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The Gambler's Edge - A Theoretical Framework to Trading Securities

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THE GAMBLER'S EDGE—A THEORETICAL FRAMEWORK TO TRADING SECURITIES

by

Matthew Haines

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in

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ABSTRACT

The Gambler's Edge—A Theoretical Framework to Trading Securities

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Like gamblers, retail investors seeking excess returns in financial markets are prone to miscalculation and their models are often misspeciffied. Forecasting asset prices is extremely difficult in the long run and nearly infeasible in the short run. Additionally, retail investors are likely to be at a disadvantage both technologically and informationally—rarely will they be ahead of the curve. With these disadvantages and the difficulty of predicting future outcomes, retail investors may come to view prices as unpredictable and random in nature, like a roll of the dice. This theoretical research explores a possible investing methodology (derived from gambling principles) should an investor choose to accept asset prices as random. It displays the possibility that investors can harness the power of compound returns if they possess statistically advantageous strategies, and if they trade as frequently as possible. This research then continues to exhibit investors can reduce their risk while simultaneously increasing their expected returns should they successfully follow this theoretical framework.

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PUBLIC ABSTRACT

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Like gamblers, retail investors seeking excess returns in financial markets are prone to miscalculation and their models are often misspeciffied. Forecasting asset prices is extremely difficult in the long run and nearly infeasible in the short run. Additionally, retail investors are likely to be at a disadvantage both technologically and informationally—rarely will they be ahead of the curve. With these disadvantages and the difficulty of predicting future outcomes, retail investors may come to view prices as unpredictable and random in nature, like a roll of the dice. This theoretical research explores a possible investing methodology (derived from gambling principles) should an investor choose to accept asset prices as random. It displays the possibility that investors can harness the power of compound returns if they possess statistically advantageous strategies, and if they trade as frequently as possible. This research then continues to exhibit investors can reduce their risk while simultaneously increasing their expected returns should they successfully follow this theoretical framework.

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Matthew Haines

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CHAPTER 1

INTRODUCTION

"Money won is twice as sweet as money earned." – Paul Newman, The Color of Money

1.1. Gambling and Trading Securities

Forecasting asset prices is extremely difficult in the long run, and nearly impossible in the short run. According to the Efficient Market Hypothesis of Fama (1965) asset prices fully reflect all available information.[1] A proposition from these models is that asset prices follow a random walk, and that profits derived from timing the markets are due entirely to chance. Therefore, like gamblers, retail investors[2] who seek excess returns in financial markets through day trading techniques often underperform simple benchmarks.[3] That does not, however, mean retail traders will not continue try to return a premium on their investments.

During the early stages of the COVID-19 pandemic many citizens were forced to stay within the confines of their homes, and many were left with copious amounts of time to themselves. In a study conducted by Chiah and Zhong (2020) they found the pandemic led to a large increase in new retail traders in stock markets across the globe. They also found trading volume disproportionately increased in economies with more gambling opportunities.[4] This puzzling correlation has interesting implications. It could imply trading financial securities (including

1. Eugene F. Fama, "The Behavior of Stock-Market Prices," *The Journal of Business* 38, no. 1 (1965): p. 34, https://doi.org/10.1086/294743.

2. Loosely defined as individual, non-professional investors.

3. Brad M. Barber et al., "The Cross-Section of Speculator Skill: Evidence from Day Trading," *Journal of Financial Markets* 18 (2014): pp. 1-24, https://doi.org/10.1016/j.finmar.2013.05.006.

4. Mardy Chiah and Angel Zhong, "Trading from Home: The Impact of Covid-19 on Trading Volume around the World," Finance Research Letters (Elsevier, September 28, 2020).

cryptocurrencies) is a substitute to gambling activities, especially when worldwide lockdowns limit the access to casinos.[5]

The substitutional relationship between gambling and trading financial securities is not an unstudied phenomenon. For instance, Gao and Lin (2014) found when the Taiwanese lottery jackpot exceeds TN$500 million, trading volume in stocks, "decreases...between 6.8% and 8.6% among lottery-like stocks."[6] Cox et. al. studied how poor performance in football betting led to "spillover" into the other stock markets; that is, they would attempt to offset their losses from gambling by turning to lottery-like stocks.[7] Similarly, Kumar (2009) researched the relationship between an individual's propensity to gamble and their propensity to invest in risky assets. He found, "there are striking similarities between the behavior of state lottery players and individual investors who invest disproportionately more in stocks with lottery features."[8] It is clear some view investing as a form of gambling. It is no surprise, then, that profit-seeking firms have stepped in to facilitate the appetites of risk-seekers.

The Securities and Exchange Commission (SEC) recently published a report regarding the trading mania involving “meme stocks” such as GameStop in early 2021; more specifically, they investigated the incentives within market structures that may have led to such sudden and massive price volatility. The SEC claims there is more market participation today partly because,

5. Paul Delfabbro et al., “Cryptocurrency Trading, Gambling and Problem Gambling,” *Addictive Behaviors* 122 (2021): p. 107021, https://doi.org/10.1016/j.addbeh.2021.107021.

6. Xiaohui Gao and Tse-Chun Lin, “Do Individual Investors Treat Trading as a Fun and Exciting Gambling Activity? Evidence from Repeated Natural Experiments,” *The Review of Financial Studies* 28, no. 7 (2014): p. 2129, https://doi.org/10.1093/rfs/hhu075.

7. Justin Cox, Adam Schwartz, and Robert Van Ness, “Does What Happen in Vegas Stay in Vegas? Football Gambling and Stock Market Activity,” *Journal of Economics and Finance* 44, no. 4 (August 2020): pp. 724-748, https://doi.org/10.1007/s12197-020-09513-9.

8. Alok Kumar, “Who Gambles in the Stock Market?,” *The Journal of Finance* 64, no. 4 (2009): pp. 1889-1933, https://doi.org/10.1111/j.1540-6261.2009.01483.x.

“commissions have fallen or been eliminated.” Not only has trading online become inexpensive, but digital engagement practices have been designed to entice retail investors to trade more often. Some examples include, “features that allow customers to discuss stocks and trades and display their trades and portfolios to others. A number of features, which broadly include behavioral prompts, differential marketing, game-like features, and other design elements or features, appear designed to engage individual investors.”[9] The structure of security markets has evolved to cater to more risk-seeking, inexperienced investors. Today, possibly more so than ever, financial markets are the world's largest casinos.

