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RELATIONSHIPS BETWEEN MIDDLE SCHOOL STUDENTS’ ADAPTIVE REASONING WHEN CREATING LEARNER-GENERATED DRAWINGS AND PARTNER TALK DURING INQUIRY-BASED MATHEMATICAL TASKS

by

Angela M. Frabasilio

A dissertation submitted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Education

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ABSTRACT

Relationships Between Middle School Students’ Adaptive Reasoning when Creating Learner-Generated Drawings and Partner Talk During Inquiry-Based Mathematical Tasks

by

Angela Frabasilio, Doctor of Philosophy

Utah State University, 2022

Major Professor: Beth L. MacDonald, Ph.D.
Department: Mathematics Education and Leadership

The purpose of this mixed methods exploratory study was to examine what adaptive reasoning indicators 7th-grade students’ evidenced within discussion and learner-generated drawings and how they adapted their reasoning when engaged in levels 2 and 3 depth of knowledge (DOK), inquiry-based mathematical tasks. The researcher observed 18 seventh-grade partners, communicating through online discussion boards, as they progressed through three inquiry-based mathematical tasks. Written descriptions of a) learner-generated drawings and b) student discussions were transcribed and structurally coded per six adaptive reasoning indicators: 1) relationships and connections, 2) justifications, 3) alternates pursued, 4) prior knowledge, 5) legitimacy determined, and 6) pattern recognition. Adaptive reasoning evidenced in these two communication modalities was examined qualitatively through descriptive statistics and field note narratives. Additionally, students completed a DOK level test which established baseline...
for student comparison. The qualitative data was quantitized using epistemic network analysis (ENA) to examine how students adapted their reasoning through visual network models and differences graphs.

Results indicated that all six indicators or strategies of adaptive reasoning were evidenced within the different modalities of drawing and discussion, approximately one-third in drawings and two-thirds in discussions. Students primarily adapted their reasoning using the indicator relationships and connection or justifications. The other four indicators were used, primarily in conjunction with these two indicators. Themes in student discussion and patterns within drawn gestures indicated different adaptive reasoning strategies. These included student communicating through such things as arguments, explanations, challenges, and drawn scribbles, erasures, and directional arrows. Distinct patterns of adaptive reasoning were identified within tasks, stages of the tasks, student working DOK level and modalities of drawing and discussion. While the group use of adaptive reasoning was generalizable, students displayed unique adaptive reasoning patterns, mediated by partner interaction. The findings show how students adapt their reasoning within mathematical tasks, in light of one component of mathematical proficiency, giving insight for researchers and teachers to enhance student learning in mathematical tasks.

(251 pages)
Relationships Between Middle School Students’ Adaptive Reasoning when Creating Learner-Generated Drawings and Partner Talk During Inquiry-Based Mathematical Tasks

Angela M. Frabasilio

Adaptive reasoning is one of five components students use to develop mathematical expertise and become mathematically proficient. When students adapt their reasoning they are logically thinking about the mathematical relationships between concepts and adapting their thinking to solve problems. Three Act Math Tasks are popular math problems used in schools in which students engage in adaptive reasoning. These types of problems are beneficial to students because they engage students in inquiry-based learning, a kind of learning where students work to pose questions, interpret data, design ways to solve the problem and present their solutions. Little is known about how students adapt their reasoning as they partake in these types of tasks. The objective of this study is to better understand what adaptive reasoning strategies seventh graders used and how they used these strategies when engaged in inquiry-based mathematical tasks.

To accomplish this, the study observed 18 seventh grade students as they worked through three mathematical tasks. The researcher observed student discussions and their drawings to see what adaptive reasoning strategies were being used by students and how the strategies were used throughout different stages of the tasks. In this way a more
complete picture of how students adapted their reasoning was obtained. The researcher analyzed student use of six different adaptive reasoning indicators, including: 1) relationships and connections, 2) justifications, 3) alternates pursued, 4) prior knowledge, 5) legitimacy determined, and 6) pattern recognition.

Results indicate that students used all six adaptive reasoning strategies. Students primarily adapted their reasoning by finding relationships and connections and making justifications. Additionally, each student demonstrated a unique pattern of adaptive reasoning strategies which was mediated by their partner. Use of the other four indicators, alternates pursued, prior knowledge, legitimacy determined and pattern recognition were used in conjunction with the two primary indicators. Additionally, different patterns of use were identified within the separate modalities of student drawings and discussions.

