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HARMONIC DELAY REDUCTION

by

Jay E. Hinnenkamp

A dissertation submitted in partial fulfillment
of requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Psychology

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2020

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ABSTRACT

Harmonic Delay Reduction

by

Jay E. Hinnenkamp, Master of Science

Utah State University, 2020

Major Professor: Dr. Gregory Madden
Department: Psychology

The concurrent-chains procedure has been used to study conditioned reinforcement for over 60 years. Delay reduction theory (Fantino, 1969) (DRT) is a model of conditioned reinforcement that accurately predicts choice under many conditions but fails to make accurate predictions when choice between fixed and variable delays is studied. The following set of three experiments tests and compares a modified version of DRT (henceforth DRT_H) to four competing concurrent-chains models. The results of Experiment 1 indicated that DRT_H makes more accurate predictions than DRT when choice between fixed and variable delays are used in the concurrent-chains procedure. The results of Experiment 2 indicated that the hyperbolic value-added model (Mazur, 2001), a model that makes similar predictions to those of DRT_H , requires abnormally small free parameter values to account for choice when long initial links are used in the concurrent-chains procedure. The results of Experiment 3 provided mixed evidence that the distribution of delays used in the initial links of the concurrent-chains procedure is important. While all models were able to account for some of the findings

across the three experiments, DRT_H made the most consistent predictions across all three experiments.

(149 pages)

PUBLIC ABSTRACT

Harmonic Delay Reduction

by

Jay E. Hinnenkamp, Master of Science

Stimuli which, during the life of an organism, acquire the ability to increase the probability of behavior that they follow are called conditioned reinforcers. The concurrent-chains procedure has been used to study conditioned reinforcement for over 60 years. During this time several different models have been developed to explain how stimuli become conditioned reinforcers. The three experiments of this dissertation introduced a new quantitative model of conditioned reinforcement (DRT_H) and tested its predictions against four existing models. The results of Experiment 1 indicated that DRT_H improves upon the accuracy of its predecessor, delay reduction theory (Fantino, 1969), when predicting choice between fixed and variable delays in the concurrent-chains procedure. The results of Experiment 2 indicated that a competing model of concurrent-chains choice, the hyperbolic value-added model, requires unique assumptions to account for choice when long initial links, a component of the concurrent-chains procedure, are studied. The results of Experiment 3 provided mixed evidence that the distribution of delays used in all components of the concurrent-chains procedure is important. While all models were able to account for some of the findings across the three experiments, DRT_H made the most consistent predictions across all three experiments.

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Jay Hinnenkamp

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CHAPTER I

LITERATURE REVIEW

A tenet of behavior analysis is that the determinants of behavior are found in the interactions between organisms and their environment (Lee, 1988). A process central to this is reinforcement. According to Lattal (2013), reinforcement is the development or maintenance of a response resulting from the response contingent presentation, or removal, of a stimulus or event. Over the past 50 years, hundreds of scientific reports have detailed the ability of reinforcement contingencies to change human and nonhuman behavior within the *Journal of the Experimental Analysis of Behavior* and the *Journal of Applied Behavior Analysis*.

Reinforcers can be classified by the histories of learning that are, or are not, necessary for their ability to establish or maintain behavior. Reinforcers such as food, water, and sexual stimulation, which require no history of learning to increase or maintain responding, are classified as primary reinforcers. Reinforcers such as money that acquire their ability to increase/maintain responding through prior learning are classified as conditioned reinforcers (Kelleher & Gollub, 1962). Although there is little doubt that consequent events, though a history of learning, can come to function as conditioned reinforcers, the specifics of how this occurs have been debated within psychology for over 50 years (Fantino, 1977; Hendry, 1969; Shahan, 2010; Williams, 1994).

Early research on conditioned reinforcers suggested that consequent events become conditioned reinforcers because they acquire features/functions of the reinforcing events with which they are paired (Hull, 1943). Alternatively, Skinner (1938) suggested

that stimuli become conditioned reinforcers when they function as discriminative stimuli; i.e., stimuli correlated with a response-reinforcer contingency. In a final hypothesis, Wyckoff (1969) suggested conditioned reinforcers derive their function from the information they supply about upcoming events. According to this latter perspective, if the environment in which an organism lives is filled with uncertainty, then any stimulus that reduces uncertainty about phylogenetically important events will function as a conditioned reinforcer.

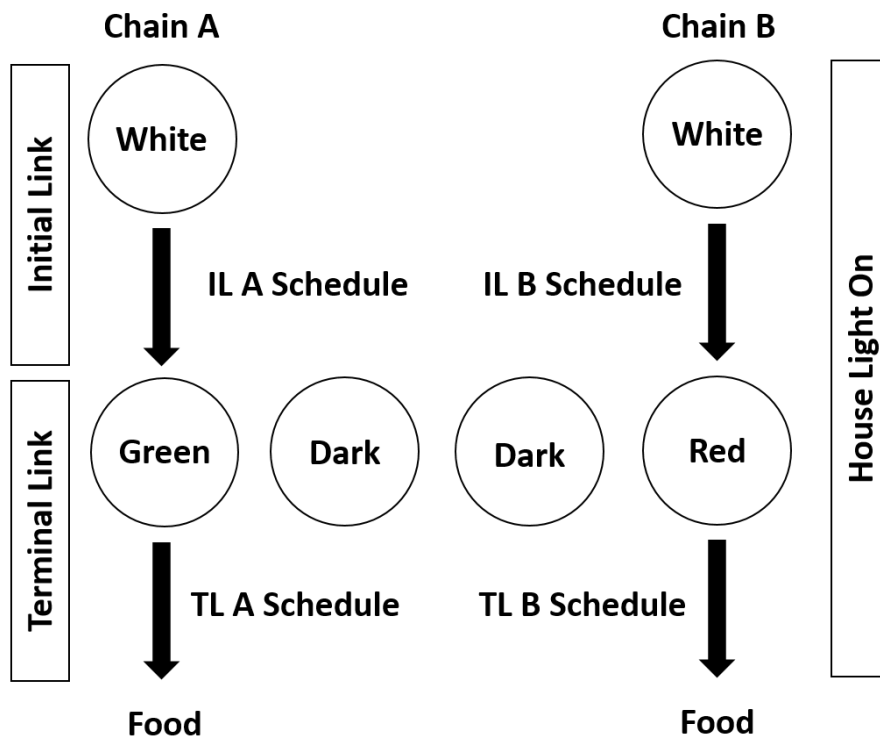
To explore the relative merits of these different hypotheses of conditioned reinforcement, a wide variety of experimental methods have been developed. These procedures include chain, multiple, and second-order schedules of reinforcement; observing-response procedures; and the concurrent-chains procedure (Hendry, 1969). Although each of these procedures may supply different insights into the factor(s) that contribute to conditioned reinforcement, the remainder of this document will focus solely on research that used the concurrent-chains procedure.

Concurrent-Chains Procedure

Autor (1960) was the first researcher to use the concurrent-chains procedure (Figure 1-1). In this procedure, organisms choose between two or more series of events. Each event is referred to as a link, and the series of links is referred to as a chain. In the simple case of a two-alternative concurrent-chains procedure, the initial links of both chains are signaled with a visual stimulus on separate response manipulandum; e.g., two colored response keys simultaneously illuminated on the wall of an operant chamber. At the start of a session, or after a food reward, the organism is free to respond to either initial-link alternative and to switch between them at any time. A separate variable-

interval (VI) schedule, each maintaining a constant probability of reinforcement (i.e., Flesher & Hoffman, 1962), is typically programmed on each initial link. The first response to an initial-link stimulus after its VI has elapsed terminates both initial-link stimuli and initiates the terminal-link stimulus and schedule on the just-selected alternative. The response that subsequently completes the terminal-link schedule requirement produces food. After food is delivered, a new initial-link value is selected for the initial-link stimulus that just produced food, while the other initial-link stimulus timer resumes timing from the point at which it was previously stopped. This cycle repeats until a predetermined number of food deliveries (from either terminal-link schedule) are delivered or until the session duration elapses.

Figure 1-1.
Concurrent-chains procedure.



The dependent variable of interest in the concurrent-chains procedure is how responding is distributed between the initial links. Such response allocation is interpreted as preference for the conditioned reinforcers presented throughout the terminal-links (Herrnstein, 1964a). These terminal-link stimuli are classified as conditioned reinforcers because their onsets maintain patterns of initial-link responding that are similar to those observed in concurrent (non-chain) schedules of primary reinforcement (Ferster & Skinner, 1957).

The decision to focus on the concurrent-chains procedure was made for three reasons. First, the concurrent-chains procedure has been extensively used to study conditioned reinforcement (Fantino, 1977, Kelleher & Gollub, 1962, Williams, 1994). Therefore, there exists a large set of findings from a diverse set of concurrent-chains arrangements that can be used to evaluate competing quantitative models of conditioned reinforcement. Second, in the concurrent-chains procedure, initial-link response allocation serves as the dependent measure of conditioned reinforcer efficacy, and this response allocation is procedurally separated from potential response-rate impacting effects of the schedules arranging primary reinforcers in the terminal link (Fantino, 1977; Fantino & Logan, 1979). For example, one could investigate conditioned reinforcers signaling the onset of either variable ratio (VR) or FI schedules of food delivery. The latter schedules will generate very different response rates during the terminal links, but these effects are procedurally separated from response allocation in the initial links. Response allocation, or preference, may then be interpreted as a metric of conditioned reinforcer efficacy without the response-rate influencing effects of the terminal-link schedules.

A final reason for selecting the concurrent-chains procedure is it does not require conditioned reinforcers to be evaluated in the absence of the primary/backup reinforcer; e.g., response acquisition using only the conditioned reinforcer. Evaluating conditioned reinforcers when separated from the primary reinforcer has been criticized because responding for the putative conditioned reinforcer might simply be an effect of extinction (e.g., extinction burst; Kelleher & Gollub, 1962).

Reinforcer Rate as a Determiner of Conditioned-Reinforcing Value

The results of Autor (1960) suggested that the rate at which reinforcers were delivered in the terminal links determined how pigeons distributed their responses in the initial links. Herrnstein (1964a) replicated and extended these findings. Collectively, these results suggested that relative rates of food delivery controlled responding in concurrent (non-chain) schedules of reinforcement (Herrnstein, 1961) and determined conditioned-reinforcing value.

However, the correspondence between controlling factors in the concurrent chains and concurrent schedules of reinforcement procedures was temporary. Later the same year, Herrnstein (1964b) arranged a concurrent-chains procedure with fixed interval (FI) and variable interval (VI) terminal-link schedules of reinforcement that shared the same overall rate of reinforcement (i.e., VI 15 s and FI 15 s). Contrary to expectation, pigeons strongly preferred the initial-link stimulus leading to the VI terminal-link. Herrnstein amended the rate hypothesis by suggesting pigeons geometrically average the aperiodic intervals within the VI schedule. The geometric mean of a set of numbers more heavily weights smaller values than the arithmetic mean. For example, the arithmetic mean of a VI 60-s schedule with 30, 60, and 90-s intervals is 60 s, while the geometric mean is 55 s.

Thus, if pigeons average aperiodic intervals geometrically, they should prefer any VI schedules to an FI schedule sharing the same arithmetic mean.

Killeen (1968) sought to quantify how animals average intervals of time in aperiodic schedules (i.e., VI and VR schedules of reinforcement). If the averaging method could be identified, then presumably the fixed duration of a concurrently available FI schedule could be set in a way that would produce indifference. Using a concurrent-chains procedure with equal initial-link VI schedules, Killeen identified pigeons' preference for a variety of terminal-link VI and FI schedules of reinforcement. These preferences were used to interpolate terminal-link VI and FI values at which pigeons should be indifferent. The values of the VI schedules, along with the interpolated indifference delays (i.e., FI schedule value at which indifference is predicted between FI and VI schedules) were entered into a generalized mean equation to identify the type of mean (arithmetic, geometric, harmonic) that best described pigeons' preferences:

$$X = \left[\frac{1}{N} \sum_{i=1}^N (y_i^r) \right]^{\frac{1}{r}} \quad (1)$$

where X is the value of the FI schedule predicted to be equivalent to the VI schedule described on the right side of the equation; there N represents the number of intervals in the VI schedule array, y_i is the i th interval in the VI schedule, and r is a parameter used to control the weighting of each interval value within the VI array. When $r = 0$, the geometric mean is supported, as smaller values are weighted more heavily than larger values. When $r = -1$ still greater weight is given to small values and the right side of the equation is the harmonic mean of the VI intervals. When $r = 1$, equal weight is given to every value, resulting in the arithmetic mean. Killeen's results were best fit when $r = -1$,

indicating that the harmonic average of the VI intervals should be used when calculating functional rates of reinforcement.

In a second experiment, Killeen (1968) tested his findings by arranging two very different VI schedules in the terminal links of a concurrent-chain procedure. In one, the VI had an arithmetic average of 80 s (VI_a 80) and a harmonic average of 12 s (VI_h 12). The other VI had an arithmetic average of 40 s (VI_a 40) and a harmonic average of 25 s (VI_h 25). If pigeons' initial-link response allocation was controlled by the proportion of the arithmetic rates of reinforcers arranged in the terminal links, the VI_a 40 (VI_h 25) alternative should be preferred, but if choice is controlled by harmonic rates, preference should favor the VI_h 12 (VI_a 80) alternative. All pigeons preferred the alternative with the higher harmonic mean rate of food, and the lower arithmetic average rate.

In the 1970s, several studies sought to build on Killeen's findings by evaluating if FI schedules were equivalent to the harmonic mean of any aperiodic schedule of reinforcement. While some findings supported the harmonic mean (MacEwen, 1972), many others did not (Davison, 1969; Davison, 1972; Duncan & Fantino, 1970; Fantino, 1967; Hursh & Fantino, 1973; Navarick & Fantino, 1972). For example, using Equation 1, some researchers obtained *r-values* of -3 (Davison, 1969) and -2 (Davison, 1972); i.e., greater weighting of brief aperiodic schedule values than predicted by the harmonic mean.

Several factors were hypothesized to account for the discrepancies in how animals averaged aperiodic intervals of time. These included the number of time intervals in the aperiodic schedules (Davison, 1969), the use of a changeover delay (COD) in the initial links (Killeen, 1970), and the size of the smallest interval in the aperiodic schedule

(Duncan & Fantino, 1970). However, when these hypotheses were tested, discrepancies still remained in how animals averaged intervals of time (Davison, 1972; Navarick & Fantino, 1972). Collectively these findings led Navarick and Fantino (1972) to suggest no method for averaging aperiodic and periodic schedules of reinforcement would be found until the context in which these schedules occurred was considered.

Context as a Determiner of Conditioned-Reinforcing Value

The preceding research assumes that when an organism compares the subjective value of two conditioned reinforcers in the concurrent-chains procedure, CR_A and CR_B , the values of these conditioned reinforcers are independent of the initial links that precede them. For example, if CR_A is preferred over CR_B by a 2:1 margin when the initial-link durations are brief (e.g., VI 60 s), preference should not change when these initial-link durations are increased (e.g., VI 240 s).

Fantino (1969) noted that existing concurrent-chains research had used a limited range of initial-link schedule durations (typically VI 60 s). When Fantino parametrically manipulated initial-link schedule values, pigeons' initial-link response allocation was sensitive to this change in reinforcement context. Specifically, as initial links were lengthened (VI 40 s \rightarrow VI 600 s) and terminal links were held constant, preference for the richer terminal-link schedule declined. This is commonly referred to as the "initial-link effect." Conversely, the "terminal-link effect" occurs when preference for the richer terminal-link schedule increases as terminal-link durations are increased, while holding constant the ratio of terminal-link reinforcement rate and initial-link durations (Grace & McLean, 2019; MacEwen, 1972; Williams & Fantino, 1978). Neither of these effects

should occur if conditioned-reinforcing value is determined solely by terminal-link reinforcement rates.

To account for these data, Fantino (1969) proposed Delay Reduction Theory (DRT):

$$\frac{B_L}{B_L + B_R} = \frac{T - t_L}{(T - t_L) + (T - t_R)} \quad (2)$$

where B_L and B_R are responses made to the left and right initial-link alternatives, t_L and t_R represent the average duration of the left and right terminal links (i.e., from onset to food), and T is the average time from onset of the initial-link stimulus to food delivery. DRT holds that, all else being equal, the conditioned-reinforcing value of a terminal-link stimulus is determined by the reduction in delay to food signaled by onset of the conditioned-reinforcing stimulus (i.e., $T - t_L$). Greater delay reductions, relative to T , establish greater conditioned-reinforcer value. At the extreme, if one terminal-link duration (t_L or t_R) exceeds the average time between successive food deliveries (T), exclusive choice of the other terminal link should occur.

Squires and Fantino (1971) amended DRT after noting it incorrectly predicted initial-link indifference when terminal-link durations are equal, irrespective of differences in initial-link durations. For example:

	Initial Link	Terminal Link
Left Key:	VI 60 s	VI 20 s
Right Key:	VI 600 s	VI 20 s

Squires and Fantino added R_L and R_R terms to reflect the rates of primary reinforcement programmed on the left and right chains¹:

$$\frac{B_L}{B_L + B_R} = \frac{R_L(T - t_L)}{R_L(T - t_L) + R_R(T - t_R)} \quad (3)$$

DRT has proven to be a powerful model for predicting choice across a variety of experimental paradigms - including studies of self-control (Navarick & Fantino, 1976), three-alternative choice (Fantino & Dunn, 1983), and observing (Fantino & Case, 1983) (for review see Fantino, Preston, & Dunn, 1993). Despite these strengths, DRT does not accurately predict choice when periodic and aperiodic schedules of reinforcement are arranged simultaneously in the concurrent-chains procedure (Fantino, Preston, & Dunn, 1993). For example, when DRT is applied to Killeen's (1968) first experiment, in which pigeons' preference for short FI and long VI terminal-link schedules were assessed, DRT inaccurately predicts that pigeons will prefer the shorter FI alternative. Interestingly, DRT's inaccurate predictions when FI and VI terminal links are used are similar to the results of Herrnstein (1964a) who found that pigeons preferred VI to FI schedules of reinforcement.

Integrating Averaging Rules and Reinforcement Context to Predict Conditioned Reinforcement

The separate research lines investigating the averaging of aperiodic reinforcement and the effects of context on choice might be profitably integrated to provide a better

¹ Primary reinforcement rate is the number of reinforcers obtained, divided by the sum of the average initial- and terminal-link durations on that chain; e.g., the reinforcement rate on a chain with a VI 60-s initial link and a VI 30-s terminal link would be: 1 food delivery / (60 s + 30 s) = .01 food deliveries per second.

version of DRT. One such integration would substitute harmonic means² (Killeen, 1968; MacEwen, 1972) for all arithmetic means within DRT (Squires and Fantino, 1971); henceforth, DRT_H.

As an initial evaluation of DRT_H, Figure 1-2 shows choice proportions from the six studies evaluated by Mazur (2001) which arranged FI and VI terminal links.³ Data were averaged across subjects in each experimental condition and are plotted against the predictions of DRT (top panel) or DRT_H (bottom panel). In both panels, 81 experimental conditions are represented along with a dashed line indicating perfect correspondence between observed and predicted choice; the solid line is the line of best fit. Spearman's rank-order correlations were conducted to determine the relationship between predicted and obtained outcomes. Both were significant but the relation was considerably stronger for DRT_H ($r_s = .89, p < .001$) than for DRT ($r_s = .23, p = .045$); likewise, the sum of squared residuals was much lower for DRT_H (1.12) than for DRT (5.31), thereby supporting the superior predictions of DRT_H.

Figure 1-3 plots data from the remaining 19 studies reviewed by Mazur (2001; 379 conditions), which arranged choice between two FI or two VI terminal links along with the data shown in Figure 1-2.⁴ As before, the predictions of DRT are shown in the

² The harmonic mean of a VI schedule is: $\frac{n}{\sum \frac{1}{x_i}}$. Where n is the number of intervals in the VI-schedule array and x_i is each of the intervals filling the array.

³ The six studies are Cicerone (1976), Davison (1969, 1972), Hursh & Fantino (1973), Killeen (1968), and Navarick & Fantino (1972). Mazur also examined a rat study by Rider (1983), but it was excluded from Figure 2 because all other data are from pigeons and rats' concurrent-chains behavior has not been thoroughly investigated.

⁴ The 24 conditions from Experiment 1 of Davison (1988) were excluded because it was the only study in which more than one food delivery was arranged in the terminal links.

Figure 1-2.

Averaged observed choice from 6 concurrent-chains studies with FI and VI terminal-link schedules of reinforcement plotted against the predictions of DRT (Top Panel) and DRT_H (Bottom Panel). The dashed line represents perfect correspondence between observed and predicted choice, while the solid line is the line of best fit.

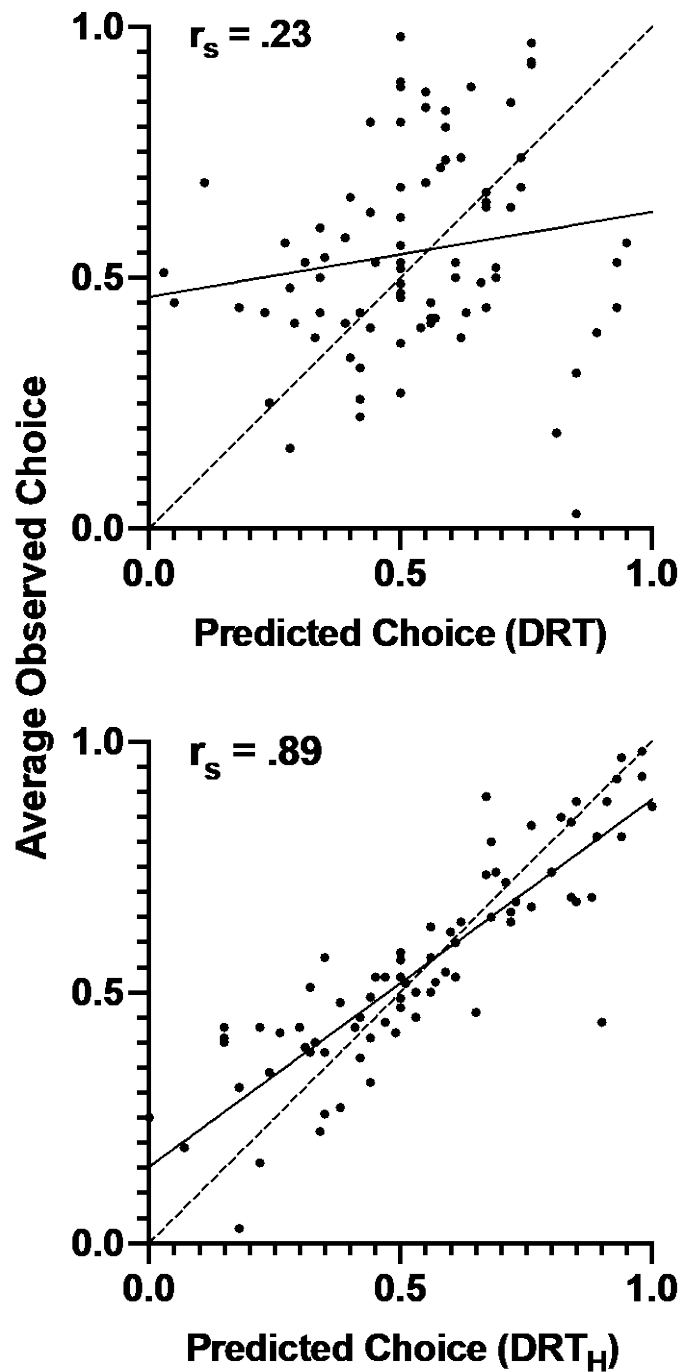
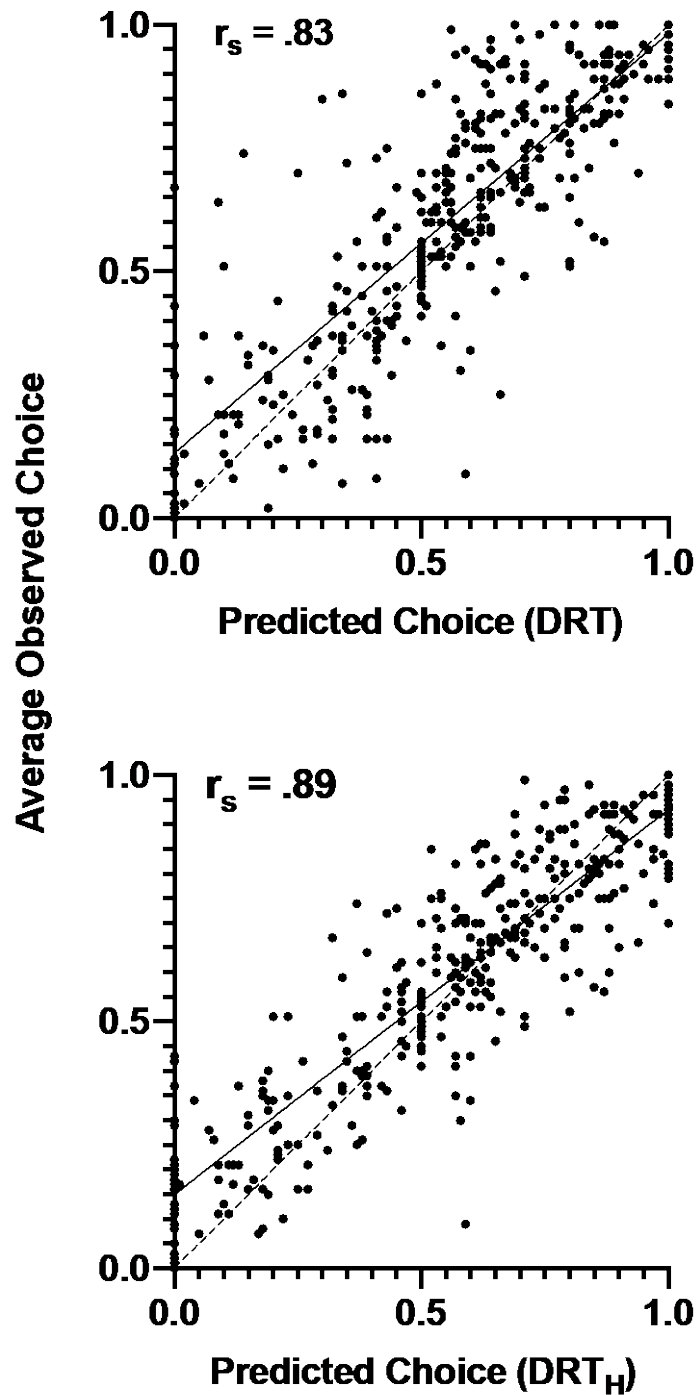


Figure 1-3.

Averaged observed choice from 19 concurrent-chains studies with two periodic or two aperiodic terminal-link schedules of reinforcement plotted against the predictions of DRT (top panel) and DRT_H (bottom panel). The dashed line represents perfect correspondence between observed and predicted choice, while the solid line is the line of best fit.



top panel and those of DRT_H appear below. In both instances, there were strong correlations between obtained choice and the predictions of DRT ($r_s = .83, p < .001$) and DRT_H ($r_s = .89, p < .001$). And as above, the sum of squared residuals is lower for DRT_H (5.9) than for DRT (9.5).

Alternative Quantitative Models of Choice in the Concurrent-Chains Procedure

Although Figures 1-2 and 1-3 suggest predictions of DRT can be improved by substituting harmonic for arithmetic means, several other quantitative models have been developed over the past 50 years to explain choice in the concurrent-chains procedure. If DRT_H is a viable model of choice in the concurrent-chains procedure, it will need to perform similarly or better than these existing models.

The contextual choice model (CCM) is a model of concurrent-chains choice proposed by Grace (1994) that builds upon the assumptions of the matching law (Herrnstein, 1961):

$$\frac{B_L}{B_R} = \left(\frac{r_{iL}}{r_{iR}} \right) \left(\frac{r_{tL}}{r_{tR}} \right)^{\frac{T_t}{T_i}} \quad (4)$$

where r_{iL} and r_{iR} are the rates of terminal link entry in the initial links, r_{tL} and r_{tR} are the arithmetic rates of primary reinforcement in the left and right terminal links, and T_t and T_i are the arithmetic average times spent in the terminal and initial links, respectively. CCM states that pigeons' sensitivity to the relative value of terminal-link stimuli is determined by the context in which those stimuli occur, with the ratio of the average time spent in the initial and terminal links determining context. When the average time spent in the terminal links is longer than the average time spent in the initial links, this results in a T_t/T_i ratio with a value greater than one, which results in an increased preference for the

shorter terminal link. When the average time spent in the initial links is longer than the average time spent in the terminal links, this results in a T_i/T_t ratio of less than one, which results in a decreased preference for the shorter terminal link. The contextual ratio is what allows CCM to predict the initial- and terminal-link effects that are typically observed in concurrent-chains research (Fantino, 1969; MacEwen, 1972).

Although CCM (Grace, 1994) is quite accurate at predicting choice in concurrent-chains procedures with two aperiodic or two periodic terminal-links, it has difficulty accounting for choice when one aperiodic and one periodic terminal link are used. To address this issue Grace (1996) replaced the ratio of terminal-link rates (r_{tL} & r_{tR}) with a ratio of terminal-link values:

$$\frac{B_L}{B_R} = \left(\frac{r_{iL}}{r_{iR}} \right) \left(\frac{v_{tL}}{v_{tR}} \right)^{\frac{T_t}{T_i}} \quad (5)$$

with the value of each terminal link determined by the following equation:

$$V = \sum_{i=1}^n p_i \frac{1}{D_i} \quad (6)$$

where D is the time from terminal-link onset to food delivery *and* p_i is the probability of encountering the i^{th} delay in each terminal link. By subsisting immediacies for rates, CCM can account for choice in the concurrent-chains procedure irrespective of what type of schedules of reinforcement are programmed (Grace 1996; Mazur, 2001).

A second model of concurrent-chains choice is the Hyperbolic Value-Added (HVA) model (Mazur, 2001). Like DRT and DRT_H , HVA assumes that the time from the initial- and terminal-link onset to food play a role in choice:

$$\frac{B_L}{B_L + B_R} = \left(\frac{r_{iL}}{(r_{iL} + r_{iR})} \right) \left(\frac{V_{tL} - V_i}{(V_{tL} - V_i) + (V_{tR} - V_i)} \right) \quad (7)$$

where r_{iL} and r_{iR} represent rates of terminal-link entry within the initial links, V_i represents the value of the food from the onset of the initial-link stimulus, and V_{iL} and V_{iR} represent the value of food from the onset of their respective terminal links. These value parameters (V_i , V_{iL} , & V_{iR}) are calculated using the hyperbolic-decay model (Mazur, 1987):

$$V = \sum_{i=1}^n P_i \left(\frac{A}{1 + kD_i} \right) \quad (8)$$

where A is the amount of the food reward, D_i is the delay to food, P_i is the probability of encountering each delay, and k is a free parameter reflecting how steeply value declines with increases in delay (in pigeon concurrent-chains experiments, k is typically estimated at 0.2). To estimate V_i in Equation 7, D_i values correspond to delays from the onset of the initial-link stimulus to food, and the delays used in calculating V_{iL} and V_{iR} are the delays from the onset of the respective terminal-link stimulus to food. HVA states that choice is determined by the relative value added by the onset of the terminal-link stimulus, hence its name, hyperbolic value-added.

The most recently developed model of concurrent-chains choice is the Cumulative Decision Model (CDM) (Christensen & Grace, 2010). CDM was initially developed to account for the acquisition of choice in the concurrent-chains procedure (Grace, Bragason & McLean, 2003; Grace & McLean, 2006) but was later applied to predict choice in steady-state conditions (Christensen & Grace, 2010). The steady-state form of CDM is:

$$\frac{B_L}{B_R} = \left(\frac{p_L r_{max} + (1 - p_L) r_{min}}{p_R r_{max} + (1 - p_R) r_{min}} \right) \quad (9)$$

where, r_{max} and r_{min} represent the maximum and minimum response strength that can be associated with each of the chains (in pigeon concurrent-chains experiments, r_{max} and r_{min} are set to 1 and .01, respectively) and p_L and p_R are the probabilities that the left and right terminal-link delays are judged short or long. These probabilities at which a delay is judged as short or long are determined by the following equation:

$$p = 1 - \phi(\log D, \log C, \sigma) \quad (10)$$

where D is the delay that is being judged as short and ϕ is a cumulative normal distribution with a mean of C and a standard deviation of σ .

Within the concurrent-chains procedure, CDM assumes that pigeons judge the delays they encounter in the terminal link as short or long by comparing them to a criterion (C) delay. The criterion delay is a geometric average of the times spent in the initial link and the times spent in both terminal links. If a just-experienced terminal-link delay is judged as shorter than C , the probability of responding on the chain that produced the delay is strengthened; if it is judged longer, responding on the alternative is weakened. The accuracy of the short/long judgment is determined by σ , with high standard deviations producing less accurate judgments and low standard deviations producing more accurate judgments.

CDM accounts for the initial-link and terminal-link effects reported in concurrent-chains research by incorporating initial-link delays into C . When terminal-link durations are held constant and the duration of initial links increases (i.e., the conditions responsible for the initial-link effect – less extreme preference as initial-link duration increases), the addition of the longer initial-link delays increases C . As C increases, the

probability that delays in both terminal links will be judged short also increases. This leads CDM to predict less extreme choice at long initial-link delays (Christensen & Grace, 2008; but see Kyonka & Grace, 2009).

When initial-links are unchanged but both terminal-link delays are lengthened, while holding constant the ratio of terminal-link reinforcement rates (i.e., the conditions responsible for the terminal-link effect – more extreme preference as terminal-link durations increase), C increases in CDM. This decreases the overall probability that terminal-link delays will be judged short; however, the probability that the longer of the terminal-link delays is judged short decreases more than for the shorter terminal link (Christensen & Grace, 2009; see Figure 8). This allows CDM to predict the terminal-link effect.

The goal of the following three experiments was to evaluate DRT_H as a quantitative model of conditioned reinforcement and choice within the concurrent-chains procedure. To assess DRT_H as a model of choice, its predictions were evaluated against those of DRT (Squires & Fantino, 1971), CCM (Grace, 1994; Grace, 1996), HVA (Mazur, 2001), and CDM (Christensen & Grace, 2010). This evaluation was conducted using Akaike Information Criterion (AICc; Akaike, 1998), Bayesian Information Criterion (BIC; Schwarz, 1978), and residual analyses (McDowell, Calvin, & Klapes, 2016). For all three of these analyses, programmed rates of reinforcement, initial link durations, and terminal-link durations were used to make all predictions. In the case of harmonic mean rates and durations, a method of estimation suggested by James Mazur (personal communication, January 11, 2017) was used. Specifics of that method are discussed below. Across the three experiments, the predictions of DRT_H were compared

to competing models' predictions across concurrent-chains conditions that varied in duration and type (aperiodic and periodic) of terminal links (Experiments 1 and 2), duration of initial links (Experiments 1 and 2), and the type of distributions composing the initial links (Experiment 3). These specific manipulations were selected to identify potential weaknesses of the different concurrent-chains quantitative models.