1.2. The Fallible Human in a Disadvantageous Setting

If investing is a valid substitution for gambling, then the same blunders affecting millions of gamblers can affect millions of investors. These cognitive blunders have been studied in detail by Daniel Kahneman who won the Nobel Prize in Economics for, "having integrated insights from psychological research into economic science, especially concerning human judgment...under uncertainty." [10] Kahneman, along with his now deceased colleague Amos Tversky, studied cognitive biases, or heuristics affecting human decision-making such as confirmation bias, loss aversion, prospect theory, and more.[11] They found humans, left to their own devices, are prone to miscalculation.

In addition to having to contend with their own cognitive biases, investors participate in financial markets which have been analyzed by millions of individuals seeking to return higher

9. “Staff Report on Equity and Options Market Structure Conditions in Early 2021,” Staff Report on Equity and Options Market Structure Conditions in Early 2021 § (2021). https://www.sec.gov/News/Press-Release/2021-212 (2021).

10. “Daniel Kahneman Facts - The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2002,” NobelPrize.org, accessed July 15, 2021, https://www.nobelprize.org/prizes/economic-sciences/2002/kahneman/facts/.

11. Daniel Kahneman, Thinking Fast and Slow (New York, New York: Farrar, Straus and Giroux, 2011).

profits.[12] Today's markets contain investors with tremendous amounts of capital allowing them to invest in technology and research. Many institutions now rely on algorithms created by "quants" who often have advanced degrees, and they utilize ultramodern computers to process massive amounts of data, and to trade at incredible frequencies.[13] With their technological and financial clout, it is no surprise large institutions have been shown to have asymmetric information advantages.[14] The investing game has and will likely continue to change at a rapid pace to the detriment of the retail investor.

In 1999 a study found 2.5 million U.S. adults were estimated to be pathological gamblers with another 3 million problem gamblers as defined by the American Psychiatric Association.[15] This study did not include investors who purchase risky assets, or day trade.[16] Later, Markovi et. al. (2012) surveyed online investors to see if addiction was prevalent among online traders. They discovered 81% of the respondents, "met the [strict] criteria for the existence of addiction," while 96% met a loose criterion for addiction.[17] The addictive nature of gambling, it seems, is present in online trading. Addiction, left unchecked, has been shown to have the potential of financial ruin for compulsive gamblers. It is unlikely retail investing are different. Because of the addictive nature of trading, human biases, and asymmetric information disadvantages it is estimated 90% of day traders

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12. Maneet Ahuja, *Alpha Masters: Unlocking the Genius of the World's Top Hedge Funds* (John Wiley, 2014).

13. Scott Patterson, *The Quants: How a Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It* (New York, New York: Crown Business, 2010).

14. Francisco Javier Forcadell, Elisa Aracil, and Fernando Úbeda, "The Impact of Corporate Sustainability and Digitalization on International Banks' Performance," *Global Policy* 11, no. S1 (2020): pp. 18-27, https://doi.org/10.1111/1758-5899.12761.

15. Melissa Schettini Kearney, "The Economic Winners and Losers of Legalized Gambling," *National Tax Journal* 58, no. 2 (January 2005): pp. 281-302, https://doi.org/https://www.nber.org/papers/w11234.

16. Day Traders, as defined by the U.S. Securities and Exchange Commission, "rapidly buy, sell and short-sell stocks throughout the day in the hope that the stocks continue climbing or falling in value...allowing them to lock in quick profits. Day trading is extremely risky and can result in substantial financial losses."

17. Markovic, Hrvoje, Nora Nikolac, Mara Tripković, Ivana Haluga-Golubović and Zaim Čustović. "Connection between Addictive Behavior and Investing on the Stock Market in Croatia." *Alcoholism and psychiatry research* 48 (2012): 69-80.

are unsuccessful.[18] Like gamblers in a casino, most retail investors are destined for failure in the long run.[19]

1.3. Financial Models and Uncertainty

Lars Peter Hansen was a corecipient of the 2013 Nobel Prize in Economics for his work regarding model uncertainty.[20] Financial and economic models have progressively increased in complexity as economic agents and policymakers attempt to understand economic systems and forecast future outcomes. When one creates a model, particularly models where imperfect agents are involved, there are two main types of uncertainty: uncertainty inside of a model, and outside.[21]

Human agents operate in economic environments in which they have imperfect knowledge. Each individual has their own unique belief and value system which are difficult to capture. To ease the complexity of such issues economists have made simplifying assumptions within their models. For example, when modeling basic consumer choice, economists assume perfect information.[22] Clearly this is unrealistic. An economic agent's entire choice set can be unknown, outcomes can be uncertain, preferences may not yet be known, and other participants' desires are a mystery. In addition to uncertainty within a model, outside factors affect the model's efficacy.

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18. Neale Godfrey, "Day Trading: Smart or Stupid?" Forbes (Forbes Magazine, June 29, 2021), https://www.forbes.com/sites/nealegodfrey/20178/07/16/day-trading-smart-or-stupid/#1839e9c81007.

19. Doojin Ryu, "The Profitability of Day Trading: An Empirical Study Using High-Quality Data," *Investment Analysts Journal* 41, no. 75 (May 2012): pp. 43-54, https://www.researchgate.net/publication/279699523_The_profitability_of_day_trading_An_empirical_study_using_high-quality_data.

20. "Bio," Lars Peter Hansen Official Website, accessed August 22, 2021, http://larspeterhansen.org/meet-lars/bio/.

21. *Consequences of Uncertainty – Lars Peter Hansen*, *YouTube* (YouTube, 2014), https://www.youtube.com/watch?v=pK0hlBTyyLk. (47 minutes, 42 seconds).

22 . Jonathan Levin and Paul Milgrom, "Introduction to Choice Theory - Stanford University," October 2004, https://web.stanford.edu/~jdlevin/Econ%20202/Choice%20Theory.pdf.

Since the 2007 Financial Crisis, the term "systematic risk" has become prevalent in economic discussions; "there is no single agreed definition of systemic risk, but it refers generally to the risk that the financial system as a whole, or important parts of it, seize up in a crisis."[23] More generally, systemic risk includes the risk of "shocks" to systems such as a crisis, new and radical tax structures, or war. There are also other outside variables that may affect a model. Culture, technology, policy, and institutions are dynamic.[24] Even if a static model is constructed perfectly, the model may become obsolete.

