, like the problems they seek to address, must be far reaching and multi-faceted and, consequently, are extraordinarily difficult to realize.

And so the inequities persist.

But the stubbornness of these difficulties hasn't prevented a cadre of mathematics teachers and education researchers from striving to relegate them to the past.

Much of the effort put forth has had the goal of publicizing the existence of inequities in U.S. schools, such as the work of people like Jeannie Oakes, in informing the education community and the public of the negative effects of tracking in schools (for a particularly influential text, see Oakes (1985)). Others have tried constructed additional supports directly focused on students, such as the Algebra Project and the Navajo Nation Math Circles (Moses and Cobb, 2001; Gilon, 2017).

Numerous other examples exist, but in spite of the best efforts of these pioneers of equity in math education, an extraordinary amount of work remains to be done, due in

no small part to the persistence of unproductive beliefs and practices among mathematics teachers themselves (Brentnall, 2016).

In an attempt to address one component of the pre-service teacher side of the equity battle, this dissertation seeks to build upon the work of these pioneers of equity in mathematics education by making issues of equity and social justice a focal point in the math content side of pre-service mathematics teacher preparation programs.

In this chapter we discuss the necessity of cultivating *social consciousness* in teachers of mathematics, which we define as awareness of and sensitivity to the social and cultural contexts in which students learn and do mathematics coupled with the belief in their importance for equitable mathematics teaching practice, in pre-service math teachers. We argue that properly trained mathematics teachers possessing acute social consciousness need not shy away from discussing sensitive topics (e.g. issues regarding race and/or politics) in the classroom, and that such "hard topics" can serve as the basis for mathematically rich, cognitively demanding tasks that can and should be incorporated into mathematics classrooms. This discussion includes a review of some of the teacher-related obstacles to social-justice pedagogy, and motivates the introduction of the concept of "Mathematics for Social Justice," a fairly old, but unfortunately still nascent, movement and philosophy that seeks to introduce issues of social justice, fairness, and equity into the mathematics classroom. We then argue that social consciousness may be inculcated or augmented in pre-service teachers when they are afforded opportunities to engage in carefully scaffolded mathematics for social justice experiences during their undergraduate teacher preparation program. Evidence suggesting how social justice mathematical modeling projects may influence the development of pre-service teacher beliefs is discussed. It is then argued that project-based learning that incorporates mathematical modeling can serve as a vehicle for introducing social justice issues into mathematics classrooms in general, and in undergraduate pre-service mathematics teacher preparation programs in particular. This paves the way for an explanation of how this dissertation provides much needed resources for the instruction of pre-service mathematics teachers—providing quality opportunities to engage in

mathematical tasks possessing high-level cognitive demand while inculcating greater social consciousness—which can serve as a model for the development of future mathematics for social justice curricular materials. The chapter closes with a statement of the specific goals of this dissertation.

1.1 The Imperative for Social Consciousness in Mathematics Teacher Candidates

In 2014, at Thomas A. Edison Career and Technical Academy, a math teacher is alleged to have told one student "'Oh yeah you don't get it because you're a female' and another, 'I know how you Jewish people are, and it's okay because I am Jewish, I know how Jewish people run business"' (Marsh and Sheehan, 2017). On June 1st of 2016, NBC News ran a story describing a math teacher at Cranford Burns Middle School in Alabama that, with the apparent intent of amusing students, gave a quiz with references to gang-related activity, prostitution, and illicit drug use. One question was, "Dwayne pimps 3 ho's. If the price is \$85 per trick, how many tricks per day must each ho turn to support Dwayne's \$800 per day crack habit?" (Folley, 2016). On the 5th of November in 2018, the Washington Post reported that fourteen teachers at Middleton Heights Elementary School "dressed up as stereotypes of Mexicans, replete with maracas, ponchos, sombreros and fake mustaches" and "as wall segments plastered with the Make America Great Again slogan." Suffice it to say, parents, particularly those of Latin*x* students, were not amused (Rosenberg, 2018).

These examples are but a handful of dozens of disturbing occurrences of blatant bigotry by school teachers documented in recent years. They are the extreme examples that come up during conversations about ridiculous teacher behavior. The vast majority of teachers, regardless of who they teach, would likely never act with such a complete lack of situational awareness.

But these are the exceptions; these are the overt cases of wrongdoing. Anyone with a small measure of common sense recognizes such comments and practices as inappropriate for any learning environment. There are, however, other ways in which teachers—even those of the highest caliber—unwittingly disenfranchise their students of opportunity, acting in

ways that create and reinforce inequities that have a profound impact on their students' futures.

1.1.1 Tracking and Inequity in Mathematics Education

Consider the competent, caring, mathematics teacher who, due to a socially learned and deeply internalized implicit racial bias, recommends a Black student for unnecessary remediation (Venzant Chambers and Spikes, 2016). This student will likely receive lower-quality instruction from less-experienced teachers due to the common practice of "teacher tracking"—where teachers "pay their dues" in the "trenches" of remedial courses before being "rewarded" with the opportunity teach "high achieving" students) (Talbert and Ennis, 1990). They are statistically more likely to be a low socioeconomic student, who tend to be "low-tracked" more frequently (Schmidt et al., 2011). As is common for students placed in remediation, they will likely never return to the "regular" track, let alone the "honors" or "accelerated" track and, consequently, they are less likely to receive academic scholarships, further hampering, and possibly eliminating, their post-secondary education options (Burris and Garrity, 2008).

Fortunately, this more subtle kind of discrimination has been recognized by many individuals in the upper echelons of mathematics teacher and teacher educator organizations, and recent publications have gone far in seeking to publicize and call for the redress of these inequities.

The National Council of Teachers of Mathematics (NCTM) 2014 publication *Principles to Actions: Ensuring Mathematical Success for All* includes an increased emphasis on equity, particularly with regard to issues of access. They condemn the practice of tracking—the placement of students on course "tracks" based on perceived ability—and call on leaders and policymakers to completely "Eliminate the tracking of low-achieving students." They insist that "when mathematics programs offer advanced courses, they must ensure that pathways to the highest-level courses exist for all students" (National Council of Teachers of Mathematics, 2014).

In the summer of 2016, TODOS: Mathematics for All (TODOS) and the National

Council of Supervisors of Mathematics (NCSM) published a joint-position statement entitled *Mathematics Education Through a Lens of Social Justice: Acknowledgment, Actions, and Accountability* wherein they make recommendations intended to facilitate the dismantling of inequitable structures in mathematics education. They state that "Three components are needed for a just, equitable, and sustainable system of mathematics education for all children. There must be acknowledgment of the unjust system of mathematics education, its legacy in segregation and other forms of institutional systems of oppression, and the hard work needed to change it. The actions taken must be driven by commitments to re-frame, re-conceptualize, intervene, and transform mathematics education policies and practices that do not serve to promote fair and equitable mathematics teaching and learning. And there must be professional accountability to ensure these changes are made and sustained." This position statement was adopted by the NCTM later that year by unanimous vote of its Board of Directors (National Council of Supervisors of Mathematics and TODOS: Mathematics for All, 2015; Larsen, 2016).

In the 2018 version of the Mathematical Association of America's (MAA) Instructional Practices Guide, a self-proclaimed "'how-to' guide focused on mathematics instruction at the undergraduate level," the authors discuss barriers to equity in college mathematics education. They argue that "We must first identify the systemic barriers inherent in higher education in general, and mathematics education specifically, and then devise strategies for removing these barriers for our students. All students deserve access to mathematics" (Mathematical Association of America, 2018).

Such barriers may have been what the authors of the Association of Teachers of Mathematics' (AMTE) *Standards for Preparing Teachers of Mathematics* (SPTM) had in mind when writing some portions of the 2017 document. A large number of the standards include numerous references to ensuring teacher candidates finish a program in possession of the experience and knowledge requisite for instructing *all* students. To this end, they assert that the training of pre-service mathematics teachers must include both the mathematical knowledge for teaching and the pedagogical content knowledge necessary for providing students

with mathematical tasks possessing a high level of cognitive demand, all while preparing them to deftly navigate the social and political pitfalls that can arise from deficiency in social consciousness (Association of Mathematics Teacher Educators, 2017).

1.1.2 The State of Equity in Mathematics Education

But is the current state of equity in math education in US schools really as dire as these publications make it sound? Does teacher preparation really need to be improved to such a degree? The National Survey of Science and Mathematics Education (NSSME) may be able to provide some insights.

Their 2013 report paints a worrisome portrait. Of the math teachers surveyed, 68 percent of those in middle schools and 77 percent of those in high schools agreed with the statement, "Students learn mathematics best in classes with students of similar abilities." It is certainly possible that this statement was not understood by survey-takers as a reference to tracking. However, when juxtaposed with some of the other results of the survey, the import of this number becomes more clear. Only 36 percent of middle school math teachers and 31 percent of high school math teachers considered themselves well prepared to, "Plan instruction so students at different levels of achievement can increase their understanding of the ideas targeted in each activity." Considered together, these two data may point to fairly widespread support of tracking among teachers of mathematics (Banilower et al., 2013).

Additional research supports this conclusion. In a 2008 survey of math teachers, 49 percent of high school teachers agreed with the statement, "My classes/classes in my school have become so mixed in terms of students' learning ability that I/teachers can't teach them" (Loveless, 2013). Furthermore, the 2013 *Brown Center Report on American Education: How Well Are American Students Learning?* indicates that tracking has not only remained popular, but that from 1990 to 2011, no fewer than 71 percent of 8th grade math students were tracked. While the actual percentage of math students in tracking in US schools does not necessarily signify teacher support, it does serve as an additional data point for consideration (Loveless, 2013).

The NSSME has more to say about teaching than solely providing evidence of widespread

teacher approval of tracking. Among middle school math teachers surveyed, only 48 percent indicated they felt well prepared to "Encourage participation of racial or ethnic minorities in mathematics," 53 percent said they could competently "Encourage participation of students from low socioeconomic backgrounds in mathematics," and 56 percent could "Encourage participation of females in mathematics." While each of these numbers do bode well for a majority of students, they also indicate that nearly half of math teachers surveyed don't feel prepared to teach diverse student populations. Among high school math teachers the numbers were even more concerning, with percentages of 39, 40, and 51, respectively. Nearly two thirds of high school math teachers surveyed claimed they were ill-prepared to teach racial or ethnic minorities (Banilower et al., 2013). These data betray a severe lack of belief in self-efficacy among US math teachers.

The impact of teacher self-efficacy beliefs on student learning outcomes is well-documented (Murrell and Foster, 2003; Buehl and Beck, 2015). However, the link of teacher self-efficacy beliefs to teacher stereotypes of student achievement (regarding race or ethnicity, SES, or students in "low" tracks) has received less attention. Existing research points to a concerning connection when juxtaposed with the reality of "teacher tracking."

A 2016 study sought to determine whether teachers' stereotypes of students' capacity to learn had any impact on their beliefs of self-efficacy as a teacher. They concluded that "The more positively teachers see their students stereotypically, the stronger are their beliefs that they can support them both individually and together with their colleagues." Consequently, at the other end of the spectrum students in schools that practice both tracking and "teacher tracking" are only further disenfranchised (Knigge et al., 2016).

How then can we help pre- and in-service teachers feel capable and comfortable when teaching racial or ethnic minority students, female students, gender non-conforming students, LGBTQIA students, and students of low socioeconomic status? How do we inculcate strong social consciousness in pre-service math teachers to improve mathematics teacher readiness vis-à-vis teaching *all* students? How can mathematics teacher preparation programs better inculcate a deep and persistent social consciousness in teacher candidates?

1.2 Mathematics for Social Justice

In the last decade, mathematics education literature, professional organizations, conferences, and workshops have seen a significant increase in emphasis on not just the ending of inequities in mathematics education, but on encouraging the planning and enactment of school mathematics lessons that require students to apply mathematics to the betterment of society and the preservation of the environment. Commonly labeled “mathematics for social justice,” this educational philosophy and social movement seeks to cultivate in students of mathematics empathy, and—perhaps most important of all—responsibility to be not only aware of, but to actively work to address the most pressing and challenging problems in society using mathematics.

1.2.1 Defining Mathematics for Social Justice

Before delving deeper into what Mathematics for Social Justice is, it is worth taking a moment to explain what it is not. It should not be confused with the struggle for social justice *in* mathematics education, though the two overlap from time to time. For example, while eliminating tracking in school mathematics is an issue of social justice in mathematics education, providing students with the opportunity to critically analyze the effects of tracking using mathematics is an example of mathematics for social justice. While training teachers to be aware of their implicit biases when planning and enacting mathematics tasks and activities is an important consideration for ensuring the progress of social justice, requiring pre-service mathematics teachers to perform a quantitative analysis of student achievement levels by race and/or socioeconomic status, and conjecture on the causes of disparities identified, or enjoining undergraduate mathematics students to perform a meta-analysis of numerical data from a series of analyses of curricular materials to ensure the included tasks are relevant to students of all backgrounds, are examples of mathematics for social justice.

Mathematics for social justice need not even require the analysis of social justice issues directly related to mathematics education. When students are using mathematics to explore issues of social justice in the world—whether it be using numerical data to identify instances

of sexism in the workplace and make predictions regarding their resolution using salary information over time, developing predictive models of the social and economic effects of climate change, exploring data disaggregated by geographic region or socioeconomic status to study the existence and identify possible causes of widespread science illiteracy— they are engaging in mathematics for social justice.

1.2.2 Mathematics for Social Justice and Equity in Mathematics Education

This approach to educating students, where they are required to answer "hard," controversial questions can serve purposes beyond merely providing students with real-world applications of mathematics.

It is in this vein that the authors of the TODOS/NCSM document *Mathematics Education Through the Lens of Social Justice: Acknowledgment, Actions, and Accountability* write, affirming the value in presenting "mathematics as an analytical tool to understand, critique, and transform the world," and proceeding to argue that "facilitating student mathematical proficiencies that transcend textbooks and promote quantitative literacy, civic engagement, as well as individual and collective agency, is a social justice act of mathematics education." That is, engaging students in mathematics for social justice serves the ends of social justice in mathematics education (National Council of Supervisors of Mathematics and TODOS: Mathematics for All, 2015).

The NCTM also appears to hold the view that mathematics for social justice should be incorporated into the mathematics classroom. The first of their "Key Recommendations" in the 2018 publication *Catalyzing Change in High School Mathematics: Initiating Critical Conversations*, includes a call for providing students with opportunities to "determine whether or not claims made in scientific, economic, social, and political arenas are valid," arguing that "Never have the broader aims of mathematics education been more important than they are today, when mathematics underlies much of the fabric of society" (National Council of Teachers of Mathematics, 2018).

Even the MAA Instructional Practices Guide includes recommendations regarding social justice pedagogies. It highlights the importance of "attending to issues of power" in

the teaching and learning of mathematics at the undergraduate levels, inviting teachers of college mathematics to consider the question, "Is this mathematics empowering students or does it maintain the status quo?" and arguing that "Challenging existing power dynamics can be achieved by exploring the use of mathematics to critique social and political issues," which they clarify by citing an example workshop at Tufts University where attendees used geometry to analyze gerrymandering in state and federal redistricting practices (Mathematical Association of America, 2018).

1.2.3 Mathematics for Social Justice and Pre-Service Mathematics Teacher Preparation

We have seen that some of the most influential organizations currently operating within the realm of mathematics education agree that mathematics for social justice should be incorporated into mathematics classrooms.

But it is worth asking whether requiring *pre-service teachers* to engage in mathematics for social justice activities that *do not* directly address issues of equity in mathematics education actually serves to inculcate or augment social consciousness. Research suggests this may be the case.

One study involving an attempt to influence teacher-candidate beliefs found that mathematics content courses that included tasks that explicitly "challenge beliefs" can serve to positively impact pre-service beliefs (Conner et al., 2011). Katz and Stupel found that it is far easier to influence pre-service teacher beliefs than in-service teacher beliefs, suggesting the time for intervention is during their teacher preparation program (Katz and Stupel, 2016). Another study found that pre-service mathematics teachers who observed innovative teaching methods which resulted in what they considered to be positive outcomes were more likely to accept their utility and effectiveness (Lloyd, 2018).

The AMTE authors of the Standards for Preparing Teachers of Mathematics also seem to agree that pre-service teachers benefit from mathematics for social justice. In Standard P.3, regarding teacher preparation program "Characteristics to Develop Candidate Knowledge, Skills, and Dispositions," that candidates should have opportunities to "closely

examine" the "social, historical, political, and institutional contexts" that "foster and constrain student access to and advancement in mathematics," to examine "the roles that power, privilege, and oppression play in schooling" as well as "effective antiracist, and social-justice pedagogies" that "disrupt institutional bias with teaching innovation, critical reflection, and social action." Standard C.4, regarding teacher candidate "Knowledge, Skills, and Dispositions" on the "Social Contexts of Mathematics Teaching and Learning," indicates that in addition to possessing productive dispositions toward issues of access and identity, that "Well-prepared beginning teachers of mathematics" can "draw on students' mathematical, cultural, and linguistic resources/strengths," "understand the roles of power, privilege, and oppression in the history of mathematics education" and are prepared to challenge inequities in "existing educational systems." They claim that candidates given such opportunities "develop deeper understandings and ethical skills sets for advocacy work in mathematics education" (Association of Mathematics Teacher Educators, 2017).

There are likely additional reasons why mathematics teacher education organizations suggest that pre-service teachers be provided with opportunities to engage in mathematics for social justice. One of these is that pre-service teachers appear to adopt the pedagogical strategies with which they are instructed "that they themselves *preferred* to be taught, or the way they think students learn *best*" (emphasis original) (Cox, 2014). That is, pre-service math teachers who receive instruction through mathematics for social justice may be more likely to incorporate social justice pedagogies into their future classrooms. Note that this has the potential to backfire. Poor planning and/or enactment may result in teacher candidates concluding that mathematics for social justice is ineffective for helping students achieve to a high degree. Another negative outcome may arise should a teacher accidentally or intentionally inject their opinion into a mathematics for social justice lesson—thus possibly distorting student conclusions, as they will think there is a specific answer they are *supposed* to obtain. Furthermore, students with opinions that differ from the teacher's may reject mathematics for social justice as ideologically motivated and therefore inappropriate for their future classrooms. Finally, due to the sometimes delicate nature of questions being

explored, teachers that guide students to a specific conclusion risk the ire of parents and administrators, and possibly even their job security (see Gutstein (2005)).

In short, the incorporation of mathematics for social justice into pre-service mathematics teacher preparation programs may support the initiatives of math education organizations, but has the potential to have the opposite effect based on the details of enactment. For the projects in this dissertation, design elements specifically intended to prevent the latter of these outcomes are incorporated into lesson and project plans, as detailed in Chapter 2.

1.2.4 Mathematics for Social Justice Can Require High Cognitive Demand

Beyond providing indirect support to teacher organizations efforts, mathematics for social justice has the potential for additional benefits due to their essential nature. Mathematics for social justice has great potential for providing mathematics teacher educators with socially relevant, and cognitively demanding mathematical tasks with which to engage and challenge their students.

For mathematics for social justice to be impactful, students must be afforded substantial autonomy in determining how to model a problem and interpret the results of calculations they perform. They require opportunities to evaluate their own work, to judge the quality and accuracy of any models they produce, and to revise and improve their model when they identify shortcomings. Students cannot be “walked” through this kind of task, as removing the challenges of precisely identifying the mathematics problem to solve, of making assumptions and identifying variables to produce a tractable problem, and of analyzing and assessing their solution, devolves the task from that of mathematically rich and cognitively demanding to one of low cognitive demand centered on procedural knowledge.

We can see that mathematics for social justice has the potential to incorporate mathematical tasks with high cognitive demand, but it is worth discussing whether requiring these kinds of inquiry-based, constructivist mathematics tasks of pre-service teachers will serve to improve math teacher preparation.

The authors of *Designing Professional Development for Teachers of Science and Mathematics* put it somewhat tragically when they wrote, “it is surprising how often the principle of constructivism is conveyed to teachers in the context of how they should help their students learn, without its being the basis for how they learn.” In other words, how can we expect in-service teachers who receive instruction during their own content courses that does not utilize inquiry-based teaching strategies? They continue, “Experiencing learning in ways that hold to constructivist principles is the only way for teachers to understand deeply why it is important for their students to learn in this way” (Loucks-Horsley, 2005).

Additional research points in this direction. Lloyd identified the same factor as necessary for the development of pre-service mathematics teacher beliefs regarding effective teaching strategies and student learning. She concluded, “In essence, to maximize retention and transfer, teacher educators must teach in the same manner that they expect pre-service teachers to teach their future students” (Lloyd, 2013).

1.2.5 Mathematics for Social Justice May Influence the Development of Teachers’ Beliefs

Studies where researchers specifically attempted to impact teacher beliefs are far fewer than those attempting to understand their impact, but what research has been done provides us with a reasonable starting place for identifying effective means to the inculcation of social consciousness.

For example, explicitly challenging the beliefs of pre-service teachers and requiring ongoing reflection seem to be integral components of effective attempts to inculcate new beliefs and alter teaching practice (Caudle and Moran, 2012; Conner et al., 2011; Turner et al., 2011). Furthermore, teacher candidates who receive instruction in ways they are expected to teach, who are witness to the successful implementation of innovative teaching practices, are more likely to use innovative teaching practices, and possess a more strongly-held belief in their own capacity to teach (Jao, 2017; Katz and Stupel, 2016; Lloyd, 2013, 2018; Loucks-Horsley, 2005).

While successful, these attempts have been relatively disparate, making comparison a

challenge and severely complicating—but also highlighting—the need to address the problem of developing an up-to-date theory of teacher belief inculcation. Only further complicating the situation is that there appears to be no common definition of "belief" across existing studies of teacher belief.

Recent efforts in cognitive neuropsychiatry provide a promising roadmap to a framework of belief that might be co-opted and leveraged by teacher educators.

Belief is only beginning to receive the consideration it is due among cognitive neuropsychologists (Bell et al., 2006). This being the case, it should come as no surprise that a complete cognitive account of mathematics teacher belief formation has yet to be suggested in the literature, though pieces, such as the impact of emotional response to stimuli, have been suggested (see Ashton and Gregoire-Gill (2003)). However, attempts by researchers to influence teacher beliefs, taken together with what information cognitive neuropsychiatry offers about belief formation, can provide us with a starting point to account for and, consequently, better inculcate productive beliefs in teachers.

For this research we adopt a specific definition of belief, namely "the mental acceptance or conviction in the truth or actuality of some idea" (Connors and Halligan, 2011, p. 1). With this definition, we can understand beliefs as motivating individuals' actions and, therefore, statements of belief as indicative of the state of their social consciousness (e.g., a math teacher who believes in equity will express negative feelings or comments about tracking when they understand the social inequities associated with the practice).

Given our acceptance of their definition, and in an attempt at systematizing our efforts of social consciousness inculcation, we can responsibly utilize the five stages of belief proposed by Connors and Halligan as an underlying framework for understanding the cognitive processes in which individuals engage when encountering new beliefs (see Figure 1.1).

By their account, belief formation begins with a *precursor*, an internal or external trigger, which is immediately followed by a *search for meaning*. During this stage, the individual explains or accounts for the precursor and then situates it within their existing "web of beliefs" (Connors and Halligan, 2011, p. 7). This process results in one or more

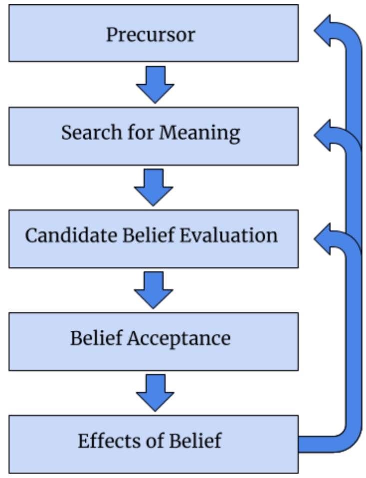


Fig. 1.1: The five stages of belief formation, with arrows indicating their interdependence. Adapted from Connors, M. H., & Halligan, P. W. (2015). A cognitive account of belief: a tentative road map. *Frontiers in psychology*, 5:1588. doi:10.3389/fpsyg.2014.01588.

"proto-beliefs" (Connors and Halligan, 2011, p. 8) intended to account for the trigger. Next comes the *candidate belief evaluation*, wherein the individual evaluates, consciously or unconsciously, these proto-beliefs in terms of their "observational adequacy" (i.e., how well it/they explain the trigger) and their "consistency with pre-existing beliefs" (Connors and Halligan, 2011, p. 8). Should a proto-belief survive this investigation, the next stage, that of *accepting and holding the belief*, is initiated, though the conviction with which the belief is held may vary significantly depending on the details of the evaluation that resulted in its acceptance. The final stage describes the *consequential effects of holding the belief*. These effects can be far reaching, as the individual will "perceive the world in a way that is consistent with the new and congruent existing beliefs" (Connors and Halligan, 2011, p. 10). In other words, just as the individual's existing beliefs prior to the precursor to the newly accepted belief influenced its formation, so too will the formation of future beliefs be influenced by the newly accepted belief (Connors and Halligan, 2011).

Well-organized mathematics for social justice experiences can take advantage of known stages of belief by providing students with repeated precursors and by serving as a source of productive, equitable candidate beliefs. In short, in addition to incorporating effective methods for influencing teacher beliefs, mathematics for social justice, properly planned

and enacted, can actually leverage the way in which beliefs are formed.

1.2.6 Mathematics for Social Justice in Action

We have seen that mathematics for social justice has strong support among professional mathematics teacher and teacher educator organizations for incorporation in mathematics courses, going so far as to argue for its allusion in standards for evaluating teacher preparation programs. We have also discussed some of the benefits to pre-service mathematics teacher preparation courses it can provide, but we have yet to see any details regarding the form mathematics for social justice lessons and projects take.

A cursory examination of the relatively small body of existing social justice mathematics lessons and projects reveals a consistent pattern. They center around a question grounded in a social justice issue, usually devoid of any mathematical context, and students are tasked to explore ways in which the problem might be explored, and in some cases solved, using mathematics (Gutstein, 2005). In cases where tasks have been planned to a greater degree, students have engaged in what could be categorized as the mathematical modeling process (Gutstein and Peterson, 2013).

Past mathematics for social justice lessons have challenged students to explore persistent racism in the housing market as an ongoing consequence of now-outlawed "redlining" practices, tasked students with evaluating government spending in the areas of defense and education, and engaged them in evaluating the cost, in both dollars and lives, of the use of sweatshops by corporate manufacturers.

With a few exceptions, past efforts in the area have been informal or spontaneous. Gutstein, a household name in the mathematics for social justice community, rarely planned and organized lessons beyond providing students with a motivating social justice question and asking students with using mathematics to find an answer, which teaching strategy appears to have served him well (Gutstein, 2005). This approach has the added benefit of allowing busy mathematics teachers, who lack preparation time but possess a desire to start teaching mathematics for social justice, with opportunities to incorporate it into their classrooms. But it does have its disadvantages.

First of these is the challenge of assessment. Without preparation, one is tasked with retroactively assigning learning objectives to the lesson, ensuring they align with any required standards, and contemplating any weights with regard to different prompts involved. The second difficulty with this kind of spontaneous task is that it can seem extraordinarily daunting for those wishing to teach mathematics for social justice, but who lack experience in the area. Without any lesson plan structure, inexperienced newcomers may struggle with incorporating the social justice tasks into their existing mathematics curriculum, with introducing the task to students, with monitoring student progress, and with orchestrating any kind of productive discussion about both the mathematics and the social justice issue.

There have been more structured approaches to mathematics for social justice, some of which that have even resulted in curricular materials available for use by teachers. The most widely known available resources for motivated teachers are books, such as *Reading and Writing the World With Mathematics: Toward a Pedagogy for Social Justice* (Gutstein, 2005), *Rethinking Mathematics: Teaching Social Justice By the Numbers* (Gutstein and Peterson, 2013), and *Teaching Mathematics for Social Justice: Conversations with Educators* (Wager and Stinson, 2012), websites like RadicalMath.org, Mathalicious.com, and the resources page at CreatingBalanceConference.org, as well as a small number of the resources at the Southern Poverty Law Center's Teaching Tolerance teacher resource website Tolerance.org. Many of these resources include only informal descriptions of successfully enacted lessons, but do not actually include materials for teacher use.

One of the most exciting efforts in this area are lessons developed for use in a mathematical modeling course intended for pre-service teachers, which is being developed by 2022 by the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools [MODULE(S2)] research action cluster of the Association of Public and Land-Grant Universities.

On top of, or perhaps as a consequence of, the dearth of existing mathematics for social justice materials, there is currently no established design framework for producing new, quality resources. That being said, the elements common to social justice mathematics

tasks provide a direction for selecting an appropriate method of delivery.

1.3 Project-Based Learning and Mathematical Modeling

Given the level of autonomy students are afforded in developing a method for addressing a social justice issue using mathematics, the requirement that students analyze and interpret abstract mathematical results in concrete terms, and the necessity of discussion of students results and their implications, the most suitable teaching strategy for incorporating mathematics for social justice into the classroom may by a combination of project-based learning and mathematical modeling.

1.3.1 Defining Project-Based Learning

An increasing number of mathematics teachers seeking to make their instructional practices more student-centric are turning to project-based learning (PBL). According to the National Middle School Association, in PBL "students engage in a common project with unclear processes but clearly identified expected outcomes." It has been found to positively correlate with greater levels of student achievement (Boaler, 1998), differentiates instruction for diverse learning needs due its student-driven nature (Bodily et al., 1998), and drives students to persist in problem-solving when the central topic is meaningful to the student on a personal level (Cross, 1996). Some research has found that project-based learning helps students retain learned material for a longer duration of time as well as understand the content more deeply than their peers who do not learn through PBL (Penuel and Means, 2000; Stepien et al., 1993), and that they perform as well, if not better, on standardized tests (Parker et al., 2013). Other research has found that students are more capable at applying their knowledge to real-life contexts due to learning through PBL (Hixson et al., 2012). Interestingly, its successful implementation has been found to hinge in large part on whether the teacher believes in the effectiveness of constructivist learning methods (Grant, 2002).

Despite the ever-growing enthusiasm for PBL among teachers, there is no generally accepted design framework for developing PBL resources, though there have been attempts

to create one, and examining some of the more recently developed PBL design frameworks will shed additional light on the nature and structure of PBL projects.

One framework, knows as the PAVE framework, argues that project-based learning should meet four criteria. It must promote discovery by tasking students with open-ended challenges providing students to “reason inductively, explore concepts and relationships, and discover connections.” Secondly, it must be an authentic experience that utilizes “real data,” requiring students to utilize the same kinds of “models and techniques used by applied mathematicians and scientists.” It must require students to analyze the “plausibility of solutions” using visualizations of both the developed model and the concrete data. Finally, it should be engaging by centering around “an accessible, original question that fits into a broader storyline” (Lewis and Powell, 2016).

Another framework, known as Gold Standard PBL, asserts that that in order to achieve the goals of “key knowledge, understanding and success skills,” PBL projects must include seven “essential design elements.” They contend that project-based learning must include a “challenging problem or question” that is presented in the form of an open-ended, “student-friendly” question. They believe that it should require “sustained inquiry,” which they clarify to mean the project that should last “more than a few days” since students should be “ask[ing] questions, find[ing] resources to help answer them, then ask[ing] deeper questions—and the process [should repeat] until a satisfactory solution or answer is developed.” In addition, PBL should incorporate “authenticity,” which they define as including one or more of the following characteristics: solving real-world problems, using authentic tools, have a real impact, or “speak[ing] to students’ own concerns, interests, cultures, identities, and issues in their lives.” They claim PBL should utilize “student voice and choice,” or autonomy, in approaching and even selecting the central problem, and that it should require “reflection,” perhaps as part of classroom discussion, but definitely in a project journal. They believe it should also allow for “critique and revision,” utilizing feedback from peers and the teacher, as well as from outside “adults and experts.” Finally, the Gold Standard PBL framework requires that students develop a “public product,” whether as a “tangible

thing" or a "presentation." Note that in the case of this last design element, they truly mean public. The developers of Gold Standard PBL believe that students are less likely to "slack off" when they are accountable to their peers and the teacher, that some "anxiety can be a healthy motivator," and that it serves to establish a "learning community" and communicate with others about "what PBL is and what it does for students" (Larmer et al., 2015).

A third PBL design framework, which is the design framework employed for the development of the mathematics for social justice project materials of this dissertation, is called HQPBL (as in High-Quality PBL). It includes six criteria "which must be minimally present in a project" to be classified as HQPBL. The first of these criteria is that the project must require "intellectual challenge and accomplishment." The framework's developers contending that "A high quality project requires students to think critically about a complex problem, question, or issues with multiple answers, and then work on the project over the course of days, weeks, and even months." Like its counterparts, this framework also includes as one of its criteria, "authenticity," arguing that students should experience "the project as 'real'." They clarify 'real' as involving the use of the "tools, techniques, and technology" found in "the world outside the school." Next they claim that a project should require a "public product" where student work is "publicly displayed, discussed, and critiqued." Next, the framework specifies that projects require "collaboration" where students work together with their peers, and "receive guidance from adult mentors and experts." Furthermore, projects should require "project management," insisting that "students learn and make use of project management processes, tools, and strategies similar to those used" by professionals. Finally, HQPBL projects involve "reflection," its developers claiming that students should be required to "reflect on, write about, and discuss the academic content, concepts, and success skills they are learning" (The Buck Institute for Education, 2018).

Project-based learning, with its real-world contexts, iterative problem-solving, and reporting of results, holds much in common with the mathematical modeling process.

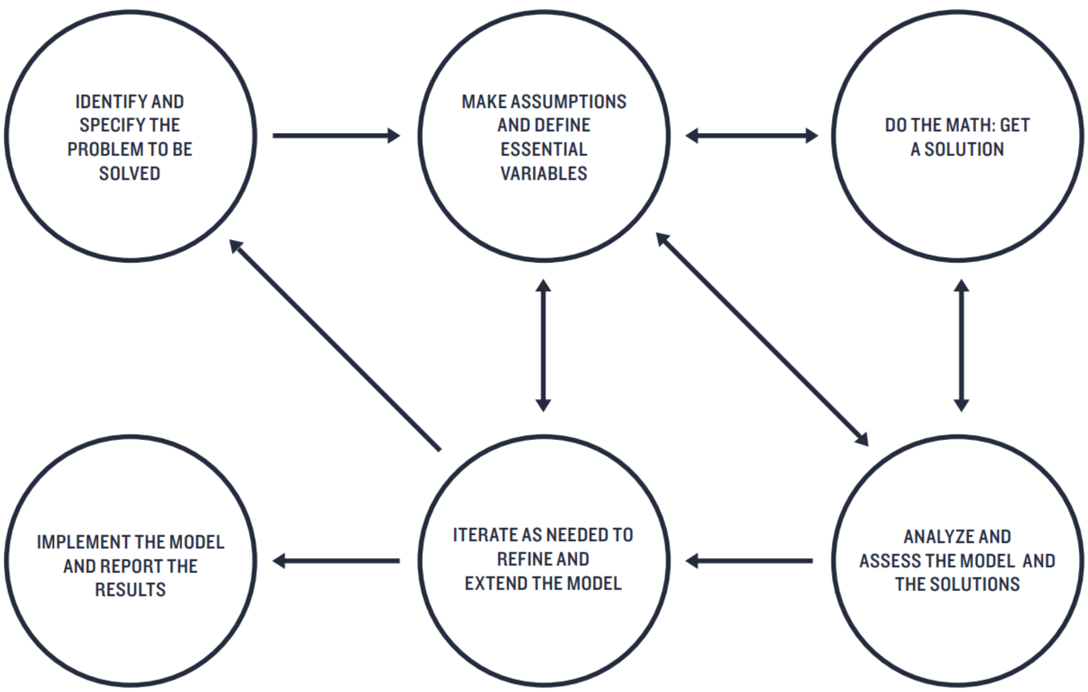
1.3.2 Defining Mathematical Modeling

There are a great many definitions for "mathematical modeling." Some of the more liberal interpretations of the phrase hold that nearly every mathematical endeavor requires some kind of abstraction with which to work, and therefore, nearly everything in mathematics is mathematical modeling. This is *not* the definition employed in this dissertation.

We understanding mathematical modeling in the manner of the Consortium for Mathematics and Its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM) as described in their joint publication *Guidelines for Assessment & Instruction in Mathematical Modeling Education* (GAIMME). They write, "Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena." To clarify what they mean, the GAIMME report serves as both a tool for distinguishing between mathematical modeling problems and other math problems and for "transforming a mathematics problem into a modeling problem." They distinguish a "math problem" from a "word problem" by the inclusion of labels, a "word problem" from an "application problem" by the inclusion of meaning, and an "application problem" from a "modeling problem" by the requirement of interpretation (Consortium for Mathematics and Its Applications and the Society for Industrial and Applied Mathematics, 2019).

To provide additional understanding of what we mean by mathematical modeling, the GAIMME report outlines the modeling process. Note that it refers to modeling as a "process" rather than a "cycle," the latter being a commonly used designation—they argue for the term "process" as there is not always a clear cycle between the different components of the process. The authors of the GAIMME report understand mathematical modeling as a process involving the following: (i) identifying and specifying the problem to be solved, (ii) making assumptions and defining essential variables, (iii) "doing the math": getting a solution, (iv) analyzing and assessing the model and the solutions, (v) iterating as needed to refine and extend the model, (vi) implementing the model and reporting the results. The process is represented in Figure 1.2.

Fig. 1.2: The Math Modeling Process. From *GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education*. Copyright ©2016 Society for Industrial and Applied Mathematics and Consortium for Mathematics and Its Applications. Reprinted with permission. All rights reserved.



Notice the existence of multiple cycles and how several, but not all, arrows are bidirectional. This is intended to better reflect the modeling process as *actually* practiced by professionals, which can bounce back and forth between different components in a number of ways. The authors of the GAIMME report explain their reasoning for the diagram's structure, writing, "We do not wish to imply that there exist an ordered number of steps that we could follow to guarantee that we have found a solution to a modeling problem. On the contrary some components happen in parallel and some are repeated as needed" (Consortium for Mathematics and Its Applications and the Society for Industrial and Applied Mathematics, 2019).

1.3.3 Social Justice Mathematical Modeling Projects

The connection between project-based learning and mathematical modeling is self-evident, and their potential as an effective vehicle for mathematics for social justice requires little explanation.

Mathematical modeling experiences, centered around authentic, socially-relevant issues, designed according to constructivist teaching principles, and expanded to require ongoing reflection are extraordinarily well-suited to mathematics for social justice. We refer to projects resulting from the combination of these mathematical content and teaching strategies as social justice mathematical modeling projects.

The culmination of all that came before is this: social justice mathematical modeling projects naturally incorporate each of the methods judged effective for influencing teacher beliefs by researchers (see Figure 1.3), and, properly planned and organized, may be able to leverage what we know about the stages of belief development to inculcate and augment social consciousness in pre-service math teachers.

1.4 The Need This Work Satisfies

We have examined evidence pointing to a need for increased social consciousness in teachers of mathematics, we have introduced mathematics for social justice and recognized a clear need for quality curricular materials for math teachers and teacher educators, we

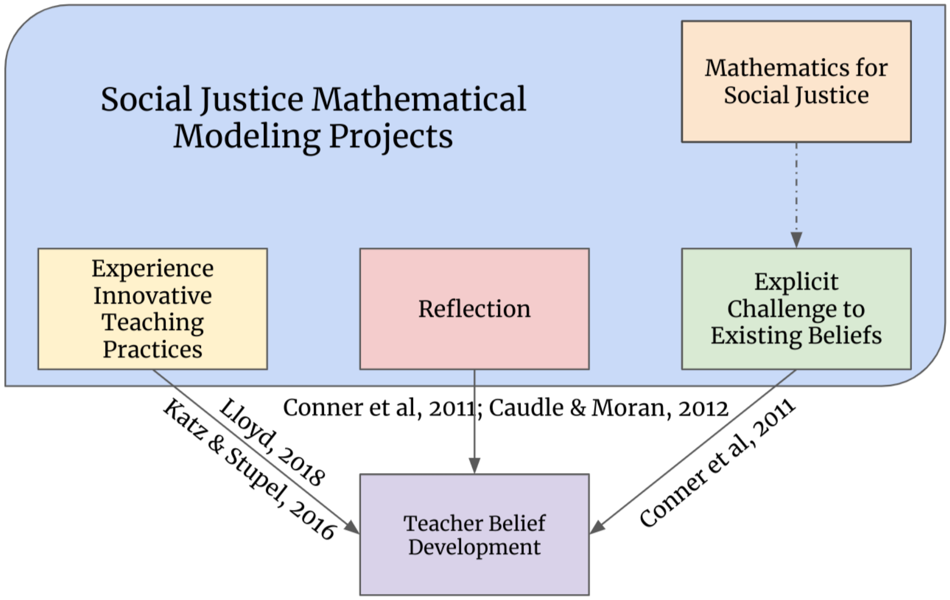


Fig. 1.3: Diagram of interactions between social justice mathematical modeling projects and research into teacher belief development. The dotted arrow indicates that mathematics for social justice can, but need not, explicitly challenge existing beliefs.

have looked at some of the recent calls for the inclusion of mathematics for social justice in mathematics classrooms—including pre-service teacher content courses—by professional mathematics teacher and teacher educator organizations, and we have explored some of the research that suggests introducing social justice content into pre-service teacher education may serve to increase social consciousness as well as prepare them to incorporate mathematics for social justice into their future classrooms.

In addition, we have argued that project-based learning that incorporates mathematical modeling is well-suited for delivering mathematics for social justice content, we have explored some PBL design frameworks and identified one for use in this dissertation, we have made clear what we mean by mathematical modeling, and we have briefly explained how social justice mathematical modeling projects combine all of these ideas into an effective whole that may positively impact the development of teacher beliefs, inculcating and augmenting social consciousness.

We have not, however, discussed the details of the intended project materials, as we will spend considerable time and space doing so in Chapter 2 when we discuss design elements.

What remains for now is a clear statement of the specific goals of this dissertation.

This dissertation intends to provide mathematics teacher educators with the much-needed curricular materials to begin incorporating mathematics for social justice into pre-service mathematics teacher preparation programs. These materials can also serve the more general purpose of providing undergraduate mathematics instructors with high-quality, mathematically rich projects that can be incorporated into existing curricula, and engage students in mathematical tasks with high-level cognitive demand.

Following initial development, each project is test-enacted in a mathematics content course at Utah State University, during which time a researcher's reflective journal is kept to document each test enactment, and student work samples are collected. Based on the results of these test-enactments, each project will undergo revision and expansion to ensure it incorporates each of the design principles outlined in Chapter 2.

Finally, the collected student work samples are monitored for evidence of an evolution of social consciousness.

1.4.1 Objectives

In summation, the objectives of this dissertation are:

1. Produce social justice mathematical modeling projects for inclusion in pre-service mathematics teacher preparation programs.
    1.1. Provide teacher preparation programs with additional supports for inculcating social consciousness in mathematics teacher candidates.
    1.2. Provide undergraduate mathematics courses with mathematics for social justice curricular materials.
    1.3. Provide undergraduate mathematics instructors with curricular materials that engage their students in mathematical tasks with high-level cognitive demand.

1.4. Provide undergraduate mathematics instructors with project-based learning curricular materials.

2. Monitor and report on the extent to which the trial implementation of mathematics for social justice projects in pre-service mathematics teacher preparation courses results in a productive evolution of social consciousness.

1.4.2 Research Questions

In order to achieve the intended objectives, the social justice mathematical modeling projects developed for this dissertation must be tested to answer the following questions:

1. What additions or modifications will better align learning activities found in the project lesson plans and student materials with the intended learning objectives?

2. What additions or modifications to the project lesson plans and student materials will help instructors smoothly and competently incorporate one of these projects into their mathematics curriculum?

3. Does student work on social justice mathematical modeling projects provide any evidence of an evolution of social consciousness?

Answers to these questions will provide vital data for improving the social justice mathematical modeling projects of this dissertation, provide insights into the impact of mathematics content courses on pre-service teacher belief development, and provide a roadmap for the development of future projects intended to maximize a productive evolution of social consciousness in pre-service mathematics teachers.

CHAPTER 2
METHODOLOGY

A clarion call for the integration of mathematics for social justice into mathematics classrooms—at all levels—by professional mathematics teacher and teacher educator organizations has sounded, and ample evidence suggests that social justice mathematical modeling projects provide a number of benefits that include improvements to pre-service mathematics teacher preparation and can serve to inculcate or augment social consciousness.

2.1 Rationale for Topic Selection and Suitability for Program Inclusion

The topics central to each proposed social justice mathematical modeling projects were not chosen at random. They represent either issues in which the target student population has or will soon have a stake, as well as more general, but widely recognized—and in one case potentially catastrophic—social justice problems in the United States and the world.

These topics were also selected in part due to the relatively small amount, and in some cases complete lack, of ambiguity in available data. In other words, in most cases there is only one position that is supported by the data—and which also happens to fall on the side of equity and social justice. However, in the era of "fake news" and "alternative facts," rather than tell students to accept statements about these controversial issues, and risk blow-back from uninformed ideologues, students located answers for themselves. Furthermore, this approach served to empower students by helping them view mathematics as a powerful tool for understanding and changing their world. With all of this in mind, we will motivate the inclusion of each of the chosen topics individually.

2.1.1 Tracking Harms Students

Tracking, "the practice of grouping students in classes on the basis of perceived ability levels" (Oakes, 1985), is widely practiced in US schools. In fact, in the US and as recently

as 2011, 61% of 4th grade students and 76% of middle school students were "tracked" in mathematics in some form (Loveless, 2013). It may come as a surprise, given its ubiquity, but tracking is frowned upon by all major mathematics teacher and teacher educator organizations.

The NCTM has come out against the practice strongly, publicizing their opposition in a number of locations. Their 2014 publication *Principles to Actions* argues that the practice is inequitable because "students in low tracks are often confronted with a narrow and fragmented mathematics curriculum," that is delivered with fewer and less effective teaching strategies than those afforded students in "high" tracks (National Council of Teachers of Mathematics, 2014).

A more recent NCTM publication reiterates these statements. They declare that tracking "is detrimental to student learning opportunities, as classes that aggregate previously unsuccessful students almost always cover less mathematical content in more time and tend to use the same didactic teaching and procedurally focused tasks that may have inhibited students' success and contributed to their disengagement in mathematics in the first place" (Boston et al., 2017).

In their newly published *Catalyzing Change*, they cite evidence that students in "high" tracks "experience mathematics instruction that cultivates their mathematical identities, conceptual understanding, and critical problem-solving and thinking skills," while those relegated to "low" tracks are engaged heavily in "rote procedures, with instruction devoting little or no attention to developing" students' positive identities as doers of mathematics. The authors of *Catalyzing Change* attribute this disparity, in part, to the practice of "teacher tracking" where more experienced teachers are assigned to teach in "upper-level mathematics courses and the least experienced teachers [are] assigned to entry-level mathematics courses" (National Council of Teachers of Mathematics, 2018).

The TODOS/NCSM, and NCTM adopted, position statement *Mathematics Education Through the Lens of Social Justice: Acknowledgment, Actions, and Accountability* calls for elimination of tracking, citing research that shows "unequal distributions of resources, course

taking opportunities, access to high cognitive demand tasks; and mathematics learning outcomes based on race, class, language, and culture" and that tracking "institutionalizes a fixed mindset about students and their capacity to learn" (National Council of Supervisors of Mathematics and TODOS: Mathematics for All, 2015).

The 2017 AMTE *Standards for Preparing Teachers of Mathematics* (SPTM) deal explicitly with issues of equity in a great number of standards. Indicator P.3.3 under Standard P.3 of the SPTM goes so far as to refer to tracking as a form of "oppression," due to the way it disproportionately harms certain student racial and ethnic demographics.

The position of these organizations is unambiguous: tracking must go. So why does it persist? That is part of the question students explored. In order that pre-service mathematics teachers might be better equipped to challenge tracking in mathematics education—advocating on behalf of their students—they were tasked with exploring tracking in school mathematics through the lens of equity.

2.1.2 Climate Change is an Existential Threat

There may be readers who do not believe that climate change is a social justice issue, so we will begin by attempting to convince them otherwise.

Letting alone for now the fact that the United Nation's Secretary General has recently referred to Climate Change as "a direct existential threat" in the newly published *Report of the Secretary-General on the 2019 Climate Action Summit and the Way Forward in 2020*, we can point to a number specific ways in which an increase in the average global temperature will disproportionately harm the impoverished, while having a significantly diminished effect on the countries and organizations responsible for the largest amount of carbon dioxide emissions (United Nations, 2019).

In point of fact, despite their relatively minuscule contribution to emissions—especially in comparison with countries like China or the United States—the countries of the African continent are predicted to experience some of the most dangerous direct effects of climate change including increases in cases of "malaria, malnutrition, diarrhea, and drownings."

Moreover, India, while contributing more to global carbon emissions than any African countries, is also expected to be disproportionately impacted by the negative health effects of climate change. Around the globe, the poorest in every country are more susceptible to food insecurity and the worsening health associated with it, inequities that are only expected to worsen as an increase in global temperatures results in a decrease in crop yields. In fact, climate change is expected to negatively impact nearly every facet of peoples' lives, even creating and exacerbating mental health problems due to the increase in global environmental disasters, and resulting in an increase of violent criminal behavior due to the increase in temperature (Melillo et al., 2014; Levy and Patz, 2015).

And for the business-minded readers, the increase in global temperature is expected to decrease worker productivity, especially in the realm of agriculture (Wuebbles et al., 2017; Hallegatte et al., 2016).

Climate change is an issue *everyone* needs to be aware of. Unfortunately, despite the fact that it has been, and will certainly continue to be, scientifically tested and verified to exist and derive in large part from human causes, climate science has been highly politicized, contributing to widespread climate science illiteracy in the United States public. The situation has worsened to the point of interested parties even challenging the inclusion of climate science in school education (Albeck-Ripka, 2018).

In order that pre-service mathematics teachers be informed about climate change, possess the mathematical background necessary for understanding and explaining climate science issues, and hold to principles of equity in mathematics and science education by advocating for climate change to be an integral part of students' education, one of the projects for this dissertation focused on the impact of global warming on the strength and amount of precipitation in tropical cyclones (e.g., Atlantic hurricanes). Students developed a vector field model of a tropical cyclone wind velocity field, explored how the Enhanced Greenhouse Effect increases ocean surface temperatures and decreases the depth of the oceanic thermocline, and discussed and reflected on issues of environmental justice related to hurricane prediction and disaster response.

2.1.3 The War on Drugs is Racist

The "War on Drugs," a coordinated effort among legislators, public policy makers, and corrections officials, initiated under US President Richard Nixon to decrease illicit drug use through harsh punishments, has not only failed, but disproportionately harms Persons of Color.

In her extraordinary work, *The New Jim Crow: Mass Incarceration in the Age of Colorblindness*, Michelle Alexander documents the myriad ways in which the United States criminal justice system disenfranchises Persons of Color of what are supposed to be inalienable rights (Alexander, 2010). While one could reasonably argue that this source represents only one political perspective and therefore may be an unrealistic representation of reality, individuals across the political spectrum have voiced similar opinions.

The Hamilton Project of the centrist Brookings Institution self-describes as an "economic policy initiative [...] guided by an Advisory Council of academics, business leaders, and former public policy makers" and which seeks to "[provide] a platform for a broad range of leading economic thinkers to inject innovative and pragmatic policy options into the national debate." They have documented that while White and Black Americans sell drugs at nearly identical rates, Black Americans are arrested, incarcerated, and receive harsher sentences than their White counterparts (Kearney et al., 2014).

In a 2017 review of the War on Drugs, Coyne and Hall of the libertarian CATO institute noted that "The unintended consequences of the War on Drugs do not affect all groups equally. In the United States, it is well documented that these policies disproportionately impact minority communities, particularly blacks and Hispanics" (Coyne and Hall, 2017, p. 11). Following an itemization of injustices resulting from the War on Drugs, they call for its end, writing, "For more than 100 years, prohibition has been the primary policy in the United States with regard to illicit substances. As the data show, however, these policies fail on practically every margin" (Coyne and Hall, 2017, p. 20).

Even the strongly conservative American Enterprise Institute has argued that "the growing cost of the Drug War is now impossible to ignore: billions of dollars wasted, blood-

shed in Latin America and on the streets of our own cities, and millions of lives destroyed by draconian punishment that doesn't end at the prison gate; one of every eight black men has been disenfranchised because of a felony conviction" (Perry, 2018).

In spite of all of this, a great number of people remain blissfully unaware of the issue, resulting in—with few exceptions, such as the 2010 Fair Sentencing Act and the 2018 First Step Act—the continuation of dated, racist policies and practices.

In order that pre-service teachers learn to thoughtfully confront and discuss instances of institutionalized racism, a problem from which schools are not immune, students explored how the "War on Drugs" was founded on principles of racism and bigotry, and how its long term effects have harmed multiple generations of Persons of Color.

Now that we have provided some insights into the thought behind the selection of social justice issues at the center of this dissertation's projects, we can start looking more directly at how these projects were organized.

Each social justice mathematical modeling project is designed in accordance with the HQPBL design framework. The framework appropriately leaves the determination of project details to the designers themselves. As such, pedagogical design principles from a number of sources were incorporated to ensure the quality of the developed project materials, of the mathematical modeling tasks they require, of the student project management and work monitoring tools they utilize, and of the assessment items and instruments they include.

Before we review the sources underpinning our design choices, we will outline the components common to all social justice mathematical modeling projects developed for this dissertation. Differences between projects do appear in the length of time in which projects are intended to be completed, in the nature of some pedagogical notes that appear—being specific to the topic of the project—in the readings required, and, in one case, in the number worksheets provided. Finally, it is important to note that the project materials described below constitute the form of project materials following their test-enactment and subsequent revision. While the initial projects were developed according to the HQPBL

design framework and the mathematics tasks development principles discussed below, they do not incorporate many details such as pedagogical notes, relevant standards, detailed sample approaches, etc.

2.2 Project Organization and Components

Following the example of mathematical modeling course materials developed for the [MODULE(S2)] RAC of the Mathematics Teacher Educator Partnership, the final form of each set of project materials was separated into three major sections: Overview, Project Details, and Resources. Each of these sections was further subdivided to allow for quick navigation to the desired content by math teachers. The intent and content of each section is discussed below.

2.2.1 Overview

Technically above the Overview section, the first page of each set of materials begins with the intended duration of the project in number and in frequency of in-class meetings dedicated to the project, as well as recommendations for students' prerequisite mathematical content knowledge.

As the name suggests, the "Overview" provides teachers with a summary of necessary information for determining whether the project could be easily and appropriately incorporated into their classroom.

The first of four subsections in the "Overview" is the "Summary." Teachers are provided with an explanation of the social justice topic students will explore, the format of the presentation and report students are expected to produce, and the issues they will reflect and write about for the duration of the project.

The second subsection provides the "Learning Objectives" the project was designed to help students in achieving. Unlike many of the general goals commonly found in lesson plan compilations and other curricular material resources, the learning objectives for these projects are specific objectives aligned to the different cognitive and affective levels at

which students engage with mathematical tasks, as defined in Cangelosi's Learning Levels (Cangelosi, 1992).

Following the learning objectives is a list of "Relevant Standards and Indicators" from the SPTM. This section is intended to provide teacher educators with the information necessary to determine whether the inclusion of a project can serve to address a specific need in their pre-service mathematics teacher preparation program.