CHAPTER II

EXPERIMENT 1

Introduction

Delay reduction theory (DRT) has been highly effective at predicting choice across a wide range of experiments (Figure 1-3) (Fantino, Preston, & Dunn, 1993). However, one limitation is its inability to predict preference in the concurrent-chains procedure when one terminal-link schedule is aperiodic and the other is periodic (Figure 1-2) (Mazur, 2001). Fantino, Preston, and Dunn (1993) give the following description of this limitation:

But it is well known that variable schedules are not functionally equivalent – as measured in choice procedures - to fixed schedules with comparable mean inter reinforcement intervals. Thus, formulations such as Equation 2 [Equation 3] should not be expected to apply to schedules other than VI in any precise fashion.

The modification to DRT proposed here, DRT_H , assumes the harmonic mean better approximates how animals evaluate variable time intervals. To illustrate how harmonic means enable DRT to make more accurate predictions when aperiodic and periodic terminal-link schedules are used, I will compare the predictions of DRT in two published experiments, Killeen (1968) and Fantino (1969). To make these predictions, five values are needed for Equation 3. The first two, t_L and t_R , are the arithmetic average times to food from the onset of the left and right terminal-link stimuli. In Killeen these values are 15 ($t_L = FI\ 15\ s$) and 54 ($t_R = VI\ 54$). The third and fourth values, R_L and R_R , are the overall rates of reinforcement on the left and right chains. In Killeen, R_L is $1/(56\ [left\ initial\ link\ (IL_L) = VI\ 56\ s] + 15\ [left\ terminal\ link\ (TL_L) = FI\ 15\ s])$ and R_R is $1/(56\ [IL_R = VI\ 56\ s] + 54\ [TL_R = VI\ 54\ s])$. The final value, T , is the average time from the onset of

the initial links to food delivery. According to Squires and Fantino (1971), an estimate of T can be obtained by summing two arithmetic averages: (1) the average time to complete either initial link schedule (one of the two concurrently operating VI 56-s schedules in Killeen may be completed every 28 s: $1 / [(1/56) + (1/56)] = 28$) and (2) the average duration of the terminal links; $(54 + 15)/2 = 34.5$; thus, $T = 28 + 34.5 = 62.5$ s. Inserting these values into Equation 3 yields a predicted initial-link response allocation strongly favoring the alternative terminating in the FI 15-s terminal-link:

$$\frac{B_L}{B_L + B_R} = \frac{\left(\frac{1}{(56 + 15)}\right) * (62.5 - 15)}{\left(\frac{1}{(56 + 15)}\right) * (62.5 - 15) + \left(\frac{1}{(56 + 54)}\right) * (62.5 - 54)} = .89$$

However, both of Killeen's pigeons preferred the VI 45-s alternative (.41 and .37, respectively).

By contrast, DRT_H , which substitutes harmonic for arithmetic means, predicts preference for the VI terminal link in Killeen (1968):

$$\frac{\left(\frac{1}{(19.6 + 15)}\right) * (21.9 - 15)}{\left(\frac{1}{(19.6 + 15)}\right) * (21.9 - 15) + \left(\frac{1}{(19.6 + 9.3)}\right) * (21.9 - 9.3)} = .31$$

The differing predictions occur because the arithmetic mean of the VI schedules in the terminal links (54 s) is longer than the FI 15-s terminal link; whereas the harmonic mean of the VI (9.3 s) is shorter than the FI 15 s.

Now consider the predictions of DRT and DRT_H for Fantino and Davison (1983), who arranged a VI 15-s terminal link instead of a FI 15-s terminal link. Substituting the

values of the initial links (VI 60-s schedules) and the other terminal link (VI 45 s) into Equation 3 yields a prediction similar to the FI 15-s condition from Killeen (1968):

$$\frac{\left(\frac{1}{(60 + 15)}\right) * (60 - 15)}{\left(\frac{1}{(60 + 15)}\right) * (60 - 15) + \left(\frac{1}{(60 + 45)}\right) * (60 - 45)} = .81$$

and DRT_H makes the same prediction:

$$\frac{\left(\frac{1}{(27 + 6.7)}\right) * (27 - 6.7)}{\left(\frac{1}{(27 + 6.7)}\right) * (27 - 6.7) + \left(\frac{1}{(27 + 6.7)}\right) * (27 - 20.2)} = .81$$

Consistent with these predictions, Fantino and Davison's pigeons allocated 86% of their responses to the VI 15-s alternative.

The inability of DRT to accurately predict choice when aperiodic and periodic terminal-link schedules are concurrently arranged suggests arithmetic means overestimate pigeons' subjective time to food in VI schedules. This overestimation has little impact on predictions of DRT when both terminal-links are VI schedules because both links are overestimated. However, when FI and VI terminal links are used, only the duration of VI terminal link is overestimated (the arithmetic and harmonic means of an FI schedule are the same) thereby skewing predictions inaccurately toward the FI alternative.

One shortcoming of the analyses presented thus far is the potential for inaccurately estimating the harmonic-means of T , R_L , and R_R . Throughout Chapter 1, estimates of these values were calculated using the method outline by Squires and Fantino (1971). However, this estimation method assumes arithmetic means, not harmonic means; this is likely to underestimate the harmonic-mean time between initial-

link and terminal-link onset. An alternative method for estimating the harmonic means was suggested by James Mazur (personal communication, January 11, 2017) and involves halving each time in the initial-link VI and adding it to each terminal-link VI and then entering each of these sums into the harmonic-mean equation. Given that the times between events during individual trials have not been reported in the literature and because average times are reported as arithmetic means, the existing literature does not allow us to evaluate the accuracy of Mazur's method for calculating the harmonic-mean times for T , R_L , and R_R .

The primary purpose of Experiment 1 was to empirically measure the harmonic-means needed by DRT_H (T , R_L , and R_R) and evaluate the accuracy of Mazur's method for calculating harmonic means. A secondary purpose was to evaluate if the results of Killeen (1968) and Fantino (1969) could be replicated using the concurrent-chains procedure of Savastano and Fantino (1996). In this procedure, each chain is randomly assigned to the left or right key following every reinforcer. This is different than the traditional concurrent-chains procedure that assigns a chain to a single key (left or right) for an entire condition. This modified concurrent-chains procedure was selected to reduce the likelihood of position bias (Mazur, 1984) and to reduce the number of sessions required to meet stability criteria (Savastano & Fantino, 1996).

In Experiment 1, five conditions were conducted: three from Killeen (1968) and two from Fantino (1969). These conditions were selected because (1) they contain initial- and terminal-link values that will allow our lab, which has not previously conducted concurrent-chains research, to replicate initial-link effects; (2) Killeen and Fantino both reported the values comprising each of their VI schedules (this is important because the

distributions of these values (i.e., arithmetic or exponential) can impact the harmonic mean); and (3) Killeen used FI and VI terminal-link schedules of reinforcement, so replicating these conditions allows an empirical test of DRT and DRT_H when they make diverging predictions.

Method

Subjects

A power analysis using the obtained data from the VI 40-s and VI 120-s initial-link conditions from Squires and Fantino (1971) ($d = 1.8$) with $\alpha = .05$ and power = .8 indicated that a sample size of five was needed for simple within-subject comparisons. To facilitate statistical analyses, 6 experimentally experienced, unsexed pigeons were used. Pigeons were housed in a colony room with a 12-hour light/dark cycle, maintained at 85% of their free-feeding weight, and had free access to water in their home cage. Experimental sessions were conducted seven days a week and the procedures were approved by the Institutional Animal Care and Use Committee at Utah State University (Protocol number: 2746).

Apparatus

Two Med Associates (St Albans, VT) ENV-007 modular operant chambers equipped with two response keys, a pellet feeder, and a house light were used. The two side keys (Med, ENV 131M), which could display shapes and colors, were located 16.5 cm above the chamber floor and 2.5 cm from the sides of the chamber. Bioserv (Flemington, NJ) 45 mg grain-based pellets were dispensed into a receptacle located below and between the two side keys, 2.5 cm from the floor. Chambers were enclosed in a sound-attenuating cubicle and white noise was presented throughout all sessions. The

house light was centered on the rear wall 26 cm from the floor. A PC controlled experimental events using MED-PC IV software.

Procedure

Preliminary Training

Because pigeons were previously used in unrelated research, no magazine training or autoshaping was required. One of the two side keys was illuminated with a color or shape that was later used as a terminal-link stimulus. The active key location (left or right) and the stimulus was randomly selected between reinforcers with the constraint that each stimulus occurred an equal number of times per session on each key. Food was initially delivered for pecking according to a VI 10-s schedule, with schedule value gradually increasing between sessions to VI 90 s. Preliminary training under the VI 90 s schedule concluded when 56 reinforcers were obtained per session across three consecutive days.

Concurrent-Chains Procedure

Next pigeons were introduced to the concurrent-chains procedure of Savanstano and Fantino (1996). Pigeons were concurrently presented with two unique initial-link stimuli, with location randomly assigned to each of the two response keys on each trial. Each initial-link stimulus was associated with an independent schedule of reinforcement terminating with a unique terminal-link stimulus. When a terminal-link stimulus was presented, the unselected initial link response-key turned black, its VI timer stopped, and responses to it had no programmed consequences. When the schedule of reinforcement associated with the selected terminal-link stimulus was completed, the terminal-link stimulus was removed, three food pellets were delivered, and then both initial-link stimuli

were re-presented. A new VI value was assigned to the just-selected alternative, while the unselected initial-link timer continued from its position on the last trial. Sessions ended when 56 food cycles were completed. The stimuli associated with each of the initial- and terminal-links was counterbalanced across pigeons.

Table 2-1 shows the common initial-link (IL) and independent terminal-link (TL) schedules of reinforcement arranged in each condition of Experiment 1, and the order in which the conditions were completed. The VI schedules are those reported by Killeen (1968) and Fantino (1969), including the individual values that populate the VI arrays. Conditions were conducted for a minimum of 16 sessions and until initial-link response allocation was stable. To evaluate stability, the proportion initial-link response- and time-allocation was averaged in three-session blocks over the previous nine sessions. Pigeons' choice was judged stable if (a) session blocks deviate by $\leq 5\%$, (b) the highest nor lowest single-session response/time proportion did not occur during the final 3 sessions, and (c) there were no increasing ($M_1 < M_2 < M_3$) or decreasing ($M_1 > M_2 > M_3$) trends in either of the measures (Savanstano & Fantino, 1996; Mazur, 2004). If a pigeon's choice proportions deviated from the group median by more than 1.5 x the interquartile range (IQR), that proportion was considered an outlier and the condition was repeated.

Table 2-1.

Experimental conditions and condition order in Experiment 1. Pigeon ID numbers are provided for the six subjects.

IL Schedules	TL A Schedule	TL B Schedule	Pigeon Number and Condition Order					
			P8	P1	P24	P23	P1270	P20
VI 56	VI 54	FI 15	4	2	3	4	5	2
VI 56	VI 54	FI 20	2	5	4	5	2	5
VI 56	VI 54	FI 25	5	3	2	3	4	3
VI 120	VI 30	VI 90	3	4	5	1	1	1
VI 40	VI 30	VI 90	1	1	1	2	3	4

Data Analysis

Data from the stable sessions of each condition were used for all analyses. An alpha level of .05 was used for all statistical analyses and all reported confidence intervals were at the 95% level. To determine our lab's ability to replicate the initial-link effect, a within-subjects t-test compared the proportion of initial-link choice responses allocated to the shorter terminal link (VI 30) in the final two conditions shown in Table 2-1, the two conditions from Fantino (1969). The ability of DRT and DRT_H to predict pigeons' choice when FI and VI terminal-links are used was evaluated by comparing the sum of the squared residuals for both models (i.e., [average obtained choice proportions – predicted choice proportions]²). To determine if the residuals from the two models differed, the squared residuals were analyzed using a within-subjects t-test. To evaluate the effectiveness of Mazur's method for estimating harmonic-mean values in the concurrent-chains procedure, a one-sample t-test was used to compare predicted and obtained harmonic-mean times and rates. In all cases, when multiple within-family comparisons were conducted, Bonferroni adjusted *p*-values were used.

Akaike Information Criterion and Bayesian Information Criterion Analyses.

To further compare the predictions of DRT and DRT_H, and to compare their predictions to three competing models of concurrent-chains choice, HVA (Mazur, 2001), CCM (Grace 1994; Grace 1996), and CDM (Christensen & Grace, 2010), Akaike Information Criterion (AICc; Akaike, 1998) and Bayesian Information Criterion (BIC; Schwarz, 1978) analyses were conducted. AICc and BIC analyses are useful for assessing quantitative models for two reasons. First, they provide objective criteria for determining if additional free parameters are needed. Recent reviews of quantitative models of

concurrent-chains choice have included between two and four (Mazur, 2001; Christensen & Grace, 2010) free parameters. Justification for these numbers of free parameters has been based on the number of free parameters used in previous reviews (Mazur, 2001) or by giving each model three parameters and only including a fourth parameter if it increases the proportion of variance accounted for by 5% (Christensen & Grace, 2010). Only including a fourth free parameter if it increases the proportion of variance accounted for by 5% is an objective start at evaluating the necessity of free parameters in quantitative models, but a more thorough method is using AICc and BIC analyses to evaluate all quantitative models of concurrent-chains choice with between zero and four free parameters.

A second reason for using AICc and BIC analyses is that they allow models with differing numbers of free parameters to be fairly compared. To account for choice in the large number and variety of experiments that have used the concurrent-chains procedure, different quantitative models will likely require different numbers and types of free parameters. Without AICc and BIC analyses, it is difficult to compare models that use different numbers of free parameters because more free parameters generally produce better fitting models. AICc and BIC analyses correct for the number of free parameters used by a model, which allow models with different numbers of free parameters to be fairly compared.

To conduct AICc and BIC analyses, two choice ratios were calculated for each condition and each pigeon in Experiment 1, one for when the shorter terminal-link was assigned to the left key (half of the reinforcers in each session) and one when it was assigned to the right key. Choice ratios were calculated this way for two reasons. First,

this method allowed analyses to determine if pigeons were biased toward one side or key of the operant chamber. Second, it doubled the number of data points that could be used in the AICc and BIC analyses. Traditionally, AICc and BIC analyses have only been used with data sets that have a wide range of values and at least five data points (Klapes, 2018).

In Experiment 1, five concurrent-chains models were fitted to each pigeons' data using ordinary least-squares regression. For all models the experimentally programmed times and rates of reinforcement were used to make predictions. When harmonic means were estimated, the method described by James Mazur (personal communication, January 11, 2017) was used. Parameter values for all models were estimated using the Solver add-in in Microsoft Excel with the constraint that free-parameter values could not be less than zero or greater than ten. The Solver add-in uses the Generalized Reduced Gradient Nonlinear Solving Method (Ladson, Waren. Jain, & Ratner, 1978) to fit free parameters. Figure 2-1 displays all five quantitative models of concurrent-chains choice evaluated in Experiment 1 in their logarithmic form.

In all models b represent bias, which is a systematic preference for one side of the chamber, one key, or one stimulus across all experimental conditions, and regardless of reinforcement parameters. In all models, the a parameter represents differential sensitivity to some feature of the concurrent-chains procedure. This includes sensitivity to the relative rates of terminal-link entry (a_i), sensitivity to terminal link delays (a_t), sensitivity to the relative rates of food delivery on each chain (a_f), and sensitivity to the time between initial-link onset and food deliveries (a_T). Delay Reduction Theory (DRT) and DRT_H have no additional free parameters. The contextual choice model (CCM) has

Figure 2-1. *Quantitative models of choice and their free parameters in the concurrent-chains procedure.*

CCM:	$\log \left(\frac{B_1}{B_2} \right) = \log(b) + a_i \log \left(\frac{r_{i1}}{r_{i2}} \right) + \left(\frac{T_t}{T_i} \right)^k \log \left(\frac{V_{t1}}{V_{t2}} \right)$ <p style="text-align: center;">where</p> $V = \sum_{i=1}^n p_i \frac{1}{D_i^{a_t}}$
CDM:	$\log \left(\frac{B_1}{B_2} \right) = \left(\log b \frac{r_1^{a_i} p_1 r_{max} + (1 - p_1) r_{min}}{r_2^{a_i} p_2 r_{max} + (1 - p_2) r_{min}} \right)$ <p style="text-align: center;">where</p> $p = 1 - \phi(k(\log D), \log C, \sigma)$
DRT*:	$\log \left(\frac{B_1}{B_2} \right) = \log(b) + a_f \log \left(\frac{R_1}{R_2} \right) + \log \left(\frac{T - (t_1 a_t)}{T - (t_2 a_t)} \right)$
DRT _H :	$\log \left(\frac{B_1}{B_2} \right) = \log(b) + a_f \log \left(\frac{R_1}{R_2} \right) + \log \left(\frac{(T a_T) - t_1}{(T a_T) - t_2} \right)$
HVA**:	$\log \left(\frac{B_1}{B_2} \right) = \log(b) + a_i \log \left(\frac{r_{i1}}{r_{i2}} \right) + \log \left(\frac{V_1 - (V a_T)}{V_2 - (V a_T)} \right)$ <p style="text-align: center;">where</p> $V = \sum_{i=1}^n P_i \frac{A}{1 + k D_i}$

Note: * The a_t parameter in DRT was placed on t (terminal-link delay) as initially done by Mazur (2001). In exploratory analyses, the a_t parameter was also placed on T (time from initial-link onset to food delivery), but this manipulation did not significantly improve the performance of DRT.

** For all Experiment 1 calculations, the value of k was set to .2. This practice is consistent with those used by Mazur (2001) in his review of the concurrent-chains literature.

one additional parameter, k , a scaling parameter applied to the ratio of time spent in the initial and terminal links. The hyperbolic value-added model (HVA) also has one additional parameter, k , which impacts how steeply food values declines with increases in its delay. Finally, it should be noted that the location of the free parameters in DRT were added not by Squires and Fantino (1971), but by Mazur (2001). Mazur did this to facilitate a comparison of all the models with an equal number of free parameters. It is possible that placing the free parameters in different locations in the model may improve its performance, but to align with previous reviews of concurrent-chains literature, the free parameter locations used by Mazur were used throughout this document for DRT.

The cumulative decision model (CDM) contains two additional parameters. The first is σ , which represents the standard deviation of the criterion delay. Larger σ values reflect less accurate timing decisions (Christensen & Grace, 2010). The second is k , which has been used two different ways by Grace and McLean (2015; 2019). Because only the second of these uses of k is relevant to the current project, I will only discuss that use here. Accordingly, when fitting data from concurrent chains containing a terminal link with variable delays to food, Grace and McLean (2019) allowed k to vary. When $k < 1$ reinforcers delivered after variable terminal-link delays were more valuable than reinforcers delivered after fixed delays; when $k > 1$ the opposite was true.

After free parameter values were determined, the resulting residual sum of squares (RSS) was used to determine the AICc and BIC values for each pigeon:

$$AICc = n \ln\left(\frac{RSS}{n}\right) + 2K \left(\frac{n}{n - K - 1}\right)$$

and

$$BIC = n \ln \left(\frac{RSS}{n} \right) + K \ln (n)$$

where K is the number of parameters fitted to an ordinary least-squares regression with n data points. In both AICc and BIC analyses, lower (more negative scores) represent better fitting and more parsimonious models. With no free parameters, AIC and BIC produce identical values, but BIC more heavily penalizes for additional free parameters.

Across subjects within the same experiment, AICc and BIC values from the same quantitative model can be summed to identify a best model (McArdle, Navakatikyan, & Davison, 2007). Although it is difficult to eliminate any quantitative model with a single experiment, the difference in AICc and BIC between two models can be used to calculate an evidence ratio (ER; Burnham & Anderson, 2002):

$$ER = e^{0.5 (\text{highest AICc} - \text{lowest AICc})} \text{ or } ER = e^{0.5 (\text{highest BIC} - \text{lowest BIC})}$$

The higher the evidence ratio, the higher the likelihood that the better performing model is superior. For example, a difference of 5 AICc (or BIC) units produces an ER of 12.2:1, indicating that there is a .92 probability ($12.2/[12.2+1] = .92$) that the model with the more negative AICc (or BIC) score is the correct model. It is generally accepted that a difference of 10 AICc (or BIC) units ($ER = 148.4$) provides convincing evidence of the superiority of one model over another, as there is a .993 probability ($148.4/[148.4+1] = .993$) that the better performing (more negative) model is the correct model (Burnham & Anderson, 2002; Navakatikyan, 2007).

Residual Analysis. The difference between a pigeon's observed choice proportion and a model's prediction is referred to as a residual. Systematic patterns in residuals indicate that a model is incorrect or fails to account for some factor that systematically impacts choice. To analyze the residuals in Experiment 1, a cubic

polynomial test for residual trend was used (McDowell, Calvin, & Klapes, 2016). In this analysis, residuals were plotted against predicted choice proportions and fit with a cubic polynomial function. A cubic polynomial function is used because it is a flexible function that can accommodate a large number of residual patterns. The effect size (R^2) from each subject's cubic function is then compared to a median R^2 value generated by the following equation developed by McDowell, et al., (2016):

$$R_{\alpha=0.50}^2 = \frac{3F_{\alpha=0.50}}{3(F_{\alpha=0.50} - 1) + n - 1} \quad (11)$$

where $F_{\alpha=0.50}$ is the value in an F distribution for sample size n , where half the distribution is in the right tail and half in the left tail. Equation 11 describes how median R^2 values change with sampling size distributions when cubic polynomial functions are fitted to randomly generated data. Subjects' R^2 values are divided into two groups: R^2 values that are less than the median value identified by McDowell, et al., and R^2 values that are more. An exact binomial test can then be conducted to determine the probability of having that many data points occur above or below the median. The cubic polynomial test was selected for use in Experiment 1 because it only requires one null-hypothesis test, which reduces the likelihood of false positives, and because it does not require that residuals are pooled, which can obscure trends in individual subjects' residuals.

Results

Individual and group averaged sessions to stability and harmonic-mean times from the onset of the initial link to food delivery (T_H) averaged across the final nine stable sessions in each condition can be seen in Table 2-2. The harmonic-mean times spent in the terminal links (t) are not shown because predicted and obtained times were nearly identical. Figure 2-2 displays the individual and average T_H times for pigeons in

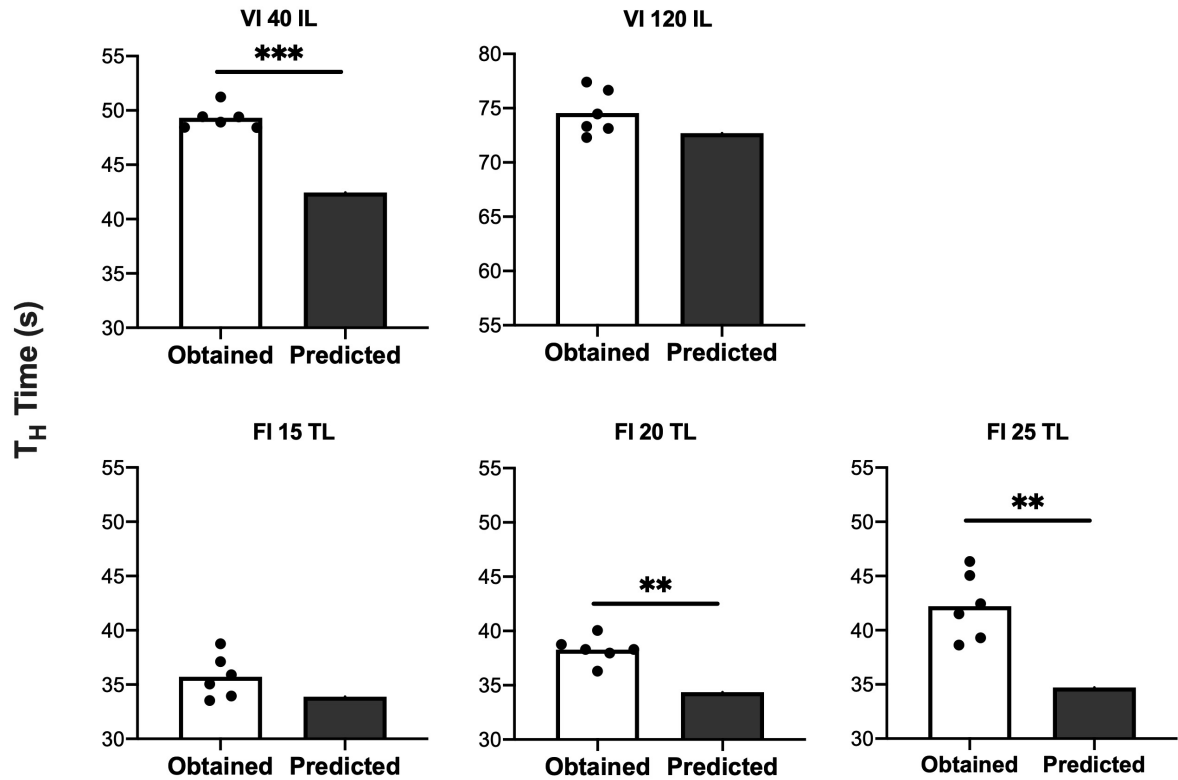
Table 2-2.

Sessions until stability, obtained T_H , R_R , R_L values, and proportion choice from Experiment 1. Values in brackets denote standard deviations around the group averages.

Pigeon	Condition	Sessions Until Stability	Obtained Harmonic T_H (sec)	Obtained Harmonic R_R	Obtained Harmonic R_L	Proportion Choice Rich
P1	IL VI 40	32	49.44	0.02	0.01	1.00
	IL VI 120	40	73.33	0.02	0.01	0.79
	TL FI 15	26	33.52	0.03	0.03	0.49
	TL FI 20	39	36.29	0.03	0.03	0.61
	TL FI 25	21	42.46	0.02	0.02	0.73
P8	IL VI 40	25	51.23	0.02	0.01	0.95
	IL VI 120	56	74.47	0.02	0.01	0.73
	TL FI 15	33	35.05	0.03	0.03	0.58
	TL FI 20	36	38.77	0.03	0.02	0.67
	TL FI 25	25	39.29	0.03	0.02	0.62
P20	IL VI 40	27	48.44	0.02	0.01	0.98
	IL VI 120	40	77.40	0.02	0.01	0.91
	TL FI 15	40	37.11	0.03	0.03	0.92
	TL FI 20	51	40.06	0.02	0.03	0.93
	TL FI 25	25	41.51	0.03	0.02	0.72
P23	IL VI 40	30	49.41	0.02	0.01	0.97
	IL VI 120	34	76.65	0.02	0.01	0.87
	TL FI 15	35	33.95	0.03	0.03	0.57
	TL FI 20	35	38.31	0.03	0.02	0.85
	TL FI 25	26	26.08	0.03	0.02	0.92
P24	IL VI 40	15	48.45	0.02	0.01	1.00
	TL FI 25	30	46.34	0.02	0.02	0.99
	TL FI 15	36	38.75	0.03	0.02	0.89
	TL FI 20	46	38.31	0.03	0.02	0.90
	IL VI 120	56	72.30	0.02	0.01	0.94
P1270	IL VI 40	36	48.93	0.02	0.01	0.99
	IL VI 120	42	73.13	0.02	0.01	0.82
	TL FI 15	52	35.91	0.03	0.03	0.45
	TL FI 20	33	37.97	0.03	0.02	0.85
	TL FI 25	28	45.05	0.02	0.02	0.97
Group Averages	IL VI 40	28 [7.23]	49.32 [1.04]	0.02 [0.001]	0.01 [0.003]	0.98 [0.02]
	IL VI 120	45 [9.18]	74.55 [2.06]	0.02 [0.001]	0.01 [0.001]	0.84 [0.08]
	TL FI 15	37 [8.67]	35.71 [1.98]	0.03 [0.001]	0.03 [0.003]	0.65 [0.20]
	TL FI 20	40 [7.04]	38.29 [1.22]	0.03 [0.002]	0.03 [0.001]	0.80 [0.13]
	TL FI 25	26 [3.07]	42.21 [3.06]	0.03 [0.002]	0.02 [0.001]	0.82 [0.15]

Figure 2-2.

Obtained and predicted harmonic mean times from initial-link onset until food (T_H). Black dots represent individual-subject data. The top two panels are data from the Fantino (1969) conditions where initial-link duration (VI 40 IL & VI 120 IL) was manipulated. The bottom three panels are data from the Killeen (1968) conditions where FI terminal-link duration (FI 15 TL, FI 20 TL, & FI 25 TL) was manipulated.



Note: ** $p < .01$, *** $p < .001$

the stable sessions along with the predicted T_H times calculated using Mazur's method.

The top two panels are T_H times from the two conditions from Fantino (1969). The bottom panels are T_H times from the three conditions from Killeen (1968). Note the different y-axis range in the VI 120-s condition.

The differences between obtained and predicted T_H times were systematic; in 10 of 11 cases from the Fantino conditions, and 10 of 11 cases from the Killeen conditions, obtained T_H times were longer than the predicted times. With a Bonferroni adjusted $p = .01$, one-sample t-tests indicated that the obtained T_H times in the VI 40-s IL ($t_{(5)} =$

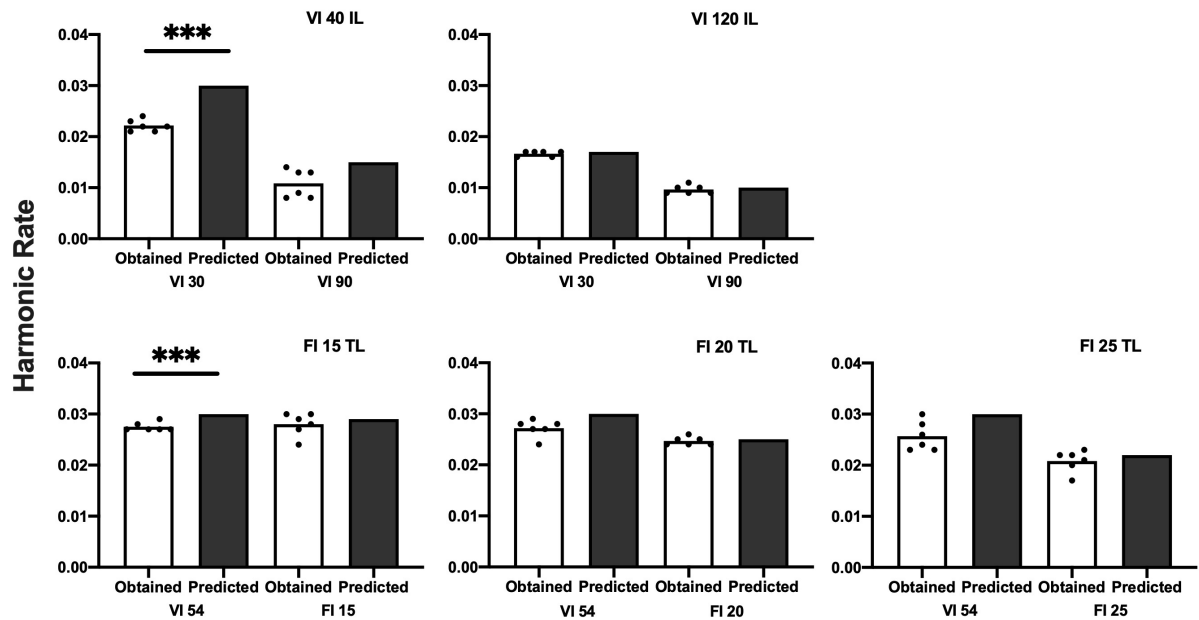
16.23, $p < .001$, CI = 48.22, 50.4), FI 20 s TL ($t_{(5)} = 7.88$, $p = .001$, CI = 37.01, 39.57), and FI 25 s TL ($t_{(5)} = 6.00$, $p = .002$, CI = 39.00, 45.42) conditions were significantly larger than the times predicted by Mazur. There was no significant difference between obtained and predicted T_H times in the VI 120 s IL ($t_{(5)} = 2.21$, $p = .08$, CI = 72.39, 76.70) or the FI 15 s TL ($t_{(5)} = 2.27$, $p = .073$, CI = 33.64, 37.79) conditions.

Individual and average obtained harmonic rates of reinforcement for each choice alternative in each condition can be seen in Table 2-2. R_R is the harmonic rate on the chain with the richer terminal link, while R_L is the harmonic rate on the chain with the leaner terminal link. In the two conditions from Killeen (1968), R_R is the harmonic mean rate of reinforcement on the chain terminating in the VI 54-s terminal link.

Figure 2-3 displays obtained harmonic rates of reinforcement for each pigeon in each condition, and the programmed rates in those conditions. In 49 of the 60 cases the obtained harmonic rate of reinforcement was lower than the programmed rate. With a Bonferroni adjusted $p = .005$, one-sample t-tests indicated that the obtained rates on the R_R chains (left-most bars within each panel) were significantly different than predicted rates in the VI 40 s IL ($t_{(5)} = 16.41$, $p < .001$, CI = 0.021, 0.023,) and FI 15 s TL ($t_{(5)} = 7.32$, $p < .001$, CI = 0.027, 0.028) conditions. There was no significant difference between obtained and predicted harmonic rates of reinforcement on the R_R chain in the VI 120 s IL ($t_{(5)} = 1.58$, $p = .175$, CI = 0.016, 0.017), FI 20 s TL ($t_{(5)} = 4.03$, $p = .01$, CI = 0.025, 0.029), or FI 25 s TL ($t_{(5)} = 3.69$, $p = .014$, CI = 0.023, 0.029) conditions. With a Bonferroni adjusted $p = .005$, one-sample t-tests indicated that the harmonic rate of reinforcement on the R_L chain was not significantly different than the programmed rates in the VI 40 s IL ($t_{(5)} = 3.66$, $p = .015$, CI = 0.007, 0.014), VI 120 s IL ($t_{(5)} = 1.0$, $p = .363$,

Figure 2-3.

Obtained and programmed harmonic rates of reinforcement on the two choice alternatives in Experiment 1. Abscissa information corresponds to the terminal links. Black dots represent individual subject data. The top two panels are data from the two Fantino (1969) conditions where initial-link duration (VI 40 IL & VI 120 IL) was manipulated. The bottom three panels are data from the three Killeen (1968) conditions where the FI terminal-link duration (FI 15 TL, FI 20 TL, & FI 25 TL) was manipulated.