This study is beneficial because it helps teachers and researchers better understand what adaptive reasoning strategies students are utilizing and the relationship between these strategies in a classroom setting. This affords teachers and researchers opportunities to develop better learning experiences and understand how students reason in light of mathematical proficiency.
ACKNOWLEDGMENTS

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Angie Frabasilio

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CHAPTER 1

INTRODUCTION

Lee Ann labeled the hypotenuse of the right triangle ‘125 ft.’ Her drawing represented the distance she travelled if she walked the shortcut route to the snack shop. She looked at the legs of her triangle. Noticing that one leg measured 150 feet, she let out a sigh of disappointment. Remembering her prior work with the Pythagorean theorem, she knew something was wrong. The hypotenuse shouldn’t be smaller than the leg. Erasing her calculation of 125, Lee Ann began to devise a new plan.

Lee Ann’s process of logically detecting an error in her drawing based on prior knowledge and developing new approach is an example of adaptive reasoning. Adaptive reasoning is a critical skill, or mathematical way of thinking, valuable for problem solving in mathematics classrooms and in everyday life (National Council of Teachers of Mathematics, 2014). The National Research Council (NRC) describes adaptive reasoning as a cognitive process and a strategy essential to 21st Century Skills; a student competency needed to succeed in future workplaces (Pellegrino & Hilton, 2013). Classroom strategies, such as having students create learner-generated drawings and discuss their reasonings, presents situations in which student’s adaptive reasoning may be observed.

This mixed methods study explored adaptive reasoning as 18 seventh graders created drawings and discussed their mathematics with partners during guided inquiry-based mathematical tasks. Qualitative data analysis involved dividing video task data into stanzas of meaning, structurally coding the data by adaptive reasoning indicators and
grouping the codes into themes. Students completed DOK level tests which were coded per level of achievement and included in the qualitative analysis as an indicator of students’ mathematical abilities or student working DOK level. Qualitative data was then quantitized using epistemic network analysis (ENA) to address the research questions.

**Background of the Problem**

In the publication, *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick et al., 2001), the National Research Council frame the learning of mathematics as *mathematical proficiency*, the skills, expertise, knowledge and ability needed for a student to learn mathematics successfully. In doing so they introduced the term adaptive reasoning. *Adaptive reasoning*, the ability for a student to think logically about the relationships between concepts and situations, is one of the five major components of mathematical proficiency. Kilpatrick et al. (2001) generalize the term adaptive reasoning to include various types of reasoning that guide students’ learning (e.g., the ability to generate conjectures, reflecting and checking the truth of a procedure, acting on intuitive reasoning, giving informal and formal explanations, justifying). Students adapt their reasoning to navigate through problems to a solution. In this way, adaptive reasoning relates to student achievement. Students adapt their reasoning, pick and choose facts, procedures, concepts, and strategies with discrimination, to reach and justify their solution.

Adaptive reasoning is important in classroom learning, yet students struggle with its use in the classroom (OECD, 2011). Students struggle because U.S. school curriculum, classroom practice, teacher preparation, instructional materials and
assessment are not focused on the development of adaptive reasoning. Instead there is an orientation towards conceptual understanding and procedural skills (Kilpatrick et al., 2001; National Council of Teachers of Mathematics [NCTM], 2014). The importance of students’ adaptive reasoning is expressed in leading documents in mathematics education and learning research including the NCTM document *Principles to Action: Ensuring mathematical success for all* (NCTM, 2014) and the eight Mathematical Practice Standards in the Common Core State Standards Initiative (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Hattie et al. (2017) analyzed 1400 meta-analysis of 80,000 studies identifying influences on student achievement. The results (Visible Learning Plus, 2017) list adaptive reasoning attributes within the *zone of desired effects* or having the greatest impact on student achievement outcomes. The developers of these documents call for adaptive reasoning instruction in the classroom; however, dominant cultural beliefs obstruct effective implementation (NCTM, 2014; Philipp, 2007).