The final subsections in the "Overview" are a "List of Materials" and "List of Supplementary Materials," the former of which includes materials that are vital to the successful enactment of the project, and the latter of which are intended to provide teachers new to mathematical modeling education or students new to mathematics modeling with a more scaffolded experience. All handouts, both vital and supplementary, are numbered from least to greatest, according to the order in which they are intended to be distributed. The numbering assumes the use of the supplementary materials.

2.2.2 Project Details

The "Project Details" section includes the bulk of materials intended to guide teachers in the management of students' project experience.

It begins with "Definitions and Notation," a section wherein any relevant vocabulary, mathematical or otherwise, are precisely defined, and conventions of notation established, to ensure the accurate transmission of the intended mathematical and social justice content to the teacher. This allows teachers of various mathematical, social, and cultural backgrounds to ensure the materials are well-suited to their classroom and course content, as well as ensure participants understand ambiguous terms the same way.

The next section is called "Preparation." It includes a short list of steps for teachers preparing to use the materials, beginning with a complete read-through of all materials and readings. It also provides recommendations regarding the use of a narrative scenario in which the project can be situated, in which students are imagined as professional researchers working for a think-tank known as the MacGuffin Institute, recommendations for

the technical tools students can use for project management, and directions on the creation of student teams prior to the introduction of the project.

The third subsection in the "Project Details" section is "Introducing the Project." This section provides teachers with instructions on introducing the social justice topic through a previously selected video clip, suggestions for introducing the mathematical modeling process through a slideshow available in supplementary materials, and directing students to begin development of their mathematical model.

The "Monitoring Student Progress" section provides teachers with recommendations for monitoring student teams' progress over the course of the project, explaining how students are likely to progress at different rates and how teachers can accommodate groups at different points in their project, including through the use of supplementary materials that can guide students through multiple revisions of their model, an in the analysis of their model. In addition, questions are provided that can be used to guide teams of students who are struggling in the development of their model. Finally, teachers are directed to monitor student progress during multiple in-class meetings and through written reflection journal entries.

The next section provides teachers with guidelines for "Orchestrating the Final Discussion." Teachers are provided with a modified version of the Smith & Stein "5 Practices for Orchestrating Productive Discussion in Mathematics," recommendations for the submission of students' technical reports prior to in-class presentations, questions that can be used during and following presentations to gauge student understanding of different aspects of the mathematical modeling process, guidelines for appropriate discourse during a final discussion of the social justice topic, and recommendations for questions that can be used as a segue into the central social justice topic as well as related issues (Smith and Stein, 2018). Teachers are also provided with pedagogical notes to help them manage the difficult conversations that may arise during the final discussion.

2.2.3 Sample Approaches and Models

This section may be viewed as the centrepiece of each set of project materials, as it

is the most expansive, mathematically descriptive, and possibly even pedagogically useful to teachers. It includes not only explanations of the approaches students are likely to take and the models they are likely to produce, but incorporates specific approaches taken by students during test-enactments into vignettes describing each step in the development of the model. It includes the incorrect approaches taken along the way, the ways in which those model weaknesses were identified, descriptions of the tools—both mathematical and technical—used in the development of a final model, and the rationale for each step taken along the way.

2.2.4 Resources

The resources section of the project materials includes any and all handouts intended for students. This includes a sample slideshow for welcoming students to the MacGuffin Institute and for introducing both the mathematical modeling process and the social justice mathematical modeling prompt. In addition, teachers can find an example of a letter to students from the Director of Policy Research at the MacGuffin Institute (i.e., the teacher overseeing the project), and which can be altered and used as a tool for communicating project-related business to students.

Projects also include reading assignments for both teachers (for the purposes of preparation) and students from actual reports, written by subject experts. Each of the projects recommended student and teacher readings, while freely available on the internet, are collected here for ease of access, and while students are encouraged to seek out their own sources of data, should teachers judge it appropriate, data sheets with information relevant to a number of modeling approaches are included in this section as well.

This section also includes handouts for the reflection journal entries, each of which include links to required readings. Associated scoring rubrics are attached. In addition, teachers will find the handouts from the Supplementary Materials section, which break the modeling process into manageable bites, recommended specification lists for technical reports and presentations, and associated scoring rubrics.

Now that we have provided a brief overview of the contents of each set of project

materials, we can discuss how learning objectives were created, and how tasks, and their corresponding rubrics, were designed to align with established learning objectives.

2.3 Designing the Mathematical Modeling Tasks

We have discussed the ways in which the project materials are organized, and we have provided a brief description of each of the components included therein. Now we will take some time to discuss the specific considerations and research underlying the creation of the mathematical modeling tasks for each project. These informed how projects incorporate tasks that measure the achievement of specific objectives, meet the criteria for an HQPBL project, and ensure that discussions of social justice issues in the mathematics classroom do not devolve into ideological debates or worse.

2.3.1 Ongoing Assessment of Student Learning

Ongoing assessment of student learning is a critical component of the mathematics classroom, but it can be extremely challenging. Thanks to the efforts of some experienced, knowledgeable, and enterprising math educators, a number of tools exist for guiding teachers in the specification of learning objectives, and the designing of mathematical tasks—with accompanying rubrics—that can be used to gain insight into student understanding of the mathematics, and therefore the degree to which students achieve the intended learning objectives.

For the purposes of this dissertation, all learning objectives were written to align with one of Cangelosi's Learning Levels (see Figure 2.1), a modification of Bloom's Taxonomy that is tailored specifically for the learning of mathematics (Cangelosi, 1992).

Table 2.1: Scheme for Categorizing Learning Levels Specified by Objectives. From Cangelosi (1992).

I. Cognitive Domain

A. Construct a Concept
Students achieve an objective at the construct-a-concept learning level by using inductive reasoning to distinguish examples of a particular concept from nonexamples of that concept.

B. Discover a Relationship
Students achieve an objective at the discover-a-relationship learning level by using inductive reasoning to discover that a particular relationship exists or why the relationship exists.

C. Simple Knowledge
Students achieve an objective at the simple-knowledge learning level by remembering a specified response (but not multiple-step process) to a specified stimulus.

D. Comprehension and Communication
Students achieve an objective at the comprehension-and-communication level by (i) extracting and interpreting meaning from an expression, (ii) using the language of mathematics, and (iii) communicating with and about mathematics.

E. Algorithmic Skill
Students achieve an objective at the algorithmic-skill level by remembering and executing a sequence of steps in a specific procedure.

F. Application
Students achieve an objective at the application level by using deductive reasoning to decide how to utilize, if at all, a particular mathematical content to solve problems.

G. Creative Thinking
Students achieve an objective at the creative-thinking learning level by using divergent reasoning to view mathematical content from unusual and novel ways.

II. Affective Domain

A. Appreciation
Students achieve an objective at the appreciation learning level by believing the mathematical content specified in the objective has value.

B. Willingness to Try
Students achieve an objective at the willingness-to-try learning level by choosing to attempt a mathematical task specified by the objective.

To this end, all mathematical tasks developed for the projects were created with the express intent of helping students achieve the listed learning objectives. The specification of learning objectives should not be misunderstood as specifying the *only* possible learning

objectives teachers may help students achieve through these projects, merely that the listed objectives guided the initial development of the projects and, therefore, lacking a specific need on the part of a teacher, constitute a list of possible, *recommended* learning objectives.

The central mathematical task(s) of each project, the challenge of producing one (or more, depending on the project) mathematical model for use in gaining insight into a social justice issue, engages students at a number of learning levels including, but not necessarily limited to, the *Comprehension and Communication*, the *Application*, and the *Creative Thinking* learning levels of the Cognitive Domain, and both the *Appreciation* and *Willingness to Try* learning levels of the Affective Domain. Beyond these, some projects include additional mathematical tasks intended to help students achieve learning objectives at other Learning Levels, such as *Algorithmic Skill* and *Discover a Relationship*.

While the Learning Levels are intended to be used in conjunction with weighted learning objectives—the student achievement of which are assessed by way of mathematical tasks which Cangelosi refers to as "miniexperiments"—the variety of approaches to assessment taken by educators requires some flexibility in this area. Furthermore, the GAIMME report recommends that assessment of student achievement of mathematical modeling learning objectives focus on students' engagement in the process, and not on the resulting model or relative "success," the latter of which is often judged by the utility of the model itself (Consortium for Mathematics and Its Applications and the Society for Industrial and Applied Mathematics, 2019). As such, while learning objectives are provided, each aligning to a specific Learning Level, specific weights are omitted. On the other hand, Learning Levels have been selected and reflection journal, technical report, and presentation scoring rubrics created carefully, which, barring the project's algorithmic skill learning objectives, would allow teachers wishing to take full advantage of Cangelosi's Learning Levels to do so without much effort.

An additional consideration in the development of mathematical tasks for these projects was with respect to ensuring the intended level of cognitive demand persists when transferred from the task as presented in curricular materials to the task as set up by the teacher,

and from the task as set up by the teacher to the task as enacted by students (Stein and Smith, 1998). To that end, initial development and test-enactment required ongoing reflection by the project facilitator on the level of cognitive demand students are experiencing.

Despite its utility in evaluating the level of cognitive demand for a given mathematical task before and during project test-enactment, Smith and Stein's taxonomy was not relied heavily upon in the creation of mathematical tasks as Cangelosi's Learning Levels provide, among other things, a refinement of the levels of cognitive demand inherent in different tasks which subsumes the taxonomy of Smith and Stein (see Cangelosi (2003)).

2.3.2 Promoting Authenticity

A number of the criteria from the HQPBL framework were easy to meet. For example, either a project requires students to produce a public product or it does not, or students are tasked to work in and self-manage groups or they are not. Other criteria are more nebulous, such as that of "intellectual challenge and accomplishment." Fortunately we have helpful tools like Cangelosi's Learning Levels for specifying and designing tasks to engage students in different kinds of "intellectual" challenges, and for assessing student "accomplishment" of learning goals associated to those challenges. Meeting the criterion of "authenticity" required some additional effort on the part of the project designer and facilitator.

"Authentic" mathematical tasks are defined for the purposes of this dissertation as those that require students to solve real problems using real tools, and in the manner employed by industry professionals. In the case of the social justice mathematical modeling projects of this dissertation, several characteristics, intended to maximize the authenticity of student experiences, were incorporated. While some aspects of projects for promoting authenticity have been discussed briefly, such as a narrative scenario, additional details are provided here.

The first project aspect for promoting authenticity in the students' learning experience is context. To provide an authentic context, students were embedded in a real-world scenario and tasked with producing an answer to a real question. In addition, when student group members discussed project-related questions with the instructor, the latter takes on the

role of project supervisor. Finally, the incorporation of project management utilities, like Trello in the case of this project enactment, and the requirement that students produce a technical report, provided additional augmentations to contextual authenticity.

A second aspect of these projects that was intended to promote an authentic experience was in the mathematical tasks and tools students were required to use. The requirement that students engage in the mathematical modeling process served not only to provide them with a mathematical task possessing a high level of cognitive demand, but in participating in the same kind of mathematical tasks as industry professionals. Furthermore, students made use of real-world tools, like curve-fitting software, to provide visualizations or to allow for the use of calculus techniques for the analysis of the data they use.

The third project aspect for promoting authenticity was in the impact of their work. Students answered hard questions. This kind of project, where students understand and model a challenging social justice issue, can serve to motivate student actions outside the classroom, whether it be in the form of political activism, practices that reflect an increased environmental consciousness, or something else entirely.

2.3.3 Ensuring Ongoing Student Reflection

The benefits of metacognition on student learning are widely documented. Indeed, research has found that students can deepen their understanding of course content by way of reflection (Sawyer, 2008), that learning is hastened when students understand both what they know as well and what remains to be learned (Davis, 2003), and that metacognitive tasks can prepare students for the workplace by instilling in them the capacity for assessing the strengths and weaknesses of their work (Di Stefano et al., 2014).

It should be no wonder, then, why HQPBL includes “Reflection” as a criterion. In order that students receive these benefits, each of the projects required students to write multiple reflection journal entries. Students explained their models, the thought that went into its creation, as well any other mathematical tasks in which they engaged. In addition, students reflected on the social justice topic central to the project. This served the additional benefit of encouraging *all* group members to contribute for the entirety of the project, as

unwillingness to do so would be documented by their fellow group members.

2.3.4 Planning Productive Discussions

Discussions in the mathematics classroom can serve a number of highly desirable purposes. Indeed, the eight Mathematics Teaching Practices from *Principles to Actions*, which are intended to act as a "research-informed framework" for "strengthening the teaching and learning of mathematics," include "facilitate meaningful mathematical discourse" (National Council of Teachers of Mathematics, 2014). Productive discussions can also engage teachers in "several other effective teaching practices—pose purposeful questions, elicit and use evidence of student thinking, use and connect mathematical representations, and support productive struggle." (Smith and Stein, 2018). However, discussions improperly facilitated can quickly devolve into something lacking even the semblance of the word "productive."

To provide teachers with a method for ensuring classroom discussions of mathematics be productive for everyone involved, Smith and Stein developed what are now known colloquially as the "5 Practices." In their book *5 Practices for Orchestrating Productive Mathematics Discussions*, the authors provide a framework for mathematics teachers to plan and facilitate productive classroom discussions. "The five practices are

1. ***anticipating*** likely student responses to challenging mathematical tasks and questions to ask students who produce them;

2. ***monitoring*** students' actual responses to the tasks (while students on the tasks in pairs or small groups);

3. ***selecting*** particular students to present their mathematical work during the whole-class discussion;

4. ***sequencing*** the student responses that will be displayed in a specific order; and

5. ***connecting*** different students' responses and connecting the responses to key mathematical ideas" (Smith and Stein, 2018).

For shorter mathematics for social justice lessons, nothing regarding the practices must necessarily change. However, for these extended social justice mathematical modeling projects, there are some ways in which the practices needed to be adapted.

For example, the monitoring of student work necessitated ongoing submissions of student work in some form. However, given the more "polished" way in which student groups are likely to submit their work, there are fewer opportunities here to locate student approaches to tasks that involved divergent thinking, thus significantly limiting the kind of material that could be selected for discussion. This problem was lessened, at least in part, by dedicating a number of meetings to group project work (in the courses where such an intervention was possible), thus allowing the teacher to monitor students *as* they work. Furthermore, students were expected to document the entirety of the model development process in their technical reports. In future iterations of project enactments, this will provide the teacher with an opportunity to review each group's approach, and plan the final discussion according to the "5 Practices." What follows is a modification of these practices better suited to the nature of these extended projects, and for use in planning a project's final discussion.

1. Review the **Sample Approaches** found in these project materials. (Anticipating)

2. Read through each team's technical report. (Monitoring)

3. Make note of unanticipated approaches or otherwise interesting approaches. Be sure to find at least one particular aspect of each team's model to point out and celebrate later during presentations and the final discussion, (Selecting)

4. Determine the order of presentations based on what you think will be pedagogically advantageous. (Sequencing)

5. Make use of the suggested questions and write additional questions that will help students recognize their mathematical modeling successes and identify ways in which their model can be improved. (Connecting)

Much more so than other classroom discussions involving mathematics, student discussions during social justice modeling projects have the potential to end disastrously, due to several difficulties beyond those normally present in facilitating mathematics discussions. Successful discussions require cooperation on the part of both the students and the instruction. They can require a significant degree of sensitivity to the kinds of speech considered “acceptable” for standard classroom discourse, and certainly necessitate the moderation of the discussion of sensitive topics by the mathematics teacher.

For these reasons, the final social justice discussion for these projects begins with the introduction of a small number of guidelines that we will borrow from Dr. Julia Aguirre (who, admittedly, wrote that she did not care to receive attribution), a researcher of equity in math education who has ample experience in facilitating difficult discussions. They include the following:

- Prepare to feel discomfort.
- Listen respectfully.
- Share the time.
- Be mindful of the intent *and* the impact of your words.
- Challenge ideas, not people.

2.3.5 Planning for Equity

The final consideration for the development of projects tasks was attendance to issues of equity. The development of any new teaching materials would be ineffective if it does not incorporate equitable teaching practices at every point of the development process.

Aside from the obvious equitable practices, one of the most effective methods for ensuring mathematical tasks are equitable is designing them to allow for multiple entry-points while preserving the complexity of the task. One might say, and many do, that equitable mathematical tasks possess a ”low floor” and ”high ceiling.”

Equity can be promoted to an even greater degree by designing mathematical tasks in such a way that students with different interests and mathematical backgrounds can contribute in different ways.

One of the benefits of social justice mathematical modeling projects, especially given that the projects are intended to be *group* projects, is their intrinsic incorporation of these equitable teaching practices. The very nature of this kind of mathematics instruction serves in and of itself to differentiate instruction, at least when facilitated by a mathematics teacher or teacher educator comfortable with the application this kind of novel mathematics teaching strategy.

Now that we have discussed the specific considerations and guiding principles that informed the development of these projects, we need to discuss how they were tested and evaluated, and subsequently refined.

We have discussed many of the details of the principles informing the initial development of social justice mathematical modeling projects, but have yet to explain other vital components of their development, namely, their testing, evaluation, and revision.

Each of these three developmental components played an integral part in the readying of these projects for use in a regular classroom. The difference between mathematical tasks as presented in curricular materials and tasks as enacted in the classroom can be stark (Stein and Smith, 1998). But that is not all. There are additional project components that needed to be evaluated.

Would the planned scenarios contribute to or detract from the authenticity they are intended to augment? How would students navigate the project management resources, and would their explanations be effective in communicating what ends up being the most integral components and aspects of their use? Were the technical reports and presentations, as planned, reasonable in the amount of additional work they required beyond what is normally expected of students? Did the other forms of student work collected provide any data for what project components were effective for engaging students in mathematical tasks possessing a high level of cognitive demand?

Other questions surrounded the content and organization of the final project materials. Would they provide the essential components and include the necessary instruction to allow for the easy integration into an existing course curriculum in terms of: relevance of the mathematical content, reasonableness in the expectations regarding the number of class meetings intended to focus on the projects, the ease of monitoring student progress by way of the project management software or other project management tools, the suitability of the teacher and student resources, and so forth?

To ensure that the results of this dissertation would not only be informed by research in their initial development, but ultimately resulted in projects which are engaging and inspiring to students while deepening their understanding of mathematics and contributing to a positive mathematical identity, and which are well-refined for the use of practicing teacher educators, each project was test-enacted and underwent a post-enactment evaluation and revision based on the results of the enactment.

2.4 Project Test Enactment

In an ideal scenario, each project under development would have undergone repeated test-enactments, with evaluation and revision between each test. In practice, this is not feasible given the amount of time and university resources necessary to undertake such an endeavor. Feasibility required a small number of—what we consider to be reasonable—concessions.

Each of the three projects underwent a single test-enactment and evaluation that provided enough data to inform not only the revision of the projects developed and tested, but of future projects as well. Furthermore, the amount of student work samples collected will provided vital insights into the development and/or evolution of social consciousness in students who engaged in the projects. These three projects—that involve exploring racism in the War on Drugs, the impact of tracking in school mathematics, and the social justice implications of anthropogenic climate change's impact on hurricanes—were test-enacted in the mathematics content and method courses to which they were considered most relevant.

The use of real classrooms, rather than test-enacting projects with small groups of

students outside of the normal classroom or course environment, circumvented a number of possible obstacles to accurate data. A simulated classroom environment involving student recruits might have distorted data intended to determine the alignment of mathematical tasks with their intended learning objectives, as it greatly increases the difficulty in ensuring the students engaging in the project possess no more than the level of mathematics experience assumed by the project. Other possibilities for distortion likely existed, but this one alone serves as a sufficient impetus leading us to conclude a normal classroom environment was preferable for the purposes of test-enactment.

The mathematical content knowledge assumed and required by the project exploring the War on Drugs was deemed best suited for testing in an entry-level calculus course, and was therefore tested in Logan Campus MATH 1210 Calculus 1 courses at Utah State University. The project tasking students with exploring the issue of tracking in school mathematics clearly fit well into a course for pre-service mathematics teachers, and was tested in MATH 4500 Methods of Secondary School Mathematics Teaching at USU. Both of these projects were tested during the Spring 2019 semester. Finally, the project exploring global warming and its connection to hurricanes was tested during a 7-week Summer semester in 2019, in MATH 2210 Multivariable Calculus.

Scheduling and organizational issues required substantial deviation from the intended time frames of two to four weeks for completing a project, the details of which are discussed in Chapter 3.

2.5 Project Evaluation and Revision

To ensure that the roles of mathematics teacher and project developer are properly delineated the test-enactment teacher kept a researcher's reflective journal for the entirety of enactment that was analyzed following test-enactment, together with samples of student work. To further reduce the possibility of confusion of roles—given that the test-enactment teacher would be the same individual developing, evaluating, and revising the materials—all analysis and sampling of student work was postponed until the completion of the projects, including grading, using the project evaluation instrument.

In the researcher's reflective journal reflection journal, the test-enactment teacher recorded their experience in facilitating each aspect of the project being tested, including introducing the project, monitoring student progress, and assessing student achievement of learning goals, the project management tools being used. In addition, an attempt was made to be transparent regarding possible influences on the progress and success of project test-enactments.

These evaluations were used in conjunction with the student work submissions for the purposes of revising each of the social justice mathematical modeling projects.

2.5.1 Project Evaluation Instrument

Following completion of a test-enactment, the project developer made use of an instrument, developed specifically for the purposes of evaluating each of the project material components, which focused on important characteristics of curricular materials and effective professional development, as well as additional prompts intended to help evaluate the effectiveness of the project management software (National Research Council of the National Academies, 2004; Loucks-Horsley, 2005).

The instrument developed includes multiple sections, the first of which focuses on the introduction of the project with its accompanying scenario, the setting up of accounts for use of the project management software, the initial encounter of students with their groups, and student groups' initial approach to engaging in the required tasks, and the pedagogical difficulties and successes experienced by the test-enactment teacher. Data for responding to the prompts in each of these sections was obtained from student work samples, student communication and collaboration information from the project management software, and the test-enactment teacher's reflection journal. The prompts for the first section of the evaluation instrument include:

- Describe students' reception of the narrative scenario as a means to introduce the project. Be sure to make note of any aspects of the project for which they expressed interest or enthusiasm, as well as those that elicited apathy or frustration.

- What difficulties (e.g., misunderstanding of instructions, technical difficulties) did students experience while setting up their user accounts for the project management software and during the usage tutorial?

- Describe, in detail, any other factors (e.g., project materials, student content knowledge and mindset, other environmental and social factors) you think facilitated or complicated the introduction of the project.

- What pedagogical guidance would make the introduction of the project easier to manage for the teacher?

The second section of the evaluation instrument focuses on the mathematical tasks in which student groups engaged over the course of the project. It includes the following prompts:

- What mathematical definitions or common notational conventions needed for the project that should be included in the project materials?

- How well did the students understand the prompts, and did they find the goals of the tasks unambiguous? Cite evidence to back up your claim.

- Did the tasks, as enacted by students, require the intended level(s) of cognitive demand? Cite evidence to back up your claim.

- Do the tasks, as enacted by students, require the expected prerequisite mathematical content knowledge?

- What student approaches to completing the task were unanticipated, but still mathematically rigorous?

- Are there any ways that this project's mathematical tasks can be improved that are not evident from your responses to the other prompts?

- What additional resources would have been helpful for students in engaging in the mathematical tasks found in this project without altering their learning level(s)?

The third section of this instrument evaluated different aspects of the use of the online project development software. It includes the following prompts:

- Describe, in detail, any ways in which the use of the project management software by student groups appears to benefit or hinder the groups' progress on completing project tasks?
- Do the benefits afforded by the use of project management software appear to outweigh the obstacles it poses (if evidence of any exists) to student progress on the project?
- Describe, in detail, what changes in the amount of scaffolding would result in an improved learning experience for students using the project management software with their group, and the effects you expect from such changes.

The fourth section of this instrument focuses on the student group technical reports and presentations with their accompanying classroom discussions. It includes the following prompts:

- What technical report requirements served best at helping students achieve the intended learning objectives?
- What technical report requirements were least effective at helping students achieve the intended learning objectives?
- How could the required technical report specifications be altered to better help students achieve the intended learning objectives?
- What difficulties (e.g. outbursts, arguments, tangential conversations) did you experience while facilitating discussions following student group presentations? What might be done to prevent these difficulties when this project is used in the future?
- Did the presentation format result in student group presentations that provided helpful insight into students' learning of mathematics and experiences doing mathematics during the project? What evidence can you cite in favor of your conclusion?

- What might be done to assist in making student group presentations more helpful to the teacher for the purposes of assessment, and more useful to the students for the purpose of achieving the specified learning objectives?
- Are there any presentation requirements that now seem unnecessary (e.g., overly complicated, cumbersome, irrelevant to learning objectives)?
- What questions posed to the presenting groups are not found in the project materials?
- Did any student group presentations incorporate additional information beyond what is required by the grading rubric that you think should be added to the requirements?

The final section of the evaluation instrument was intended to serve as a final, general assessment of the project materials. It includes the following prompts:

- What aspects of this project are particularly strong?
- How might the strengths of this project be generalized to allow for incorporation into new social justice mathematical modeling projects?
- What aspects of this project are in greatest need of improvement?
- How might future projects avoid the weaknesses you identified in the previous prompt?

2.5.2 Post-Evaluation Revision Projects

Revision of the projects was informed by the contents of completed evaluation instruments, which were, in turn, informed by a researcher's reflective journal of each project's test enactment as well as student work samples from each. In addition to revisions of the existing content of the project materials, sections were expanded, as planned, to incorporate the design criteria and organization principles outlined in this chapter.

2.5.3 Measuring Impact on Beliefs & Social Consciousness

At the outset of this dissertation, the secondary objective of monitoring student work for evidence of an evolution of social consciousness seemed overly ambitious. It was only

following test-enactments and during the subsequent project evaluations that the potential of social justice mathematical modeling was truly recognized. The interesting and promising results appearing in some student work samples, a number of which can be found in Chapter 4, necessitated the adoption of a, at the very least tentative, system for *evaluating* whether students were undergoing an evolution of social consciousness during the project.

Unfortunately, measuring changes in student beliefs poses a significant challenge. Given the complexity of the process involved, and the tendency among humans to put on the social mask they deem most suitable to a given situation, the use of quantitative methods relying on surveys are—at least at this early stage in the development of this theory—unsuitable, especially given the lack of a detailed understanding of the precise mechanisms involved.

For this preliminary evaluation, student work submissions were utilized as data sources for determining student affect toward the social justice topic of the mathematical modeling project. For this preliminary review of student work, three codes were utilized to characterize the entirety of a single student work submission. If the student exhibited a disposition toward the social justice topic that can be clearly characterized as pro-human rights, pro-science/mathematical modeling, or pro-equity (for the Racism in the "War on Drugs," Global Warming & Hurricanes, and "Tracking" in School Mathematics projects, respectively) in their writings, the sample is coded as positive. If the student exhibits a disposition biased against the social justice topic, the sample was coded as negative. In the case that the response either did not address the social justice topic or included both positive and negative dispositions, the sample is coded as indeterminate. Students whose submissions' coding changed over the course of the project to positive are understood as having experienced a positive evolution of social consciousness. Note that in this characterization, students who begin a project with dispositions coded as positive cannot undergo a productive evolution of social consciousness, the intent being that students already possessing productive beliefs have no need for such an evolution. Table 2.2 includes examples from student work for each of the three codes.

The excerpt from student work in the first row of Table 2.2 suggests the student not only

ID: 2210-01 **Assignment**: Student Reflection Journal 2 **Code**: +
"I think that the model is a great way to go from writing equations down on paper to applying this math in a real-life situation. I think a lot of the time we go through so many classes that we forget the purpose of the stuff we are learning and that the math we have learned has a real-world application *that can make a difference*." (Emphasis Added)

ID: 4500-34 **Assignment**: Student Reflection Journal 1 **Code**: -
"I don't agree with the organizations that claim tracking creates and reinforces social inequalities and oppresses students. *I believe that tracking schools actually help students develop math skills at their own level.* If they need remedial help for mathematics then they should take remedial math classes to help them catch up to the regular level students are at. Then when they no longer need remedial classes they should be able to take regular classes. When the students' skills surpass their peers in regular classes then they should take advance honors math classes. Tracking [schools'] goals are to improve each student's mathematical skills and to teach each student at their own level." (Emphasis Added)

ID: 1210-001-57 **Assignment**: Student Reflection Journal 1 **Code**: I
"The most interesting thing I have learned is that the War on Drugs was essentially started because of the hippie movement, and that the government tried to sway public opinion by associating marijuana with hippies."

Table 2.2: Examples from student work to illustrate the coding system used for evaluating whether students underwent a productive evolution of social consciousness.

recognizes the utility of mathematics in general, but understands, and therefore possibly believes, that it can be used to affect positive change in the world. This led to their work being coded as evidence of productive, pro-science/mathematical modeling beliefs.

While not necessarily overtly inequitable on the face, the student quote in the second row of Table 2.2 demonstrates that the student's views are not in alignment with what are considered equitable teaching practices among math teacher and teacher educator organizations, leading to this work sample being coded as indicative of beliefs that are not pro-equity.

The excerpt in the third row appears to suggest that the student recognizes the War on Drugs was intended to target specific groups of people. However, there is insufficient evidence in the statement to conclude that the student believes this to be a bad thing, let alone that they recognize that the people harmed most directly are Persons of Color.

Therefore, this work submission was coded as indeterminate, since it did not provide any *clear* insight into whether the student holds pro-human rights beliefs in relation to the social justice topic.

The complete results of the review of student work samples using this system of coding is located in Chapter 4.

CHAPTER 3

PROJECT EVALUATION AND REVISION

This dissertation's primary objective is the development, test enactment, and revision of mathematics for social justice modeling projects, while the secondary objective was to monitor and evaluate student work submissions to determine whether and the degree to which any evolution of social consciousness occurred. This chapter documents the results of the former.

During the Spring semester of 2019 at Utah State University, two projects, "Tracking" in School Mathematics and Racism in the "War on Drugs," underwent test enactment in Methods of Secondary School Mathematics Teaching and Calculus 1, respectively. Two months following test-enactment completion, the third project, Global Warming & Hurricanes, was trial run in an accelerated seven-week semester of Multivariable Calculus (see Table 3.1 for a summary of each test-enactment).

Project	Semester	Notable Characteristics	Participants
Racism & the "War on Drugs"	Spring 2019	Remotely facilitated; no presentations; no final discussions	Sampling of 20 students from 166 research subjects taking MATH 1210 Calculus
Global Warming & Hurricanes	Summer 2019	7-week semester; weekly in-class time for project work	13 STEM and STEM education majors taking MATH 2210 Multivariable Calculus
"Tracking" in School Mathematics	Spring 2019	No presentations; all participants were pre-service math teachers	12 pre-service math teachers taking MATH 4500 Methods of Secondary School Mathematics Teaching

Table 3.1: Summary of characteristics of each test enactment and description of participating students. All test-enactments occurred at Utah State University.

For the duration of each test enactment, and in accordance with the methods described in Chapter 2, a researcher's reflective journal was used to document test-enactment details relevant to the project evaluation instrument, and to engage in self-critical reflection to serve as evidence for and against changes to project materials.

Important conclusions from each evaluation are provided below. Each evaluation is broken into lesson learned from the test-enactment for relevant section of the project materials. Lessons learned from individual enactments are used to inform revisions to all projects. The last evaluation is followed by a summary description of general revisions to project materials.

3.1 Racism & the "War on Drugs" Project Enactment Results

This was by far the most challenging of the three project enactments to manage. With a total number of 255 students, spread across multiple course sections and recitation courses, came a number of additional challenges in logistical and pedagogical complexity. As a result, several of the planned aspects of the project were unable to be realized, including the planned in-class introduction of the project, in-class meetings allowing students to work on the project and receive face-to-face interaction with and immediate feedback from the project facilitator, and presentations followed by a final discussion of the social justice topic. These substantive deviations from the planned project structure impacted the results in ways both negative and positive.

Given the lack of contact between the project facilitator and students, opportunities to challenge students on their model assumptions and ask guiding questions intended to reveal flawed model components were restricted to written feedback. These face-to-face interactions were intended vital instances wherein the facilitator can communicate accurate and ethical candidate beliefs to students who were experiencing cognitive dissonance and attempting to situate new information within their existing web of beliefs. Additionally, while the remote project facilitation still made use of the MacGuffin Institute materials and narrative scenario, the facilitator was unable to gauge student response beyond what was revealed in technical reports.

On the other hand, this method of remote project facilitation required a greater measure of reliance on the quality of the project material handouts than the other two enactments as in-class expressions of confusion at handouts or other project characteristics could not be immediately clarified by the facilitator. Thus, while data relevant to the scenario, presentations and the final discussion, and the impact of the project on students' social consciousness may be sparse, this enactment provided data that served to inform revision for the written components in all of the projects.

The strongest aspect of this project is certainly the subject matter. Many students had never heard of the "War on Drugs" prior to this project and expressed surprise and bewilderment upon learning about its history and the associated numerical data demonstrating that it is indeed racist. For example:

> MATH 1210-001-51 (Student Reflection Journal 1)
> Before starting this project, I didn't know that the War on Drugs was a legitimate thing. So the most interesting thing that I have learned from this project so far is the history behind it.

> MATH 1210-002-25 (Student Reflection Journal 2)
> I learned that African Americans and Hispanics are arrested more often [than] Caucasians and often have longer sentences. I found this interesting because when beginning this project, I did not think that there would be that significant of a difference in incarceration rates between races.

> MATH 1210-001-63 (Student Reflection Journal 3)
> It was both interesting and deeply saddening to find evidence supporting the idea that the War on Drugs is racist.

> MATH 1210-001-18 (Student Reflection Journal 2)
> After doing some research on the number of people arrested per race per capita on basis of drug abuse, I was actually surprised by how big the difference was between white and black people. It seems that significantly more black people

> are arrested than white people which does indicate that there might just be some racism or other factors involved.
>
> MATH 1210-002-2 (Student Reflection Journal 1)
>
> I think what I found most interesting as we have done research is the disproportionality of drug use and race, compared to incarceration and race. I was surprised to find that in all of the research it says that white people use drugs the most but are much less likely to be put in prison for that.
>
> MATH 1210-001-16 (Student Reflection Journal 3)
>
> There were a few things that I found interesting over the course of this project. I really enjoyed the research aspect. I learned a lot from the type of research we did. The results of our project were also interesting and surprising to me. I thought that the war on drugs was racist before this project, but I never realized how racist. According to our model it is very racist.
>
> MATH 1210-003-28 (Student Reflection Journal 3)
>
> I think the most important thing I've learned with this project is the seriousness and scope of the issue. I knew on a basic level that there was a discrepancy between black and white incarcerations due to drug-use, but I was unaware of the scale of that discrepancy. Seeing the graphs and data for the issue was shocking, and [it has] motivated me to be more aware of the issue.
>
> MATH 1210-002-11 (Student Reflection Journal 1)
>
> [I've] been really surprised at how long this has been going on and how little attention it has gotten. I've been shocked at some of the numbers [I've] seen that have to do with drug use and race and incarceration.

These are only a few of the many statements made by students who, prior to engaging in this project, knew little to nothing about the War on Drugs.

This is far from the only lesson learned from this project's test enactment. The sections below contain important takeaways used to improve the listed component of the project materials, and to inform the respective sections in the other projects' materials.

3.1.1 Lessons for *Learning Objectives*

It was anticipated that development of a mathematical model is evidence of students achieving a learning objective at the Application cognitive process level. This seems to have been borne out in enactment, a fact stated by the project facilitator upon completion of enactment, though there were some reservations about the content relevance to the course.

> MATH 1210 - April 31, 2019 (Researcher's Reflective Journal)
> With regard to learning objectives, I think the modeling component of the project did in fact lead students to achieve an objective at the application cognitive learning level (but the calculus tasks not being as central to the project is still problematic).

Students appear to have achieved a learning objective at the Algorithmic Skill learning level based on their completion of the calculus-based analysis of the data provided. All of the technical reports included students' results of following the steps of the algorithms for locating extreme values and inflection points. Students were encouraged to take advantage of professional computing software to perform these computational tasks (for authenticity and because the values associated with the complicated functions they obtained would have been unrealistic to compute by hand). That being said, it is possible that students took advantage of the computing software without understanding the steps involved in finding extreme values or inflection points. As an aside, an associated learning objective ought to be broken into two objectives, one that focuses on whether they performed the required computations and another on their interpretation and explanation of the results of their calculations.

Finally, the technical report appears to have been effective at helping students understand and communicate mathematics. There was one reservation expressed in the reflection

journal. Students were directed to include the names of a group member on their cover page only if that individual contributed to the group's work.

> MATH 1210 - April 31, 2019 (Researcher's Reflective Journal)
> Assuming that all of the students listed on the cover pages actually contributed, I believe I can conclude that the technical report did lead students to achieve the expected learning goal at the expected learning level (comprehension and communication).

It is possible that students who did help with the project did not participate in the writing of the report. They would have had their names included even though they never did any of the writing for the report, meaning they did not need to communicate their mathematics in the same way as those who did the writing. Perhaps students could be encouraged, or even required, to split this writing task among all of the group members to ensure every student has the opportunity to demonstrate achievement of the learning objective.

There is no indication from student work or from project facilitator reflections that the project involved any unexpected prerequisite mathematical content knowledge. However, there were concerns expressed by both students and the project facilitator regarding the relevance of much of the mathematical content to the course in which the project was enacted.

In fact, a primary concern held by the project facilitator, which was expressed at a number of junctures during this test-enactment, was about how the calculus tasks ended up being peripheral to the project. While the calculus-based analysis was envisioned as a major component of the project, in practice it appears to have taken a back seat to the remaining tasks. Upon completion of the project enactment, the project facilitator wrote:

> MATH 1210 - April 31, 2019 (Researcher's Reflective Journal)
> I feel like the calculus ended up being kind of artificially injected. While this was by design, in retrospect I think it would have been better to design tasks in such a way as to having calculus principles arise naturally during this project. I'm not entirely sure how this would be accomplished, but I have an idea relating

> to population density functions and the availability of methadone in the current opioid crisis. I wonder whether the project in its current form would be better suited to a course that covers proportions.

Many students appeared to have related concerns during a significant portion of the project. For example, near the beginning of the project, one student wrote:

> MATH 1210-001-37 (Student Reflection Journal 1)
> I'm still a little confused as to what the end-result of this project is supposed to look like and why we are even doing it. So far it seems more like a project I'd do in an ethic class rather than a math class, and I'm just having a hard time seeing how this project fits into my calculus class (I'm just being honest).

Near the end of the project, the same student appeared to hold the same concerns, though appears to have at least appreciated some aspects of the project:

> MATH 1210-001-37 (Student Reflection Journal 3)
> I would've have [*sic*] liked a bit more connection with the curriculum of my calc class. For the most part I still learned a lot, and while it wasn't calculus based in particular, it did show me a different way of tackling issues like this.

On the other hand, there were students who did appear to appreciate the calculus component of the project. One student wrote:

> MATH 1210-001-51 (Student Reflection Journal 2)
> I love the concept of teaching students the applications of math. This is a great idea for that, however, I don't feel that it was implemented well.
> Thank Patrick for giving us this glimpse into how one can use calculus in the real world!

That last comment likely assumed the course instructor would be reading the reflection journal entries, and not the project facilitator.

An alternative to completely omitting any calculus component might be to include the calculus-based activities as extra activities that can be introduced if there is sufficient time.

This kind of idea was mentioned in one of the final entries of the researcher's reflection journal:

> MATH 1210 - April 31, 2019 (Researcher's Reflective Journal)
>
> Another option would be to still include the calculus tasks in the project materials, but to label them as some kind of extra or "enrichment activity" (to use the common school textbook language) for students with a calculus background. Then again, this leads back to the question of whether this project is appropriate for a calculus course.

While this entry did point out a problem posed by including calculus material, this might be avoided by providing instructions on how to use computing software to perform all calculations. The additional "enrichment" would be understood as learning to use professional tools to analyze a model (and not necessarily to artificially inject calculus tasks into the project). The next version of this project will attempt to do exactly this.

3.1.2 Lessons for *Definitions & Notation*

No unanticipated *mathematical* definitions or notational conventions appeared during this project. However, there are relevant definitions that, in retrospect, ought to have been provided by the project facilitator. Students were tasked with researching the "War on Drugs" for themselves, as well as defining mathematically what it would mean for it to be racist.

On the surface this might not seem problematic, but appears to have resulted in some unexpected deviations from the desired direction for the project since students' definitions of the "War on Drugs" did not all agree. Just over one month into the project, the facilitator highlighted, among other things, this issue:

> MATH 1210 - March 1, 2019 (Researcher's Reflective Journal)
>
> My remaining concerns are with regard to students' knowledge of what the "War on Drugs" is. I thought it was common knowledge. Apparently that is not the case. I have had several students who think that it only refers to attempts to

> prevent drug trafficking across the southern border of the US. I think the next iteration of this project will include a given definition of the War on Drugs. I am also leaning toward providing students with a single definition of racism.

It was expected that students would all agree on the "War on Drugs" as referring to (something like) the criminal justice initiatives started under Nixon and expanded under Reagan that greatly expanded efforts to curb drug use by way of harsh minimum sentencing requirements for illicit drug use and distribution. To ensure everyone is on the same page next time, a common definition of the "War on Drugs" will be provided.

Note also, the reference in the quote above to a common definition of racism. This may seem like a less ambiguous term, but it raises some concerns that were initially unanticipated by the project developer. The definition of racism for the much of the general populace differs from that of civil rights groups. In the former, racism is seen as any form of discrimination or bigotry on the basis of race, while the latter argues that racism, given its embedding in past and current societal power structures, requires power. In the case of the latter, those who discriminate on the basis of race and hold or benefit from sociopolitical or economic power structures are racist, while those who discriminate on the basis of race but lack such power are bigoted and discriminatory, but not racist.

In anticipation of these differing views among different groups of people to which students likely belong, a single definition of racism will be provided for the next version of this project, namely, the definition used by civil rights groups. This may increase claims of "pushing an agenda" by students participating in this project. To counteract these claims, an additional change will be made involving the narrative scenario. Rather than evaluating the validity of nebulous "concerns raised by the NAACP and ACLU," students will evaluate a specific claim found in a video publicized by the Drug Policy Alliance. Students will watch this video clip during class and be provided with the definitions of racism and the "War on Drugs" mentioned above they will be expected to use to inform the development of their model.

3.1.3 Lessons for ***Preparation***

Preparation to use the project management software required considerable time and effort, as each of the boards used needed to be set up prior to launching the project. As noted by the project facilitator, while the smaller section courses were not problematic, the large section course added a substantial amount of work:

> General Reflections - January 7, 2019 (Researcher's Reflective Journal)
> Thinking of how long it took to create boards for a single course, time was not prohibitive except for in the case of the single large lecture section (1210-001) of Calculus 1, which had a total of 28 boards to create (the other calculus sections were 8 and 9 boards, respectively).

It is worth pointing out that the groups' size for this project enactment was 5-6 students. These numbers proved to be too large, with many students reporting scheduling difficulties due to having so many people in their group. When considered together with the initial challenge of setting up boards for each group, the mandated use of specific project management software is not recommended for future versions of this project.

A more reasonable alternative may be to *recommend* students take advantage of free collaboration software of their choice. The next version of this project will not require the use of specific project management software, but the pedagogical note will be included to let the project facilitator know to encourage students' use of technological tools to simplify group communication and collaboration.

Project management software may be utilized more effectively if at least one individual is appointed to act as "manager," coordinating the efforts of the group. While it would certainly add to the authenticity of the task to designate one member of the group for this role, the additional amount of effort required would, considered together with the existing amount of work associated with a standard mathematics content course, be excessive. An alternative option would be for the project facilitator to act as group project manager for each group, allocating tasks and coordinating student work.

However, this would likely be feasible only in the case of a small teaching load coupled with a smaller class size. For a standard calculus course at Utah State University, with a roster generally containing no fewer than 40 students, this would likely be prohibitive due to the amount of work involved.

A small number of students appear to have enjoyed learning to use the project management software as a tool for collaborating with their peers. One expressed their feelings as follows:

> MATH 1210-001-51 (Student Reflection Journal 1)
>
> I also am glad that this project introduced me to Trello. It is a really cool application that I have started using to help me manager [*sic*] my homework.

However, most students expressed frustration with the project management software. One wrote:

> MATH 1210-001-13 (Student Reflection Journal 1)
>
> I think it is kind of a weird website and it is kinda [*sic*] confusing but i [*sic*] eventually learned how to work with it and communicate with the other members of my group.

Most feelings were expressed in stronger terms. One student expressed frustration with all technical tools they used to collaborate with their team, recommending to future students to establish norms of communication early on:

> MATH 1210-001-92 (Student Reflection Journal 3)
>
> I would also recommend that the person meet early and often with his/her team members. Trying to work together via email/text/trello cards is extremely difficult.

Other groups abandoned the project management software, opting instead to rely on other tools to plan meetings and coordinate their work. For example, one group decided they needed to incorporate group texting to communicate with each other about the project (suggesting they were unsatisfied with the required software):

MATH 1210-001-37 (Student Reflection Journal 1)

We only had two team members who didn't show up to the meeting. To make sure that all team members will be there for the next meeting, we made a group-chat over text, and included everyone who we've contacted over Trello.

In summary, while a small number of students appear to have appreciated the use of project management software, the vast majority appear to have abandoned it at their earliest convenience.

3.1.4 Lessons for *Introducing the Project*

As mentioned above, for this project, the project facilitator was unable to introduce the project during class time, relying solely on remote communication using Canvas Announcements and Inbox. This made it substantially more difficult to gauge students' reactions to the narrative scenario. The only real information received in this regard comes from the few in-person student-facilitator interactions and any information gleaned from student technical reports and reflection journals, the latter of which revealed nothing, at least in the sample of student work analyzed. A comprehensive analysis of student work for all 93 students who consented to having their work available for use in this research project might reveal more in this regard, though the existence of any relevant student writings ought to have been noted in the reflective journal. This needs to be performed in future research using this data set.

In the case of this project, many students experienced significant frustration with setting up their user accounts for the project management software. Many students deleted the Trello board invitations instead of using them to set up their accounts and get started on the first set of homework tasks, as noted in the Researcher's Reflective Journal:

MATH 1210 - January 31, 2019 (Researcher's Reflective Journal)

The first assignment is due tomorrow, but a number of students have contacted me to notify me that they deleted the email inviting them to their respective Trello board.

> MATH 1210 - February 1, 2019 (Researcher's Reflective Journal)
> A large number of students (about half) still haven't accepted the Trello board invitation.

This confusion resulted in a substantial increase in the amount of time and effort required to facilitate the project. This was noted at the time by the project facilitator as a potential reason for excluding this software component of the project:

> MATH 1210 - January 31, 2019 (Researcher's Reflective Journal)
> I have had to spend additional time re-sending board invitations yesterday and today. This is something important to remember during evaluation of the project management software component of the project.

> MATH 1210 - February 1, 2019 (Researcher's Reflective Journal)
> I opted to send out reminder messages to all of the students who had yet to begin the assignment. It has taken about four hours to notify all of the students. The response has been good so far and, with about thirty minutes until the assignment deadline, most students have now completed the assignment. There are still likely to be a significant number who don't finish.

This challenge persisted beyond the first due date associated with this project:

> MATH 1210 - February 4, 2019 (Researcher's Reflective Journal)
> I have received more communications from students that deleted their Trello board invitations. Something is going to have to change in the next iteration of this project.

There are a number of factors that complicated the introduction of the project. As mentioned elsewhere, the lack of face-to-face interaction appears to have introduced significant, unnecessary challenges to a smooth introduction of the project. The project was introduced entirely through Canvas:

> MATH 1210 - January 24, 2019 (Researcher's Reflective Journal)
>
> The project was introduced through a message sent to all students in sections 001, 002, and 003 of MATH 1210, introducing myself as project facilitator, explaining that I would communicate with students through Canvas, explaining the reflection journal and technical report components of the project, notifying them of the need to complete the Trello registration as well as accept the invitation to their respective group's Trello board.

Had the project facilitator been able to guide students through the project management software account set up there likely would have been far fewer instances of students accidentally deleting or missing email invitations. This also would have ensured that fewer students, who either do not check their preferred email listed on Canvas, or who do not have their notification settings properly set up and do not regularly check their Canvas Inbox or their course's Canvas page for announcements.

In addition to these challenges, many students experienced considerable frustration surrounding the central project task of building a mathematical model due to concerns about what this project actually entails or what it means to engage in the mathematical modeling process. Several students alluded to these concerns in their initial reflection journal. For example,

> MATH 1210-001-57 (Student Reflection Journal 1)
>
> We are still a little confused by what we are expected to do, but that might go away as we move forward.

> MATH 1210-002-2 (Student Reflection Journal 1)
>
> If we could get a better explanation for what kind of model you want or are expecting, that would be much appreciated.

> MATH 1210-003-12 (Student Reflection Journal 1)
>
> I would just like more clarity on how we are to use math, specifically calculus, to solve such a complex ethical question like this.

This too may have been diminished had the project been introduced in person, and the project facilitator been able to field questions about the objectives of the project and relevance to their class.

All this serves as additional evidence that students would have benefited from an in-class explanation of the mathematical modeling process, including a guided modeling experience involving a simple model. This could serve to illustrate both the modeling process and the different forms mathematical models can take. This would likely have prevented much of the confusion and frustration expressed by students in their reflection journals. This view was echoed by a number of students in their reflection journals. For example,

> MATH 1210-001-51 (Student Reflection Journal 2)
>
> I feel that there was too much confusion on what we were/are supposed to do for this project. I realize that there is a lot in organizing this project. A few suggestions would be an in-class explanation of the project that would allow students to ask questions (or a video posted on canvas would suffice). Also showing us an example would have helped me see roughly what we are to do for this project.

Future iterations of this project should include an in-class explanation and illustration of the mathematical modeling process.

Should teachers wish to require the use of project management software throughout the enactment of this project, they are likely to need email addresses for students (unless they decide to leave everything up to the students, which may or may not be effective). If the course is overseen with an online learning management system (LMS)—as was the case for this test enactment, the software being Canvas—it is imperative that the facilitator inform students well before the introduction of the project to update their preferred email address as listed on their LMS, and that they do so multiple times.

Teachers hoping to incorporate this project into their classroom would benefit from—perhaps more than anything else—an extensive explanation of how to introduce students to the mathematical modeling process. An ideal scenario might augment this explanation

with additional project materials intended to help teachers orchestrate an in-class discussion centered on the development of a simple mathematical model (e.g., the GAIMME report's explanation of gas station problem). This would provide the project facilitator with a simple way to explain what modeling is, and is not, while provide students with an example of the development of a model that includes at least one subsequent revision.

The project should also include reassurances that students will inevitably feel somewhat overwhelmed (at least initially) given the amount of autonomy they are afforded. This should be understood as a normal part of the modeling process, that excessive attempts to ease students' fears by providing scaffolds or explaining potential modeling approaches would completely alter the nature of the project, dramatically decreasing the amount of cognitive demand, and that teachers should trust students' ability to come up with ideas.

Another challenge that appeared at multiple junctures during test enactment involved student groups recognizing how the lists of factors they decided are important for identifying racism necessarily implies an approach. That is, students created their lists because they saw a relationship between the central project question and the list item. In spite of this, many students either did not recognize the fact that they were on the cusp of a model or had misconceptions about how a mathematical model should look.

> MATH 1210 - March 31, 2019 (Researcher's Reflective Journal)
>
> I have been contacted by, and subsequently met with, a few groups who expressed some concerns about whether they were taking the "right" approach to completing the project. The meetings, three in total, were nearly identical. I first told them that there is no single "right" way to construct a model to answer the project question. Then each group also insisted that they had not come up with a mathematical model. I responded to this claim with a series of questions to identify what their group had discussed so far for the project. Each of the groups responded with an explanation that they wanted to compare proportions of different populations as the primary means of identifying the existence of racism in the War on Drugs. I responded that they did indeed have a model,

> and that they had just explained it to me. They appeared surprised that their explanation of comparing proportions constituted a mathematical model. They all seemed to think that a model must be a single function or equation that, when the right numbers are substituted in, spits out an answer to the question. I explained that while this may be the case in some models, it need not be so in general.

This suggests that the project materials, in anticipation of this kind of occurrence and in addition to the previously mentioned explanation of how to introduce the mathematical modeling process, ought to include a series of questions intended to help students recognize how the creation of their lists held an implicit direction to take for developing a model.

3.1.5 Lessons for *Monitoring Student Progress*

Most groups completed the task in exactly the way anticipated by the project developer, namely, a proportional reasoning argument. The only deviations came from one group that attempted to calculate a correlation coefficient between different relevant sets of data and another that attempted to compare the definite integrals for the polynomial trendlines for different populations over time (they did not provide a rationale for doing so, nor did they explain what the resulting number indicates in real-world terms).

It is also important to make a note of the approaches which were anticipated in part, but not mathematically rigorous. Among all of the remaining groups that used a proportional reasoning argument, several compared proportions in a manner that would produce an erroneous conclusion. In particular, they compared the proportion of different number of people in different racial demographics imprisoned (or, in some cases, imprisoned for drug-related crimes) to either the total US population or the total US prison population. Both of these results can, and in some cases did, lead conclusion that racism does exist, but against White people rather than the marginalized populations who actually are disproportionately targeted. For example, one group wrote the following in their technical report (though they did at least acknowledge that their inability to locate some relevant data means their

conclusions are extremely limited):

> MATH 1210-003-10,13,18,30,32 (Student Technical Report)
> The data we found shows that the War on Drugs is not racist against people of color. We can see that there were great differences in total percentages of drug related arrests between White people and men and women of color. In the past 20 years, White men and women have been arrested at a higher rate for drugs than the other races, combined... Because we were unable to find exact population breakdowns for those years, we cannot conclude if it truly is not racist based solely on one variable. We would need the population of each ethnic group before coming to a better conclusion. But from our data, our results lead us to believe that the War on Drugs is not racist against people of color.

This erroneous conclusion comes, of course, from the fact that the majority of US citizens and the majority of the US prison population are both White; a correct application of proportional reasoning would have compared proportions of imprisoned populations of a given racial or ethnic demographic to the total number of people in the US of that racial or ethnic demographic (which is what most groups actually did).

Given the goal of this project is, in part, to combat racism, this kind of result was deeply concerning to the project facilitator:

> MATH 1210 - April 31, 2019 (Researcher's Reflective Journal)
> I am very concerned about the few groups that concluded that the War on Drugs is not racist, or that it is somehow racist against white people. Students were supposed to have read the excerpts from the relevant articles by the Cato and Brookings Institutes that both concluded that the War on Drugs is, in fact, racist. My intent was for students to revise their models after learning about some of the factors studied by professional researchers. This appears not to have had the intended effect across the board. I wrote a long message yesterday that I posted as an announcement wherein I explained the problems with the approaches taken by those groups (without singling them out; I don't want to

> make anybody feel bad). I also took a moment to point out that the professionals that actually research the issue have concluded that the War on Drugs is racist. The final in-class discussion would have been helpful for addressing this kind of thing, and it definitely has to be included in future versions of this project. It also would have been helpful to be able to provide students with ongoing feedback during in-class meetings dedicated to working on the project.

This was not the sole concern related to combating racism noted by the project facilitator, though the others were, perhaps, less severe. Even among the many groups whose model did result in a conclusion that the "War on Drugs" is racist, some other issues appeared. Earlier in the project, some students employed language used to refer to a different racial population using antiquated, unacceptable terms:

> MATH 1210 - March 1, 2019 (Researcher's Reflective Journal)
> My second concerns are with regard to students employing now-antiquated language to refer to certain racial demographics (e.g., I have had to correct more than one student for referring to Asian people as "Oriental"; I also provided them with a link to an article explaining why that term is viewed as inappropriate).