Then there is the ambiguity the model designers, themselves, must traverse. Should two economists create the same model, one may have full confidence in the outputs whereas the other may not. With uncertainty stacked upon uncertainty, it is no wonder Hansen is concerned with, "overconfidence in a particular model or perspective of the economic system."[25] A retail investor attempting to create a model in which they earn a premium will likely misspecify their model, and like a gambler, their overconfidence may result in failure. Friedrich Hayek, another Nobel Laureate, may have put it best when he stated in his Nobel Lecture:

> "While in the physical sciences it is generally assumed...that any important factor...will itself be directly observable and measurable, in the study of such complex phenomena as the market, which depend on the actions of many individuals, all the circumstances which will determine the outcome of a process...will hardly ever be fully known or measurable."[26]

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23. Douglas J. Elliot, "Systemic Risk and the Asset Management Industry" (Brookings Institution, May 2014), https://www.brookings.edu/wp-content/uploads/2016/06/systemic_risk_asset_management_elliott.pdf.

24. Gérard Roland, "Fast-Moving and Slow-Moving Institutions," *Institutional Change and Economic Behaviour*, 2008, pp. 134-159, https://link.springer.com/chapter/10.1057%2F9780230583429_7.

25. Lars Peter Hansen, "Uncertainty in Economic Analysis and the Economic Analysis of Uncertainty," *KNOW: A Journal on the Formation of Knowledge* 1, no. 1 (2017): pp. 171-197, https://doi.org/https://www.journals.uchicago.edu/doi/10.1086/692519.

26. "Friedrich Von Hayek Prize Lecture," NobelPrize.org, accessed July 2, 2021, https://www.nobelprize.org/prizes/economic-sciences/1974/hayek/lecture/.

1.4. Nothing More than a Game of Chance

Section 1.1 shows evidence of a modern relationship between trading financial securities and gambling. Section 1.2 highlights the innate heuristics humans have when making judgements leading miscalculations. Lastly, Section 1.3 reviews the imperfections of economic models involving fallible humans. Given the extreme difficulty of precisely forecasting security prices it may be prudent to adhere to Confucius' claim, "real knowledge is to know the extent of one's ignorance."[27] In accepting their own ignorance retail investors may come to view trading as a game of chance, and they may come to see themselves as the disadvantaged gamblers. Given this, why should they not approach trading in the same manner as a composed, calculative gambler who ends with a profit? The remainder of this study explores a possible methodology for ambitious retail investors to seek excess returns when viewing asset prices as random in nature.

27. Lily Ng, "'Real Knowledge Is to Know the Extent of One's Ignorance'," Teachers College - Columbia University, May 10, 2012, https://icccr.tc.columbia.edu/real-knowledge-is-to-know-the-extent-of-ones-ignorance-confucius-says-the-effects-of-culture-on-contradiction-and-conflict-management.

CHAPTER 2

GENERAL FORMULATIONS

"Compound interest is the eighth wonder of the world." - Albert Einstein

2.1. General Structure

The stock market has returned a yearly rate of 10% on average.[28] To put this into perspective, if a worker were to invest $500 at the end of every month for 40 years into a Roth IRA, the future value would be around $3,162,040. This calculation assumes the worker buys and holds onto their assets until the end of the 40 years. They never sold throughout that period; but what if they had?

Assume Investor A buys $10,000 worth of assets with a guaranteed return of 10% per year. They choose to sell those assets after a year leading to a final balance of $11,000. The change in their starting balance and their ending balance is linear. Now, assume Investor B buys the same assets, but they choose to sell them after six months. After this halfway sale Investor B will have made 5% leading to a balance of $10,500. Then, with that new balance, they immediately repurchase the same assets. If they sell again at the end of the year their final balance will be $11,025 which is greater than Investor A's $11,000.

To further exaggerate the disparity, say Investor B was able to participate in these transactions weekly for ten years. Figure 1 displays the results.

28. "The Reality of Investment Risk," The Reality of Investment Risk | FINRA.org, accessed November 15, 2021, https://www.finra.org/investors/learn-to-invest/key-investing-concepts/reality-investment-risk.

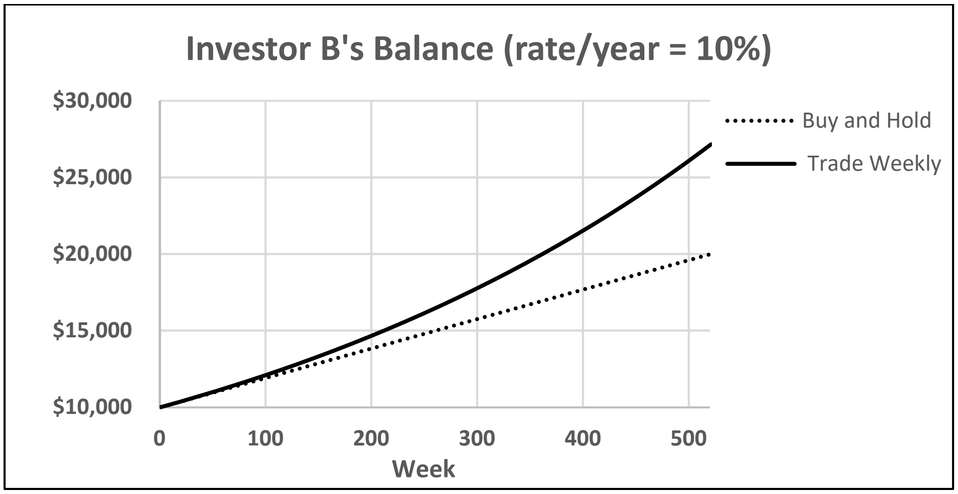

Figure 1. Positive Strategy: Buy and Hold vs. Weekly Trade Example

An easy way to view this is compound interest accruing in a traditional savings account. Each new balance following an interest payment has more money earning a return for the following period. This same principle can be applied to investing strategies such as day trading. It is critical to note, however, that the expected rate of return was a positive 10% per year in this example. What if the expected rate of return were negative 10% per year? Figure 2 displays the results.