Procedural understanding strategies still dominate classroom learning (Corrêa, 2018; NCTM, 2014). Recent empirical studies find students’ adaptability of reasoning in learning situations unsatisfactory (Csíkos, 2016; Heinze et al., 2009; Stylianides et al., 2017; Torbeyns et al., 2017). The mathematics education community views adaptive reasoning as important; however, it is poorly utilized by students. There is a gap between the community’s learning expectations and student performance. One step in understanding this gap is to explore, in more detail, how adaptive reasoning is presently being used with more in-depth insight to inform classroom practice. To gain more insight, it would be important to use a qualitative methodology.
More insight into how students use adaptive reasoning is needed; however, adaptive reasoning can be difficult to view since much reasoning occurs within students’ minds. Empirical classroom research on adaptive reasoning is predominately geared towards measuring adaptive reasoning in reference to an intervention (Samuelsson, 2010; e.g., Kaasila et al., 2010) or a given a condition (e.g., Heinze et al., 2018). Many of these studies measure adaptive reasoning under the broad stroke of mathematical proficiency rather than specifically observing adaptive reasoning. Newer studies that focus primarily on adaptive reasoning (e.g., Awofala, 2017; Yulian & Wahyudin, 2018) help researchers identify adaptive reasoning indicators and classroom instances of adaptive reasoning occurrence, such as during inquiry-based learning (Kuhlthau et al., 2015), throughout mathematical tasks (Corrêa, 2018), and during exploratory talk (Barnes, 2008; Boyd, & Kong, 2017; Wells & Ball, 2008).

Exploratory talk is when partners engage critically and constructively with each other’s ideas in order to come to a joint solution (Barnes, 2008). In addition, researchers use learner-generated drawings to provide children a “voice” that may assist researchers in their interpretation of the students’ thoughts; thoughts that would otherwise be difficult to convey (Theron et al., 2012; Thomson, 2009). In problem solving, learning strategies often include “make a drawing” to assist in the solution (Lesh & Zawojewski, 2007). Learner-generated drawings support dialogue such as partner exploratory talk (NCTM, 2014), resulting in a richer understanding of adaptive reasoning of students (Arcavi, 2003; Mair & Kierans, 2007).
Problem Statement

Adaptive reasoning, an important component of mathematical proficiency, is not used well by students in the classroom setting. Findings delineate how educators should structure students’ discourse and engage students in learner-generated drawings; these classroom strategies evidence adaptive reasoning but are not being considered in light of how students adapt their reasoning when developing mathematical proficiency.

Purpose of the Study and Research Questions

The purpose of this mixed methods, exploratory study (Teddlie & Tashakkori, 2009) was to examine how seventh graders adapt their reasoning as evidenced through their drawings and discussions and the relationships between uses of adaptive reasoning indicators during guided inquiry-based mathematical tasks in the classroom. The following research questions were developed to examine this phenomenon:

1. What adaptive reasoning indicators do 7th grade students’ evidence throughout their discussions and solutions during guided inquiry-based mathematical tasks?
2. What adaptive reasoning indicators do 7th grade students’ evidence throughout their drawings and solutions during guided inquiry-based mathematical tasks.
3. How do adaptive reasoning indicators relate to 7th grade students’ engagement with level 2 and 3 depth of knowledge tasks?
Study Significance

Researchers (Cai et al., 2019) recommend that studies should not just “advance the field’s knowledge and understanding of the teaching and learning of mathematics” but that significant research in mathematics education “can, and perhaps should, be much closer to the classroom and aim to directly impact practice” (p.1). By examining students drawing and discussing in a classroom situation, detailed insight will be gained into how and when students adapt their reasoning during a mathematical task. Findings from this study have the potential to explain how different indicators of adaptive reasoning relate to one another as students problem solve. This multi-faceted approach provides the mathematics education field with more complete understandings of students’ adaptive reasoning. Most significantly, findings from this study have the potential to inform classroom practice and provide evidence for teachers determining when and how students adapt their reasoning.