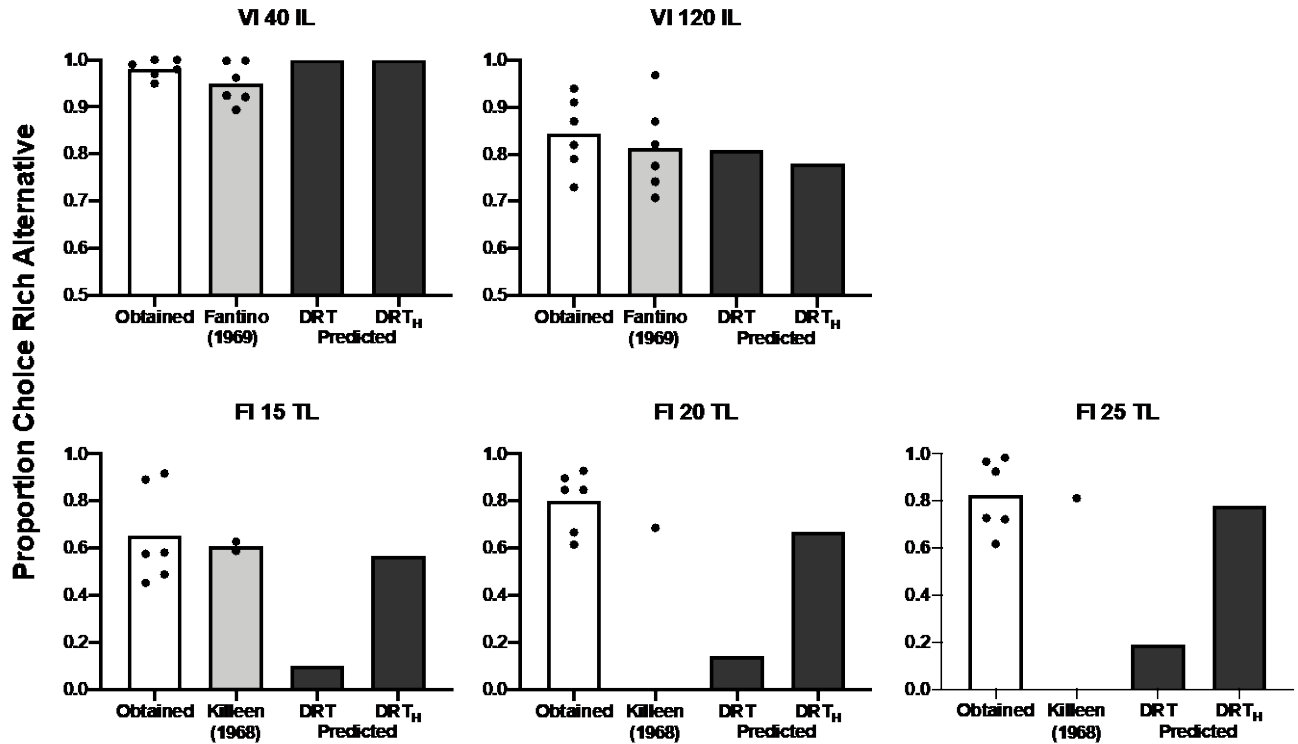
Note: *** $p < .001$

CI = 0.009, 0.011), FI 15 s TL ($t_{(5)} = 1.07$, $p = .332$, CI = 0.026, 0.030), FI 20 s TL ($t_{(5)} = 1.0$, $p = .363$, CI = 0.024, 0.026), or in the FI 25 s TL ($t_{(5)} = 1.34$, $p = .239$, CI = 0.019, 0.023) conditions.

The individual and average proportion of initial-link responses allocated towards R_R in each condition are shown in Table 2-2. Average choice proportions, along with the obtained choice proportions from Fantino (1969) and Killeen (1968), and the predictions of DRT and DRT_H are plotted in Figure 2-4. Inset data points are individual subject data. Choice was comparable between the present experiment and the one conducted by Fantino (Fantino's VI 40 IL: $M = .95$, $SD = .04$; VI 120 IL: $M = .81$, $SD = .10$).

Figure 2-4.

Average choice proportions from Experiment 1 (Obtained), data reported by Fantino (1969) and Killeen (1968), model and predictions of DRT and DRT_H. Black dots represent individual subject data. The top two panels are data from the Fantino (1969) conditions where initial-link duration (VI 40 IL & VI 120 IL) was manipulated. The bottom three panels are data from the Killeen (1968) conditions where the FI terminal-link duration (FI 15 TL, FI 20 TL, & FI 25 TL) was manipulated. Note: Killeen (1968) included just one pigeon in the FI 20 and 25 TL conditions; hence, no bar corresponding to the average is provided.



Consistent with the initial-link effect, my pigeons' preference for R_R was significantly higher when the initial link was shorter (M difference = .14, SD = .07), $t_{(5)} = 4.63$, $p = .006$. Thus, Experiment 1 replicated the initial-link effect observed by Fantino (1969) when the Savanstano and Fantino (1996) procedure was used.

In the lower panels of Figure 2-4, the choices of Killeen's pigeons, in all cases, fell within the range of the obtain proportions in Experiment 1 (note that a single pigeon completed the FI 20 and 25 TL conditions). To assess the accuracy of the predictions

made by DRT and DRT_H in the Killeen (1968) conditions, a within-subjects *t*-test compared the sum of the squared residuals (i.e., observed choice – predicted choice). This sum was 7.19 for DRT and 0.86 for DRT_H when no free parameters were included in the models; $t_{(17)} = 10.03$, $p < .001$. Thus, DRT_H made more accurate predictions than DRT when FI and VI terminal link schedules of reinforcement are used in the concurrent-chains procedure.

Additional Model Comparisons

The results of the AICc and BIC analyses are presented in Tables 2-3 and 2-4, respectively. Each table displays AICc and BIC scores with 0-3 free parameters for each model, with the free parameter(s) providing the best (lowest) scores included in parentheses. The AICc and BIC scores for all possible free parameters, parameter values, and percent variance accounted for by each model are provided in Appendix A. Because the initial-links durations and distributions were identical in each condition of Experiment 1, the a_i parameter (sensitivity to differences in the initial-link schedules) parameter was not fitted to any of the models. The best AICc and BIC score for each model is marked with an asterisk.

For all models except CCM and CDM, the AICc analysis supported the use of one free parameter when accounting for pigeons' choices. The AICc analysis indicated that no free parameters should be included in CCM and that two free parameters should be included in CDM. The results of the BIC analyses were identical to those of AICc, with one exception: the best fitting form of DRT_H had no free parameters. The lowest (best) AICc and BIC scores were produced by HVA with one free parameter (a_T). Tables 2-3 and 2-4 also display the difference, in AICc and BIC units, between this best-scoring

Table 2-3.

Best AICc values for all quantitative models analyzed in Experiment 1. Lower (more negative) scores represent better fitting and more parsimonious models. The $\Delta AICc$ column is the AICc difference between each model and the best performing model (HVA), while the Probability column displays the probability that the overall best performing model (HVA) is better than the best version of each model.

Model	0 Free Parameters	1 Free Parameter	2 Free Parameters	3 Free Parameters	$\Delta AICc$	Probability
CCM	-62.25*	-52.60 (a_t)	-47.14 (b / a_t)	-60.19 ($b / a_t / k$)	-27.34	1.00
CDM	-	-38.83 (σ)	-63.58* (σ / k)	-45.99 ($\sigma / b / k$)	-26.01	1.00
DRT	18.56	-13.54* (a_t)	4.88 (b / a_t)	22.84 ($b / a_t / a_i$)	-76.05	1.00
DRT _H	-77.63	-86.30* (a_T)	-65.36 (b / a_T)	-45.81 ($b / a_T / a_i$)	-3.29	0.84
HVA	-78.68	-89.59* (a_T)	-73.89 (b / a_T)	-	-	-

*Note: * indicate the lowest (most negative) AIC score for a model.*

Table 2-4.

Best BIC values for all quantitative models analyzed in Experiment 1. Lower (more negative) scores represent better fitting and more parsimonious models. The $\Delta BICc$ column is the BICc difference between each model and the best performing model (HVA), while the Probability column displays the probability that the overall best performing model (HVA) is better than the best version of each model.

Model	0 Free Parameters	1 Free Parameter	2 Free Parameters	3 Free Parameters	ΔBIC	Probability
CCM	-62.25*	-43.03 (a_t)	-32.29 (b / a_t)	-46.49 ($b / a_t / k$)	-17.77	1.00
CDM	-	-29.26 (σ)	-48.74* (σ / k)	-32.29 ($\sigma / b / k$)	-31.29	1.00
DRT	18.56	-3.97* (a_t)	19.73 (b / a_t)	36.54 ($b / a_t / a_f$)	-76.05	1.00
DRT _H	-77.63*	-76.73 (a_T)	-45.41 (b / a_T)	-32.11 ($b / a_T / a_f$)	-2.39	0.77
HVA	-78.68	-80.02* (a_T)	-59.04 (b / a_T)	-	-	-

*Note: * indicate the lowest (most negative) BIC score for a model.*

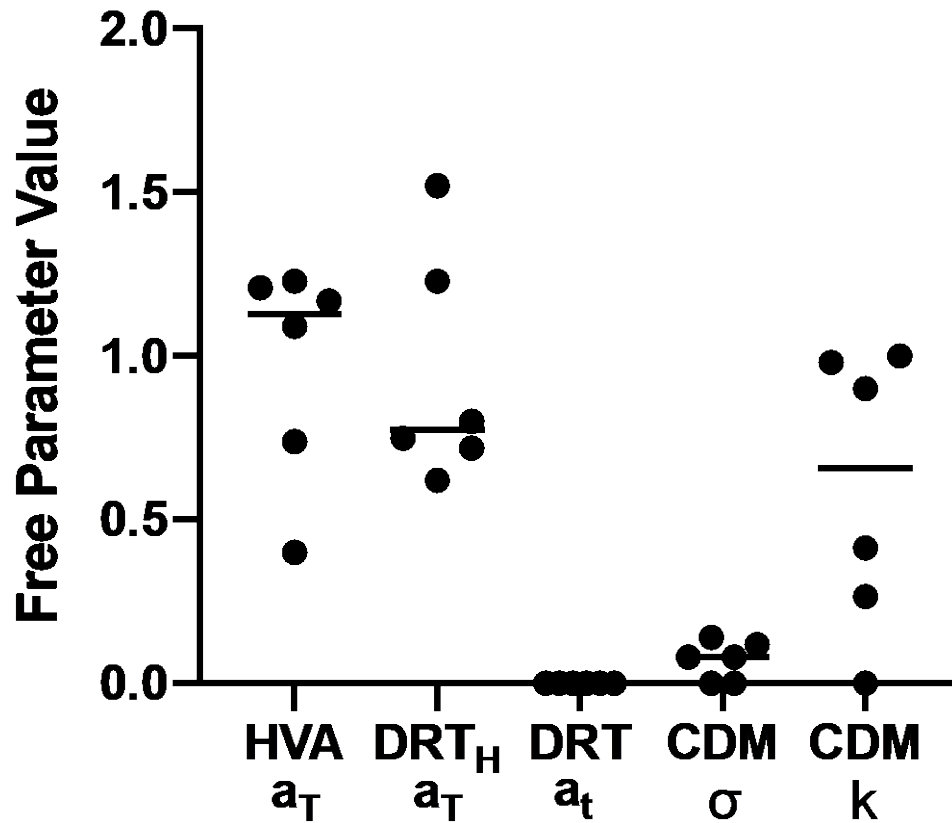
version of HVA and the best version of the remaining models along with the probability that the overall best performing model is better than the best performing version of each model. As previously noted, it is generally accepted that a $\Delta AICc$ or ΔBIC of 10 or more provides definitive evidence for the superiority of one model over another (Burnham & Anderson, 2002). Thus, HVA was convincingly better than all models except DRT_H.

Figure 2-5 displays the median free parameter values for each model that produced the lowest (best) AICc and BIC scores. Free parameter values for CCM are not displayed because the best performing version of the model included no free parameters. The a_T and a_t parameters used by HVA, DRT, and DRT_H are all multiplicative parameters; therefore, a value of 1.0 would indicate that pigeons were perfectly sensitive to the differences in delays to food in the terminal links (DRT) or to the differences in time to food after initial-link onset (DRT_H and HVA). The median a_T value for HVA was 1.13; this indicates that pigeons were slightly over-sensitive to differences in delays between initial-link onset and food. The median a_T value for DRT_H was 0.77; indicative of under-sensitivity to this difference. Finally, the median σ and k values for CDM were 0.07 and 0.66, respectively. These values are smaller than those reported by Christensen and Grace (2010; σ) and Grace and McLean (2019; k) and indicate that pigeons made very accurate short/long time judgments and that they preferred the variable-interval schedules over the fixed-interval schedules.

Figures 2-6 through 2-11 plot, respectively, obtained vs. predicted log choice ratios for each of the best versions of CDM with two free parameters (best AICc & BIC), DRT and HVA with one free parameter (best AICc & BIC), DRT_H with one free parameter (best AICc), DRT_H with no free parameters (best BIC), and CCM with no free parameters (best AICc & BIC). The dotted line represents perfect correspondence between obtained choice and model predictions. Consistent with AICc and BIC scores, DRT provided the worst predictions (Figure 2-8), with most predictions at indifference (log 0). Although DRT made accurate predictions in the two Fantino (1969) conditions, it predicted that pigeons would prefer the FI terminal link over the VI terminal link in the

Figure 2-5.

Individual subject and median free parameter values for HVA, DRT_H , DRT, and CDM that produced the highest AICc and BIC values.



three conditions from Killeen (1968). To adjust for these incorrect predictions, the sensitivity parameter (a_t) was driven to zero, which resulted in DRT's indifference predictions. The third worst performing model, CDM, had a similar pattern of predicting indifference in the data of P1 and P20. By contrast, in better performing models, like HVA and DRT_H , obtained choices were more equally distributed around the dashed line.

Figure 2-6.

CCM predictions (no free parameters) plotted against obtained choice ratios from Experiment 1.

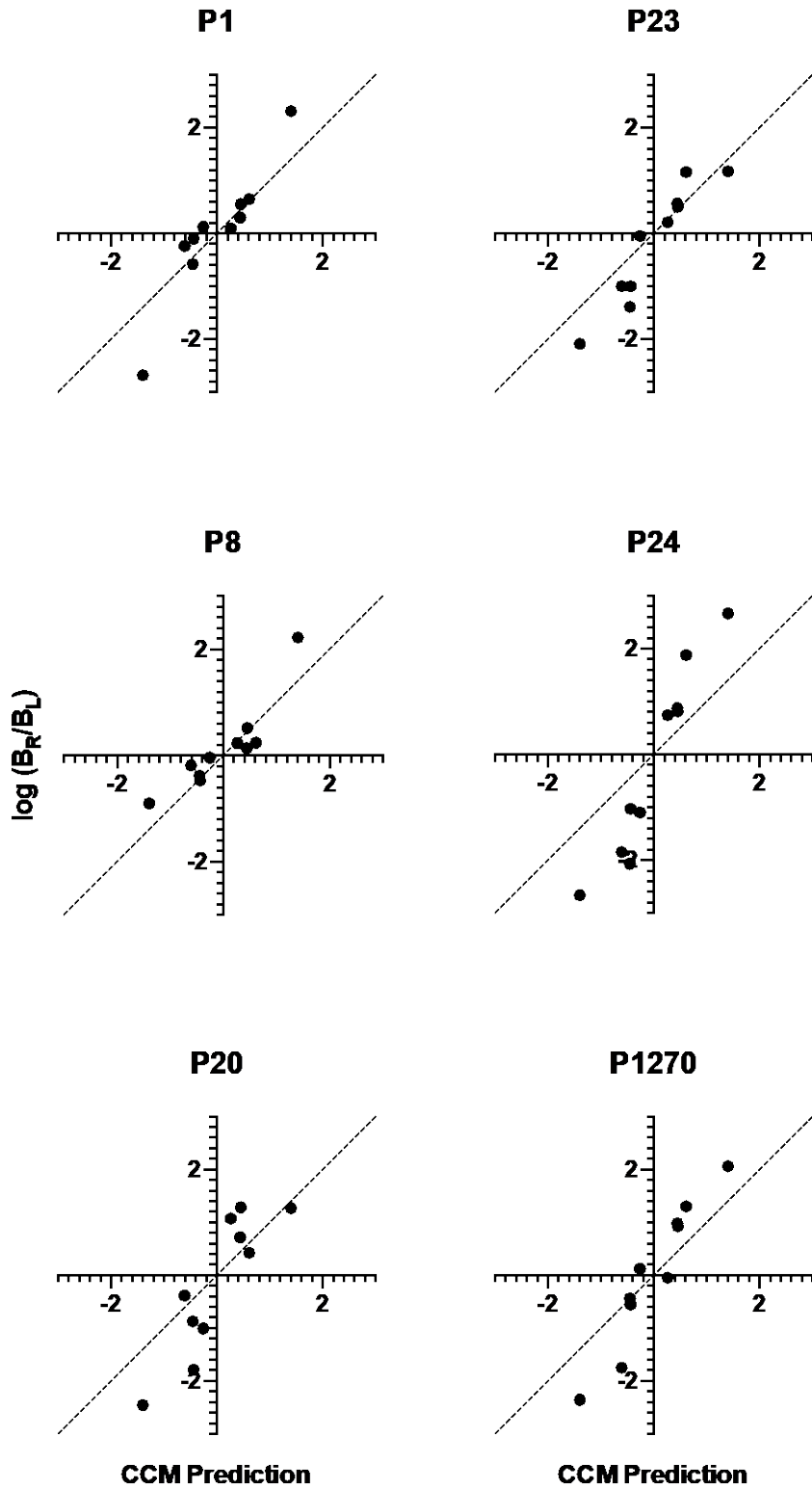


Figure 2-7.

CDM predictions (σ & k parameters) plotted against obtained choice ratios from Experiment 1.

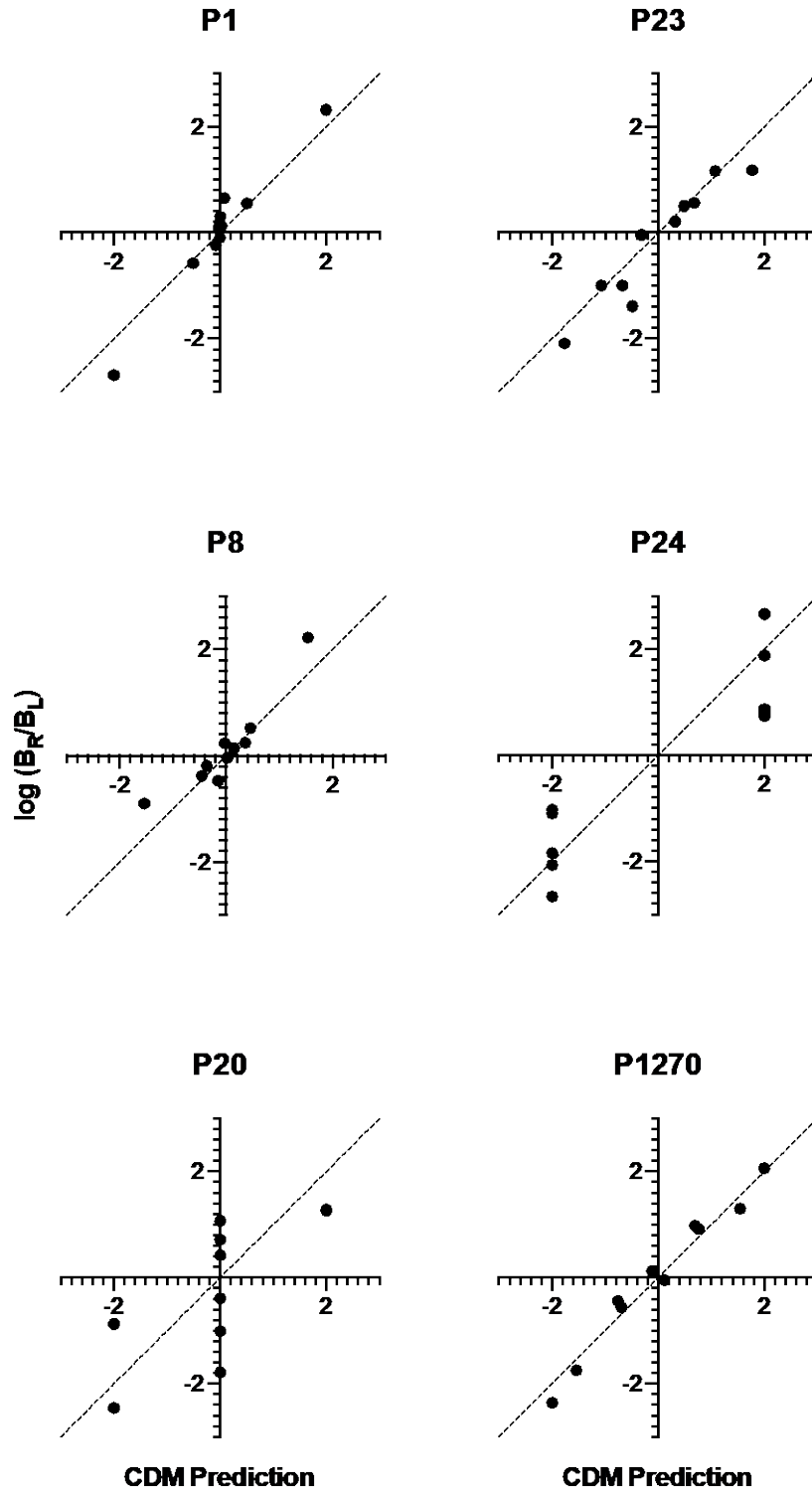


Figure 2-8.

DRT predictions (a_i parameter) plotted against obtained choice ratios from Experiment 1.

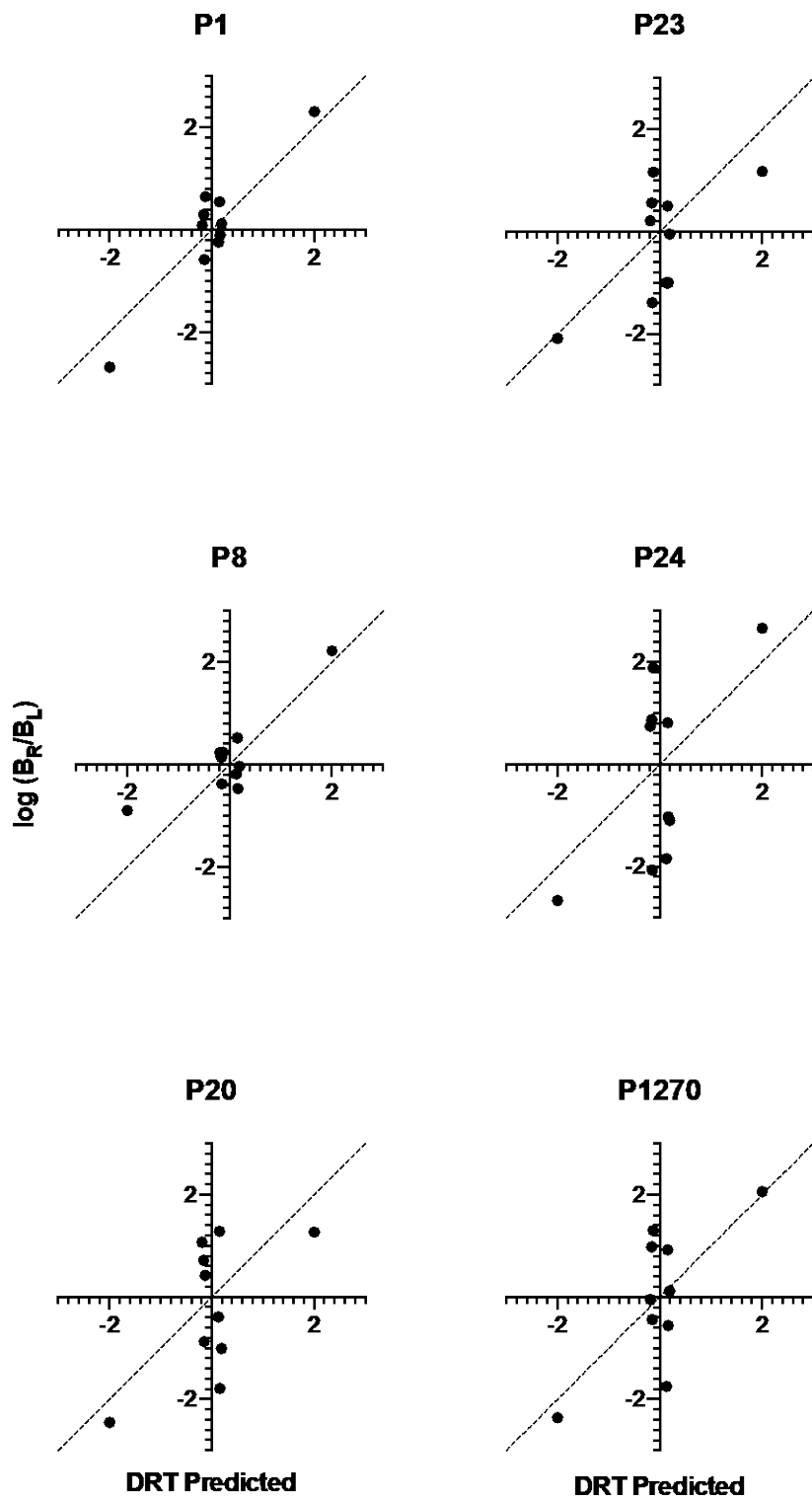


Figure 2-9.

DRT_H predictions (no free parameters) plotted against obtained choice ratios from Experiment 1.

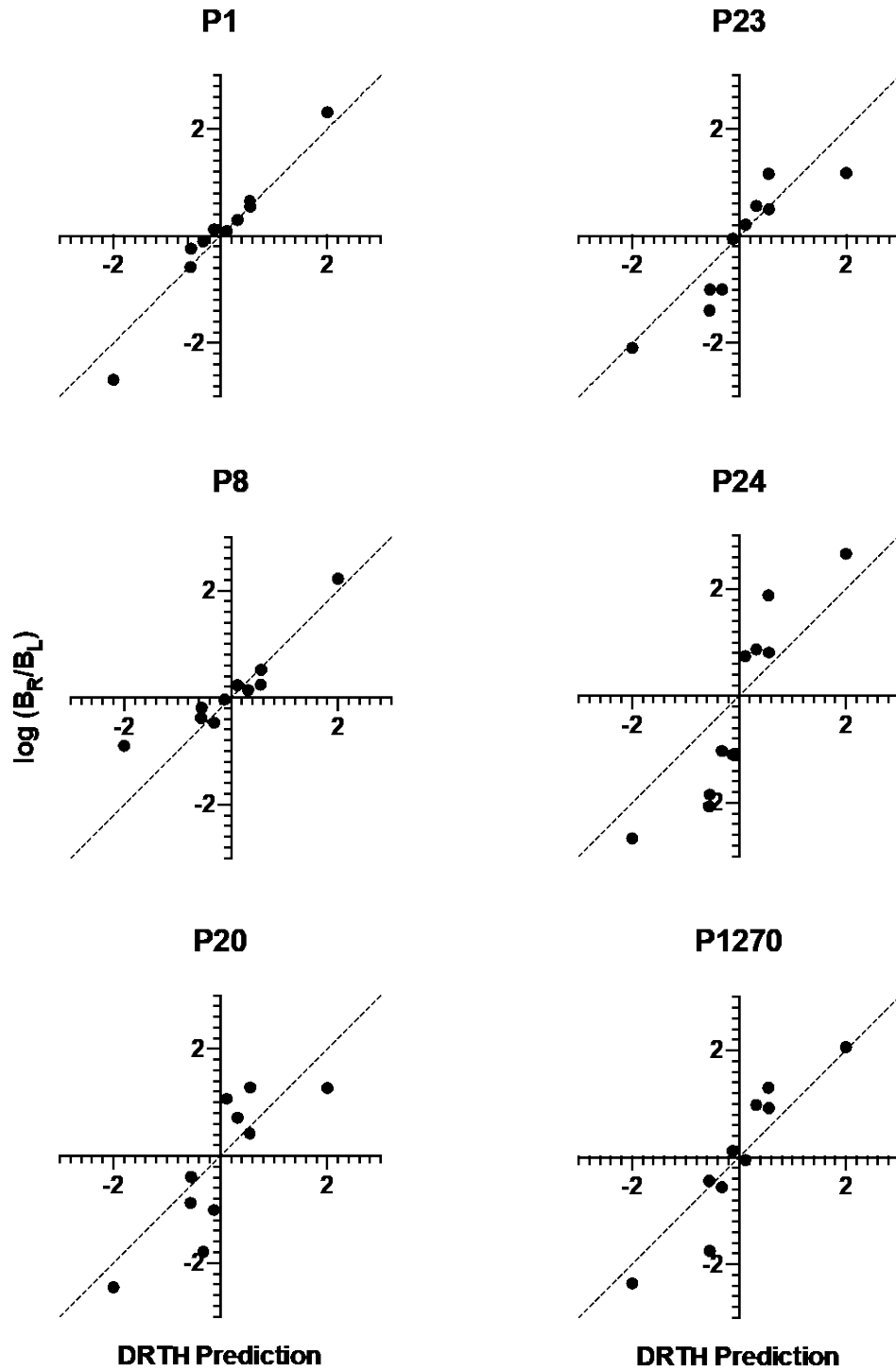


Figure 2-10.

DRT_H predictions (a_T parameter) plotted against obtained choice ratios from Experiment 1.

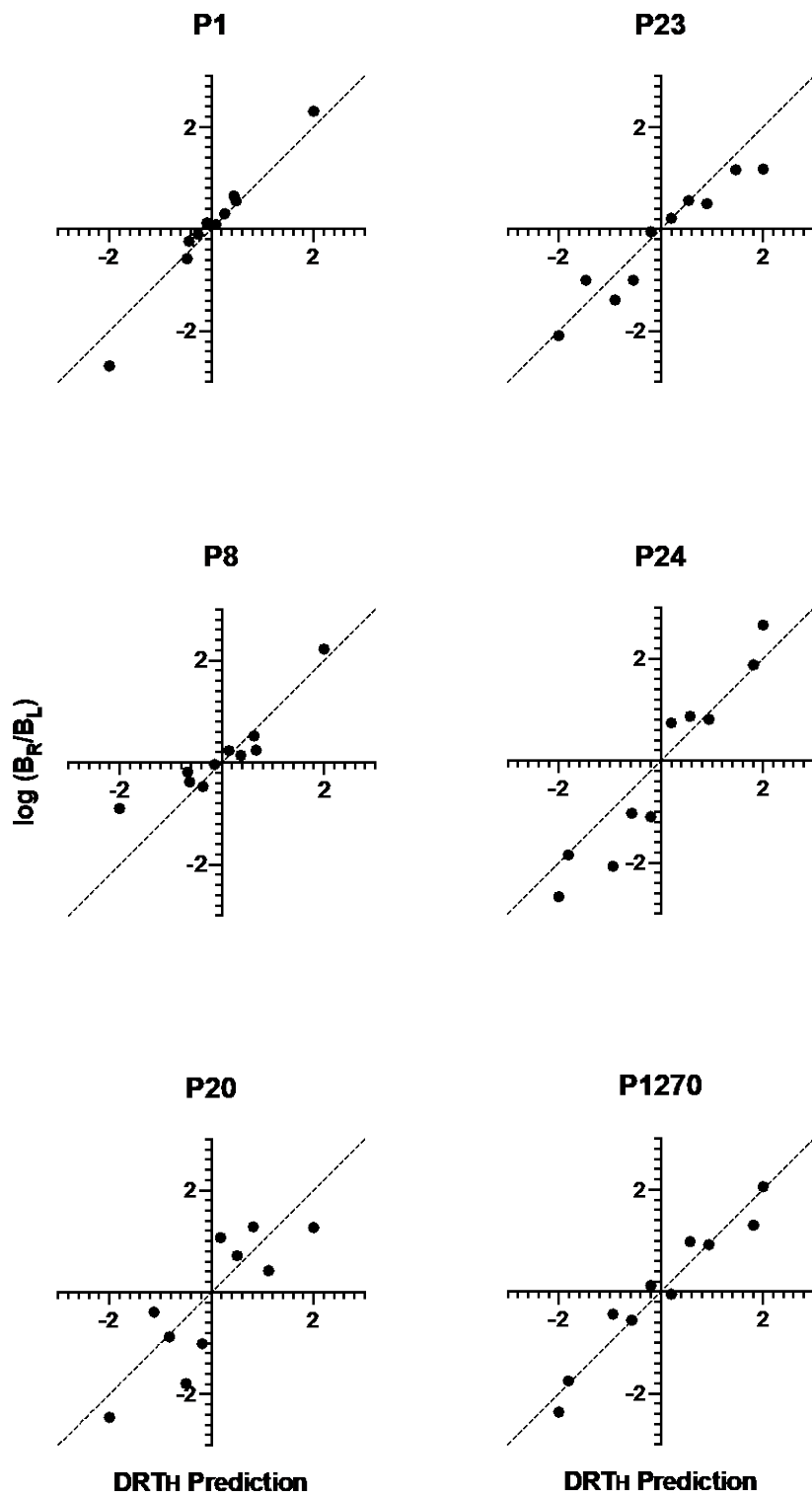
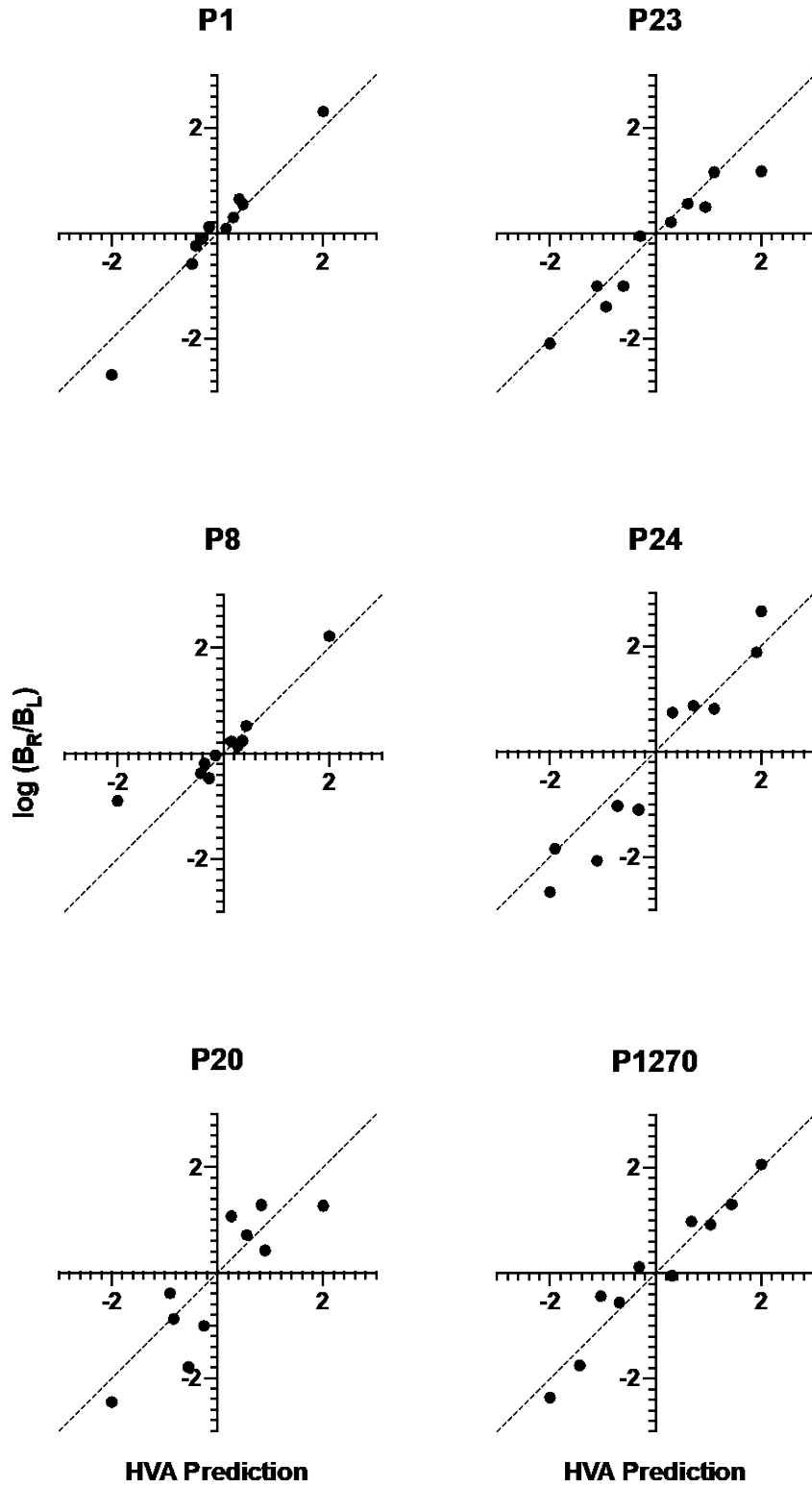


Figure 2-11.