Taken together, these results highlight the possibility of project enactments wherein a poorly prepared project facilitator fails to challenge isolated instances of racism in reflection journals, model assumptions and conclusions, or to highlight inconsistencies in models that produce erroneous results that lead to incorrect, racist or otherwise bigoted conclusions, would be unethical. In fact, in spite of a concerted effort to do exactly these things, the project facilitator still holds to concerns regarding the ethics of facilitating a test-enactment without the in-class elements (given the possibility, and likely actuality, of students failing to read feedback on Canvas).

It is therefore imperative that future versions of this project are introduced *face-to-face*, involve ongoing *in-person* feedback, and include the final in-class discussion wherein

poorly conceived models or results that rely on bigoted stereotypes can be challenged by the project facilitator.

3.1.6 Lessons for *Sample Approaches and Models*

While students relied almost exclusively on proportional reasoning in the development of their models, some of the approaches taken can still provide important insights into how the sample approaches provided can be improved. This is because, in addition to serving as explanations for how students might develop a useful model, the sample approaches are intended to provide teachers with students' rejected models along the way to the development of a final model.

Some of the student groups who relied on proportional reasoning compared incorrect proportions for determining whether the War on Drugs is racist. For example, one group used data from the Bureau of Justice Statistics to produce trendlines for each of four racial/ethnic categories (see Figure 3.1)

From this graphical representation, the students argued that the War on Drugs must be racist against both White and Black Americans since the number of arrests is higher for those two demographics. What this group's model fails to account for is the normalization with respect to the total population for each different demographic group. Had the students looked at the ratio of arrests for a given demographic to the total US population for that demographic, they would have seen that White Americans are arrested far less frequently than their counterparts among Persons of Color. This group is far from the only one to take this approach. In order that teachers might better anticipate and respond to this kind of approach, sample approaches will include intermediate models that are ultimately rejected after the hypothetical students recognize the error in their reasoning. This may have been a result of excessive reliance reasoning from graphs, rather than the algebraic expressions on which they rely. The fact that none of the groups sampled included formal declaration of variables or algebraic expressions in their reports may be evidence of this disconnect.

This also suggests that the **Sample Approaches** section of the project materials would benefit from an increase in mathematical formalism, as it can provide teachers with

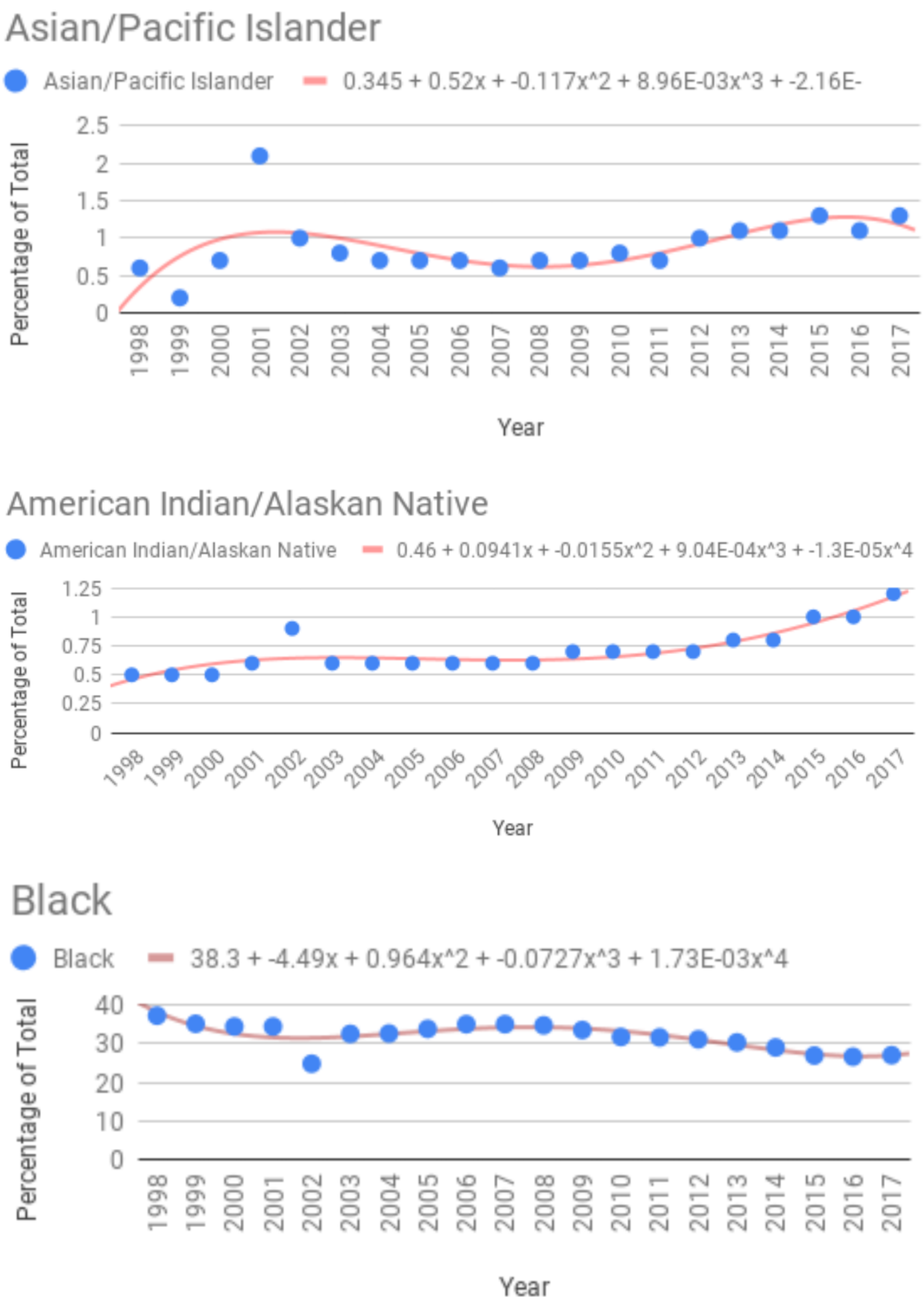


Fig. 3.1: MATH 1210-003-10,13,18,30,32 (Student Technical Report). Students used Google Sheets to produce polynomial trendlines using Drug Abuse Violations data for the years 1997 through 2017 from the FBI Criminal Justice Information Services Division.

both examples of how to formalize this kind of proportional reasoning model as well as a tool to inform feedback for students. The following simple model provides an example of what that might look like.

> Let A be the total number of Black Americans, B be the total number of White Americans, a be the total number of Black Americans arrested for drug-related crimes, and b be the total number of White Americans arrested for drug-related crimes. We are defining racism in the War on Drugs as a difference between the ratio of a to A and b to B of greater than .02. That is, the War on Drugs is racist if $\left|\frac{a}{A} - \frac{b}{B}\right| > .02$.

Note that while the number .02 was selected arbitrarily for this example, a sample approach would provide a rationale for why the hypothetical students settled on it.

3.1.7 Lessons for *Resources*

The reflection journal prompts all appear to have been understood completely. The explanations on how to use Google Sheets to create polynomial trendlines also appears to have been clear. There was no evidence of confusion in any of the student work submissions, and no students sought clarification from the project facilitator on either of these.

That being said, the project facilitator did have concerns regarding aspects of questions intended to elicit student affect toward the social justice topic of the project, specifically with regard to determining an appropriate level of "directness." This concern persisted throughout enactment, and remained during the test-enactment for the Global Warming & Hurricanes project, but the project facilitator never came to a conclusion as to whether more or less direct questioning was the most appropriate path to take.

Based on this enactment, there are two changes in the level of scaffolding that may be beneficial. The first is with regard to providing detailed instructions for the use of computing software—to allow students lacking calculus experience to participate in the model analysis—mentioned above. The second is the incorporation of the previously mentioned instruction about the mathematical modeling cycle.

There are three technical report requirements, listed below, that played vital roles in helping students achieve the affective goals of this project, namely, to recognize racism in the "War on Drugs" and to empathize with those it disenfranchises.

> *What assumptions did your team make when developing your model, and why? Be sure to include how your team is defining racism mathematically (i.e., the determination of what numbers in what places would indicate racism or non-racism is a central assumption of your model).*

> *What can you not conclude from your model, and why?*

By requiring students to acknowledge these aspects of the modeling process in their report (through which they develop analytic capacities expected of professional modelers), the project facilitator is better positioned to use the in-class final discussion to highlight and challenge instances of intentional or unintentional racism and the use of deficit or essentialist language to describe different racial and ethnic groups. For example, in the event that a group of students comes to an erroneous conclusion about the "War on Drugs," they must acknowledge the ways—or the existence of possible ways should they fail to produce a list of specifics—in which assumptions may have divorced their results from reality. Finally, it allows the project facilitator to point out different ways in which student groups understand the concept of racism, allowing for the possibility of additional correction. The third of these helps achieve these objectives in a slightly different way. It states:

> *Finally, recall that this project began when the MacGuffin Institute was contracted by the Office of National Drug Control Policy and the Federal Bureau of Prisons. What policy recommendations do you have for them regarding the War on Drugs? Should anything change? If so, what and what do you anticipate the consequence of the change would be (both good and bad)? If not, justify why you think it is working as is.*

Rather than asking students to make claims that hinge directly on their model, this prompt elicits students' opinions. This provides students with an opportunity to advance their own

ideas. In addition to providing the project facilitator with another window into a group's collective morality, this prompt allows even those students that, through poor reasoning or unusual selection of relevant factors, produce a mathematical model with unusual or unrealistic results to share their feelings. For example, the group report quoted earlier, that concluded that the "War on Drugs" is racist against white people, ended their report with a heartening call for treating drug use as a health issue rather than a criminal act (it also revealed some additional research they performed on drug treatment centers):

> MATH 1210-003-10,13,18,30,32 (Student Technical Report)
> From the data that we have gained, we came to the conclusion that the best approach to help the problem would [be] to help the current rehab centers for drug abuse. For Salt Lake City, the Public Defender's office is the main source in helping people go through the drug court system and into a rehab center such as the Odyssey House. However, overcrowded jails and [wait lists] to help people find room at a rehab center can make it difficult for drug abusers to get the support they need. With an [ever] increasing problem, the solutions would be best found in state sponsored drug and alcohol rehab centers[.]

Nowhere else in their report was there any indication of the group's empathy toward people harmed by drug abuse. Without the associated prompt, students' empathetic suggestion of treating the issue as a health concern never would have appeared.

With regard to the mathematical comprehension and communication goal associated with the technical report, four requirements stand out as particularly important:

> *All figures or tables are referred to in the main body of the text.*

> *Figure numbers and descriptions are included below each figure.*

> *What computations, tables, graphics, or other mathematical information did your team perform when using your model? Be sure to include them, with explanations of their significance to your work, in your report!*

These requirements, in particular, task students with synthesizing the meaning of the different mathematical objects into a cohesive whole.

3.2 Global Warming & Hurricanes Project Enactment Results

In a departure from the other two project enactments, the researcher acted in dual roles, as both course instructor and project facilitator. Indeed, this multiplicity of roles served as one motivation for the documenting of enactment in a self-critical researcher's reflective journal, as part of attempts to isolate the work of one from the other. A second difference in the case of this test enactment was a more complete incorporation of the project into the existing course curriculum. The project tested in Calculus 1 was essentially extracurricular, due to its remote facilitation. Similarly—but to a lesser extent, given its in-class introduction and two additional class visits by the project facilitator—students in Methods of Secondary School Mathematics engaged in project tasks primarily outside of classroom time.

This project's test enactment took place during a 7-week summer semester, included participation by students at distance sites. The course was organized to accommodate blended learning, requiring students to actively participate in regular class meetings and to complete additional mathematics tasks on the course's Canvas page. The project was interwoven with existing curriculum by locating appropriate points to introduce project-related content. For example, vector fields were introduced earlier by using the discussion of the gradient of a function as a segue, rather than at the beginning of the unit on vector calculus as is traditional. Furthermore, in the case of this project, students were, with the exception of writing the technical report, allotted classroom time to complete every required project task.

A second distinguishing characteristic of this project was the nature of the modeling task involved. For both of the other two projects, students were given complete autonomy in how to approach the modeling tasks. While almost every group ended up employing expected mathematical tools (e.g., proportional reasoning), they were nowhere instructed to employ a specific method. In the case of this project, students were informed from the

beginning that their model must necessarily take the form of a 2- or 3-dimensional vector-field. The details of how that field would be constructed was up to students, but they were not afforded the same level of independence as students in the other two courses.

Whether this topic was appropriate for a course where students already intended to enter a STEM field, and were therefore presumably more amenable to the statements of scientific authorities, was a question that was answered immediately following the submission of participants' first reflection journal entry.

As expected, many of the students who engaged in this project began with views that aligned with the scientific consensus, either because they were already aware of the evidence and likely outcomes or felt confident they could trust the scientific community to provide accurate information. For example,

> MATH 2210-2 (Student Reflection Journal 1)
>
> I think climate change is a major problem, but I think there is a lot of misconception surrounding the issue. Humans won't go extinct by 2050 and our planet won't turn into a massive fireball. But as greenhouse gas concentration increases, the planet will inevitably warm as a response. This shift in available energy will be felt on a global scale; altering precipitation patterns, wind currents, and extreme heat events, among other things. The number of climate related deaths and liabilities will grow quickly, and soon thereafter spiral out of control. For this reason, it is important to protect our climate before an irreversible shift occurs. While the public may be divided, I think it's a good idea to trust the science on this one.

> MATH 2210-11 (Student Reflection Journal 1)
>
> Climate change is obviously real and a big threat to the planet as a whole. I don't understand why or how people think that it isn't a real issue when the consequences are happening right in front of everyone's eyes. I also don't get why humans can be so arrogant as to believe that they know better than trained professionals who do climate science as their living. It's all quite dumb. I think

> that it will have a huge impact in the years to come, especially if nothing more is done to correct it. There are already guaranteed consequences at this point, such as less livable land, less farmable land, less diversity in ecosystems, and many others, so we'll just have to hope that we can keep it from getting worse. So, yeah, I'm pretty concerned about it.

One student expressed their frustration with climate-science deniers in a particularly amusing fashion:

> MATH 2210-9 (Student Reflection Journal 1)
>
> Yeah, I agree. To disagree would be to say that I know more than 97% of climate scientists. I probably can't even tie me shoes as well as 1% of climate scientists. When people deny human caused climate change it just pisses me off. I hate it when people are like, "Science? Nah, I obviously know more than science. My brain was imparted with the power of infinite wisdom upon my birth and no amount of these foolish 'facts' or 'evidence' can ever change my convictions."

Reading responses such as these may lead one to conclude that the topic is not particularly important for students taking this course. However, there were several students who, while not necessarily possessing doubts regarding the use of the scientific method as a means of ascertaining truth, or even regarding climate change itself, expressed concerns regarding the politicization of the issue.

> MATH 2210-3 (Student Reflection Journal 1)
>
> I personally believe that the earth naturally goes through cycles where it heats and cools over long periods of time, which is why there's evidence that there have been multiple ice ages. However, I also believe that anthropogenic climate change is also real and that it's making the earth heat up faster and probably more than it otherwise would, so it's possible that it is doing irreversible damage. I think it is a real and important issue[.]

In one rather striking case, a student expressed suspicions as to whether the scientific community is actually a trustworthy source of information about the climate.

> MATH 2210-4 (Student Reflection Journal 1)
>
> After doing a short amount of research the topic on climate change still seems to be more of a controversial one than a set-in stone agreed upon issue. The different effects of climate change, or so it is said, can be: environmental, economic, and health damaging. Yet, according to the consensus, most scientists agree that climate change will severely affect these three things. Yet who is to decide what is severe? How do we know that these scientists aren't just saying this to stay employed? Or that they are being payed to say this?

Upon reading these and other responses, it became apparent that the topic was appropriate for students in this course.

Another major component of this project which appears to have been particularly successful was that of tasking students with producing a vector field. Students overwhelmingly expressed serious concerns regarding their ability to engage in the mathematical modeling process early on, but changed their tune substantially by the end, concluding that they could indeed successfully engage in the mathematical modeling process to produce something they felt was completely out of their reach. This is precisely the kind of result one would hope for in a good modeling task. Individual stories of student growth in this area can be found in the relevant section in Chapter 4.

A third aspect of this project which appears to have helped to quickly extend students' sense of self-efficacy is the vector fields worksheet. While a topic of some uncertainty with regard to the appropriate level of scaffolding, the project facilitator found it to be effective, if possibly flawed. Somewhat challenging the facilitator's negative conclusions, there was at least one student that found the task particularly helpful in getting started, so much so that they felt compelled to mention exactly this in their final reflection journal entry. This student, who was among those most apprehensive about the modeling process eventually

came to develop highly positive feelings toward modeling, more so than many of their peers. They wrote the following about the vector fields tasks:

> MATH 2210-7 (Student Reflection Journal 3)
>
> I thought the vector assignment at the beginning was the perfect way to prime the pump so that our group could take off.

Of course, this is only a single student, but, when taken in conjunction with the other writings of the project facilitator about the (possible) successful aspects of the vector fields worksheet packet—which were written well before this student expressed their approval of its use—it appears substantially more compelling.

While the topic of this project is certainly one of its strengths, attempts to impact student affect through reflection journal prompts about climate change must be improved.

During the MATH 1210 project enactment, the project facilitator faced accusations from students of attempting to promote a (presumably) political agenda. Following the completion of the enactment, reservations about this were discarded, at least temporarily, given the recognition for pointed questions as an important method for getting students to engage critically with the central topic, rather than avoiding discussion of anything controversial. During the MATH 2210 project, motivated by the contents of some of students' first reflection journal submissions, the facilitator determined to expand the purview of the social justice topic to include general science literacy rather than focusing solely on controversies surrounding climate science.

> MATH 2210 - August 10, 2019 (Researcher's Reflective Journal)
>
> I have now had the chance to read the final reflection journal entries, and have taken away a few things. I don't believe I mentioned this earlier, but in response to the blow-back I received from a considerable number of students, with numerous accusations of "pushing an agenda," and given that I was unable to figure out a way to incorporate a discussion of the social justice implications of increased numbers of hurricanes due to climate change without completely derailing what was happening in the class, I tried to broaden the kind of changes

> to social consciousness from solely focusing on anthropogenic climate change to the more broad topics of science literacy and accepting scientific results while maintaining a healthy skepticism (emphasis on the healthy part).

In retrospect, this may have been a mistake. While clear outcomes related to student affect toward the mathematical modeling process, and their ability to mathematically model appeared in student work, fewer insights came into focus regarding student views on both climate change and the more general topic of science literacy.

In light of these results, future versions of this project should avoid trying to impact more general student sensibilities, as broader questioning necessarily allows for the introduction of unnecessary vagueness on the part of the participating students.

Additional improvements are necessary, primarily to the introductory Enhanced Greenhouse Effect modeling activities. Rather than directing students to complete a series of tasks, which resulted in a high degree of facilitator intervention, future versions of this project may benefit from a guided discussion of the initial derivation and development of the two models.

3.2.1 Lessons for *Learning Objectives*

The anxiety and frustration expected to arise as part of a mathematics task possessing a high degree of cognitive demand certainly appeared. As evinced by the student work shared elsewhere, nearly every student expressed some level of concern regarding their ability to complete the tasks required for this project. One student, who identified in a later reflection journal entry as a future math teacher, expressed their anxieties in a particularly colorful, if somewhat fatalist, manner:

> MATH 2210-7 (Student Reflection Journal 1)
>
> Each step in the mathematical modeling process seems overwhelming. I have a difficult time remembering all the steps because at each step I feel as though someone has put a living elephant in front of me and told me to eat it. I wouldn't know what to do. The first step is obvious but that doesn't mean I want to do

> it, and even if I do it what am I going to do with the dead elephants? I feel as though everything's just going to rot and stumble around me and I certainly don't want to present that to the world.

Of course, anxiety alone is insufficient evidence for determining cognitive load. Only when a student is afforded resources that allow them to overcome the initial anxiety can one begin to decide whether a task required a specific level of cognitive demand. Fortunately, this project appears to have been successful, at least in part, in overcoming this challenge. Upon completion of the project, this same student had the following to share:

> MATH 2210-7 (Student Reflection Journal 3)
> Mathematical modeling is an exhilarating process involving a lot of shooting blind. I found it very satisfying to create a model of something in the real world. It was incredibly daunting at first, but I feel a lot more confident in my ability to engage in the mathematical modeling process. I feel like it's less like being told to eat a live elephant in front of me, and more like being handed a bag of clay and being told to make a sculpture out of it with certain parameters. It is a lot less daunting when you view it as something that you're creating, versus something that only has one right answer. If I did this project over again, I would definitely panic a lot less. The beginning of the project was hard because I felt kind of lost. Even so I would not trade the experience, I've learned a lot. I wouldn't want to [lose] the moments of realization and excitement in discovering new things. I definitely want my students to experience the success and excitement of mathematical modeling.

In spite of this and other similar results, the project facilitator's concerns regarding the possibility of excessive scaffolding persisted to the end of the test enactment.

> MATH 2210 - August 8, 2019 (Researcher's Reflective Journal)
> While most of the models began in a similar state, they took a variety of forms in the end. That being said, they did all appear to include some combination

> of $(-x-y)\mathbf{i}+(x-y)\mathbf{j}$, likely due to the tasks that were required in the vector fields [worksheet]. I hope I can think up a way of revising the [worksheet] that will be helpful to students without necessarily suggesting a specific approach. On the other hand, leading the students—by way of a series of vector field tasks of increasing complexity—to a simplistic model, and then encouraging them to creatively alter an improve upon that model, also sounds kind of appealing.

In addition to this uncertainty regarding the vector fields worksheet, the project facilitator questioned the cognitive level at which students engage with the mathematical content. Initially supposing a mathematical modeling task must necessarily fall into the category of *application* (barring the report and presentation components of course), the completion of the project left the facilitator uncertain in this regard.

> MATH 2210 - August 8, 2019 (Researcher's Reflective Journal)
> As anticipated, the technical report and presentation components of the project led students to achieve objectives at the comprehension and communication cognitive learning level. That being said, while I had initially classified the modeling component of this project as leading students to achieve a learning objective at the application learning level, I'm not entirely sure this is true. On the one hand, they were experimenting with different functions, tweaking them, and deciding how to incorporate them into their model (usually as scaling factors for some unit vector field they came up with). On the other hand, they took this approach because I suggested it as an option, not because they, upon considering multiple approaches to obtaining a vector field possessing the desired characteristics, decided this would be a productive method. On the other *other* hand, they engaged in an exploration of multivariable functions where they were looking for specific characteristics. Would I characterize this as (to quote Cangelosi) "using deductive reasoning to decide how to utilize, if at all, a particular mathematical content to solve problems"? Perhaps. I will have to think more about this. (emphasis original)

This uncertainty persists even to the present, but it has been determined that the initial modeling tasks will be revised to act as part of a teacher-guided illustration of the modeling process, and that the vector-fields worksheet will remain unchanged. This, of course, may change again after a second test enactment.

3.2.2 Lessons for *Definitions & Notation*

While not necessarily vital to the success of the project, there are a number of definitions, related to fluid flows, from which teachers benefit. The reason these may be beneficial is they afford insights into appropriate approaches taken by students. In this project, every group attempted to incorporate the phenomena of increasing wind velocity—as distance to the eyewall decreases, viewed from the exterior—and of the hurricane eye acting as a source, into their model. Groups accomplished this in a number of ways, relying on the use of a scalar function that increases as the distance to the origin decreases.

For example, the group that produced the following model likely obtained the factor $\frac{1}{\sqrt{x^2+y^2}}$ when normalizing vector fields they then proceeded to sum (with the intent of preserving the sink- and vortex-like characteristics of the summand vector fields), and then multiplied by the factor of $\frac{1}{1+x^2+y^2}$ to ensure the vector magnitudes increase as distance to the origin decreases.

$$\mathbf{f} = \frac{1}{1+x^2+y^2}\left\langle \frac{-x-y}{\sqrt{x^2+y^2}}, \frac{x-y}{\sqrt{x^2+y^2}} \right\rangle$$

This near-ubiquitously taken approach, of using a radially symmetric scaling function of two variables to produce a desired characteristic, provides teachers with motivation for an introductory discussion of certain aspects of two-dimensional fluid flows. Specifically, they allow for a discussion of the concepts of vortex flow and uniform sink flow, both of which incorporate an assumption of vector magnitudes inversely proportional to distance to a central point and constant along concentric circles centered around that point.

Notice the striking similarity between the student-produced model above with the sum of a 2-dimensional uniform sink flow and a 2-dimensional vortex flow (with appropriately

chosen scalars and centered around the origin):

$$\frac{1}{\sqrt{x^2+y^2}}\left\langle \frac{-x-y}{\sqrt{x^2+y^2}}, \frac{x-y}{\sqrt{x^2+y^2}} \right\rangle.$$

It should be noted that these similarities are partially by design—due to the nature of the vector fields worksheet that introduced vector fields and their properties. In fact, the project facilitator expressed concerns as to whether students had been provided with excessive scaffolding, though this may have come from vocal suggestions by the facilitator rather than deficiencies in the vector fields worksheet prompts:

> MATH 2210 - July 19, 2019 (Researcher's Reflective Journal)
>
> Because of last week's task sheet—I hope that I did not provide excessive scaffolding—most groups now have a unit vector field that has a sink at the origin and flows either clockwise or counterclockwise. I think I gave away too much by suggesting students begin with a unit vector field and then scale it by some scalar function that will result in the desired distribution of vector magnitudes. Some groups almost immediately came up with something $\left(\text{like } \frac{1}{x^2+y^2} \text{ or } \frac{1}{\sqrt{x^2+y^2}}\right)$ by which they they scaled their unit vector field to make the vector magnitudes increase and approach infinity as the distance to the origin approached zero. That being said, there were a couple of groups who spent the remainder of the day trying to come up with something this characteristic that didn't also result in one or more lines through the origin where their vector field was undefined or zero—I saw a few attempts, such as $\frac{1}{x+y}$ and $\frac{1}{|xy|}$. I suppose I will have to wait until next week to determine whether the other groups, that quickly came up with a scaling factor, will still have an interesting challenge for the remainder of the project. I hope I didn't ruin the sense of exploration and discovery for them by making it too easy, and I *really* hope that I didn't alter the learning level or inordinately decrease the level of cognitive demand required by this component of the project. (emphasis original)

These concerns were somewhat diminished during the following meeting.

> MATH 2210 - July 26, 2019 (Researcher's Reflective Journal)
>
> I am still unsure as to whether I should have provided the direction I mentioned in last week's entry—I think it may be appropriate for individual groups that aren't getting anywhere after considerable time, but perhaps not something to say to the whole class. On the other hand, I am less concerned about ruining students' opportunity to discover, as the groups that came up with their scaling factors quickly have moved onto more difficult characteristics of the hurricane. They each want to incorporate an eyewall and, while they both recognize that they could just sort of declare one by restricting the domain of their vector field, they want to figure out a way that accomplishes this in a more natural way. I am excited to see what they come up with. I also pointed groups toward a 3-dimensional vector field plotting applet for GeoGebra and encouraged them to try and take their model further if they come to decide their 2-dimensional model was "good enough."

Regardless, pointing out these similarities can serve to both introduce students to the basics of the specialized language used in fluid dynamics (and more generally in continuum mechanics) as well as afford them a sense of accomplishment and achievement that comes with recognizing how their intuition aligns closely with the approach taken by professionals.

The recognition of these possibilities suggests that the project materials may benefit from the inclusion of the following mathematical and fluid dynamical definitions. It should be noted that these definitions differ somewhat from those used by industry professionals; this is to minimize the amount of necessary prerequisite knowledge for understanding the concept they define.

Vector Field: A vector field $\mathbf{F}$ in $\mathbb{R}^n$ is an assignment of an n-dimensional vector $\mathbf{F}(x_1, x_2, \ldots, x_n)$ to every point $(x_1, x_2, \ldots, x_n)$ of some subset D of $\mathbb{R}^n$. In this case, D is called the domain of $\mathbf{F}$.

Most important for this project are the following cases:

A vector field $\mathbf{F}$ in $\mathbb{R}^2$ is an assignment of a 2-dimensional vector $\mathbf{F}(x, y)$ to every point (x, y) in some subset D of $\mathbb{R}^2$.

A vector field $\mathbf{F}$ in $\mathbb{R}^3$ is an assignment of a 3-dimensional vector $\mathbf{F}(x, y, z)$ to every point (x, y, z) in some subset D of $\mathbb{R}^3$.

Flow Velocity: A flow velocity is a vector field $\mathbf{F}(x_1, x_2, \ldots, x_n)$ that describes the velocity of a fluid at each point in its domain.

Ideal Fluid: An ideal fluid is a theoretical fluid which is both incompressible (i.e., has constant density) and non-viscous (i.e., has frictionless flow).

Steady State Fluid: Let $\mathbf{F}(x_1, x_2, \ldots, x_n)$ be the flow velocity of a fluid. If the fluid, whose motion $\mathbf{F}$ describes, is in a steady state, then the vector $\mathbf{F}(x_1, x_2, \ldots, x_n)|_P$ assigned to each point $P(x_1, x_2, \ldots, x_n)$ in the domain of $\mathbf{F}$, remains constant over time.

Vortex Flow: A vortex flow in an ideal fluid is a 2-dimensional flow velocity $\mathbf{F}(x, y)$ where vectors are tangent to concentric circles centered around a singular point, where the magnitude of a vector is inversely proportional to the distance between its initial and the singular point. In rectangular coordinates, $\mathbf{F}(x, y) = \frac{Q}{\sqrt{x^2+y^2}} \langle \frac{-y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}} \rangle$, for some constant Q. Q is usually replaced with $\frac{q}{2\pi}$, but the rationale behind this scaling by a factor of $\frac{1}{2\pi}$ is not necessarily one at which students will naturally arrive over the course of their model's development.

Uniform Sink Flow: A uniform sink flow in an ideal fluid is a 2-dimensional flow velocity $\mathbf{F}(x, y)$ where vectors point directly toward a central point, called a sink. Approaching the sink, the flow area decreases, resulting in an increase in fluid velocity. The magnitude of a vector in a uniform sink flow is inversely proportional to its distance from the sink and remains constant at all points equidistant from the sink. In rectangular coordinates, $\mathbf{F}(x, y) = \frac{-K}{\sqrt{x^2+y^2}} \langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \rangle$, for some constant K (K is usually replaced with $\frac{k}{2\pi}$), but the rationale behind this scaling by a factor of $\frac{1}{2\pi}$ is not necessarily one at which students will naturally arrive over the course of their model's development.

3.2.3 Lessons for *Preparation*

For this project, students were verbally directed to setup and utilize whatever collaboration software, mobile or otherwise, they preferred. this was based on the poor results of the two previous test enactments surrounding the use of Trello as a project management and collaboration tool:

> General Reflections - April 27, 2019 (Researcher's Reflective Journal)
>
> I have come to a number of important conclusions regarding the projects based on the enactments thus far. Foremost among them, students overwhelmingly disliked the requirement of using Trello during both the MATH 4500 and MATH 1210 projects, and ceased using it is soon as they were permitted to do so. Some made use of other collaboration tools, with several teams reporting transitioning to the mobile device application, GroupMe. In the MATH 2210 project, I am removing the project activities which required students to use Trello, but will encourage them to take advantage of collaboration and project management tools (e.g., Trello, Slack, GroupMe)

While the use of Trello or Slack was recommended as helpful tools for collaboration, it appears most students opted to rely entirely on either group texting or email. It should also be pointed out that while all of the students in this course have smart mobile devices, this need not—and likely is not—the case everywhere. This could be problematic in some locations with less affluent students.

Based on some communication issues witnessed among students with group members spread across campus distance sites, easy-to-use collaboration or communication software should be encouraged, if not necessarily mandated for student use. One alternative to Trello which is significantly more widely used, and therefore may be familiar to students, is Slack. That being said, while Utah State University permits students to use Slack free of charge, its use is not free in general. USU allows students to use Slack for free, but this may not be true elsewhere, nor may other institutions have the budget to allow it. Another more

simple possibility would be mandated use of GroupMe, a free application for mobile devices (though this faces the same problem of assuming students possess a mobile smart device).

Barring any sudden epiphanies in the near future, the most effective way to improve the project materials in this regard is the inclusion of a list of—preferably free—project management collaboration software utilities that students can choose from.

3.2.4 Lessons for *Introducing the Project*

Students overwhelmingly found the framing narrative to be a humorous, engaging introduction to the group project, even among those students who expressed immediate frustration at the thought of participating in a *group* project.

Given that the project was introduced during the first week of the 7-week course, there was little time for the project facilitator to build rapport with students, perhaps suggesting that the change in attitude witnessed was due less to trust in the instructor and more due to actual interest or—at the least—entertainment. Students were aware of the general aspects of the project prior to its formal introduction, including the fact that they would be situated as employees of some organization engaging in research. In spite of this, nearly all 17 participants elicited positive nonverbal feedback to their formal letter of congratulation regarding their recent hire at the MacGuffin Institute:

> MATH 2210 - June 28, 2019 (Researcher's Reflective Journal)
> I began by distributing letters informing students of their recent "hire" at the MacGuffin Institute, which, as best I could tell based on the laughter that ensued, they found entertaining. This includes those students who had expressed vocal frustration with the idea of a group project only moments before receiving their letter. I think that the frame narrative does not necessarily add to the task's authenticity, but it does seem to have a positive effect on student engagement.

As stated above in the researcher's reflection journal entry, there was no immediate, overt indication noticed by the project facilitator that students suddenly viewed themselves as

professionals-in-training or that they took the project any more seriously, likely suggesting the scenario did not lend any additional *true* authenticity to the project experience. However, the immediate shift in attitude among those most vocally apprehensive about engaging in group work, from frustrated cynic to reluctant but amused participant, provides support for retention of the narrative frame in future versions of the project materials (something of which was unclear following the previous two project enactments).

In spite of the relative successes regarding the narrative scenario as a means to introduce the project, the introduction, while planned to a much greater degree than those for the MATH 1210 and MATH 4500 projects, experienced a number of complicating factors, primarily due to the plan of having students incorporate the Enhanced Greenhouse Effect into their future hurricane models (by way of one or more parameters). Achievement of this goal was planned to begin with a guided exploration of the Enhanced Greenhouse Effect. In retrospect, this approach may have served primarily to enhance student fears about their ability to engage in the mathematical modeling process.

Students had already expressed trepidation at the goal of a vector-field hurricane model, which they viewed as well out of reach for individuals with their level of preparation:

> MATH 2210 - June 28, 2019 (Researcher's Reflective Journal)
>
> I continued by explaining the ultimate goal of producing a vector field model of a hurricane. Several students appeared apprehensive about the idea. I hope that I can convince them it is within their capacity to do.

The first two weeks of the project involved the development of qualitative climate forcing models, first a zero-dimensional model that relies on the Stefan-Boltzmann law and a calculation of Intensity that relies on the assumption of (nearly) straight lines of solar radiation due to the distance between the Earth and Sun. Facilitation of this component of the project did not go as planned, with each group experiencing significant unproductive struggle:

> MATH 2210 - June 28, 2019 (Researcher's Reflective Journal)
>
> Following explanation of their goal, and reassuring students that I would be

> [there] to help them along by providing them with a guided modeling experience involving the Greenhouse Effect. I distributed the associated task sheet took time to visit each group to discuss their questions and concerns. I think I may have picked too challenging a task. I thought that providing students with the assumptions regarding straight-line solar radiation and the Earth emitting radiation like a physics "Black Body" would be enough to get them going. Eventually, after what I would consider to be excess intervention by the facilitator (me), students were able to complete the task sheet. I think I might review the development of this model during next week's meeting.

As suggested in this entry, the project facilitator did begin the next meeting with an overview of the development of the model. This appeared to ease students' fears somewhat:

> MATH 2210 - July 5, 2019 (Researcher's Reflective Journal)
> Today's project discussion began with review of the guided modeling task from last week. I took the time to explain my thought process for responding to each of the prompts on the task sheet. The students appeared to be much more comfortable with the derivation of the model after that.

Following this partial success, the project facilitator provided students with a diagram describing the qualitative aspects of the single-layer atmospheric model, and tasked them with explaining the interactions at each of the layers shown (the top and bottom of the atmosphere and the surface of the Earth). This appears to have been far too complicated a task, at least with the given level of scaffolding, leading the project facilitator to express serious concerns about this component of the project:

> MATH 2210 - July 5, 2019 (Researcher's Reflective Journal)
> On the other hand, the revised model that incorporates a single-layer atmosphere (for which I provided students a diagram and encouraged them to try and figure out the details) appears to have been an enormous challenge. I am having serious

> reservations about this inclusion of this portion of the project in future iterations. I am strongly considering a relaxation of the plans I had for the incorporation of parameters intended to connect this Enhanced Greenhouse Effect model with their (to-be-developed) hurricane velocity field models, or at least diminishing the derivation of the models from this and last week's meetings.

In summation, the Greenhouse Effect modeling component of the project—at least in its current form—appears to be ill-suited for retention in future versions of this project. That being said, some student reflections specifically pointed to aspects of the Enhanced Greenhouse Effect model as some of the most interesting things they had learned early on in the course.

> MATH 2210-8 (Student Reflection Journal 1)
> I think the most interesting thing ive [*sic*] learned is just the process you use. At least for the global warming one it was cool to be able to isolate systems and understand what goes in and out of them.

Another student, while possibly betraying a misunderstanding of the shortwave radiation re-emitted by the Earth, provides additional evidence that some aspect of this project component may be worth preserving.

> MATH 2210-1 (Student Reflection Journal 1)
> The most interesting thing I have learned so far is that the earth emits its own energy. I understood that the sun brought energy into the earth system but did not understand the role the earth played in putting energy into that system.

While these tasks may be excessively challenging for students at this level to undertake on their own, an alternative to complete removal of these models may be to incorporate them into an introductory whole-class discussion that introduces the modeling process and provides an interesting and relevant example of its use in action.

There are several pieces of information that would likely benefit teachers who wish to incorporate this project into their curriculum. The first has to do with the time in which

specific topics are introduced. This project necessarily fits well into a course that covers the topic of vector fields. This means that a teacher that intends to incorporate this project as more than a few-days learning activity will likely need to reorganize the order in which content is introduced. The course in which this project was test-enacted began with a comprehensive review of vector-valued functions and the associated calculus. This allowed the project facilitator to relate the introduction of vector fields to the recently discussed vector-valued function topics. Furthermore, the gradient of a function is often introduced well before any discussion of vector calculus begins. This served as a useful entry-point in the curriculum in which vector fields could be reasonably introduced without seeming unnatural. While the timing did not line up perfectly, it appears to have been successful. As noted in the researcher's reflective journal:

> MATH 2210 - July 12, 2019 (Researcher's Reflective Journal)
>
> Today students completed a worksheet packet [that] introduced and allowed them to explore 2-dimensional vector fields with their groups. I created the prompts with the intent of familiarizing students with simple vector fields, and the construction of more complex vector fields by taking advantage of linearity. The intent was to counter the fact that I am technically introducing this material much earlier in this course than our text suggests. We recently introduced the gradient as a vector-valued function, and then discussed how, unlike our previously introduced vector-valued functions that described space curves, this one can be understood as assigning a vector to every point in $\mathbb{R}^3$.
>
> Today, right before handing out the worksheet, I reminded students of our gradient example in hopes that this material wouldn't seem like a complete deviation from the rest of the course. I think this was effective, but perhaps only because students had already grown accustomed to having a change of pace on days when we worked on our modeling project.

A second, related point that could be included in a pedagogical note is with regard to students' sense of self-efficacy. As mentioned elsewhere, students initially expressed uncer-

tainty as to whether they believed they would be able to finish the project, especially near the beginning. While some students experienced a sense of relief upon the introduction of the vector-fields group task, a lack of self-efficacy persisted among a number of students far into the project. The project facilitator initially thought that students worries were put to rest following the introduction of the vector fields group worksheet, as evinced below, but retracted this conclusion after reading the first reflection journal entries.

> MATH 2210 - July 12, 2019 (Researcher's Reflective Journal)
>
> I think today was largely successful. I overheard a few students express to their fellow group members their increase in confidence in their ability to come up with a model. I think prior to today's meeting, and prior to learning what vector fields and linearity are, the students were skeptical that they would gain the mathematical toolkit necessary to get anywhere. Then of course, there were the (what I now recognize as) overly complex models we explored in our two previous meetings. I'm sure that those did not instill any level of confidence in students' self-confidence for this project.

> MATH 2210 - July 18, 2019 (Researcher's Reflective Journal)
>
> A second thing to note is that students are still expressing a lot of trepidation about the complexity of the project. It appears that the students expressing more confidence last week were not a representative sample.

As noted in the researcher's reflection journal, a number of students expressed concerns with regard to their ability to engage in the mathematical modeling process. For example:

> MATH 2210-2 (Student Reflection Journal 1)
>
> I have very limited experience with mathematical modeling, and said limited experience is very basic. I definitely feel intimidated by the process, but am interested in what I will learn.

> MATH 2210-3 (Student Reflection Journal 1)
>
> I am a bit nervous still about mathematical modeling. I've never really done

> it before outside of the examples we did in class, so I get the general idea and process, but I don't feel super confident in my own abilities to come up with a model that would be useful for anything. I have a lot of vague ideas and could potentially be useful working with someone who knows what they're doing, but by myself I think I'll be very lost without a lot of guidance.
>
> MATH 2210-5 (Student Reflection Journal 1)
> Currently I am not to [*sic*] sure about mathematic [*sic*] modeling, I still feel a little confused by the topic as it can seem overwhelming periodically.
>
> MATH 2210-7 (Student Reflection Journal 1)
> I'm a bit discouraged but will work hard to learn and succeed.

It is important that teachers begin the project with realistic expectations for how overwhelmed students may feel, but also have insight as to when their students may begin to feel more confident in their ability to complete the project. With the other projects—where students still likely felt overwhelmed—participants already possessed mathematical tools sufficient to develop an appropriate model prior to starting the course in which the project was enacted. This project, however, requires new mathematical content knowledge.

To this end, the next version of these project materials should include pedagogical notes that explain how students are likely to respond to the introduction of the project with a greater measure of fear and uncertainty than should be expected in the other projects. Furthermore, this kind of note may also help teachers determine whether and when to provide additional scaffolding for groups which are struggling unproductively to make progress on the development of their model.

3.2.5 Lessons for *Orchestrating the Final Discussion*

This was the sole project test enactment that included both the presentation and discussion components. However, unlike the other topics, which center on issues outside of what is generally considered "hard science," this project's central focus is slightly less

political a topic and, given its placement in a course taken almost solely by students pursuing a STEM degree, fewer students were expected to disagree with any assertions about the existence of climate change and its impact on tropical cyclones. As such, outbursts and arguments were not expected to occur. Expectations were met in this regard.

However, one should not presume that students' views regarding anthropogenic climate change be uniformly favorable. As seen elsewhere, there were students who began this project expressing a substantial level of suspicion regarding the motivations of climate scientists, and it is possible that these views persisted into the final discussion. As such, it may be important that the project facilitator maintain a personal distance from the central claim unless they make sure to point to evidence as the rationale for any personal views shared. Assertions devoid of a discussion of evidence, or at the very least an appeal to evidence, should be avoided. Each of these points may serve to inform the inclusion of important pedagogical notes for ensuring the final discussion remain productive and centered on scientific evidence.

Students were given substantial leeway in the determination of how to present the development of their mathematical model, the primary direction being to share the contents of their technical report as it was required to explain not just the final result, but each step in the development of their model. This appears to have been, in some measure, successful.

Even so, students may benefit from some additional guidance in regard to what aspects of their model's development to focus on during their presentation, and a specific order in which to introduce the different topics they cover. Given that some groups only addressed the assumptions inherent in their model when prompted by the project facilitator, it seems reasonable to provide students with a list of topics and an order in which to address them. This would likely serve to benefit students by providing additional supports for their achievement of the comprehension and communication-related learning objectives.

Furthermore, it may be appropriate to provide students with the technical report specifications earlier in the project, perhaps even at the very beginning. This will ensure students have realistic expectations for what will be required of them. Given the trepidation with

which many students began the project, it is vital that the specifications list be purged of spurious content so as to prevent unnecessary anxiety in students. The report may seem daunting even after a group has completed the development of their model. There is no need to exacerbate the issue.

The following constitute clarified versions of the questions asked by the project facilitator following groups' presentations (in the case they were not already addressed during the presentation itself):

- How does the domain of your vector field hurricane model differ from that of a real hurricane?
- What hurricane characteristics does your model not incorporate? If you had more time to improve your model, which of these characteristics would you attempt to incorporate into your model, and how would you try to accomplish this?
- Can you please explain your model's parameter(s) in terms of the real-world phenomena they represent?

In addition to these, the following question could provide important insights into students' understanding of the role of parameters in mathematical modeling, and would constitute an important addition to the project materials.

- How sensitive are your model's parameters to change? In other words, how much of an impact does a small change to a parameter's value have on the entire model, and in what way is the model affected?

Additional helpful questions are likely to arise in future presentations and discussions. As such, it should be expected that this component of the project materials evolve with some degree of regularity.

3.2.6 Lessons for *Sample Approaches and Models*

All groups but one employed variations of the expected approach to completing this modeling task. One group engaged in an interesting exploration of multivariable functions

for the purposes of producing a scaling function that would not require the artificial restriction of its domain to produce some form of an eyewall. They began with a familiar formula, scaling a unit vector field exhibiting a counterclockwise flow by a scaling factor of $\frac{1}{x^2+y^2}$ to produce the desired characteristic of increasing velocity as distance to the singular point decreases:

$$\mathbf{f}(x,y) = \left\langle \frac{-y}{(x^2+y^2)^{3/2}}, \frac{x}{(x^2+y^2)^{3/2}} \right\rangle$$

They provided the following description of their exploration:

> MATH 2210-6,7,8 (Student Technical Report)
>
> We knew that a critical factor that we wanted to incorporate into our model was the eye of the hurricane. We set out to find an equation where the magnitude of the vectors increased as they approached the origin, but would then drop off as they approached the center, allowing for the formation of an eye. We began experimenting with a variety of functions to see what we could find. Knowing that exponential functions increase as they approach the origin, at least from one direction, we play around with the function e^x, to see if we could create an eye.
>
> One of our first attempts involved surrounding the origin with manipulated functions of e^x. ... We ended up with four different equations that would all need to be parameterised [*sic*]. It seemed infeasible to use all four equations to constrain our vector field, so we began experimenting with other variations instead.
>
> After a week of trying to figure this out an epiphany hit. An equation that was undefined between -1 and 1 would create a natural gap in are [*sic*] vector field. It also became apparent that we could incorporate the equation for a circle into our equation to allow the eye to be circular. Therefore, instead of merely having the equation e^x, we could incorporate and x^2+y^2 somewhere. This result in a scaling equation of $\frac{1}{e^{x^2+y^2}}$.

> However, this equation is not undefined between −1 and 1. In order to create an eye we incorporated a square root in the exponent of e, resulting in $\frac{1}{e^{\sqrt{x^2+y^2-1}}}$.

This group later attempted to incorporate an eye in a different manner, involving the use of logarithms, but returned to this model when their attempts at implementing this approach did not produce the desired result. This approach, of a natural domain restriction will be included in the project materials for this project as a sample approach.

3.2.7 Lessons for *Resources*

As mentioned elsewhere, students found the initial tasks, involving the guided modeling experience exploring the Enhanced Greenhouse Effect, to be overwhelming. The project facilitator felt compelled to step in multiple times to provide students with clarifications and suggestions throughout. This does not necessarily indicate that the prompts on those task sheets were ambiguous. It may be that students merely felt overwhelmed given their mandated "head-first" dive into a complex mathematical modeling challenge. On the other hand, while some initial clarifications were requested during group work on the vector fields worksheet, students found the tasks included far more accessible to the point that the project facilitator expressed concerns as to whether excessive scaffolding had been provided.

All of this being understood, it is worth pointing out that, unlike the case with the "War on Drugs" project, no students expressed bewilderment at the goal of this project. They instead expressed doubt in their ability to successfully complete the project. A balance between the appropriate levels of scaffolding and autonomy is clearly the most desirable end, but the path to achieving such a balance remains unclear. Additional enactments may be required to further illuminate this challenge.

Unlike the previous cases, this project enactment did not assume a prior knowledge of the content expected to be most central to the development of their model. Students received guidance through all aspects of the initial Enhanced Greenhouse Effect models that relied on results they may not have been aware of prior to taking the course (e.g., the Stefan-Boltzmann Law). While those tasks have been deemed excessively challenging for students'

first modeling experience, the bulk of the challenge lay in the collective lack of modeling experience among participants rather than in some other prerequisite mathematical content knowledge.

The vector fields worksheet includes a series of tasks of increasing complexity intended to lead students to recognize the linear properties of vector fields without necessarily introducing the relevant vocabulary. One way in which the mathematics tasks of this project might be improved is through a revision of this worksheet. In particular, while linearity was introduced informally, students may benefit from a more formal introduction after they have had the chance to explore these properties for themselves.

While the additive aspect of linearity was introduced, students never explored scalar multiplication in conjunction with additivity. This might seem tangential in some sense, but this has the potential to improve students' understanding of how to incorporate parameters into their project. Each group incorporated a parameter into their project that was to be understood as dependent on the Enhanced Greenhouse Effect. However, they each relied solely on a scalar that is applied to the model in its entirety, rather than considering whether to differentiate between impacts on, say, vortex and sink flow aspects separately.

In addition to altering the vector fields worksheet to help students better appreciate linearity, the mathematics tasks need to incorporate some method for helping students recognize assumptions in their model, as well as better appreciate the role of assumptions in the mathematical modeling process. The project facilitator expressed concerns in this area following student groups' in-class presentations.

> MATH 2210 - August 8, 2019 (Researcher's Reflective Journal)
> Changing gears, I think it was pretty clear from their explanations of assumptions that there is considerable work to do in that area. Most acknowledged an assumption about geographic location, or recognized some of the components of their model which were not realistic (e.g., no eyewall, wind velocity that instantaneously decreases to zero upon breaching the eyewall from the exterior). Those that did not attempt to produce a 3-dimensional model all acknowledged

> the obvious difference between their 2-dimensional model and a real hurricane wind velocity field. The students did, however, all miss some important things. No groups appeared to recognize that their model is defined over the entire plane minus the origin (or unit circle for those who included an eyewall at that distance from the origin), which certainly is not reflective of anything in reality. Nor did those groups that attempted to create 3-dimensional models acknowledge the analogous domain problems (i.e., a hurricane of infinite diameter and height). Finally, but less surprisingly given we did not spend time explaining uniform sink flows, no groups thought to question that, given that air is rushing toward the center of the hurricane, that it has to be going somewhere (i.e., leaving the hurricane).

In light of these concerns, it appears that the project materials would benefit from pedagogical guidance intended to help the project facilitator ask questions that help students recognize their model assumptions—not only during final presentations, but from the beginning of the project.

Upon completion of this project, it became clear that in addition to the mathematical learning objectives, both cognitive and affective, this project should incorporate an objective that focuses on students' understanding of the hurricane development and their connection to climate change.

In their current form, the technical report specifications already elicit student thinking which may provide insights into these topics. Specifically, the prompt in the technical report specifications asks students to explain the following:

> *[Your] report should include a description of how rising global temperatures may contribute to an increase in the strength and/or frequency of hurricanes (or, more generally, tropical cyclones).*

Of the two reports by groups consisting of students who all consented to having their work available for use in research, both responded to this prompt with explanations that at

the very least suggest the group members may understand the general mechanisms behind hurricanes and their relation to climate change.

> MATH 2210-1,2,10 (Student Technical Report)
>
> While there are several factors that ultimately lead to stronger, more frequent cyclone formation with rising global temperatures. Of these, our team has selected two distinct factors. The first will be the rise in ocean temperatures. As explored in our model, the threshold for cyclone formation is 26 degrees Celsius. Higher ocean temperatures will lead to an increased possibility for cyclones and allow for more intense storms.
>
> The second factor involves the water holding capacity of our atmosphere and the potential for water to evaporate. As temperatures increase, more water will evaporate into the atmosphere. The water stored in our atmosphere with the potential to condense into clouds is known as precipitable water. As precipitable water increases, the potential for cyclones to form and to be more destructive increases too.

This group appears to consider as separate two very interrelated ideas. The former, which may be a reference to an increase in the depth of the ocean thermocline, obviously serves as a source for the latter—that is, the precipitable water. That being said, the students do appear to understand the basic idea of how increasing temperatures can affect hurricanes. The other group explained this same phenomenon in the following manner:

> MATH 2210-6,7,8 (Student Technical Report)
>
> Climate change has the potential to increase both the frequency and intensity of hurricanes. Rising global temperatures lead to warmer ocean temperatures overall. This means that there are potentially more areas hurricanes can form, and that the circumstances necessary for them to form can occur more often. Additionally, as hurricanes draw their energy from warm ocean waters, warmer ocean temperatures will likely lead to more hurricanes of higher intensity.

Because technical reports are a collaborative effort, teacher's wishing to draw hard conclusions about individual students' understanding should proceed with extreme caution. Students lacking understanding of these mechanisms *may* learn from their peers, but not necessarily. In the end, however, this prompt serves to encourage participating students to develop, at the very least, a basic understanding of the connection between climate change and hurricanes.

Finally, students made use of existing GeoGebra applets to plot their 2- and 3-dimensional vector fields. While these utilities were sufficiently sophisticated to allow students to quickly visualize the impact of altering different aspects of their model, their limits were quickly revealed during attempts to analyze the impact of altering parameter values.

The applets used do not incorporate the option of parameters, and so any adjustment requires directly altering the vector field components, rather than changing a single value either with direct input or, as is common with GeoGebra applets, with a slider. In addition to the difficult of parameters are issues related to unclear labeling conventions and, in the case of the 3-dimensional plotting applet, a prohibitively restricted domain that complicates the issue of viewing qualitative components of a vector field model.

Students would greatly benefit from a more user-friendly graphical user interface, a domain which can be altered by the user, and the ability to easily incorporate and explore the impact of model parameters. While not necessarily vital for the next version of this project, a future endeavor to develop such an applet seems like a worthwhile endeavor.

3.3 "Tracking" in School Mathematics Project Enactment Results

The characteristic of this enactment that distinguishes it the most from the other, aside from having the fewest students (only 12), is that each student in this course, Methods of Secondary School Mathematics Teaching, is planning to become a Middle or High School math teacher. While each of the other two courses is required for mathematics teacher certification through this institution, they are also required for a large number of other majors.

Another challenge faced by the project facilitator involved the tension between students'

normal classroom obligations and the additional work required by the project. In spite of the fact that it was required, there is evidence that students took the project less seriously than the rest of the responsibilities, especially earlier in the semester. In the case of the initial tasks students were directed to complete outside of class, 3 days after its due date, multiple students still had not completed the required tasks, leading the project facilitator to wonder whether the project would be viewed as a priority by the students. That being said, fewer challenges appeared among these students than with those taking the MATH 1210 course:

> MATH 4500 - March 3, 2019 (Researcher's Reflective Journal)
> I have a few students that still haven't completed the rest of the initial task sheet. I sent out individual reminders to complete everything as soon as possible. Unsurprisingly, given that this is an upper level course, this group of students appears much more willing to engage in the project than many of the calculus students. I only hope there are no challenges in balancing their other work, which includes clinical experiences at local schools, with completion of the project-related tasks.