Definition of Terms

Following are terms related to the study and their identified usage within this dissertation:

Mathematical proficiency. The skills necessary for students to learn mathematics well, viewed in a holistic way to include student conceptual understanding, strategic competence, procedural fluency, adaptive reasoning and productive disposition (Kilpatrick et al., 2001).
**Adaptive reasoning.** Adaptive reasoning is the ability for a student to think logically about the relationships between concepts and situations. Indicators of adaptive reasoning include: 1) the development of relationship or connections between concepts and situations, 2) posed disagreements, arguments and/or alternatives considered, 3) justification, explanation of reasonings or proofs, 4) integration and transfer of prior knowledge, 5) recognition of patterns, 6) determination of legitimacy of correctness or appropriateness of strategies (Baroody, 2003; Hatano & Inagaki, 1986; Kilpatrick et al., 2001).

**Learner-generated drawings.** Learner-generated drawings are drawings created by the student for the purpose of learning (Van Meter & Firetto, 2013). In the study, drawings that represent the situation of the task, or situational drawings, are called *story drawings*. Drawings that represent mathematical ideas, or representational drawings, are called *math drawings*.

**Guided inquiry-based mathematical tasks.** Guided inquiry-based mathematical tasks promote learning through student investigation. Effective guided inquiry-based mathematical tasks engage and challenge students to work together and to problem solve. Inquiry-based tasks used in this study are called Three-Act Math Tasks. Divided into three parts called ‘acts,’ these tasks generally last 20 to 30 minutes and prompt student inquiry to solve real-world problems. During Three-Act Math Tasks, students work in whole-class and partner groups and engage in exploratory talk (Redmond-Sanogo et al., 2018).

**Exploratory talk.** Exploratory talk is a type of talk where partners engage critically and constructively with each other’s ideas. Exploratory talk is characterized
with the establishment of dialogue ground rules that promote listening, reasoning, and sharing of ideas (Barnes, 2008).

**Assumptions**

The researcher assumes that knowledge is constructed through learning experiences (Clements & Battista, 1990). In mathematics class, deep learning occurs when students are engaged in rich sensory experiences. These assumptions are derived through theorists such as Piaget (1964), Vygotsky (1978) and other constructivists, backed by more modern cognitive neuroscientific research (Medina, 2014). Productive cognitive dissonance, created within a mathematical task of genuine and rich inquiry, motivates students to engage in problem solving (Deci et al., 1991).

In addition, students conceptual understanding of mathematics is important when participating in guided inquiry-based mathematics tasks (Smith & Stein, 2011). Classroom features that affect conceptual understanding and student motivation include the explicit attention of both teachers and students on important mathematical concepts and the allowance of student struggle with these concepts (Hiebert & Grouws, 2007; Rugutt & Chemosit, 2009). It is assumed that this culture exists within the classroom in which the study takes place for the affordance of adaptive reasoning.

**Conclusion**

The use of adaptive reasoning is important to students’ mathematical proficiency development yet adaptive reasoning is not understood fully in the mathematics educational field. For instance, educators are still exploring how to teach mathematics to
promote students’ adaptive reasoning or assess students’ adaptive reasoning. While studies exist on adaptive reasoning, a better understanding of its use in inquiry-based learning situations is needed to create classroom strategies that focus on adaptive reasoning.
CHAPTER TWO
LITERATURE REVIEW

Exploratory talk and learner-generated drawings are not new classroom strategies; however, they are not being considered in light of adaptive reasoning when developing student mathematical proficiency. The purpose of this study was to examine how seventh graders adapt their reasoning as studied through their discussions and drawings during inquiry-based mathematical tasks. Adaptive reasoning is defined as the ability of a student to “think logically about the relationships among concepts and situations” (Kilpatrick et al., 2001, p. 130). Students adapt their reasoning as they problem solve. They wonder about their approaches, paradigms, justifications, and explanations. They make decisions about their solution path, choosing between different strategies, determining whether a solution is possible or implausible, and directing and redirecting their thoughts considering alternate solutions (Kilpatrick et al., 2001).