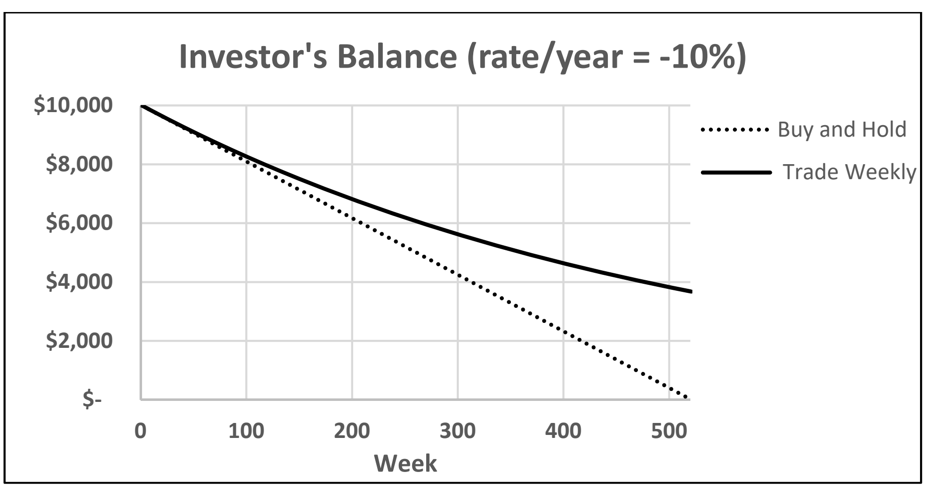

Figure 2. Negative Strategy: Buy and Hold vs. Weekly Trade

Should a strategy with a negative expected return be utilized, the investor's balance will decrease. What is interesting and possibly counterintuitive, though, is the buy-and-hold strategy decreases faster than the trading weekly strategy. This is because after each weekly trade, there is less of a balance to lose in terms of the negative 10% yearly rate. Given the exact same parameters, it seems the buy-and-hold strategy is strictly dominated by the strategy of trading frequently. This statement is powerful, but it can be misleading.

Strategies that provide fixed returns are easy to comprehend and monitor. Many investors operate in environments of extreme uncertainty, and as previously shown, humans are prone to miscalculation. Retail investors may not even know the expected rate of return of their strategies, or they may not have consistent strategies at all. They may buy and sell on a whim like a gambler at the slots.

The Gambler's Edge (TGE) is an investing methodology which aims to provide retail investors with a regimented, calculative approach to trading that allows them to harness the power of compounding if prices are assume to be random. Below are the basic steps in TGE:

1. Hunt for Winning Strategies
2. Combine Winning Strategies into a Portfolio
3. Calculate Expected Balance of the Portfolio
4. Calculate the Probability the Portfolio Will Result in a Net Profit

2.2. Hunting for Winning Strategies

Prior to searching for strategies, it is important to highlight the famous work of Eugene Fama. As previously mentioned, Fama pioneered the theory of efficient markets which has long been utilized in the field of Finance. He, "demonstrated that stock price movements are impossible to predict in the short-term and that new information affects prices almost immediately, which means that the market is efficient."[29] If a system is efficient, then there will be no advantageous strategies within that system that a trader could implement that will give them an edge.

In contrast, "Robert Shiller discovered that stock prices can be predicted over a longer period, such as over the course of several years...Robert Shiller's conclusion was therefore that the market is inefficient."[30] Today's competitive markets had made information readily available supporting Fama's claims. Given this, if there were advantageous strategies they would be exceptionally difficult to find, yet the market is filled with fallible humans. Likewise, algorithms are also created by these fallible humans. Even though markets are often considered efficient, they are imperfect like their participants. Knowing markets will never truly be perfect, especially in the long-run, there must be advantageous strategies that can be found.

As shown in the previous section, an investor can harness the power of compounding if they have a strategy with a positive, constant rate of return. Unfortunately, gamblers and traders deal with uncertain returns making the analysis of strategies more difficult. Uncertain returns are expressed in terms of averages or expected rates of returns. In both gambling and in investing expected rates of returns can be expressed as follows:

$$E(r_s) = p(w_s)(\overline{r_s}|w) - p(l_s)(\overline{r_s}|l) \tag{1}$$

$Where$:

$r = rate\ of\ return\ per\ hit$[31]
$s = the\ s^{th}\ strategy\ out\ of\ a\ set\ of\ strategies$
$w = win$
$l = loss$
$p(w_s) = \frac{Winning\ Hits_s}{Total\ Hits_s}$, and $p(l_s) = \frac{Losing\ Hits_s}{Total\ Hits_s}$

29. "The Nobel Prize: Eugene F. Fama Facts," NobelPrize.org, accessed May 21, 2021, https://www.nobelprize.org/prizes/economic-sciences/2013/fama/facts/.

30. "The Nobel Prize: Robert J. Shiller Facts," NobelPrize.org, accessed May 21, 2021, https://www.nobelprize.org/prizes/economic-sciences/2013/shiller/facts/.

31. A hit is defined as a completed transaction or iteration. The parameters of this definition are determined by the trader.

The output can roughly be interpreted as the expected, or average rate of return per iteration. A wise gambler understands this formula in its entirety. They know a positive output indicates an advantageous strategy, whereas a negative output indicates a disadvantageous strategy. A simple average could be utilized, but this formula provides additional insights into the nature of the expected return. For example, if a strategy has a 99% of making a dollar, and 1% of losing $90 hey can expect to make around $.10 per iteration. They have the advantage. They also understate they will win often, but when they lose it will be significant. This equation is an integral piece to discovering advantageous strategies.

There are thousands of different trading strategies, many of which have been studied extensively.[32] Regardless of the strategy type, Equation (1) would highlight if that strategy was truly advantageous. Given a retail investor has access to the necessary data, they could conduct a back-test to see if a certain strategy would, indeed, be significantly advantageous. If the results from the back-test lead to a significantly positive output for Equation (1) then that strategy might be advantageous. For the remainder of this essay the term "winning strategy" will be utilized to describe strategies that have positive expected returns.

2.3. Combining Winning Strategies into a Portfolio

Once winning strategies are found and verified, they can be combined into a portfolio of strategies analogous to a casino's floor full of advantageous games. Each game on its own is advantageous, but by combining many types of games a casino is able to reach more players and play more often. Assuming each strategy has a different expected return, the trader can take the weighted average of the strategies' returns based on how many hits each strategy provides in their

32. Markos Katsanos, Intermarket Trading Strategies (John Wiley & Sons, 2009).

portfolio. The result is an expected return for the overall portfolio represented by π. Below is the formula for a portfolio's weighted average of the expected return per hit.

$$E(R_\pi) = \sum_{s=1}^{n} \frac{[E(r_s)]m_s h_s}{H} \tag{2}$$

$Where$:

$R = rate\ of\ return\ per\ hit\ for\ a\ portfolio$
$n = total\ number\ of\ strategies$
$m = margin\ or\ leverage\ associated\ with\ the\ s^{th}\ strategy\ (if\ none, m = 1)$
$$H = \sum_{s=1}^{n} \bar{h}_s$$

A note should be made regarding H. $\bar{h}_i$ is the number of hits a strategy provides after it is combined with the other strategies in a portfolio. This is an important distinction because there is the possibility hit opportunities from different strategies occur simultaneously. For Equation (2) it is assumed an investor chooses to place their balance into only one hit, and the other is ignored. Given this stipulation, it would be wise for an investor to always choose to trade the strategy with the higher $E(r_s)$ should they be forced to decide between strategies.