These results suggests that teachers might benefit form additional guidance on how to best incorporate the project into their course. The more students view the project as an integral component of their course, the less likely they may be to leave project-related homework for the last minute (which, given the importance of ongoing reflection, would likely lessen the impact of the project on the development of students' beliefs).

The social justice topic may be categorized as both a strength and a weakness of this project. The topic is clearly a strength as tracking is inequitable, a majority of teachers support tracking policies, and so pre-service teachers would benefit from experiences that clearly outline the inequities associated with tracking during their teacher preparation program. On the other hand, many students who participate in a project intended for math teachers in training likely had positive experiences in their own likely-tracked mathematics classrooms and, as a consequence, may view tracking in a positive light. While the goal

is to convince individuals such as these that the cons of tracking ultimately outweigh any perceived benefits, there is the risk of them convincing their peers that tracking is a net-positive practice. Alternatively, there remains the risk of students coming to the "wrong" conclusion on tracking based on the results of their model. This may seem like an odd statement given that students are given complete autonomy in developing their models, but this risk does not exist to the same degree in the other projects, which both focus on topics that are considered "settled" in some sense. The scientific consensus on climate change is overwhelming and scholars across the political spectrum agree that the War on Drugs has been a failure that has disproportionately harmed persons of color (which is why the readings associated with the MATH 1210 project can draw from sources on both ends of the spectrum). On the other hand, despite the near-consensus among mathematics education and teacher education organizations, tracking is viewed positively be a great number of people, including academics. A number of researchers that explore education for "gifted" students continue to publish papers that give evidence that tracking benefits students in accelerated courses because it correlates positively with AP test-scores. In short, unlike the other two projects, students that search scholarly sources for information on tracking are likely to find at least a handful of sources that claim tracking is beneficial.

As mentioned previously, one group of students applied simple linear regression to scatter plot data they obtained from the NAEP *Nation's Report Card* website. They plotted NAEP scores, percent of students who scored a 3 or greater on AP tests (they don't indicate *what* AP test), child poverty levels, and AP participation of high school graduates, against the percentage of students tracked in 8th grade. Their intent was to determine whether schools where students are "more" tracked have higher levels of student achievement, the latter of which they define as AP course participation, higher AP test scores, and higher NAEP assessment scores. Perhaps unsurprisingly, these students found their data held conflicting interpretations.

MATH 4500-15,21,33,46 (Student Technical Report)

Altogether, there is insufficient data to conclude strictly one way or the other as

> far as is tracking equitable or not, especially in specific reference to mathematics. In some cases, ..., we are led to believe that tracking is equitable as equal opportunities are presented throughout the low tracking to the high tracking rates.

The students appear to be suggesting that given that the results appear similar in schools with and without tracking, tracking must be equitable as it is producing the same result as students would have experienced in a non-tracked school. They continue:

> MATH 4500-15,21,33,46 (Student Technical Report)
>
> In other cases, we see that it is not equitable as we see an increase in AP and NAEP scores as an increase in tracking occurs.

These students argue that because children at tracked schools were more likely to score high on NAEP and AP tests, that tracking is inequitable, as it appears that students from non-tracked schools are not offered the same opportunities. They go on to conclude:

> MATH 4500-15,21,33,46 (Student Technical Report)
>
> [W]e see that tracking is not as inequitable as we had initially thought it to be.

This conclusion would likely shock individuals familiar with the issues surrounding tracking, as it did the project facilitator—at least until they understand that the students making the claim did not consider the impact of tracking on different student demographics. Ultimately, the group never appears to have considered, at least in their discussions as revealed through their technical report, the question of tracking from the perspective of race or other social identity. Fortunately, the students appear to have expressed views contradicting this conclusion in their later reflections. For example,

> MATH 4500-21 (Student Reflection Journal 2)
>
> The most important thing that I learned is that there are a lot of people that, regardless of whether tracking is equitable in its entirety or not, feel that they have been wronged by tracking.

Another appears to be nearing the idea of looking at data disaggregated by things like race and ethnicity, though never makes entirely clear what they mean by "different groups":

> MATH 4500-15 (Student Reflection Journal 2)
>
> My only concern is that students benefit well from interacting with others who think differently than them. Tracking may prevent that socialization. In order to challenge inequitable structures in a future career, I plan on being observant to the systems in place. If I notice (or read or hear) something that challenges the system, I'd hope to conduct an investigation. I would try to analyze how the inequitable structure affects different groups.

Another student from this group appears to demonstrate some understanding of how tracking has effects on the mathematical identity of students placed in different tracks:

> MATH 4500-33 (Student Reflection Journal 2)
>
> This will sound silly, but the most important thing that I have learned about tracking in school mathematics is that it exists!! I honestly didn't understand the concept until we talked about it in methods when we began this project. I knew that I took honors classes, but I had never much considered the effect that that had on me and on other students who were in different classes.

The project facilitator expressed some relief at these statements, as recorded in the researcher's reflective journal:

> MATH 4500 - April 21, 2019 (Researcher's Reflective Journal)
>
> I feel some small amount of relief now that I have read students' most recent reflection journals. Many students, even those from the group that concluded tracking is helpful, appear to accept that tracking is harmful in spite of the results of their model.

Still concerned that students finish the project with the wrong conclusions, the project facilitator provided students with an in-depth explanation of some of the ways in which

tracking harms students, which was followed by the final in-class discussion of the social justice topic and the writing of students' individual action plans.

> MATH 4500 - April 26, 2019 (Researcher's Reflective Journal)
> Following this, we reviewed our definition of equity (Gutierrez's definition), and I outlined one of several common scenarios involving improper placement of students in tracking programs due to the implicit biases of decision-makers. I felt the need to do this to make sure that, even if groups' models did not conclude this, tracking is definitely an inequitable system (I understand that not everybody agrees with this, but I have approached this entire research study with that as an assumption and, therefore, motivating factor for the selection of tracking as [an] appropriate topic of study).

This group's approach and conclusions have revealed how some of the most important takeaways from this project can be missed by students that do not consider data that has been disaggregated by things like race, ethnicity, and gender, and serves as something of a cautionary tale for future project facilitators. While this enactment did not appear to result in an increase in support for tracking among students, the potential for this negative outcome remains. This, in turn, has multiple implications for the project materials. First of all, it suggests that the project materials may benefit from additional sample questions the teacher can use to bring students attention back to more distinct population demographics. Second, it may be evidence that the readings assignments should be spread out across multiple assignments and started near the beginning of the project, so that, rather than serving as a final correction to students approaches, the readings inform even the initial development of the model.

The next iteration of this project will provide students with the readings from the Brookings Institute near the beginning of the project. They include important statistics that can be used to inform assumptions for the development of a model. In addition, they include data that might lead one to conclude that tracking is a net positive if considered

devoid of a greater context, allowing the project facilitator to begin asking critical questions. Subsequent readings provide greater emphasis on the racial and class injustices of tracking.

3.3.1 Lessons for *Learning Objectives*

Students were expected to engage with this mathematics content primarily at the application learning level, as they were supposed to identify appropriate mathematical tools for themselves, at the comprehension and communication level for their technical report and their reflection journal entries. Following a review of students' submitted technical reports, the project facilitator concluded that the learning levels appear to have been correctly identified, still expressing uncertainty as to whether data sources ought to be provided, but concluding that the level of cognitive demand required by the project remained unaffected.

> MATH 4500 - April 21 , 2019 (Researcher's Reflective Journal)
>
> I think that the technical report did help students achieve an objective at the comprehension and communication cognitive learning level, and that the challenge of producing their own model using whatever approach they felt best did lead students to achieve a learning objective at the application learning level (I don't remember whether I wrote this for the calculus project, but I came to the same conclusion with that project, excluding the interpolation software and directions and related calculus tasks). With regard to the Smith/Stein Mathematics tasks Framework, I absolutely would characterize the work done by these students as *Doing mathematics tasks*, as I hoped they would. While I did provide those who asked with directions to different data sources (after allowing them to try it on their own for a while; I'm still unsure as to whether I will provide students a list of all sources of data I am aware of, and then just allowing them to pick through it), and I did ask questions to help groups recognize that they did indeed have a model, I don't think this diminished the level of cognitive demand. The project still did (among other things) "Require students to analyze the task and actively examine task constraints that may limit possible solution

> strategies and solutions" and "involve some level of anxiety for the student due to the unpredictable nature of the solution process required."

This reflection journal entry suggests students may have achieved the intended objectives. However, hindsight revealed the need for the alteration of some existing objectives and the inclusion of other, additional learning objectives. In particular, whereas the original materials implicitly categorized groups' presentations as subsumed by the technical report objective, these two tasks are distinct, necessitating an individual learning objective for each. As mentioned elsewhere, due to time constraints, students were unable to present their groups' models, but they still achieved the intended learning objective. This clearly points to the conclusion that the report and classroom presentation are sufficiently distinct as learning activities to demand their separate assessment and, therefore, learning objectives.

Further reflection has also led the researcher to conclude that application may be the incorrect learning level for mathematical modeling tasks in general. In a course specifically dedicated to modeling, this would likely be the appropriate classification, but in courses where the project is added to complement existing curriculum, and where students are unlikely to possess modeling experience, the learning level may be *willingness to try*. While this is, admittedly, an affective objective, which are traditionally unweighted for the purposes of assigning a grade, a substantial amount of the challenge to mathematical modeling lies in the requirement of tenacity on the part of students. Students who persist in engaging in the modeling process, in spite of their frustrations and in the face of set-backs, are succeeding. This of course says nothing as to whether their final model holds any practical utility, but at this level of modeling education, the process is likely to be more important than the result, a conclusion echoed in assessment chapter of the GAIMME report (SIAM & COMAP, 2019).

3.3.2 Lessons for *Definitions & Notation*

There were no specific mathematical definitions that appeared during the project test enactment that should be added to the project materials. That being said, the sample ap-

proaches provided for teacher reference would likely benefit from a measure of mathematical formalism.

Given the mathematical goals of the project—of helping students learn and engage in the modeling process—establishing expectations for the teacher seems like a useful addition to the materials. Sample approaches that lack in things like precision language may lead teacher educators to believe mathematical formalism is unimportant. As the tasks are intended to possess some degree of authenticity, students should be expected to produce authentic, professional-looking results, and teachers should be encouraged to see that they do so. An additional benefit of encouraging rigorous mathematics in students' models comes from the greater ease of generalizing that results from this approach.

Students would also likely benefit from a single, shared definition of equity in school mathematics. Future version of this project will adopt one part of the three-part definition of equity proposed by Gutierrez in 2002:

Equity in School Mathematics: Erasure of the ability to predict students' mathematics achievement and participation based solely on characteristics such as race, class, ethnicity, sex, beliefs and creeds, and proficiency in the dominant language (Gutierrez, 2002)

In addition to a shared definition of equity, both teachers and students will benefit from the inclusion of definitions of ambiguous or unfamiliar terminology (e.g., some sources conflate the definitions of ability grouping and tracking, while others do not). Furthermore, some writings on tracking focus primarily on the negative outcomes for students disproportionately placed in "lower" tracks, paying far less attention to those negatively impacted by "higher" course tracking. To provide teachers with additional resources for addressing these situations, the project materials will also include the following definitions:

Tracking: Tracking is the practice of placing students in course "track" based on their perceived ability.

Ability Grouping: Often used synonymous with "tracking," ability grouping is an umbrella term for any practices that groups students based on their perceived ability, whether in the same or different classes.

Essentialism: A sociological theory that posits any entity such as an individual, group, object, or concept has innate and universal qualities. Essentialism has been, and continues to be, used as a justification for racism, imperialism, and bigotry.

Stereotype Threat: Stereotype threat refers to the negative impact on students' achievement that results from the fear of confirming negative stereotypes about some component of their identity.

Model-Minority Stereotype: The model-minority stereotype is a harmful, essentialist generalization that, among other things, claims that Asian students have a natural aptitude for science and mathematics.

While not necessarily appearing explicitly at any time during the project test-enactment, students' reliance on proportional reasoning arguments suggest the following mathematical definitions may be beneficial additions to the project materials.

Directly Proportional: Two quantities a and b are said to be directly proportional to one another if $a = kb$, where k is a constant.

Inversely Proportional: Two quantities a and b are said to be inversely proportional to on another if $a = \frac{k}{b}$, where k is a constant.

3.3.3 Lessons for *Preparation*

This project's introduction was distinct from both of the others. Whereas the project enacted in MATH 1210 was introduced remotely through Canvas and the project enacted in MATH 2210 was introduced in class by the course instructor—as the project facilitator held dual roles during that enactment—the project introduced into MATH 4500 was introduced by the project facilitator, in person, but in a course taught by a different instructor.

In light of challenges experienced during the initial phases of the MATH 1210 project, introduced online only four weeks prior to the beginning of this project, the project facilitator determined to require students to create their Trello accounts during class rather than prior.

MATH 4500 - February 26, 2019 (Researcher's Reflective Journal)

> Based on the debacle that was getting students registered for Trello and added to the appropriate board for their team, I am planning on allocating some time during the in-class launch in MATH 4500 to get everyone signed up. I don't anticipate this will take up much time so long as I wait until class to send the board invites and notify students of the email address associated with the invitations.

This appears to have smoothed many of the challenges faced with getting students registered for the calculus courses.

> MATH 4500 - February 27, 2019 (Researcher's Reflective Journal)
> I oversaw registration with Trello and made sure all students were added to their respective Trello boards. This went far more smoothly than the debacle with the calculus project.

This may, however, be a biased assessment given that there were only 12 students for whom to prepare Trello accounts for this course, which is a far cry from the hundreds necessary for the calculus courses, a fact highlighted by the project facilitator.

> MATH 4500 - February 27, 2019 (Researcher's Reflective Journal)
> It probably would not have been possible in a large lecture course, though individual recitations may have allowed it.

On the other hand, for a Methods course, which is unlikely to be offered in large lecture sections, this approach to helping students set up project management software accounts is possible.

As was the case with the MATH 1210 project, students abandoned the project management software as a collaboration tool as soon as they were permitted to do so. Unlike the other course, these students did not express their frustrations surrounding Trello in their personal writings, so little more can be concluded about how the project management software helped or hindered student progress, though the quick transition away from the software suggests that students did no find it particularly helpful or enjoyable.

That being said, there are additional factors which may have led students away from the project managements software. For example, most of the students in this course already knew each other due to their shared courses as part of their degree programs. They may have not needed to use tools intended for remote collaboration since they were not remote.

Furthermore, requiring the teacher of a methods course, such as this, to spend the additional time required for *effective* project management through the project management software is likely unrealistic, even for a course with students as few as this. As such, future versions of this project will probably benefit most from providing students with a list of tools at the beginning of the project, from which they can select (or not) the option best suited to their collaboration needs.

3.3.4 Lessons for *Introducing the Project*

Students were notified by the course instructor of what they would be expected to do for the project some time before its formal introduction, but did not receive a detailed explanation of how they would be imagining themselves as members of a think tank. Overall, the students may have found the narrative scenario mildly amusing, a point mentioned in the researcher's reflective journal.

> MATH 4500 - February 27, 2019 (Researcher's Reflective Journal)
> Participants appeared to have appreciated, or at least been amused by, the narrative scenario to a mild degree (smirks and glances at neighbors all around). I can't help but wonder if their lack of familiarity with me, this being our first interaction, is making their responses more reserved out of discomfort or because they are unsure of how I would respond. That, of course, just may be me wishing they had been more enthusiastic.

Note the reflection on whether students would have responded differently had they already been familiar with the project facilitator. The more overt responses witnessed during the introduction of the MATH 2210 project suggest this may be the case.

Overall, the narrative scenario, while not vital to the mathematical, and possibly even affective, goals of the projects, may increase student engagement, and will therefore be retained for future versions of the project. Furthermore, the project materials would benefit from an in-depth explanation for how teachers should use the narrative scenario as a tool for introducing the project, as well as a description of the kind of response to its use they should expect from students.

Prior to the project's in-class introduction, the students had already learned about the mathematical modeling process. This unexpected preparation, due in part to the course instructor's own interest in modeling and to the fact that the course instructor is the principle investigator for this research and has an interest in a successful enactment, certainly smoothed the introduction of the project. The previous introduction of modeling allowed for additional in-class time to be spent on other issues, without which time might have been insufficient for some particularly important discussions to take place.

Another factor which impacted the introduction of the project, in ways both positive and negative, was the initial task sheet: *Getting Started with Mathematical Modeling*.

Positive outcomes resulted from having students immediately start engaging in the mathematical modeling process by listing factors they believe would be important to determining whether tracking is an equitable practice. Perhaps even more important was the second prompt, wherein students attempted to identify their own biases regarding the issue of tracking. Student responses to this prompt revealed important insights into their affect. However, the wording needs to be modified to ensure student responses are unambiguous. For example, one student wrote:

> MATH 4500-46 (Student Initial Task Sheet)
> I feel like I might be biased towards tracking because I was always tracked and placed on the higher track.

It is unclear from this statement whether "biased towards" is indicative of approval or disapproval. Contextually, it appears that the student is indicating a favorable view of tracking,

but the possible ambiguity substantially complicates the process of assessing student affect over the course of the project—this response was coded as Indeterminate.

A negative outcome resulting from the use of the initial task sheet came from a printing error made by the project facilitator.

> MATH 4500 - February 27, 2019 (Researcher's Reflective Journal)
> I accidentally only printed one side of the task sheet, so I provided a digital version on Canvas and made the rest of the sheet's prompts an assignment that is due tomorrow.

The fourth prompt, which required students to create a Trello card for the purposes of sharing their lists and familiarizing them with the use of intended project management software, was not negatively impacted in any noticeable way. Nor was the fifth, which directed students to set up a meeting time outside of class with their groups. However, the third prompt, which required students to list assumptions related to their model appears to have introduced unintentional ambiguity. Rather than helping students recognize assumptions in their model and the role those play in the modeling process—as it was intended to do—the prompt did not lead students to recognize the interaction between the development and limitations of their model and the assumptions they make in creating their model and selecting appropriate data. Students appear to have recognized implicit assumptions in the selection of their data, but did not appear to recognize how those assumptions would ultimately limit the applicability of their model. Had the prompt been printed, as intended, this deficiency in the project materials could have been addressed during class, allowing students to receive immediate feedback on how to think about their model's assumptions.

One particular group included a member who, unlike many, if not all, of the other students registered for this course, experienced some of the more (obviously) negative consequences of tracking. This student chose to share their experience with their peers, which appears to have had an impact. It is worth noting that the result of this group's model implementation was inconclusive for determining whether tracking ought to be ceased or more widely implemented. They relied on comparisons of ACT/SAT takers among students from

institutions they classified, based on the existence of advanced course offerings, as tracking and non-tracking schools. While their results may have been ambiguous, one member of this group pointed specifically to the sharing of their peers' experience as the reason for their conclusion that tracking can be harmful.

> MATH 4500-1 (Student Reflection Journal 2)
> I think the most important thing I have learned, not necessarily from the data, but from our discussions was that tracking doesn't always help students. It may even hinder them. I went to a tracking school when I was in high school. I was in the advanced track, and really did enjoy it. I didn't realize how those in the regular or remedial tracks may have been negatively affected by tracking, as [4500-42] related [their] own experience.

The project facilitator noted how this kind of sharing of personal experiences may have a profound impact on listeners.

> MATH 4500 - February 27, 2019 (Researcher's Reflective Journal)
> Many students indicated that they were unaware of tracking, in spite of the fact that they were tracked themselves. One group included a member that was placed in a remedial track, and who related their experience with stigma that came with this placement. The other students in the group appeared to empathize a lot. I think these kinds of personal experiences could do a lot to influence the development of other students' beliefs on a topic.

Future versions of this project would likely benefit from specific guidance for teachers to encourage students to share their own experiences with tracking.

Perhaps the most important guidance for a teacher seeking to incorporate this project into their course, at least with regard to introducing the project, would be to direct teachers to encourage students to share their own experiences involving tracking. Given the ubiquity of the practice in US schools, teachers will more than likely have multiple students possessing first-hand experience with tracking.

If students have negative experiences they are willing share, this may have a profound impact on their peers' perceptions of tracking, regardless of their own experiences. Should all participating students relate positive experiences, the teacher should make a point of highlighting the near universal condemnation of the practice by math teacher educator organizations and following up with the obvious question of "If everyone is having a 'positive' experience, why would these organizations speak out against the practice in such strong terms?" This will serve to return the discussion to a focus on equity, and the inequities created and reinforced by tracking.

A final conclusion regarding appropriate changes revolves around students' understanding of the mathematical modeling process itself. Students held a variety of inaccurate and unrealistic expectations for what they should expect to produce over the course of the project. It should be pointed out that while the modeling process was introduced by the course instructor rather than the project facilitator, nothing in the project facilitator's plans for introducing the process would have prevented any of the confusion students expressed, which were primarily regarding what a model should look like. These misconceptions persisted far into the semester, and were only resolved when the project facilitator was able to meet with each group during a second in-class meeting dedicated to working on the project.

> MATH 4500 - April 3, 2019 (Researcher's Reflective Journal)
>
> I spent time with each of the three individual groups. All three groups initially claimed they had yet to settle on a mathematical model to answer the question. As you might expect given the amount of time already expended on this project, this was very concerning to me. I followed up by asking questions that merely restated the prompts the students were directed to respond to in the development of their model (e.g., "What would it mean, in terms of numerical data, for tracking to be inequitable?"). Following my questioning, all three groups eventually came to the same conclusion, namely, that they did indeed have a model. It appears they all assumed that a mathematical model must necessarily take the form of a single equation that consolidates all of the relevant information,

> and produces a single definitive result. I had to emphasize, repeatedly, that mathematical modeling is a process, and that a model is just the mathematical formulation of the question we are trying to answer. That seemed to put most everyone at ease.

The challenge of ensuring students understand the wide variety of forms a mathematical model can take and, perhaps most importantly for this project, that it need not be a differential equation or some other kind of single equation, was also experienced during the MATH 1210 project. As with those project materials, so also would these benefit from an introduction of the modeling process that includes a guided example. The "gas station" problem from the GAIMME report could serve this purpose well.

3.3.5 Lessons for *Monitoring Student Progress*

There is no indication from student work, nor from the reflections of the project facilitator, that students faced any mathematical challenge for which they lacked the expected prerequisite mathematical content knowledge.

That being said, students did appear to either struggle with mathematically formalizing their models, or perhaps they never attempted to do so. The available data is inconclusive in this regard. Whatever the case may be, two groups relied on direct comparisons of different proportions of population groups and the third calculated relied on simple linear regression applied to a number of National Assessment of Educational Progress (NAEP) data sets. However, none of the groups produced any sort of formal mathematical expressions, equations or inequalities, that serve as generalizations of the specific case they used when implementing their model. They did each explain, to varying degrees of specificity, what their understanding of equity means in terms of numerical data, as well as the relationships that demonstrate how the data they used implied the conclusion they reached. Given additional feedback, they may have been able to produce a more general result that could be applied to a wider range of situations.

The implications of this fact for the improvement of project materials lie primarily in

the previously mentioned guided exploration of the modeling process centered on the "gas station" problem, and in the need for additional questions intended to assist in monitoring student progress. To this end, relevant questions will be added to the *Monitoring Student Progress* section of the materials, intended to help students recognize the ways in which their work can be generalized and applied to a wider range of situations and, if possible, related problems.

Furthermore, the examples located in the *Sample Approaches* section should, of course, include the kind of mathematical formalism the course instructor should expect from students. The inclusion of formal mathematical language will serve as both a tool for providing appropriate feedback on the initial development and subsequent improvement of their model—by making visible the thought process involved in the development of a rigorous model—as well as a road-map for teachers with students that would benefit from additional challenge.

As originally planned, this project intended that students revise their model at least once—beyond merely altering their selection of variables due to challenges they faced in locating relevant data. As enacted, this revision was supposed to take place following the reading assignments that specifically criticize tracking. It was intended that groups or students which had not considered race, ethnicity, or gender identity important considerations in the development of their model would recognize the need to do so. Unfortunately, what the project facilitator assumed would be the obvious takeaways were, in practice, not. There are, of course, a number of reasons why students may have not revised their models to address the issues brought up in the reading assignments.

First, their model may have already incorporated the relevant information regarding racial/ethnic demographics, as was the case with one group.

Another explanation may be that the students never completed the reading assignment. Given their other responsibilities in the clinical teaching experience in which they were engaging at the time, they may have viewed project-related tasks as less of a priority. While their second reflection journal entry could be cited as evidence as to whether a

student completed the reading assignment, they may have come across similar information in other ways (e.g., during a discussion with their fellow group members). If they never completed the required readings, they may have not come across data that would suggest the need to revise their model.

A third reason students recounted no revision on their technical reports may be a limited understanding of the modeling process. As stated elsewhere, many students held unrealistic views of the mathematical modeling process. One view expressed by a number of students appears to suggest that, rather than beginning with a basic model and slowly adding complexity to account for additional factors, they believe the first model they develop should already incorporate everything.

> MATH 4500-15 (Student Reflection Journal 1)
>
> I am surprised at how difficult it is to transform an everyday question into a model in general. It's difficult sifting variables into the language of mathematics. There are way too many variables out there, limited data, and thus we have to make many assumptions. I guess I am surprised how much critical judgment is needed for mathematical modeling.

> MATH 4500-42 (Student Reflection Journal 1)
>
> I find it interesting how many variables, assumptions, and extra information we need to take into account. In [another course] I found that analyzing and evaluating qualitative data could be done without extensive work. However, this model seems to need much thought on variables and assumptions, on top of that it is difficult to evaluate data for a person [*sic*] perceptions, class structure, and etc due to the fact that there are so many details and variables that have to be taken into account.

Whatever the case, the importance of iteration in the modeling process—and in particular, revision—dictates the need for a change to project materials to ensure students both understand that importance and also have the opportunity to revise their model, preferably

multiple times. The guided "gas station" problem introduction to modeling does demonstrate, to some degree, how revision plays a pivotal, and continual, role in mathematical modeling. To further emphasize this lesson, additional changes are warranted. Additional pedagogical guidance will be provided to teachers, including questions that can be asked repeatedly throughout the modeling project, to help students recognize the need for revision as well as possible directions to take for a model's improvement.

In addition to increasing students' appreciation—and participation in—the process of model revision, students would also benefit from additional guidance in the role of assumptions in modeling. This is not to say, however, that students did not recognize how assumptions help create, but also limit, mathematical models. In fact, each group appeared to recognize that assumptions placed strict limits on the conclusions that can be drawn from a model's implementation. One student, in at least partial recognition of how assumptions can serve to replace specific data at the cost of limiting a model's applicability, wrote:

> MATH 4500-34 (Student Reflection Journal 1)
>
> The most surprising thing that I have learned while doing this project was how little data there is on tracking and non-tracking schools. There are a lot of assumptions that we need to make for this project.

One group appeared to recognize that limitations resulted from their assumptions, but did not provide any further explanation of what those limitations might be.

> MATH 4500-1,34,42,48 (Student Technical Report)
>
> Our assumptions limited the application of our model because they did not account for the numerous variables that were either difficult to gauge and/or could not be found in the studies.

Another group, the group that relied on simple linear regression, acknowledged that their assumption that standardized test scores accurately reflect student understanding of concepts may not accord with reality.

> MATH 4500-15,21,33,46 (Student Technical Report)
>
> For test scores, we assume that the tests actually measure the degree of student

> mastery of learning objectives (i.e. internal validity). We are also assuming that tests aren't growth-mindset based (i.e. you only have one shot of tasking the test, and that's your test score).

On the other hand, they did not appear to recognize some of the more important assumptions implicit in their model. For example, they assumed that a school that practices tracking, and that has a relatively high percentage of AP test-takers or scores on AP exams, reflects for the entire student body. In other words, their model may indicate that tracking is equitable, when in reality the disproportionate representation of different students in AP courses is indicative of the exact opposite.

In light of these and other concerns, the student materials for the next iteration of this project must include a greater emphasis on understanding implicit assumptions and should provide the teacher with questions that can be used to guide students to recognizing them.

Finally, students would benefit from a greater understanding of the use of parameters for both generalizing and analyzing existing models. In fact, this may be the most important goal for the next iteration of project materials. This is because a thorough understanding of model parameters requires understanding of both model assumptions and the importance of revision and iteration. Specifically, parameters are, in essence, algebraic representations of model assumptions. Altering the values of parameters and assessing how this changes the model serve as tools for understanding the ways in which a model can be improved.

3.3.6 Lessons for *Orchestrating the Final Discussion*

While this enactment did not allow for individual group presentations, it did include a final discussion of the social justice topic. As to whether this discussion can provide evidence that the decision to apply the 5 Practices for Orchestrating Productive Discussion in Mathematics, the answer is likely no. The change in format makes evaluation a challenge, as noted by the project facilitator:

> MATH 4500 - April 26, 2019 (Researcher's Reflective Journal)
>
> Today was the final day of the project. Unfortunately, due to time constraints,

> student groups were unable to present their models and conclusions. In place of presentations, I opted to briefly explain the results and approaches taken by each group. I tried to incorporate the Five Practices (by the sequence in which I introduced project approaches) to guide the discussion, but I don't know that the results were particularly effective in this case. Perhaps in-class presentations would have worked better?

The difficulties experienced, however, are almost certainly not the result of student factors. Rather, they stem from the challenge of incorporating a project into a course which is already filled to the brim, so to speak, with content.

3.3.7 Lessons for *Sample Approaches and Models*

As with the other projects, a lack of mathematical formalism was apparent in student technical reports. Neither of the two groups who relied on proportional reasoning made use of mathematical symbolism to describe their model:

> MATH 4500-8,17,39,41 (Student Technical Report)
> In our model we are defining equity as there is proportional representation of students of different demographics (specifically race) among all academic tracking programs.

> MATH 4500-1,34,42,48 (Student Technical Report)
> From the important attributes listed above, we decided that an answer to the project question "looked like" a change, or no change, in the percentage of ACT/SAT participants among tracking and non-tracking schools.

This result clearly indicates the project materials need to include the kind of formal mathematical expressions teachers should hope students produce.

The fact that one group attempted make use of simple linear regression should not be overlooked. Students in an upper level course for pre-service teachers, like the one in which this project was tested, are likely to have experience with statistics and may default

to statistics as the tool of choice. Unfortunately, available data is quite scarce, severely complicating the use of regression techniques. Given the likelihood of reliance on regression techniques, and the lack of appropriate data to which regression techniques can be applied, the project materials may benefit from including an intermediate model by a hypothetical group that takes exactly this approach which, upon recognizing the scarcity of relevant data and weakness of certain correlations, opts to return to a proportional reasoning method. This would allow the teacher to anticipate this approach and provide students with feedback that could lead them to recognize the limitations of this approach. All this being said, the future may bring additional data sources, subverting any need to discourage regression techniques.

3.3.8 Lessons for *Resources*

As mentioned elsewhere, students overwhelmingly held misconceptions about what constitutes a mathematical model. In addition to their initial frustrations in this regard, many students experienced significant challenge procuring relevant data necessary for implementation of their models. For example,

> MATH 4500-34 (Student Reflection Journal 1)
>
> The most surprising thing that I have learned while doing this project was how little data there is on tracking and non-tracking schools.

In addition, two groups contacted the project facilitator in hopes of receiving guidance in this regard. This difficulty, taken together with the same difficulty experienced by MATH 1210 students, led the project facilitator to question whether students should not be provided with data, rather than tasking them with locating their own.

> MATH 4500 - April 10, 2019 (Researcher's Reflective Journal)
>
> Two groups have contacted me outside of class time with concerns about locating data with which to implement their model. Both groups insisted they were unable to locate any relevant data. I decided to point them toward the 2013 and 2016 Brown Center Report on American Education, as well as the Civil

> Rights Data Collection on the US Department of Education's website. Based on the similar struggle expressed by students in the MATH 1210 course, I am having second thoughts about asking them to locate their own data. I did not wish to bias the development of their model by pointing them toward data from the beginning, which could certainly imply the use of one method over another. Supplying data sources from the beginning would also take away some of the challenge and frustration experienced by professional modelers. In short, I don't know which approach would be better at this point. I only hope that my providing data for these groups does not alter their model.

Ultimately, students will likely benefit most from something of a balance between simply providing them with sources and letting them seek out their own. In this vein, future versions of this project will provide students with resources, but only after certain milestones have been reached. Specifically, the resources section of this project will include a list of data sources that will be distributed to students following the class meeting wherein the project is introduced. While this potentially removes a significant challenge faced by practicing mathematical modelers, the documents and other sources provided do still require a substantial amount of parsing on the part of students. This should serve to preserves a significant degree of authenticity, while diminishing students' frustrations, as they will know that, while there is a lot of information through which they must sift, that the data sources do indeed contain relevant data.

As mentioned elsewhere, students avoided the use of formal mathematical language, perhaps due to the lack of such language in the project materials. For example,

> MATH 4500-1,34,42,48 (Student Technical Report)
>
> From the important attributes listed above, we decided that an answer to the project question "looked like" a change, or no change, in the percentage of ACT/SAT participants among tracking and non-tracking schools.

The use of the scare quotes around "looked like" is a direct reference to language used in the project materials, which asked students to describe what a solution would look like.

The fact that none of these groups use formal mathematical language is likely a result of the lack of direction to do so in the project materials.

To address this kind of deficiency in the project materials, the handouts with questions, originally posed as a list students should use to inform their discussions, will be expanded to require students to answer each question, and additional questions will be added to require the use of formal mathematical language.

Students would likely benefit from at least one additional opportunity to write a reflection journal regarding their learning experience. In addition, the language used in prompts should be modified to be more direct. While this risks alienating students should the language be poorly phrased, it should prevent students from avoiding thinking deeply about the social justice topic, which would undermine one of the primary goals of the project. The researcher's reflection journal entry immediately following the final discussion included the following related note:

> MATH 4500 - April 26, 2019 (Researcher's Reflective Journal)
> As an aside, I think that an additional reflection journal, after all of the presentations and in-class discussion, might be a more effective way of getting students to think critically about the issue, though I will probably keep the action plan as well. As I did in the MATH 1210 case, I can't help but wonder whether reflection journal questions should have been more pointed. I was attempting to minimize the Hawthorne effect by posing questions in such a way as to gather information about students' beliefs in a roundabout way. I now wonder whether that is the right approach. On the other hand, there were a lot of calculus students who accused me of pushing a liberal or leftist agenda, so I don't know how more pointed I could be. I will have to think on this some more.

Since then, and with an additional project enactment to learn from, it has been determined that questions will, in fact, be more "pointed." Finally, the inclusion of an additional reflection journal entry following the final discussion, during which students will grapple

directly with the social justice topic, will serve both the ends of the project goals as well as provide additional data point for the researcher for analyzing future project enactments.

As one might expect, the requirements directing groups to explain their model assumptions, and associated limitations, provided valuable insights into student understanding of assumptions.

A clear trend has appeared across all of the projects regarding which technical report requirements were least effective at helping students achieve the intended learning objectives, namely, those which include directions for things that may be categorized by some as overly pedantic.

For example, a requirement that directs students to avoid the use of first-person singular tense, while obviously important for a group effort, is likely unnecessary, as students will more than likely naturally make use of other acceptable tenses, such as first-person plural.

Another requirement that now seems completely unnecessary involves the use of appendices, specifically in regards to proper labeling with capital letters or Roman numerals. Since none of the groups in *any* of the projects included appendices of any kind—opting to include figures and tables in the body of the text—the requirement is likely superfluous.

As with the other two projects, the most important alteration to the report specifications must be the use of more concise language in some requirements, the condensation of other multiple requirements to fewer, and the removal of what are now judged to be superfluous requirements (e.g., requirements for tense and labeling of appendices).

3.3.9 Summary of Project Revisions

Each project has undergone a significant revision, and the materials for each have been greatly expanded. Table 3.2 provides a summary of the changes to project materials.

Project Component	Description of Improvements
Initial Prompt	Initial project prompts have been altered to provide teachers with a responsible distance from the entity making the social justice claim. For example, rather than directly asking, "Is the 'War on Drugs' racist?", students are tasked with evaluating the Drug Policy Alliance claim that the War on Drugs is racist.
Learning Objectives	Learning objectives have been altered where the intended Learning Level differed from the actual Learning Level that appeared during test-enactments. Some objectives have been split into multiple objectives and additional objectives have been added where it was deemed appropriate.
Definitions & Notation	Mathematical definitions with accepted notation have been added, as well as definitions relevant to the social justice topic, as understood by professionals of the related disciplines.
Pedagogical Notes	Pedagogical notes have been added based on enactments to ease facilitation for teachers using the materials, including recommendations for facilitating discussions of the social justice topic.
Sample Approaches & Models	Sample approaches have been added that include intermediate models, incorporate models employed by students, and models that can reasonably be obtained by students with the intended level of prerequisite knowledge.
Reflection Journal Entries	Reflection journal prompts have been altered to be more direct, lessening the possibility of students avoiding discussing the social justice issue, and required readings have been incorporated into each of the reflection journal assignments.

Table 3.2: Summary of improvements to project materials based on the results of test-enactments.

The *Overview* section includes a more descriptive summary, a new list of related standards and indicators from the AMTE Standards for Preparing Teachers of Mathematics, a list of revised learning objectives, a list of all required materials as well as a list of supplementary materials for teachers hoping for a more scaffolded mathematical modeling experience. Following the lists of materials is the section with new, more helpful, definitions, mathematical and otherwise.

The *Project Overview* sections for preparation, introducing the project, and orchestrating the final discussion include numerous pedagogical notes for ensuring the project runs smoothly, as well as providing helpful questions to guide students while they progress, check understanding during their presentation, and to engage in thoughtful discussions of the social justice topic.

The *Sample Approaches and Models* section now includes vignettes outlining the development of possible models. Vignettes are based in large part on the approaches taken by students during test enactment, providing teachers with realistic expectations for how students will engage with different aspects of the modeling process for each project. The projects *"Tracking" in School Mathematical* and Global Warming & Hurricanes both include two vignettes, while *Racism in the "War on Drugs"* includes a single.

The only project to undergo a major transformation in the intended mathematical content was *Racism in the "War on Drugs"*. Based on the results of the test-enactment, the calculus-based tasks have been removed. This is due to the fact that, while they did require students to interpret some data in real-world terms, the bulk of the tasks involved were procedural in nature, and led students away from the mathematical modeling process

The *Resources* section for each project has also been altered. The number of Reflection Journal entries has been increased to three in each case, and each reflection journal entry requires one or more accompanying readings related to the social justice topic of the project.

In addition, existing task sheets have been reorganized to act more as worksheets students can use to help organize their engagement in the modeling process. It is these reorganized worksheets that are intended to serve as additional scaffolding for teachers new

to mathematical model, for those with students who lack modeling education experience, and for those with little time for additional preparation time.

Inevitably, future enactments will reveal additional ways in which the project materials can be improved. For the time being, however, they can serve not only as resources for teachers and math teacher educators wishing to incorporate modeling into their classroom, but as templates for future projects.

CHAPTER 4

IMPACT OF PROJECTS ON SOCIAL CONSCIOUSNESS

Since its initiation, this project included a secondary objective of monitoring student work for evidence of an evolution of social consciousness. This effort was motivated by the results of a small number of research studies involving attempts to influence the development of teachers' beliefs that relied on action research, reflection, and Constructivist teaching strategies (see Caudle & Moran, 2012; Conner, Edenfield, Gleason, & Ersoz, 2011; Turner, Warzon, & Christensen, 2011; Jao, 2017; Katz & Stupel, 2016; Lloyd, 2013; Lloyd, 2018; Loucks-Horsley, 2010), and from general insights from Conners' and Halligan's model of belief development (Connors & Halligan, 2015). Since that time, student work has revealed that the potential impact of social justice mathematical modeling projects on student beliefs may be far greater than anticipated. In response to this unexpected potential, the initial review of student work samples was significantly expanded. This chapter includes preliminary results from an analysis of student work in light of Conners' and Halligan's model of the stages of belief development (Connors & Halligan, 2015). Recall that in their model, the stages of belief begins when an individual encounters a precursor, which initiates a search for meaning within the individual's existing web of beliefs, and then the either conscious or unconscious evaluation of candidate beliefs intended to explain the precursor. Following this evaluation, one of these candidate beliefs may be accepted, which has long term effects for the interpretation of future precursors, searches for meaning, and candidate belief evaluation (Connors & Halligan, 2015).

Student writing assignments for each individual were read and situated within the five stages of belief formation to gain insights into the evolution of students' beliefs and, therefore, of social consciousness. Evaluation of student work was performed by the researcher/project facilitator, and the course instructor and researcher reviewed and reflected on the in-class discussions associated with the project. As this is a preliminary study, no

formal instrument was utilized for evaluating student work with respect to changes in belief.

A substantial number of participants appear to have experienced some degree, however small, of productive evolution of social consciousness; though, as is the case with any research relying on a form of self-reporting and given that participants were aware that their writings would be used for research purposes, we cannot discount the possibility that changes in participant writings regarding a project's social justice topic mathematics may be due to the Hawthorne Effect, that is the tendency of the participants to behave differently than they normally would because they know they are taking part in a study (Corsini, 2001; McCambridge, Witton, & Elbourne, 2014). In addition, it must be pointed out that while we situate students' written submissions within the five stages of beliefs formation described above, it is likely that participants experienced multiple iterations of stages of belief formation, with numerous beliefs, and at multiple times throughout the project. That is, even if the statements found in the student work examples below do in fact align with the stage of belief development we suggest, they may come from entirely different iterations of belief development, or from the development of different beliefs altogether.

The creators of the stages of belief framework, recognizing the limits of their account of belief, advise caution operationalizing their model, though they do "suggest that a complete theory of belief will need to account for at least these five stages" (Connors & Halligan, 2015, p. 10). Following their recommendation, while we attempt to align student work with the framework to gain insight into the process of pre-service teachers' belief development, we recognize the complexity of belief formation and the limits of the stages of belief framework and, therefore, do not employ the framework as a strict coding system.

In the paragraphs that follow we provide an account of possible changes to student beliefs over the course of each project. Each section begins with an explanation of the data collection tools. This is followed by a sampling of individual student stories, as told through their own writings, regarding how their beliefs changed over the course of the project.

4.1 Racism in the "War on Drugs"

Data collection for this project made use of three reflection journal entries intended

to provide insights into students' beliefs about the existence of systemic racism in the US criminal justice system.

The first reflection journal included one prompt:

1. What is the most interesting thing you have learned during this project so far?

The second reflection journal included two prompts.

1. Write down something you learned while engaging in this part of the project that really stuck out to you and explain why it did so.

2. Did the results of your model line up with what you believe (or believed) about the War on Drugs before starting this project? Explain.

The third reflection journal included one prompt.

1. Write down something interesting you learned about the War on Drugs over the course of this project, and why you found it interesting.

Of the 255 students who engaged in the project, 166 consented to having their work used for this research. A random number generator was used to select 20 students—a number of students comparable to the numbers from the other projects—from among those 166.

Of the 20 students selected, 19 submitted all 3 reflection journal entries, and the remaining student submitted the first 2. By far, this collection had the greatest number of student response that avoided talking about the social justice topic. This may be due to discomfort participants felt about openly discussing issues of race, an anxiety which appears to have diminished somewhat over the course of the project.

4.1.1 Student Stories About Racism in the "War on Drugs"

Of the 20 research subjects, 3 appear to have begun the project expressing criticism for the War on Drugs due to racism, all of whom appeared to retain that view in their second entry, and two of whom did the same in their third—the other student's final reflection journal entry was ambiguous on the issue.

Of these two, one began with an unclear statement about correlations, but ultimately concluded their statement by agree that the War on Drugs is, in fact, racist, resulting in the work sample being coded as indicative of productive beliefs about the social justice issue.

> MATH 1210-001-77 (Reflection Journal 1)
>
> Before we began this project, I had a general idea that [the] numbers of those being arrested for drug use didn't correlate to the numbers being arrested for drug use based on race. However, many factors were very surprising to me as far as the extent to which the war on drugs is in fact very racist.

This student appears to have begun the project already believing the War on Drugs to be racist. In other words, the precursors in the project directing the student to consider the question of whether the War on Drugs is racist already found fertile ground in the student's existing web of beliefs for immediate accommodation. This belief appears to have been further strengthened over the next part of the project, as evinced by the student's next reflection journal entry.

> MATH 1210-001-77 (Reflection Journal 2)
>
> This part of the project was very telling. The part that really stuck out to me was the percentages of people arrested for drugs. Not just the differences in races of who gets arrested but the amount of people that are arrested for drug use or distribution. Even though it's not directly related to the question, it's crazy to me that most people in prison are there for drugs. I think that this represents a huge flaw in our countries [*sic*] priorities in the justice system. Rather than being helped for drug addictions, we are criminalizing, which I think it [*sic*] counterproductive and just as concerning as the War on Drugs itself.

The student has apparently begun to evaluate, and quite possibly accept, a novel belief about the criminal justice system at large. This is evidence of how existing beliefs can impact the development of new beliefs. The student's work in examining data sources served as precursors for new beliefs, the acceptance or rejection of which were likely impacted by their

strengthened belief about injustice in the War on Drugs. This (apparent) belief regarding mass imprisonment in the US appears to have been retained for the remainder of the project.

> MATH 1210-001-77 (Reflection Journal 3)
>
> While there were many interesting things, one that I find the most surprising and interesting [is the] fact that the war on drugs has created the largest prison population on earth. While this isn't exactly related to the question, I think that this represents the bigger problem, that creates the disparity that we calculated.

It would have been helpful to understand what mechanisms the student believes are involved in mass incarceration, as well as more precisely how mass incarceration relates to racism in the War on Drugs. In any case, their last statement—through the causal relationship they describe—serves as evidence of the interrelationship between the development beliefs about racism in the War on Drugs and mass incarceration as a problem in US society.

Of the remaining participants, 1 expressed agreement with stereotypical representations of drug users as impoverished.

> MATH 1210-002-25 (Reflection Journal 1)
>
> The most interesting thing I have learned in this project so far is that socioeconomic status can be tied to drug usage. I think this is interesting that that stereotype is in some cases accurate.

It is important to note that this actually provides no evidence either way of whether this student believes the War on Drugs to be racist in any way. It does reveal a small amount of information about their existing web of beliefs, given that this particular fact was the one the student decided was most interesting. The fact that the student latched onto this piece of information they located, among the vast amount of others they entertained by this point of the project, may be an externalization of their attempt to accommodate the precursor into their existing web of beliefs, while retaining the belief that racism is not a major factor in the War on Drugs. In other words, in an attempt to avoid accepting racism as a major factor, the student may be offering a more acceptable—at least to the student—explanation

that identifies class, rather than race, as the characteristic against which the US criminal justice system discriminates.

By their next journal entry, the student appears to have started considering the possibility of racism in the War on Drugs.

> MATH 1210-002-25 (Reflection Journal 2)
>
> One of my main roles in this section of the project was researching incarceration rates for drug use and possession based on race. I learned that African Americans and Hispanics are arrested more often that [*sic*] Caucasians and often have longer sentences. I found this interesting because when beginning this project, I did not think that there would be that significant of a difference in incarceration rates between races. Now I have seen a variety of statistics and reports that show that there is a difference that needs to be considered when completing our model.

From this, it appears that the initial assessment was correct. That is, the student did not believe race would be an issue in the War on Drugs. While it appears that the student is evaluating the candidate belief that the War on Drugs is, in fact, racist, their writings later in this entry suggest they retain their resistance. They write:

> MATH 1210-002-25 (Reflection Journal 2)
>
> Our model demonstrated that people of African American or Hispanic race are incarcerated for drug possession and use more frequently and often for longer sentences. We believed at the start that the War on Drugs may have been unequal in race but not necessarily racist. Our model demonstrates that race does play a part in the War on Drugs but is not the only factor that influences sentences.

In spite of the individualistic language (i.e., the use of the word “you”) employed in the prompt eliciting this response, the student rephrases their initial belief as shared by the rest of the group, and then argues that race is only one of several factors influencing sentencing

practices—this is certainly true as all issues are multifaceted, but there is an argument to be made that if race is a factor in unequal sentencing, then sentencing practices are, in fact, racist.

In their last reflection journal entry, the student concludes:

> MATH 1210-002-25 (Reflection Journal 3)
> I learned a lot about prison incarceration rates for white and for African Americans, however I found it interesting that there was not as much information readily available about Hispanic people and people of other races, if we can only access information about a couple specific races, that sets up the war on drugs to be racist in the first place.

This final statement could have multiple meanings. For example, the student may be asserting that lack of recording of incarceration statistics for different races empowers those with a racist agenda. Based on their previous writings, it may be that the student has conceded that the data does point to racism in the War on Drugs, but contends that the data is incomplete and so no conclusions on the issue can be certain, thus attempting to hold onto the previous belief that the War on Drugs is not racist. Or perhaps this student is attempting to exhibit a healthy skepticism, a vital characteristic of any successful researcher. Unfortunately, we cannot know for certain. In any case, the second and third entries were ambiguous enough to warrant being coded as indeterminate.

The remaining 16 students either expressed ambiguous statements about race and the War on Drugs in their first entry, or avoided touching the issue of race entirely. Of those 16 students, 5 continued to either avoid discussing the issue of race or gave ambiguous statements necessarily coded as indeterminate in their remaining entries, 6 expressed views critical of War on Drugs due to racism in both of their remaining journal entries, 3 expressed views that characterize the War on Drugs as specifically *not* racist in their second entries and ambiguous statements in their third, and 2 expressed ambiguous views in the second entry, one of whom did not submit their third journal entry and the other of whom expressed views critical of the War on Drugs due to racism in their third entry.

One interesting result came from a student in a group that apparently all looked at different data and came to different conclusions. Their first entry, as stated above, is ambiguous, as they avoid discussing race entirely. By the second entry, the student, now more willing to discuss issues of race shares the following:

> MATH 1210-003-28 (Reflection Journal 2)
>
> Interestingly, our team conclusions varied by the variable we looked at. Some variables led our group mates to one conclusion, while another led to a different one. For my own, I chose to look at incarcerations on a state level and found a shocking discrepancy in black incarcerations versus white in the years following drug enforcement legislation. I looked specifically at Louisiana because it was the state with the highest rate of incarcerations, and the ratio of black to white prisoners was 4:1. I initially thought this could have been skewed due to demographics of Louisiana, however according to the census, Louisiana is only 32.6% black. This was in line with what I had previously thought, as I believed there were some variables that would show this discrepancy.

This student describes at least two candidate beliefs they entertained over the course of this part of the project, though they inform us of their existing earlier belief that there is racism in the War on Drugs. Ultimately, they find that the evidence they explore supports and strengthens the hold of their existing belief, and the other candidate belief—that the prison population demographics are proportional to the state populations demographics and therefore the War on Drugs is not racist—is rejected.

By the third entry, nothing appears to have changed, though they do appear to suggest that, in spite of the concerns of the project facilitator, the project was well-suited to the calculus course in which it was enacted:

> MATH 1210-003-28 (Reflection Journal 3)
>
> This project has taught me so many different things, however it was most exciting to see how we could relate math we had learned in Calculus to real world issues that affect people. However, I think the most important thing I've learned

> with this project is the seriousness and scope of the issue. I knew on a basic level that there was a discrepancy between black and white incarcerations due to drug-use, but I was unaware of the scale of that discrepancy. Seeing the graphs and data for the issue was shocking, and it's motivated me to be more aware of the issue.

This student appears to have begun the project with a high level of social consciousness. This characteristic appears to have was retained—and likely even enhanced based on their statements regarding the new information they learned—over the course of the project.

Student ID	Reflection Journal 1	Reflection Journal 2	Reflection Journal 3
1210-001-77	+	+	+
1210-003-35	+	+	+
1210-002-02	+	+	I
1210-001-16	I	+	+
1210-001-18	I	+	+
1210-001-37	I	+	+
1210-001-63	I	+	+
1210-002-24	I	+	+
1210-003-28	I	+	+
1210-001-67	I	+	I
1210-002-11	I	I	+
1210-001-13	I	I	I
1210-001-57	I	I	I
1210-001-92	I	I	I
1210-002-05	I	I	I
1210-003-23	I	I	N/A
1210-001-23	I	-	I
1210-001-51	I	-	I
1210-003-13	I	-	I
1210-002-25	-	I	I

Table 4.1: Student affect throughout the *Racism in the "War on Drugs"* project. Work samples coded as positive are denoted by +, negative by −, and indeterminate by I. Rows for students who underwent a productive evolution of social consciousness are highlighted.

Table 4.1 provides a summary of the coding of student work samples for this project. Recall that in this case, a + indicates the student expressed views that can be characterized as pro-human rights, a − indicates the student expressed views that are either racist or hos-

tile to criminal justice reform, and I indicates the researcher was unable to determine the student's disposition toward the social justice issue. Notice how the number of indeterminately coded student work samples decreases somewhat as the project progresses. Whereas 16 first reflection journals were coded indeterminate, only 7 second reflection journal entries and 10 third reflection journal entries were coded the same way. This may be evidence that students became more comfortable discussing issues of race over time.

Note that highlighted rows in indicate the associated student experienced and evolution of social consciousness according to the characterization described in Chapter 2. By this characterization, 8 students underwent a productive evolution of social consciousness of the course of the project. Note that students above the colored band began the project expressing beliefs coded as positive, and therefore may have been unable to experience and evolution of social consciousness.

4.2 Global Warming & Climate Change

The decisions made by the researcher to expand the scope of the affective goals to focus on appreciation for science literacy may have been detrimental to the success of the affect goals associated with this project because prompts lacked sufficient specificity to elicit precise answers from students regarding their thoughts and feelings about climate change. So while the first reflection journal included a prompt that specifically asked students to share their opinions on the issue, the attempt at being seen as an unbiased arbiter resulted in unclear data for assessing student affect in the remaining reflection journal entries.

This, in turn, led to the conclusion reiterated time and again in Chapter 3 of providing students with more pointed questions in their reflection journals. It also necessitates an alteration in what kind of affective changes could be evaluated from the student work submissions associated with this project.

Fortunately, the prompts did elicit ongoing student perspectives on the mathematical modeling process and their ability to engage in it. Therefore, the impact of the project on student beliefs, for this specific project, will focus on student affect toward mathematical modeling. In particular, the researcher primarily focused on understanding how students

described *their ability* to engage in the modeling process, as students in a multivariable calculus course certainly almost all appreciate the importance of mathematical modeling in the abstract, and on assessing student affect toward mathematics as a whole.

Data collection for this project made use of three reflection journal entries. The first reflection journal included two prompts intended to provide insights into their beliefs about climate change and the modeling process.

1. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?

2. What is the most interesting thing you have learned so far?

The second reflection journal included these same two prompts, and the third included the modeling prompt.

Of the 17 students enrolled in the course, 13 consented to having their work used for this research, all of whom submitted all 3 reflection journal entries. Each entry provided helpful insights into student perspectives on modeling. This may be due to the fact that students were repeatedly asked the same prompt, something which ought to inform the selection of prompts for future project reflection journals.

4.2.1 Student Stories About Hurricanes and Mathematical Modeling

Of the 13 research subjects, 5 began the project with confidence in their ability learn and engage in the modeling process, a sentiment which appears to have been reinforced by the project for at least 4 of them. The remaining student of those five responded in the third reflection journal in a way that was coded as indeterminate. 3 students began with clear negative statements about their ability to engage in the modeling process, one whose second and third reflection journal entries were coded indeterminate, and the remaining 2 who appeared to have gained confidence in their ability to model. The remaining 5 students began the project with indeterminate statements, 3 of whom exhibited a positive affect toward their ability to model by the end of the project, and 2 of whom gave responses

coded as indeterminate in the remaining two reflection journals. In summation, 4 students' final responses were coded as indeterminate, and the remaining 9 were positive.