HVA predictions (a_T parameter) plotted against obtained choice ratios from Experiment 1.



Deviations from these model predictions may be evaluated visually in Figures 2-12 (CCM), 2-13 (CDM – 2 free parameters), 2-14 (DRT– 1 free parameter), 2-15 (DRT_H – no free parameters), 2-16 (DRT_H – 1 free parameter), and 2-17 (HVA– 1 free parameter). In each panel, residuals (y-axis) are plotted against predictions of the best version of each model (x-axis), along with the polynomial functions and R^2 values for each pigeon. Systematic deviations from these model predictions were evaluated with a residual analysis, the results of which are shown in Table 2-5. According to McDowell et al. (2016), who fitted cubic polynomial functions to randomly generated residuals, the median R^2 value for a cubic polynomial function fitted to 10 random residuals (the number of data points in the figures) is .307. A one-tailed exact binomial test evaluated the hypothesis that obtained R^2 values were significantly greater than this expected value, an outcome that would indicate the model had non-random residuals. To achieve significance, all 6 pigeons' R^2 values had to exceed .307. As shown in Table 2-5, only CCM had significant non-random residuals.

Discussion

Experiment 1 replicated the findings reported by Fantino (1969) and Killeen (1968), thereby demonstrating that our lab could establish systematic control of concurrent-chains behavior in pigeons. Further, it demonstrated that this control could be achieved when using the Savastano and Fantino (1996) procedure in which the location of the richer terminal link is randomly assigned following every reinforcer. The ability to replicate existing findings using the Savastano and Fantino (1996) procedure is important because that procedure offers three distinct advantages. First, because chains are assigned to one-of-two locations, it alleviates the necessity to run multiple conditions with each

Figure 2-12.

CCM (no free parameters) residuals plotted against CCM model predictions. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

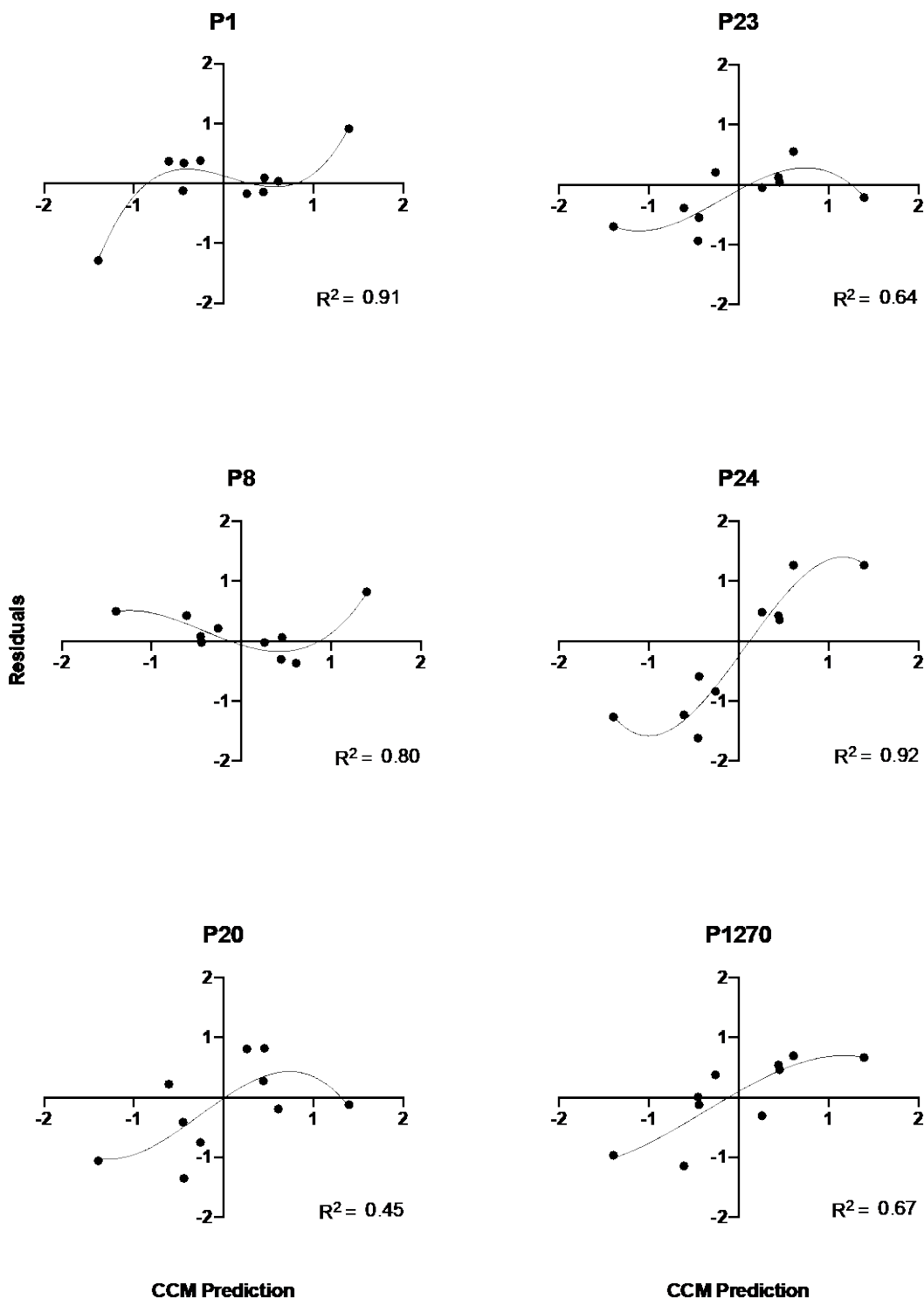


Figure 2-13.

CDM (σ & k parameters) residuals plotted against CDM model predictions. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

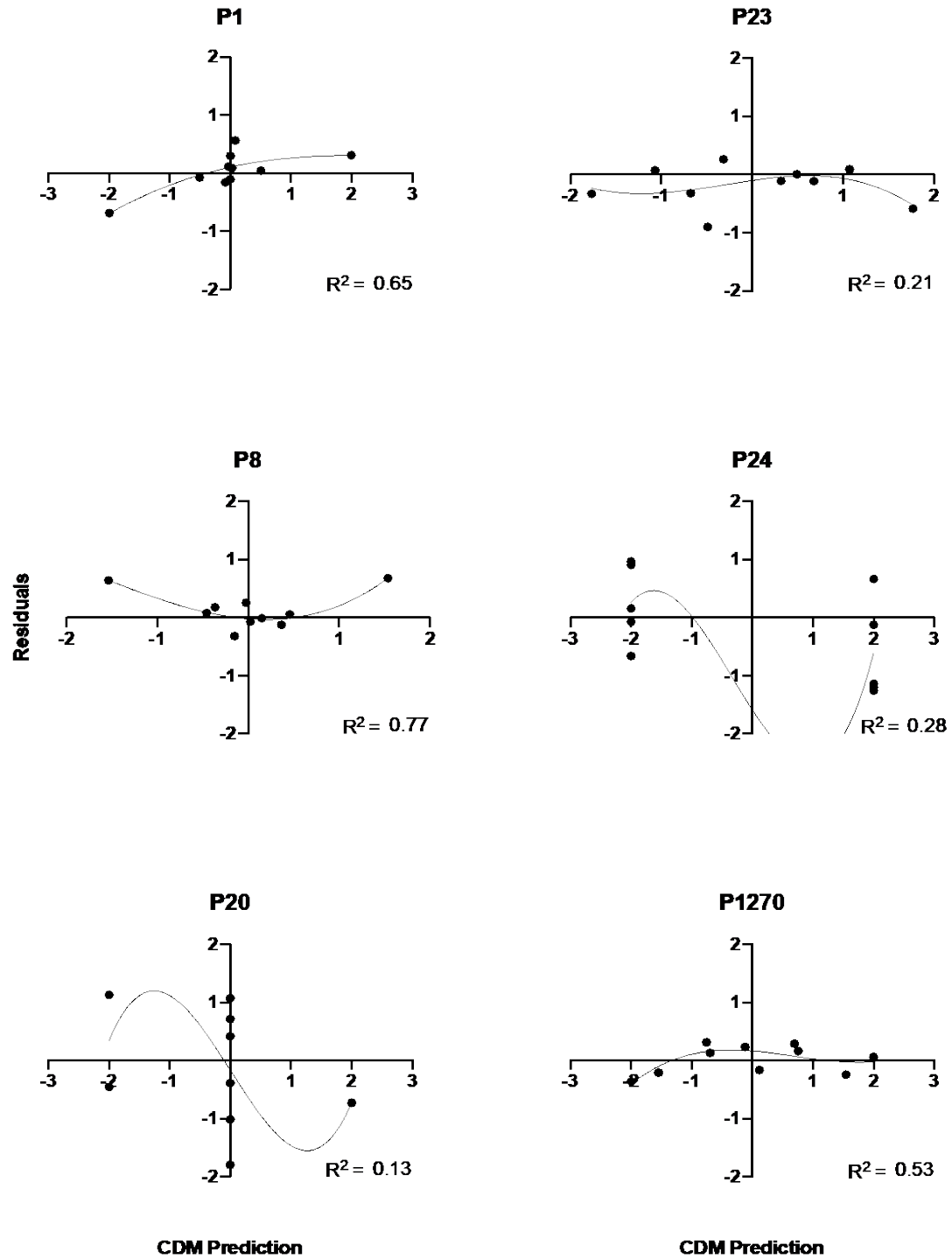


Figure 2-14.

DRT (a_t parameter) residuals plotted against DRT model predictions. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

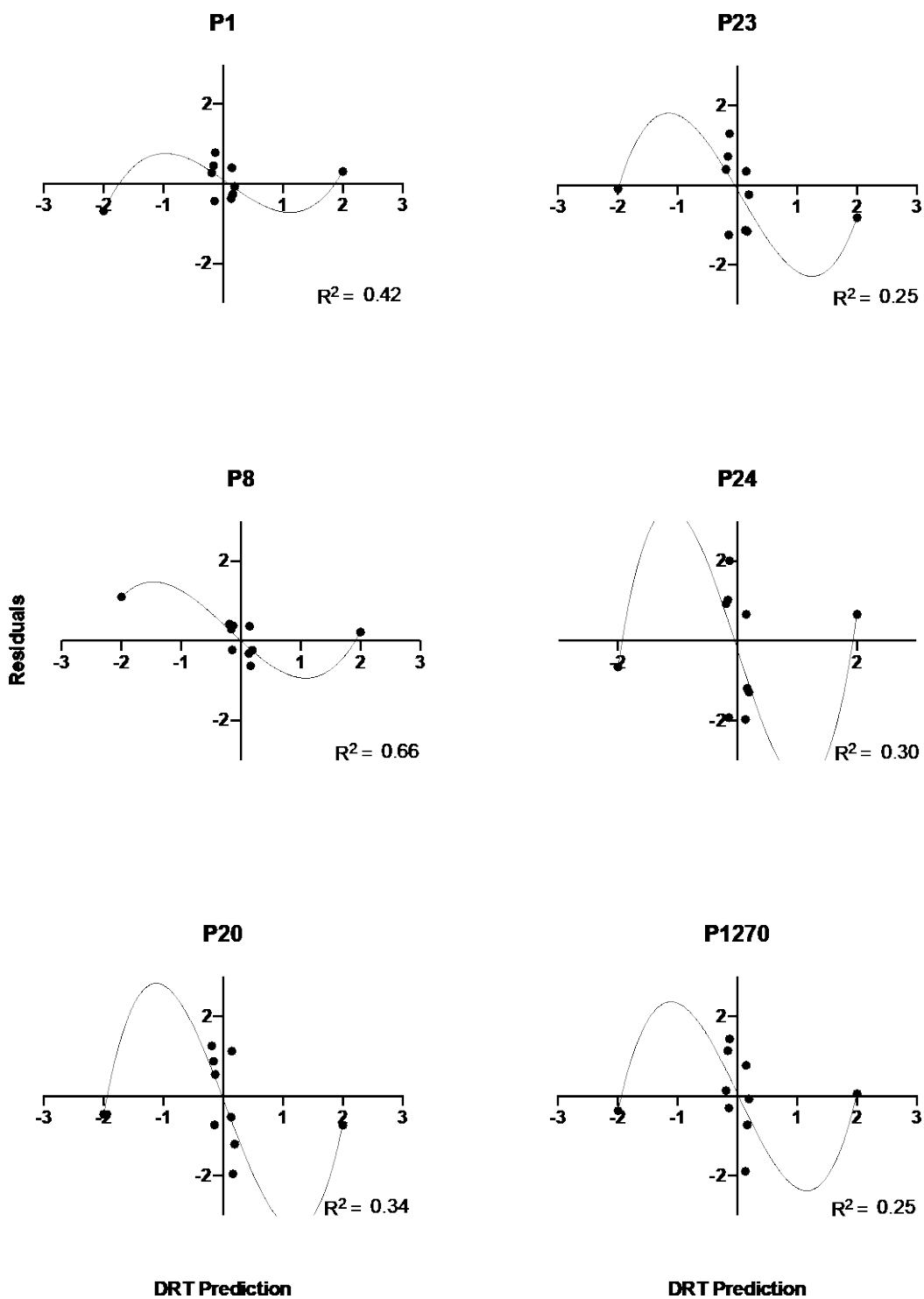


Figure 2-15.

DRT_H (no free parameter) residuals plotted against DRT_H model predictions. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

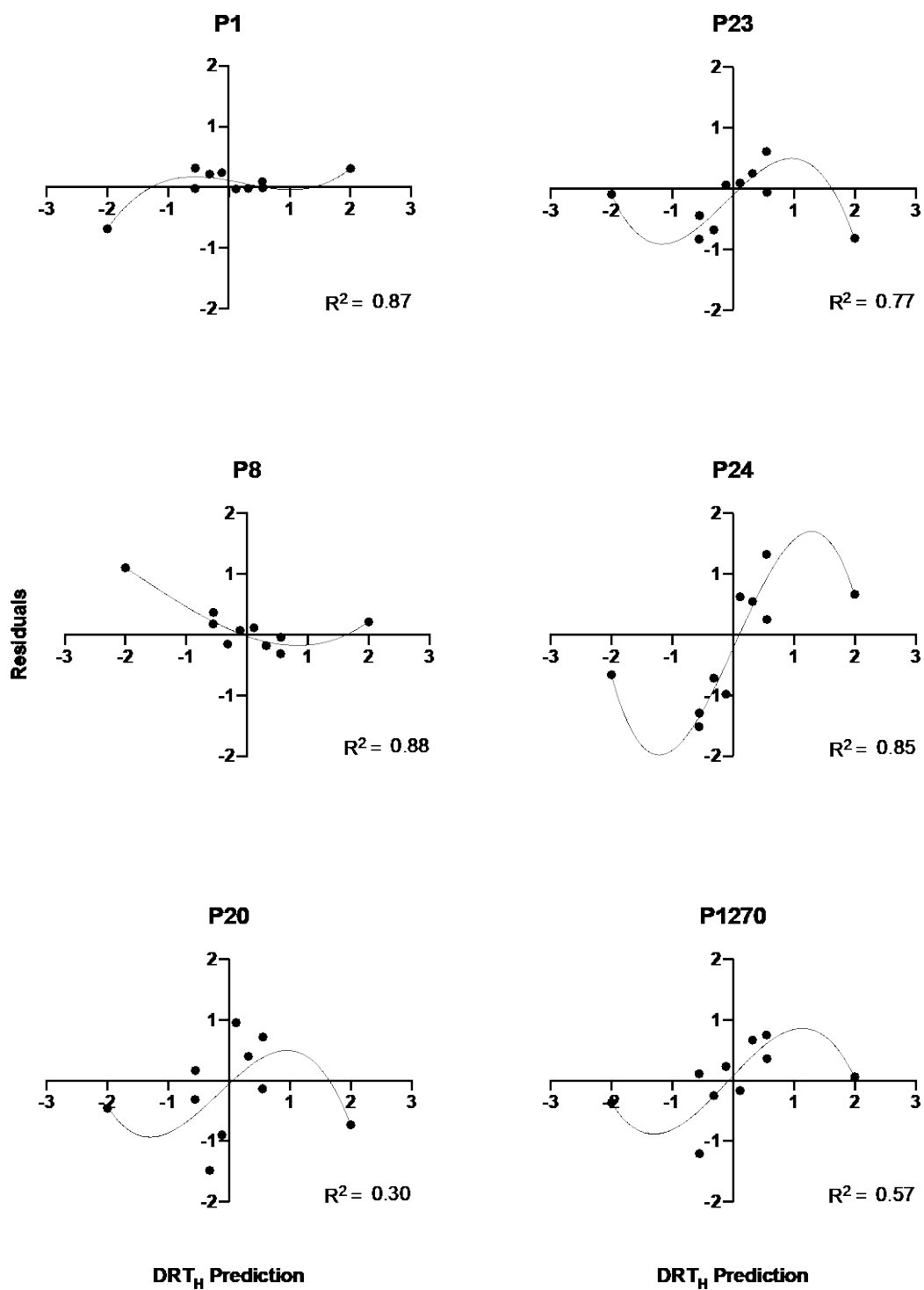


Figure 2-16.

DRT_H (a_T parameter) residuals plotted against DRT_H model predictions. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

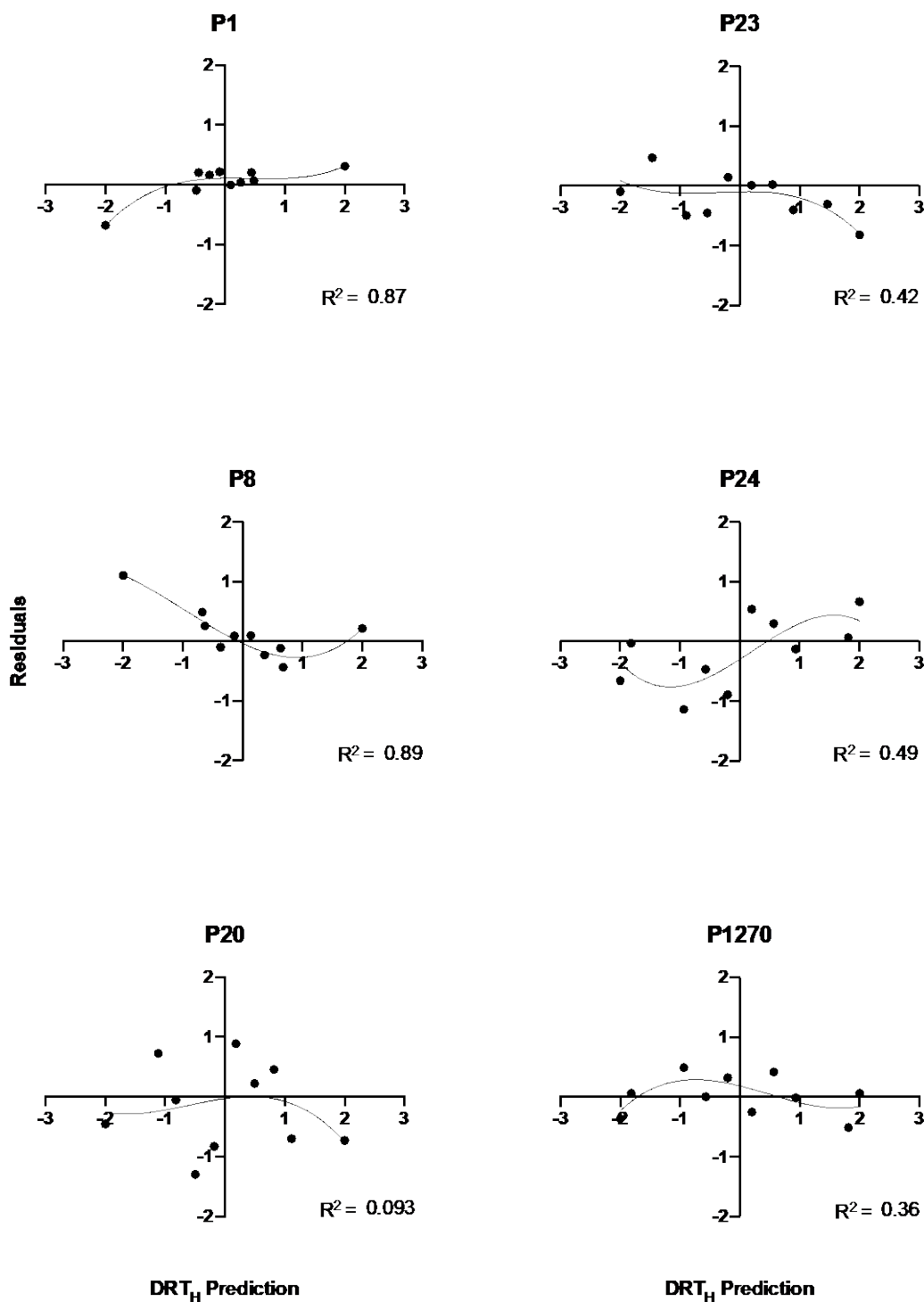


Figure 2-17.

HVA (a_T parameter) residuals plotted against HVA model predictions. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

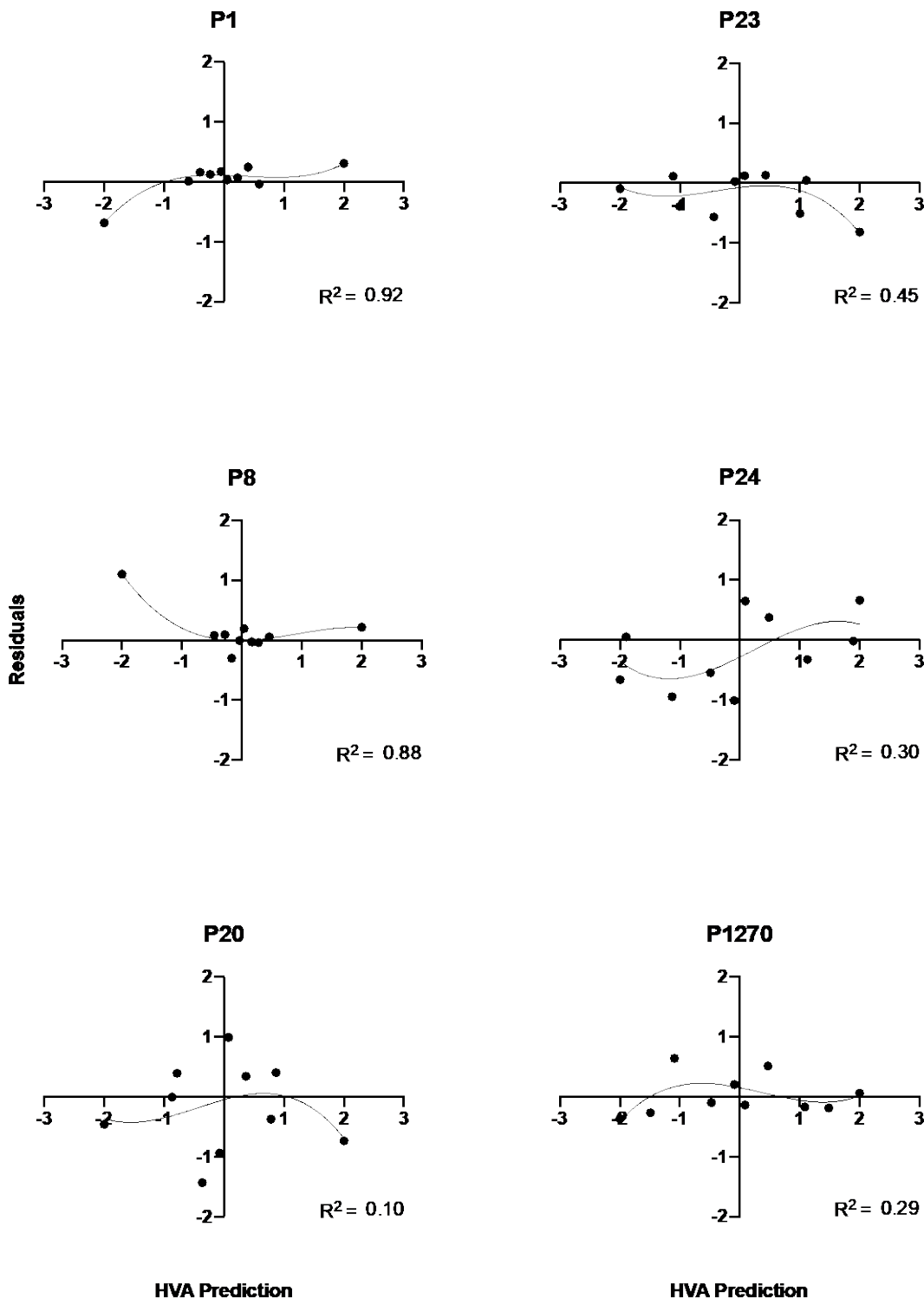


Table 2-5.

Results of the cubic polynomial test for residual trends. Column two displays the number of R^2 values produced by the polynomial test that were greater than the expected R^2 for 10 random residuals ($M = .307$).

Model	$R^2 > M$	p -value
CCM	6	$p = .016^*$
CDM	3	$p = .656$
DRT	4	$p = .344$
DRT _H - no	5	$p = .109$
DRT _H - a _T	5	$p = .109$
HVA	4	$p = .344$

*Note: $*p < .05$*

chain at each location. This shortens the amount of time needed to run multiple conditions. A second advantage of presenting chains at different locations in the operant chamber is that it generates two separate choice proportions, one when the chain is on the left side of the chamber and one when the chain is on the right side of the chamber. These choice proportions can be compared to determine if pigeons are systematically biased toward either the left or right side of the chamber. Finally, presenting stimuli at two different locations has also been shown to reduce the likelihood that position biases develop in the concurrent-chains procedure (Savanstano & Fantino, 1996)

Experiment 1 also evaluated the relative adequacy of DRT (Squires & Fantino, 1971), DRT_H, and three other models of concurrent-chains choice: CCM, CDM, and HVA. These models were compared primarily using AICc and BIC analyses. These analyses evaluated the ability of the five models to predict choice in the current

experiment, in which initial-link effects were evident and, in three conditions in which periodic and aperiodic terminal links were arranged. AICc and BIC outcomes indicated that DRT_H was better than DRT at accounting for pigeons' choices in the latter three conditions. These results support the preliminary analysis conducted in Chapter 1 (Figure 1-2) and are not surprising as DRT's inability to account for choice under these conditions has been acknowledged (Fantino, Preston, & Dunn, 1993). These results also support the findings of Killeen (1968) which indicated that the harmonic mean, and not the arithmetic mean, best describe how pigeons equate variable and fixed delays.

Comparing the remaining four models, HVA (Mazur, 2001) proved to be the best of these models; however, the fits provided by DRT_H (with one or fewer free parameters) could not be ruled out based on the differences in AICc or BIC values. By contrast, the fits provided by the best versions of CCM (Grace 1994; Grace 1996), and CDM (Christensen & Grace, 2010) had much higher (worse) AICc and BIC values, allowing me to rule them out as viable models of choice in Experiment 1.

Experiment 1 also evaluated the free parameter values of models with the best AICc and/or BIC outcomes. Analyzing the values of free parameters in these models is important because it allows us to evaluate how reasonable the values are, given what the specific free parameter corresponds to. For example, DRT returned a_t parameter values approximating zero when periodic and aperiodic terminal links were arranged. This means that pigeons were insensitive to the relative durations of the terminal links. This conclusion is unreasonable because research has repeatedly shown that pigeons are sensitive to relative terminal-link durations (Mazur, 2001; Christensen & Grace, 2010), as they were in this experiment. The obtained sensitivity parameter (a_T) values in HVA

(*median* = 1.13) and DRT_H (*median* = 0.77) were not significantly different from 1.0, a value reflecting perfect sensitivity to differences in times to food, following initial-link onset; therefore, these parameter values may be regarded as reasonable.

Experiment 1 included an analysis of the residuals; i.e., the differences between predicted and obtained initial-link choice for each of the five models (McDowell et al., 2016). Only the residuals produced by CCM (Grace 1994; Grace 1996) were nonrandomly distributed, with CCM systematically underpredicting preference for the richer terminal link. As discussed above, it is difficult to rule out the viability of CCM as a model of concurrent-chains choice with only the results from Experiment 1. Therefore, these findings, along with those from the other two other experiments in this document, will be considered in the General Discussion to determine which of the five models made the most accurate predictions across the three experiments. Future research and analyses of existing concurrent-chains research should continue to evaluate the residuals of concurrent-chains models in tandem with AICc and BIC analyses.

Finally, Experiment 1 allowed me to evaluate the accuracy of estimates of times (T_H , R_L , & R_R) needed by DRT_H to make predictions about choice in the concurrent-chains procedure. The method suggested by Mazur (personal communication, January 11, 2017) was reasonably accurate in predicting T_H (the harmonic mean time between successive foods) and R_L/R_R (the harmonic mean rate of reinforcement on the two chains). The T_H predictions tended to be smaller than the observed times in the conditions from Fantino (1969) and Killeen (1968), with average deviations ranging from 1.96 s in the FI 15 s TL condition from Killeen (1968) to 6.87 s in the VI 40 s IL condition from Fantino (1969). Although these deviations are small, they make it difficult

to evaluate the predictions made by DRT_H and HVA, as they are most often quite similar, as in Experiment 1. Thus, before these models are compared using the published concurrent-chains literature, continued evaluation of the accuracy of Mazur's method should be conducted and, if necessary, an alternative method should be proposed and evaluated in future research.

To summarize, the core findings of Experiment 1 were (1) DRT_H makes more accurate predictions than DRT when one periodic and one aperiodic terminal link are used in the concurrent-chains procedure; (2) when predicting harmonic means of intervals in the concurrent-chains procedure (intervals needed to make predictions with DRT_H), the method proposed by Mazur (personal communication, January 11, 2017) significantly under-predicted the duration of T_H values; (3) although DRT_H made more accurate predictions than DRT, CCM, and CDM, it made predictions that were comparable to, but not quite as good as HVA. These findings led me to conduct Experiment 2 which assessed the predictions of HVA and DRT_H under conditions where the two models made diverging predictions.

CHAPTER III

EXPERIMENT 2

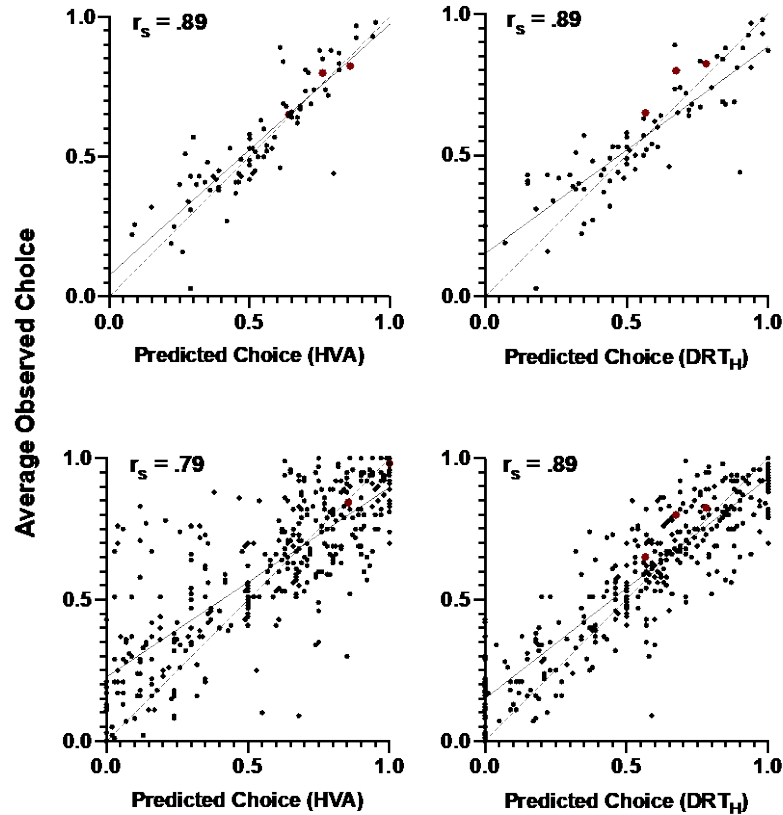
Introduction

DRT and DRT_H predict that the conditioned reinforcing effectiveness of a terminal-link stimulus is determined by the differences in two factors: time from onset of the initial-link stimulus to food (T) and time from the onset of the terminal-link stimulus to food (t_L and t_R). The two models differ only in how they average these time intervals. A third model of concurrent-chains choice, the Hyperbolic Value-Added (HVA) model (Mazur, 2001) (Equation 7), also indicates that the times from the initial- and terminal-link onset to food play important roles in determining choice in the concurrent-chains procedure.