To do this they could rank each strategy by $E(r_s)$. Then they could evaluate how many hits would occur from a portfolio of strategies organized by rank. This prevents the double counting of hits which would positively bias the equations in the later sections. Additionally, it is essential all calculations are dealt with in the same time frame. For example, if the investor wants to know R per year, then all variables must be in terms of per year.

2.4. Calculating the Expected Balance of the Portfolio

Recall section 2.1 which highlights the compounding nature of TGE given a portfolio of strategies has a positive expected rate of turn. If an investor wanted to estimate the balance of their portfolio after a certain amount of hits, they could sum each individual hit. This can become tedious—especially if there are dozens of strategies with hundreds of hits. Rather, an equation can be formulated to allow for one single calculation. In fact, it's simply the Future Value formula using the variables from an investor's portfolio of strategies:

$$B_H = B_0[1 + E(R_\pi)]^H \tag{3}$$

$Where$:

$B_H = the\ expected\ balance\ at\ a\ certain\ H$

Equation (3) offers the investor insight into what they can expect from their portfolio by the h^{th} hit, but an important factor has been ignored: risk. Expecting the expected balance, ironically, would be unwise. The following chapter explores TGE's risk in detail.

CHAPTER 3

PROBABILISTIC OUTCOMES

"Take calculated risks. That is quite different from being rash." – George S. Patton

3.1. Explanatory Example

Prior to exploring the risk of TGE, a hypothetical example may assist in expressing the logic found in the following sections. For this example, assume: 1) a certain gambler has one strategy with a 75% change of winning; 2) they begin with $10; 3) they can only bet $1 per iteration; 4) they win or lose a dollar per bet; 5) each iteration's outcome is independent of previous outcomes; 6) the gambler can only play one iteration. Given these assumptions, call this Scenario 1. Figure 3. is a table summarizing Scenario 1 where W signifies a win for a hit, and L signifies a loss:

Scenario 1

Outcome Space	
W	L

	Ways this can Occur	Respective Probabilities	Final Probability
p(Win)	1	0.75	75%
p(Loss)	1	0.25	25%

Figure 3. Scenario 1 Summary

The table shows the gambler has a 75% chance of having more money than they began with. Now, holding assumptions 1-5, assume the gambler decides they will play two, three, or four rounds. Call these scenarios Scenario 2, Scenario 3, and Scenario 4, respectively. Figure 4. summarizes Scenarios 2-4:

Scenario 2

Outcome Space	
WW	LL
WL	LW

	Ways this can Occur	Respective Probabilities	Final Probability
p(All Win)	1	(.75)(.75)	56%
p(W & L)	2	(.75)(.25)	38%
p(All Lose)	1	(.25)(.25)	6%

Scenario 3

Outcome Space	
WWW	LLL
WWL	LLW
WLW	LWL
LWW	WLL

	Ways this can Occur	Respective Probabilities	Final Probability
p(All Win)	1	$(.75)^3$	42%
p(2W & L)	3	$(.75)^2(.25)$	42%
p(W & 2L)	3	$(.75)(.25)^2$	14%
p(All Lose)	1	$(.25)^3$	2%

Scenario 4

Outcome Space	
WWWW	LLLL
WWWL	LLLW
WWLW	LLWL
WLWW	LWLL
LWWW	WLLL
WWLL	LLWW
WLWL	LWLW
WLLW	LWWL

	Ways this can Occur	Respective Probabilities	Final Probability
p(All Win)	1	$(.75)^4$	32%
p(3W & L)	4	$(.75)^3(.25)$	42%
p(2W & 2L)	6	$(.75)^2(.25)^2$	21%
p(W & 3L)	4	$(.75)(.25)^3$	5%
p(All Lose)	1	$(.25)^4$	0%

Figure 4. Scenarios 2-4 Summary

In Scenario 2 there are four possible outcomes. The gambler could win consecutively (WW), they could break even (LW & WL), or they could lose consecutively (LL). In Scenario 2 the probability the gambler will have a net loss is only 6%.[33] This is much less than the probability of 25% found in Scenario 1.[34] Regarding Scenario 4, the gambler has a 74% chance of having a net gain, but only a 5% chance of a net loss. The probability of having a net loss is nearly five times less in Scenario 4 than Scenario 1. Were iterations added to this example it would become more

33. Probability of losing money = (possible negative outcomes) [respective probabilities] = (1)[(.25)(.25)] = 6.25%

34. Probability of breaking even = (possible outcomes) [respective probabilities] = (2)[(.25)(.75)] = 37.5%

evident the risk of a net loss decreases with every additional iteration. This is because the strategy is a winning strategy with a positive expected rate of return of \$0.50 per iteration.[35]

An important distinction must be made, though: in Scenario 4 it is true the gambler has a 5% chance of losing money before they begin. If, however, the gambler has already played three iterations the probability the fourth iteration results in a net loss depends on what has previously transpired. Each iteration is an independent event. The gambler's total balance would change every iteration. To calculate these probabilities, one could utilize Markov Chains, but this goes beyond the intended scope of this study. What is essential to glean from Scenarios 1-4, is the risk of a net loss decreases the more a gambler is expected to play given their strategy's expected return is positive. This same logic can be applied to TGE. An investor’s risk of a net loss decreases as opportunities for positive hits increase if they have a portfolio of winning strategies.

3.2. TGE’s Risk Equation

It is possible to derive an equation that allows an investor to calculate the probability their portfolio will have a net gain after a definite amount of hits. Referring to Scenarios 1-4 in the previous section, the outcome space grows exponentially with every additional iteration; in fact, it grows at a rate of 2^H. Managing the possible outcomes within the outcome space becomes overbearing. As an example, a trader who expects 50 hits using their portfolio has 1.126 quadrillion possible outcomes. Fortunately, a hit is either a win or a loss, or binary.