However, it must be noted that, during coding of student work for this project, it became apparent to the researcher that the binary coding system, of classifying student work as exhibiting a positive or negative affect (and Indeterminate otherwise), may be insufficiently refined a system for assessing student affect toward mathematical modeling. On the other hand, the cognitive processes underlying even these ambiguous responses does provide a greater degree of illumination of student affect, if only slightly, when evaluated against the stages of belief development. For example,

> MATH 2210-5 (Reflection Journal 1)
> Currently I am not [too] sure on [mathematical] modeling, I still feel a little confused by the topic as it can seem overwhelming periodically. I do enjoy learning this process as I feel it is something that I will be using on a frequent basis after I am through with college.

The student clearly lacks confidence in their ability to model, but seems to express a positive affect toward learning the process, so the response was coded as indeterminate. On the other hand, viewed through the lens of belief development, the precursor, in this case the reflection journal prompt, has initiated the student's search for meaning in their existing web of beliefs. The student's current beliefs appear to be in conflict with one another, if not diametrically opposed. The student views modeling as something outside of their capacity to do, but holds other beliefs that help the student recognize not only the joy they experience in learning the modeling process, but also the important practical motivations. In other words, the student's existing web of beliefs appears to allow for the possibility of evaluating and accepting new candidate beliefs about their ability to mathematically model. The acceptance or rejection of such a candidate belief would necessarily have the secondary impact of either reinforcing or weakening the strength of the student's belief in their own ability to learn new things.

The next reflection journal entry suggests that the student has come to hold, if only weakly, a belief in their ability to engage in the modeling process.

> MATH 2210-5 (Reflection Journal 2)
>
> As we have learned more in class about parameterizing equations and solving integrals and volumes of 3d objects I feel a lot more comfortable with coming up with a model for our work. In the beginning of our class I wasn't sure as to how we were going to be able to come up with a model that would accurately represent a hurricane. Since we have added more and learned more about the way that mathematical modeling works I feel more comfortable with coming up with a model to give a good representation of a hurricane.

This entry was coded as indicative of positive student affect regarding their ability to engage in the modeling process and toward mathematics in general (the latter is certainly less surprising at this level of mathematics).

Obviously, statements about the strength with which a belief is held are highly subjective, especially when gleaned from a written statement, but it appears from their third reflection journal as if the student's belief in their modeling capabilities is further strengthened over the remainder of the project, as have their feelings toward modeling in general.

> MATH 2210-5 (Reflection Journal 3)
>
> I feel a lot more confident about mathematic [*sic*] modeling after our final project, I think the overall concept of the project became more and more clear as we progressed through the project. I have a better opinion of modeling now that it doesn't have to be a perfect representation at the beginning but it is something that you constantly work towards and refine to make it more understandable as you learn more.

This was also coded as indicative of a positive affect toward the modeling process. It also reveals that the student has come to understand modeling as the process it is.

This challenge of an insufficiently refined coding system appeared even across responses from the same student. For example, another student wrote the following in their first reflection journal entry:

> MATH 2210-9 (Reflection Journal 1)
>
> I feel like mathematical modeling can be a good way to look at the effects and impacts of things like [hurricanes and climate change]. Honestly, I'm not super enthused about having to take part in it. I'd rather this class just be composed of quizzes and tests and the usual classroom stuff, but there's not anything I can do about that either.

This response was coded as indeterminate, as it appears that the student sidestepped the question of how they feel about their own ability to engage in the process, opting instead to express their frustration with the nature of the course. Viewed through the lens of belief development, this response may be evidence that the precursor has been immediately accommodated by the student's existing web of beliefs, specifically by beliefs about learning and the appropriate format of a mathematics course. Since this project both presents and requires a method of learning outside of what the student believes to be appropriate forms of instruction, the belief appears to be rejected out of hand.

Their next entry reveals little has changed.

> MATH 2210-9 (Reflection Journal 2)
>
> I'm still not overly enthused about having to take part in mathematical modeling, but I think our vector field model is pretty good and hopefully it doesn't get much harder than that.

Does this student have confidence in their ability to mathematically model? It is possible, but in no way clear. Given the ambiguity, this response was coded as indeterminate. As a window into the student's other beliefs, this response is suggestive that the student retains a strongly-held belief about how learning should take place, but also that the student's resolve may be weakening slightly, as they express a small degree of pride in their work.

The student, clearly tiring of the repeated precursors and unwilling to provide details about how they view their ability to model, writes the following in their last entry:

> MATH 2210-9 (Reflection Journal 3)
>
> My level of enthusiasm about mathematical modeling has not changed since the last reflection journal entry.

Fortunately, many responses were not so ambiguous, which gave insights into some profound changes in student affect, the understanding of which is further informed by the stages of belief formation.

One student, who self-identified as a pre-service math teacher in their work submissions, began the project with a significant degree of trepidation. Some of their work has been shared in Chapter 3, but a more complete picture of their story appears here.

> MATH 2210-7 (Reflection Journal 1)
>
> Overall mathematical modeling seems like a valuable yet incredibly daunting task. I am a huge fan a graphs or diagrams explain [*sic*] what is going on, but I'm learning that I don't even know what factors to consider when trying to put together a simple diagram. It seems to me as though years of research would be required to effectively mathematically model these issues.

They continue return to the topic further on in the entry:

> MATH 2210-7 (Reflection Journal 1)
>
> Each step in the mathematical modeling process seems overwhelming. I have a difficult time remembering all the steps because at each step I feel as though someone has put a living elephant in front of me and told me to eat it. I wouldn't know what to do. The first step is obvious but that doesn't mean I want to do it, and even if I do it what am I going to do with the dead elephants? I feel as though everything's just going to rot and stumble around me and I certainly don't want to present that to the world.

It might be difficult to find a response more clearly indicative of lack of confidence in one's ability to engage in the modeling process than this. The precursor appears to have had a strong emotional impact on the student, resulting in some interesting associations that may suggest the student's existing web of beliefs include a belief about their own inadequacy in the face of a major perceived challenge. This possibility receives additional support later in this same (lengthy) journal entry.

> MATH 2210-7 (Reflection Journal 1)
>
> I love math. The most interesting thing I've learned so far in this class is that applying math to real-world problems beyond simple physics and chemistry is really hard. This has been a bitter pill to take. learning that I don't know how to apply math to real-world situations without them being previously contrived to fit the math, is difficult to accept. It [has] been an eye-opener for me. I want to understand mathematical modeling because I would like to include that in my teaching. One of the biggest complaints students have about math is that they don't see when they will ever use it. I want them to see how math is applicable to real-world things, and I want them to get excited about solving actual problems. But I don't see how I can do this without spoon-feeding them the answer I think they should get, if I don't understand ... mathematical [modeling] myself. I'm a bit discouraged but will work hard to learn and succeed.

The student clearly wants to succeed, but appears unable to conjure any belief in their own ability to support that hope, so they rely instead on their belief that effort leads to success.

Fortunately, the next entry strongly suggests the student has started to believe in their ability to learn and engage in the modeling process.

> MATH 2210-7 (Reflection Journal 2)
>
> Up until the last time we met with our groups the mathematical modeling process it's been frustrating and daunting. After the last meeting with our group breakthrough about how to incorporate an equation that would allow our Vector field to have an eye. I was thrilled [about] this discovery and felt

> accomplished. I'm so excited to keep going, and can't wait to see what we'll figure out next. My view on the mathematical modeling process is a lot more positive now. It's difficult at the beginning of the process because you don't know what you're doing but now I know it's like that for everyone. It is an exciting Adventure to discover more accurate ways of modeling the problem at hand.

They continue further on:

> MATH 2210-7 (Reflection Journal 2)
>
> Now that I understand a little bit more about the mathematical modeling process I feel more confident in my ability to engage in that process. I'm not sure I could handle anything more complex but I am enjoying this.

The student provides one additional insight into some of the social/environmental precursors that led to this change in attitude.

> MATH 2210-7 (Reflection Journal 2)
>
> I think the most interesting thing I've learned so far is that merely because students are quiet does not mean they feel confident about what is going on. At first I thought I was the slow one in the group because I asked a lot of questions, but now I sincerely feel that I have been able to contribute to the group. We have been able to work together in solving this problem[.]

Additional successes strengthened this student's belief in their own self-efficacy, specifically with regard to mathematical modeling. Following completion of the project, this student wrote:

> MATH 2210-7 (Reflection Journal 3)
>
> Mathematical modeling is an exhilarating process involving a lot of shooting blind. I found it very satisfying to create a model of something in the real world. It was incredibly daunting at first, but I feel a lot more confident in my ability to engage in the mathematical modeling process. I feel like it's less like

> being told to eat a live elephant in front of me, and more like being handed a bag of clay and being told to make a sculpture out of it with certain parameters. If is a lot less daunting when you view it as something that you're creating, versus something that only has one right answer.
>
> If I did this project over again, I would definitely panic a lot less. The beginning of the project was hard because I felt kind of lost. Even so I would not trade the experience, I've learned a lot. I wouldn't want to [lose] the moments of realization and excitement in discovering new things. I definitely want my students to experience the success and excitement of mathematical modeling.

This project appears to have had a profound impact on this student's belief in their own self-efficacy and about how mathematical modeling kind provide a rich and enriching learning experience.

Student ID	Reflection Journal 1	Reflection Journal 2	Reflection Journal 3
2210-04	+	+	+
2210-08	+	+	+
2210-10	+	+	+
2210-11	+	+	+
2210-12	+	+	I
2210-01	I	+	+
2210-05	I	+	+
2210-07	-	+	+
2210-13	-	+	+
2210-02	I	-	+
2210-03	I	I	I
2210-09	I	I	I
2210-06	-	I	I

Table 4.2: Student affect throughout the *Global Warming & Hurricanes* project. Work samples coded as positive are denoted by +, negative by −, and indeterminate by I. Rows for students who underwent a productive evolution of social consciousness are highlighted.

A summary of the coding of student work samples for this project is provided in Table 4.2. Recall that in this case, a + indicates the student's disposition toward mathematical

modeling was positive, a $-$ indicates a negative disposition, and I indicates the researcher was unable to determine the student's disposition.

Rows are highlighted to indicate the associated student experienced and evolution of social consciousness according to the characterization described in Chapter 2. By this characterization, 5 students underwent a productive evolution of social consciousness over the course of the project. Note again that students above the colored band in Table 4.2 could not experience an evolution of social consciousness according the definition established in Chapter 2.

4.3 "Tracking" in School Mathematics

Data collection for this project made use of the following written assignments: an initial task sheet; two reflection journals, one written near the beginning of the project and one following the submission of the pre-service teacher team's technical report; and an action plan written immediately following the final in-class discussion of the modeling approaches taken by different teams and the social justice implications of tracking.

The initial task sheet included a prompt asking students to reflect on and list their own perspectives and biases regarding the issues of tracking. The first reflection journal included prompts addressing multiple facets of the project, including one intended to elicit student responses for gaining insight into their beliefs.

1. Do you agree with the organizations that claim tracking creates and reinforces social inequities and is actually oppressive? Why or why not?

The second reflection journal included two such prompts.

1. Write down what you consider to be the most important thing you have learned about the practice of tracking in school mathematics.

2. Write down a specific plan you intend to enact to ensure that you are working to challenge inequitable structures like tracking in your future career.

The action plan involved a single prompt, issued verbally by the project facilitator, to write down, in two or three sentences, a plan for how they will promote equity in their future classroom and challenge inequitable structures within their own school.

Of the 12 pre-service teacher participants, eight submitted the initial task sheet, 10 submitted the first reflection journal, 11 submitted the second reflection journal, and 11 were present to complete the action plan writing assignment. In addition, it is unclear what number of students completed the reading assignment, and therefore the degree to which they incorporated the ideas found therein in a revised mathematical model.

Furthermore, the initial task sheet prompt, wherein participants were asked to list their own perspectives on and biases regarding the issue of tracking, lacked clear insights into students' views of tracking. The reflection journal entries provided the clearest insights into the students' changing (or unchanging) beliefs over the course of the project. The action plan provided some evidence of an evolution of social consciousness in the participating pre-service teachers by way of the commitments they chose to make. However, none of the participants made specific mention of the issue of tracking itself, opting instead to focus on specific related issues like maintaining high expectations for all students, advocating on behalf of students, and challenging the practice of teacher tracking, i.e., the widespread practice of placing inexperienced teachers in charge of remedial mathematics courses.

4.3.1 Student Stories About Tracking

Of the 12 participants, three appeared to have begun the project already believing that tracking creates and perpetuates inequities, and so no belief inculcation may have been necessary or possible. For example, one student wrote in their first journal entry that, "I don't actually think that the benefits that it [tracking] provides are enough to make up for the bad effects on the other students." Their existing web of beliefs may have either already included the belief that tracking is harmful, or was well-equipped to quickly situate the precursor, quickly leading to the acceptance of the new belief.

In one of the starker examples, one participant admitted early on that their past experiences in school mathematics have contributed to their holding negative stereotypical

views of students in normal or remedial course tracks.

> MATH 4500-48 (Student Initial Task Sheet)
> Students in AP Classes/Honors are smarter, work harder, are better. Students in Remedial classes aren't as smart, hard working, or dedicated as those in AP classes. Students who get dropped down [from] AP or honors classes didn't try hard enough. Students at the 'normal' level are equivalent to those in remedial classes.

The participant provides us with some valuable insights into their "web of beliefs" on the nature of how they understand success in school mathematics. Four weeks later, the same participant wrote,

> MATH 4500-48 (Student Reflection Journal 1)
> When I was first introduced to this topic, I thought that tracking was a good thing, because it challenged students. As the class and I have engaged in discussion, I was able to get a different view from the experiences of my classmates. It made me think critically about the downsides of tracking. As to whether I agree or not with the claims that tracking reinforces social inequalities and is actually oppressive, the verdict is still out for me. I hope that it is a good thing but based on the data I've seen and the experiences my classmates have shared, [I'm] starting to believe that it may not be [a] good thing.

The participant appears to be evaluating proto-beliefs as possible explanations for the concerning data they encountered and is at the same time reevaluating the original unproductive beliefs about tracking. An additional three weeks later, the student appears to have accepted one of the proto-beliefs, concluding,

> MATH 4500-48 (Student Reflection Journal 2)
> I think the most important thing that I have learned about tracking is how harmful it can be. I've never considered it to be harmful, but I was always in the 'advanced' classes. This project helped me see the opposite side, when

> students are placed in ‘remedial’ classes, and the damage that it can do to the students. It helped me to see the side that I was unfamiliar with and has actually helped me see that tracking is harmful to many students. I thought it was a good thing when we started this project, but now I think that it is a harmful thing to most students, remedial and/or advanced.

One week later, in their action plan written immediately following our final classroom discussion of tracking, this pre-service teacher expressed their intention to “be an advocate for students who aren’t receiving the support they need.” Without a longitudinal study we will be unable to determine the conviction with which the newly accepted belief is (or may be) held, and therefore whether it will actually shape the actions and future beliefs of the participant. However, this response may indicate what form consequential effects of holding the belief might take.

Another two participants, while possibly exhibiting a weakening of resolve in their conviction that tracking benefits students more than it harms them, never appear to have accepted a productive belief about tracking. In their first reflection journal entry, one of these students wrote,

> MATH 4500-34 (Student Reflection Journal 1)
>
> I don’t agree with the organizations that claim tracking creates and reinforces social inequalities and oppresses students. I believe that tracking schools actually help students develop math skills at their own level. If they need remedial help for mathematics then they should take remedial math classes to help them catch up to the regular level students are at. Then when they no longer need remedial classes they should be able to take regular classes. When the students’ skills surpass their peers in regular classes then they should take advance [*sic*] honors math classes. Tracking [schools’] goals are to improve each student’s mathematical skills and to teach each student at their own level.

This participant appears to have accommodated the information of the precursor, i.e., that tracking creates and perpetuates inequities, within their existing system of beliefs. We

cannot be certain whether the participant evaluated other candidate beliefs or immediately situated the new information within their web of beliefs in an initial search for meaning. Their second reflection journal entry appears to show no change in belief, with the student writing,

> MATH 4500-34 (Student Reflection Journal 2)
> The most important thing I have learned about the practice of tracking in schools is that it is supposed to help students learn at their own level. Every student is at a different level of learning and some students need more help than others so tracking is a way of incorporating equity in schools. This purpose is to give students an opportunity to learn math concepts at their own level.

The participant may have evaluated other beliefs, but none appear to have been accepted. In the same entry the participant appeared to exhibit at least a willingness to remain open to alternative conclusions regarding tracking, expressing their commitment to "gather data from student performance based on this knowledge and after years of teaching and collaborating with teachers of different schools that are tracking or non-tracking, I will come to a conclusion about the effectiveness of tracking." The student does appear to have undergone some evolution of social consciousness, however slight, with the student expressing their intent to employ "differentiated learning techniques" to accommodate students of varied achievement levels.

Of the remaining six participants, two began with a neutral view of tracking and later concluded the practice is harmful, three began with a neutral view of tracking and retained that perspective for the remainder of the project (though two of them committed to differentiating instruction for diverse learners while the other to continuing a critical analysis of tracking in their action plans), and one only submitted the action plan wherein they made a commitment to hold all students to high expectations.

Table 4.3 provides a summary of the coding of student work samples for this project. Recall that in this case, a + indicates the student expressed views that can be characterized

as pro-equity , a $-$ indicates the student expressed views in opposition to what are considered equitable teaching practice and policies, and I indicates the researcher was unable to determine the student's disposition toward the social justice issue.

Rows are highlighted to indicate the associated student experienced and evolution of social consciousness according to the characterization described in Chapter 2. By this characterization, 6 students underwent a productive evolution of social consciousness over the course of the project.

Due to the nature of an Action Plan assignment, wherein students are required to write down a specific plan for how they will incorporate equitable teaching practices into their future careers, it is unlikely that such a submission would ever be coded as negative. So while students 4500-33, 4500-46, and 45-21 would have undergone a productive evolution of social consciousness according to the characterization described in Chapter 2, it may be more responsible to take a more conservative view and count only students 4500-08, 4500-01, and 4500-48 as having experienced the desired change in beliefs. For this reason, only the rows corresponding to these three students were highlighted.

Student ID	Initial Task Sheet	Reflection Journal 1	Reflection Journal 2	Action Plan
4500-39	+	+	I	+
4500-41	+	+	I	+
4500-42	+	I	+	+
4500-08	I	+	+	+
4500-01	I	I	+	+
4500-48	-	I	+	+
4500-33	I	I	I	+
4500-46	I	N/A	N/A	+
4500-21	N/A	N/A	I	+
4500-15	I	I	I	I
4500-17	I	I	I	N/A
4500-34	I	-	I	I

Table 4.3: Student affect throughout the *"Tracking" in School Mathematics* project. Work samples coded as positive are denoted by +, negative by −, and indeterminate by I. Rows for students who underwent a productive evolution of social consciousness are highlighted.

4.4 Final Thoughts

Over the course of this analysis of student work, additional insights appeared regarding the specific stage(s) of belief which may be impacted by different project activities and components. Whereas at the initiation of this dissertation, the beginnings of these interrelationship between social justice mathematical modeling projects and the stages of belief formation were partially understood and hinted at, additional time and reflection served to illuminate some of the details of this relationship.

The initial project launch, the social justice-related content found in each of the readings, the prompts for reflection journal entries, the quantitative data used to inform the development of their models, the discussions students have with their team (especially in-class discussions monitored by the teacher), the final presentations of their peers, and the final discussion of the social justice topic, are all locations where a student may encounter precursors. In other words, social justice mathematical modeling projects include a large number of precursors intended to challenge a students' existing beliefs.

As students encounter new ideas in the required readings, as they see quantitative data supporting a specific conclusion regarding the social justice issue, as they discuss the issue with their team, as they see other groups' presentations and engage with the ideas they share, and throughout the final discussion of the social justice topic, students are being provided with candidate beliefs that they can use to explain the precursors they keep encountering throughout the project.

Finally, the required reflection journal entries, the group technical report, group discussions, and the final discussion of the social justice topic provide students with opportunities to externalize their evaluation of the productive candidate beliefs students encounter during the project, and safe forums in which to consciously evaluate them.

In short, post-enactment reflection revealed that participants in social justice mathematical modeling projects are provided with repeated opportunities to engage with and adopt pro-student, pro-science, and pro-human beliefs.

CHAPTER 5
CONCLUSION

A lot of ground has been covered, so much so that we will likely lose sight of the forest for the trees without taking stock of everything discussed in the preceding pages.

While the results of this work can be viewed from a number of perspectives, one of particular interest and import is an evaluation of this work in light of its written objectives and associated research questions. In other words, we need to evaluate the degree to which each objective was achieved and each research question answered.

Secondly, additional insights into the interplay between social justice mathematical modeling projects and the development of pre-service teacher beliefs, unanticipated in the objectives and questions but of which we must still review, have appeared over the course of this work. Finally, it is also important that the implications of this work for a variety of stakeholders be discussed, including math teachers, math teacher educators, and researchers of the psychology of mathematics and the development of teacher beliefs.

5.1 Objectives and Research Questions Revisited

At the outset of this research project, two dissertation objectives were identified and three research questions formulated, the latter of which for the purposes of achieving the specified objectives. Completion of this project requires an honest reckoning of what aspects of each question were answered, and of the degree to which objectives were achieved.

This primary objective is the distillation of 4 more specific objectives, each aimed at addressing the need for high-quality curricular materials that can help students prepare to become empathetic and socially conscious teachers of mathematics.

1. Produce social justice mathematical modeling projects for inclusion in pre-service mathematics teacher preparation programs.

1.1. Provide teacher preparation programs with additional supports for inculcating social consciousness in mathematics teacher candidates.

1.2. Provide undergraduate mathematics courses with mathematics for social justice curricular materials.

1.3. Provide undergraduate mathematics instructors with curricular materials that engage their students in mathematical tasks with high-level cognitive demand.

1.4. Provide undergraduate mathematics instructors with project-based learning curricular materials.

The achievement of Objective 1.1 hinged on whether the developed project materials incorporate research-backed methods of positively influencing the development of pre-service teacher beliefs. As these materials did, even in their untried form, we can reasonably conclude that the revised materials do constitute such "additional supports." However, as we saw throughout Chapter 4, the impact on student outcomes was not uniform. Thus, while this objective was achieved, this conclusion is held with the caveat that the quality of these additional supports can—and ought to—be improved upon through future test enactments, evaluations, and revisions.

The creation of any set of project-based learning materials, done in accordance with accepted best practices for curricular design, *ought* to result in the achievement of Objectives 1.2, 1.3, and 1.4. However, the differences between planned and enacted curriculum can, as we have seen in Chapter 3, be stark. To mitigate this oft-noted disparity, Research Questions 1 and 2 were formulated:

1. What additions or modifications will better align learning activities found in the project lesson plans and student materials with the intended learning objectives?

2. What additions or modifications to the project lesson plans and student materials will help instructors smoothly and competently incorporate one of these projects into their mathematics curriculum?

The answers to these two questions have been extensively documented in Chapter 3. Learning activity prompts objectives have been clarified, added to, or otherwise altered so the mathematical activities in which students engage lead them to achieve the intended learning objectives—as informed by those that students were observed to have achieved during test-enactments. Project materials have been greatly improved upon, in the inclusion of relevant information, pedagogical notes, and realistic sample approaches and models, to make facilitation of the projects easier for teachers wishing to incorporate them into their classroom (see Table 3.2 for a more precise summary of improvements). As a result of these alterations, the revised materials can be judged as possessing the characteristics outlined in Objectives 1.2, 1.3, and 1.4 and, therefore, we can conclude that these objectives have been achieved. As with Objective 1.1, there is still room for improvement. Future enactments will likely reveal that the revised materials, while greatly superior to those test-enacted, need further improvement.

In summation, Objective 1 of this dissertation has been achieved, but future enactments should be used to identify additional ways in which the project materials can be improved.

The secondary objective of this work was in regards to whether engaging in social justice mathematical modeling projects can positively influence the development of pre-service teachers' beliefs. In particular, the objective expressed the intent to:

2. Monitor and report on the extent to which the trial implementation of mathematics for social justice projects in pre-service mathematics teacher preparation courses results in a productive evolution of social consciousness.

This objective led to the formulation of the third research question:

3. Does student work on social justice mathematical modeling projects provide any evidence of an evolution of social consciousness?

As Objective 2 was solely focused on observation and documentation of results, we can reasonably conclude that it has been achieved. In retrospect and based upon the results of the enactment—as detailed in Chapter 4—it might have been possible to formulate a more

ambitious objective. Regardless, the success in this component of the dissertation project contributed much to a number of additional insights into appropriate project structure and facilitation to ensure a greater positive impact on student social consciousness.

5.2 Additional Insights and Their Implications

The achievement of the written objectives are not the only outcomes of this dissertation. There are additional insights into project design and into the relationship between social justice mathematical modeling and the stages of belief development that appeared during test-enactments, evaluations, and revisions of each project, all of which have implications for mathematics teachers, teacher educators, and teacher belief researchers.

5.2.1 Vital Components of Project Design

As noted throughout Chapter 3, some design characteristics had a demonstrably greater impact on the project enactments than others. And as mentioned above, new projects ought to be developed, and math teachers and teacher educators can use the results of this work as a rough framework for the development of their own projects.

Ensuring mathematics relevant to the course remains central throughout the project appeared as a challenge over the entirety of the *Racism & and "War on Drugs"* enactment. This was likely due to selecting a course in which to test the project prior to fully developing the project and noting what kind of mathematical content naturally arises when developing a model. In practice, this means that a social justice topic should only be selected if the relevant mathematical content arises naturally in its study. Of course, another way this outcome can be achieved—at least for teachers with the option of selecting the courses they will teach—is by first developing a project, identifying what mathematics arise naturally along the way, and then enacting it in a relevant mathematics course. This latter option may also be possible for teachers who participate in a professional learning community and who share curricular materials. Approaching the mathematical content in this manner, and then designing tasks, prompts, and assessment items in accordance with Cangelosi's Learning Levels will serve to ensure that level of cognitive demand is preserved and student

learning can be appropriately assessed.

Another insight into project design that appeared along the way is the enormous importance of ongoing reflection. The inclusion of reflection in projects was initially motivated by its role as a criterion for HQPBL, and to a lesser degree in light of the research mentioned in Chapter 2 supporting its use. In the aftermath of the project evaluations, the role of reflection was revealed to be far more vital than was anticipated. The impact of reflection might have been substantially less pronounced had the duration of the projects had been significantly shorter, as it appears that students needed substantial time not just to think about the mathematical content, but also to reflect on how they view themselves in relation to the modeling process as well as their personal beliefs about the social justice topic. Project developers should take the time to seek out compelling readings and formulate stimulating questions to ensure that students have multiple opportunities to engage in these important activities, but should also be sure to afford students sufficient time to do so. Project developers hoping to positively impact their students' social consciousness and mathematical identity should not underestimate the importance of enacting the project over substantial time and requiring ongoing reflection. As such, attempts to develop projects intended to be enacted over the course of only one or two weeks, while understandable given the challenges of today's classroom, should be discouraged unless the topic is one around which controversy does not revolve.

One final design choice appears to have played an important role in project enactments, at least for the *"Tracking" in School Mathematics* and *Global Warming & Hurricanes* projects. The use of the modified Smith & Stein 5 Practices for orchestrating Productive Discussions in Mathematics allowed for an engaging, organized, informative, and interesting discussion of the mathematics that appeared in students' models and the social justice topic. Project developers are strongly encouraged to incorporate their use into their own project materials, and teachers facilitating projects should be sure they do not go neglected during enactments.

In summation, teachers of mathematics can—with some effort—positively impact their

students' mathematical identities and challenge their existing inequitable beliefs, all while providing them with opportunities to engage in mathematical tasks possessing a high level of cognitive demand, through social justice mathematical modeling.

5.2.2 The Interplay Between Social Justice Mathematical Modeling and Belief Development

Beyond identifying components vital to the successful enactment of a social justice mathematical modeling project, the test-enactments and subsequent evaluations of student work samples provided some additional insights into how specific project components interact with individual stages of belief. While it was suggested in the introductory chapter that the projects can leverage what is known about the development of beliefs to positively impact student social consciousness, the specifics of many such interactions remained unclear prior to the post-enactment evaluations of student beliefs. These evaluations revealed that not only do project components interact with belief development, but that they, in some cases, interconnect with multiple stages of belief development (see Figure 5.1).

The first thing that became more clear in the aftermath of the projects was that the instances wherein students were presented with precursors to initiate the development of new beliefs ended up being far greater in number than what was planned. At the outset, it was intended that the project introduction, readings, and reflection prompts could serve to prompt the development of new beliefs, but the many student writings that referred to statements of fellow team members and in-class discussions serve as compelling evidence that team discussions, classroom discussions, and model presentations and results also act as additional precursors.

It was hoped that the formulation of proto-beliefs, a major component of the search for meaning, would be influenced by the results of the mathematical model. That is, students would settle the controversy around the social justice issue, at least in their own minds, by exploring one or more of its quantitative aspects. It has since become more evident that students are exposed to candidate beliefs not only from the modeling experience, but during required readings, during discussions with their teams and class, and during final

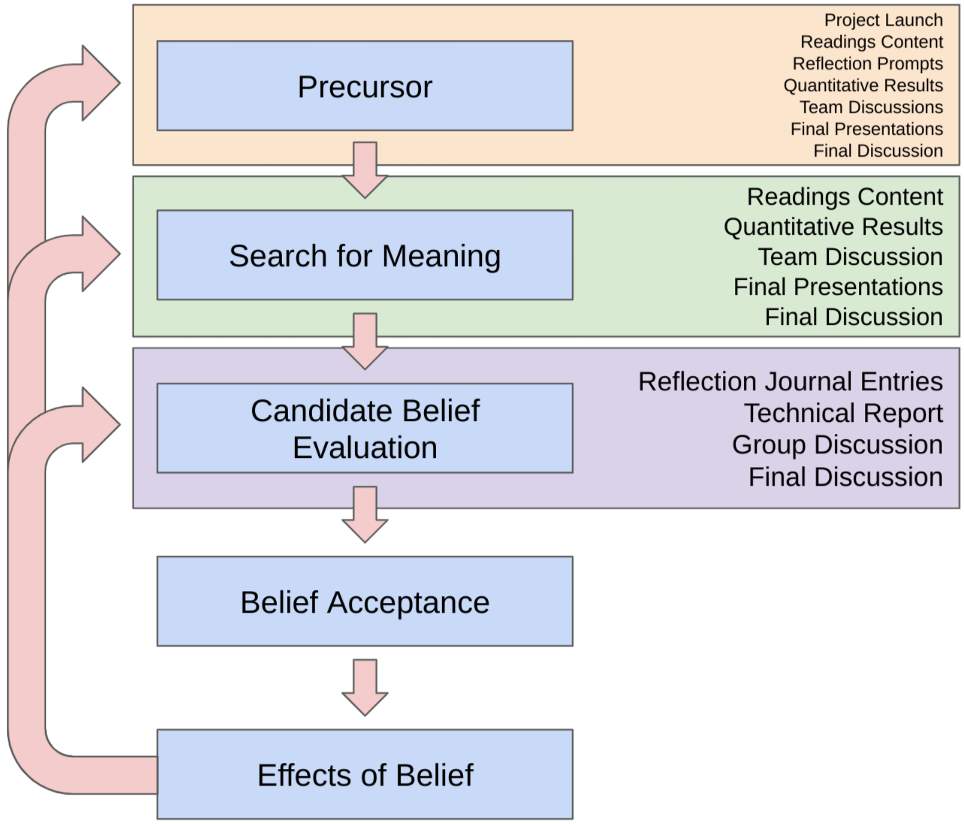


Fig. 5.1: Diagram showing the specific stages of belief development leveraged by different components of social justice mathematical modeling projects.

presentations. While it would be particularly difficult, if not impossible, to determine whether candidate beliefs students encounter during the final presentations and discussion are ever adopted, it still serves as one final opportunity to challenge students' existing inequitable beliefs.

Finally, during the analysis of student writings, some of the students appeared to be evaluating the issues as they were writing. While this may be evidence that they were merely attempting to provide an unbiased analysis of the issue, which is of course a central goal of these projects, the mere act of doing so required the *conscious* some aspects of the social justice issues. Given that candidate belief evaluation need not be a conscious act, the opportunities for conscious evaluation required by the reflection journal entries, parts of the required technical report, and during group and final discussions, serve a vital

purpose for ensuring candidate belief evaluation be less influenced by unconscious biases. This externalization of the evaluation process also provides the project facilitator with vital insight into the evolution of social consciousness students may be experiencing. In addition, the external evaluation of candidate beliefs can provide important insights into how projects can be improved to further augment their impact on student social consciousness.

At its inception, this dissertation began with an idea that social justice mathematical modeling can take advantage of efforts to influence the development of teacher beliefs, and possibly even successfully leverage what is known about different stages of belief development.

In hindsight we can now see much more precisely how the stages of belief formation serve not only as a tool for understanding the development of pre-service teacher beliefs, but also as evidence supporting the precise structure and format of the projects themselves. Indeed, perhaps the most important upshot of this work for efforts to influence the development of pre-service teacher beliefs, and student beliefs in general, is that social justice mathematical modeling successfully takes advantage of what little is truly known about the development of new beliefs. The extended duration of each project, the repeated precursors coupled together with productive, equitable candidate beliefs, and the requirement for ongoing reflection on the part of the students, all leverage what is known of non-pathological belief formation.

There are a number of takeaways from these insights for researchers of teacher beliefs.

First among these is that social justice mathematical modeling, as envisioned in this work, serves as an additional data point supporting the use of the intervention employed for the purposes of influencing teachers' beliefs.

Secondly, while there appears to have been an impact on student beliefs due to their participation in the project enactments, the true measure of that impact remains unclear. One reason for this ambiguity may be the indirect nature of the reflection journal prompts. While the intent was to minimize the effect of observation on the views expressed by students, the indirect prompts allowed students uncomfortable with discussing the social justice

topic to avoid doing so entirely. Therefore, teacher belief researchers seeking to use similar methods for assessing the state of student beliefs, and by extension student social consciousness, should make prompts direct, but stated in such a way as to allow the teacher facilitating the project a "responsible distance" from the views students are tasked with evaluating. For example, rather than asking students to answer the question, "Is the 'War on Drugs' racist?", task them with evaluating the Drug Policy Alliance's claim that the "War on Drugs" is racist. This allows for the exploration of controversial topics will minimizing the injection of, as well as accusations of the injection of, teachers' biases into discussions.

Finally, given the wide range of approaches taken by teacher belief researchers to influence the development of pre- and in-service teacher beliefs, one or more updated models of teacher belief development would greatly benefit the research community as a whole. In particular, the successes of this dissertation project suggest that existing models of teacher belief development need to be updated to include the more recent results from cognitive psychology and neuropsychiatry regarding non-pathological belief formation.

5.3 Directions for Future Work

A great deal of uncertainty appeared in students work submissions. This is indicative of at least two things.

The first of these is that reflection journal prompts may have been worded in such a way as to allow students, who felt uncomfortable discussing the social justice topic, to avoid doing so in their reflections. To decrease instances of this phenomenon in future enactments, reflection journal prompts were altered to be more direct. The impact of these changes necessarily require additional test-enactments.

Secondly, the coding system employed for evaluating student work may have been too coarse to capture the nuance that appeared in student writings, so much so that the majority of student work samples provided no insight into the state of individuals' social consciousness. A research-based coding refinement is an important next-step in validating the use of social justice mathematical modeling as a method of influencing belief development.

Whatever the case may be, even the existing coding system provided important glimpses into the minds of the students who participated in the projects. This being the case, an immediate direction for future work can be the evaluation of the remaining 146 student participants in the Racism and the "War on Drugs" project using the existing coding system. This evaluation will serve well to inform the refinement of the coding system.

Regardless, once a more sophisticated coding system has been established and projects have undergone additional revision to maximize the helpful information—for the purposes of assessing the state of their beliefs—provided in student work samples, a formal instrument for accurate measurement needs to be developed and validated. The existence of such an instrument would go far to establish social justice mathematical modeling as a useful pedagogical tool and method of intervention.

The coding system used for deciding whether students experienced any kind of evolution of social consciousness needs considerable refinement. This could reasonably be the next step in a much larger effort of developing a full theory of the interplay between social justice mathematical modeling and the development of students' beliefs about learning mathematics, their own mathematical identities, as well as their beliefs about the focal social justice topic itself.

Furthermore, the revised, refined coding system should be used to reevaluate existing data, and by additional coders, for the purposes of comparison with existing results and evaluating the coding system's consistency when employed by different individuals. This will serve to either refute or bolster existing results, and provide important insights that can be used to initiate validation of a measurement instrument.

Another effort worth making is in the improvement of the project evaluation instrument developed for the purposes of this dissertation. Some items provided greater insight than was anticipated, others provided less. Some might provide better insights if they are separated into multiple items, while others could be eliminated. Further development of this instrument will, by extension, result in better project materials. Should this direction prove particularly beneficial, the instrument could be further developed in preparation for

undergoing formal validation.

Finally, the most obvious direction for future work is in the further refinement of existing project materials and development of new projects. Test-enactment of existing materials will provide new student work samples which can either be evaluated using the existing method of coding, or to inform the development of the refined system of coding. This will lead to further improvements of the newly revised materials which will, in turn, provide additional insights into how new projects can be developed in such a way as to minimize the amount of test-enactment that must take place before it is ready for use by practicing teachers.

Social justice mathematical modeling projects are a novel idea, and one that needs extensive classroom testing before it will be established as a teaching strategy and as a form of intervention in pre-service math teacher programs.

And even if the prospective teachers who participated in these projects want to incorporate similar activities into their own classrooms, they will be unable to do so without sufficiently easy-to-use project materials and professional development to train them in their use.

The future of this work is large enough to be intimidating, but is an effort worth making.

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APPENDICES

APPENDIX A

IRB Approval

Institutional Review Board

Exemption #3
Certificate of Exemption

From: Melanie Domenech Rodriguez, IRB Chair

Nicole Vouvalis, IRB Director

To: ***Brynja Kohler***

Date: ***April 17, 2019***

Protocol #: ***10006***

Title: ***Development and Evaluation of Social Justice Mathematical Modeling Projects for Teacher Preparation***

The Institutional Review Board has determined that the above-referenced study is exempt from review under federal guidelines 45 CFR Part 46.104(d) categories 1 and 2:

> *Research, conducted in established or commonly accepted educational settings, that specifically involves normal educational practices that are not likely to adversely impact students' opportunity to learn required educational content or the assessment of educators who provide instruction. This includes most research on regular and special instructional strategies, and research on the comparison among instructional techniques, curricula, or classroom management methods.*
>
> *Research that only includes interactions involving educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures, or observation of public behavior (including visual or auditory recording) if at least one of the following criteria is met: (i) The information obtained is recorded in such a manner that the identity of the human subjects cannot readily be ascertained, directly or through identifiers linked to the subject; (ii) Any disclosure of the responses outside the research would not reasonably place the subjects at risk of criminal or civil liability or be damaging to the subjects' financial standing, employability, educational advancement, or reputation, or (iii) the information obtained is recorded by the investigator in such a manner that the identity of the human subjects can readily be ascertained, directly or through identifiers linked to the subjects, and the IRB conducts a limited IRB review to make required determinations.*

This exemption is valid for three years from the date of this correspondence, after which the study will be closed. If the research will extend beyond three years, it is your responsibility as the Principal Investigator to notify the IRB **before** the study's expiration date and submit a new application to continue the research. Research activities that continue beyond the expiration date without new certification of exempt status will be in violation of those federal guidelines which permit the exempt status.

As part of the IRB's quality assurance procedures, this research may be randomly selected for audit during the three-year period of exemption. If so, you will receive a request for completion of an Audit Report form during the month of the anniversary date of this certification.

In all cases, it is your responsibility to notify the IRB **prior** to making any changes to the study by submitting an Amendment request. This will document whether or not the study still meets the requirements for exempt status under federal regulations.

Upon receipt of this memo, you may begin your research. If you have questions, please call the IRB office at (435) 797-1821 or email to irb@usu.edu.

The IRB wishes you success with your research.

APPENDIX B

Project Evaluation Instrument

Social Justice Mathematical Modeling Project Evaluation Instrument

*For use by the researcher following the completion of the project in its entirety, including grading by the test-enactment teacher. Provide evidence from student work submissions, student communication and collaboration information from the project management software, and the test-enactment teacher reflection journal to justify responses to the prompts below.

1. **What project is this for?**
2. **Initiating the Project**
   - Describe students' reception of the narrative scenario as a means to introduce the project. Be sure to make note of any aspects of the project for which they expressed interest or enthusiasm, as well as those that elicited apathy or frustration.
   - What difficulties (e.g., misunderstanding of instructions, technical difficulties) did students experience while setting up their user accounts for the project management software and during the usage tutorial?
   - Describe, in detail, any other factors (e.g., project materials, student content knowledge and mindset, other environmental and social factors) you think facilitated or complicated the introduction of the project.
   - What pedagogical guidance would make the introduction of the project easier to manage for the teacher?
3. **Mathematical Tasks**
   - What mathematical definitions or common notational conventions needed for the project that should be included in the project materials?
   - How well did the students understand the prompts, and did they find the goals of the tasks unambiguous? Cite evidence to back up your claim.
   - Did the tasks, as enacted by students, require the intended level(s) of cognitive demand? Cite evidence to back up your claim.
   - Do the tasks, as enacted by students, require the expected prerequisite mathematical content knowledge?
   - What student approaches to completing the task were unanticipated in the task rubric and project materials, but still mathematically rigorous?
   - Are there any ways that this project's mathematical tasks can be improved that are not evident from your responses to the other prompts?
   - What additional resources would have been helpful for students in engaging in the mathematical tasks found in this project without altering their learning level(s)?
4. **Project Management Software**
   - Describe, in detail, any ways in which the use of the project management software by student groups appears to benefit or hinder the groups' progress on completing project tasks?
   - Do the benefits afforded by the use of project management software appear to outweigh the obstacles it poses (if evidence of any exists) to student progress on the project?
   - Describe, in detail, what changes in the amount of scaffolding would result in an improved learning experience for students using the project management software with their group, and the effects you expect from such changes.
5. **Technical Reports and Presentations**
   - What technical report requirements served best at helping students achieve the intended learning objectives?
   - What technical report requirements were least effective at helping students achieve the intended learning objectives?

- How could the required technical report specifications be altered to better help students achieve the intended learning objectives?
- What difficulties (e.g. outbursts, arguments, tangential conversations) did you experience while facilitating discussions following student group presentations? What might be done to prevent these difficulties when this project is used in the future?
- Did the presentation format result in student group presentations that provided helpful insights into students' learning of mathematics and experiences doing mathematics during the project? What evidence can you cite in favor of your conclusion?
- What might be done to assist in making student group presentations more helpful to the teacher for the purposes of assessment, and more useful to the students for the purpose of achieving the specified learning objectives?
- Are there any presentation requirements that now seem unnecessary (e.g., overly complicated, cumbersome, irrelevant to learning objectives)?
- What questions posed to the presenting groups are not found in the project materials?
- Did any student group presentations incorporate additional information beyond what is required by the grading rubric that you think should be added to the requirements?

6. **General Conclusions**

   - What aspects of this project are particularly strong?
   - How might the strengths of this project be generalized to allow for incorporation into new social justice mathematical modeling projects?
   - What aspects of this project are in greatest need of improvement?
   - How might future projects avoid the weaknesses you identified in the previous prompt?

APPENDIX C

Selections from Revised Project Materials

Racism in the "War on Drugs"

Duration: 4-6 weeks

In-class Meetings: 4

Prerequisite Content: Proportional Reasoning

Overview

Summary

In this project, students explore how institutional racism has led to a disproportionately large number of Persons of Color incarcerated for drug-related offenses. This project positions students as researchers in a think tank focusing on education policy that are exploring a claim by the Drug Policy Alliance. Their goals are to produce a mathematical formulation of the concept of racism in the context of the US War on Drugs and to develop a mathematical model they can use to analyze the DPA's claim. Students present their model, field questions regarding the development of their model, and participate in a discussion of the social justice issues surrounding racism in the US criminal justice system. Students engage in ongoing reflection about the mathematical modeling process and how they view themselves in relation to mathematics and the modeling process. Students read documents produced by actual think tanks across the political spectrum discussing racism in the War on Drugs, write about their and thoughts and beliefs about institutional racism, and reflect on the social justice implications of criminal justice reform.

Relevant SPTM Standards and Indicators

Indicator C.1.2. Demonstrate Mathematical Practices and Processes -

> "[Well-prepared beginning teachers] can apply their mathematical knowledge to real-world situations by using mathematical modeling to solve problems appropriate for the grade levels and the students they will teach."

Indicator P.2.2. Build Mathematical Practices and Processes -

> "Mathematical modeling—using mathematics to analyze real-world situations—receives continued attention throughout the program."

Indicator P.2.3. Provide Sustained, Quality Experiences -

> "[M]athematics content courses should be taught using teaching methods that serve as models of effective instruction."

ML.1. Essential Understandings of Mathematics Concepts and Practices -

> "[Well-prepared beginning] teachers understand algebraic thinking as (a) the study of structures in the number system, including those arising in arithmetic; (b) the study of patterns, relations, and functions; and (c) the process of mathematical modeling."

ML.4. Meaningful and Interdisciplinary Contexts -

> "Well-prepared beginning teachers of mathematics at the middle level understand how to engage middle level learners in meaningful and interdisciplinary contexts, including the use of mathematical modeling."

HS.1. Essential Understandings of Mathematics Concepts and Practices in High School Mathematics -

> "Well-prepared teachers have substantive experiences engaging in mathematical modeling so that they can understand mathematical modeling and its potential place in the curriculum."

Learning Objectives. In this project, students will:

A. Engage in the mathematics modeling process to develop a model for identifying and analyzing the racism in the "War on Drugs." (willingness to try)

B. Incorporate parameters into their model allowing them to explore the effects of altering known or unknown values. (algorithmic skill)

C. Write a technical report detailed the development of their model, including the real-world meaning of components and parameters. (comprehension and communication)

D. Present the contents of their report to their peers and field questions about their modeling process and decisions. (comprehension and communication)

E. Explain how racism was involved in the initiation of the War on Drugs, and how it persists in US drug policy. (comprehension and communication)

F. Write reflection papers on the mathematical modeling process, the state of racism in US drug policy, and how the former can be used to understand the latter. (comprehension and communication)

G. Recognize the relevance of mathematics as a tool for understanding the world and making predictions. (appreciation).

List of Materials

Handout numbers are to indicate distribution order for teachers intending to engage students in a more scaffolded modeling experience. Regardless of whether teachers choose this route, they should make use the reflection journal and action plan tasks.

- MacGuffin Institute Letter Template
- Handout 3: Reflecting on Your Experience (Reflection Journal 1)
- Handout 5: Reflecting on Your Experience (Reflection Journal 2)
- Handout 10: Taking Action
- Handout 11: Reflecting on Your Experience (Reflection Journal 3)

- Computers with internet access (at least 1 per group)
- Classroom computer with projector for sharing digital content and for student use during presentations.
- Word processing software capable of writing mathematical formulas (e.g., Overleaf, Microsoft Word, Google Docs)

Supplementary Materials

Supplementary materials are provided for teachers intending to provide students with a more scaffolded modeling experience.

- Mathematical Modeling & Project Introduction Slideshow
- Handout 1: Defining the Problem & Selecting Appropriate Variables
- Handout 2: Making Appropriate Assumptions
- Handout 4: Implementing Your Model
- Handout 6: Iterating the Modeling Process
- Handout 7: Analyzing Your Model
- Handout 8: Reporting Your Results - Technical Report Specifications
- Handout 9: Presenting Your Results - Presentation Requirements

Project Details

Definitions and Notation

Directly Proportional: Two quantities a and b are said to be directly proportional to one another if and only if $a = kb$, where k is a constant.

Essentialism: A sociological theory that posits any entity such as an individual, group, object, or concept has innate and universal qualities. Essentialism has been used as a justification for racism, imperialism, and bigotry. In the context of this project, we promptly reject any and all essentialist explanations for observed differences in mathematical achievement levels.

Implicit Bias: The unconscious attitudes, stereotypes and unintentional actions (positive or negative) towards members of a group merely because of their membership in that group.

These associations develop over the course of a lifetime beginning at a very early age through exposure to direct and indirect messages. When people are acting out of their implicit bias, they are not even aware that their actions are biased. In fact, those biases may be in direct conflict with a persons explicit beliefs and values. (Institute for Democratic Renewal, 1998).

Institutional Racism: Anonymous, subtle and systemic discrimination based on race, in legal instruments, as well as in private organizations and professions (educational, legal, healthcare, political, religious, etc.), private businesses and public decision-making bodies.

Because this form of racism is anonymous and built into standard institutional practice, individuals often resist acknowledging its existence and deny, consciously or unconsciously, their complicity in maintaining it.

Examples of institutionalized racism include policies and practices that: arbitrarily govern a person's credit-worthiness; determine what information, positive or negative, is presented in the media about individuals involved in newsworthy events; or place undue value on selective educational experiences or qualifications in establishing promotion criteria in jobs and schools. (Institute for Democratic Renewal, 1998)

Inversely Proportional: Two quantities a and b are said to be inversely proportional to one another if and only if $a = \frac{k}{b}$, where k is a constant.

Prejudice: A pre-judgment or unjustifiable, and usually negative, attitude of one type of individual or group toward another group and its members. Such negative attitudes are typically based on unsupported generalizations (or stereotypes) that deny the right of individual members of certain groups to be recognized and treated as individuals with individual characteristics. (Institute for Democratic Renewal, 1998)

Race: The classification of humans based on arbitrary physical characteristics such as skin color, facial form and/or eye shape. (Institute for Democratic Renewal, 1998)

Racism: An ideological system of oppression and subjugation, held consciously or otherwise, based upon unfounded beliefs about racial and ethnic inequality. This system of oppression is based on a view that an arbitrary set of physical characteristics, such as skin color, facial form or eye shape, are associated with or even determine behavior, culture, intellect, or social achievement. (Institute for Democratic Renewal, 1998)

War on Drugs: The effort in the United States since the 1970s to combat illegal drug use by greatly increasing penalties, enforcement, and incarceration for drug offenders. (Britannica)

Preparation

1. Prior to starting this project, read through all handouts and suggested teacher and student readings, and watch the introductory video clip (see **Introducing the Project**).

> **Pedagogical Note.** Some teachers may be unfamiliar with definition of "racism" as defined by scholars of critical race theory. It is important that teachers prepare to respond to inevitable questions and comments about the disproportionate representation of Persons of Color among the US prison population for drug-related offences, a phenomenon explained by institutional racism. The teacher must be comfortable discussing this topic and model appropriate communication.

2. Teachers employing the recommended narrative scenario as a frame for the project should write a brief introductory letter congratulating students on their recent hire at the MacGuffin Institute and introducing them to their roles as a policy researchers who will use mathematical modeling to determine whether tracking in school mathematics is an equitable practice. A MacGuffin Institute letter template can be found under **Resources**. While the letters can be provided to students the day of project launch, it is recommended that students receive their letter prior to introduction of the project to avoid unnecessary distraction. In addition, it is recommended that resources be distributed digitally to minimize paper waste.

> **Technical Tool Tip.** Teachers intending to follow the recommendation to distribute project resources digitally should ensure prior to launching the project that students' preferred forms of contact (e.g., email) are up-to-date.

3. While it is recommended that teachers direct students to take advantage of the organization and communication tools of their liking, teachers wishing to make use of project management software to distribute materials and monitor student progress should set aside a substantial amount of time to setup these electronic resources for students.

> **Technical Tool Tip.** Students often prefer tools that integrate easily with smart devices. Some easy-to-use project management and collaboration resources with mobile support include Slack, Trello, and GroupMe.

4. Student groups should be in place prior to introducing the project, and should ideally consist of no more than four members. Unless the teacher has reason to do otherwise, student groups should be created randomly. Teachers may wish to designate a group leader to manage groups' in-class work and oversee organization and planning of out-of-class meetings.

> **Technical Tool.** Some online learning management systems (e.g., Canvas) can generate random groups of students as well as randomly assign a group leader, allowing teachers to avoid introducing unconscious bias into group and leader selection.

Introducing the Project

1. Begin by showing students the following video clip (the first 32 seconds at a minimum) of Richard Nixon announcing the beginning of the War on Drugs: President Nixon Declares Drug Abuse "Public Enemy Number One". Next, show students the following video clip from the Drug Policy Alliance: Drug War Update: Overincarceration

> **Pedagogical Note.** Students new to mathematical modeling will likely benefit from a guiding modeling task. Slides are provided for just such an introductory task and provides a segue into the modeling project including an appropriate location to share the video clips.

2. Direct students to begin development of a mathematical model in order to determine whether "tracking" in school mathematics is an equitable practice.

> **Pedagogical Note.** Teachers seeking to provide students with a more scaffolded modeling experience should make use of Handout 1 as both an initial project prompt and a guide for students on how to get started.

Monitoring Student Progress

As with any mathematical modeling experience, students will initially feel overwhelmed with the amount of autonomy they are afforded given the open-ended nature of the mathematical task. To ensure the authentic modeling experience does not devolve into a situation of learned

helplessness, students work should be monitored during multiple class meetings dedicated to the modeling project and through their responses to prompts for their reflection journal entries.

Student teams should be expected to progress at different rates. However, teachers should ensure that all students have the experience to engage in at least one iteration of the modeling process. Teams that progress quickly should be strongly encourages to revisit aspects of their model, incorporate model parameters, and further analyze their model. Students should be provided with the Reflection Journal handouts no more frequently than one week apart to ensure they are afforded time sufficient to actually reflect on the process in which they are engaging.

Pedagogical Note. For teachers making use of the supplementary materials, the remaining worksheets are intended to be distributed at the discretion of the teacher and based on students' progress. Students lacking mathematical modeling experience can reasonable be expected to complete the tasks outlined in Handouts 1 through 5 and 8 through 11 at a minimum. Groups progressing quickly through the tasks should be provided with Handouts 6 and 7, which will require them to revisit and generalize their model. Handouts 6 and 7 can be repeatedly used by students to revise, and analyze their group's model.

The questions listed below for use during the final discussion can also be used throughout the modeling project to guide student teams who are struggling to progress.

Finally, assessment should depend primarily on student engagement in the modeling process, and not on the model they produce. The Supplementary Materials include suggestions for technical report and team presentation specifications and associated scoring rubrics.

Orchestrating the Final Discussion

Presentations and the final discussion should be organized using the following modified version of the 5 Practices for Orchestrating Productive Discussions in Mathematics (Smith & Stein, 2018):

1. Review the **Sample Approaches** found in these project materials. (Anticipating)
2. Read through each team's technical report. (Monitoring)
3. Make note of unanticipated approaches or otherwise interesting approaches. Be sure to find at least one particular aspect of each team's model to point out and celebrate later during presentations and the final discussion, (Selecting)
4. Determine the order of presentations based on what you think will be pedagogically advantageous. (Sequencing)
5. Make use of the suggested questions and write additional questions that will help students recognize their mathematical modeling successes and identify ways in which their model can be improved. (Connecting)

Technical reports should be submitted prior to the day of presentations to allow the teacher to read through each report and list questions and comments for use during presentations. In addition to checking the report specifications against the technical report specifications list, the project facilitator should place groups in sequence for presentations based on their model approaches.

The following questions can be used following student presentations and to guide the final discussion.