Figure 3-1 displays the predictions of HVA (left column) and DRT_H (right column) for the same data sets analyzed in Chapter 1. These data sets include six published experiments in which terminal links contained one periodic and one aperiodic schedule (top two panels); data are averaged across subjects. They also include the predictions for 19 published experiments that arranged conditions in which both terminal links were periodic or both aperiodic (bottom two panels). The results from Experiment 1 (red dots) have also been included in their respective panels. For these predictions two points are worth noting. First, Mazur's method for estimating the time from initial-link onset to food delivery was used for both HVA (V_i) and DRT_H (T_H). Although the results of Experiment 1 suggest that Mazur's method may slightly underpredict these times, if errors occur, they should occur equally in both models' predictions. Furthermore, there currently exists no better alternative method for calculating these times and this method

Figure 3-1.

Averaged observed choice from 6 concurrent-chains studies with FI and VI terminal-link schedules of reinforcement (top two panels) and 19 concurrent-chains studies with two periodic or two aperiodic terminal-link schedules of reinforcement (bottom two panels) plotted against the predictions of HVA (left panels) and DRT_H (right panels). Red dots are the results from the Killeen (1968) and Fantino (1969) conditions in Experiment 1.



has been previously used in the most extensive review of concurrent-chains literature (Mazur, 2001). Second, the bias and sensitivity parameters in DRT_H and HVA were set to 1 (i.e., no bias and perfect sensitivities to rates of terminal-link entry (a_i), rates of food delivery on each chain (a_f), and terminal link delays (a_t)) and the k parameter within HVA was set equal to 0.2. Although these assumptions are likely incorrect, free parameters were omitted to facilitate comparison of HVA and DRT_H , all else being equal.

The model predictions in Figure 3-1 are quite similar. The sum of squared residuals for the experiments displayed in the top panels in Figure 3-1 were 0.84 and 1.1 for HVA and DRT_H , respectively. These values indicate that HVA made more accurate predictions when one periodic and one aperiodic terminal link schedule were programmed in concurrent-chains procedure. The sum of squared residuals for the experiments displayed in the bottom two panels in Figure 3-2 were 13.3 and 5.9 for HVA and DRT_H , respectively. These values indicate that DRT_H made more accurate predictions when two periodic or two aperiodic schedules were programmed in the concurrent-chains procedure.

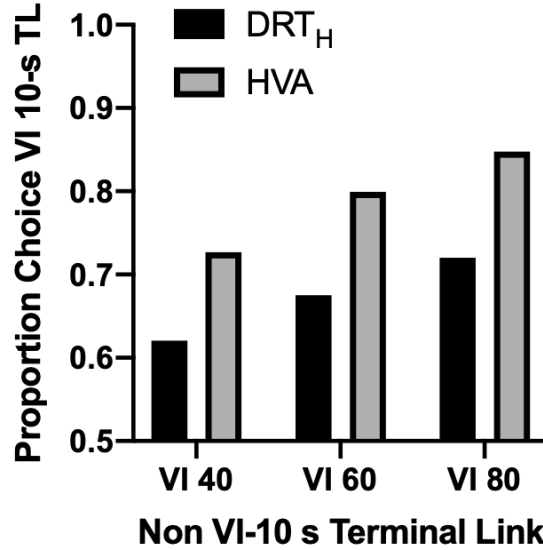
Differentiating the Models

In seeking to differentiate the predictions of these two models, several parameters of the initial- and terminal-links schedules were explored. One condition under which the two models make divergent predictions is when initial links are long (e.g., VI 300 s) and at least one of the terminal links is short (e.g., VI 10).

Figure 3-2 displays the predictions of DRT_H and HVA (no free parameters) for the concurrent-chains conditions shown in the inset table. Both models predict preference for the richer VI 10-s terminal link will increase as the duration of the lean terminal link increases, but DRT_H predicts weaker preferences than HVA. HVA's more extreme predictions are noteworthy because they exceed the average choice proportions obtained in previous concurrent-chains research using initial links ≥ 250 s (Davison, 1988; Fantino, 1969; Mazur, 2004). HVA makes these more extreme predictions because V_i (the discounted value of food; discounted because of the delay from initial-link onset to food delivery) is small when long initial links are used. When a small V_i is subtracted

Figure 3-2.

Predicted choice proportion for the VI 10-s terminal link made by DRT_H and HVA when the duration of the second terminal link is increased from VI 40 s to VI 80 s. All terminal links are preceded by a VI 300-s initial link.



Condition		Initial Link	Terminal Link
1	Left	VI 300	VI 10
	Right	VI 300	VI 40
2	Left	VI 300	VI 10
	Right	VI 300	VI 60
3	Left	VI 300	VI 10
	Right	VI 300	VI 80

from V_{tL} or V_{tR} , it has little effect on their value-addition, upon terminal-link entry. When V_{tL} and V_{tR} are unaffected by V_i , HVA predicts that conditioned-reinforcing value is determined by the relative value of the terminal-link stimuli:

$$\frac{B_L}{B_L + B_R} = b \left(\frac{a_t V_{tL}}{(a_t V_{tL}) + (a_t V_{tR})} \right)$$

Given that limited concurrent-chains research (3 conditions across 3 experiments) has been conducted with initial links ≥ 250 s, and because only one of the three conditions used VI terminal links (Fantino, 1969), Experiment 2 investigated pigeons' preference for VI terminal-links schedules with VI 300-s initial links.

Method

Subjects and Apparatus

A power analysis using obtained data from the VI 120 s initial-link conditions from Fantino and Davison (1983) ($d = 1.62$), with $\alpha = .05$ and power = .8, indicated that a sample size of six pigeons was needed for simple within-subject comparisons. Therefore, six experimentally experienced, unsexed pigeons were used. Housing, feeding, scheduling, experimental chambers and Institutional Animal Care and Use Committee protocol numbers were identical to those in Experiment 1.

Procedure

Table 3-1 shows the sequence of conditions that were completed by each pigeon. The procedure was identical to that used in Experiment 1 with three exceptions. First, the values of the initial- and terminal-link schedules differed. Second, the distributions of VI schedules that composed the initial-link schedules were created using the method outlined by Catania and Reynolds (1968). This distribution is geometric but has a larger lowest-value than Fleshler and Hoffman (1962) distributions. This larger lower-value increased the harmonic mean of the VI distribution and allowed the predictions of DRT_H and HVA to be evaluated with shorter session durations. For the same reason, the third procedure change was that sessions were terminated after 40 food deliveries.

Table 3-1.

Experimental conditions and condition order in Experiment 2. Pigeon ID numbers are provided for the six subjects.

IL Schedules	TL A Schedule	TL B Schedule	Condition Order					
			P2331	P1	P24	P8	P4740	P23
VI 300 s	VI 10 s	VI 40 s	1	2	3	1	3	2
VI 300 s	VI 10 s	VI 60 s	2	3	2	3	1	1
VI 300 s	VI 10 s	VI 80 s	3	1	1	2	2	3

Data Analysis

Data from the stable sessions of each condition were used for all analyses. An alpha level of .05 was used for all statistical analyses and all reported confidence intervals were at the 95% level. The effects of the manipulated terminal-link duration (TLB in Table 3-1) on choice in the stable sessions was assessed using a one-way ANOVA. The accuracy of Mazur's method for predicting harmonic-mean times and rates in the concurrent-chains procedure was evaluated using one-sample t-tests. The ability of DRT_H and HVA to account for choice when given no free parameters was assessed in two ways. First, differences between predicted and obtained choice proportions were evaluated using a one-sample t-test. Second, the sum of the squared residuals was calculated for both models, and compared using a within-subjects t-test. In all cases, when multiple within-family comparisons were conducted, Bonferroni adjusted p-values were used.

To assess the ability of the five quantitative models of concurrent-chains choice (Figure 2-1) to account for the results of Experiment 2, AICc (Akaike, 1998) and BIC (Schwarz, 1978) analyses, and residual analyses were conducted. These analyses were conducted identically to those described in Experiment 1 with one exception, instead of using a .2 value for k in HVA (Mazur, 2001), the k value was allowed to vary as a free parameter. This modification was made to give HVA all possible ways to account for choice in Experiment 2.

Results

Individual and group averaged sessions to stability and harmonic-mean times from the onset of the initial link to food delivery (T_H) averaged across the final nine

stable sessions in each condition can be seen in Table 3-2. The harmonic-mean times spent in the terminal links (t) are not shown because predicted and obtained times were nearly identical. Obtained T_H values are displayed graphically in Figure 3-3. The absolute differences between obtained and predicted T_H times ranged from 0.6 to 17.1 s. In 14 of the 18 cases, average obtained T_H times were smaller than the predicted times but one-sample t-tests, with a Bonferroni adjusted $p = .017$, revealed no significant differences between programmed and obtained T_H times in the VI 40 s TL ($t_{(5)} = 2.80, p = .038, CI = 76.41, 91.09$), VI 60 s TL ($t_{(5)} = 2.52, p = .053, CI = 84.62, 97.21$), or VI 80 s TL ($t_{(5)} = 0.22, p = .836, CI = 90.98, 104.31$) conditions.

Individual and average obtained harmonic rates of reinforcement for each choice alternative in each condition can be seen in Table 3-2. R_R is the harmonic rate on the chain with the VI 10-s terminal link, while R_L is the harmonic rate on the chain with the lean (i.e., VI 40, VI 60, or VI 80) terminal link. Figure 3-4 displays the obtained harmonic rate of reinforcement for each pigeon in each condition and the programmed rates in those conditions. With a Bonferroni adjusted $p = .008$, one-sample t-tests indicated that the obtained rates on the R_R chains (left-most bars within each panel) were not significantly different than predicted rates in the VI 40 s TL ($t_{(5)} = 0.93, p = .93, CI = 0.012, 0.015$), VI 60 s TL ($t_{(5)} = 0.28, p = .794, CI = 0.012, 0.014$) or the VI 80 s TL ($t_{(5)} = 1.04, p = .347, CI = 0.012, 0.014$) conditions. With a Bonferroni adjusted $p = .008$, one-sample t-tests indicated that the obtained rates on the R_L chains (right-most bars within each panel) were significantly larger than predicted rates in the VI 40 s TL ($t_{(5)} = 4.75, p = .005, CI = 0.010, 0.011$), but were not significantly different from the predicted

Table 3-2.

Sessions until stability, obtained T_H , R_R , R_L values, and choice proportions from Experiment 2. Values in brackets denote standard deviations around the group averages.

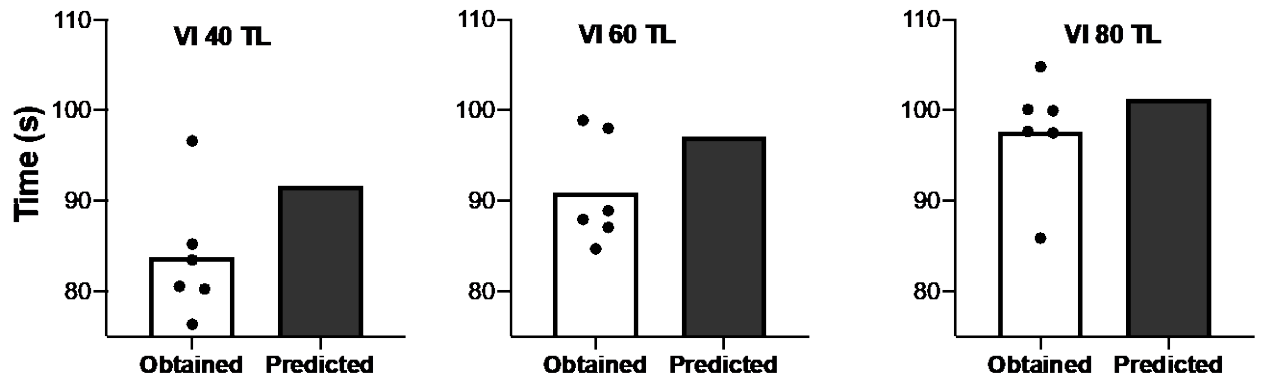
Pigeon	Condition	Sessions Until Stability	Obtained Harmonic T_H (sec)	Obtained Harmonic R_R	Obtained Harmonic R_L	Proportion Choice VI 10-s Terminal Link
P1	VI 40 TL	28	80.26	0.015	0.009	0.62
	VI 60 TL	38	98.88	0.012	0.008	0.68
	VI 80 TL	22	104.80	0.011	0.007	0.67
P8	VI 40 TL	28	85.23	0.013	0.010	0.62
	VI 60 TL	19	88.89	0.013	0.009	0.54
	VI 80 TL	26	99.94	0.012	0.007	0.59
P23	VI 40 TL	25	76.37	0.014	0.011	0.66
	VI 60 TL	16	87.06	0.013	0.009	0.66
	VI 80 TL	20	85.86	0.014	0.008	0.56
P24	VI 40 TL	39	83.46	0.013	0.010	0.72
	VI 60 TL	28	84.71	0.015	0.008	0.77
	VI 80 TL	24	97.50	0.013	0.007	0.73
P2331	VI 40 TL	31	96.62	0.010	0.010	0.74
	VI 60 TL	24	87.93	0.013	0.008	0.53
	VI 80 TL	31	97.66	0.013	0.007	0.72
P4740	VI 40 TL	21	80.57	0.014	0.010	0.57
	VI 60 TL	39	98.00	0.012	0.008	0.76
	VI 80 TL	44	100.11	0.013	0.007	0.65
Group Averages	VI 40 TL	29 [6.09]	83.75 [7.00]	0.013 [0.001]	0.010 [0.001]	0.66 [0.06]
	VI 60 TL	27 [9.59]	90.91 [6.00]	0.013 [0.001]	0.008 [0.001]	0.66 [0.11]
	VI 80 TL	28 [8.77]	97.65 [6.35]	0.013 [0.001]	0.007 [0.001]	0.65 [0.07]

rates in the VI 60 s TL ($t_{(5)} = 2.39$, $p = .062$, CI = 0.008, 0.009) or VI 80 s TL ($t_{(5)} = 3.10$, $p = .026$, CI = 0.007, 0.008) conditions.

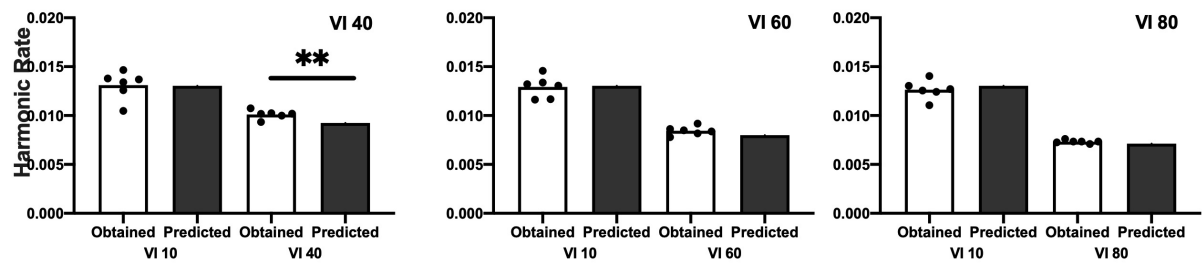
Individual and averaged obtained choice proportions for the VI 10-s terminal link in each condition are shown in Figure 3-5 (see also Table 3-2). The figure also shows the parameter-free predictions of HVA (red) and DRT_H (green). A one-way repeated-measures ANOVA indicated that choice did not significantly change across the three conditions ($F_{(2,5)} = 0.003$, $p = .98$). To determine if obtained choice proportions differed

Figure 3-3.

Obtained and predicted harmonic mean times from initial-link onset until food (T_H). Black dots represent individual-subject data

**Figure 3-4.**

Obtained and programmed harmonic rates of reinforcement on the two choice alternatives in Experiment 2. Abscissa information corresponds to the terminal links. Black dots represent individual subject data.



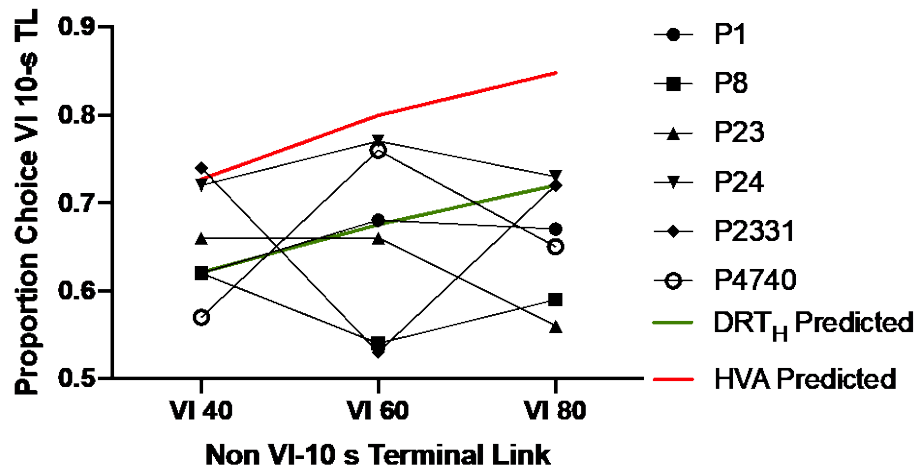
Note: ** $p < .01$

from the predictions made by DRT_H and HVA, six one-sample t-tests were conducted.

With a Bonferroni adjusted $p = .008$, post-hoc comparisons indicated the obtained choice proportions were less than those predicted by HVA in the VI 80 s condition ($t_{(5)} = 6.815$, $p = .001$, CI = 0.58, 0.72), but the difference between obtained and HVA predicted choice was not significant in the VI 40 s ($t_{(5)} = 2.656$, $p = .045$, CI = 0.59, 0.72) or VI 60 s ($t_{(5)} =$

Figure 3-5.

Individual obtained choice proportions and model predictions from Experiment 2. The green line represents the predictions of DRT_H , while the red line represents the predictions of HVA.



3.342, $p = .021$, CI = 0.55, 0.77) conditions. None of the choice proportions predicted by DRT_H were significantly different than the obtained choice proportions in the VI 40 s ($t_{(5)} = 1.35$, $p = .235$, CI = 0.59, 0.72), VI 60 s ($t_{(5)} = 0.448$, $p = .673$, CI = 0.55, 0.77) or VI 80 s ($t_{(5)} = 2.335$, $p = .067$, CI = 0.58, 0.73) conditions. Squared residuals between observed and predicted choice proportions were significantly larger for HVA (sum = 0.48) than for DRT_H (sum = 0.14); $t_{(17)} = 3.219$, $p = .005$. Thus, the parameter-free version of DRT_H made more accurate predictions than the parameter-free version of HVA when long initial-link are used in the concurrent-chains procedure.

Tables 3-3 and 3-4 display AICc or BIC scores, respectively, when the five models were allowed 0-3 free parameters. Asterisks identify the version of the model providing the best (lowest) score, and the free parameter allowed to vary in that model is identified in parentheses. The AICc and BIC scores for all possible free parameters,

Table 3-3.

Best AICc values for all quantitative models analyzed in Experiment 2. Lower (more negative) scores represent better fitting and more parsimonious models. The $\Delta AICc$ column is the AICc difference between each model and the best performing model (DRT_H), while the Probability column displays the probability that the overall best performing model (DRT_H) is better than the best version of each model.

Model	0 Free Parameters	1 Free Parameter	2 Free Parameters	3 Free Parameters	$\Delta AICc$	Probability
CCM	-115.98*	-111.29 (a_t)	-92.49 (b / a_t)	-48.81 ($b / a_t / k$)	-5.78	0.95
CDM	-	-104.18* (σ)	-94.39 (σ / k)	-56.02 ($\sigma / b / k$)	-17.58	1.00
DRT	-119.37*	-114.83 (a_f)	-98.30 (b / a_f)	-40.49 ($b / a_t / a_f$)	-2.39	0.77
DRT_H	-121.76*	-119.10 (a_f)	-105.80 (b / a_T)	-49.05 ($b / a_T / a_f$)	-	-
HVA	-75.92	-118.00* (k)	-103.61 (b / k)	-51.57 ($b / a_T / k$)	-3.76	0.87

Table 3-4.

Best BIC values for all quantitative models analyzed in Experiment 2. Lower (more negative) scores represent better fitting and more parsimonious models. The $\Delta BICc$ column is the BICc difference between each model and the best performing model (DRT_H), while the Probability column displays the probability that the overall best performing model (DRT_H) is better than the best version of each model.

Model	0 Free Parameters	1 Free Parameter	2 Free Parameters	3 Free Parameters	ΔBIC	Probability
CCM	-115.98*	-107.79(a_t)	-97.49 (b / a_t)	-92.30 ($b / a_t / k$)	-5.78	0.95
CDM	-	-100.68* (σ)	-99.39 (σ / k)	-99.52 ($b / \sigma / k$)	-21.08	1.00
DRT	-119.37*	-111.33 (a_f)	103.30 (b / a_f)	-83.98 ($b / a_t / a_f$)	-2.39	0.77
DRT_H	-121.76*	-115.60 (a_f)	-110.80 (b / a_f)	-92.55 ($b / a_T / a_f$)	-	-
HVA	-75.92	-114.50* (k)	-108.60 (b / k)	-95.07 ($b / a_T / k$)	-7.26	0.97

parameter values, and percent variance accounted for by each model are provided in Appendix B. Because the initial-link durations and distributions were identical in each condition of Experiment 2, the a_t parameter (sensitivity to differences in the initial-link schedules) parameter was not fitted to any of the models.

For all models except CDM and HVA, AICc and BIC analysis indicated that no free parameters should be included in the models. AICc and BIC analyses indicated that CDM made optimal predictions when σ was included as a free parameter and that HVA made optimal predictions when k was included as a free parameter. Tables 3-3 and 3-4

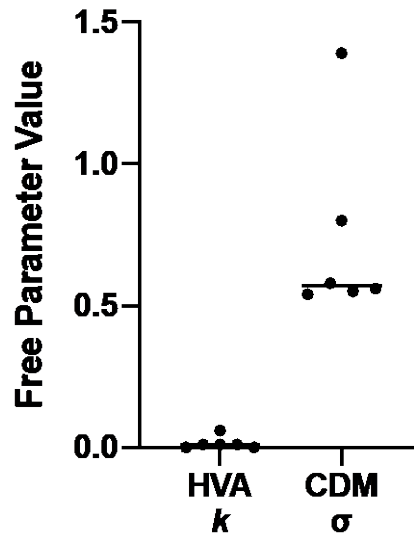
also display the difference, in AICc and BIC units, between the no-free parameter version of DRT_H and the AICc- or BIC-determined best version of the remaining models along with the probability that the overall best performing model is better than the best performing version of each model. It is generally accepted that a $\Delta AICc$ or ΔBIC of 10 or more provides definitive evidence for the superiority of one model over another (Burnham & Anderson, 2002). By this metric, DRT_H was convincingly better than CDM, but not any of the other models.

Figure 3-6 displays the free parameter values for the two models that required these parameters - HVA and CDM. The k parameter in HVA is a multiplicative discounting-rate parameter. Derived k -values ranged from 0.00 to 0.06 ($Mdn = .01$), which is lower than the constant value used for this parameter by Mazur (2001); i.e., $k = 0.2$. The median value of σ (0.57) in CDM was larger than the values from Experiment 1 ($Mdn = .08$) and those reported by Christensen and Grace (2010) ($Mdn = .24$). Higher σ values indicated a larger standard deviation around the criterion delay (C) and this suggests pigeons were less accurate at discriminating if the delays encountered in each terminal link were smaller than the criterion delay.

Figures 3-7 through 3-11 plot obtained vs. predicted log choice ratios for the best versions of the five models. The dotted line represents perfect correspondence between obtained choice and model predictions. Percent variance accounted for by the models ranged from 56% (CCM) to 67% (DRT_H and HVA). Because the low k parameter in HVA was unusual for the literature, I also fit the data using HVA with k held constant at 0.2. When no free parameters were included in HVA, its median percent variance

Figure 3-6.

Median free parameter values for HVA and CDM that produced the highest AICc and BIC values. Black dots represent individual subject data.



accounted for fell to -0.09; negative variance accounted for indicates that the mean of the data provides a better prediction of choice than HVA.

The residuals (y-axis) plotted against each model's predictions (x-axis), along with the polynomial functions and R^2 values, for each pigeon can be seen in Figures 3-12 (CCM), 3-13 (CDM), 3-14 (DRT), 3-15 (DRT_H), and 3-16 (HVA). Systematic deviations from the model predictions were evaluated with a residual analysis, the results of which are shown in Table 3-5. According to McDowell et al. (2016), who fitted cubic polynomial functions to randomly generated residuals, the expected median R^2 value for a cubic polynomial function fitted to six data points is .630. The number of R^2 values greater than this median can be seen in Table 3-5. One-tailed exact binomial tests revealed that none of the models had nonrandom residuals.

Figure 3-7.

CCM predictions (no free parameters) plotted against obtained choice ratios from Experiment 2.

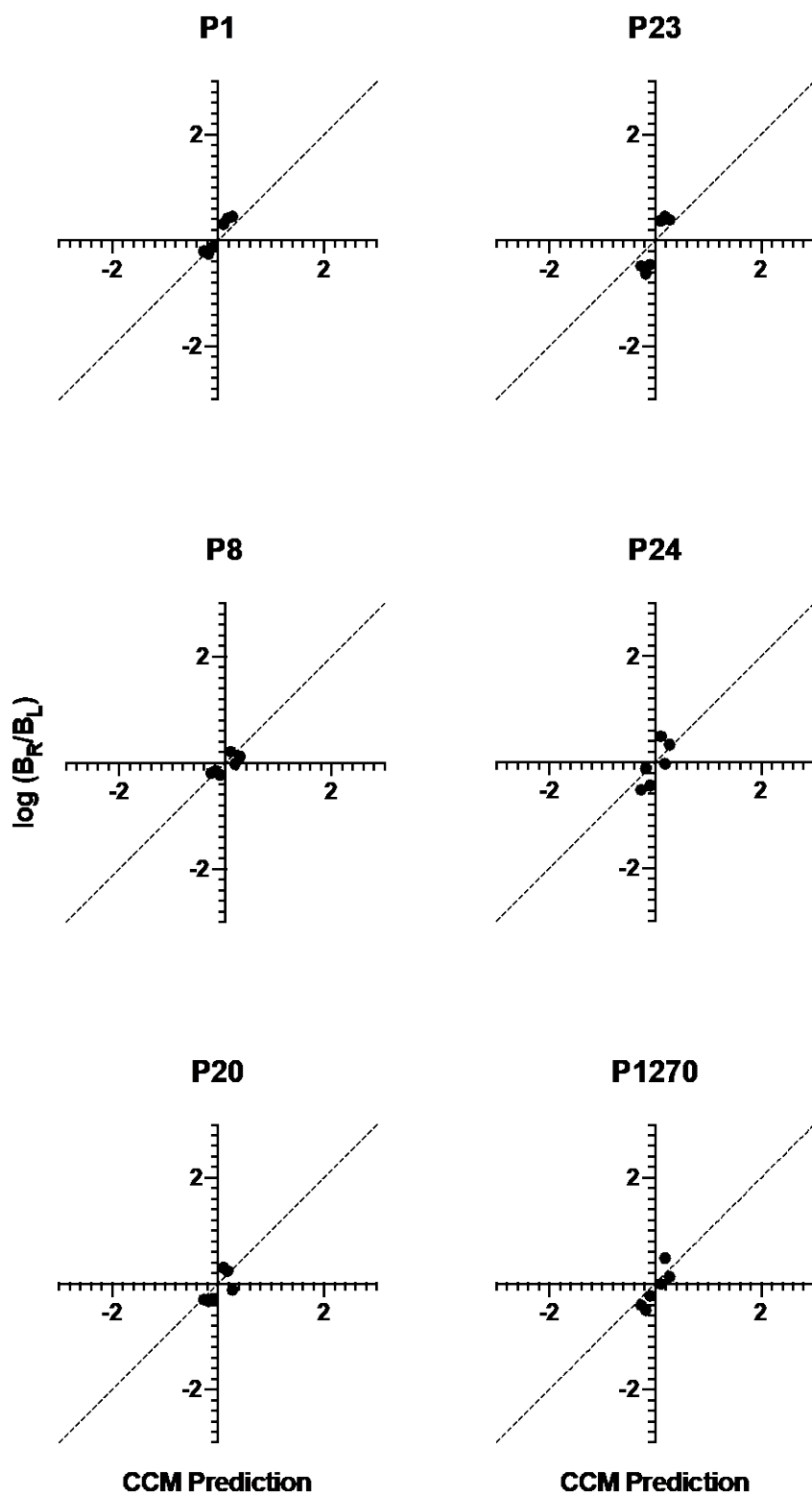


Figure 3-8.

CDM predictions (σ parameter) plotted against obtained choice ratios from Experiment 2.

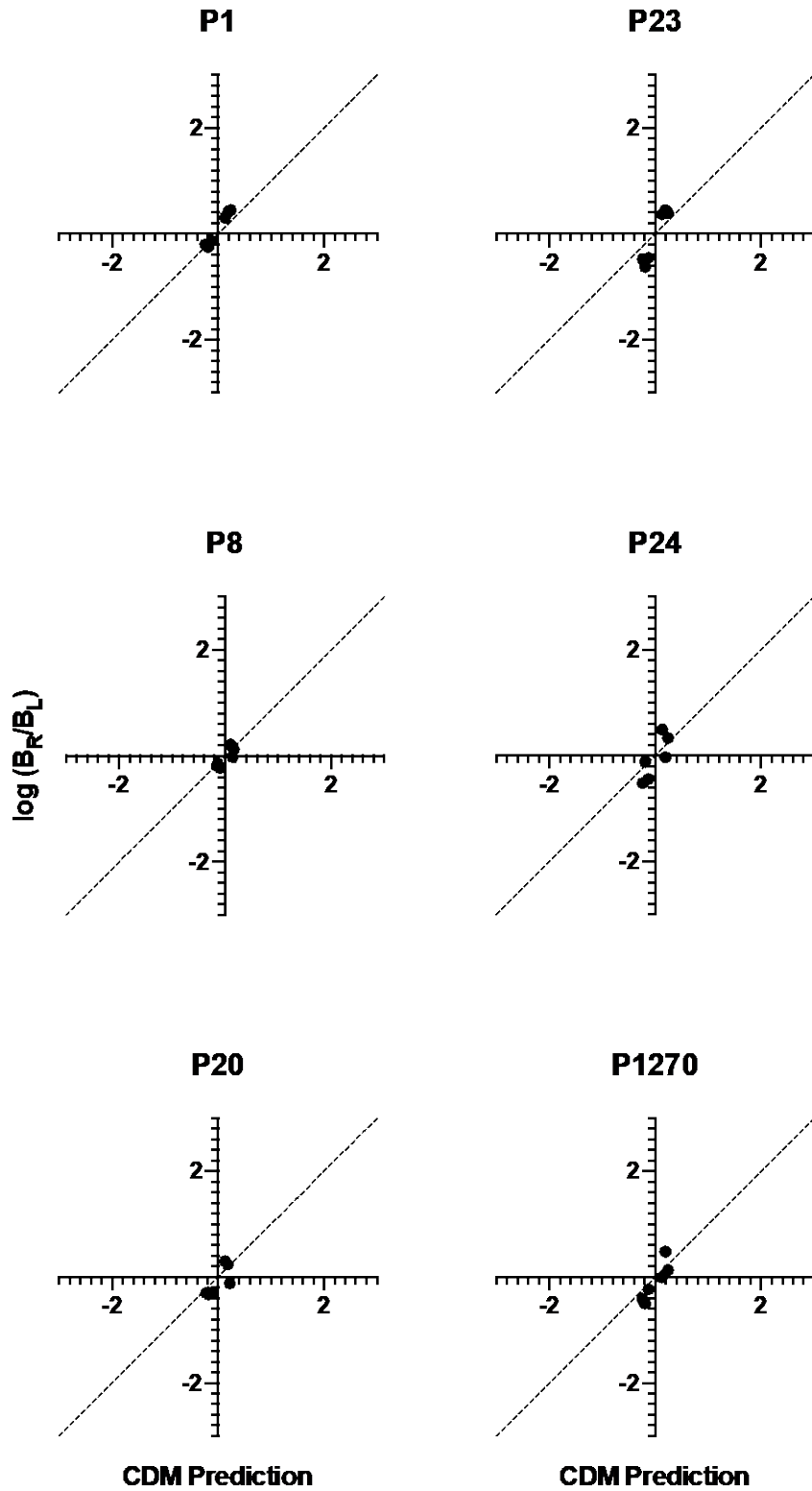


Figure 3-9.

DRT predictions (no free parameters) plotted against obtained choice ratios from Experiment 2.