French mathematician Blaise Pascal publicized what is now known as Pascal's Triangle, greatly reducing the complexity of such large, binary outcome spaces.[36] Pascal's triangle can be viewed as a catalog for the coefficients calculated in the “Ways this can Happen” column in Figure

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35. $E(r) = (.75)(\$1) - (.25)(\$1) = \$0.50$

36. Karl J. Smith, “Pascal's Triangle,” *The Two-Year College Mathematics Journal* 4, no. 1 (1973): pp. 1-13.

4. Using binomial coefficients, the number of ways a certain outcome can occur is easily found. For example, to find how many of the possible outcomes would contain 5 wins out of five hits (WWWWW)[37] the coefficient would be written as $\binom{5}{5}$. In Pascal's Triangle on the 5th row, 5th column there is only one possible outcome in which five wins occur with five hits.

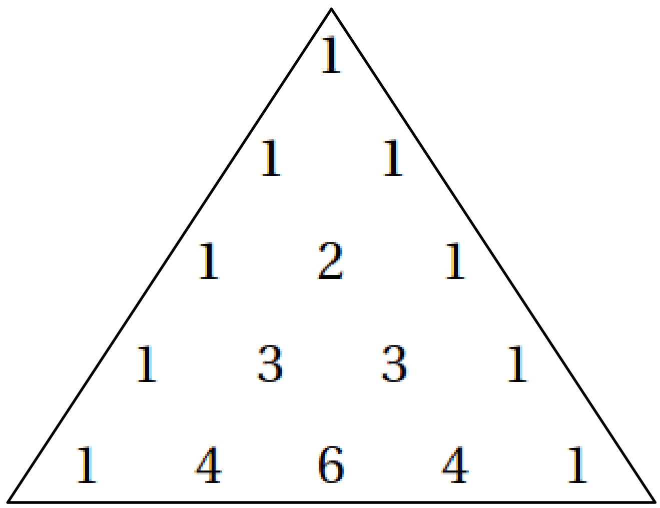

Figure 5. Pascal's Triangle

The binomial coefficient essential for calculating the probability of a net gain, but an issue arises: breakeven outcomes. At the end of their activities an investor can either have a net loss, a net gain, or they can break even. There is, however, a simplifying assumption that can be made—consider breakeven and losing outcomes as one category. An investor can either have a net gain, or they cannot. With Pascal's Triangle and outcomes categorized, the below equation can be derived:

$$p(\alpha) = 1 - \sum_{i=0}^{i=\frac{H}{2}} \binom{H}{i} \left(p(W)^{i}\right)\left(p(L)^{H-i}\right) \tag{4}$$

37. The order the wins occur does not matter in this case.

$Where$:

$$p(W) = \frac{Winning\ Hits\ in\ Portfolio}{H}$$
$$p(L) = \frac{Losing\ Hits\ in\ Portfolio}{H}$$
$\alpha = portfolio\ balance\ ends\ with\ a\ net\ gain$

$i = i^{th} possible\ win\ within\ the\ outome\ space$

$Note\ 1: when\ \frac{H}{2} \neq integer, round\ UP\ to\ the\ nearest\ integer$[38]

$Note\ 2: for\ simplicity\ it\ is\ assumed\ (\bar{R}|W) = (\bar{R}|L) = 1$

$Note\ 3: p(L) < 1.\ A\ rational\ trader\ shouldn't\ consider\ a\ strategy\ with\ a\ 100\%\ of\ loss.$

As an example, assume an investor finds their portfolio gives them:

$$H = 5$$
$$p(W) = .75$$
$$(\bar{R}|W) = (\bar{R}|L) = 1$$[39]

Given these conditions:

$$p(\alpha) = 1 - \sum_{i=0}^{i=\frac{5}{2}} \binom{5}{i} \left(p(.75)^{i}\right)\left(p(.25)^{5-i}\right)$$
$$= 1 - [\ (.25^5) + (5)(.75)(.25^4) + (10)(.75^2)(.25^3)]$$
$$= 1 - [.0001 + .0147 + .0879]$$
$$= 1 - .1035$$
$$= .8965$$

Meaning there is an 89.65% chance this investor will have a net gain after five hits. Figure 6. displays this example to the 50th hit:

38. This ensures breakeven outcomes are counted with the losing outcomes.

39. Further discussion on expected rates of return and risk can be found in Section 4.1 of this research.

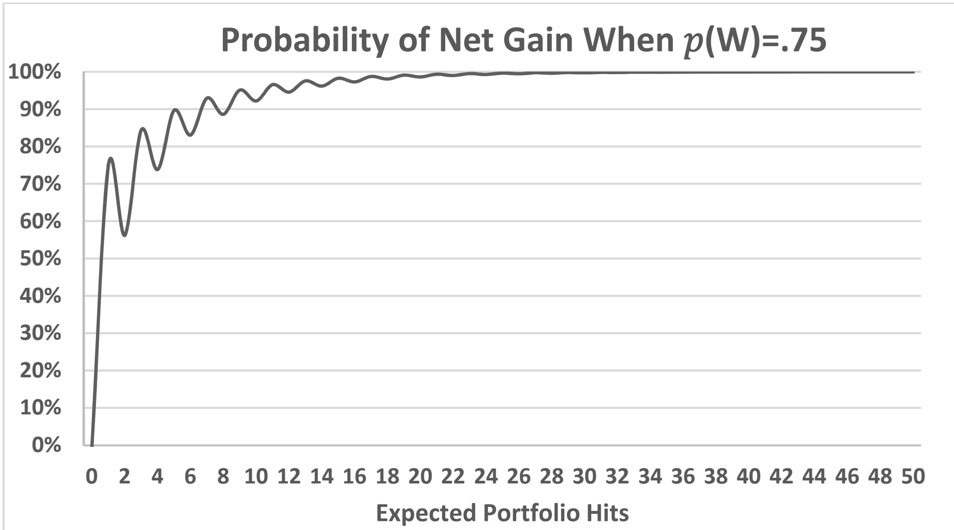

Figure 6. Example: Probability of a Net Gain When p(W)=.75[40]

3.3. The Power of Repeated Play

As previously mentioned, an investor's risk of a net loss decreases as their portfolio's opportunities for positive hits increase. Intuitively, many casinos offer American Roulette on their floors which gives casinos an expected rate of return of 5.26% edge or more per round. One strategy a gambler could use is to bet on all red slots which has a 47.37% chance of winning with a payout of 1:1.[41] From the casino's perspective, there is a 47.37% chance they will lose their entire bet. To some this may be considered risky. Yes, the casino has the statistical advantage, but a worker nearing retirement would not invest in such a strategy despite the advantage. What allows casinos to confidently place these seemingly risky bets is the knowledge that thousands of gamblers will

40. The oscillating nature of this graph stems from the breakeven scenarios which are possible every even hit.

41. Yibi Huang, "Stat 22000 Lecture Slides Random Variables" (Department of Statistics, University of Chicago), accessed August 23, 2021, https://statistics.uchicago.edu/~yibi/teaching/stat220/17aut/Lectures/L09.pdf.

play thousands of rounds allowing the Law of Large numbers to elicit the advantage; or in other words, the casino will have thousands of hits nearly guaranteeing a net profit.