To examine adaptive reasoning more closely, this review is separated into six sections. The first section describes the conceptual framework. The second section explores adaptive reasoning, focusing on studies that define adaptive reasoning indicators. The third section discusses theory and studies about learner-generated drawings. The fourth section discusses findings related to partner dialogue, focusing on exploratory talk. The fifth section looks at current research on guided inquiry-based mathematical tasks, focusing on Three-Act Math Tasks. The sixth and concluding section summarizes the literature in light of research initiatives and findings.
Conceptual Framework

Bergman (2010) contends that ontological and epistemological considerations of mixed methods research cross theoretical boundaries, making it difficult to ground mixed methods research. Rather than attempting to fit this type of research into a preconceived ideology, Bergman suggests letting the research guide the paradigm. Following Bergman’s suggestion, this researcher chose a constructivist paradigm. Constructivist learning, developed by Piaget (1964) and others, posits that students build their knowledge by connecting new ideas and experiences with prior knowledge. The researcher chose this paradigm based on the following considerations. First the observed actions of seventh-graders evidence constructed adaptive reasoning, which consists of students changing and constructing knowledge through adaptation. Second, guided inquiry-based mathematical tasks afforded opportunities to construct their knowledge, also aligning with constructivist theory of learning (Buell et al., 2017). In addition, this study uses Epistemic Network Analysis (ENA) to more closely observe and analyze student thought processes, which works well within a constructivist framework (Shaffer, 2017). ENA is a method of conversational analysis that identifies and quantifies connections between student use of adaptive reasoning indicators in a detailed visual model. ENA is further explained in Chapter 3.

Although Bergman’s recommendation is used to guide paradigm selection, it is also important for researchers to state their theoretical orientation when dealing with complex research context (Poth, 2018). This researcher positions herself as a constructivist, assuming that students construct their own knowledge. Students construct
knowledge by reflecting on actions with which to use in their knowledge construction and link existing knowledge with this new information to develop conceptual knowledge (Baroody, 2003). This includes generative learning theories, the idea that students generate or construct knowledge through the active integration of new knowledge with existing knowledge (Hanke, 2012). Wittrock (1992) described the generative learning theory as the building of relationships between stimuli and prior experiences. As a constructivist the building of procedural knowledge is important in the development of mathematical proficiency. While recent research suggests procedural knowledge is created from conceptual knowledge (Baroody et al., 2007), both permit a flexibility in the invention and use of mathematical strategies or procedures, a flexibility that relies upon the nature of instruction (Baroody, 2003). This study explores the instructional strategy of student-generated drawings and discussion through the lens of constructivism in order to explore adaptive reasoning of students as they progress through guided inquiry-based mathematical tasks. As students draw and discuss, their construction of knowledge and refinement knowledge construction is evidenced through changes made within their drawings and directional changes in thought expressed through dialogue.

This study’s conceptual framework involves four concepts: (1) adaptive reasoning, (2) learner-generated drawing (3) partner dialogue, and (4) inquiry-based mathematical tasks (see Figure 2.1). All four concepts represent strategies or processes grounded in constructive and generative learning theories. As shown in Figure 2.1, students evidence adaptive reasoning by engaging in the concepts “partner dialogue” and “learner-generated drawings”. Dialogue and drawings situated in inquiry-based mathematical tasks provide a learning platform conducive to students’ ability to engage
in and evidence adaptive reasoning. In inquiry-based mathematical tasks, students engage in exploratory talk, they look critically at relationships between concepts or ideas and build upon each other’s ideas in a critical and constructive manner (Pierce & Gilles, 2008). When students create drawings, they are afforded opportunities to recognize differences between their drawn images and their thoughts; they self-regulate and generate new knowledge (Van Meter & Firetto, 2013).

Figure 2.1

*Conceptual Framework, Created by the Researcher*

Indicators of Adaptive Reasoning

Alan Schoenfeld (personal communication, August 5, 2019) explains that indicators of adaptive reasoning are difficult to observe, are dependent upon what the individual knows, and are more apparent when individuals engage in challenging
problems. This study was designed to engage students by placing them in challenging inquiry-based mathematical tasks. To make adaptive reasoning observable, the researcher included strategies that employed visible means of communication: drawing and partner dialogue.

The NRC (Kilpatrick et al., 2001) describe indicators (see Figure 2.2) to evidence adaptive reasoning. Those indicators include: 1) changes and constructs knowledge through adaptation, relationship or connections between concepts and situations, 2) considers alternatives, 3) justifies or offers informal explanations, reasonings or proofs, 4) integrates and transfers prior knowledge, 5) uses deductive, inductive or intuitive reasoning, 6) poses disagreements or arguments, 7) recognizes and uses patterns, analogies or metaphors, 8) determines legitimacy of correctness or appropriateness of strategies. For the purpose of this study, indicators were summarized into six indicators.