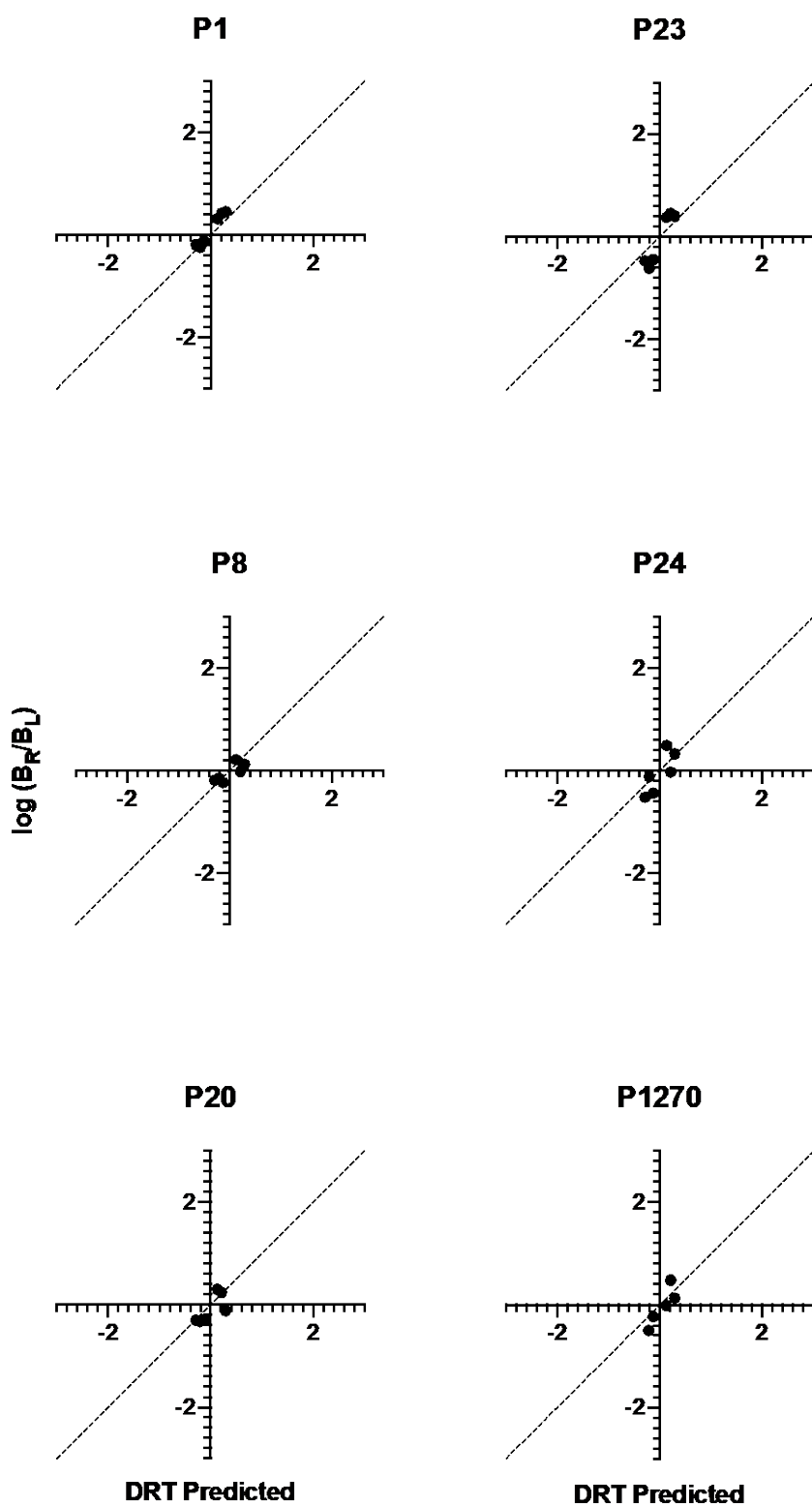


Figure 3-10.

DRT_H predictions (no free parameters) plotted against obtained choice ratios from Experiment 1

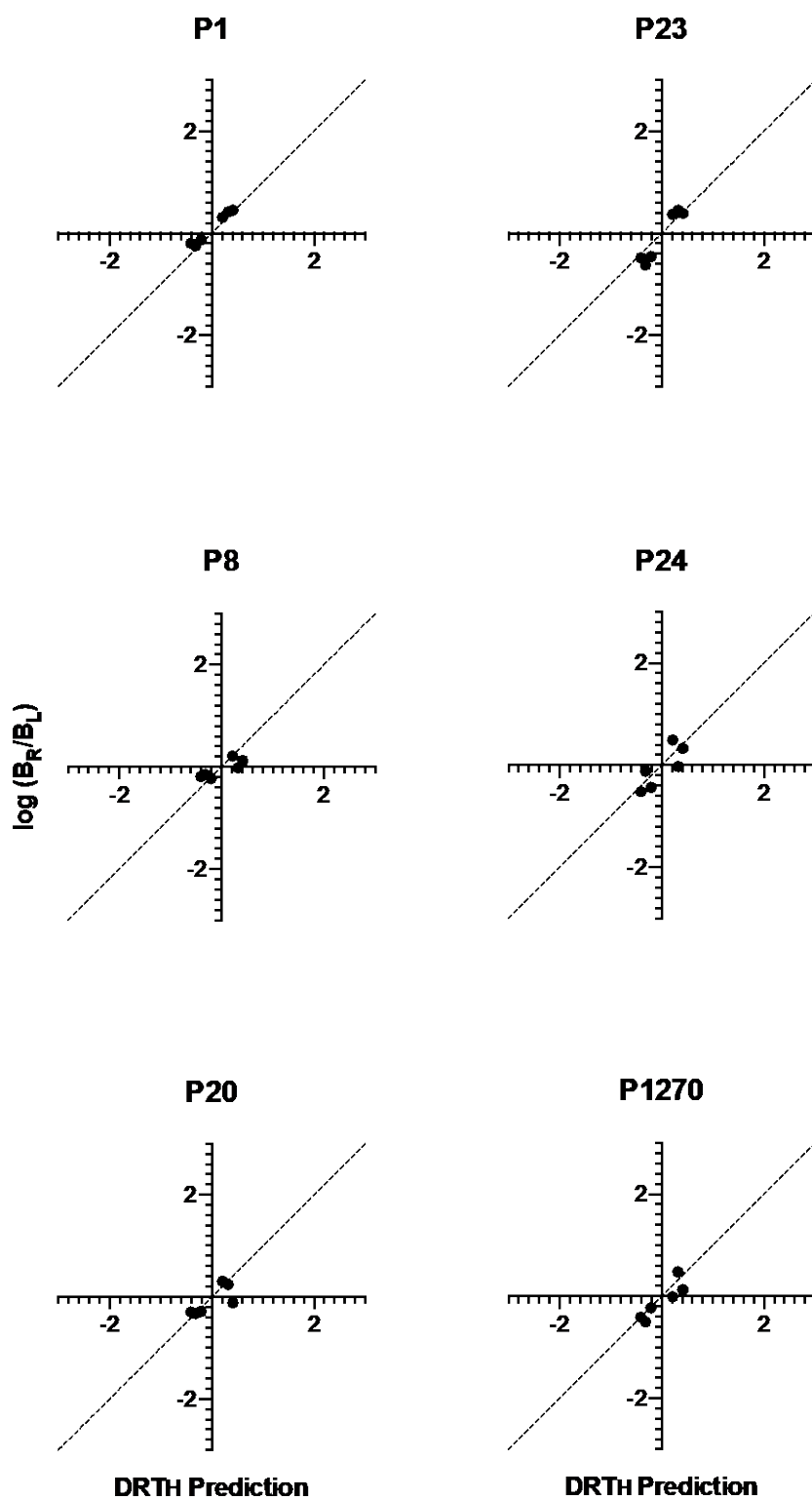


Figure 3-11.

HVA predictions (k parameter) plotted against obtained choice ratios from Experiment 2

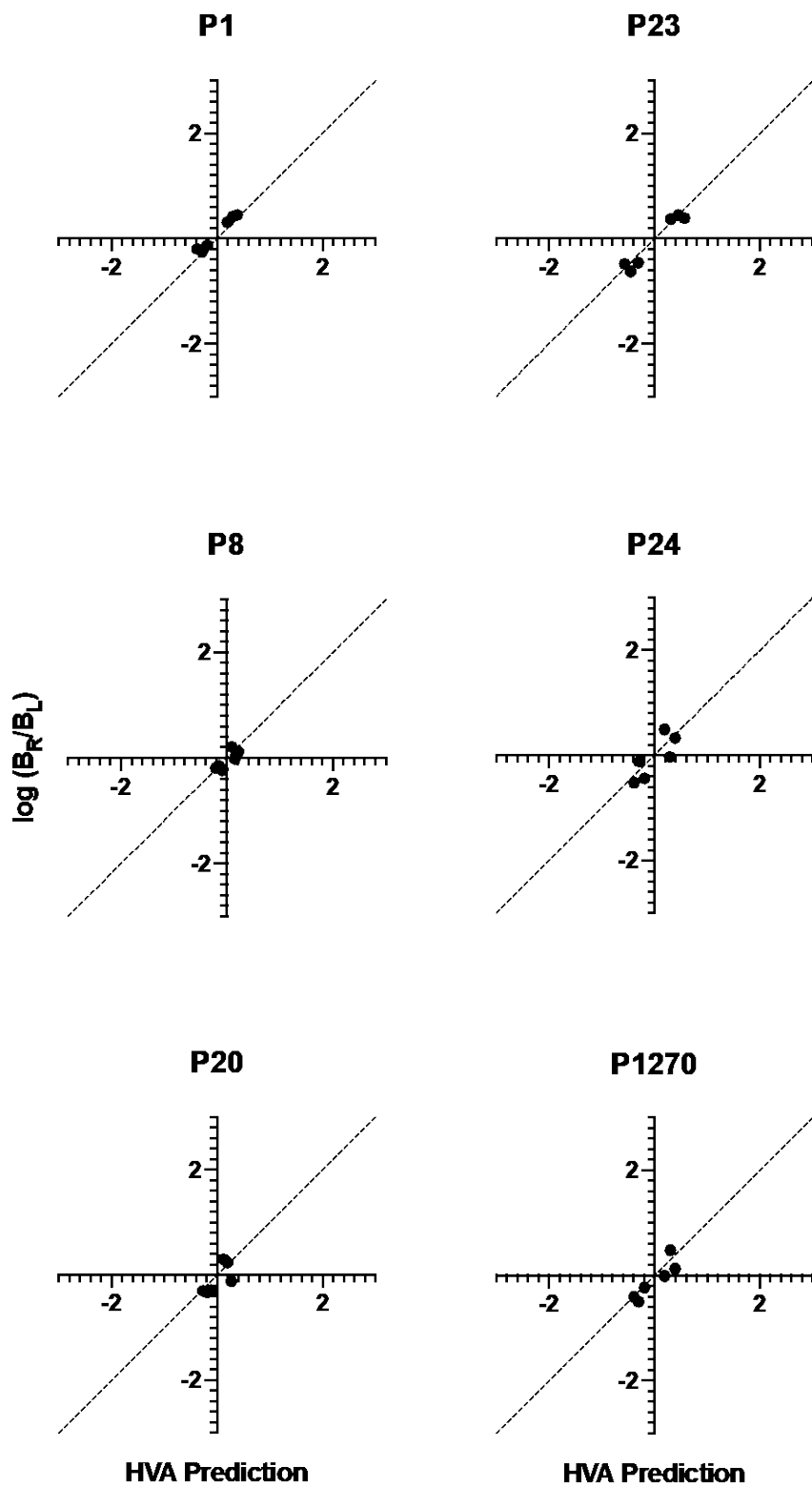


Figure 3-12.

CCM (no free parameters) residuals plotted against CCM model predictions for Experiment 2. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

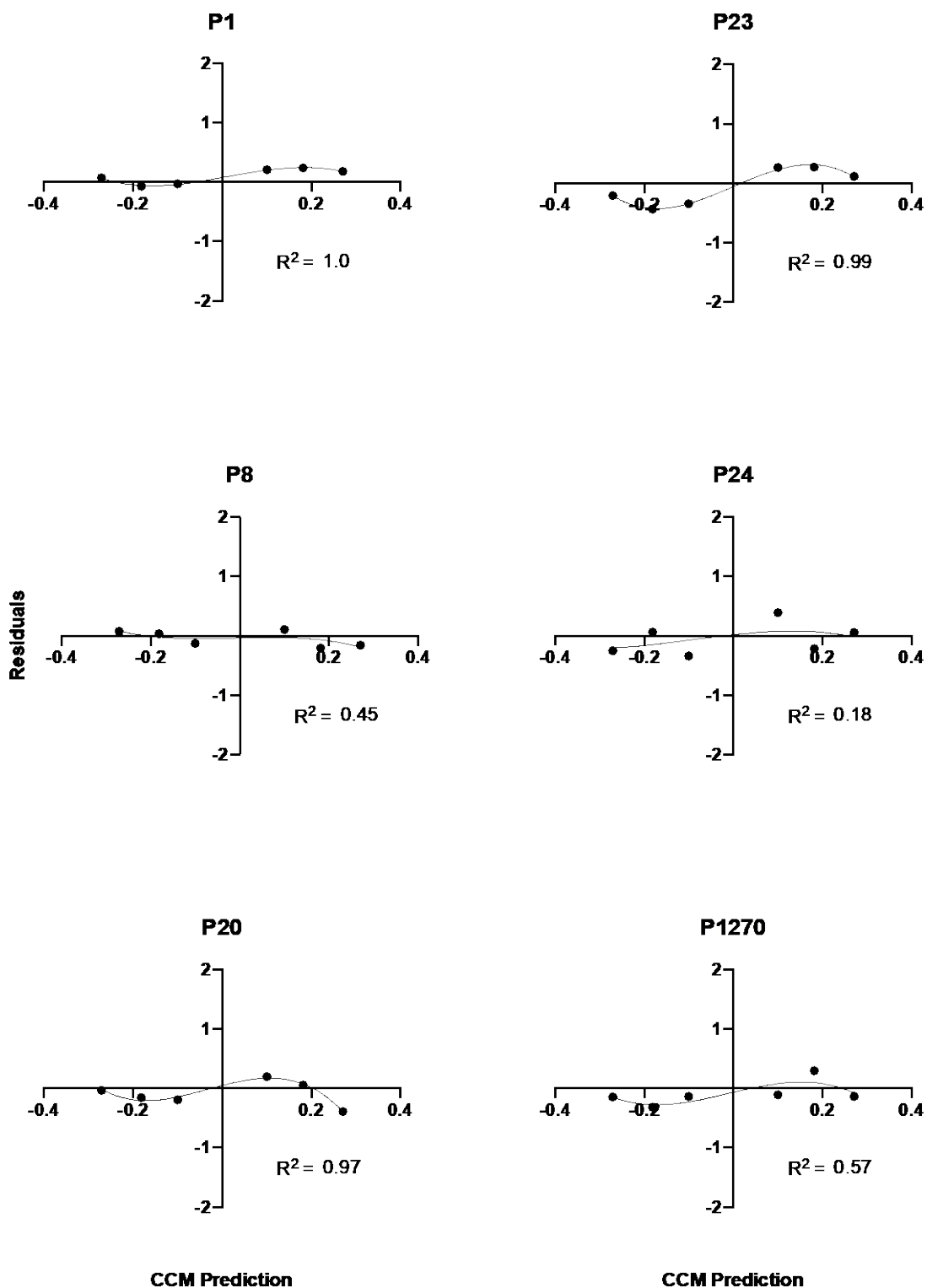


Figure 3-13.

CDM (σ parameter) residuals plotted against CDM model predictions for Experiment 2. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

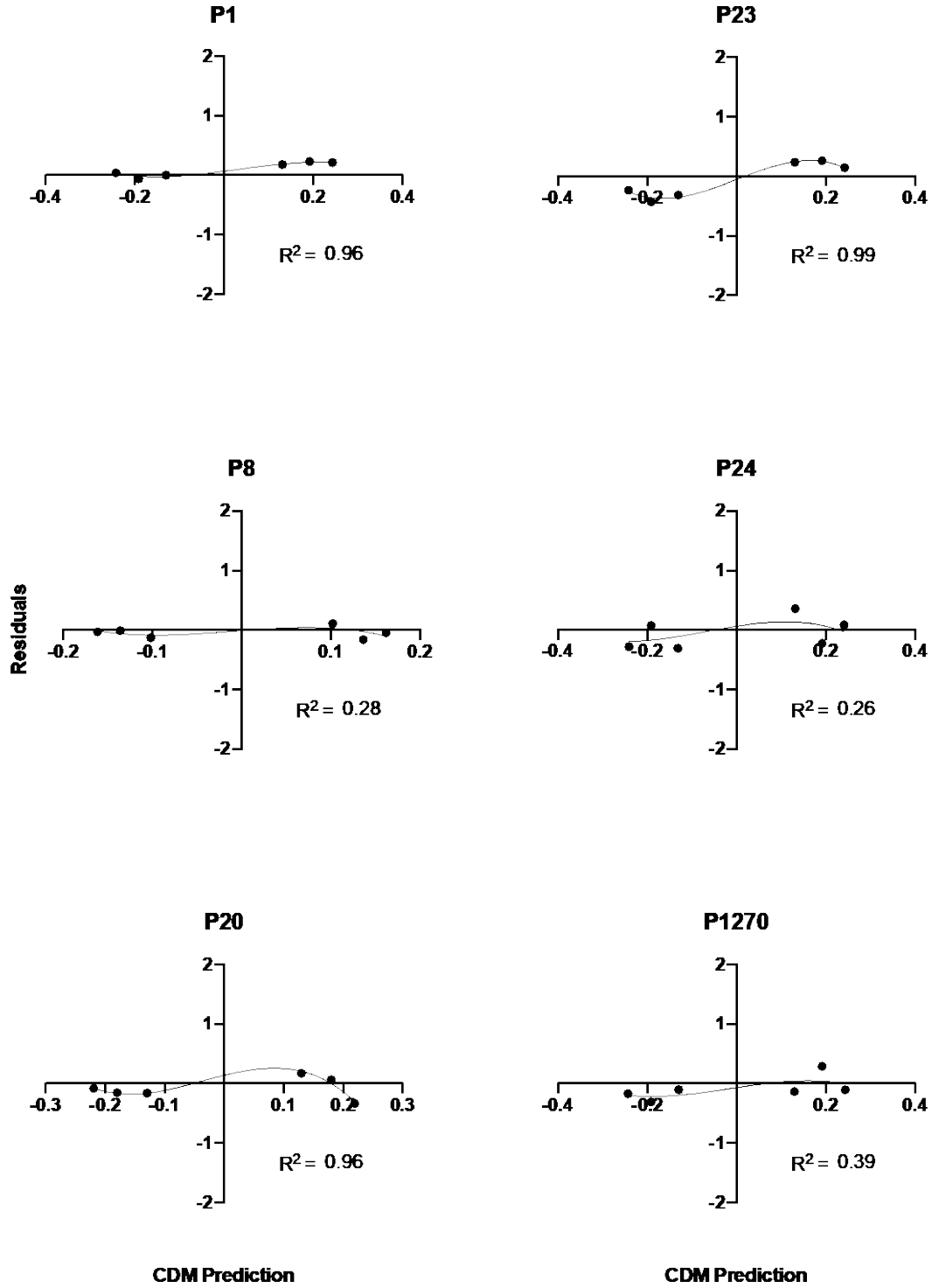


Figure 3-14.

DRT (no free parameters) residuals plotted against DRT model predictions for Experiment 2. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

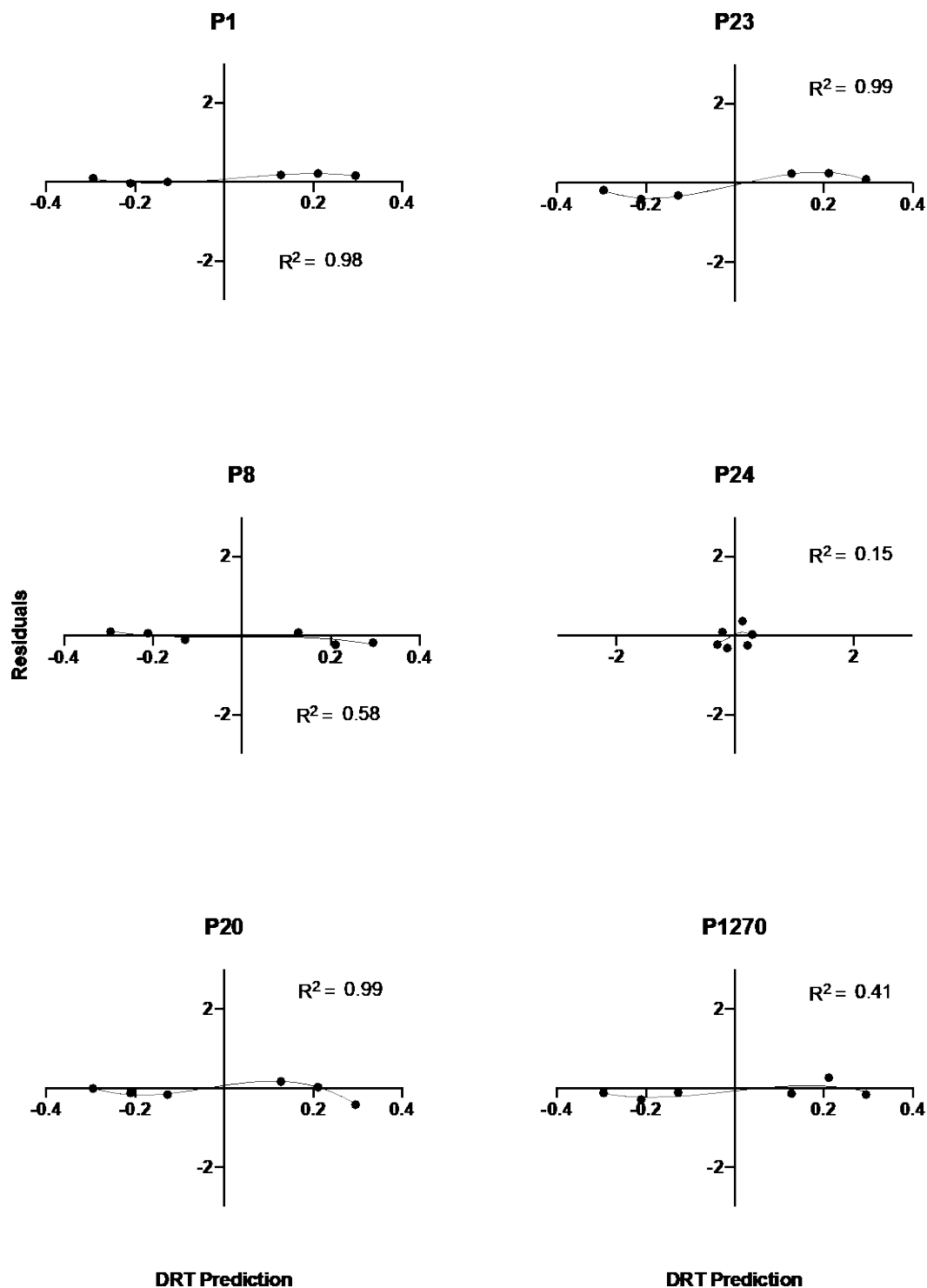


Figure 3-15.

DRT_H (no free parameter) residuals plotted against DRT_H model predictions for Experiment 2. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

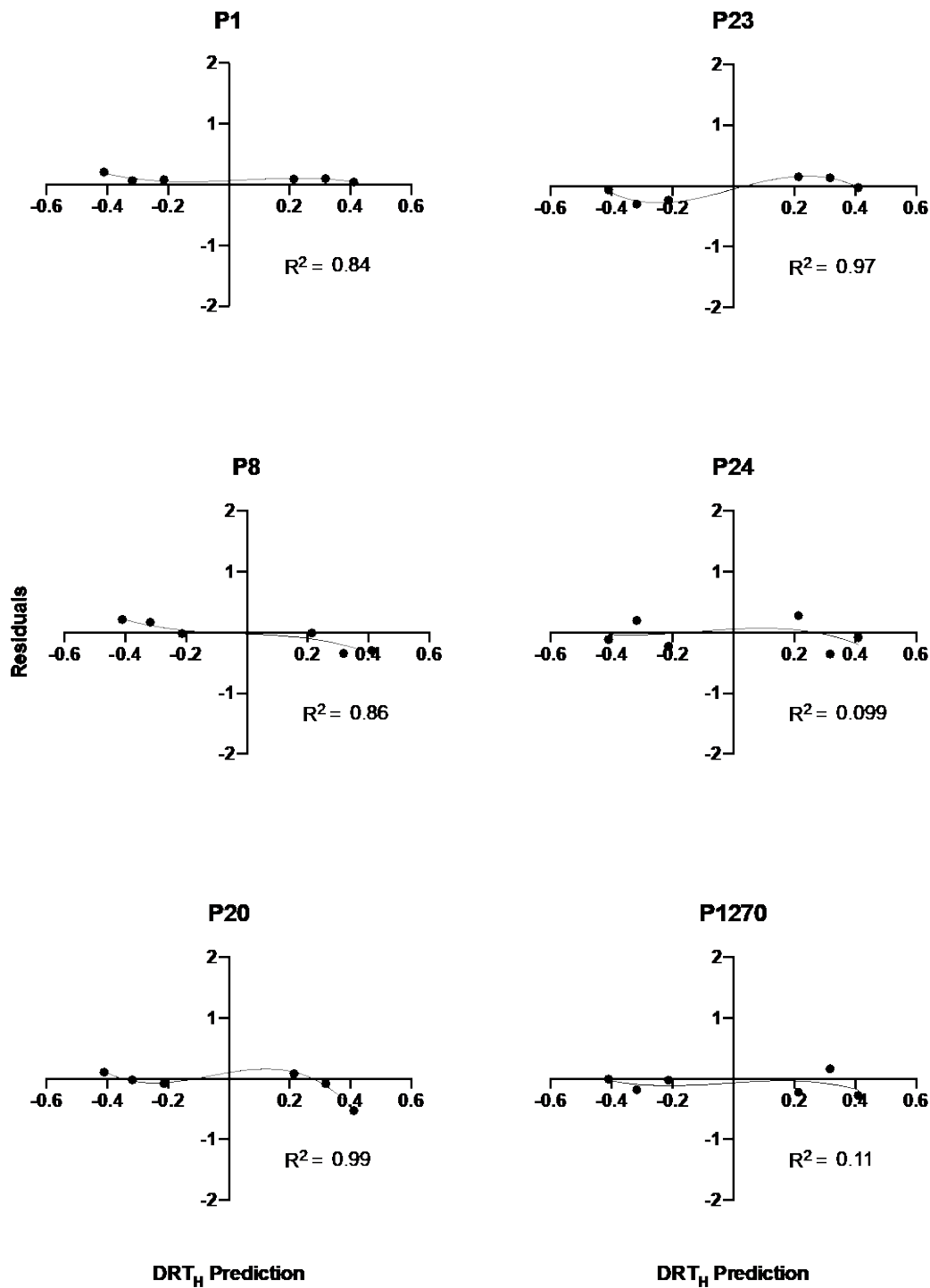


Figure 3-16.

HVA (k parameter) residuals plotted against HVA model predictions from Experiment 2. The black line represents a cubic polynomial function fit to the residuals to detect systematic patterns in residuals.

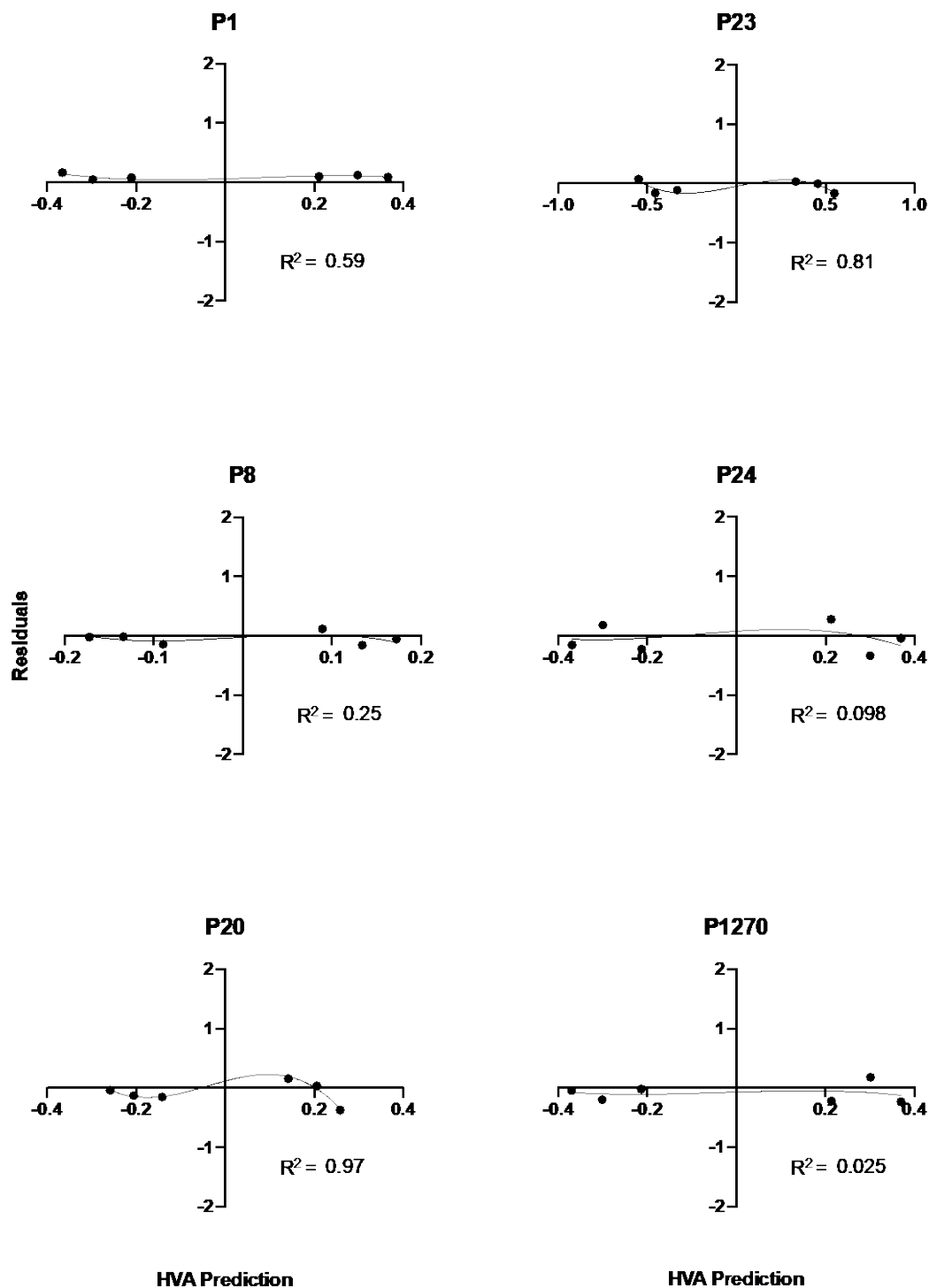


Table 3-5.

Results of the cubic polynomial test for residual trends. Column two displays the number of R^2 values produced by the polynomial test that were greater than the expected R^2 for 6 random residuals ($M = .630$).

Model	$R^2 > M$	p -value
CCM	3	$p = .656$
CDM	3	$p = .656$
DRT	3	$p = .656$
DRT _H	4	$p = .344$
HVA	2	$p = .344$

Discussion

The results from Experiment 2 indicate that when long initial-links are programmed in the concurrent-chains procedure, DRT_H and HVA, the two best-performing models from Experiment 1, make comparably accurate predictions. The AICc and BIC analyses conducted in Experiment 2 indicated that the best version of CDM was convincingly less accurate than all of the other models. The residuals between predicted and obtained choices were not significantly different than chance for any of the models.

Three aspects of the findings are worthy of further discussion. First, unexpectedly, preference for the VI 10-s terminal link did not increase as the duration of the alternative terminal-link increased from VI 40 s to VI 80 s. Although each of the five models make unique predictions about what proportion of initial-link responses will be allocated to the VI 10-s terminal link, they all predict that preference for the VI 10-s terminal link should increase as the longer terminal link is increased from VI 40 s to VI 80 s. Fantino and Davison (1983) used similar terminal link durations and found that preference

systematically shifted to the richer alternative as the other terminal link became leaner (this finding informed the power analysis that preceded Experiment 2). One procedural difference between their experiment and mine is that their initial links were shorter (VI 120 s) than mine (VI 300 s). It has been previously shown that as initial-link duration increases, while terminal-link duration is held constant, preference shifts towards indifference (Grace & McLean, 2019; MacEwen, 1972; Williams & Fantino, 1978). Thus, the long initial-links used in Experiment 2 may have contributed to the observed insensitivity to changes in the lean terminal-link schedule. It is also possible that a condition-order effect may have obscured the anticipated effect, but it is not possible to evaluate this with the sample sizes used. Thus, Experiment 2 may have been underpowered to detect small effects of increasing the duration of the lean terminal link.

The second noteworthy aspect of Experiment 2 is that the obtained times from the onset of the initial link to food delivery did not differ significantly from those predicted by Mazur's method (personal communication, January 11, 2017). This is in contrast to the finding of Experiment 1, in which Mazur's method significantly underestimated these times. Moreover, when obtained and predicted harmonic rates were compared across the three conditions of Experiment 2, only 1 of 3 harmonic rates on the rich chain (R_R) and none of the rates on the lean chains (R_L) differed significantly from those predicted by Mazur. It is unclear why Mazur's method is more likely to predict or underpredict harmonic mean times and rates in Experiment 1, but predict or overpredict them in Experiment 2. One potential answer is the long initial links used in Experiment 2. However, given the limited data collected thus far, it is hard to make any definite statements regarding the factor(s) contributing to the accuracy of Mazur's predictions.

The final noteworthy aspect of these findings is that to account for the choice proportions observed in Experiment 2, the k parameter in HVA, which reflects how steeply food value declines with increasing delays, assumed a median value of 0.01. This is an unusually low value for pigeons, and perhaps it is an unreasonable value. In discrete-trial choice research with fixed and variable delays, Mazur (1984, 1986) set k to a constant value of 1.0. In discrete-trial choice research on the effects of varying intertrial intervals, Mazur et al. (1985) set the pigeons' k to a constant of 0.4. Similarly, when pigeons' discrete-trial choices were evaluated with delayed and probabilistic food, Mazur (1991) used a k -value of 0.3, and in research using the adjusting-delay procedure with different delays, amounts, and rates of food, Mazur (2000) used a k -value of 0.7. In their review of the pigeon concurrent-chains literature, Christensen and Grace (2010) set $k = 0.2$, allowing it to vary when doing so “substantially improved” HVA's performance (these judgments were not based on AICc or BIC analyses). Of the 18 studies they reviewed, only 6 required k to deviate from 0.2, and in only one of those did k assume a median value lower than 0.05. The study requiring the low k value, Dunn and Fantino (1982), had only three conditions, but one of those conditions involved long (VI 240 s) initial links. Another relevant experiment is Mazur (2004). In that study, initial-link duration varied from 15 to 225 s and the terminal links were FI 2-s and FI 12-s. Once again, when HVA was fitted to the obtained choice proportions, k values were unusually low, ranging from 0.03 to 0.25 ($Mdn = 0.1$) across four pigeons. Thus, to predict choice when long initial links are used, HVA requires unusually low k values.