Following the same logic as a profit-seeking casino with a floor of advantageous games, a trader with a winning portfolio of strategies should also have a guaranteed profit should they place thousands of trades. To investigate, let the number of hits in Equation (4) go to infinity.

$$\lim_{H\to\infty} p(\alpha) = \lim_{H\to\infty}\left[1 - \sum_{j=0}^{j=\frac{H}{2}} \binom{H}{j} \left(p(W)^j\right)\left(p(L)^{H-j}\right)\right]$$

For simplicity let $\lim_{H\to\infty}\left(p(L)^{H-j}\right)$ which is a part of Equation (4). Also, let $L_t =$ the total losing hits in a portfolio:

$$\lim_{H\to\infty} p(L)^{H-j} = \lim_{H\to\infty}\left(\frac{L_t}{H}\right)^{H-j} = \lim_{H\to\infty}\left(\frac{L_t}{H}\right)^{H}\left(\frac{L_t}{H}\right)^{-j} = \left[\lim_{H\to\infty}\left(\frac{L_t}{H}\right)^{H}\right]\left[\lim_{H\to\infty}\left(\frac{L_t}{H}\right)^{-j}\right] = 0$$

Therefore:

$$\lim_{H\to\infty}\left[1 - \sum_{j=0}^{j=\frac{H}{2}} \binom{H}{j} \left(p(W)^j\right)(0)\right] = 1 - 0 = 1$$

This can be interpreted as follows: as an investor increases their portfolio's hits to infinity the probability of a net gain will be 100% assuming $p(W) > p(L)$. Should an investor's advantageous portfolio allow them to trade often, their risk of a net loss theoretically disappears.

Traditionally it is assumed lower risk is associated with lower expected returns.[42] Today the rate of return for a 1-year U.S. Treasury Bond, which is often considered to be nearly risk-free,

42. Raphael Amit and Joshua Livnat, "Diversification and the Risk-Return Trade-Off," Academy of Management Journal 31, no. 1 (1988): pp. 154-166, https://doi.org/https://www.jstor.org/stable/256502?seq=1#metadata_info_tab_contents.

is close to .09%.[43] Contrarily, TGE shows it is possible to expect compounding returns while lowering one's risk at the same time. Recall Equation (3): $B_H = B_0(1+R)^H$. As H increases the expected balance increases. Now, recalling Equation (4), as hits H increases the probability of losing money decreases. It is in this inverse relationship where the power of repeated play becomes evident. As an investor adds winning hits to their portfolio they exponentially increase their expected returns while eliminating their risk of a net loss.

3.4. Creating a Probability Distribution

Recall Equation (3) function finds the expected balance of a portfolio after a certain amount of hits. It is a single point found on along an exponential function. Also recall Equation (4) which provides the probability an investor will have a net gain (breakeven or greater) after a certain amount of hits. With this information a normal distribution can be derived allowing an investor to answer many questions.

As an example, say Investor C would like to know the probability they will double their balance given their portfolio. Also assume Investor C has found the following given their portfolio:

$$H = 25$$
$$p(\alpha) = 85\%$$
$$E(R_\pi) = .2\%$$
$$B_{25} = \$16{,}406$$
$$(\bar{R}|W) = (\bar{R}|L) = 1$$

Figure 7. displays their Expected Balance function on the following page:

43. "U.S. Department of the Treasury," Daily Treasury Yield Curve Rates, November 12, 2021, https://www.treasury.gov/resource-center/data-chart-center/interest-rates/pages/textview.aspx?data=yield.

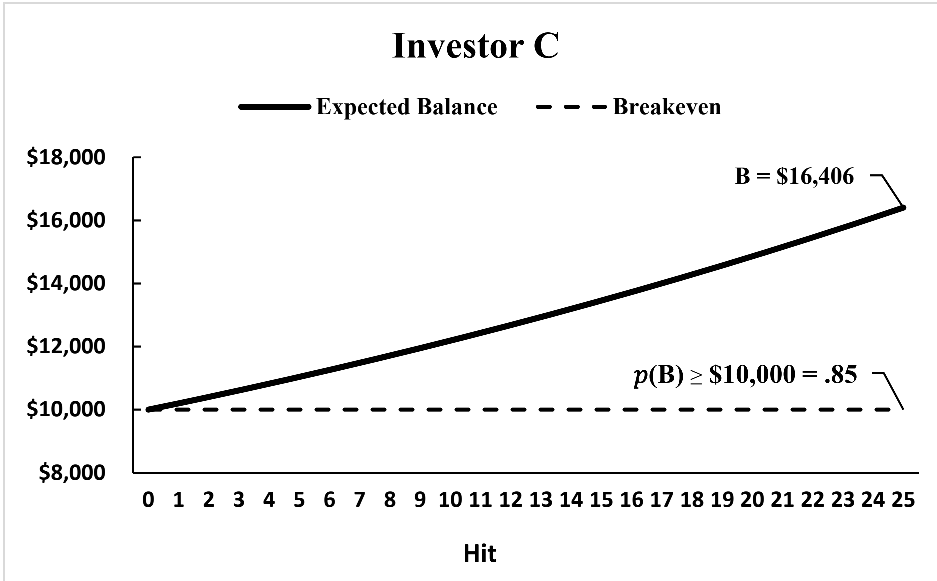

Figure 7. Example: Investor C

Holding H constant at $H = 25$ the investor can derive a normal distribution. Below is the equation for finding a Z-score:

$$Z = \frac{X-\mu}{\sigma} \tag{5}$$

$where$

$X = random\ variable$
$\mu = Mean$
$\sigma = Standard\ Deviation$

The first thing an investor must do is calculate the standard deviation of the distribution. Rearranging Equation (5) yields the following:

$$\sigma = \frac{X-\mu}{Z} \tag{6}$$

Using $1 - p(\alpha)$ the investor finds Z using a Z table or calculator, and then uses $X = B_0, \mu = B_H$ to insert into Equation (6) yielding:

$$\sigma = \frac{B_0 - B_H}{Z} \tag{7}$$

Below is the standard deviation for Investor C's distribution at $H = 25$.

$$\sigma = \frac{\$10{,}000 - \$16{,}406}{1.04} = \$6{,}160$$

Now the investor is able to create a normal distribution at $H = 25$ for their expected balance.