- How did your team translate our definition of racism to mathematics?
- What real-world phenomena placed restrictions on what values your variables and parameters could take?
- Why did you decide to use that specific value for a parameter?
- What happens to your model when you alter the values you input? How much do you have to alter them by to obtain that result?
- What does this mean in real-world terms?
- How did you translate this concept to mathematics?

Following presentations, but prior to beginning a discussion of the social justice topic, it is important to establish norms for appropriate classroom discourse. The following list has been successful during previous discussions of socially charged topics (My thanks to Dr. Julia Aguirre for the list!):

- Prepare to feel discomfort.
- Listen respectfully.
- Share the time.
- Be mindful of the intent *and* the impact of your words.
- Challenge ideas, not people.

Students will be unable to avoid discussing race during this project, a fact that some of them may be uncomfortable with, and some will resist acceptance of the ubiquity of institutional racism in the criminal justice system. However, the teacher should not allow students to avoid discussing the issue, and should redirect off-topic comments and discussions back to the issue of race. The questions below can serve as a useful segue into a discussion of the long term effects of incarceration on former inmates rehabilitation into society, such negative impact on job prospects and revocation of their right to vote.

- What is the goal of the US penal system?
- To what degree is the US penal system achieving that goal?
- In what ways have minimum sentencing guidelines impacted the US prison population.
- What consequences might you expect from former inmates being denied the right to vote?

Students should respond to the Action Plan prompts during class immediately following the Final Discussion. Reflection Journal 3 should be sometime following the final class meeting dedicated to the modeling project.

Sample Approaches and Models

After sharing their different lists of what factors are most important to understanding tracking in terms of equity, Group 1 settles on the following list:

- Total number of people in the US
- Total number of people in the US who are charged with drug-related crimes from each racial/ethnic demographic (i.e, White, Black, Latin$x$, Asian, Pacific Islander, Native American)

Now that they have settled on their list of important factors, the group begins discussing how they will mathematize the concept of racism in the War on Drugs. After a few minutes of discussion, one students suggests that compare the ratios between the each of the numbers of people charged with drug-related crimes for the different racial/ethnic identity on their list and the total population of the United States. If they compare all of the ratios to each other (pairwise), the student claims, they can determine whether one group is being charged with drug-related crimes more frequently than another. They assign labels for each of their 12 important factors (C_i with $i \in \{1,2,3,4,5,6\}$ for the number of people charged with drug-related crimes from racial/ethnic demographic i, and T for the total population of the United States), and write down the following model:

Mathematical Model. Let C_i be the number of people in the US from demographic i who are charged with drug-related crimes, and let T be the total population of the US. Then the War on Drugs is racist if and only if:

$$\frac{C_i}{T} \not\approx \frac{C_j}{T}, \text{ for every } i, j \in \{1,2,3,4,5,6\}.$$

Next they decide to explore the available data, but come to an unfortunate, and somewhat frustrating, conclusion. The data available is fairly sparse, and they can only locate information about the Hispanic, non-Hispanic Black, and non-Hispanic White racial/ethnic demographics. In addition, one student points out that their model is only equipped to compare populations in a given year. Another student responds that this limitation could also serve as a tool if the War on Drugs is racist, because it would allow them to determine whether it is becoming more or less racist over time.

Following some additional discussion, the team makes some significant alterations—and simplifications— to their model:

Mathematical Model. Let C_i be the number of people in the US from demographic i, $i \in \{1,2,3\}$, who are charged with drug-related crimes, and let T be the total population of

the US. Then the War on Drugs is racist if and only if:

$$\begin{aligned} &\frac{C_1}{T} \not\approx \frac{C_2}{T}. \\ \text{OR } &\frac{C_1}{T} \not\approx \frac{C_3}{T} \\ \text{OR } &\frac{C_2}{T} \not\approx \frac{C_3}{T}. \end{aligned}$$

Satisfied with their new model, the students plan to show their teacher during the next class meeting dedicated to the project.

Upon reviewing their work, the asks whether they have tested out their model with any actual data and, if so, what it indicates about any disparities in drug-related crime charges between Black and White males. Responding that they had not yet done so, the teacher asks them to consider the following scenario.

Suppose, for the sake of argument, that the population of the US is 100 people, 30 of whom are Black and 70 of whom are White. Now suppose that 20 people are charged with drug-related crimes, 10 from each racial demographic. What does your model suggest about racism in this hypothetical?

They decide to make the following assignment of indices to racial/ethnic demographics: $1 \mapsto$ Black, $2 \mapsto$ White. Plugging in the appropriate numbers in the appropriate places, the students obtain the following values: $C_1 = 10$, $C_2 = 10$, and $T = 100$. This results in the following ratios:

$$\frac{C_1}{T} = \frac{10}{100} = \frac{1}{10} \qquad \frac{C_2}{T} = \frac{10}{100} = \frac{1}{10}.$$

The students respond that their model indicates that there is no racism in this hypothetical, to which the teacher asks whether they think one group is being targeted more than another.

One of the students, nodding with comprehension, responds that $\frac{1}{3}$ of the Black population was charged with a crime, but only $\frac{1}{7}$ of the White population. The rest of the group now understanding the flaw in their model, they return to their table and decide to revise their model. They write down their new and improved model (Again assuming they will only have data for the Hispanic, non-Hispanic Black, and non-Hispanic White demographics):

Final Mathematical Model. Let C_i be the number of people in the US from demographic i, $i \in \{1,2,3\}$, who are charged with drug-related crimes, and let T_i be the total number of people in US from demographic i, $i \in \{1,2,3\}$. Then the War on Drugs is racist if and only if:

$$\begin{aligned} &\frac{C_1}{T_1} \not\approx \frac{C_2}{T_2}. \\ \text{OR } &\frac{C_1}{T_1} \not\approx \frac{C_3}{T_3} \\ \text{OR } &\frac{C_2}{T_2} \not\approx \frac{C_3}{T_3}. \end{aligned}$$

Applying their new model to the teacher's hypothetical scenario, the students conclude that it makes sense.

Resources

Teacher Readings

The following readings are *necessary* preparation for teachers preparing to incorporate this project into a methods or content course.

- Drug Policy Alliance: The Drug War, Mass Incarceration and Race
- Four Decades and Counting: The Continued Failure of the War on Drugs
- The Sentencing Project: State-by-State Data
- Twelve Facts about Incarceration and Prisoner Reentry
- Bureau of Justice Statistics: Jail Inmates in 2017
- Bureau of Justice Statistics: Prisoners in 2017
- Bureau of Justice Statistics: Correctional Populations in the United States, 2016
- Pew Research Center: The gap between the number of blacks and whites in prison is shrinking
- Drug Policy Alliance: New Solutions for Drug Policy

The following readings are not free and, as such, are only *recommended*:

- The New Jim Crow: Mass Incarceration in the Age of Colorblindness

Student Readings

The following readings are *strongly recommended* for students engaging in this project. As such, they have been incorporated directly into the Reflection Journal Handouts.

- Four Decades and Counting: The Continued Failure of the War on Drugs
- Drug Policy Alliance: The Drug War, Mass Incarceration and Race
- The Sentencing Project: State-by-State Data
- Twelve Facts about Incarceration and Prisoner Reentry
- Bureau of Justice Statistics: Jail Inmates in 2017
- Bureau of Justice Statistics: Prisoners in 2017
- Bureau of Justice Statistics: Correctional Populations in the United States, 2016
- Pew Research Center: The gap between the number of blacks and whites in prison is shrinking
- Drug Policy Alliance: New Solutions for Drug Policy

References

Institute for Democratic Renewal. (1998). *A Community Builder's Tool Kit: 15 Tools for Creating Healthy, Productive Interracial/Multicultural Communities*. pp. 32-34. Available for free at https://www.racialequitytools.org/resourcefiles/idr.pdf.

Reflecting on Your Experience

The sentencing project is an organization that collects data on the US prison population and advocates for criminal justice reform. Take a few minutes to explore the data for the state in which you reside.

- The Sentencing Project: State-by-State Data

Reflection Journal 1

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. What is your knee-jerk response to claims regarding racism in War on Drugs? Explain.
2. What did you find most interesting or surprising among the Sentencing Project data you explored?
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project so far?
5. Is there anything you want your teacher to know?

Reflection Journal 1 Scoring Rubric

1. Student describes response to racism claims. (3 points)

• The student explains their reaction to claims of racism in the War on Drugs.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

2. Student describes Sentencing Project Data interests. (3 points)

• The student describes what they learned about their own state from the Sentencing Project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

Total _______/14

Reflecting on Your Experience

The Drug Policy Alliance has made a number of serious accusations regarding the US criminal justice system. Review their claims in the following document:

- Drug Policy Alliance: The Drug War, Mass Incarceration and Race

The DPA is considered to be a liberal-leaning organization. It's worth asking whether, and to what, degree their views are shared by those of a different political affiliation Read the section entitled "The War on Drugs and Racial Bias in the United States" in this article from the conservative-leaning CATO institute:

- Four Decades and Counting: The Continued Failure of the War on Drugs

Reflection Journal 2

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. What do the analyses of the DPA and the CATO Institute have in common? Where do they disagree?
2. Explain your team's mathematical definition of racism in War on Drugs.
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project so far?
5. Is there anything you want your teacher to know?

Reflection Journal 2 Scoring Rubric

1. Student compares DPA and CATO reports. (3 points)

• The student compares the DPA and CATO reports on racism in the War on Drugs.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

2. Student explains mathematical definition of racism. (3 points)

• The student explains their team's mathematical definition of racism in the War on Drugs.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

Total ________/14

Taking Action

Having an informed opinion is important, but substantive, persistent change only follows from taking action.

In recent years, criminal justice has found bipartisan support. This has resulted in legislation like the 2010 Sentencing Act and the 2018 First Step Act, but society has quite a long way to go before the reparations will have been paid for the long-term effects of institutional racism in the US criminal justice system.

The scale of these changes must be large, much more-so than what any individual can accomplish. But that doesn’t mean we as individuals can’t have an impact.

Writing an Action Plan

1. Write down at least two specific actions you will take, whether in your own community or another, to challenge systemic racism and to help repair the damage done to individuals harmed by the US criminal justice system.

Reflecting on Your Experience

Accusations of racism in the criminal justice system are serious, but they are unlikely to have much of an impact without reasonable recommendations for change. Spend a few minutes reviewing what the Drug Policy Alliance claims is the best way forward:

- Drug Policy Alliance: New Solutions for Drug Policy

Reflection Journal 3

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. What specific actions did the DPA recommend? Do you agree with them? Why or why not?
2. Did the results of your mathematical model conflict with what you thought about the War on Drugs at the beginning of this project? Explain.
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project?
5. Is there anything you want your teacher to know?

Reflection Journal 3 Scoring Rubric

1. Student reviews DPA policy recommendations. (3 points)

• The student describes and evaluates the policy recommendations of the Drug Policy Alliance.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

2. Student discusses model results. (3 points)

• The student explains whether model results aligned with their views on the War on Drugs.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

Total _______/14

Global Warming & Hurricanes

Duration: 6 weeks

In-class Meetings: 6

Minimum Prerequisite Content: Vector-Valued Functions

Overview

Summary

In this project, students explore the relationship between global warming and predictions of an increase in more damaging tropical cyclones as a result of anthropogenic climate change. This project positions students as scientific researchers tasked with producing a simplified, qualitative vector-field model of a fluid velocity field for a tropical cyclone. Their goal is to understand, and produce an accessible explanation of, how human accelerated global warming is expected to result in an increase in the frequency of tropical cyclones with greater levels of precipitation. Central to this explanation is a student-developed 2- or 3-dimensional vector field possessing qualitative characteristics of a tropical cyclone, such as: a direction of rotation dictated by geographic location and due to the Coriolis Force, eye-ward air flow due to a central low-pressure zone, an eyewall, and so forth. Students present their model, field questions regarding the development of their model, and participate in a discussion of the social justice implications of an increase in tropical cyclone with heavier levels of precipitation. This final discussion centers around data contrasting those who produce the most Greenhouse Gas emissions with those most negatively impacted by tropical cyclones. Students engage in ongoing reflection about engaging in the mathematical modeling process, their thoughts and beliefs about climate change, how they view themselves in relation to mathematics and the modeling process, and the social justice implications of inaction regarding climate change.

Relevant SPTM Standards and Indicators

Indicator C.1.2. Demonstrate Mathematical Practices and Processes -

> "[Well-prepared beginning teachers] can apply their mathematical knowledge to real-world situations by using mathematical modeling to solve problems appropriate for the grade levels and the students they will teach."

Indicator P.2.2. Build Mathematical Practices and Processes -

> "Mathematical modeling—using mathematics to analyze real-world situations—receives continued attention throughout the program."

Indicator P.2.3. Provide Sustained, Quality Experiences -

> "[M]athematics content courses should be taught using teaching methods that serve as models of effective instruction."

ML.1. Essential Understandings of Mathematics Concepts and Practices -

> "[Well-prepared beginning] teachers understand algebraic thinking as (a) the study of structures in the number system, including those arising in arithmetic; (b) the study of patterns, relations, and functions; and (c) the process of mathematical modeling."

ML.4. Meaningful and Interdisciplinary Contexts -

> "Well-prepared beginning teachers of mathematics at the middle level understand how to engage middle level learners in meaningful and interdisciplinary contexts, including the use of mathematical modeling."

HS.1. Essential Understandings of Mathematics Concepts and Practices in High School Mathematics -

> "Well-prepared teachers have substantive experiences engaging in mathematical modeling so that they can understand mathematical modeling and its potential place in the curriculum."

Learning Objectives. In this project, students will:

A. Plot 2-dimensional vector fields. Match the algebraic and graphical representations of a vector field. Predict the qualitative characteristics of a vector field written as a linear combinations of familiar vector fields. (algorithmic skill)

B. Engage in the mathematics modeling process to develop a 2- or 3- dimensional vector field (or fields) possessing qualitative characteristics of a hurricane (e.g., vortex flow, sink flow, eyewall). (willingness to try)

C. Incorporate parameters into their model allowing them to explore perturbations to sink flow and vortex flow components of their model through partial-derivative parameter sensitivity analysis. (algorithmic skill)

D. Write a technical report detailed the development of their vector field model, including the real-world meaning of components and parameters. (comprehension and communication)

E. Present the contents of their report to their peers and field questions about their modeling process and decisions. (comprehension and communication)

F. Explain how anthropogenic climate change may result in an increase in tropical cyclone strength and frequency. (comprehension and communication)

G. Recognize the relevance of mathematics as a tool for understanding the world and making predictions. Increase appreciation and respect for the work of professional scientists and the scientific method (appreciation).

List of Materials

Handout numbers are to indicate distribution order for teachers intending to engage students in a more scaffolded modeling experience. Regardless of whether teachers choose this route, they should make use the reflection journal and action plan tasks.

- MacGuffin Institute Letter Template
- Handout 1: An Introduction to Vector Fields
- Handout 4: Reflecting on Your Experience (Reflection Journal 1)
- Handout 6: Reflecting on Your Experience (Reflection Journal 2)
- Handout 11: Taking Action
- Handout 12: Reflecting on Your Experience (Reflection Journal 3)
- Computers with internet access (at least 1 per group)
- Classroom computer with projector for sharing digital content and for student use during presentations.
- Word processing software capable of writing mathematical formulas (e.g., Overleaf, Microsoft Word, Google Docs)

List of Supplementary Materials

Supplementary materials are provided for teachers intending to provide students with a more scaffolded modeling experience.

- Mathematical Modeling & Project Introduction Slideshow
- Handout 2: Defining the Problem & Selecting Appropriate Variables
- Handout 3: Making Appropriate Assumptions
- Handout 5: Implementing Your Model
- Handout 7: Iterating the Modeling Process
- Handout 8: Analyzing Your Model
- Handout 9: Reporting Your Results - Technical Report Specifications
- Handout 10: Presenting Your Results - Presentation Requirements

Project Details

Definitions and Notation

Vector Field: A vector field $\mathbf{F}$ in $\mathbb{R}^n$ is an assignment of an n-dimensional vector $\mathbf{F}(x_1, x_2, \ldots, x_n)$ to every point $(x_1, x_2, \ldots, x_n)$ of a some subset D of $\mathbb{R}^n$. In this case, D is called the domain of $\mathbf{F}$.

Most important for this project are the following cases:

A vector field $\mathbf{F}$ in $\mathbb{R}^2$ is an assignment of a 2-dimensional vector $\mathbf{F}(x, y)$ to every point (x, y) in some subset D of $\mathbb{R}^2$.

A vector field $\mathbf{F}$ in $\mathbb{R}^3$ is an assignment of a 3-dimensional vector $\mathbf{F}(x, y, z)$ to every point (x, y, z) in some subset D of $\mathbb{R}^3$.

Ideal Fluid: An ideal fluid is a theoretical fluid which is both incompressible (i.e., has constant density) and non-viscous (i.e., has frictionless flow).

Steady State Fluid: A fluid in a steady state is one in which the velocity of the fluid at every point remains constant over time.

Vortex Flow: A vortex flow in an ideal fluid is a vector field where vectors are tangent to concentric circles centered around a singular point. The magnitude of a vector is inversely proportional to its distance from, and constant at points equidistant from, the central singular point.

Uniform Sink Flow: A uniform sink flow in an ideal fluid is a vector field where vectors point directly toward a central point, called a sink. Approaching the sink, the flow area decreases, resulting in an increase in fluid velocity. The magnitude of a vector in a uniform sink flow is inversely proportional to its distance from the sink and remains constant at all points equidistant from the sink.

Preparation

1. Prior to starting this project, read through all handouts and suggested teacher and student readings, and watch the introductory video clip (see **Introducing the Project**).

> **Pedagogical Note.** Many teachers may be unfamiliar with some of the more specialized terminology like "uniform sink flow" and concepts like an "ideal fluid." It is important that teachers prepare to respond to questions such as, "If all the fluid is flowing to the center of the hurricane, doesn't it have to escape somehow? Otherwise it seems like we would just have an ever-growing body of fluid at the center of the hurricane."
>
> In addition, teachers new to mathematical modeling education are encouraged to make use of the Supplementary Materials. They are numbered to indicate the order in which they ought to be distributed to students in tandem with the regular project materials.

2. Teachers employing the recommended narrative scenario as a frame for the project should write a brief introductory letter congratulating students on their recent hire at the MacGuffin Institute and introducing them to their roles as a policy researchers who will use mathematical modeling to determine whether tracking in school mathematics is an equitable practice. A MacGuffin Institute letter template can be found under **Resources**. While the letters can be provided to students the day of project launch, it is recommended that students receive their letter prior to introduction of the project to avoid unnecessary distraction. In addition, it is recommended that resources be distributed digitally to minimize paper waste.

> **Technical Tool Tip.** Teachers intending to follow the recommendation to distribute project resources digitally should ensure prior to launching the project that students' preferred forms of contact (e.g., email) are up-to-date.

3. While it is recommended that teachers direct students to take advantage of the organization and communication tools of their liking, teachers wishing to make use of project management software to distribute materials and monitor student progress should set aside a substantial amount of time to setup these electronic resources for students.

> **Technical Tool Tip.** Students often prefer tools that integrate easily with smart devices. Some easy-to-use project management and collaboration resources with mobile support include Slack, Trello, and GroupMe.

4. Student groups should be in place prior to introducing the project, and should ideally consist of no more than four members. Unless the teacher has reason to do otherwise, student groups should be created randomly. Teachers may wish to designate a group leader to manage groups' in-class work and oversee organization and planning of out-of-class meetings.

> **Technical Tool.** Some online learning management systems (e.g., Canvas) can generate random groups of students as well as randomly assign a group leader, allowing teachers to avoid introducing unconscious bias into group and leader selection.

Introducing the Project

1. Begin by showing students the following video clip: Engines of Destruction: The Science of Hurricanes!.

> **Pedagogical Note.** Students new to mathematical modeling will likely benefit from a guiding modeling task. Slides for just such an introductory task, and that explains some of the mechanism underlying the Enhanced Greenhouse Effect, can provide a segue into the modeling project. They can be found in the Supplementary Materials.

2. Inform students that they will work with their team to develop a qualitative model of a hurricane.

3. Distribute **Handout 1: An Introduction to Vector Fields** and direct students to respond to the prompts it contains, working individually until the final prompt. The final prompt and exploring the data sources provide should use up any remaining time during the first meeting.

> **Note.** Teachers seeking to provide students with a more scaffolded modeling experience should make use of Handout 2, which may serve as a more formal guide for students on how to get started.
>
> In addition, visualization can greatly reduce student' anxiety with this task. Teachers are strongly encouraged to point students toward software that can plot vector fields, such as Maple. Teachers looking for a free option might consider the GeoGebra applet at the following link: https://www.geogebra.org/m/QPE4PaDZ

Monitoring Student Progress

As with any mathematical modeling experience, students will initially feel overwhelmed with the amount of autonomy they are afforded given the open-ended nature of the mathematical task. To ensure the authentic modeling experience does not devolve into a situation of learned helplessness, students work should be monitored during multiple class meetings dedicated to the modeling project and through their responses to prompts for their reflection journal entries.

Student teams should be expected to progress at different rates. However, teachers should ensure that all students have the experience to engage in at least one iteration of the modeling process. Teams that progress quickly should be strongly encourages to revisit aspects of their model, incorporate model parameters, and further analyze their model. Students should be provided with the Reflection Journal handouts no more frequently than one week apart to ensure they are afforded time sufficient to actually reflect on the process in which they are engaging.

Pedagogical Note. For teachers making use of the supplementary materials, the remaining worksheets are intended to be distributed at the discretion of the teacher and based on students' progress. Students lacking mathematical modeling experience can reasonable be expected to complete the tasks outlined in Handouts 2 through 6 and 9 through 12 at a minimum. Groups progressing quickly through the tasks should be provided with Handouts 7 and 8, which will require them to revisit and generalize their model. Handouts 7 and 8 can be repeatedly used by students to revise, and analyze their group's model.

The questions listed below for use during the final discussion can also be used throughout the modeling project to guide student teams who are struggling to progress.

Finally, assessment should depend primarily on student engagement in the modeling process, and not on the model they produce. The Supplementary Materials include suggestions for technical report and team presentation specifications and associated scoring rubrics.

Orchestrating the Final Discussion

Presentations and the final discussion should be organized using the following modified version of the 5 Practices for Orchestrating Productive Discussions in Mathematics (Smith & Stein, 2018):

1. Review the **Sample Approaches** found in these project materials. (Anticipating)
2. Read through each team's technical report. (Monitoring)
3. Make note of unanticipated approaches or otherwise interesting approaches. Be sure to find at least one particular aspect of each team's model to point out and celebrate later during presentations and the final discussion, (Selecting)
4. Determine the order of presentations based on what you think will be pedagogically advantageous. (Sequencing)
5. Make use of the suggested questions and write additional questions that will help students recognize their mathematical modeling successes and identify ways in which their model can be improved. (Connecting)

Technical reports should be submitted prior to the day of presentations to allow the teacher to read through each report and list questions and comments for use during presentations. In addition to checking the report specifications against the technical report specifications list, the project facilitator should place groups in sequence for presentations based on their model approaches.

The following questions can be used following student presentations and to guide the final discussion.

- How does the domain of your vector-field hurricane model compare with an actual hurricane?
- What real-world phenomena placed restrictions on what values your variables and parameters could take?
- Why did you decide to use that specific value for a parameter?
- What happens to your model when you alter the values you input? How much do you have to alter them by to obtain that result?
- What does this mean in real-world terms?
- How did you translate this concept to mathematics?

Prior to beginning a discussion of the social justice topic, it is important to establish norms for appropriate classroom discourse. The following list has been successful during previous discussions of socially charged topics:

- Prepare to feel discomfort.
- Listen respectfully.
- Share the time.
- Be mindful of the intent *and* the impact of your words.
- Challenge ideas, not people.

Given the way it has been politicized, many students may be hesitant to discuss their thoughts and opinions regarding the global climate crisis and its relation to an expected increase in Category 4 and 5 hurricanes. The following questions can serve as a useful segue into a discussion of environmental justice as it relates to the science of tropical cyclone modeling.

- How does more accurate and timely hurricane prediction relate to human rights and environmental justice?
- What actions by public officials and private citizens, if any, do you think should be taken in anticipation of an increase in Category 4 and 5 hurricanes?

Students should respond to the Action Plan prompts during class immediately following the Final Discussion. Reflection Journal 3 should be sometime following the final class meeting dedicated to the modeling project.

Sample Approaches and Models

Sample Approach #1

One of the students in Group 1 has already taken a course on vector calculus and quickly guides the rest of the group through the prompts in *Handout 1: An Introduction to Vector Fields*. They

quickly come to the following model:

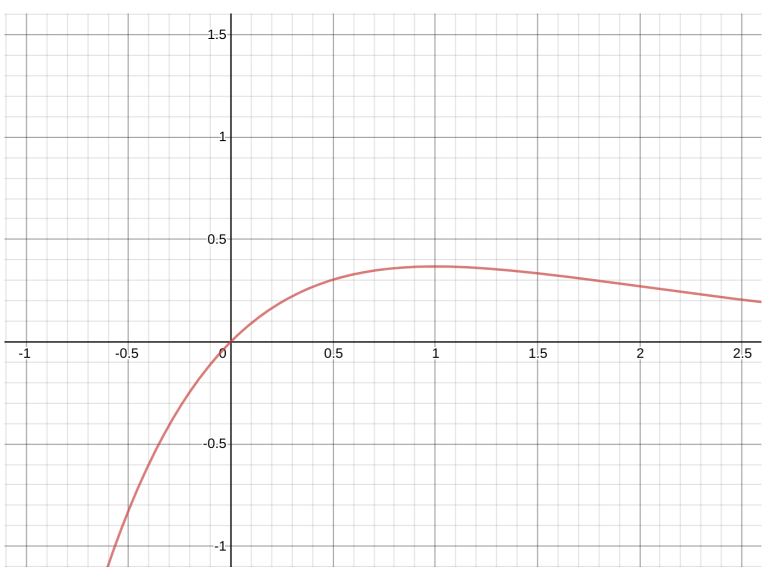
$$\mathbf{F}(x,y) = \frac{1}{\sqrt{x^2+y^2}}\left\langle \frac{-x-y}{\sqrt{2x^2+2y^2}}, \frac{x-y}{\sqrt{2x^2+2y^2}} \right\rangle.$$

During a group discussion about what else to include in their model, the students in Group 1 determine to incorporate and eyewall. One of them points out that, if they consider the inside of, say, the unit circle to be the eye of their hurricane, they can accomplish this easily by just assert a restriction of the domain of their vector field to the unit circle's exterior, arguing that this approach could save them a great deal of work. They question the project facilitator as to whether this is allowed, and receive a response in the affirmative. While deliberating this decision, one group member points out that restricting the domain would result in a model of a hurricane where the wind instantaneously decreases to zero upon breaching the eye from the exterior, which is not very realistic. After some grumbling about unnecessarily complicating their work, the remaining group members agree they will try to find a more natural way of modeling the eyewall.

During their next meeting and after trying a number of unsuccessful attempts to scale their current model by some clever function, one member reminds the rest of the group that the way they came up with their current model was starting with a unit vector field,

$$\mathbf{F}(x,y) = \left\langle \frac{-x-y}{\sqrt{2x^2+2y^2}}, \frac{x-y}{\sqrt{2x^2+2y^2}} \right\rangle,$$

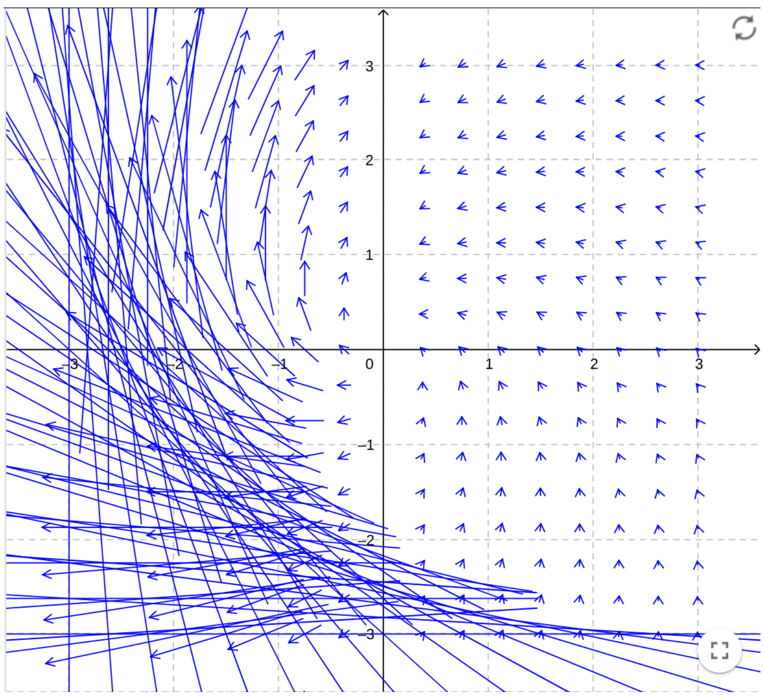
and scaling it by a function which approaches infinity as x and y approach zero, namely, $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$. This student suggests they try and find other functions that, instead of necessarily approaching infinity as x and y approach zero, have maxima along the unit circle, and then quickly decreases inside the unit circle. Unsure where to begin with this approach, the group decides to start with functions of a single variable. After some experimentation on Desmos, one student exclaims they may have found something, pointing excitedly to the graph they produced:



Upon noticing their peers bewilderment, the student points out that the graph kind of looks like one half of a sideways cross section of a hurricane (if you ignore rain bands). The group decides to try scaling their vector field of unit vectors by the function associated with this graph, $f(x) = xe^{-x}$, to obtain a new vector field

$$\mathbf{F}(x,y) = xe^{-x}\left\langle \frac{-x-y}{\sqrt{2x^2+2y^2}}, \frac{x-y}{\sqrt{2x^2+2y^2}} \right\rangle.$$

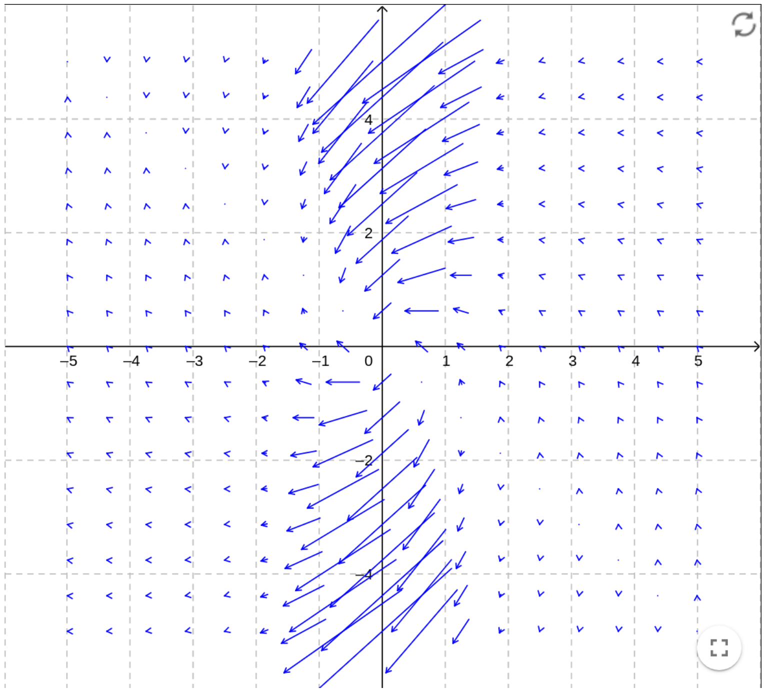
Immediately upon plotting the new vector field, the group concludes this was not the right approach:



During a discussion about what went wrong, one student points out that the along the y-axis, all vectors will be zero and another realizes that the negative exponential term was making the vectors with negative x-component blow up. One student points out that squaring the x term in the exponential function will prevent the magnitudes from blowing up, but another points out that that still leaves the problem of zero vectors along the y-axis. The group settles on replacing xe^{-x} with $(x+y)e^{-x^2}$, to address both problems simultaneously. The resulting model is

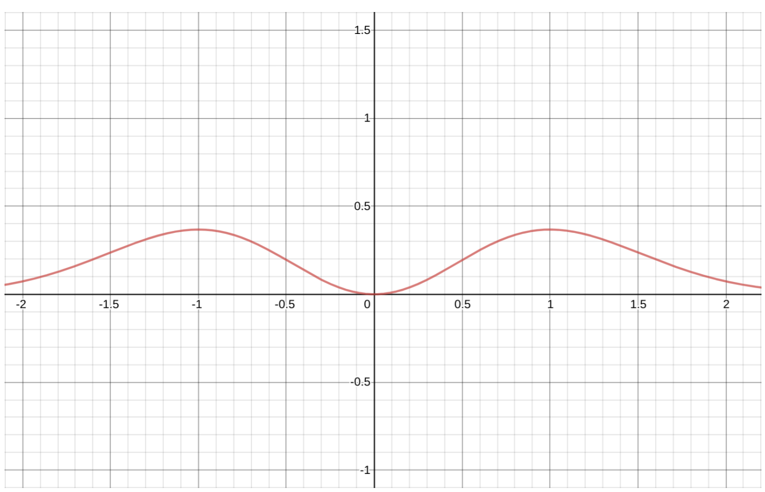
$$\mathbf{F}(x,y) = (x+y)e^{-x^2}\left\langle \frac{-x-y}{\sqrt{2x^2+2y^2}}, \frac{x-y}{\sqrt{2x^2+2y^2}} \right\rangle,$$

with the following plot:



Someone immediately points out that they forgot to account for the fact that vectors will be zero along the line $y = -x$, notes that the vortex structure is gone as well.

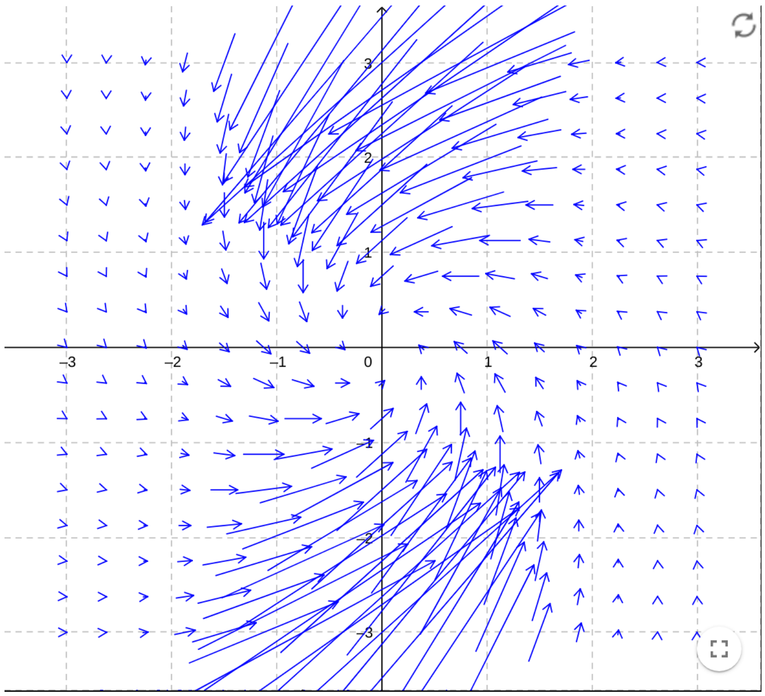
Immediately upon beginning their next day of in-class project work, one student enthusiastically informs the rest of the group that they have been thinking about their conundrum, and decided to spend some more time looking at plots of functions of one variable. Instead of $y = e^{-x}$, they decided to try plotting $y = x^2 e^{-x^2}$ since it never blows up, and is only zero at the origin, suggesting they may be able to extend the idea behind this function to one of both x and y. The plot looked promising:



The students eventually noticed that while $x + y = 0$ when $y = -x$, $x^2 + y^2 = 0$ only when both $x = 0$ and $y = 0$. They decide to try altering their scaling factor again. The result is

$$\mathbf{F}(x, y) = (x^2 + y^2)e^{-x^2}\left\langle \frac{-x - y}{\sqrt{2x^2 + 2y^2}}, \frac{x - y}{\sqrt{2x^2 + 2y^2}} \right\rangle,$$

which has the following plot:

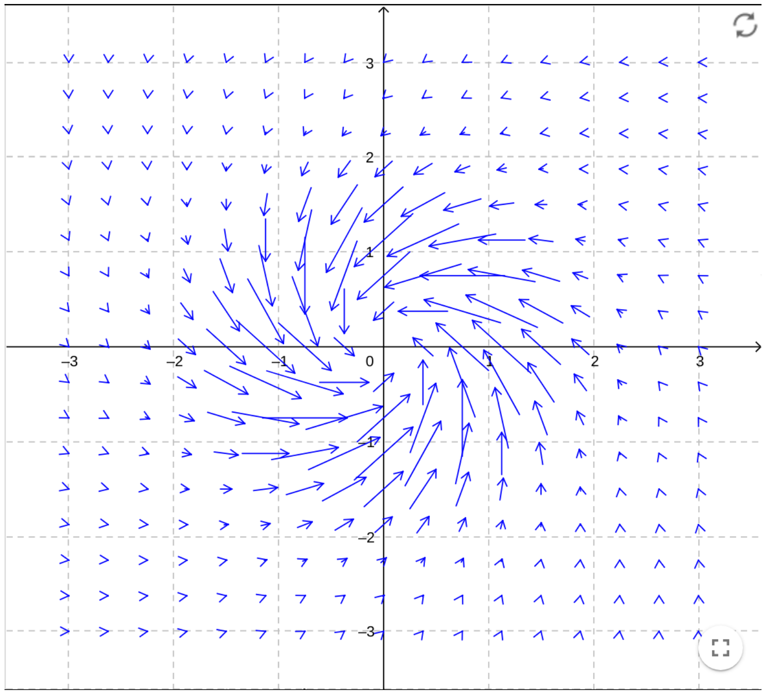


The students notice that near the x-axis, as x approaches zero from either side, the magnitude of the vectors first increase and then decrease, just as they were hoping it would. However, they also have the obvious problems that appear as y grows in magnitude. One student points out that when y is close to zero the y^2 term is small, but as y grows in magnitude, so does y^2.

They ask each other what was different in the case of the x terms that prevent this from happening. One student reminds the rest of the group that the e^{-x^2} term shrinks really fast—faster than x^2 grows. This, they conclude, is probably what is preventing the vectors from blowing up in magnitude as x gets further from zero. They resolve to try a similar tactic, multiplying by an additional term of e^{-y^2}, to diminish the impact of y^2 as y grows in magnitude. They write down their final mathematical model:

$$\begin{aligned}\mathbf{F}(x,y) &= (x^2+y^2)e^{-x^2}\cdot e^{-y^2}\left\langle \frac{-x-y}{\sqrt{2x^2+2y^2}}, \frac{x-y}{\sqrt{2x^2+2y^2}}\right\rangle \\ &= (x^2+y^2)e^{-(x^2+y^2)}\left\langle \frac{-x-y}{\sqrt{2x^2+2y^2}}, \frac{x-y}{\sqrt{2x^2+2y^2}}\right\rangle,\end{aligned}$$

which has the following plot (magnitudes have been lengthened to emphasize how they change as $(x,y)\to(0,0)$):

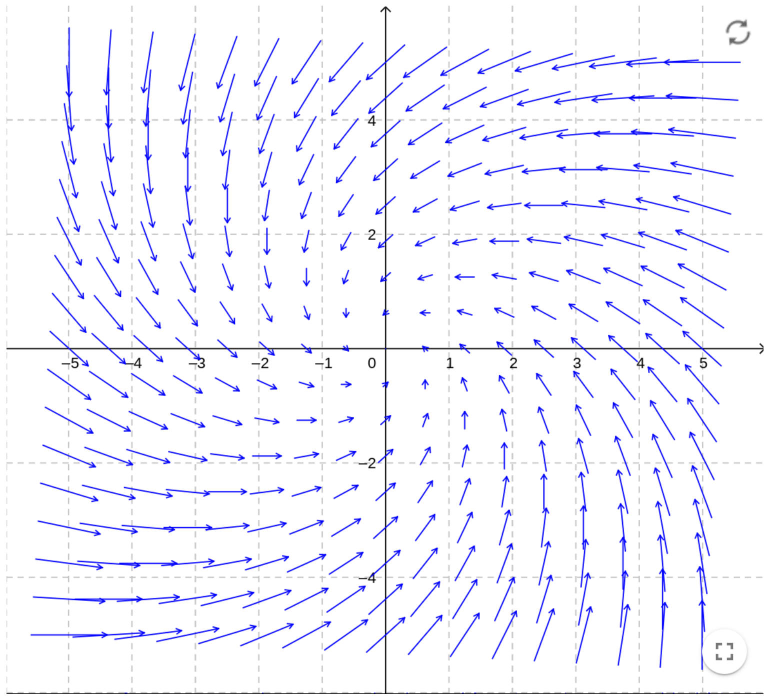


Sample Approach #2

After some discussion, tinkering with the GeoGebra applet to which the teacher provided a link, and looking back at the tasks in **Handout 1: An Introduction to Vector Fields**, the students decide to experiment with their newly learned concept of linearity of vector fields, and look at the following sum:

$$\begin{aligned}\mathbf{F}(x,y) &= \langle -x,-y\rangle + \langle -y,x\rangle \\ &= \langle -x-y, x-y\rangle\end{aligned}$$

which has the following plot:

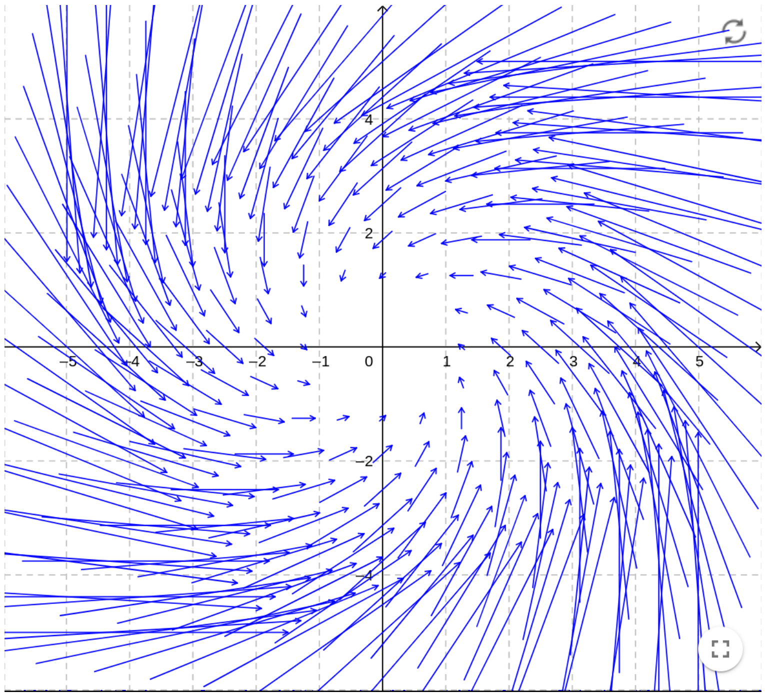


Each member of the group agrees that they would like to somehow incorporate an eye in their hurricane, but they would rather it be through a restriction on the natural domain of their vector field rather than solely a declaration that the vector field is undefined in some central circular region. They decide they will have the inside of the unit circle be their hurricane eye and start discussing how they could produce the desired characteristic.

After some discussion, one student says they think it should be possible to somehow incorporate the equation of the unit circle, but they are not entirely sure of how they could force the vector field to be undefined inside the circle. Some additional time and thought leads another team member to suggest that maybe they could use a the square root function somehow. After a little more tinkering, the students recognize that $\sqrt{x^2+y^2-1}$ is only defined for (x, y) such that $x^2+y^2 \geq 1$ (which is only true on and outside of the unit circle). They decide to try out the following vector field:

$$\mathbf{F}(x, y) = \sqrt{x^2+y^2-1}\langle -x-y, x-y\rangle,$$

which has the following plot:

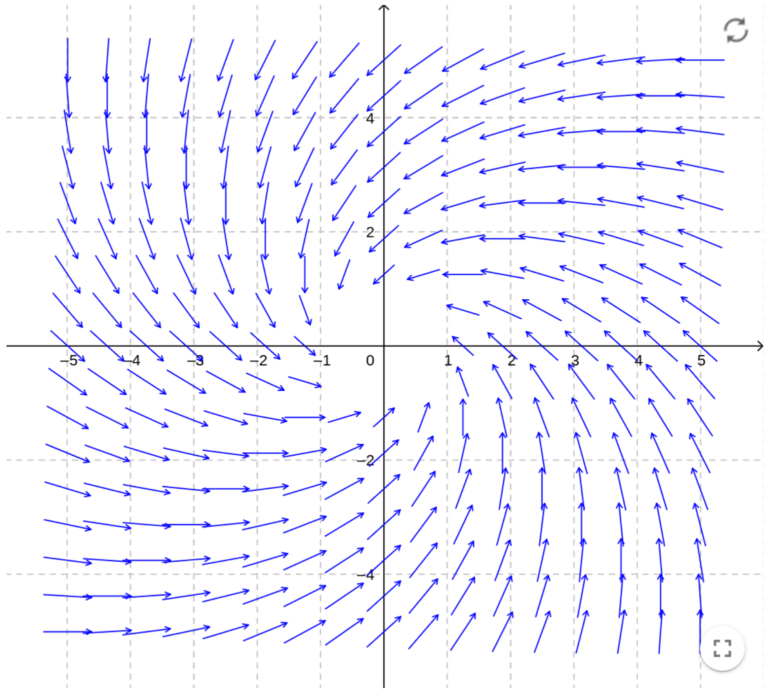


They celebrate their success at restricting the domain, but are understandably concerned that the vector magnitudes appear to be growing as the distance from the eye increases, which is the opposite of what they want. They agree to each spend some time before their next meeting exploring how they could ensure the magnitudes grow on approach to the eyewall.

One of the team members decides to use Desmos to explore single-variable functions that increase as x approaches 0. They notice that $\frac{1}{x^2}$ increases from both sides as x approaches 0, and decides to see what happens if they scale their vector field by $\frac{1}{x^2+y^2}$:

$$\mathbf{F}(x,y) = \frac{\sqrt{x^2+y^2-1}}{x^2+y^2}\langle -x-y, x-y\rangle.$$

They obtain the following plot:



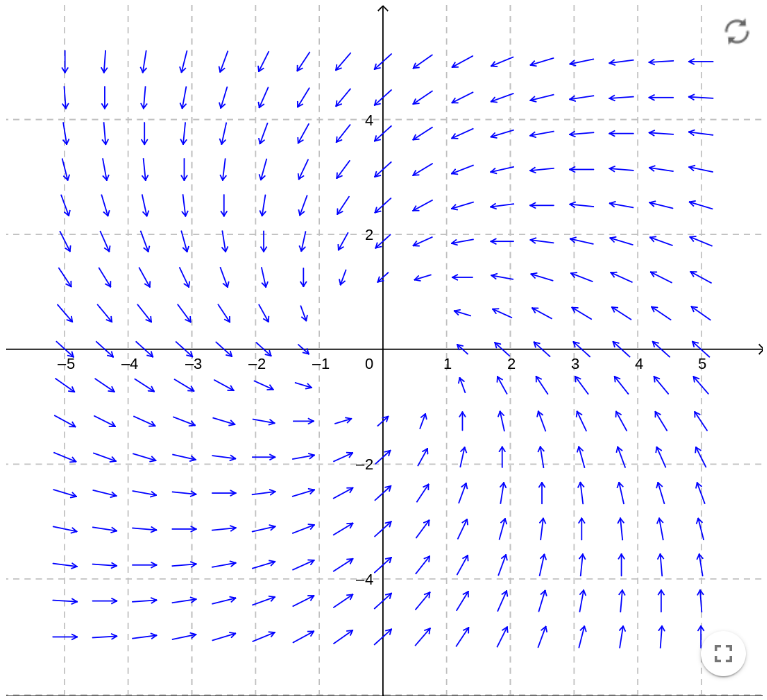
Noting how they vectors all appear to be the same length, they decide to try and normalize their vector field prior to introducing any scaling factor. Normalizing their original vector field, the student obtains:

$$\mathbf{F}(x, y) = \frac{1}{\sqrt{2x^2 + 2y^2}} \langle -x - y, x - y \rangle,$$

And then reintroducing their original scaling factor they obtain:

$$\mathbf{F}(x, y) = \frac{\sqrt{x^2 + y^2 - 1}}{\sqrt{2x^2 + 2y^2}} \langle -x - y, x - y \rangle,$$

which has the following plot:



Noticing how similar the previous this vector field is to their previous try, the student determines to seek feedback from the rest of the group during their next meeting.

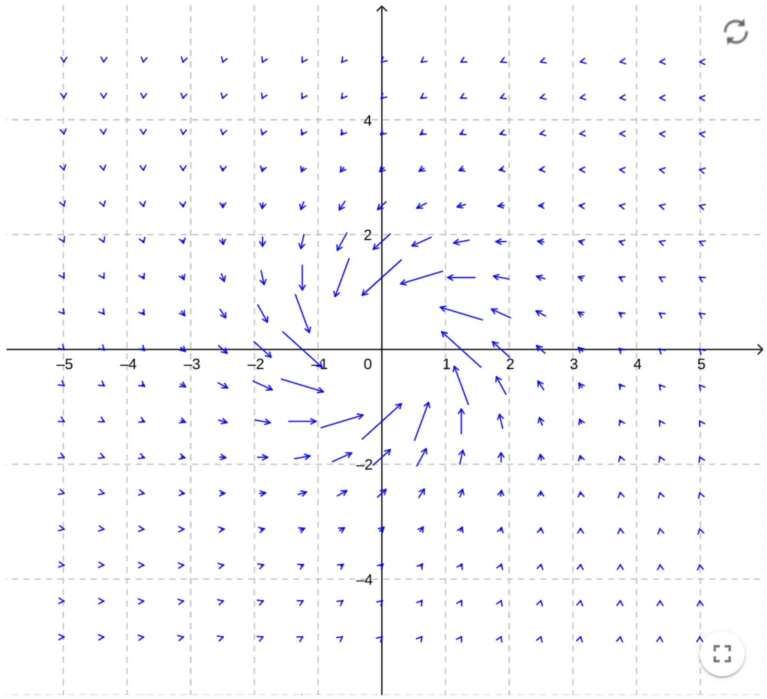
By the next meeting, they are still unsure of how to appropriately scale their vector field, but have conclude that whatever they do, they should begin with a unit vector field and work from there.

At the beginning of their next meeting, the student shares their thoughts. The students agree that it makes sense to scale all of the vectors to be length one prior to coming up with an appropriate scaling factor.

Another student shares that they have been looking at the function $y = e^{-(x^2+y^2)}$, pointing out that while it does not have a natural eye, it does increase uniformly from every direction as (x, y) approaches $(0, 0)$. Another student suggests they try and put all of these disparate ideas together, starting with a unit vector field, and then incorporating a negative exponential and a square root function somehow. After some experimentation, they group settles on the following vector field for their final model:

$$\mathbf{F}(x, y) = \frac{e^{-\sqrt{x^2+y^2-1}}}{\sqrt{2x^2+2y^2}}\langle -x-y, x-y\rangle.$$

which has the following plot:



Resources

Teacher Readings

The following readings are *necessary* preparation for teachers preparing to incorporate this project into a methods or content course.

- UNISDR: Economic Losses, Poverty & DISASTERS
- Hurricanes: Improved Track and Intensity Predictions (pp. 1)
- Hurricane Matthew: Haiti south '90% destroyed'
- CO2 Emissions by Country
- Comparing the Winds of Sandy and Katrina
- Development of tropical cyclone wind field for simulation of storm surge/sea surface height using numerical ocean model (Figure 8; pp.9)
- Hindcast of Waves and Currents in Hurricane Katrina (Figure 2; pp.3)

The following readings are not free and, as such, are only *recommended*. In particular, they provide an introduction to Blackbody radiation and the Greenhouse Effect (which are discussed in the introductory modeling activity from the Supplementary Materials slides), as well as providing a more detailed explanation of qualitative characteristics of tropical cyclones:

- Atmospheric Science: An Introductory Survey (Section 4.3 - Blackbody Radiation)
- Atmospheric Science: An Introductory Survey (Section 8.4 - Tropical Cyclones)

Student Readings

The following readings are *strongly recommended* for students engaging in this project. As such, they have been incorporated directly into the Reflection Journal Handouts.

- UNISDR: Economic Losses, Poverty & DISASTERS
- Hurricanes: Improved Track and Intensity Predictions (pp. 1)
- Hurricane Matthew: Haiti south '90% destroyed'
- CO_2 Emissions by Country
- Comparing the Winds of Sandy and Katrina
- Development of tropical cyclone wind field for simulation of storm surge/sea surface height using numerical ocean model (Figure 8; pp.9)
- Hindcast of Waves and Currents in Hurricane Katrina (Figure 2; pp.3)

An Introduction to Vector Fields

What even is a "Vector Field"?

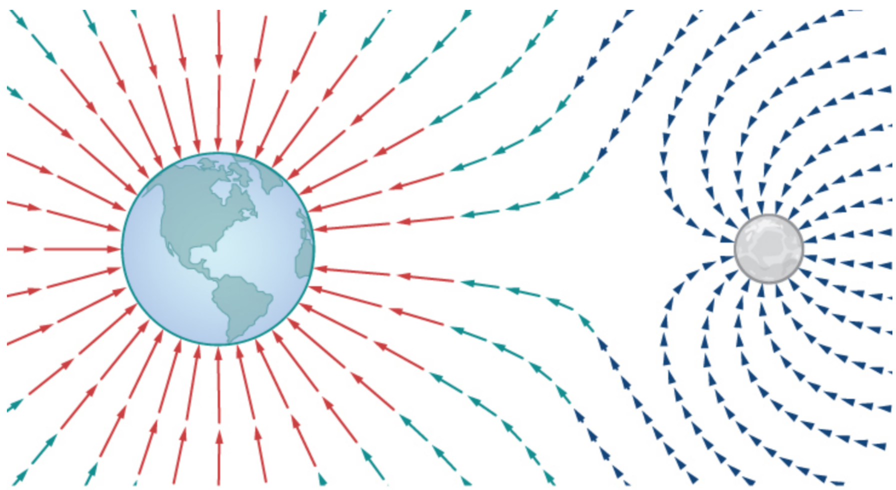
Definition 1 (Vector Field)**.** *A vector field* $\mathbf{R}$ *in* $\mathbb{R}^n$ *is an assignment of an* n*-dimensional vector* $\mathbf{F}(x_1, x_2, \ldots, x_n)$ *to each point* $(x_1, x_2, \ldots, x_n)$ *of a subset* D *of* $\mathbb{R}^n$*. The subset* D *is the domain of the vector field.*

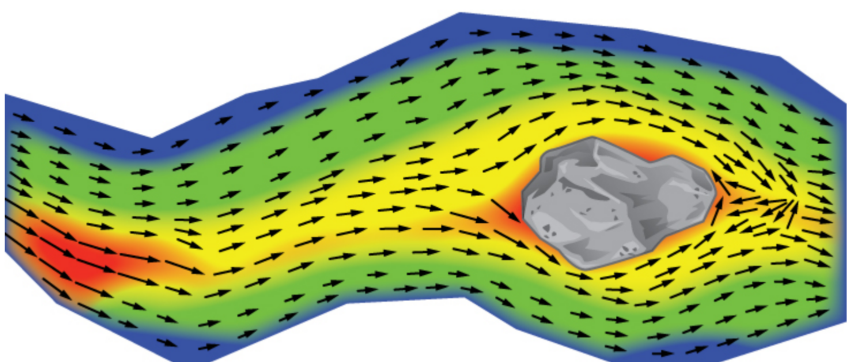
Most relevant to our course,

A vector field $\mathbf{F}$ *in* $\mathbb{R}^2$ *is an assignment of a two-dimensional vector* $\mathbf{F}(x, y)$ *to each point* (x, y) *of a subset* D *of* $\mathbb{R}^2$*. The subset* D *is the domain of the vector field.*

A vector field $\mathbf{F}$ *in* $\mathbb{R}^3$ *is an assignment of a three-dimensional vector* $\mathbf{F}(x, y, z)$ *to each point* (x, y, z) *of a subset* D *of* $\mathbb{R}^3$*. The subset* D *is the domain of the vector field.*

Among other things, vector fields can represent gravitational fields (top image) and fluid flows (bottom image) .