A question arises as to why smaller k values are required in HVA when long initial links are programmed in the concurrent-chains procedure. To answer this question,

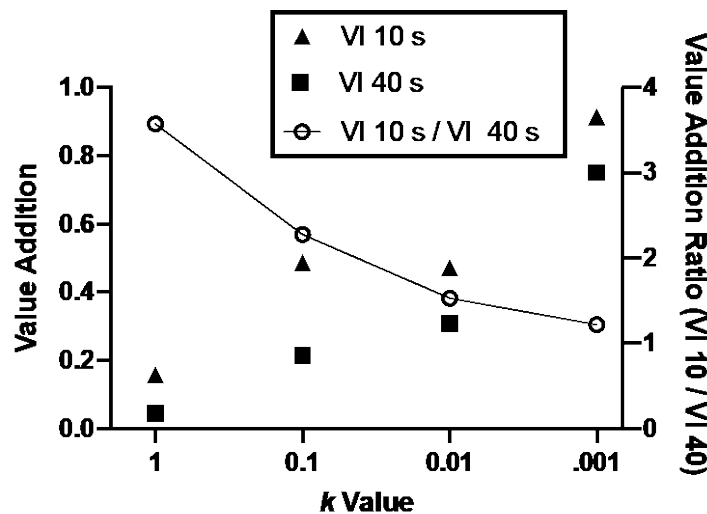
it is necessary to consider how k interacts with the $1+$ term in the denominator of Equation 8 (reproduced and highlighted here for reader ease).

$$V = \sum_{i=1}^n P_i \left(\frac{A}{1 + kD_i} \right)$$

When the multiplier k is very small, the delay term becomes negligible relative to the added 1. As a result, the ratio of value addition $((V-V_R) / (V-V_L))$ signaled by the two terminal links approaches 1, which will yield choice predictions that approach indifference. This effect of k on value addition is illustrated in Figure 3-17 using schedules arranged in one condition of Experiment 2 (other conditions yield comparable functions). As k decreases from 1 to 0.001, the ratio of value addition (open data points and right y-axis) decline, toward 1. Thus, to account for the less extreme preferences for the richer alternative, when long initial links are arranged, k must be smaller than is typical for pigeon choice research.

Figure 3-17.

Value addition (left y-axis) and ratio of value addition (right y-axis) signaled by VI 10 s and VI 40 s terminal links with differing k values. Closed symbols represent value addition, while open symbols represent the ratio of value addition.



But is it reasonable to assume that pigeons change the extent to which they discount delayed food (k) in response to long initial links? Before answering that question, it may be instructive to further consider the $1+$ term in Equation 8. Mazur (1987) included this term to prevent value (V) from approaching infinity as delays approach zero. This rationale is reasonable when brief delays are investigated but is not easily justified in Experiment 2, when just 1 of the 270 programmed delays was less than 2 s, and none were less than 1 s. If the $1+$ term is removed from the denominator of Equation 8, the value of k has no impact on the predictions of HVA.

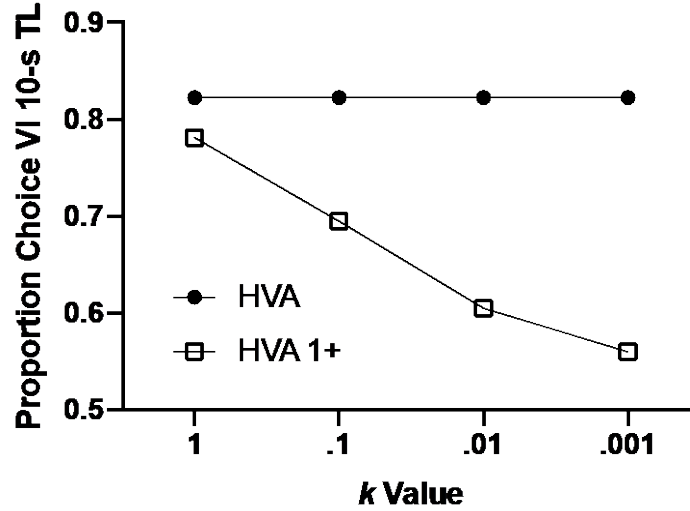
To illustrate this, Figure 3-18 displays the choice predictions of HVA from the same condition of Experiment 2, with and without the $1+$ term in the denominator of Equation 8. When $k = 1$, HVA predicts a strong preference for the VI 10 s TL over the VI 40 s TL regardless of the status of the $1+$ term. However, as k decreases, only the version of HVA that includes the $1+$ term makes different predictions. This happens because k is a multiplicative term on each value-impacting portion of the equation in HVA (i.e., V , V_R and V_L). When k impacts the value in one term of HVA it impacts value in all terms, and the result is that predicted choice remains constant.

To appreciate this, consider the function that determines the value addition of terminal-link stimuli in HVA when the $1+$ term is omitted. For simplicity, the portion of the equation that determines how rates of conditioned reinforcement impact choice (which does not interact with k) and the b and a_i free parameters have been removed:

$$\frac{B_R}{B_R + B_L} = \left(\frac{\frac{1}{k * D_{tR}} - \frac{1}{k * D_i}}{\left(\frac{1}{k * D_{tR}} - \frac{1}{k * D_i}\right) + \left(\frac{1}{k * D_{tL}} - \frac{1}{k * D_i}\right)} \right)$$

Figure 3-18.

Predictions for the VI-40 s terminal link condition from Experiment 2 made by HVA with (open symbols) and without (filled symbols) the 1+ term in the denominator of the hyperbolic-decay model (Equation 8).



where B_R and B_L represent the number of initial-link responses allocated to the rich and lean chains, D_R and D_L represent the delays to food from the onset of each of the rich and lean terminal links, and D_i represents the delay to food from the onset of the initial link. Now consider the predictions when $k = 0.2$ and the delays from the VI 40-s condition of Experiment 2 are entered into the equation:

$$\frac{B_R}{B_R + B_L} = \left(\frac{\frac{1}{.2 * 10_{tR}} - \frac{1}{.2 * 175_i}}{\left(\frac{1}{.2 * 10_{tR}} - \frac{1}{.2 * 175_i} \right) + \left(\frac{1}{.2 * 40_{tL}} - \frac{1}{.2 * 175_i} \right)} \right) = .83$$

HVA predicts that pigeons will allocate 83% of their response to the VI 10-s chain. Now consider HVA's predictions when $k = 0.001$:

$$\frac{B_R}{B_R + B_L} = \left(\frac{\frac{1}{.001 * 10_{tR}} - \frac{1}{.001 * 175_i}}{\left(\frac{1}{.001 * 10_{tR}} - \frac{1}{.001 * 175_i} \right) + \left(\frac{1}{.001 * 40_{tL}} - \frac{1}{.001 * 175_i} \right)} \right) = .83$$

and when $k = 1000$:

$$\frac{B_R}{B_R + B_L} = \left(\frac{\frac{1}{1000 * 10_{tR}} - \frac{1}{1000 * 175_i}}{\left(\frac{1}{1000 * 10_{tR}} - \frac{1}{1000 * 175_i} \right) + \left(\frac{1}{1000 * 40_{tL}} - \frac{1}{1000 * 175_i} \right)} \right) = .83$$

No matter what value k is set equal to, without a $1+$ in the denominator, HVA makes identical predictions. Therefore, the ability of HVA to predict the results of Experiment 2 appears to be an artifact of the interaction between a procedurally dependent low k value and the $1+$ term in the denominator of Equation 8; it is not the result of value addition signaled by terminal links, which predict more extreme choice.

To answer the question raised earlier, it is not unreasonable to assume that pigeons change the extent to which they discount delayed food (k) in long initial links; however, when considering this potential explanation, it is worth considering two points. First, HVA has been shown to make accurate predictions across a wide range of initial-link conditions when $k = 0.2$ (Mazur, 2001). If k systematically changes with initial-link duration, it seems that allowing it to vary should improve HVA's predictions when different initial-link lengths are studied. However, there are numerous published experiments in which this is not true (Alsop & Davison, 1988; Davison, 1976, 1988; Fantino, 1969; Preston & Fantino, 1991; Wardlaw & Davison, 1974). For example, Davison (1976) varied initial-link durations from 27 s to 181 s and when Christensen and Grace (2010) analyzed these results and allowed k to vary, it did not significantly improve the performance of HVA.

Second, if pigeons systematically change the rate at which they discount delayed food as initial-link duration is increased in the concurrent-chains procedure, then this prediction should be represented mathematically within HVA. In its current form, k does

not just impact the delays associated with the initial link, it impacts all delays. As shown above, this ultimately leads to k having no impact on the choice predictions made by HVA when the $1+$ term is omitted. If k truly represents how steeply food value declines with increasing delays, then changes in k within HVA should lead to changes in predicted preference without depending on the $1+$ term it uses to calculate value. Until these modifications are included in HVA, I contend that it is unreasonable to allow a procedurally dependent discounting parameter to take on the low values required to fit the data in Experiment 2, and the data reported by Mazur (2004) and Dunn and Fantino (1982).

In addition to DRT_H and HVA, two other models (CCM and DRT) performed comparably well at accounting for the choice proportions observed in Experiment 2. As discussed in Chapter 1, DRT does well at predicting choice in the concurrent-chains procedure when two aperiodic or two periodic terminal links are used. Therefore, it is not surprising that DRT made accurate predictions in Experiment 2. Like DRT and DRT_H , CCM also predicts that when long initial links are used in the concurrent-chains procedure pigeons' choices will be less extreme than the predictions of HVA. This prediction is a result of the T_t/T_i exponent in CCM. When the average time spent in the initial link is longer than the average time spent in the terminal links, the T_t/T_i ratio is less than 1, which reduces preference for the shorter terminal link.

To summarize, there are three core findings from Experiment 2. First, when the length of the longer terminal link was increased from VI 40 s to VI 80 s in Experiment 2, there were no significant changes in pigeons' choices. Second, Mazur's method for predicting harmonic means (T_H , R_R , and R_L) was reasonably accurate, as these

predictions were not significantly different from obtained times. Third, when long initial-links are programmed in the concurrent-chains procedure, all of the models, four of the five models made comparable predictions with one (HVA) or no (CCM, DRT, DRT_H) free parameters; CDM made less accurate predictions. A caveat to the preceding statement is that the free parameter value required by HVA to account for choice in Experiment 2 was unusually low when compared to k values in other choice research using pigeons. These low k parameters appear to be the result of inaccurate predictions made by HVA when long initial links are used in the concurrent-chains procedure.

CHAPTER IV

EXPERIMENT 3

Introduction

The five concurrent-chains models discussed throughout Experiments 1 and 2 calculate the conditioned-reinforcing value of terminal-link stimuli in the concurrent-chains differently (see Figure 2-1). These models also make differing predictions about how factors such as rate of primary reinforcement (i.e., rate of food delivery) and rate of conditioned reinforcement (i.e., rate of terminal-link entry) will impact choice in the concurrent-chains procedure. CCM (Grace, 1994; Grace, 1996), CDM (Christensen & Grace, 2010) and HVA (Mazur, 2001) predict that in addition to the conditioned-reinforcing value of terminal-link stimuli, choice in the concurrent-chains procedure should also be influenced by the relative rates of conditioned reinforcement (i.e., r_{i1} & r_{i2}). That is, according to CCM, CDM and HVA, pigeons should prefer a chain with a higher arithmetic rate of terminal-link entry over a chain with a lower arithmetic rate of terminal-link entry.

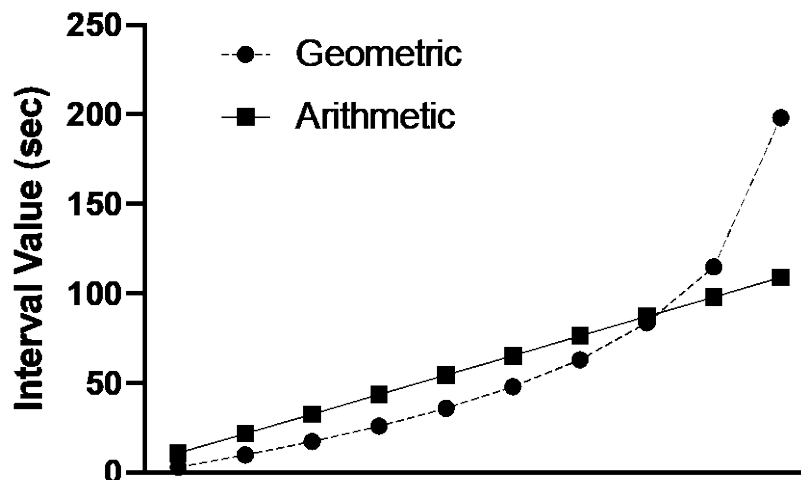
Unlike CCM, CDM, and HVA, DRT does not predict that rate of conditioned reinforcement will directly impact choice; instead, DRT predicts that choice will be influenced by the relative arithmetic rates of primary reinforcement on the two chains in the concurrent-chains procedure (Squires & Fantino, 1971). Building upon DRT, DRT_H also predicts that the relative rates of primary reinforcement arranged on the two chains should influence choice, but DRT_H calculates this rate using a harmonic mean, not an arithmetic mean. This method of calculating rates of reinforcement, relevant when

variable delays are arranged, is unique to DRT_H ; the predictive accuracy of this different averaging method was tested in Experiment 3.

To reiterate, all four models except DRT_H calculate reinforcement rates (conditioned or primary) using arithmetic averages. One potential problem with the use of arithmetic averages is that findings from discrete-trial and adjusting-delays research have consistently shown that the distribution of delays to food matters in both rats (Brunner & Gibbon, 1995) and pigeons (Brunner, Gibbon, & Fairhurst, 1994; Mazur, 1984; 1986; McDiarmid & Rilling, 1965). Because CCM, CDM, HVA, and DRT use arithmetic averages, their predictions will be unaffected by the distribution of initial-link VI-schedule values, as long as it does not affect the arithmetic average of the distribution. For example, a VI 60-s schedule populated with 10 interval values can be created using several different methods. This is illustrated in Figure 4-1, which displays 10 values of two different VI distributions, both with an arithmetic average of 60: an arithmetic distribution and a geometric distribution, the Fleshler and Hoffman (1962) method. In arithmetic distributions a constant duration is added to each sequential value to populate the VI schedule. In geometric distributions, the duration added increases with each sequential value. Where HVA, CCM, CDM, and DRT predict the composition of these two initial-link schedule distributions will not change initial-link choice when the time to food in the terminal links is identical, DRT_H predicts choice will favor the distribution of initial-link values with the lower harmonic mean. In Figure 4-1, the harmonic mean of the arithmetic distribution is 37.2 s, whereas the harmonic mean of the geometric distribution is 16.4 s. Therefore, DRT_H predicts preference for the initial link populated with

Figure 4-1.

Two VI 60-s schedules composed of arithmetic or geometric (Fleshler & Hoffman, 1962) progressions.

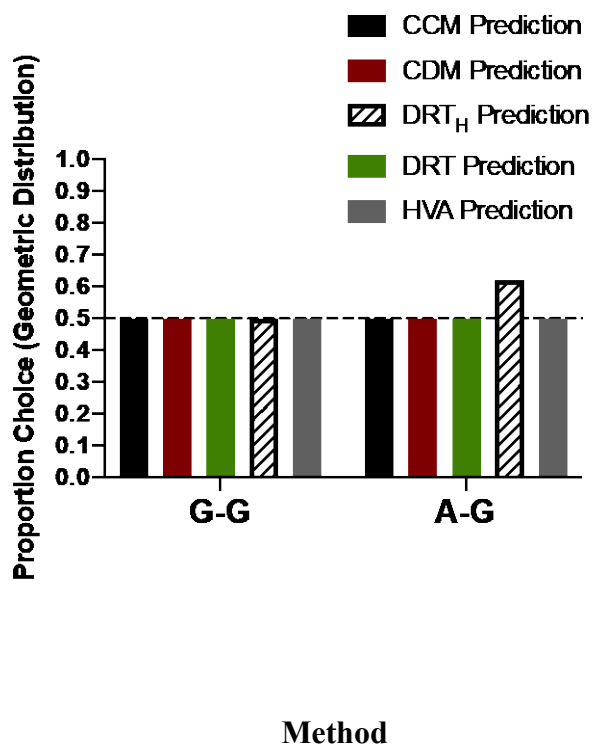


geometrically distributed values because it uses a harmonic mean which more heavily weights smaller delays.

In Experiment 3, the Savanstano and Fantino (1996) concurrent-chains procedure was used to investigate choice when both of the initial links were always programmed with VI 60-s schedules and both terminal links were always programmed with FI 2-s terminal links. In one condition, the initial-link VI 60-s distributions were both geometric (G-G on the x-axis of Figure 4-2). Here, because the overall rates of terminal-link entry and the distribution of delays in the initial links are identical, all five models predict indifference. In a second condition, one of the VI 60-s initial links was changed to an arithmetic distribution (A-G). Under these conditions DRT_H uniquely predicts that pigeons will prefer the chain with the geometric distribution in the initial link.

Figure 4-2.

Predicted choice proportions for the chain containing the initial link with the Flesher and Hoffman (1962) VI distribution for Experiment 3



Subjects and Apparatus

A power analysis using the obtained data from the VI 30-s and VI 90-s initial-link conditions in Mazur (2004) ($d = 1.4$) with $\alpha = .05$ and power = .8 indicated that a sample size of 8 was needed for single within-subject comparisons. To facilitate statistical analyses, 8 experimentally experienced, unsexed pigeons were used. Housing, feeding, scheduling, experimental chambers and Institutional Animal Care and Use Committee protocol numbers were identical to those in Experiments 1 and 2.

Procedure

The procedure and stability criteria used in Experiment 3 were identical to those described in Experiment 2, with three exceptions. First, geometric (Fleshler & Hoffman,

1962) and arithmetic distributions composed the VI 60-s initial links. Second, FI 2-s schedules of reinforcement were programmed in both terminal links. Short FI terminal links were used to increase the likelihood that pigeons' choices would be sensitive to the differences in the initial-link distributions. Third, the stimuli present during initial- and terminal-links were different in the G-G and A-G conditions. Condition order was counterbalanced across pigeons, with P1, P20, P23 and P4740 completing the A-G condition first and pigeons P8, P24, P1270 and P2331 completing the G-G condition first.

Data Analysis

Data from the stable sessions of each condition were used for all analyses. Choice in the A-G condition is reported as the proportion of initial-link responses allocated to the geometric initial link. In the G-G condition, one of the chains was randomly designated at the beginning of the experiment as the numerator schedule in the calculation of the choice proportion. An alpha level of .05 was used for all statistical analyses and all reported confidence intervals were at the 95% level. Predicted and programmed times (T_H) and harmonic rates of reinforcement on the arithmetic VI (R_A) and the geometric VI (R_G) schedules were compared using one-sample t-tests. Stable choice proportions were analyzed using a 2 x 2 (condition x condition-order) mixed ANOVA. In all cases, when multiple within-family comparisons were conducted, Bonferroni adjusted p-values were used.

Results

Individual and group averaged sessions to stability and harmonic-mean times from the onset of the initial link to food delivery (T_H) averaged across the final nine (stable sessions) in each condition can be seen in Table 4-1. The harmonic-mean times

Table 4-1.

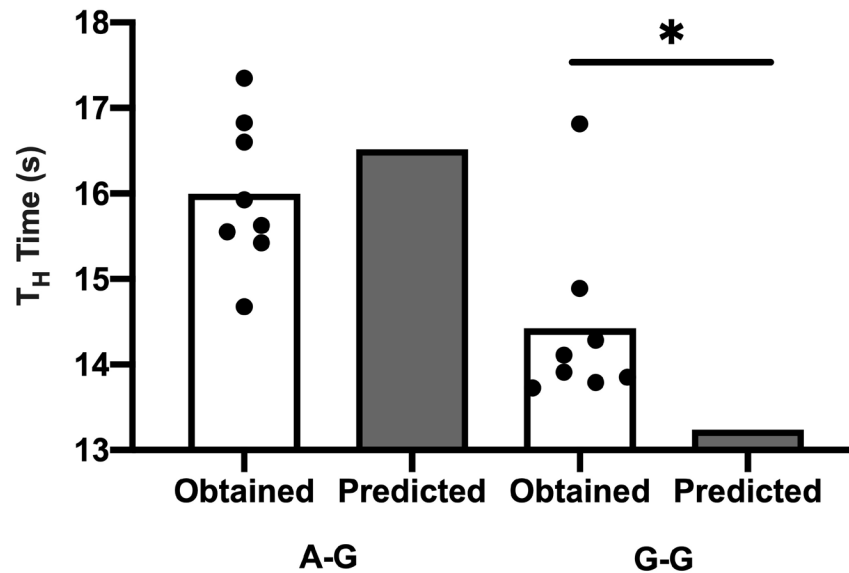
Individual subjects' condition order, sessions until stability, obtained T_H , R_G , R_A values, and proportion choice from Experiment 3. Values in brackets denote standard deviations around the group averages.

Pigeon	Condition	Sessions Until Stability	Obtained Harmonic T_H (sec)	Obtained Harmonic R_G	Obtained Harmonic R_A or R_G	Proportion Choice Geometric IL
P1	G-A	25	16.60	0.07	0.05	0.56
	G-G	19	13.85	0.07	0.07	0.48
P20	G-A	28	17.35	0.07	0.04	0.62
	G-G	16	14.28	0.07	0.07	0.48
P23	G-A	27	15.42	0.07	0.06	0.56
	G-G	21	13.73	0.07	0.08	0.50
P4740	G-A	38	16.83	0.07	0.05	0.62
	G-G	24	14.89	0.07	0.07	0.54
P8	G-A	23	14.68	0.07	0.07	0.50
	G-G	23	13.79	0.08	0.07	0.48
P24	G-A	18	15.55	0.08	0.05	0.50
	G-G	21	13.91	0.07	0.07	0.54
P1270	G-A	22	15.93	0.07	0.06	0.55
	G-G	23	14.11	0.07	0.08	0.55
P2331	G-A	19	15.63	0.08	0.06	0.48
	G-G	17	16.82	0.06	0.07	0.55
Group	G-A	25 [6.33]	16.00 [0.87]	0.07 [0.00]	0.06 [0.01]	0.55 [0.05]
Averages	G-G	20 [2.93]	14.42 [1.04]	0.07 [0.00]	0.07 [0.00]	0.53 [0.03]

spent in the terminal links (t) are not shown because predicted and obtained times were nearly identical. Figure 4-3 displays the individual and average T_H times, along with those predicted by Mazur's method (filled bars; personal communication, January 11, 2017). Obtained T_H times in the G-G conditions were significantly larger than the predicted time ($t_{(7)} = 3.23$, $p = .014$, $CI = 13.56, 15.29$); obtained T_H times did not differ significantly from the predicted time in the A-G condition ($t_{(7)} = -1.69$, $p = .134$, $CI = 15.27, 16.73$). Exploratory analyses revealed no significant effect of condition order on the accuracy of these predictions.

Figure 4-3.

Obtained and predicted harmonic mean times from initial-link onset until food (T_H). Black dots represent individual-subject data. A-G and G-G x-axis labels represent data from the arithmetic-geometric and geometric-geometric initial-link conditions, respectively. Note that the y-axis range does not extend to zero.



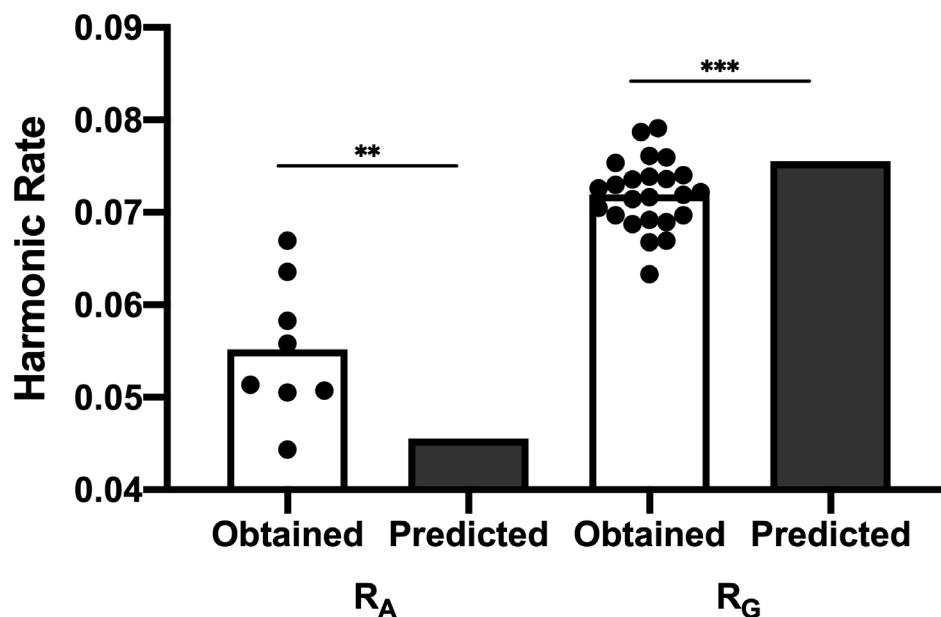
Note: * $p < .05$

Individual and group averaged obtained and predicted harmonic rates of reinforcement (R_G and R_A) may be found in Table 4-1. Because there was no significant effect of condition on the accuracy of the predicted values of R_G , Figure 4-4 displays these obtained and predicted harmonic rates of food, collapsing R_G values across condition-order. Obtained R_A values were significantly larger than the predicted rate ($t_{(7)} = 3.65$, $p = .008$, $CI = 0.050, 0.061$), whereas obtained R_G values were significantly smaller than the predicted rate ($t_{(23)} = -4.69$, $p < .001$, $CI = 0.070, 0.074$).

The predicted and obtained proportions of initial-link responses allocated towards the geometric chain in the A-G condition is shown in the left panel of Figure 4-5 (see also

Figure 4-4.

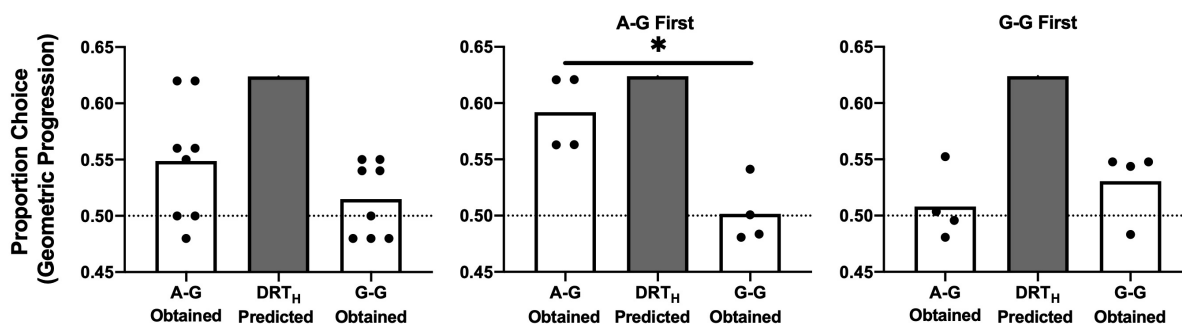
Obtained and predicted harmonic-mean rates of reinforcement on the arithmetic (R_A) and geometric (R_G) chains. Black dots represent individual-subject data. R_A and R_G x-axis labels represent data from the arithmetic and geometric chains, respectively.



Note: ** $p < .01$, *** $p < .001$

Figure 4-5.

Average choice proportions from Experiment 3. Black dots represent individual-subject data. The left panel displays aggregate data from the A-G condition (condition order collapsed). The middle panel displays data from pigeons who first completed the A-G condition, while the right panel displays data from pigeons who first completed the G-G condition.



Note: * $p < .05$

Table 4-1). The middle and right panels separate data by condition order. The 2 x 2 ANOVA revealed no significant main effect of condition order ($F_{(1,6)} = 2.44, p = .169$). However, the main effect of condition was significant ($F_{(1,6)} = 6.60, p = .042$), with greater preferences for the geometric initial link in the A-G conditions than the G-G conditions. However, there also was a significant condition x order interaction ($F_{(1,6)} = 18.24, p = .005$)⁵. Specifically, preference for the geometric initial link in the A-G condition was significantly more pronounced among those pigeons that completed the A-G condition first ($t_{(6)} = 4.84, p = .006$). Condition order did not impact choice in the G-G condition ($t_{(5)} = 1.20, p = .548$). Thus, preference for the geometric initial link in the AG condition was dependent on condition order.

Discussion

The results from Experiment 3 offer mixed support for the prediction of DRT_H that pigeons should prefer a chain with a geometrically distributed initial link over a chain with an arithmetically distributed initial link. This prediction held among all four of the pigeons that completed the A-G condition first, but among only one of four pigeons that completed the G-G condition first. Therefore, before any definite answers regarding pigeons' preferences for geometric initial-link distributions over arithmetic initial-link distributions can be given, it will be necessary to understand why pigeons who completed the A-G condition second did not demonstrate a consistent preference for the geometric over the arithmetic initial link.

⁵ Exploratory analyses indicated that the condition x order interaction was significant regardless of which geometric chain would have been randomly selected to serve as the numerator in the calculations of proportion choice.

In an attempt to identify a potential explanation for the order effect observed in Experiment 3, exploratory analyses were conducted looking for differences across the two condition-orders in (a) the number of sessions required to reach stability, (b) the obtained harmonic rates of reinforcement (i.e., R_A & R_G), and (c) obtained harmonic T_H times. Of these analyses, only a 2 x 2 mixed ANOVA (condition x condition-order) conducted on sessions to stability revealed any significant effects. In this analysis, there was a significant main effect of condition ($F_{(1,6)} = 15.19, p = .008$), no main effect of condition order ($F_{(1,6)} = 2.67, p = .154$), but a significant condition x order interaction ($F_{(1,6)} = 18.75, p = .005$). Specifically, the number of sessions to reach stability in the A-G condition was significantly larger among pigeons that first completed the A-G condition ($t_{(6)} = 5.18, p = .002$). Condition order did not impact choice in the G-G condition ($t_{(5)} = 0.31, p = .999$). Thus, like preference for the geometric over the arithmetic initial link, sessions until stability in the AG condition was dependent on condition order. Although these results align with the proportion choice results, they do not explain why pigeons that completed the A-G condition second did not consistently come to prefer the geometric initial link. As no explanation is readily available for these order effects, future research might address the question raised in Experiment 3 using a between-subjects design. Such an experiment would allow for an assessment of the replicability of the preference for the geometric initial link among pigeons that completed the AG condition first.

A final noteworthy aspect of Experiment 3 was that obtained T_H times did not differ from those predicted by Mazur's method (personal communication, January 11, 2017) in the A-G condition, but they were larger than those predicted in the G-G

condition. These results are similar to those from Experiment 1, which suggests that when Mazur's method makes errors estimating T_H times, it tends to underpredict times. Obtained T_H times that are lower than predicted times would lead DRT_H to overpredict preference for the shorter of two terminal links.

The obtained harmonic rates on the arithmetic (R_A) and geometric (R_G) chains were also significantly different from those predicted using Mazur's method. Specifically, obtained harmonic rates of reinforcement were smaller than programmed rates on the geometric chain and larger than the programmed rate on the arithmetic chain. Together, these rate-estimate inaccuracies led DRT_H to predict more extreme preferences for the geometric chain than would have been made from obtained harmonic rates. Given the mixed accuracy of Mazur's method for correctly estimating harmonic T_H , R_A , and R_G times in the three experiments of this dissertation, future research should continue to collect and report these values, while evaluating other, more accurate estimation methods.

To summarize, the results from Experiment 3 offer mixed support for the hypothesis that pigeons prefer a chain with a geometrically distributed initial link over a chain with an arithmetically distributed initial link that share the same arithmetic mean. An order effect precludes providing a more definitive answer. Future research should attempt to replicate the current experiment with a between-subjects experimental design. If that experiment revealed that pigeons prefer the initial link with a geometric distribution of values, this would raise questions about the use of arithmetic rates in contemporary models of concurrent-chains choice. If the results from Experiment 3 cannot be replicated, this would raise questions about DRT_H 's assumption that all intervals of time in the concurrent-chains procedure are averaged harmonically and it

would require DRT_H to justify when arithmetic and harmonic averaging should be used in the concurrent-chains procedure.

CHAPTER V

GENERAL DISCUSSION

The goal of the three experiments presented here was to evaluate a new form of DRT (Squires & Fantino, 1971), DRT_H , which substitutes harmonic for arithmetic means. To evaluate DRT_H , it was compared to several competing models of concurrent-chains choice across three experiments. A summary of the results from these three experiments is presented in Table 5-1. In the table, an “+” indicates the model was supported by the analysis indicated in the row, and a “-” indicates the model was either not supported (general trends, systematic residuals, unreasonable free-parameter values) or performed convincingly worse than the support models ($AICc$ & BIC). In the table, the *General Trends* row indicates if obtained choice proportions varied systematically in the direction predicted by each model (e.g., choice shifts towards indifference as the length of the initial link increases). The *AICc* & *BIC* rows refer to the accuracy of the choice predictions made by the best version of each model, as assessed in $AICc$ (Akaike, 1998) and BIC (Schwarz, 1978) analyses in Experiments 1 and 2. The *Residuals* row refers to the residual analysis (McDowell, Calvin, & Klapes, 2016) performed on the differences between obtained and predicted choice proportions in Experiments 1 and 2. The *Free Parameter Values* row indicates if the mean free-parameter values needed by each model were reasonable or unreasonable given the value of this parameter in the published literature. Given that the mixed outcomes of Experiment 3 did not clearly support or refute any model, that experiment is not represented in Table 5-1. To summarize the table, across the two experiments and eight metrics, DRT_H was the model that made the most viable predictions.

Table 5-1.*Results for Experiments 1, 2, and 3 and the models they support (+) or refute(-).*

Experiment Number Test Condition	CCM	CDM	DRT	DRT _H	HVA
Experiment 1: General Trends	+	+	-	+	+
Experiment 1: AICc & BIC	-	-	-	+	+
Experiment 1: Residuals	-	+	+	+	+
Experiment 1: Free Parameter Values	+	+	-	+	+
Experiment 2: General Trends	-	-	-	-	-
Experiment 2: AICc & BIC	+	-	+	+	+
Experiment 2: Residuals	+	+	+	+	+
Experiment 2: Free Parameter Values	+	+	+	+	-

Experiment 1 tested whether the substitution of harmonic for arithmetic means in DRT (Squires & Fantino, 1971) could improve its predictions when choice between aperiodic and periodic terminal links were investigated in the concurrent-chains procedure. All models besides DRT correctly predicted trends in pigeons' preference in Experiment 1. DRT failed to predict pigeons' preferences for the aperiodic over periodic terminal links in the three conditions from Killeen (1968). The substitution of harmonic for arithmetic means within DRT allowed DRT_H to make predictions that were comparable to HVA, as determined by AICc and BIC analyses. The latter analyses

suggested the remaining models (DRT, CCM, and CDM) made less accurate predictions. An analysis of residuals around the model predictions (McDowell et al., 2016) revealed nonrandom residuals for CCM only. Systematic patterns in residuals can indicate that a model fails to account for a factor that systematically impacts choice. Finally, when the free-parameter values used by each of the models were evaluated against what might be considered reasonable values from the published literature, the sensitivity parameter used by DRT (a_t) was the only one found to be unreasonable; in this case, unreasonably low ($Mdn = 0.0$). Such a value suggested pigeons were insensitive to terminal-link delays, which is at odds with previously published findings (Mazur, 2001; Christensen & Grace, 2010), and the results of Experiments 1 and 2.