**Figure 2.2**

*Relationship Between Mathematical Proficiency, Adaptive Reasoning and Adaptive Reasoning Indicators (Kilpatrick et al., 2001)*
Adaptive Reasoning Evidenced in Guided Inquiry-Based Mathematical Tasks

Inquiry-based learning is a pedagogy based on active learning theory (Cattaneo, 2017) where students are challenged to construct their knowledge by taking a dynamic and energetic role in their learning process (Petress, 2008). Guided inquiry-based activities are teacher-guided and student-centered; students begin with a question and follow-up by investigating solutions. They are “creating new knowledge as information is gathered and understood, discussing discoveries and experiences, and reflecting on newfound knowledge” (Savery, 2006, p. 16).

The researcher chose Three-Act Math Tasks as the type of guided inquiry-based mathematical task for this study. Designed by Dan Meyer (2010) and set up in three acts, or three sections. The researcher chose this type of task because its structure is set in three sections that delineate evidence of adaptive reasoning. The first act presents a challenging and engaging problem. The second act provides a space for students to seek, identify information, and construct and present solutions, and the third act reveals one possible solution. Within the task, students adapt their reasoning; they pose questions, look for relationships and patterns, justify their reasoning and determine the legitimacy of their solutions. To integrate the use of learner-generated drawings into the task, the tasks are broken down into four stages in this study. These stages are further explained in Chapter 3.

Students engaged in Three-Act Math Tasks participate in exploratory talk. In addition, the researcher coupled the Three-Act Math Tasks strategy with a learner-generated drawing strategy. This combination provides a structure for guided inquiry-
based learning that evidences adaptive reasoning as examined in the next two subsections.

**Adaptive Reasoning Evidenced in Learner-Generated Drawings**

When students adapt their reasoning, constructive learning occurs as the relationships between mathematical concepts and situations become more evident, allowing students to consider alternative relationships and more readily be able to explain or justify strategies and solutions (Kilpatrick et al., 2001). Learner-generated drawings is a constructive classroom strategy that generates learning through self-regulated processes, described in the cognitive model of drawing construction (CMDC, Van Meter & Firetto, 2013). In this strategy, students draw elements that represent the task situation, called *situational drawings*, and elements that represent the mathematics within the situation, called *mathematical drawings* (Rellensmann et al., 2017). When students become aware of inconsistencies between drawn elements and prior knowledge, cognitive dissonance (Festinger, 1962) occurs and students engage in resolution (Van Meter & Firetto, 2013). Students adapt their reasoning when the resolution requires a cognitive change (Prain & Tytler, 2012; Vosniadou, 1994). For instance, a student might set out to create a situational drawing of a fireman climbing a ladder that leads to a window on the second floor of a schoolhouse. Students set standards for their drawings. They discern needed elements, the amount of detail, and the orientation of the elements. Upon drawing, the student may encounter conflicting spatial considerations, or students might plan to label elements of a drawing and find conflict in prior knowledge, similar to
Lee Ann and her task to find the short-cut distance to the snack shop, as mentioned in Chapter 1.

Students evidence adaptive reasoning during the drawing task by changing their drawings, erasing, and making alterations. Students also evidence their reasoning during engagement dialogue as they create and adjust their individual drawing.

**Adaptive Reasoning Evidenced in Partner Dialogue**

Within the mathematical tasks, students are paired with a partner and engage in exploratory talk. *Exploratory talk* is a mode of conversation that students use in inquiry learning situations that is critical but constructive (Barnes & Todd, 1977). For instance, students engaged in exploratory talk offer their thoughts to others to see how their ideas fit with another’s thoughts. Students challenge proposals, describe their reasoning, and offer alternative ideas. When ideas are contradicted, students are afforded the opportunity to adapt their reasoning (Barnes, 2008). Students, seeking agreement, progress jointly through the task to develop a solution. This type of dialogue evidences adaptive reasoning as students engage in sensemaking talk, helping students experience and learn from other’s processes and construct meaning (Cervetti et al., 2014).