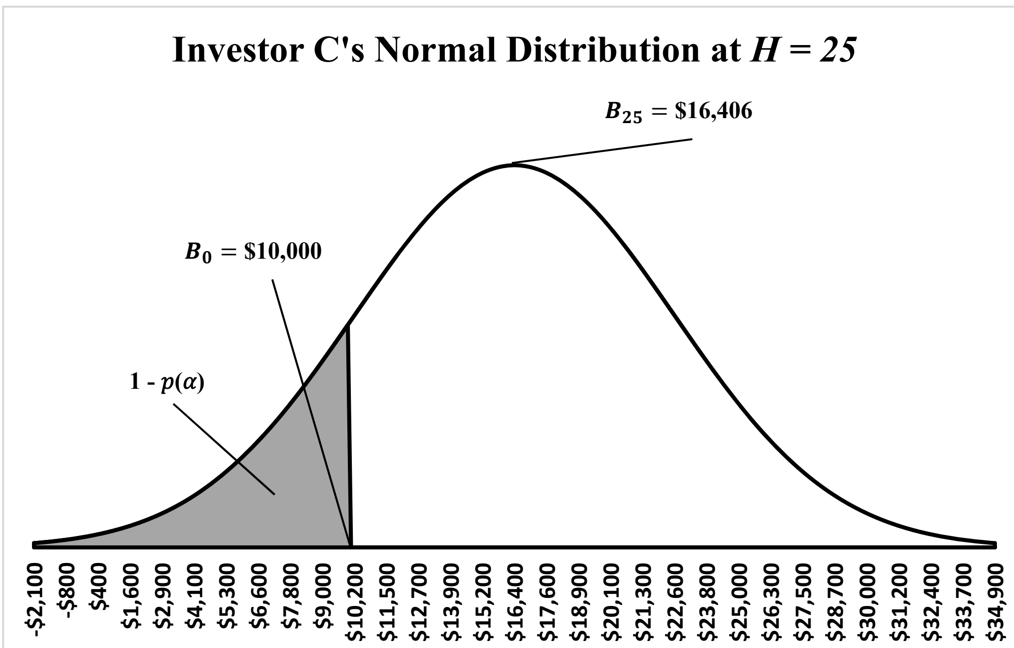

Figure 8. Example: Investor A's Distribution at H=25

Investor C is now able to calculate the Z score associated with any X variable they desire. If they want to know the probability of doubling their balance, or more, they would find:

$$Z = \frac{\$32{,}812 - 16{,}406}{\$6{,}160} = 2.66 \rightarrow\ .9961$$

Finding the area above $32,812:

$$1 - .9961 = .39\%$$

CHAPTER 4

CONCLUSION

"Keep it simple, stupid." – Kelly Johnson

4.1. Limitations and Further Research

Throughout this research many simplifying assumptions were made. Perhaps the most critical is the assumption that should a winning strategy be found, it will remain advantageous in the future. As discussed in Section 1.3 the world is dynamic, and as such, the value of assets and human sentiments are likely to evolve. Further research could highlight how likely certain strategies are to remain advantageous in the short-run and long-run. In practice, an investor should continually monitor their strategies and only utilize them as they remain positive.

Concerning Equation (4), the assumption $(\bar{R}|W) = (\bar{R}|L) = 1$ allowed for simplicity when displaying the property of risk reduction within TGE. In practice, an investor's portfolio of strategies may not return the exact amount given a win or a loss. Studying the effects of unequal rates of return given a win or a loss on the probably of a net gain would increase accuracy. Despite this limitation, if $E(R_\pi) > 0$ the claim, "as an investor adds winning hits to their portfolio they exponentially increase their expected returns while eliminating their risk of a net loss" holds. Additionally, further research in the areas of Markov Chains, Monte Carlo simulations, and an in-depth analysis of the consistency of financial time-series data would benefit an investor attempting to better understand their portfolio of strategies.

4.2. Concluding Remarks

The 4th century Logician William of Ockham coined the principle now known as Ockham's Razor. The principle states, "entities should not be multiplied unnecessarily."[44] When a certain process, belief, model, etc. lends little assistance it is unnecessary and, therefore, should be omitted. Investing institutions and professional investors are able to obtain excess returns by ferociously contending for quicker information, purchasing the latest technology, and hiring the greatest minds. Even then, many fail. Millions of models using hundreds of different explanatory variables have been constructed to describe markets, but a perfect model is far from being constructed. The market often proves to be too complex to return a large, consistent premium. It would be prudent for the small, disadvantaged retail investor to adhere to Ockham's advice.

Rather than chasing newfangled techniques, myopic advice, or the trendiest products a retail investor can admit to their own shortcomings. They could objectively cut their processes, beliefs, and models with Ockham's Razor. They may also come to accept that price movements (especially in the short run) are seemingly random like a roll of the dice. They could then reach the conclusion that if prices are random, they could approach investing like a profitable gambler or a profit-seeking casino would at the tables. This is what TGE allows the retail investor to do.

Like a casino hunting for advantageous games, the first and most arduous step of TGE is hunting for investment strategies that provide a statistical edge. Once strategies are found the second step highlights how a set of positive strategies can be combined into a portfolio like a casino offering multiple games on their floor. Step three shows the retail investor how to calculate their portfolio's expected balance, and step four shows the investor how create a probability distribution for their portfolio under certain assumptions. This paper shows that if an investor were to find

44. Sugihara Hiroshi, "What Is Occam's Razor?" (Univeristy of California - Riverside, 1996), https://math.ucr.edu/home/baez/physics/General/occam.html.

advantageous strategies and implement them as TGE suggests, they can expect exponential gains in the long run. It also highlights an inverse relationship between risk and return that TGE can provide as investors increase their opportunity to trade similar to a casino betting millions on their slightly advantageous games.

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