Experiment 2 tested the ability of the five concurrent-chains models to predict choice when long initial links were programmed in the concurrent-chains procedure. Long initial links were investigated because under these conditions HVA predicted more extreme choice than DRT_H . Although all models predicted that pigeons would increase their preference for the VI 10-s terminal link as the average delay to food on the alternative terminal link increased, none of the pigeons demonstrated this preference. Given the limited concurrent-chains research using long initial links it is difficult to determine if this prediction made by all models is inaccurate or is the result of an uncontrolled factor, such as the order that the pigeons completed the conditions. The AICc and BIC analyses of model predictions was able to rule out only CDM. The accuracy of the best version of the remaining four models were comparable, and residuals of all models were unsystematic and, therefore, no single model emerged as the one best able to account for the results of Experiment 2. Further evaluation of the four viable

models considered the free-parameter values needed to make predictions. On this count, only HVA fell short. Specifically, HVA required unusually low values of k (the delay-discounting parameter) to account for obtained choice proportions. Of further concern was the fact that these low k values themselves did not allow HVA to predict the choices observed in Experiment 2. Instead, k needed to interact with the $1+$ term in the hyperbolic decay function (Equation 8) which is imbedded throughout HVA. When the $1+$ term was excluded from HVA, k was rendered inert and HVA was unable to accurately predict the choices observed in Experiment 2. This raises questions about the validity of HVA when long initial links are used in the concurrent-chains procedure. In sum, Experiment 2 offered the most unequivocal support for CCM, DRT, and DRT_H (the latter two making divergent predictions only when periodic and aperiodic schedules are arranged).

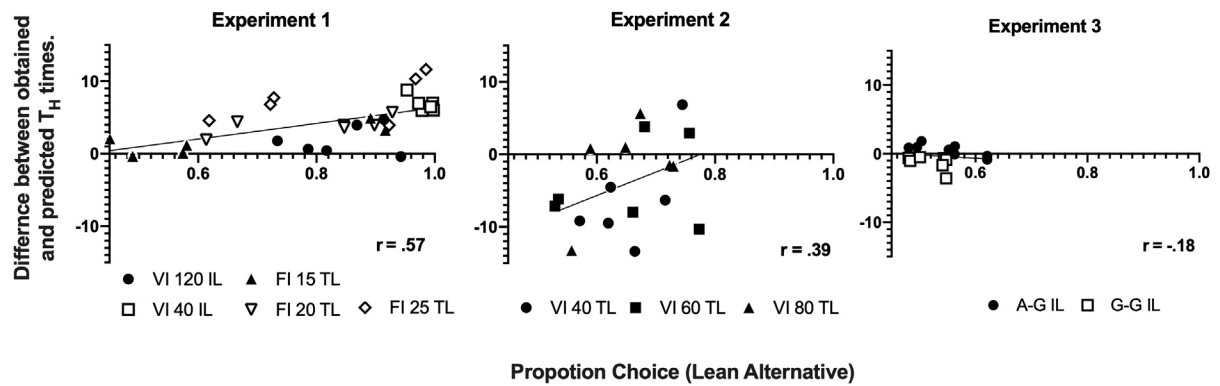
Experiment 3 attempted to further differentiate the predictions of the five models by arranging initial-link VI schedules with different distributions of values, but with identical arithmetic means. Among the five models, DRT_H uniquely predicted that pigeons should prefer a geometrically distributed initial link, relative to an arithmetically distributed initial link. This prediction is due to DRT_H's use of harmonic, instead of arithmetic means. All four of the pigeons that choose between geometric and arithmetic initial-links in the first phase demonstrated a preference for the geometric distribution. However, for pigeons that completed this phase second (following a phase in which they chose between two identical geometric distributions), only one of four pigeons preferred the geometric initial link. This experiment should be conducted again using a larger sample size and a between-subjects experimental design, as this will allow an evaluation of the unique prediction of DRT_H without a potential carry-over effect. If this prediction

proves false, this would require DRT_H to substitute arithmetic rates of primary reinforcement for harmonic rates of reinforcement. If this substitution were made, it would be necessary to reevaluate DRT_H 's predictions in Experiments 1 and 2, along with its predictions of existing concurrent-chains research (see Chapter 1).

A secondary function of the three experiments of this dissertation was to evaluate James Mazur's (personal communication, January 11, 2017) method for estimating the three harmonic mean durations needed by DRT_H to make predictions in the concurrent-chains procedure (T_H , R_L , R_R). Across the three experiments, significant deviations from these predictions were detected in 4 of 10 conditions; in each of these conditions, Mazur's method under-predicted the obtained durations. These significant deviations tended to occur when pigeons strongly favored one of the two chains. Mazur's method assumes that as soon as a terminal-link entry is programmed to occur, it will occur. However, terminal-link entries are response-dependent and if choice strongly favors the other chain, it may be some time before the less-preferred key is pecked to initiate the transition to the terminal link. The three panels of Figure 5-1 show, for each experiment, a scatterplot with preference on the x-axis (range 0.5-1.0) and the difference between obtained and predicted T_H values on the y-axis. Open symbols represent conditions in which obtained T_H times were significantly different than predicted T_H times. In Experiment 1 (left panel), there was a tendency for deviations to increase as choice proportions increased ($r_{(28)} = .57, p = .001$). This was not true in Experiment 2 ($r_{(16)} = .39, p = .108$), or 3 ($r_{(14)} = -.182, p = .518$), but choice was less extreme in these experiments. Although large differences in predicted and obtained T_H times also occurred in Experiment 2 (middle panel), these differences were not systematic, with Mazur's

Figure 5-1.

Differences between predicted and obtained T_H times across Experiments 1, 2, and 3 for individual subjects plotted against obtained choice proportions. Open symbols indicate conditions in which obtained T_H times differed significantly from predicted times.



method both over- and underpredicting obtained T_H times. Thus, one way that Mazur's predictions might be improved is to incorporate predicted choice proportions into the calculations, with more extreme choice proportions producing longer T_H times. Moreover, given the potential problems with calculating the harmonic means used in T_H , R_R , and R_L , a goal of future concurrent-chains research should be to empirically investigate the conditions under which Mazur's method for estimating harmonic mean times produces accurate and inaccurate predictions. Once this has been done, the adequacy of DRT_H as a quantitative model of choice can be further assessed by comparing its predictions to the results of existing concurrent-chains research.

Although the experiments presented in this document were conducted with nonhumans under controlled laboratory settings, the results of the studies have the potential to inform and improve applied practices. For example, a recent review of translational token economy research (Hackenberg, 2018), an applied technique which

utilizes conditioned reinforcers, states that practitioners can create conditioned reinforcers by pairing neutral stimuli with primary reinforcers or by delivering reinforcement for a response in the presence of a specific stimulus (i.e., training a discriminated operant). While these suggestions are not incorrect, they may fail to utilize the wealth of information on conditioned reinforcement that is summarized by the five quantitative models presented in this document, if the predictions of these models can also be shown to adequately predict human choice. For example, CCM, CDM, DRT_H and HVA all predict that the distribution of delays to primary reinforcement in the presence of the conditioned reinforcer matter. If applied practitioners were to utilize this information, by occasionally delivering immediate primary reinforcers, or other backup reinforcers, in the presence of the conditioned reinforcer, it may allow practitioners to more effectively and efficiently create and maintain conditioned reinforcers. Therefore, a goal of future research should be to test the accuracy of modern quantitative models of conditioned reinforcement in humans and to attempt to incorporate and evaluate their predictions in applied settings.

In conclusion, the results of the three experiments of this dissertation support modifying DRT (Squires & Fantino, 1971) by substituting harmonic means for arithmetic means. Across 10 different experimental conditions, which varied initial-link duration, terminal-link duration, and types of terminal-link schedules of reinforcement, the modified DRT_H made predictions as accurate (HVA), or more accurate (CCM, CDM), than other contemporary models of concurrent-chains choice.

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APPENDICES

APPENDIX A

Table A-1.

Subject, model, free-parameter values (b , a_i , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 1.

Subject	Model	b	a_i , a_T or σ	a_i or a_f	k	% VAC
P1	CCM	-	-	-	-	0.78
	CCM	1.10	-	-	-	0.79
	CCM	-	1.25	-	-	0.81
	CCM	1.10	1.51	-	-	0.93
	CCM	1.10	1.27	-	1.32	0.99
	CDM	-	0.08	-	-	0.92
	CDM	1.10	0.08	-	-	0.92
	CDM	-	0.07	-	0.98	0.92
	CDM	1.10	0.07	-	0.98	0.93
	DRT	-	-	-	-	0.52
	DRT	1.25	-	-	-	0.53
	DRT	-	0.00	-	-	0.85
	DRT	-	-	0.00	-	0.65
	DRT	1.25	0.00	-	-	0.86
	DRT	1.25	-	0.00	-	0.65
	DRT	1.25	0.00	0.00	-	0.88
	DRTH	-	-	-	-	0.94
	DRTH	1.25	-	-	-	0.95
	DRTH	-	1.16	-	-	0.95
	DRTH	-	-	0.72	-	0.94
	DRTH	1.25	1.16	-	-	0.95
	DRTH	1.25	-	0.72	-	0.95
	DRTH	1.25	2.27	2.08	-	0.95
	HVA	-	-	-	-	0.94
	HVA	1.25	-	-	-	0.94
	HVA	-	0.74	-	-	0.95
	HVA	1.25	0.74	-	-	0.95

Table A-2.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 1.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P8	CCM	-	-	-	-	0.78
	CCM	1.37	-	-	-	0.81
	CCM	-	1.19	-	-	0.83
	CCM	1.37	1.19	-	-	0.86
	CCM	1.37	1.23	-	0.92	0.86
	CDM	-	0.14	-	-	0.79
	CDM	1.37	0.14	-	-	0.82
	CDM	-	0.13	-	0.90	0.83
	CDM	1.37	0.13	-	0.90	0.86
	DRT	-	-	-	-	-0.18
	DRT	1.02	-	-	-	-0.18
	DRT	-	0.00	-	-	0.62
	DRT	-	-	0.00	-	0.11
	DRT	1.02	0.00	-	-	0.62
	DRT	1.02	-	0.00	-	0.11
	DRT	1.02	0.00	0.00	-	0.67
	DRTH	-	-	-	-	0.75
	DRTH	1.02	-	-	-	0.75
	DRTH	-	1.51	-	-	0.78
	DRTH	-	-	0.08	-	0.77
	DRTH	1.02	1.51	-	-	0.78
	DRTH	1.02	-	0.08	-	0.77
	DRTH	1.02	1.66	1.16	-	0.78
	HVA	-	-	-	-	0.70
	HVA	1.02	-	-	-	0.70
	HVA	-	0.40	-	-	0.78
	HVA	1.02	0.40	-	-	0.78

Table A-3.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 1.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P20	CCM	-	-	-	-	0.67
	CCM	0.67	-	-	-	0.69
	CCM	-	0.92	-	-	0.68
	CCM	0.67	0.92	-	-	0.70
	CCM	0.67	0.77	-	1.47	0.76
	CDM	-	0.00	-	-	0.45
	CDM	0.67	0.04	-	-	0.58
	CDM	-	0.00	-	1.00	0.45
	CDM	0.67	0.04	-	1.00	0.58
	DRT	-	-	-	-	-0.25
	DRT	0.85	-	-	-	-0.25
	DRT	-	0.00	-	-	0.33
	DRT	-	-	0.00	-	-0.07
	DRT	0.85	0.00	-	-	0.33
	DRT	0.85	-	0.00	-	-0.07
	DRT	0.85	0.00	0.00	-	0.42
	DRTH	-	-	-	-	0.66
	DRTH	0.85	-	-	-	0.66
	DRTH	-	0.77	-	-	0.68
	DRTH	-	-	3.18	-	0.71
	DRTH	0.85	0.82	-	-	0.69
	DRTH	0.85	-	3.18	-	0.71
	DRTH	0.85	0.93	2.84	-	0.71
	HVA	-	-	-	-	0.73
	HVA	0.85	-	-	-	0.73
	HVA	-	1.08	-	-	0.74
	HVA	0.85	1.08	-	-	0.74

Table A-4.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 1.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P23	CCM	-	-	-	-	0.80
	CCM	0.64	-	-	-	0.83
	CCM	-	0.97	-	-	0.80
	CCM	0.64	0.97	-	-	0.84
	CCM	0.64	0.86	-	1.29	0.87
	CDM	-	0.12	-	-	0.58
	CDM	0.64	0.05	-	-	0.62
	CDM	-	0.12	-	0.27	0.87
	CDM	0.64	0.12	-	0.27	0.90
	DRT	-	-	-	-	-0.27
	DRT	0.75	-	-	-	-0.26
	DRT	-	0.00	-	-	0.35
	DRT	-	-	0.00	-	-0.06
	DRT	0.75	0.00	-	-	0.36
	DRT	0.75	-	0.00	-	-0.05
	DRT	0.75	0.00	0.00	-	0.43
	DRTH	-	-	-	-	0.78
	DRTH	0.75	-	-	-	0.79
	DRTH	-	0.74	-	-	0.86
	DRTH	-	-	3.48	-	0.87
	DRTH	0.75	0.77	-	-	0.89
	DRTH	0.75	-	3.48	-	0.88
	DRTH	0.75	0.79	1.96	-	0.89
	HVA	-	-	-	-	0.81
	HVA	0.75	-	-	-	0.82
	HVA	-	1.17	-	-	0.87
	HVA	0.75	1.17	-	-	0.88

Table A-5.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 1.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P24	CCM	-	-	-	-	0.64
	CCM	0.67	-	-	-	0.65
	CCM	-	0.94	-	-	0.64
	CCM	0.67	0.94	-	-	0.65
	CCM	0.67	0.59	-	2.04	0.88
	CDM	-	0.00	-	-	0.56
	CDM	0.67	0.00	-	-	0.57
	CDM	-	0.00	-	0.00	0.76
	CDM	0.67	0.00	-	0.00	0.77
	DRT	-	-	-	-	0.03
	DRT	0.60	-	-	-	0.05
	DRT	-	0.00	-	-	0.39
	DRT	-	-	0.00	-	0.14
	DRT	0.60	0.00	-	-	0.40
	DRT	0.60	-	0.00	-	0.15
	DRT	0.60	0.00	0.00	-	0.46
	DRTH	-	-	-	-	0.70
	DRTH	0.60	-	-	-	0.71
	DRTH	-	0.73	-	-	0.88
	DRTH	-	-	6.46	-	0.86
	DRTH	0.60	0.73	-	-	0.89
	DRTH	0.60	-	6.46	-	0.87
	DRTH	0.60	0.74	03.48	-	0.91
	HVA	-	-	-	-	0.73
	HVA	0.60	-	-	-	0.74
	HVA	-	1.23	-	-	0.87
	HVA	0.60	1.23	-	-	0.89

Table A-6.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 1.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P1270	CCM	-	-	-	-	0.77
	CCM	1.05	-	-	-	0.77
	CCM	-	1.05	-	-	0.80
	CCM	1.05	1.05	-	-	0.77
	CCM	1.05	0.79	-	1.63	0.90
	CDM	-	0.08	-	-	0.65
	CDM	1.05	0.08	-	-	0.65
	CDM	-	0.06	-	0.41	0.97
	CDM	1.05	0.06	-	0.41	0.97
	DRT	-	-	-	-	0.05
	DRT	1.16	-	-	-	0.06
	DRT	-	0.00	-	-	0.51
	DRT	-	-	0.00	-	0.21
	DRT	1.16	0.00	-	-	0.51
	DRT	1.16	-	0.00	-	0.21
	DRT	1.16	0.00	0.00	-	0.57
	DRTH	-	-	-	-	0.83
	DRTH	1.16	-	-	-	0.83
	DRTH	-	0.73	-	-	0.94
	DRTH	-	-	3.46	-	0.89
	DRTH	1.16	0.74	-	-	0.95
	DRTH	1.16	-	3.46	-	0.89
	DRTH	1.16	0.74	0.34	-	0.96
	HVA	-	-	-	-	0.85
	HVA	1.16	-	-	-	0.85
	HVA	-	1.21	-	-	0.94
	HVA	1.16	1.21	-	-	0.94

APPENDIX B

Table B-1.

Subject, model, free-parameter values (b , a_i , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 2.

Subject	Model	b	a_i , a_T or σ	a_i or a_f	k	% VAC
P1	CCM	-	-	-	-	0.73
	CCM	1.26	-	-	-	0.84
	CCM	-	1.47	-	-	0.83
	CCM	1.26	1.47	-	-	0.94
	CCM	1.26	0.39	-	0.00	0.99
	CDM	-	0.55	-	-	0.75
	CDM	1.26	0.55	-	-	0.86
	CDM	-	10.0	-	8.52	0.88
	CDM	1.26	10.0	-	8.52	0.99
	DRT	-	-	-	-	0.79
	DRT	1.26	-	-	-	0.90
	DRT	-	1.29	-	-	0.84
	DRT	-	-	2.07	-	0.85
	DRT	1.26	1.29	-	-	0.95
	DRT	1.26	-	2.07	-	0.96
	DRT	1.26	0.27	3.85	-	0.97
	DRTH	-	-	-	-	0.86
	DRTH	1.26	-	-	-	0.97
	DRTH	-	1.32	-	-	0.87
	DRTH	-	-	0.87	-	0.87
	DRTH	1.26	1.32	-	-	0.98
	DRTH	1.26	-	0.87	-	0.98
	DRTH	1.26	10.0	1.39	-	0.98
	HVA	-	-	-	-	-1.48
	HVA	1.26	-	-	-	-1.37
	HVA	-	0.00	-	-	-0.45
	HVA	1.26	0.00	-	-	-0.34
	HVA	-	-	-	0.01	0.86
	HVA	1.26	-	-	0.01	0.97
	HVA	-	0.00	-	0.05	0.87
	HVA	1.26	0.00	-	0.05	0.98

Table B-2.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 2.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P8	CCM	-	-	-	-	0.36
	CCM	0.90	-	-	-	0.43
	CCM	-	0.65	-	-	0.54
	CCM	0.90	0.65	-	-	0.62
	CCM	0.90	0.18	-	0.00	0.73
	CDM	-	1.39	-	-	0.64
	CDM	0.90	1.39	-	-	0.72
	CDM	-	10.0	-	4.64	0.65
	CDM	0.90	10.0	-	4.64	0.72
	DRT	-	-	-	-	0.25
	DRT	0.90	-	-	-	0.33
	DRT	-	0.47	-	-	0.58
	DRT	-	-	0.00	-	0.54
	DRT	0.90	0.47	-	-	0.66
	DRT	0.90	-	0.00	-	0.62
	DRT	0.90	0.26	1.45	-	0.66
	DRTH	-	-	-	-	-0.77
	DRTH	0.90	-	-	-	-0.69
	DRTH	-	3.00	-	-	0.15
	DRTH	-	-	0.12	-	0.58
	DRTH	0.90	10.00	-	-	0.41
	DRTH	0.90	-	0.12	-	0.66
	DRTH	0.90	10.00	0.59	-	0.70
	HVA	-	-	-	-	-13.94
	HVA	0.90	-	-	-	-13.86
	HVA	-	0.00	-	-	-9.01
	HVA	0.90	0.00	-	-	-8.94
	HVA	-	-	-	0.00	0.54
	HVA	0.90	-	-	0.00	0.61
	HVA	-	0.00	-	0.01	0.60
	HVA	0.90	0.00	-	0.01	0.68

Table B-3.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 2.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P23	CCM	-	-	-	-	0.38
	CCM	0.82	-	-	-	0.48
	CCM	-	0.92	-	-	0.38
	CCM	0.82	0.92	-	-	0.48
	CCM	0.82	0.27	-	0.00	0.65
	CDM	-	0.80	-	-	0.50
	CDM	0.82	0.80	-	-	0.61
	CDM	-	10.0	-	6.43	0.54
	CDM	0.82	10.0	-	6.46	0.64
	DRT	-	-	-	-	0.41
	DRT	0.82	-	-	-	0.51
	DRT	-	0.80	-	-	0.43
	DRT	-	-	0.55	-	0.42
	DRT	0.82	0.80	-	-	0.53
	DRT	0.82	-	0.55	-	0.52
	DRT	0.82	0.27	2.33	-	0.54
	DRTH	-	-	-	-	0.24
	DRTH	0.82	-	-	-	0.35
	DRTH	-	2.00	-	-	0.43
	DRTH	-	-	0.43	-	0.46
	DRTH	0.82	10.00	-	-	0.60
	DRTH	0.82	-	0.43	-	0.56
	DRTH	0.82	10.00	0.91	-	0.60
	HVA	-	-	-	-	-3.95
	HVA	0.82	-	-	-	-3.85
	HVA	-	0.00	-	-	-2.24
	HVA	0.82	0.00	-	-	-2.14
	HVA	-	-	-	0.00	0.43
	HVA	0.82	-	-	0.00	0.53
	HVA	-	0.00	-	0.02	0.49
	HVA	0.82	0.00	-	0.02	0.59

Table B-4.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 2.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P24	CCM	-	-	-	-	0.60
	CCM	0.88	-	-	-	0.62
	CCM	-	2.19	-	-	0.86
	CCM	0.88	2.19	-	-	0.87
	CCM	0.88	0.59	-	0.00	0.97
	CDM	-	0.56	-	-	0.62
	CDM	0.88	0.56	-	-	0.64
	CDM	-	8.56	-	10.0	0.95
	CDM	0.88	8.56	-	10.0	0.96
	DRT	-	-	-	-	0.67
	DRT	0.88	-	-	-	0.68
	DRT	-	1.69	-	-	0.85
	DRT	-	-	4.24	-	0.89
	DRT	0.88	1.69	-	-	0.87
	DRT	0.88	-	4.24	-	0.91
	DRT	0.88	0.27	6.03	-	0.91
	DRTH	-	-	-	-	0.85
	DRTH	0.88	-	-	-	0.87
	DRTH	-	0.61	-	-	0.90
	DRTH	-	-	1.59	-	0.93
	DRTH	0.88	0.61	-	-	0.92
	DRTH	0.88	-	1.59	-	0.94
	DRTH	0.88	10.00	2.06	-	0.95
	HVA	-	-	-	-	0.51
	HVA	0.88	-	-	-	0.52
	HVA	-	0.00	-	-	0.78
	HVA	0.88	0.00	-	-	0.80
	HVA	-	-	-	0.06	0.93
	HVA	0.88	-	-	0.06	0.94
	HVA	-	0.00	-	0.18	0.94
	HVA	0.88	0.00	-	0.18	0.96

Table B-5.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 2.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P2331	CCM	-	-	-	-	0.53
	CCM	0.89	-	-	-	0.55
	CCM	-	1.46	-	-	0.59
	CCM	0.89	1.46	-	-	0.60
	CCM	0.89	0.39	-	0.00	0.67
	CDM	-	0.58	-	-	0.55
	CDM	0.89	0.58	-	-	0.57
	CDM	-	10.0	-	8.57	0.64
	CDM	0.89	10.0	-	8.57	0.66
	DRT	-	-	-	-	0.57
	DRT	0.89	-	-	-	0.59
	DRT	-	1.29	-	-	0.60
	DRT	-	-	2.03	-	0.60
	DRT	0.89	1.29	-	-	0.62
	DRT	0.89	-	2.03	-	0.62
	DRT	0.89	0.26	3.83	-	0.63
	DRTH	-	-	-	-	0.62
	DRTH	0.89	-	-	-	0.64
	DRTH	-	1.35	-	-	0.63
	DRTH	-	-	0.88	-	0.62
	DRTH	0.89	1.35	-	-	0.64
	DRTH	0.89	-	0.88	-	0.64
	DRTH	0.89	10.00	1.35	-	0.65
	HVA	-	-	-	-	-0.90
	HVA	0.89	-	-	-	-0.88
	HVA	-	0.00	-	-	-0.22
	HVA	0.89	0.00	-	-	-0.20
	HVA	-	-	-	0.01	0.62
	HVA	0.89	-	-	0.01	0.63
	HVA	-	0.00	-	0.05	0.63
	HVA	0.89	0.00	-	0.05	0.65

Table B-6.

Subject, model, free-parameter values (b , a_b , a_T , a_i , a_f , σ , k) and percent variance accounted for (%VAC) in Experiment 2.

Subject	Model	b	a_b , a_T or σ	a_i or a_f	k	% VAC
P4740	CCM	-	-	-	-	0.62
	CCM	0.81	-	-	-	0.69
	CCM	-	1.50	-	-	0.70
	CCM	0.81	1.50	-	-	0.78
	CCM	0.81	0.48	-	0.15	0.81
	CDM	-	0.54	-	-	0.63
	CDM	0.81	0.54	-	-	0.70
	CDM	-	0.98	-	1.41	0.74
	CDM	0.81	0.98	-	1.41	0.81
	DRT	-	-	-	-	0.67
	DRT	0.81	-	-	-	0.74
	DRT	-	1.31	-	-	0.71
	DRT	-	-	2.15	-	0.72
	DRT	0.81	1.31	-	-	0.78
	DRT	0.81	-	2.15	-	0.79
	DRT	0.81	0.27	3.93	-	0.80
	DRTH	-	-	-	-	0.73
	DRTH	0.81	-	-	-	0.80
	DRTH	-	1.26	-	-	0.73
	DRTH	-	-	0.89	-	0.73
	DRTH	0.81	1.26	-	-	0.81
	DRTH	0.81	-	0.89	-	0.80
	DRTH	0.81	10.00	1.37	-	0.81
	HVA	-	-	-	-	-1.10
	HVA	0.81	-	-	-	-1.03
	HVA	-	0.00	-	-	-0.29
	HVA	0.81	0.00	-	-	-0.22
	HVA	-	-	-	0.01	0.73
	HVA	0.81	-	-	0.01	0.80
	HVA	-	0.00	-	0.05	0.74
	HVA	0.81	0.00	-	0.05	0.81

CURRICULUM VITAE

Jay E. Hinnenkamp

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Education

2020	Behavior Analysis	Utah State University	Ph.D.
2013	Behavior Analysis	University of North Texas	M.S.
2005	Biology	University of Minnesota-Morris	B.A.
2005	Psychology	University of Minnesota-Morris	B.A.

Teaching & Professional Experience

2020	Introduction to Psychology Instructor, Department of Psychology, Utah State University (Spring 2020) <ul style="list-style-type: none"> • Lecture, grade, facilitate discussion in an undergraduate course
2019 – 2020	Instructor, Autism Support Services: Education, Treatment & Training, Department of Special Education and Rehabilitation, Utah State University <ul style="list-style-type: none"> • Provide 1-on-1 early intensive behavioral intervention to children diagnosed with autism
2018 – 2019	Native American Student Mentorship Program, Utah State University <ul style="list-style-type: none"> • Mentor students completing an animal learning project
2017 – 2019	Analysis of Behavior: Basic Principles Instructor, Department of Psychology, Utah State University (Fall 2017, Spring 2018, Fall 2018 & Spring 2019) <ul style="list-style-type: none"> • Lecture, grade, facilitate discussion in an undergraduate course • Supervise and mentor 7-9 undergraduate Tas in accompanying undergraduate animal laboratory

- 2013 – 2019** Analysis of Behavior: Basic Principles Online Instructor, Department of Psychology, Utah State University (Fall 2013, Spring 2014, Summer 2014, Fall 2014, Spring 2015, Summer 2015, Fall 2015, Spring 2016, Summer 2016, Fall 2016, Spring 2017, Summer 2017, Summer 2018, Summer 2019, Fall 2019)
- Lecture, grade, facilitate discussion in an online undergraduate course
- 2011** Theory & Philosophy Teaching Assistant, Department of Behavior Analysis, University of North Texas
- Grade, provide feedback, and answer questions regarding graduate level course
- 2010** Quantitative Analysis Teaching Assistant, Department of Behavior Analysis, University of North Texas
- Grade, provide feedback, and answer questions regarding graduate level course
- 2010 – 2013** Behavior Principles I & II Teaching Fellow, Department of Behavior Analysis, University of North Texas
- Lecture, grade, and facilitate discussion in an undergraduate course
- 2009 – 2013** Distance Learning Teaching Assistant, Department of Behavior Analysis, University of North Texas
- Grade, provide feedback, and create material for Techniques in Applied Behavior Analysis, Research and Application in Behavior Analysis, and Current Issues in Behavioral Treatment of Autism courses
- 2005 – 2008** Behavior Therapist, Partners in Excellence, North St. Paul, Minnesota
- Construct, modify, and direct early intensive behavioral intervention to children diagnosed with autism

Research Experience

- 2019 – 2020** Research Assistant, Dr. Higbee, Department of Special Education, Utah State University
- Research on behavioral variability in play in children diagnosed with autism and research on computerized staff training
- 2014 – 2020** Research Assistant, Dr. Madden, Department of Psychology, Utah State University
- Research on delay discounting in rats, behavioral economic research on suboptimal choice in pigeons, and concurrent chains research in pigeons and rats
- 2008 – 2013** Research Assistant, Dr. Vaidya, Department of Behavior Analysis, University of North Texas
- Research on stimulus equivalence and symmetrical responding in pigeons
- 2008** Research Assistant, Dr. Ingvarsson, Child Study Center, Fort Worth, Texas
- Research on echoic, tact, and textual prompts in children diagnosed with autism

Presentations

Hinnenkamp, J. & Madden, G. (2018). *Harmonic delay reduction*. Symposium presented at the Winter Conference on Animal Learning and Behavior, Logan UT. Presenting Author.

Hinnenkamp, J. & Madden, G. (2016). *The effects of probabilistic outcomes on nonhuman choice*. Symposium presented at the 40th annual convention of the Association for Behavior Analysis, Chicago, IL. Presenting Author.

Hinnenkamp, J. & Madden, G. (2015). *Suboptimal choice: A unit-price analysis*. Poster presented at the 38th annual meeting of the Society for Quantitative Analyses of Behavior, San Antonio, TX. Presenting Author.

Hinnenkamp, J. & Vaidya, M. (2013). *Derived relational responding in nonhumans: New directions in the study of symmetry and equivalence*. Symposium presented at the 39th annual convention of the Association for Behavior Analysis, Minneapolis, MN. Presenting Author.

Hinnenkamp, J. & Vaidya, M. (2012). *Recent developments in the study of derived relational responding with nonhumans*. Symposium presented at the 38th annual convention of the Association for Behavior Analysis, Seattle, WA. Presenting Author.

Hinnenkamp, J. & Vaidya, M. (2011). *Symmetry in the pigeon? A systematic replication of Urcuioli (2008).* Symposium presented at the 6th annual ABAI international conference, Granada, Spain. Presenting Author.

Hinnenkamp, J. & Vaidya, M. (2011). *Symmetry in the pigeon? A systematic replication of Urcuioli (2008).* Symposium presented at the 37th annual convention of the Association for Behavior Analysis, Denver, CO. Presenting Author.

Ingvarsson, E. T., Le, D. D., Johnson, K. J., & **Hinnenkamp, J.** (2010). *The effectiveness of and preference for echoic, tact, and textual prompts for establishing intraverbal responding in children with autism.* Symposium presented at the 36th annual convention of the Association for Behavior Analysis, San Antonio, TX.

Tucker, K., **Hinnenkamp, J.**, Morford, Z., Kalafut, K., & Rosales-Ruiz, J. (2009). *Assessing and expanding the behavior of primates in captivity.* Poster presented at the 35th annual convention of the Association for Behavior Analysis, Phoenix, AZ.

Publications

Hinnenkamp, J. E. (Manuscript in Progress) Concurrent Chains: A systematic review and evaluation of current quantitative models.

Hinnenkamp, J. E. & Madden, G. J. (Manuscript in Progress) The effects of initial-link duration on rats responding under concurrent-chains schedules.

Peck, S., Rung, J. M., **Hinnenkamp, J. E.**, & Madden, G. J. (accepted pending revisions). Reducing impulsive choice: VI: Delay-exposure training reduces aversion to delay-signaling stimuli. *Psychology of Addictive Behaviors*.

Rung, J. M., Peck, S., **Hinnenkamp, J. E.**, Preston, E., & Madden, G. J. (2019). Changing delay discounting and impulsive choice: Implications for addictions, prevention, and human health. *Perspectives on Behavioral Science*, 121.

Renda, R. C., Rung, J. M., **Hinnenkamp, J. E.**, Lenzini, S. N., & Madden, G. J. (2018). Impulsive choice and preexposure to delays: iv. effects of delay-and immediacy-exposure training relative to maturational changes in impulsivity. *Journal of the experimental analysis of behavior*, 109(3), 587-599.

Hinnenkamp, J. E., Shahan, T. A., & Madden, G. J. (2017). How suboptimal is suboptimal choice?. *Journal of the experimental analysis of behavior*, 107(1), 136-150.

Stein, J. S., Renda, C. R., **Hinnenkamp, J. E.**, & Madden, G. J. (2014). Impulsive choice, alcohol consumption, and pre-exposure to delayed rewards: II. Potential mechanisms. *Journal of the experimental analysis of behavior*, 103, 33-49.

Hinnenkamp, J. (2011). The 2011 Presidential Scholar's Essay: Applied Behavior Analysis Outside of the Research Setting. *Inside behavior analysis*, 2, 10-11.

Hinnenkamp, J. (2009). Teens "Click" Their Smoking Habits. *Behavior analysis digest international*, 21, 8-9.

Grant Writing Experience

2016	Innovative Student Research Grant (Not Awarded) Society of the Advancement of Behavior Analysis “Conditional delay discounting: The role of probability in impulsivity.”
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Honors

2020	Graduate Student Teacher of the Year
2016	Graduate Student Travel Award
2015	Ray Alvord Scholarship Recipient
2015	Graduate Student Travel Award
2011	ABAI Student Committee’s Presidential Scholar Recipient
2011	Outstanding Graduate Student in Behavior Analysis Award
2008 – 2010	Donald L. Whaley Memorial Scholarship Recipient

Professional Memberships

2009 – Present	Society for the Quantitative Analysis of Behavior
2008 – Present	Association for Behavior Analysis International
2008 – 2013	Texas Association for Behavior Analysis