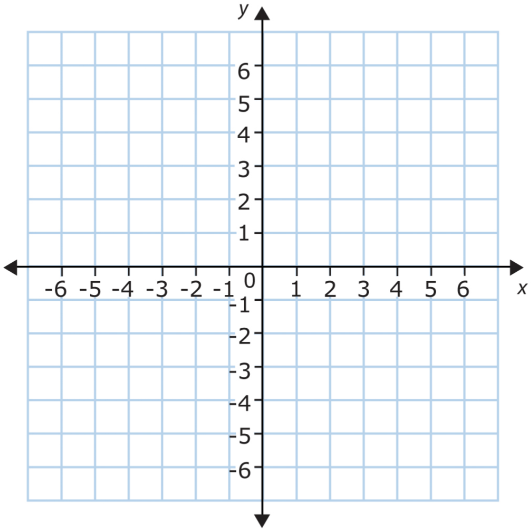


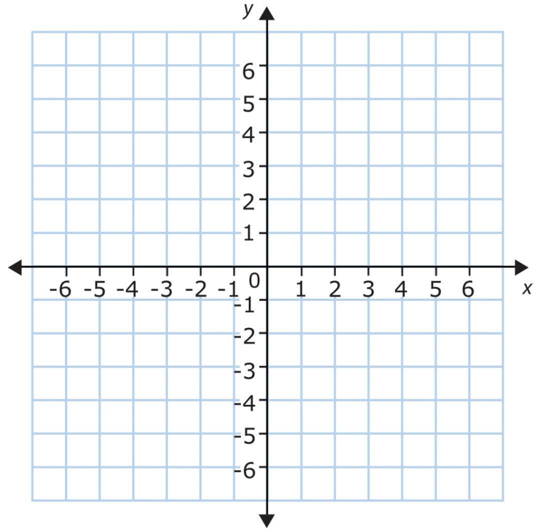


Images from Volume 3 of the OpenStax Calculus series.
Access for free at https://openstax.org/books/calculus-volume-3/pages/1-introduction

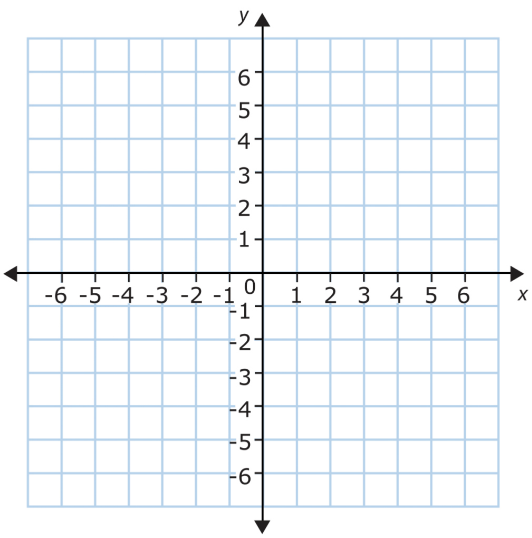
Visualizing Vector Fields

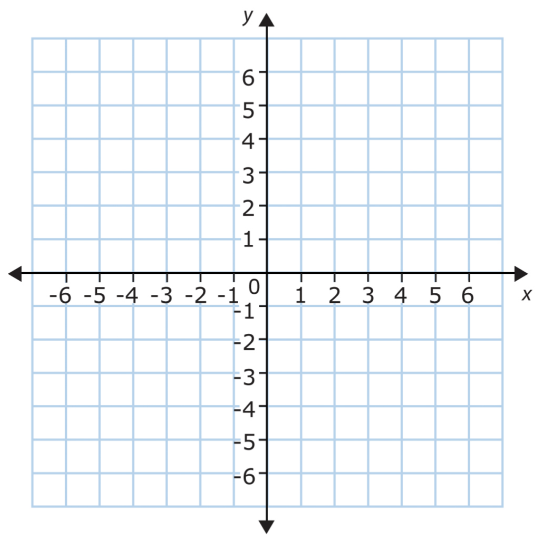
On the coordinate planes below, sketch the vector fields $\mathbf{F}_1(x,y) = \langle 0, x \rangle$ and $\mathbf{F}_2(x,y) = \langle y, 0 \rangle$.





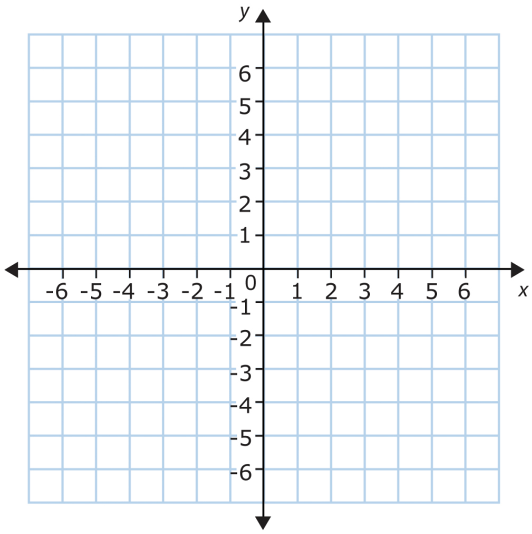
Just as with regular vectors, we add vector fields *component-wise*. On the coordinate plane below, sketch the vector fields $\mathbf{F}_1 + \mathbf{F}_2$ and $\mathbf{F}_1 - \mathbf{F}_2$ using the vector fields from the preceding exercise.

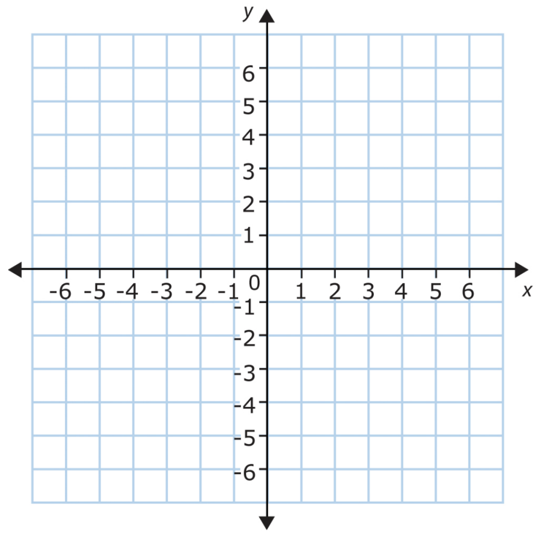




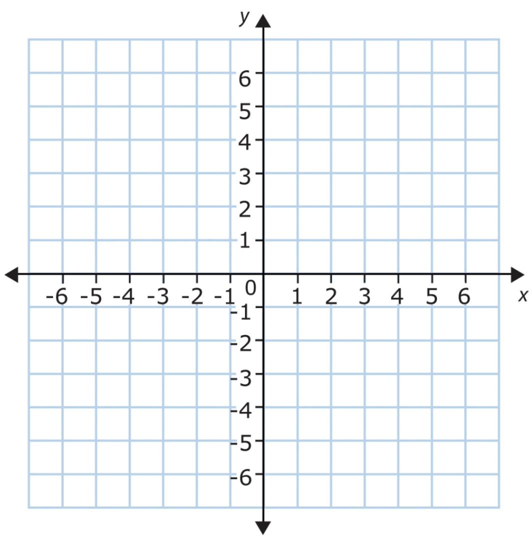
How do the plots of the sum and difference of $\mathbf{F}_1$ and $\mathbf{F}_2$ compare to their individual plots?

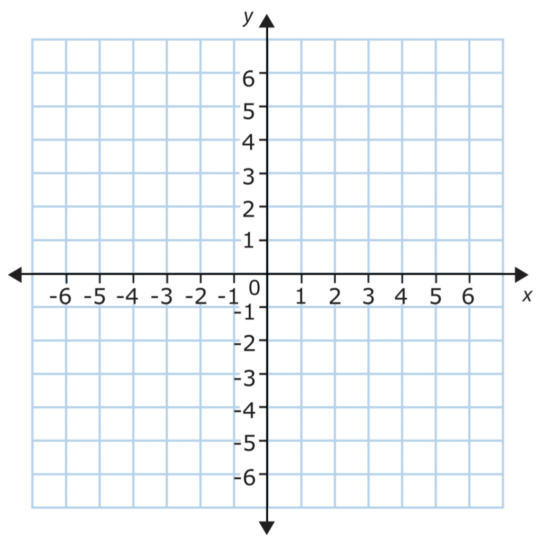
Scalar multiplication also still works with vector fields! On the coordinate plane below, sketch the vector fields $2 \cdot \mathbf{F}_1$ and $3 \cdot \mathbf{F}_2$ using the vector fields from the preceding exercises.





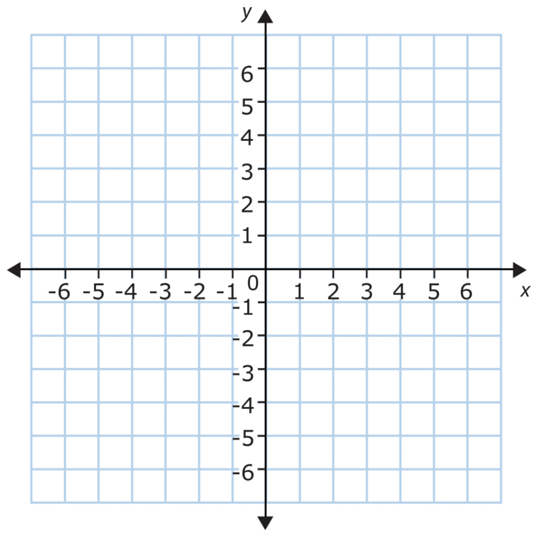
But what about the distribution of scalar multiplication across a vector sum? Does that still work? On the On the coordinate plane below, sketch the vector fields $2 \cdot \mathbf{F}_2 + 2 \cdot \mathbf{F}_1$ and $2 \cdot (\mathbf{F}_2 + \mathbf{F}_1)$ using those same vector fields from the preceding exercises.





Let $F(x, y) = f_1(x, y)\mathbf{i} + f_2(x, y)\mathbf{j}$ and $G(x, y) = g_1(x, y)\mathbf{i} + g_2(x, y)\mathbf{j}$ be vector fields, and let $\alpha, \beta \in \mathbb{R}$. In the space below, write down a conjecture about a formula for $\alpha\mathbf{F} + \beta\mathbf{G}$.

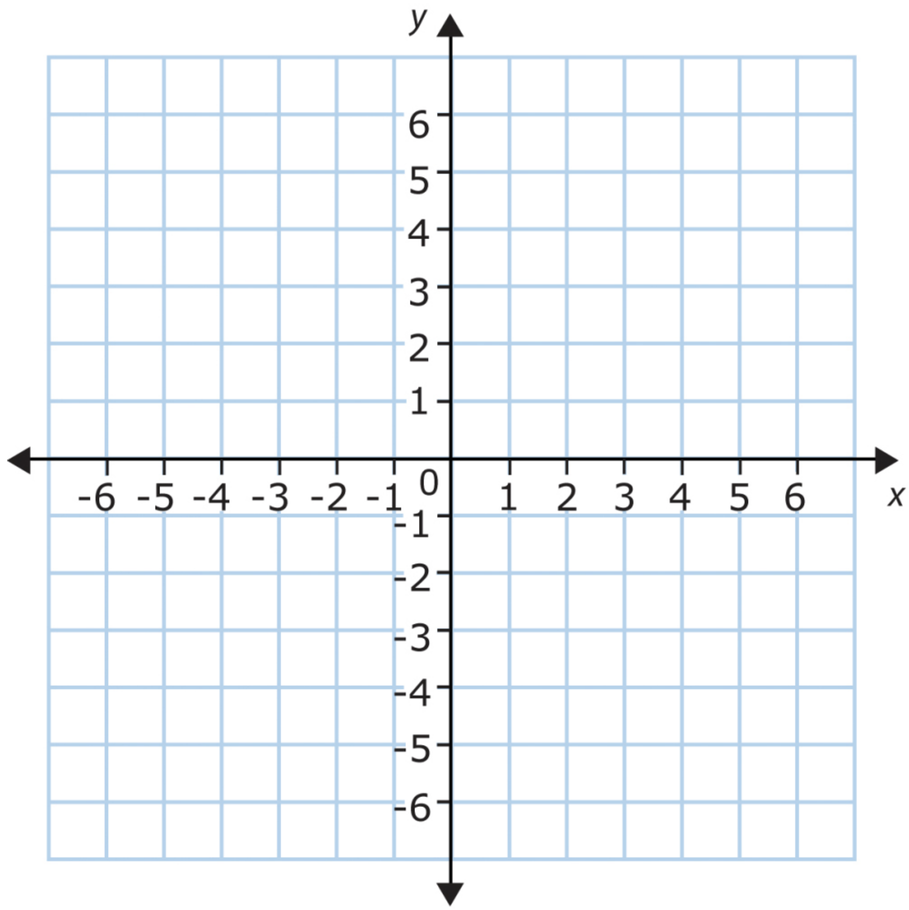
On the coordinate plane below, draw a vector field that where every vector is a unit vector that points away from the origin.



Now let's try going the other direction, from a graphical to an algebraic description. Try to write down the equation for the vector field you drew in the preceding exercise. It might help to think about how you might could write the vectors as a linear combination of the standard unit vectors $\hat{\mathbf{i}} = \langle 1, 0 \rangle$ and $\hat{\mathbf{j}} = \langle 0, 1 \rangle$.

Now see if you can come up with an equation for a vector field where every vector is a unit vector that lies tangent a circle centered at the origin. Again, it might help to think about how you might could write the vectors as a linear combination of the standard unit vectors $\hat{\mathbf{i}} = \langle 1, 0 \rangle$ and $\hat{\mathbf{j}} = \langle 0, 1 \rangle$.

One the coordinate plane below, sketch a 2 dimensional vector field possessing one or more characteristics of a hurricane.



Share your sketch with the rest of your team. What characteristics do your plots share? How are they different? With the rest of your team, settle on some hurricane characteristics that you want your model to possess, and start trying to write down an equation for the plot you want to produce.

Reflecting on Your Experience

In addition to be interesting weather phenomena, hurricanes carry a real cost. The United Nations Office for Disaster Risk Reduction recently looked into some of the damage costs associated with hurricanes. Read Box 4 on page 22 (page 24 of the PDF document) of a report they compiled in 2017 about some of the economic costs of hurricanes and other large storms:

- UNISDR: Economic Losses, Poverty & DISASTERS

The enormous cost associated with hurricanes has motivated efforts to improve the accuracy of tracking, and predicting the intensity of, hurricanes. The National Oceanic and Atmospheric Administration spends considerable time and resources in this area. In 2005, NOAA made a committment to reduce the amount of error in their predictions. Take a minute to read their plan, and then look at their data tables regarding changes in error in hurricane predictions in recent years (Note that you are welcome to seek clarification on the meaning of the different table data from your team or your teacher):

- Hurricanes: Improved Track and Intensity Predictions (pp. 1)
- National Hurricane Center Forecast Verification

Reflection Journal 1

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. What were you most surprised to learn in the UN report section you just read?
2. Do you think NOAA achieved the goal they outlined? Explain.
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project so far?
5. Is there anything you want your teacher to know?

Reflection Journal 1 Scoring Rubric

1. Student shares most surprising thing learned in UN report. (3 points)

• The student shares what they found most interesting in the UN report.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

2. Student evaluates NOAA's achievement of goals. (3 points)

• The student explains whether they think NOAA achieved their listed goals.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

Total _______/14

Reflecting on Your Experience

The cost of insufficiently precise hurricane prediction and tracking is not just economic in nature. Take a minute to examine Figure 5 and its description on page 9 (page 11 of the PDF document) of that same UNISDR report.

- UNISDR: Economic Losses, Poverty & DISASTERS

Sometimes it is difficult to wrap our head around these numbers. Take a minute to read the following article on the impact of Hurricane Matthew in Haiti back from 2016. Be sure to take a moment to look at the images:

- Hurricane Matthew: Haiti south '90% destroyed'

Take a few minutes to compare the infographics on pages 4 and 5 (pages 6 and 7 in the PDF) from the UNISDR report, with national contributions to Greenhouse Emissions as recorded by Worldometers:

- CO2 Emissions by Country

Reflection Journal 2

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. What did you notice when comparing the information from the UNISDR and Worldometers?
2. Explain how global warming may result in more destructive hurricanes.
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project so far?
5. Is there anything you want your teacher to know?

Reflection Journal 2 Scoring Rubric

1. Student describes their observations from data comparison. (3 points)

• The student compares countries harmed by hurricanes with highest polluters.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

2. Student explains how global warming impacts hurricanes. (3 points)

• The student explains how global warming may impact future hurricanes.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

Total _______/14

Taking Action

Understanding the social and economic impact of hurricanes is an important first step for developing an appropriate plan for the future of hurricane response, but few of us will probably enter professions where that understanding can have a direct influence.

However, that doesn't mean that there is nothing that we as individuals can do!

Writing an Action Plan

1. Write down at least two specific actions you plan to take in your own community which can contribute the the global effort to reduce the impact of climate change and to improve the the national and international response to hurricane catastrophes.

Reflecting on Your Experience

You and your team have accomplished a lot. Take a minute to look at the vector field models of some professional modelers and atmospheric scientists. Before you go any further, be aware that many professionals draw their vector fields using vectors of the same length. They represent the vector magnitude using a "heat map," which assigns different colors to regions based upon the magnitude of vectors located there.

As you look at the following graphics, make note of the differences and similarities between your model and those you examine. Some of these are based on actual weather readings, while others are mathematical models. Figure and page numbers are provided to make them easier to locate.

- Comparing the Winds of Sandy and Katrina
- Development of tropical cyclone wind field for simulation of storm surge/sea surface height using numerical ocean model (Figure 8; pp.9)
- Hindcast of Waves and Currents in Hurricane Katrina (Figure 2; pp.3)

Reflection Journal 3

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. How was your team's model similar to those you just examined? In what ways were they different?
2. Some people and organizations claim that the hurricane response is a human rights issue. Do you agree? Explain.
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project?
5. Is there anything you want your teacher to know?

Reflection Journal 3 Scoring Rubric

1. Student comments on models examined. (3 points)

• The student describes similarities and differences between professional and team's model.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

2. Student discusses hurricane response as a human rights issue. (3 points)

• The student explains whether whether they think hurricane response is a human rights issue.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

Total _______/14

“Tracking” in School Mathematics

Duration: 4 weeks

In-class Meetings: 4

Prerequisite Knowledge: Proportional Reasoning

Overview

Summary

In this project, students explore the practice of “tracking” in school mathematics, where students are placed in course tracks based on their perceived ability. This project positions students as researchers in a think tank focusing on education policy that are exploring a claim by the Association of Mathematics Teacher Educators, in their Standards for Preparing Teachers of Mathematics, that tracking is inequitable (they go so far as to label it a form of ”oppression”). Their goals are to produce a mathematical formulation of the concept of equity in the context of school mathematics and tracking and to develop a mathematical model they can use to analyze the AMTE’s claim. Students present their model, field questions regarding the development of their model, and participate in a discussion of the social justice issues surrounding tracking. Students engage in ongoing reflection about the mathematical modeling process and how they view themselves in relation to mathematics and the modeling process. Students read documents produced by mathematics education organizations discussing tracking, write about their and thoughts and beliefs about—and any personal experiences with—tracking, and reflect on the social justice implications of embracing or eliminating tracking.

Relevant SPTM Standards and Indicators

Indicator C.1.2. Demonstrate Mathematical Practices and Processes -

> “[Well-prepared beginning teachers] can apply their mathematical knowledge to real-world situations by using mathematical modeling to solve problems appropriate for the grade levels and the students they will teach.”

Indicator C.4.1. Provide Access and Advancement -

> “Access includes ensuring that students have opportunities to learn important mathematics taught by qualified teachers. Well-prepared beginners recognize that access involves structures in schools and in classrooms and recognize classroom practices that threaten access. Access becomes particularly important in the placement of students into higher level courses, in which the focus is on doing mathematics rather than practicing procedures.”

Indicator C.4.4. Understand Power and Privilege in the History of Mathematics Education -

> “Well-prepared beginning teachers of mathematics understand the roles of power, privilege, and oppression in the history of mathematics education and are equipped to question existing educational systems that produce inequitable learning experiences and outcomes for students. . . . [W]ell-prepared beginners might ask how

students are recommended and placed in gifted and remedial/intervention programs, whether the placement of students across various programs is representative of the school population"

Indicator P.2.2. Build Mathematical Practices and Processes -

> "Mathematical modeling—using mathematics to analyze real-world situations—receives continued attention throughout the program."

Indicator P.2.3. Provide Sustained, Quality Experiences -

> "[M]athematics content courses should be taught using teaching methods that serve as models of effective instruction."

Indicator P.3.3. Address the Social Contexts of Teaching and Learning -

> "An effective mathematics teacher preparation program embeds opportunities for candidates to learn about the social, historical, political, and institutional contexts that affect mathematics teaching and learning. By closely examining these contexts and the structures, policies, and practices that foster and constrain student access to and advancement in mathematics, candidates develop deeper understandings and ethical skill sets for advocacy work in mathematics education."

ML.1. Essential Understandings of Mathematics Concepts and Practices -

> "[Well-prepared beginning] teachers understand algebraic thinking as (a) the study of structures in the number system, including those arising in arithmetic; (b) the study of patterns, relations, and functions; and (c) the process of mathematical modeling."

ML.4. Meaningful and Interdisciplinary Contexts -

> "Well-prepared beginning teachers of mathematics at the middle level understand how to engage middle level learners in meaningful and interdisciplinary contexts, including the use of mathematical modeling."

HS.1. Essential Understandings of Mathematics Concepts and Practices in High School Mathematics -

> "Well-prepared teachers have substantive experiences engaging in mathematical modeling so that they can understand mathematical modeling and its potential place in the curriculum."

Learning Objectives. In this project, students will:

A. Engage in the mathematics modeling process to develop a model for analyzing how "tracking" in school mathematics creates and reinforces social inequities. (willingness to try)

B. Incorporate parameters into their model allowing them to explore the effects of altering known or unknown values. (algorithmic skill)

C. Write a technical report detailed the development of their model, including the

real-world meaning of components and parameters. (comprehension and communication)

D. Present the contents of their report to their peers and field questions about their modeling process and decisions. (comprehension and communication)

E. Explain how tracking in school mathematics creates and reinforces social inequities. (comprehension and communication)

F. Reflect on the mathematical modeling process, the state of equity in school mathematics, and how the former can be used to understand the latter. (comprehension and communication)

G. Recognize the relevance of mathematics as a tool for understanding the world and making predictions. (appreciation).

List of Materials

Handout numbers are to indicate distribution order for teachers intending to engage students in a more scaffolded modeling experience. Regardless of whether teachers choose this route, they should make use the reflection journal and action plan tasks.

- MacGuffin Institute Letter Template
- Handout 3: Reflecting on Your Experience (Reflection Journal 1)
- Handout 5: Reflecting on Your Experience (Reflection Journal 2)
- Handout 10: Taking Action
- Handout 11: Reflecting on Your Experience (Reflection Journal 3)
- Computers with internet access (at least 1 per group)
- Classroom computer with projector for sharing digital content and for student use during presentations.
- Word processing software capable of writing mathematical formulas (e.g., Overleaf, Microsoft Word, Google Docs)

Supplementary Materials

Supplementary materials are provided for teachers intending to provide students with a more scaffolded modeling experience.

- Mathematical Modeling & Project Introduction Slideshow
- Handout 1: Defining the Problem & Selecting Appropriate Variables
- Handout 2: Making Appropriate Assumptions
- Handout 4: Implementing Your Model
- Handout 6: Iterating the Modeling Process

- Handout 7: Analyzing Your Model
- Handout 8: Reporting Your Results - Technical Report Specifications
- Handout 9: Presenting Your Results - Presentation Requirements

Project Details

Definitions and Notation

Ability Grouping: Often used synonymous with "tracking," ability grouping is an umbrella term for any practices that groups students based on their perceived ability, whether in the same or different classes.

Directly Proportional: Two quantities a and b are said to be directly proportional to one another if and only if $a = kb$, where k is a constant.

Equity: There are multiple understandings of the term, but we will define a equity in education as "being unable to predict student patterns (e.g., achievement, participation, the ability to critically analyze data or society) based solely on characteristics such as race, class, ethnicity, sex, beliefs and creeds, and proficiency in the dominant language" (Gutierrez, 2002).

Essentialism: A sociological theory that posits any entity such as an individual, group, object, or concept has innate and universal qualities. Essentialism has been used as a justification for racism, imperialism, and bigotry. In the context of this project, we promptly reject any and all essentialist explanations for observed differences in mathematical achievement levels.

Implicit Bias: The unconscious attitudes, stereotypes and unintentional actions (positive or negative) towards members of a group merely because of their membership in that group.

These associations develop over the course of a lifetime beginning at a very early age through exposure to direct and indirect messages. When people are acting out of their implicit bias, they are not even aware that their actions are biased. In fact, those biases may be in direct conflict with a persons explicit beliefs and values. (Anti-Defamation League).

Institutional Racism: Anonymous, subtle and systemic discrimination based on race, in legal instruments, as well as in private organizations and professions (educational, legal, healthcare, political, religious, etc.), private businesses and public decision-making bodies.

Because this form of racism is anonymous and built into standard institutional practice, individuals often resist acknowledging its existence and deny, consciously or unconsciously, their complicity in maintaining it.

Examples of institutionalized racism include policies and practices that: arbitrarily govern a person's credit-worthiness; determine what information, positive or negative, is presented in the media about individuals involved in newsworthy events; or place undue value on selective educational experiences or qualifications in establishing promotion criteria in jobs and schools. (Institute for Democratic Renewal, 1998)

Inversely Proportional: Two quantities a and b are said to be inversely proportional to one another if and only if $a = \frac{k}{b}$, where k is a constant.

Model-minority Stereotype: The model-minority stereotype is a harmful, essentialist generalization that, among other things, claims that Asian students have a natural aptitude for

science and mathematics.

Prejudice: A pre-judgment or unjustifiable, and usually negative, attitude of one type of individual or group toward another group and its members. Such negative attitudes are typically based on unsupported generalizations (or stereotypes) that deny the right of individual members of certain groups to be recognized and treated as individuals with individual characteristics. (Institute for Democratic Renewal, 1998)

Race: The classification of humans based on arbitrary physical characteristics such as skin color, facial form and/or eye shape. (Institute for Democratic Renewal, 1998)

Racism: An ideological system of oppression and subjugation, held consciously or otherwise, based upon unfounded beliefs about racial and ethnic inequality. This system of oppression is based on a view that an arbitrary set of physical characteristics, such as skin color, facial form or eye shape, are associated with or even determine behavior, culture, intellect, or social achievement. (Institute for Democratic Renewal, 1998)

Stereotype Threat: Stereotype threat refers to the the negative impact on students' achievement that results from the fear of confirming negative stereotypes about some component of their identity.

Tracking: Tracking is the practice of placing students in a course "track" based on their perceived ability.

Preparation

1. Teacher should read through all handouts and suggested teacher and student readings, and watch the introductory video clip (see **Introducing the Project**).

> **Pedagogical Note.** Many teachers may be unfamiliar with some of the more specialized terminology like "stereotype threat" and concepts like the "model-minority stereotype." It is important that teachers prepare to respond to inevitable questions and comments about the disproportionate representation of different student racial and ethnic demographics in different mathematics courses. The teacher must be comfortable discussing the topics and model appropriate communication.
>
> In addition, teachers new to mathematical modeling education are encouraged to make use of the Supplementary Materials. They are numbered to indicate the order in which they ought to be distributed to students in tandem with the regular project materials.

2. Teachers employing the recommended narrative scenario as a frame for the project should write a brief introductory letter congratulating students on their recent hire at the MacGuffin Institute and introducing them to their roles as a policy researchers who will use mathematical modeling to determine whether tracking in school mathematics is an equitable practice. A MacGuffin Institute letter template can be found under **Resources**. While the letters can be provided to students the day of project launch, it is recommended that students receive their letter prior to introduction of the project to avoid unnecessary distraction. In addition, it is recommended that resources be distributed digitally to minimize paper waste.

> **Technical Tool Tip.** Teachers intending to follow the recommendation to distribute project resources digitally should ensure prior to launching the project that students' preferred forms of contact (e.g., email) are up-to-date.

3. While it is recommended that teachers direct students to take advantage of the organization and communication tools of their liking, teachers wishing to make use of project management software to distribute materials and monitor student progress should set aside a substantial amount of time to setup these electronic resources for students.

> **Technical Tool Tip.** Students often prefer tools that integrate easily with smart devices. Some easy-to-use project management and collaboration resources with mobile support include Slack, Trello, and GroupMe.

4. Student groups should be in place prior to introducing the project, and should ideally consist of no more than four members. Unless the teacher has reason to do otherwise, student groups should be created randomly. Teachers may wish to designate a group leader to manage groups' in-class work and oversee organization and planning of out-of-class meetings.

> **Technical Tool.** Some online learning management systems (e.g., Canvas) can generate random groups of students as well as randomly assign a group leader, allowing teachers to avoid introducing unconscious bias into group and leader selection.

Introducing the Project

1. Begin by showing students the following video clip: The Resurgence of Ability Grouping and Persistence of Tracking. Be sure to point out to students the ways the speaker conflates the terms "ability grouping" and "tracking" when discussing middle schools. Being aware of the ambiguity surrounding the issue prior to locating data sources may reduce some frustration.

> **Pedagogical Note.** Students new to mathematical modeling will likely benefit from a guiding modeling task. Slides for just such an introductory task, and that provide a segue into the modeling project are found in the Supplementary Materials.

2. Direct students to begin development of a mathematical model in order to determine whether "tracking" in school mathematics is an equitable practice.

> **Note.** Teachers seeking to provide students with a more scaffolded modeling experience should make use of Handout 1 as both an initial project prompt and a guide for students on how to get started.

Monitoring Student Progress

As with any mathematical modeling experience, students will initially feel overwhelmed with the amount of autonomy they are afforded given the open-ended nature of the mathematical task. To ensure the authentic modeling experience does not devolve into a situation of learned

helplessness, students work should be monitored during multiple class meetings dedicated to the modeling project and through their responses to prompts for their reflection journal entries.

Student teams should be expected to progress at different rates. However, teachers should ensure that all students have the experience to engage in at least one iteration of the modeling process. Teams that progress quickly should be strongly encourages to revisit aspects of their model, incorporate model parameters, and further analyze their model. Students should be provided with the Reflection Journal handouts no more frequently than one week apart to ensure they are afforded time sufficient to actually reflect on the process in which they are engaging.

> **Pedagogical Note.** For teachers making use of the supplementary materials, the remaining worksheets are intended to be distributed at the discretion of the teacher and based on students' progress. Students lacking mathematical modeling experience can reasonable be expected to complete the tasks outlined in Handouts 1 through 5 and 8 through 11 at a minimum. Groups progressing quickly through the tasks should be provided with Handouts 6 and 7, which will require them to revisit and generalize their model. Handouts 6 and 7 can be repeatedly used by students to revise, and analyze their group's model.

The questions listed below for use during the final discussion can also be used throughout the modeling project to guide student teams who are struggling to progress.

Finally, assessment should depend primarily on student engagement in the modeling process, and not on the model they produce. The Supplementary Materials include suggestions for technical report and team presentation specifications and associated scoring rubrics.

Orchestrating the Final Discussion

Presentations and the final discussion should be organized using the following modified version of the 5 Practices for Orchestrating Productive Discussions in Mathematics (Smith & Stein, 2018):

1. Review the **Sample Approaches** found in these project materials. (Anticipating)
2. Read through each team's technical report. (Monitoring)
3. Make note of unanticipated approaches or otherwise interesting approaches. Be sure to find at least one particular aspect of each team's model to point out and celebrate later during presentations and the final discussion, (Selecting)
4. Determine the order of presentations based on what you think will be pedagogically advantageous. (Sequencing)
5. Make use of the suggested questions and write additional questions that will help students recognize their mathematical modeling successes and identify ways in which their model can be improved. (Connecting)

Technical reports should be submitted prior to the day of presentations to allow the teacher to read through each report and list questions and comments for use during presentations. In addition to checking the report specifications against the technical report specifications list, the project facilitator should place groups in sequence for presentations based on their model approaches.

The following questions can be used following student presentations and to guide the final discussion.

- How did your team translate our definition of equity to mathematics?
- What real-world phenomena placed restrictions on what values your variables and parameters could take?
- Why did you decide to use that specific value for a parameter?
- What happens to your model when you alter the values you input? How much do you have to alter them by to obtain that result?
- What does this mean in real-world terms?
- How did you translate this concept to mathematics?

Following presentations, but prior to beginning a discussion of the social justice topic, it is important to establish norms for appropriate classroom discourse. The following list has been successful during previous discussions of socially charged topics (My thanks to Dr. Julia Aguirre for the list!):

- Prepare to feel discomfort.
- Listen respectfully.
- Share the time.
- Be mindful of the intend *and* the impact of your words.
- Challenge ideas, not people.

A common approach among students who are uncomfortable discussing issues of race will be to explore the issue from a socioeconomic-status perspective. In addition to pointing to the oppressive nature of tracking, the questions below can serve to make students aware of concepts like implicit bias, and connect in students' minds socioeconomic status and systemic racism (the latter of which can serve as a segue into a discussion of "redlining"):

Pedagogical Note. Teachers must be sure to strongly challenge any responses that rely on racism or other kinds of prejudice.

- How can we account for the clear disproportionate representation of historically marginalized racial/ethnic groups in remedial and advanced mathematics courses?
- Socioeconomic status is well-correlated with race and ethnicity. How can we account for this connection?

Pedagogical Note. Multiple data sources, read uncritically, can serve to reinforce deficit narratives about historically marginalized students. Teachers should be sure to model appropriate communicate when speaking about any data, especially that which is presented in a way that reinforces deficit narratives.

Students should respond to the Action Plan prompts during class immediately following the Final Discussion. Reflection Journal 3 should be sometime following the final class meeting dedicated to the modeling project.

Sample Approaches

Sample Approach #1

After sharing their different lists of what factors are most important to understanding tracking in terms of equity, Group 1 settles on the following list:

- Number of students from each racial/ethnic demographic (i.e., White, Black, Latin$x$, Asian, Pacific Islander, Native American) in US public schools who are placed in "accelerated" or "honors" tracks
- Number of students from each racial/ethnic demographic (i.e, White, Black, Latin$x$, Asian, Pacific Islander, Native American) in US public schools who are placed in "remedial" tracks

Now that they have settled on their list of important factors, the group begins discussing how they will mathematize the concept of equity. After a few minutes of discussion, one students suggests that compare the ratios between the each of the numbers of students on their list and the total number of students in US schools. If they compare all of the "accelerated" ratios to each other (pairwise), and the "remedial" ratios to each other (pairwise), the student claims, they can determine whether one group is being "accelerated" or put in "remediation" more than they should. They assign labels for each of their 12 important factors (A_i with $i \in \{1,2,3,4,5,6\}$ for "accelerated" student population i, R_i with $i \in \{1,2,3,4,5,6\}$ for "remedial" student population i, and T for the total population of students in US public schools,), and write down the following model:

Mathematical Model. Let A_i be the number of US public school students from demographic i placed in accelerated courses, R_i be the number of US public students from demographic i placed in remedial courses, and T be the total number of US public school students.

$$\text{Tracking is inequitable if } \frac{A_i}{T} \not\approx \frac{A_j}{T}, \text{ for every } i,j \in \{1,2,3,4,5,6\}$$
$$\text{OR if } \frac{R_i}{T} \not\approx \frac{R_j}{T}, \text{ for every } i,j \in \{1,2,3,4,5,6\}.$$

One student points out an immediate problem. What is meant by "$\approx$"? In other words, how close do two ratios have to be in order to be "close enough" in this model? A second student points out a second, perhaps even more concerning, problem: they are looking at the ratio of these populations to the total US public school population, but they should be looking at the ratio of these populations to the total US public school population *of that racial/ethnic demographic*. Otherwise their ratios will be meaningless, or if incorrectly represented, enormously misleading and irresponsible.

Suppose for the sake of simplicity, this student says, that there are one hundred students in US public schools. Now suppose that 20% of a group of students whose total racial or ethnic demographic accounts for only 5% of the total US public school student population, is tracked into a "remedial" course. 5% of our one hundred students is 5 students, and 20% of those 5—1 student to be exact—is tracked into remediation. Now suppose that that 5% of students whose racial or ethnic demographic group accounts for 60% of the total US public school student population, is tracked into a "remedial" course. 60% of our one hundred students is 60 students,

and 5% of those 60—3 students to be exact—is placed in remediation. The ratio of students tracked from the first group to the total population is $\frac{1}{100}$, or 1%, while the ratio of the students tracked from the second group to the total school population is $\frac{3}{100}$, or 3%. Comparing those ratios directly, the current model would suggest that tracking may be inequitable (depending on how we define $\approx$), but for the wrong reason. The group that is tracked at a rate 4 times greater than the other appears to be less impacted by tracking! It makes much more sense to compare the 20% of students tracked in one demographic to the 5% of students tracked in the other demographic. The student concludes by reiterating they should be looking at the ratio of these racial/ethnic populations to the total US public school population *of that specific racial/ethnic demographic*.

Convinced by this argument, the rest of the group agrees to alter their model, changing T, which previously represented the total US public school population to $T_i, i \in \{1,2,3,4,5,6\}$, where T_i represents the total US public school population for demographic i.

They write down their new and improved model:

Mathematical Model. Let A_i be the number of US public school students from demographic i placed in accelerated courses, R_i be the number of US public students from demographic i placed in remedial courses, and T_i be the total number of US public school students from demographic i.

$$\text{Then tracking is inequitable if } \frac{A_i}{T_i} \not\approx \frac{A_j}{T_j}, \text{ for every } i,j \in \{1,2,3,4,5,6\}$$
$$\text{OR if } \frac{R_i}{T_i} \not\approx \frac{R_j}{T_j}, \text{ for every } i,j \in \{1,2,3,4,5,6\}.$$

The same student from before reminds the rest of the group that they still haven't nailed down what is meant by $\approx$ for the purposes of this model.

Another student is concerned that there are multiple ways in which inequity could manifest itself with this model since their model says the ratio comparisons must be close (whatever that means) *for every* pairwise inequality. If one or more group is "high" or "low" tracked a disproportionately large number of times, each of those instances would be evidence of inequity. A third student points out that this is a good thing because they want their model to be able to identify any of the possible cases of inequity mentioned the other student mentioned.

While they still haven't determined how they should define $\approx$ for the purposes of this model, the group decides that before their next in-class group project meeting they will explore available data in hopes that some concrete numerical information will help to illuminate this idea.

After a perusal of the data found in the recommended readings and a wider internet search, the students realize that much of the precise data they hoped for cannot be found. They approach the project facilitator to ask what what they should if their model requires information that is unavailable. The project facilitator responds that precise data on tracking can be difficult to locate and that many school districts and states do not keep it at all. The project facilitator follows up by pointing out that this kind of situation is one of the reasons mathematical modelers use assumptions, and asks them to consider whether any assumptions could simplify or modify their model in such a way as to allow the use of the data in the recommended readings or any other data they have located.

After taking some time to reflect on the data they *do* have, as well as on the current state of their model, one student points out that while they do not have precise numbers about how many students from each racial/ethnic demographic are placed in "higher" or "lower" tracks, they do have 2013 data from the National Center for Education Statistics about the *highest courses* taken by students of different racial/ethnic demographics. Maybe they could use that instead?

One student questions whether that substitution of data makes sense, arguing that they cannot know from that data alone whether the students were "tracked" into the courses they ended up taking. Another student responds by pointing out the statistic they learned from the 2013 Brown Center Report on American Education, namely that almost 80% of students are tracked in mathematics in Grade 8 (at least as of 2011). It is reasonable to conclude, the second student claims, that while not all of the data reported by the NCES describes students that were tracked, the vast majority likely do, so they can reasonably use the course enrollment data as a replacement for actual tracking data. The first student responds that the numbers may have changed significantly since 2011 and, furthermore, that outliers in non-tracked schools could completely skew their results.

They take their conflict to the project facilitator who informs them that is up to them to decide what to do, and that they can make whatever assumptions they like so long as they 1) precisely document their reasons for doing so and 2) transparently explain how each assumption limits or increases the uncertainty in their model.

After some group deliberation, they decide to assume, with some reservations for which they provide a detailed written explanation, that the NCES data is representative of "tracked" schools. They modify their model to incorporate percentages rather than actual population number and to change their list of racial/ethnic demographics to White, Black, Hispanic, Asian, and Two or More Races (to match the available data set), and decide to omit the data for students who took no since they were unsure of why that would ever happen. They write down the latest iteration of their model:

Mathematical Model. Let C_i be the percentage of US public school students of demographic i whose highest course was Calculus, P_i be the percentage of US public school students of demographic i whose highest course was Precalculus, A_i be the percentage of US public school students of demographic i whose highest course was Algebra 2, G_i be the percentage of US public school students of demographic i whose highest course was Geometry, a_i be the percentage of US public school students of demographic i whose highest course was Algebra 1, B_i be the percentage of US public school students of demographic i whose highest course was "Below Algebra 1," and O_i be the percentage of US public school students of demographic i whose highest course was some other math course.

Then tracking is inequitable if for any $i, j \in \{1, 2, 3, 4, 5\}$, at least one of the following is true:

$$C_i \not\approx C_j,$$
$$P_i \not\approx P_j,$$
$$A_i \not\approx A_j,$$
$$G_i \not\approx G_j,$$
$$a_i \not\approx a_j,$$
$$B_i \not\approx B_j,$$
$$O_i \not\approx O_j.$$

A group member points out that every model they have considered still suffers from the flaw of having to decide how close is close enough. Another member suggests that maybe representing the relationships between these quantities in a different way will help, offering up the idea of looking at the differences between each of the quantities, and trying to see if any differences are noticeably larger than the others. Perhaps they could use a to-be-determined threshold, say ϵ, such that differences greater than ϵ in magnitude are classified as evidence of inequity. Another student suggests they use the absolute value of the difference since they are most interested in magnitude and so they do not have to worry about the order in which quantities are subtracted. The rest of the group agrees and they settle on the following model:

Mathematical Model. Let $\epsilon = k$ for some to-be-determine constant k (we'll come back to this later), C_i be the percentage of US public school students of demographic i whose highest course was Calculus, P_i be the percentage of US public school students of demographic i whose highest course was Precalculus, A_i be the percentage of US public school students of demographic i whose highest course was Algebra 2, G_i be the percentage of US public school students of demographic i whose highest course was Geometry, a_i be the percentage of US public school students of demographic i whose highest course was Algebra 1, B_i be the percentage of US public school students of demographic i whose highest course was "Below Algebra 1," and O_i be the percentage of US public school students of demographic i whose highest course was some other math course.

Then tracking is inequitable if, for any $i, j \in \{1, 2, 3, 4, 5\}$, at least one of the following inequalities true:

$$|C_i - C_j| > \epsilon,$$
$$|P_i - P_j| > \epsilon,$$
$$|A_i - A_j| > \epsilon,$$
$$|G_i - G_j| > \epsilon,$$
$$|a_i - a_j| > \epsilon,$$
$$|B_i - B_j| > \epsilon,$$
$$|O_i - O_j| > \epsilon.$$

In order to get an idea of what they should pick for ϵ, the group decides to start looking at the absolute differences in the data they intend to use to gauge their magnitude in relation to one another. They decide to make the following assignment of indices to racial/ethnic demographics:

$1 \mapsto$ Two or More Races, $2 \mapsto$ Asian, $3 \mapsto$ Hispanic, $4 \mapsto$ Black, $5 \mapsto$ White. Then they distribute the workload across group members and obtain the following tables of data:

Demographic Differences in Percent of Students With Calculus as Highest Course

$C_i - C_j$ (row i, column j)	C_1	C_2	C_3	C_4	C_5
C_1	0	-32	1	5	-7
C_2	32	0	35	39	27
C_3	-1	-35	0	4	-8
C_4	-5	-39	-4	0	-12
C_5	7	-27	8	12	0

Demographic Differences in Percent of Students With Precalculus as Highest Course

$P_i - P_j$, (row i, column j)	P_1	P_2	P_3	P_4	P_5
P_1	0	-6	-1	0	-6
P_2	6	0	5	6	0
P_3	1	-5	0	1	-5
P_4	0	-6	-1	0	-6
P_5	6	0	5	6	0

Demographic Differences in Percent of Students With Algebra 2 as Highest Course

$A_i - A_j$, (row i, column j)	A_1	A_2	A_3	A_4	A_5
A_1	0	16	3	3	5
A_2	-16	0	-13	-13	-11
A_3	-3	13	0	0	2
A_4	-3	13	0	0	2
A_5	-5	11	-2	-2	0

Demographic Differences in Percent of Students With Geometry as Highest Course

$G_i - G_j$, (row i, column j)	G_1	G_2	G_3	G_4	G_5
G_1	0	7	-6	2	2
G_2	-7	0	-13	-5	-5
G_3	6	13	0	6	6
G_4	-2	5	-6	0	0
G_5	-2	5	-6	0	0

Demographic Differences in Percent of Students With Algebra 1 as Highest Course

$a_i - a_j$, (row i, column j)	a_1	a_2	a_3	a_4	a_5
a_1	0	-	0	0	2
a_2	-	0	-	-	-
a_3	0	-	0	0	2
a_4	0	-	0	0	2
a_5	-2	-	-2	-2	0

Demographic Differences in Percent of Students With Highest Course Below Algebra 1

$B_i - B_j$, (row i, column j)	B_1	B_2	B_3	B_4	B_5
B_1	0	-	0	-2	0
B_2	-	0	-	-	-
B_3	0	-	0	-2	0
B_4	2	-	2	0	2
B_5	0	-	0	-2	0

Demographic Differences in Percent of Students With Other Math as Highest Course

$O_i - O_j$, (row i, column j)	O_1	O_2	O_3	O_4	O_5
O_1	0	8	2	-7	2
O_2	-8	0	-6	-15	-6
O_3	-2	6	0	-9	0
O_4	7	15	9	0	9
O_5	-2	6	0	-9	0

After taking some time to look at and interpret the different numbers in each table they produced, the students start to make note of any differences of unusually large magnitude. They first notice the large percentage of Asian students whose final course was Calculus. They also see that, while the difference is not so large, there are still significantly more White students who take Calculus than Multiracial, Hispanic, or Black students. They also see that this difference still exists for students whose final course was Precalculus, though the differences were less pronounced. The notice that there are significantly fewer Asian students whose final math course was any of the remaining courses, but quickly recognize that this is because 67% of Asian students' final math course is already accounted for in Calculus and Precalculus. Next, they notice that far more Hispanic students' final math course was Geometry. Finally, they see that a far greater number of Black students' final math course was some other kind of math (which, as they soon discover from the footnotes, includes integrated math, trigonometry, Algebra 3, probability and statistics, and non-calculus AP or IB courses).

Based on their observations, the students conclude that while a "natural" choice of a fixed value for ϵ may exist, the current data provides relatively little guidance as to what it might be. They decide to look at the course with the smallest difference across all student racial and ethnic demographics.

One student points out that the largest difference in percentages they see for both the "Algebra 1" and "Course Below Algebra 1" tables has magnitude 2, and concludes that $k = 2$ is the most reasonable choice. Another student points out that the data is incomplete for those course, as it lacks information about the representation of Asian students, and suggests $k = 6$ as an alternative (the largest difference for the Precalculus course, arguing that Precalculus has the least largest difference while still have data for each student demographic.

The others are convinced by this argument, though one student expresses reservations about picking any value for k in this way since they will necessarily conclude tracking is inequitable unless they pick the course with the largest difference. They decide to look back at their data and see what looks to be the "average" final math course taken (by which they mean the largest number of students from as many demographics as possible, concluding it is the "other math" category course. The largest difference for their newly identified "average course" is 15. They decide to pick $k = 15$ and they settle on the following final model:

Final Mathematical Model. Let C_i be the percentage of US public school students of demographic i whose highest course was Calculus, P_i be the percentage of US public school students of demographic i whose highest course was Precalculus, A_i be the percentage of US public school students of demographic i whose highest course was Algebra 2, G_i be the percentage of US public school students of demographic i whose highest course was Geometry, a_i be the percentage of US public school students of demographic i whose highest course was Algebra 1, B_i be the percentage of US public school students of demographic i whose highest course was "Below Algebra 1," and O_i be the percentage of US public school students of demographic i whose highest course was some other math course.

Then tracking is inequitable if, for any $i, j \in \{1, 2, 3, 4, 5\}$, at least one of the following

inequalities true:

$$\begin{aligned} |C_i - C_j| &> 15, \\ |P_i - P_j| &> 15, \\ |A_i - A_j| &> 15, \\ |G_i - G_j| &> 15, \\ |a_i - a_j| &> 15, \\ |B_i - B_j| &> 15, \\ |O_i - O_j| &> 15. \end{aligned}$$

Since $|C_2 - C_4| = 39 > 15$ (as are a number of other differences), they conclude that tracking is inequitable.

Sample Approach #2

The students from Group 3 include the following items on their list of important factors for determining whether "tracking" is 6an equitable practice:

- Percentage of students with low-socioeconomic status who are placed in accelerated and remedial courses.
- Percentage of students with regular- or high-socioeconomic status who are placed in accelerated and remedial courses.

Based on their reading of the Brown Center Reports, they decide it is reasonable to assume that all of the schools they look at have some form of tracking (though they note that since only about 3/4 of schools practice tracking, they might be wrong).

After exploring the available data on the US Department of Education's Civil Rights Data Collection, they decide to look at schools with and without Title I status, and look at which of the two have more students enrolled in higher math courses. They first decide to look at schools in the same district, but ultimately struggle to find two such schools, and decide to just make sure they are in the same state (and therefore under the jurisdiction of one state board of education). They also decide that they will have to assume that the two schools they will look at are representative of all Title I and non-Title I schools in the US (they write down their reservations with that assumptions, but decide they are unsure of how they would proceed otherwise).

Finally, they also decide, without much of a reason beyond one of the team thinking that a 2% difference is reasonable, that a difference of greater than 0.2 between the ratio between calculus taking students to the whole student population at the two schools would be indicative of tracking. They write down their first model:

Mathematical Model. Let M_p be the total population of students at Title I school M, let M_c be the total number of students enrolled in calculus in school M, let N_p be the total population of students at non-Title I school N, and let N_c be the total number of students

enrolled in calculus in school N. Tracking is inequitable if

$$\left|\frac{M_c}{M_p} - \frac{N_c}{N_p}\right| > .02.$$

Before they can find two such schools in the same state, one student points out the grossly disproportionate representation of difference racial/ethnic demographics in calculus courses, and comments that maybe they are looking at the wrong thing.

During their next meeting, one of the group members says they think the approach should change from focusing solely on differences between Title I-classified and non-Title I classified schools to focus on race. Some of the other group members are hesitate about this change, insisting that socioeconomic status probably the biggest factor since (they claim) society is less racist than it used to be. The first student responds that they spent some time since their last meeting reading about the practice of "redlining," which deprived Persons of Color of opportunities for home ownership, which has had a long-term negative effects on their ability to accumulate wealth, and that much of what we see as socioeconomic inequality can be traced in many cases to be a result of systemic racism against people who have been historically marginalized by society.

Some students still express discomfort with the idea of focusing on race, but ultimately the group decides they will produce a new model, but realize this will require a significant alteration to their existing model. They believe they will no longer be able to compare different schools. Rather, they will need to compare overall school or district enrollment with enrollment in specific courses in that *same* school or district.

They look at the demographic breakdown for the local school districts to determine what groups of students it will be possible to compare. There are a total of seven groups they can compare using the Civil Rights Data Collection. They decide to explore only enrollment in Gifted & Talented programs and in Calculus will be the focus of their exploration, concluding that since many schools now have integrated mathematics curriculum—wherein algebra, geometry, etc. are all studied in tandem—the demographic breakdown of enrollment in Algebra 1 in eighth grade might not be relevant. They write down their current model:

Mathematical Model. Let D be a school district in the US, let P_i be the percentage of students of demographic i enrolled in district D, G_i be the percentage of students of demographic i enrolled in Gifted & Talented programs in district D, and let C_i be the percentage of students of demographic i enrolled in Calculus in district D.

Then tracking is equitable if and only if, for every $i \in \{1, 2, 3, 4, 5, 6, 7\}$, the following equations hold:

$$P_i = G_i$$
$$P_i = C_i.$$

They index the demographic groups as follows: $1 \mapsto$ American Indian/Alaska Native, $2 \mapsto$ Asian, $3 \mapsto$ Black, $4 \mapsto$ Hispanic, $5 \mapsto$ Native Hawaiian/Other Pacific Islander $6 \mapsto$ Two or More, $7 \mapsto$ White. They decide to use the data for a local district, Cache Country

School District, to test out their model. They observe the following:

$$P_1 = 0.3 \neq 0.0 = G_1$$
$$P_1 = 0.3 \neq 0.0 = C_1$$

$$P_2 = 0.7 \neq 1.5 = G_2$$
$$P_2 = 0.7 \neq 0.0 = C_2$$

$$P_3 = 0.7 \neq 1.0 = G_3$$
$$P_3 = 0.7 \neq 0.0 = C_3$$

$$P_4 = 8.6 \neq 2.5 = G_4$$
$$P_4 = 8.6 \neq 5.1 = C_4$$

$$P_5 = 0.5 \neq 0.0 = G_5$$
$$P_5 = 0.5 \neq 0.0 = C_5$$

$$P_6 = 1.8 \neq 2.0 = G_6$$
$$P_6 = 1.8 \neq 5.1 = C_6$$

$$P_7 = 87.3 \neq 92.9 = G_7$$
$$P_7 = 87.3 \neq 89.9 = C_7$$

They notice a few things, first among those being that perhaps requiring that the percentages be equal is a bit too restrictive, since literally none of the proposed equations held. They determine to figure out a reasonable difference between the percentages they are comparing.

One student also suggests that perhaps they should change their equal signs to $\leq$ signs (with P_i being on the left of the symbol), claiming that we shouldn't be opposed to a greater percentage of students enrolled in Gifted Talented programs or in Calculus during high school. Another student points out a problem with that idea: since the percentages have to, by definition, add up to 100%, changing the symbols to $\leq$ will only result in true mathematical statements if every single inequality was, in fact, an equality, since a strict inequality would result in percentages that add up to something greater than 100%.

Rejecting the idea of an inequality, the students decide to figure out a range of acceptable difference for percentages. Noting the small percentages involved in the Cache County School District for some of the different demographics, one student suggests they use a number smaller than 1%. They think about this, looking at the data for their chosen school district for awhile, until another student finally says they disagree.

First the student points out that in the Cache County School District, only 395 of the 17455 students are enrolled in the Gifted & Talented programs, or about 2%. This student points out that in a hypothetical district with only 500 students, with 499 students of demographic i, and 1 of demographic j, demographic j makes up 0.002% of the student population. If 2% of students

in this district were enrolled in Gifted & Talented programs, that we be a total of 10 students, and if the single student from demographic j were among those 10, they would account for 10% of all students enrolled. The student concludes by agreeing that they need to come up with an acceptable difference, but they need to be cautious when looking at districts with few students from other racial/ethnic demographics as it can skew the numbers, and suggests they try and find a district with a more diverse student population and try to come up with an acceptable difference from there.

A quick search leads them to the Hurst-Euless-Bedford Independent School District, ranked the most diverse school district in the US by Niche, a site that catalogues data and produces ratings different aspects of US schools, based on factors explained here:
https://www.niche.com/about/methodology/most-diverse-school-districts/.

Using the data for this district with their most recent model, they observe the following:

$$
\begin{aligned}
P_1 &= 0.5 \neq 0.5 = G_1 \\
P_1 &= 0.5 \neq 0.0 = C_1 \\
\\
P_2 &= 6.6 \neq 12.0 = G_2 \\
P_2 &= 6.6 \neq 34.0 = C_2 \\
\\
P_3 &= 17.7 \neq 6.5 = G_3 \\
P_3 &= 17.7 \neq 9.6 = C_3 \\
\\
P_4 &= 30.7 \neq 19.0 = G_4 \\
P_4 &= 30.7 \neq 12.8 = C_4 \\
\\
P_5 &= 2.3 \neq 1.2 = G_5 \\
P_5 &= 2.3 \neq 0.0 = C_5 \\
\\
P_6 &= 4.9 \neq 5.7 = G_6 \\
P_6 &= 4.9 \neq 6.4 = C_6 \\
\\
P_7 &= 37.4 \neq 55.1 = G_7 \\
P_7 &= 37.4 \neq 37.2 = C_7
\end{aligned}
$$

They determine to look at the differences between percentages for this school to decide on a

reasonable difference. They calculate the following differences:

$$\begin{aligned}
P_1 - G_1 &= 0.5 - 0.5 = 0.0 \\
P_1 - C_1 &= 0.5 - 0.0 = 0.5 \\
P_2 - G_2 &= 6.6 - 12.0 = -5.4 \\
P_2 - C_2 &= 6.6 - 34.0 = -27.4 \\
P_3 - G_3 &= 17.7 - 6.5 = 11.2 \\
P_3 - C_3 &= 17.7 - 9.6 = 8.1 \\
P_4 - G_4 &= 30.7 - 19.0 = 11.7 \\
P_4 - C_4 &= 30.7 - 12.8 = 17.9 \\
P_5 - G_5 &= 2.3 - 1.2 = 1.1 \\
P_5 - C_5 &= 2.3 - 0.0 = 2.3 \\
P_6 - G_6 &= 4.9 - 5.7 = -0.8 \\
P_6 - C_6 &= 4.9 - 6.4 = -1.5 \\
P_7 - G_7 &= 37.4 - 55.1 = -17.7 \\
P_7 - C_7 &= 37.4 - 37.2 = 0.2
\end{aligned}$$

One student points out that positive numbers show under-representation of a group and negative numbers show over-representation, and suggests that these differences could play a role in their model. After some additional discussion, one student suggests they use the mean of the differences as a choice for a reasonable difference. Calculation reveals the mean to be approximately 0.09. Another student points out that while the sign of the difference does reveal whether a demographic is over- or under-represented in Gifted & Talented programs or Calculus enrollment, they are trying to decide what *size* of difference is appropriate, and suggests they use the mean of the absolute value of the differences. The others think this makes sense, and a quick calculation is performed, with a result of 7.5. Some team members hold some reservations about using a number this large, arguing that a difference of 7.5% is rather large, but ultimately relent when they can't decide another way of deciding an appropriate difference. The team agrees to use an absolute difference in percentage of 7.5 as the largest possible absolute difference indicating equity, and anything greater as indicative of inequity. They also realize that by incorporating an absolute value into their model, they can use an inequality for comparisons, which they decide is more reasonable than an equality. The write down their new model:

Mathematical Model. Let D be a school district in the US, let P_i be the percentage of students of demographic i enrolled in district D, G_i be the percentage of students of demographic i enrolled in Gifted & Talented programs in district D, and let C_i be the percentage of students of demographic i enrolled in Calculus in district D.

Then district D's implementation of tracking is equitable if and only if, for every $i \in \{1, 2, 3, 4, 5, 6, 7\}$, the following inequalities hold:

$$|P_i - G_i| <= 7.5$$
$$|P_i - C_i| <= 7.5.$$

Applying their new model to their original data set from the Cache County School District, they

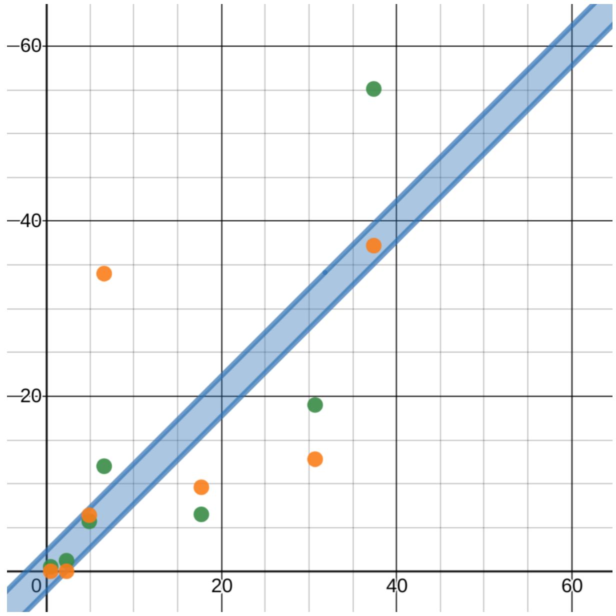
see that none of the absolute differences are greater that 7.5, suggesting that Cache Valley School District's implementation of tracking is equitable.

During an in-class meeting, they show their current model to their teacher, who asks to see the data for the district they used to calculate their number of 7.5. Upon reviewing the relevant data, the teacher asks the group whether they would consider the district to be providing their students with an equitable mathematics learning experience. The students respond in the negative, pointing to the multiple differences which are much greater than 7.5 in magnitude. The teacher responds by pointing out that, by definition, in calculating their mean with this district, they will have had at least one absolute difference greater than the mean, and that this would be the case with any district they use, meaning the district they use will necessarily be one which is inequitable according to their model. The teacher continues by pointing out that if tracking is an inequitable practice, then they are using differences that might be indicative of inequity in quantifying what is equitable, and suggests they do some more research to try before they settle on this value of K.

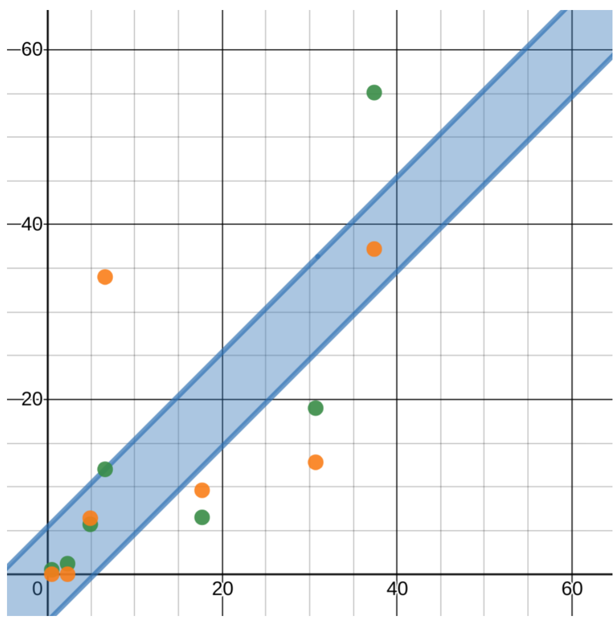
After the teacher leaves, the students spend some time discussing their definition of equity. One student reminds the others that were supposed to determine whether tracking is a form of "oppression." This student points out that if some student demographics are more restricted in their access to certain mathematics like Calculus, then tracking is obviously oppressive. The student suggests they stop trying to decide the location of the fuzzy line dividing equity and inequity, contending they return to the use of equal signs in their model, because any inequity, no matter how small, is still an inequity. They finish by stating that they all already know tracking probably inequitable since non-White, non-Asian students in every district they have looked at are underrepresented in "Gifted & Talented" program enrollment.

Another student responds that they agree that tracking is inequitable based on what they have seen, but they still need to come up with a mathematical model, and argues that going back to equal signs will not account for the possibility that students select different mathematics course paths for themselves, rather than due to tracking. The students decide to leave the "acceptable" absolute difference as a model parameter, K, and use Desmos to explore the regions $|y - x| \leq K$, for various K. They plan to associate P_i with y and both G_i and C_i with x (deciding to color points (P_i, G_i) in green and (P_i, C_i) in orange to differentiate between the two), in hopes that the visualization will reveal something they have yet to notice.

For Hurst-Euless-Bedford Independent School District they allow K to vary between the minimum and maximum absolute differences of 0.0 and 27.4, and determine to make note of any plots they find interesting. For $K = 2.3$, not even one 9th of the way to the maximum absolute difference, they note that half of the points already lie in the shaded band:

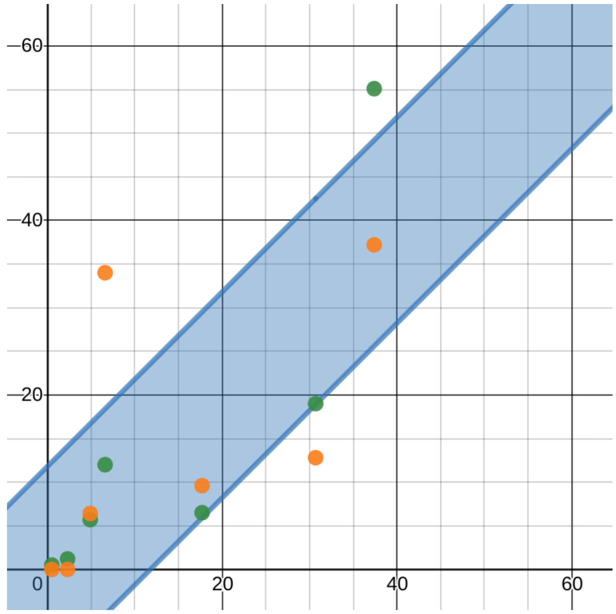


For $K = 5.4$, an additional point lies in the band, making the number of points within the band a majority:

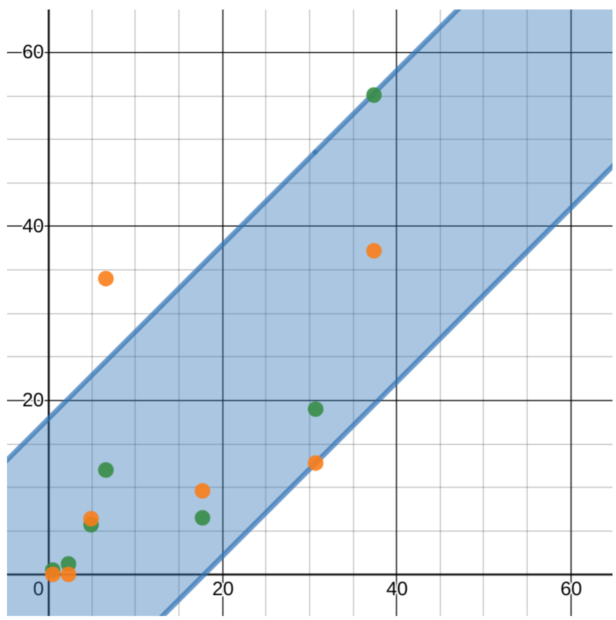


The students are struck by how much K needed to increase just to ensure the band encompassed a single additional point, and wonder whether the existence of large increases in K in order to encompass widen the band to envelope additional points might be helpful in selecting an "acceptable" value of K.

They decide to continue varying K to see if they notice anything else. For $K = 11.7$, three-fourths of the points sit within the shaded band:



For $K = 17.9$, all but one of the points lies in the shaded band:



They decide to look at the increases in K necessary to include an additional point for $n = 1$ up to $n = 14$, the total number of points. They order the points by how far they are from the line $y = x$ (points the same distance are placed are given separate indices), letting $K_0 = 0.0$ and K_i be the value of K_i that ensures point i is in the band $|y - x| \leq K$, and $\Delta K_i = K_i - K_{i-1}$, where $i \in \{2, 3, \ldots, 14\}$. They obtain a table of data (on the next page).

From the table they finally recognize what their graphs were suggesting, but they were unable to articulate, namely, that there are clusters in the different distances of the points from the line $y = x$. They notice that the increases in K are all less than one until point 8, where K suddenly has to jump by over 3. From this, they decide to let $K = 2.3$ be the value of K they will use (still

i	K_i	ΔK_i
1	0.0	0.0
2	0.2	0.2
3	0.5	0.3
4	0.8	0.3
5	1.1	0.3
6	1.5	0.4
7	2.3	0.8
8	5.4	3.1
9	8.1	2.7
10	11.2	2.8
11	11.7	0.5
12	17.7	6
13	17.9	0.2
14	27.4	9.5

recognizing the limitations of reducing equity to a numerical concept). They write down their final model.

Final Mathematical Model. Let D be a school district in the US, let P_i be the percentage of students of demographic i enrolled in district D, G_i be the percentage of students of demographic i enrolled in Gifted & Talented programs in district D, and let C_i be the percentage of students of demographic i enrolled in Calculus in district D.

Then district D's policy of tracking is equitable if and only if, for every $i \in \{1, 2, 3, 4, 5, 6, 7\}$, the following inequalities hold:

$$|P_i - G_i| <= 2.3$$
$$|P_i - C_i| <= 2.3.$$

Resources

Teacher Readings

The following readings are *necessary* preparation for teachers preparing to incorporate this project into a methods or content course:

- AMTE Standards for Preparing Teachers of Mathematics: Indicator C.4.1 Provide Access and Advancement
- AMTE Standards for Preparing Teachers of Mathematics: Indicator P.3.3 Address the Social Contexts of Teaching and Learning

- AMTE Standards for Preparing Teachers of Mathematics: ML.7 Equitable Structures and Systems in Middle Schools
- AMTE Standards for Preparing Teachers of Mathematics: HS-3 Supporting Each and Every Students Opportunity to Learn Mathematics 2013 Brown Center Report on Education Part II: The Resurgence of Ability Grouping and the Persistence of Tracking
- 2016 Brown Center Report on Education Part II - Tracking and Advanced Placement
- Ability Grouping and Gifted Children
- Are We Breaking Down Barriers to Student Learning?
- Initiating Critical Conversations on the Discontinuation of Tracking
- Mathematics Education Through the Lens of Social Justice

The following readings are not free and, as such, are only *recommended*:

- *Beyond Tracking: Multiple Pathways to College, Career, and Civic Participation*
- *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (pp. 15 - 24)
- *Principles to Actions: Ensuring Mathematical Success for All* (pp. 59 - 69)

Student Readings

The following readings are *strongly recommended* for students engaging in this project. Suggestions for appropriate locations in the project for inclusion are provided in the Monitoring Student Progress of the Project Details section:

- 2013 Brown Center Report on Education Part II: The Resurgence of Ability Grouping and the Persistence of Tracking
- 2016 Brown Center Report on Education Part II - Tracking and Advanced Placement
- AMTE Standards for Preparing Teachers of Mathematics: Indicator C.4.1 Provide Access and Advancement
- AMTE Standards for Preparing Teachers of Mathematics: Indicator P.3.3 Address the Social Contexts of Teaching and Learning
- AMTE Standards for Preparing Teachers of Mathematics: ML.7 Equitable Structures and Systems in Middle Schools
- AMTE Standards for Preparing Teachers of Mathematics: HS-3 Supporting Each and Every Students Opportunity to Learn Mathematics
- Ability Grouping and Gifted Children
- Are We Breaking Down Barriers to Student Learning?
- Initiating Critical Conversations on the Discontinuation of Tracking
- Mathematics Education Through the Lens of Social Justice

References

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Gutierrez, R. (2002). Enabling the Practice of Mathematics Teachers in Context: Toward a New Equity Research Agenda. *Mathematical Thinking and Learning*, 4(2-3), 145-187. doi:10.1207/s15327833mtl04023_4

The MacGuffin Institute

Si Hoc Legere Scis
Nimium Eruditionis Habes **Teacher Name,Dir. of Policy Research**

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July 20, 2020

Hello,

Once again, I would again like to congratulate you on your recent hire here at MacGuffin! We are certain you will play a vital role on one of our policy research teams, and we look forward to seeing the novel ways in which you contribute to our mission of provided data-drive answer to some of society's most difficult questions.

We were recently contracted by the U.S. Department of Education to explore some questions regarding the use of "tracking" in school mathematics. Tracking, the practice of placing students in course and curriculum "tracks" based on their perceived ability, finds strong support among parents, teachers, and school administrators, but groups like the NCTM and the AMTE assert that it creates and perpetuates inequities in mathematics education.

To better understand this issue and to the end of providing policy recommendations, your team will explore the topic of tracking by developing a mathematical model that can be used to evaluate whether it is an equitable practice. The results of your work will be compiled in a technical report that you and your team will submit, and present the contents of, on [DATE]. This being your first assignment at MacGuffin, I understand if you might feel a bit unsure on how to proceed. I will be providing you and your fellow team members with some additional guidance in the form of a series of task sheets (the first of which I have attached to this letter) to ensure your continual progress toward the completion of your project.

Sincerely,

Teacher Name,
Dir. of Policy Research

The MacGuffin Institute

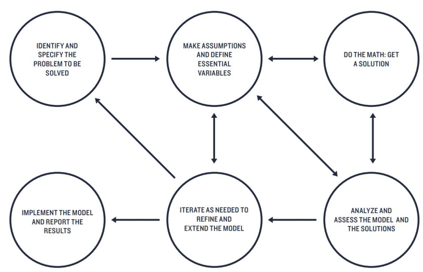
Overview

1. An Introduction to Mathematical Modeling
2. The Modeling Process in Action
3. Project Introduction
4. Questions?

An Introduction to Mathematical Modeling

Mathematical Modeling is a Process

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.



The Modeling Process in Action

To Illustrate, We Will Develop a Simple Model

Here's our prompt:

Gas prices change on a nearly daily basis, and not every gas station offers the same price for a gallon of gas. The gas station selling the cheapest gas may be across town from where you are driving. Is it worth the drive across town for less expensive gas? Create a mathematical model that can be used to help understand under what conditions it is worth the drive.

Think About It: What are your initial reactions to this prompt?

Think About It: What are your initial reaction(s) to this prompt?

MY REACTION:

THERE'S NOT ENOUGH INFORMATION TO ANSWER THE QUESTION...

This is precisely the point of modeling!

Attempt #1

1. Research
   a. We look at my own commute, identify a gas station along the way (Station A) and perform a quick internet search to learn the cost of gas at Station A. We write down the price of $3.59/gallon.
   b. Then we look for the cheapest gas at other stations elsewhere and choose one (Station B), which costs only $3.44/gallon.
   c. We decide that for a fair comparison, we should purchase the same amount at each station, and arbitrarily decide on 15 gallons as a reasonable amount.
2. Crunch the Numbers
   a. Cost of gas at Station A = ($3.59/gallon)*15 gallons = $53.85
   b. Cost of gas at Station B = ($3.44/gallon)*15 gallons = $51.60
   c. Savings by purchasing gas at Station B = $53.85 - $51.60 = $2.25
3. Therefore I should go to Station B...

Attempt #1

1. Research
   a. We look at my own commute, identify a gas station along the way (Station A) and perform a quick internet search to learn the cost of gas at Station A. We write down the price of $3.59/gallon.
   b. Then we look for the cheapest gas at other stations elsewhere and choose one (Station B), which costs only $3.44/gallon.
   c. We decide that for a fair comparison, we should purchase the same amount at each station, and arbitrarily decide on 15 gallons as a reasonable amount.
2. Crunch the Numbers
   a. Cost of gas at Station A = ($3.59/gallon)*15 gallons = $53.85
   b. Cost of gas at Station B = ($3.44/gallon)*15 gallons = $51.60
   c. Savings by purchasing gas at Station B = $53.85 - $51.60 = $2.25
3. Therefore I should go to Station B… or should I?

Question: What did we not consider?

Think-Pair-Share:

With your neighbor, discuss our model. In particular, what does our model ***NOT*** consider?

For reference, here is the prompt we were supposed to respond to:
Gas prices change on a nearly daily basis, and not every gas station offers the same price for a gallon of gas. The gas station selling the cheapest gas may be across town from where you are driving. Is it worth the drive across town for less expensive gas? Create a mathematical model that can be used to help understand under what conditions it is worth the drive.

Question: What did we not consider?

Think-Pair-Share:

With your neighbor, discuss our model. In particular, what does our model ***NOT*** consider?

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Question: What did we not consider?

Several things!... but perhaps most glaring is the omission of the extra cost incurred by driving across town.

Attempt #2

1. More Research
   a. We look up the distance between the gas stations (6 miles) so we can figure out how much it will cost to drive the extra distance.
   b. We look up fuel economy for my car (35 miles/gallon)
2. Crunch the Numbers
   a. (Cost to drive 12 Miles) = (12 Miles)/(35 miles/gallon) * ($3.44/gallon) = $1.18
   b. Savings from driving to Station B = $53.85 - $51.60 - $1.18 = $1.07
3. Therefore I should go to Station B...

Attempt #2

1. More Research
   a. We look up the distance between the gas stations (6 miles) so we can figure out how much it will cost to drive the extra distance.
   b. We look up fuel economy for my car (35 miles/gallon)
2. Crunch the Numbers
   a. (Cost to drive 12 Miles) = (12 Miles)/(35 miles/gallon) * ($3.44/gallon) = $1.18
   b. Savings from driving to Station B = $53.85 - $51.60 - $1.18 = $1.07
3. Therefore I should go to Station B... or should I?

Question: What potential problems can you identify?

Think-Pair-Share:

One more time, with your neighbor, discuss our model. In particular, can you see any problems?

For reference, here is the prompt we were supposed to respond to:
Gas prices change on a nearly daily basis, and not every gas station offers the same price for a gallon of gas. The gas station selling the cheapest gas may be across town from where you are driving. Is it worth the drive across town for less expensive gas? Create a mathematical model that can be used to help understand under what conditions it is worth the drive.

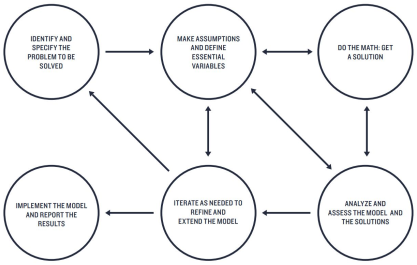
Question: What did we not consider?

Among other things, we technically only answered a very specific case of the question.

We need to generalize!

REMEMBER: Mathematical Modeling is a *Process*

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.



What do we have so far?

Defining the Problem
Determine which costs less

- Purchasing gas at Station A, which is on our planned route, or
- Traveling out of our way to Station B (which sells gas at a cheaper rate) to purchase gas there.

What do we have so far?

Defining the Problem
Determine which costs less

- Purchasing gas at Station A, which is on our planned route, or
- Traveling out of our way to Station B (which sells gas at a cheaper rate) to purchase gas there.

Assumptions

- Gas costs less at the station that is out of our way.
- The fuel economy of the car remains constant.
- If we choose to go out of our way, we will consider the added cost of the mileage between the gas station we would have gone to and the further station, and back.
- We will purchase the same amount of gas, no matter which gas station we choose.

What do we have so far?

Defining the Problem
Determine which costs less

- Purchasing gas at Station A, which is on our planned route, or
- Traveling out of our way to Station B (which sells gas at a cheaper rate) to purchase gas there.

Assumptions

- Gas costs less at the station that is out of our way.
- The fuel economy of the car remains constant.
- If we choose to go out of our way, we will consider the added cost of the mileage between the gas station we would have gone to and the further station, and back.
- We will purchase the same amount of gas, no matter which gas station we choose.

Defining Variables

- Let m be the number of miles between Station A and Station B
- P1, the price of gasoline at Station A, in dollars per gallon
- P2, the price of gasoline at Station B, in dollars per gallon
- f, the fuel economy of the car, in miles per gallon
- n, the number of gallons of gas to be purchased
- T, the cost to travel to and from Station B, in dollars
- S, the difference in money paid for purchasing the gasoline at Station B versus Station A (accounting only for the purchase of gasoline; not including the travel costs), in dollars

What do we have so far?

Our Resulting Model

- S = (P1 - P2)*n
- T = (P2 * 2m)/f
- If S > T, then it is better to drive across town to purchase gas at Station B
- If S <= T, then we should not drive across town to purchase gas.

What else can we determine from our model?

Our Resulting Model

- S = (P1 - P2)*n
- T = (P2 * 2m)/f
- If S > T, then it is better to drive across town to purchase gas at Station 2
- If S <= T, then we should not drive across town to purchase gas.

One (of several) Questions to Consider

- How far away would Station 2 have to be in order for the cost to break even? In other words, for what values of P1, P2, n, m, and f does S = T?

What else can we determine from our model?

One (of several) Questions to Consider

- How far away would Station 2 have to be in order for the cost to break even? In other words, for what values of P1, P2, n, m, and f does S = T?

If we know all values except the distance, we could answer this question. For example, we might write

(P1 - P2)*n = (P2 * 2m)/f
($3.59-$3.44)*15 gallons = ($3.44*2*m)/(35 miles/gallon)
Solving for m, we see that m = 11.45 miles

What else can we determine from our model?

Additional Questions to Consider

- Should we consider the price differential when determining whether it is worth it to drive to Station 2?
  - We could solve the equation (P1 - P2)*n = (P2 * 2m)/f for P2!
- Would it make more sense to see how many gallons of gas one would have to buy in order to make it worth driving across town for gas?

Analysis and Model Assessment

Questions to ask ourselves:

- Is the sign of our answer right?
- Is our answer off by orders of magnitude?
- Does our model behave appropriately when we increase and decrease the input variables?
- Are our assumptions reasonable and defensible?
- Are our assumptions relevant?
- Does our model follow from our assumptions? Is our model completely explained by our assumptions, or do we need to add more assumptions?
- What if our assumptions are wrong? How does that impact our answer?
- What if our scenario changes a little? Do our results change a little or a lot?

Reporting Our Results

It is vitally important that we share our results so that other social scientists and mathematical modelers can learn from both our successes and failures.

In other words, we need to report our results.

NOTE: A good report will detail each of the things we discussed along the way to producing our finished model, including any attempts that we ultimately discarded along with explanations for why we did so.

That's it. We are done.

But not really...

No model is perfect, and *every* model can be improved.

For example, the following diagram suggests we may wish to revisit our assumptions involving the added distance when traveling to:

Project Introduction

37

Let's talk about *your* task.

38

What is "tracking" in school mathematics?

39

What are critics saying about it?

The focus of this project:

The Association of Mathematics Teachers Educators claims tracking in school mathematics is inequitable.

40

This is where you come in...

You and your team will:

1) Produce a mathematical model that can be used to determine whether tracking is inequitable, using the definition of equity provided.
2) Analyze the AMTE's claims using the model you developed.
3) Produce a technical report documenting the development and analysis of your model, and the results of applying your model to analyzing the AMTE's claims.
4) Present the results of your work to the class.

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What questions do you have?

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References

The “Gas Station Problem” modeling task, and the associated interpretations, models, and diagrams for this set of slides was taken from the 2016 GAIMME created by SIAM and COMAP.

GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education, Second Edition, Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia, 2019. View the entire report, available freely online, at https://www.siam.org/Publications/Reports/Detail/guidelines-for-assessment-and-instruction-in-mathematical-modeling-education

Defining the Problem & Selecting Appropriate Variables

Every mathematical model begins with a question, but to do so you will have to translate it into mathematics. But that’s not entirely accurate. There are actually two questions that are going to inform everything you do from here on out. They are:

1. How can I represent this real-world concept using mathematics?
2. What do these mathematical statements mean in real-world terms?

We are going to begin by considering the former.

Recall that you are supposed to determine whether “tracking” in school mathematics is an equitable practice. For reference, here are our definitions of “tracking” and “equity”:

> **Tracking** is the practice of placing students in course “tracks” based on their perceived ability.
>
> **Equity** in school mathematics is “being unable to predict student patterns (e.g., achievement, participation, the ability to critically analyze data or society) based solely on characteristics such as race, class, ethnicity, sex,beliefs and creeds, and proficiency in the dominant language” (Gutierrez, 2002).

Taking Stock of Our Personal Biases

As with any other topic, we approach the issue of tracking with our own biases, many of which stem from our own experiences in the mathematics classroom. A good mathematical modeler acknowledges and accounts for their biases when developing a model. Take a few minutes to reflect on your own feelings—both positive and negative—about tracking. Write them down in the space provided.

Over the course of our individual educations, we have developed beliefs about how mathematics should be taught and how students should learn. In the space below, describe what you would consider to be the “ideal” mathematics classroom. Be sure to include descriptions the kinds of teaching strategies and learning activities you would expect to see there.

Ensuring We Understand the Problem

Take some time to think about what specific data you will need to explore the question "Is 'tracking' in school mathematics and equitable practice?" In the space below, list any and all data you think would be important for determining whether tracking is equitable and explain how you think it would do so.

Share your list items, and your justifications for their selection, with your team. Compile your lists into a single "master" list. Write down your team's master list in the space provided. Be sure to include explanations for how each factor would help your team accomplish the task you have been given.

Now that you have a starting place, spend some time with your group doing some research on the available data. Here are a few data sources to get you started, but know that you are welcome to seek out and use others. There is a lot of information to digest here, so you will probably want to divide this research effort among your team members and then come back together to discuss what each person found.

- The 2013 Brown Center Report on American Education: How Well Are American Students Learning? *With sections on the latest international tests, tracking and ability grouping, and advanced math in 8th grade*
- The 2016 Brown Center Report on American Education: How Well Are American Students Learning? *With sections on Reading and Math in the Common Core Era, Tracking and Advanced Placement (AP), and Principals as Instructional Leaders*
- Status and Trends in the Education of Racial and Ethnic Groups 2018
- US Department of Education Civil Rights Data Collection

Making Appropriate Assumptions

By now you and your team have given considerable thought to the challenge of determine whether tracking in school mathematics is an equitable practice. You've made a list of important factors and had time to do some research into some of the available information is out there.

> Take a moment to review your team's master list of relevant data along with the explanations you provided for how each list item would help determine whether tracking is equitable. Hidden in your list and in your explanations are some assumptions about this problem. For example, your list reveals something about how your team decided to quantify the concept of equity in school mathematics. In the space provided, write down any assumptions you notice. Write them out as clearly as possible.

Ensuring the Problem is Tractable

Assumptions play a vital role in the mathematical modeling process. In addition to helping us quantify terms like equity, we can use assumptions to reduce the number of variables in our model and to help us move forward even when we are lacking in relevant data. There are two caveats of which you must be aware:

1. You must provide a reasonable justification for any assumption you make.
2. Assumptions can limit, sometimes substantially, the applicability of your model.

Once again, recall that you are supposed to determine whether "tracking" in school mathematics is an equitable practice. For reference, here are our definitions of "tracking" and "equity":

> **Tracking** is the practice of placing students in course "tracks" based on their perceived ability.
>
> **Equity** in school mathematics is "being unable to predict student patterns (e.g., achievement, participation, the ability to critically analyze data or society) based solely on characteristics such as race, class, ethnicity, sex,beliefs and creeds, and proficiency in the dominant language" (Gutierrez, 2002).

In the space provided, write down precisely how your team is quantifying our definition of equity in terms of the variables you selected.

Write down additional assumptions could your team could make to reduce the number of factors in your model while retaining as much of its applicability as possible. Justify the making of each assumption and explain how you believe it will limit the applicability of your model.

Believe it or not, you now have a mathematical model! You have a list of variables, a list of assumptions to ensure the question is tractable, and a quantifiable definition of equity that incorporates the variables you selected.

Now try and formalize it.

In the space below, formally define each of your variables (e.g., "Let P be the...") and write out precisely what relationships between them would indicate that tracking is or is not equitable. For example, "If [insert explanation and relevant mathematical expression(s) here], then tracking is an inequitable practice.".

Reflecting on Your Experience

Teachers of mathematics are expected to complete their teacher preparation programs with certain competencies and attitudes. The Association of Mathematics Teacher Educators (AMTE) has created the Standards for Preparing Teachers of Mathematics (SPTM), which, they claim, should be used by schools to assess the quality of their math teacher preparation programs and their math-teachers-in-training. Take a few minutes to read the following SPTM indicators and elaborations:

- AMTE Standards for Preparing Teachers of Mathematics: Indicator C.4.1 Provide Access and Advancement
- AMTE Standards for Preparing Teachers of Mathematics: Indicator P.3.3 Address the Social Contexts of Teaching and Learning
- AMTE Standards for Preparing Teachers of Mathematics: ML.7 Equitable Structures and Systems in Middle Schools
- AMTE Standards for Preparing Teachers of Mathematics: HS-3 Supporting Each and Every Student's Opportunity to Learn Mathematics

Reflection Journal 1

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. In various locations in the standards you just read, the AMTE condemns the practice of tracking both implicitly and explicitly, going so far as to suggest it is a form of oppression (see Indicator P.3.3). Do you agree with this characterization? Why or why not?
2. Did you experience tracking at any point in your elementary or secondary education? If so, what was it like? If not, what is your impression of the practice?
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project so far?
5. Is there anything you want your teacher to know?

Reflection Journal 1 Scoring Rubric

1. Student evaluates AMTE claim about tracking. (3 points)

• The student expresses and justifies their opinion on the AMTE's claims about tracking.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

2. Student shares tracking experience. (3 points)

• The student describes their experience with tracking.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

OR

• The student describes their impression of tracking.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

Total ________/14

Implementing Your Model

Running the Numbers

By now, you and your team have developed a mathematical definition of equity in school mathematics and have determined how you will evaluate whether tracking is an equitable practice using that definition. Now it's time to try out your model. Using data about the variables used in your model, answer the following questions.

What does you model indicate about the equity of tracking in school mathematics? Explain.

What can you not conclude from this implementation of your model about the equity of tracking in school mathematics?

Based on this implementation of your model, what would you recommend to education policymakers about the practice of tracking going forward?

Reflecting on Your Experience

Some individuals in the math education community believe tracking is an equitable practice, pointing to what they view as benefits for students they characterize as "gifted." Read the following brief article by an advocate of tracking:

- Ability Grouping and Gifted Children

Members and leaders of the National Council of Teachers of Mathematics (NCTM) have written extensively about tracking, criticizing the practice and calling for its cessation. Read the following letters by former and the current presidents of the NCTM:

- Are We Breaking Down Barriers to Student Learning?
- Initiating Critical Conversations on the Discontinuation of Tracking

Reflection Journal 2

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. Do you think the benefits of tracking listed in the first article outweigh the drawbacks discussed in the two book excerpts? Explain.
2. Explain your team's mathematical definition of equity in school mathematics.
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project so far?
5. Is there anything you want your teacher to know?

Reflection Journal 2 Scoring Rubric

1. Student evaluates pros and cons of tracking. (3 points)

• The student explains whether advantages of tracking outweigh its drawbacks.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

2. Student explains mathematical definition of equity. (3 points)

• The student explains their team's mathematical definition of equity.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student's response is given in complete sentences and with correct grammar.	0	1	

Total ________/14

Iterating the Modeling Process

Recognizing That Every Model Can Be Improved

Now that you have developed and tested a model, you and your team need to revisit your list of variables and assumptions. Respond to the following prompts. They will help you identify ways in which your list of variables and assumptions can be altered in order to improve your model.

Based on your model implementation, which variables ended up being the most and least useful for determining whether tracking in school mathematics is equitable. Explain each item you list.

Are there variables that your model did not include, but that you think would have been useful for determining whether tracking in school mathematics is equitable. Explain how they would improve your model

Can you think of any way to remove the less helpful variables from your model and add the newly identified variables you listed above? Explain.

Based on your model implementation, which assumptions would you say played the most and least important roles in your model. Explain each item you list.

Are there assumptions not included in your model that now seem like they would be beneficial? Explain.

Introducing Model Parameters to Generalize Your Model

You have a model that can be applied to *one* situation. The best models can be applied to a *variety* of situations. The following prompts will help you identify parameters that you can incorporate into your model.

In the space below, list all constant values your model includes. (For example, if you assumed that a certain difference in percentage of student demographics for enrollment in a course should be expected, that difference in percentage is a constant in your model.)

Putting it All Together

For each constant you identify, replace the number in your model with an arbitrary constant. This will allow you to explore how your model might given different results for different values of the constant—more on this later. Write down your newly revised model, including any changes to variables or assumptions and the inclusion of your new model parameters.

Analyzing Your Model

Exploring the Impact of Model Parameters

Now that you have a more general model it’s time to see a little bit more of what it can, and cannot, do. To assist in the analysis of your model, answer the following questions with your team:

> Write down all of the ways in which real-world considerations restrict the range of possible values for each parameter in your model.

How does the alteration of a value for a single parameter affect the rest of your model? What does the alteration of the parameter value mean in real-world terms? Answer these questions for each of your model parameters.

Identifying Variable Domains and Their Implications

The impact of parameters is important to analyze, but you also need to understand the the precise role of each parameter in your model.

> Write down all of the ways in which real-world considerations restrict the range of possible for values each variable in your model.

How does the alteration of a value for a single variable affect the rest of your model? What does the alteration of the value of each variable mean in real-world terms? Answer these questions for each of your variables.

Reporting Your Results - Technical Report Specifications

General Specifications Checklist

Your team’s technical report must adhere to the following general specifications.

- ☐ Title page with participating team members’ names.
- ☐ Times New Roman, 12pt font.
- ☐ Length does not exceed 10 pages (not including title or reference page), double-spaced.
- ☐ All figures and tables referred to in the body of the text.
- ☐ Figure numbers and descriptions included below each figure.
- ☐ Tables and graphs are titled and axes are labeled.
- ☐ Equations are input using an equation editor.
- ☐ Ideas are introduced in the order in which they arose during model development.
- ☐ Sources are given attribution.

Documentation of the Mathematical Modeling Process Checklist

Use the following checklist of questions to ensure your document includes a comprehensive explanation of your mathematical modeling experience.

- ☐ What question is your team trying to answer?
- ☐ How did your team translate the concept of equity to mathematics?
- ☐ What variables and parameters did you use?
- ☐ Did you have to alter your variables or parameters at any time? Why?
- ☐ What sources of information did you rely on to learn about your variables?
- ☐ What assumptions did you make about the variables and parameters you used?
- ☐ Did your assumptions ever change of the course of your model’s development? How and why?
- ☐ How do your assumptions limit the applicability of your model?
- ☐ What mathematical relationships connect the variables and parameters you used?
- ☐ What are the possible ranges of values for your variables and parameters, as constrained by the real-world factors they represent?
- ☐ How do changes to your parameter values impact your model, and what does that impact mean in real-world terms?
- ☐ What do these mathematical relationships indicate about the question your team is answering?
- ☐ What did your model’s implementation indicate about equity and tracking in school mathematics?
- ☐ How should these implications translate to changes to mathematics education policy?

Technical Report Scoring Rubric

General specifications. (9 points)

• Report includes title page with participants names.	0	1	
• Report is written in 12 pt Times New Roman font.	0	1	
• Length does not exceed 10 pages double-spaced text (not including title or reference pages).	0	1	
• Any figures and tables referred to in the body of the text.	0	1	
• Figure numbers and descriptions included below any figures.	0	1	
• Any tables and graphs are titled and axes are labeled.	0	1	
• Equations are input using an equation editor.	0	1	
• Ideas are introduced in the order in which they arose during model development.	0	1	
• Sources are given attribution.	0	1	

Documentation of the mathematical modeling process. (21 points)

• Report explains what question the team is trying to answer.	0	1	
• Report includes explanation of team's quantitative definition of equity.	0	1	2
• Report lists and explains all variables and parameters used, explaining any later alterations.	0	1	2
• Report lists and explains any assumptions made, explaining any later alterations.	0	1	2
• Report explains how assumptions limit applicability of model.	0	1	2
• Report explains mathematical relationships connecting variables and parameters.	0	1	2
• Report explains natural limitations on possible values of variables and parameters.	0	1	2
• Report includes analysis of model parameters, explaining impact of small perturbation to values.	0	1	2
• Analysis of model parameters includes explanation of real-world implications.	0	1	2
• Report includes team's conclusions regarding the equity of tracking in school mathematics.	0	1	2
• Report includes policy recommendations based on model implementation results.	0	1	2

Total ________/40

Presenting Your Results - Presentation Requirements

General Presentation Guidelines Checklist

Use the following checklist of prompts to plan your presentation.

- ☐ Plan your presentation to be between 10 and 15 minutes long.
- ☐ Create appropriate visuals, such as a poster or slideshow, to present the development and results of your model.
- ☐ Make sure your presentation is easy to follow, even for someone without prior knowledge of what you will present.
- ☐ Be prepared to respond to questions in a thoughtful and honest way.
- ☐ Plan to have each team member contribute to and participate in your presentation.

Presentation of the Mathematical Modeling Process Checklist

Your presentation should include responses to each of the following questions.

- ☐ What question is your team trying to answer?
- ☐ How did your team translate the concept of equity to mathematics?
- ☐ What variables and parameters did you use?
- ☐ Did you have to alter your variables or parameters at any time? Why?
- ☐ What sources of information did you rely on to learn about your variables?
- ☐ What assumptions did you make about the variables and parameters you used?
- ☐ Did your assumptions ever change of the course of your model's development? How and why?
- ☐ How do your assumptions limit the applicability of your model?
- ☐ What mathematical relationships connect the variables and parameters you used?
- ☐ What are the possible ranges of values for your variables and parameters, as constrained by the real-world factors they represent?
- ☐ What do these mathematical relationships indicate about the question your team is answering?
- ☐ What did your model's implementation indicate about equity and tracking in school mathematics?
- ☐ How should these implications translate to changes to mathematics education policy?

Model Presentation Scoring Rubric

Presentation format (8 points)

• Presentation is between 10 and 15 minutes in duration.	0	1	
• Presentation includes appropriate visuals (e.g., slideshow poster, etc.).	0	1	
• Presentation organization is easy to follow.	0	1	2
• Team members respond thoughtfully and honestly to any questions they are asked.	0	1	2
• All team members participate in presentation.	0	1	2

Presentation of mathematical modeling process and results. (17 points)

• Students explain what question their team is trying to answer.	0	1	
• Students explain team's quantitative definition of equity.	0	1	2
• Students list and explain all variables and parameters used, explaining any later alterations.	0	1	2
• Students list and explains any assumptions made, explaining any later alterations.	0	1	2
• Students explain how assumptions limit applicability of model.	0	1	2
• Students explain mathematical relationships connecting variables and parameters.	0	1	2
• Students explain natural limitations on possible values of variables and parameters.	0	1	2
• Students explain team's conclusions regarding the equity of tracking in school mathematics.	0	1	2
• Students provide policy recommendations based on model implementation results.	0	1	2

Total ________/25

Taking Action

Having an informed opinion is important, but substantive, persistent change only follows from taking action. In your future careers as math teachers, attempts to affect change in your school will likely be faced with resistance from parents, school and district administrators, and even some of your fellow teachers. In spite of this, it is possible to preserve the integrity and collegiality of these relationships while advocating for positive change.

Planning Ahead

It is often best to plan ahead how you will respond in a difficult situation to ensure that all parties are respected, while promoting the best interests of your students.

1. How would you respond to a parent who claims that their child is not a math person and should be placed in a remedial course because they are falling behind?
2. How would you respond to a colleague or administrator who claims that teaching higher level, accelerated, or honors math courses should only be allowed for the more experienced teachers.

Writing an Action Plan

In addition to planning for specific scenarios, we can make more general plans.

1. Write down two specific actions you will take as a teacher of mathematics to advocate for you students and challenge inequitable structures in mathematics education.

Reflecting on Your Experience

In 2015, two math education organizations, TODOS: Mathematics for ALL and the National Council of Supervisors of Mathematics (NCSM), published a joint position paper about social justice issues in mathematics education. Take a few minutes to read their complete statement:

- Mathematics Education Through the Lens of Social Justice: Acknowledgment, Actions, and Accountability

Reflection Journal 3

In 1 full page of double-spaced text, written in Times New Roman 12pt font, respond to the following prompts.

1. Do you agree with the joint position paper's statements about tracking? Why or why not?
2. What did you mathematical model indicate about tracking in school mathematics and equity? Does the result align with your views of tracking? Explain.
3. What are your thoughts/feelings/opinions about mathematical modeling and your ability to engage in the modeling process?
4. What is the most interesting thing you have learned while engaging in this project?
5. Is there anything you want your teacher to know?

Reflection Journal 3 Scoring Rubric

1. Student evaluates TODOS/NCSM position paper. (3 points)

• The student expresses their views oo the TODOS/NCSM position paper’s tracking claims.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

2. Student discusses model results. (3 points)

• The student explains whether model results aligned with their views on tracking.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

3. Student shares mathematical modeling experience. (5 points)

• The student expresses their thoughts/feelings/opinions on mathematical modeling.	0	1	2
• The student discusses their ability to engage in the modeling process.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

4. Students describes most interesting thing learned. (3 points)

• The student describes the most interesting thing they have learned during the project.	0	1	2
• The student’s response is given in complete sentences and with correct grammar.	0	1	

Total ________/14

APPENDIX D

Curriculum Vitae

Curriculum Vitae For

Patrick L. Seegmiller

Contact Information

Office: USU Animal Science 221; Logan UT, 84321
Email: patrick.seegmiller@gmail.com

Education

Expected May 2020	Ph.D. in Mathematical Sciences *Dissertation*: Social Justice Mathematical Modeling for Pre-Service Teacher Preparation *Committee*: David Brown, James Cangelosi, Brynja Kohler (chair), Zhaohu Nie, Jessica Shumway	**Utah State University**
May 2015	M.S. in Mathematics *Thesis*: Explicit Construction of First Integrals for the Toda Flow on a Classical Simple Lie Algebra *Committee*: Ian Anderson, Nathan Geer, Zhaohu Nie (chair)	**Utah State University**
May 2012	B.S. in Mathematics *Major*: Mathematics *Minor*: Computer Science	**Utah State University**

Professional History

Aug. 2016 – Present	Graduate Teaching Assistant (Ph.D.)	**Utah State University**
Jan. 2016 – May 2016	Adjunct Professor	**Utah State University**
Aug. 2013 – Dec. 2014	Graduate Teaching Assistant (M.S.)	**Utah State University**
Aug. 2012 – May 2014	Graduate Research Assistant (M.S.)	**Utah State University**

Teaching Honors

- Department of Mathematics and Statistics Excellence in Teaching | 2019 (USU)

Teaching Experience

Undergraduate Recitation Leader Training Supervisor
Facilitate recitation leader training on equitable teaching of mathematics and effective pedagogical strategies. Developed and utilized instrument for observation and evaluation of undergraduate recitation leader classroom instruction and interactions with students. (Fall 2019)

Course Instructor
Responsible for all aspects of course management, instruction, and assessment. Course descriptions pro-

vided below.

Algebraic Thinking and Number Sense for Elementary Education School Teachers - MATH 2010
Pre-service elementary school teacher content course exploring foundations of algebra and numeration systems. (Spring 2020)

Multivariable Calculus - MATH 2210
Partial differentiation, multiple integration, and vector calculus, covering all fundamental theorems. (Summer 2019, Fall 2017, Spring 2017)

Geometry and Statistics for Elementary Education School Teachers - MATH 2020
Pre-service elementary school teacher content course exploring Euclidean geometry and elementary statistics. (Spring 2019)

Calculus 1 - MATH 1210
Limits and fundamental theorems and applications of differential and integral calculus. (Fall 2018, Summer 2018, Spring 2018, Fall 2014, Fall 2013)

Trigonometry - MATH 1060
Introduction to trigonometric functions, equations, identities, and their applications. (Fall 2016)

Pre-Calculus Recitation Course - MATH 1050
Functions, graphs, transformations, combinations, and inverses. Linear systems and matrix equations. (Spring 2016; 2 Sections)

Intermediate Algebra - MATH 1010
Linear equations, inequalities, polynomial, exponential, rational, and radical expressions. (Summer 2014)

Math Skills Review of Calculus - MATH 0923
All major theorems, problem-solving strategies, and applications of single variable calculus, infinite sequences and series, vector-valued functions and their derivatives, and space curves. (Summer 2013)

Math Skills Review of Pre-Calculus and Trigonometry - MATH 0922
All major theorems and problem-solving strategies from pre-calculus and trigonometry. (Summer 2013)

Calculus Techniques Recitation Course - MATH 1100
Problem-solving techniques in differential and integral calculus, applied optimization, and introduction to partial derivatives. Emphasis on applications in business, social science, and natural resources. (Fall 2011)

Teaching Assistant
Responsibilities and classroom roles ranged from grader to facilitator for semester project.

Methods of Secondary School Mathematics Teaching - MATH 4500	Spring 2019
Calculus 1 - MATH 1210	Spring 2019
Foundations of Analysis - MATH 4200	Fall 2016
Theory of Linear Algebra - MATH 5340	Fall 2016
Modern Algebra 1 - MATH 5310	Spring 2013

Research

Research Interests

- Pre-Service Mathematics Teacher Education
- Pre-Service Mathematics Teacher Belief & Attitude Formation
- Mathematics for Social Justice

Current Projects

Social Justice Mathematical Modeling Project Development and Evaluation
Designed, piloted, and evaluated, and revised project materials for social justice mathematical modeling projects for first-semester calculus students, vector-calculus students, and a methods course for pre-service teachers of mathematics. Empowered students as learners and doers of mathematics, provide socially and culturally relevant pedagogy, and differentiate instruction for all learners. Impact on development of pre-service math teacher beliefs assessed and used to inform a new theory of mathematics teacher belief development. Utah State University (with Brynja Kohler).

Model of Mathematics Teacher Belief Development
Qualitative metasynthesis of research into influencing teacher beliefs, the effects of teacher beliefs on teaching practice and student outcomes, the situated cognition learning theory, and cognitive neuropsychiatric theories of the development of nonpathological beliefs. This research builds on insights into student beliefs gleaned from student the reflection journals written during social justice modeling project enactments. The results of this research inform the refinement of existing—as well as the development of new—mathematical modeling projects.

Algebraic Thinking and Number Sense Textbook
Creation of new primary text for a mathematics content course geared toward pre-service elementary school teachers (MATH 2010 Algebraic Thinking and Number Sense for Elementary Education School Teachers). Content and presentation aligns with guidelines of Mathematical Education of Teachers II and the Association of Mathematics Teachers Educators' Standards for Preparing Teachers of Mathematics. Utah State University (with Will Tidwell and Derrick Harkness).

MODULE(S2): Mathematical Modeling
The Mathematics of Doing, Understanding, Learning, and and Educating for Secondary Schools [MODULE(S2)], supported by an NSF-IUSE grant, aims to improve prospective secondary teachers' opportunities to develop mathematical knowledge in and for teaching (MKT) as they learn mathematics in undergraduate mathematics content courses in at least 30 mathematics faculty members' courses, in a variety of undergraduate institutions, by 2022. Association of Public Land-Grant Universities, Mathematics Teacher Educator Partnership.

Publications

Seegmiller, P., & Kohler, B. (2019). Inculcation of Pre-Service Mathematics Teacher Social Consciousness Through Social Justice Mathematical Modeling Projects. In W. M. Smith, B. R. Lawler, J. F. Strayer, & L. Augustyn (Eds.), *Proceedings of the eighth annual Mathematics Teacher Education Partnership conference*. Washington, DC: Association of Public & Land-grant Universities.

Seegmiller, P., "Explicit Construction of First Integrals for the Toda Flow on a Classical Simple Lie Algebra" (2015). *All Graduate Theses and Dissertations*. 4699. https://digitalcommons.usu.edu/etd/4699

Conference Presentations

Seegmiller, P. & Tidwell, W. (Nov. 2019). *Preparing Teachers to Educate Through Mathematics for Social Justice.* Presentation for K-16 Teachers, 2019 School Science & Mathematics Association Convention, Salt Lake City, Utah.

Kohler, B. & **Seegmiller, P.** (Sept. 2019). *Mathematical Modeling for Planetary Thinking.* Poster Session for USU Faculty, Planetary Thinking Sustainability Workshop, Logan, Utah.

Seegmiller, P. & Kohler, B. (Jul. 2019). *Inculcation of Pre-Service Mathematics Teacher Social Consciousness Through Social Justice Mathematical Modeling Projects.* Presentation for Teacher Educators, 8th Annual Mathematics Teacher Educator Partnership Conference, St. Louis, Missouri.

Seegmiller, P. (Aug. 2019). *Equity in the Mathematics Classroom.* Presentation for USU Faculty & Graduate TAs, Together We Teach Conference, Logan, Utah.

Seegmiller, P. (Aug. 2019). *A Canvas Page in 50 Minutes.* Workshop for USU Faculty & Graduate TAs, Together We Teach Conference, Logan, Utah.

Seegmiller, P. & Kohler, B. (Aug. 2018). *Influencing Pre-Service Mathematics Teacher Beliefs on Tracking Practices.* Poster Session, USU Department of Mathematics & Statistics Summer Research Poster Session, Logan, Utah.

Seegmiller, P. (Aug. 2018). *A Quick-Start Canvas Guide for Mathematics Teachers.* Workshop for USU Faculty & Graduate TAs, Together We Teach Conference, Logan, Utah.

Ongoing Education and Professional Development

Mathematical Immersion for Secondary Teachers **Aug. 2018 – Present**
Collaborative learning experience with in-service middle and high school teachers of mathematics. Teachers engage in active-learning mathematics tasks and submit lesson plans and student work samples from their own classroom. Facilitated by the Education Development Center.

Together We Teach Professional Development **Jan. 2018 – Present**
Ongoing professional development exploring on learning theories, mathematical cognition, specification of appropriate learning objectives, research-based formative and summative assessment practices, development of lesson plans, and incorporation of technology into the mathematics classroom. Facilitated by the Utah State University Department of Mathematics and Statistics.

Critical Issues in Mathematics Education **February 2018, March 2019**
2019 Theme: *Mathematical Modeling in K-16 – Community and Cultural Contexts.* Received instruction on the existing state of mathematical modeling education, participated in discussion with industry professionals on existing shortfalls of modeling education, and received training on effective modeling instruction.

2018 Theme: *Access to Mathematics by Opening Doors for Students Currently Excluded From Mathematics.* Received training on existing barriers to equity in mathematics education, pedagogical strategies for differentiating instruction students of every race, ethnicity, gender identity, and socio-economic status, and participated in planning sessions for actively promoting equity in the classroom at all levels of education. CIME is hosted by the Mathematical Sciences Research Institute.

Additional Honors & Awards

- Hansen Scholarship Recipient | 2012 (USU)
- Fine Arts Talent Scholarship Recipient | 2009 (DSU)
- Academic Scholarship Recipient | 2007 (DSU)

Professional Organization Affiliations

- Association of Mathematics Teacher Educators
- Mathematical Association of America
  SIGMAA on Research in Undergraduate Mathematics Education
- National Council of Teachers of Mathematics
- School Science and Mathematics Association
- Society for Industrial & Applied Mathematics
  SIAG on Applied Mathematics Education
  SIAG on Mathematics of Planet Earth
- TODOS: Mathematics for ALL

List of References

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