The interaction of the four constructs, adaptive reasoning, student dialogue, learner-generated drawings and mathematical tasks, forms the conceptual framework for which this study is framed. The study examined how students adapt their reasoning as they create drawings, engage in exploratory talk and participate in active learning during inquiry-based mathematical tasks. The following sections explore the theory and literature of each of these concepts.
Adaptive Reasoning

This study examined the construct adaptive reasoning. The following describes 1) the origins of the term adaptive reasoning and its definition, 2) empirical studies that utilize indicators of adaptive reasoning, and 3) connections to the study.

Origins of the Term and Definition of Adaptive Reasoning

In the early 1900’s, researchers used the term adaptive reasoning to explain how students integrated conceptual knowledge and procedural knowledge where conceptual knowledge underlies procedural innovations and leads to the invention of new procedures (Baroody, 2003). The term is closely aligned to adaptive expertise, a type of expertise described by Hatano and Inagaki that differed from routine expertise (1986), and described as the ability for a student to think logically to solve problems.

The definition of adaptive reasoning has changed over the years. In 1985, Alan Schoenfeld characterized adaptive reasoning as strategic mathematical thinking. Presently most studies ground their adaptive reasoning definitions in the National Resource Council definition: “the capacity for logical thought, reflection, explanation and justification” (Kilpatrick et al., 2001, p. 137) and “refers to the ability to think logically about the relationships among concepts and situations” (p. 150).

The term adaptive reasoning surfaced approximately twenty years ago in the Adding it Up: Helping Children Learn Mathematics (Kilpatrick et al., 2001); a document that serves as a settlement between the heavily debated controversy over how mathematics should be taught and what constitutes as mathematical proficiency. The authors represented differing areas of the mathematics community and presented five
strands or components which explained underlying characteristics of mathematical proficiency (e.g., adaptive reasoning, productive disposition, strategic competence, procedural fluency, and conceptual understanding). Kilpatrick et al. explained that these five strands provided insight as to how students’ mathematical expertise, knowledge, and abilities were needed to learn mathematics successfully (see Figure 2.2). Kilpatrick et al. drew from Brownell’s (1935) work and contend that both computational and conceptual skills are important when developing mathematical reasoning, as well as other skills.

The five components of mathematical proficiency relate to one another and are used together. For example, when a student uses strategic competence to formulate and solve a problem, conceptual understanding provides the student with a base of understanding. The student may then use adaptive reasoning to choose a strategy in which to solve the problem and justify the solution correctness. Productive disposition may project a student along their trajectory, while adaptive reasoning channels a student’s thoughts into creative combinations. All of the components are important; however, adaptive reasoning is seen as the component that holds the others together (Kilpatrick et al., 2001). How a student engages with each component, allows researchers the opportunity to observe a student’s unique fingerprint of mathematical proficiency.

Adaptive Reasoning Indicators Empirical Studies

The following studies are divided into two groups: 1) studies that measure adaptive reasoning in determination of mathematical proficiency and 2) studies that examine changes in students adaptive reasoning given an intervention.
Adaptive Reasoning: Measuring Mathematical Proficiency

Awofala (2017) assessed 400 secondary school students in Nigeria in an attempt to quantitatively measure mathematical proficiency with students’ gender in relation to academic achievement. To assess adaptive reasoning, Awofala developed a mathematics adaptive reasoning checklist (MARC) based on Kilpatrick et al. definition of adaptive reasoning. Awofala (2017) describes his list as a measurement of “capacity for logical thought, reflection, explanation and justification” through the observation of student’s “deduction, statement of facts, comparison, abstraction, and application” (p. 493). Each indicator in this study was coded as a (1) for attribute present and (0) for attribute absent. Awofala computed internal consistency reliability for MARC using Kuder-Richardson formula 20 (Kuder & Richardson, 1937). In the analysis, the researcher found that all strands of mathematical proficiency and student gender were predictors of academic achievement.

Awofala’s (2017) method of adoption of indicators based on Kilpatrick et al. (2001) descriptions of adaptive reasoning is repeated in a majority of the studies that measure mathematical proficiency (Groth, 2017; Lepak et al., 2018; e.g., Corrêa, 2018). For example, Corrêa (2018) posited that adaptive reasoning is less likely to occur in a classroom strategy grounded in procedural knowledge. So, she examined mathematical proficiency in inquiry learning, grounded in conceptual knowledge construction. She qualitatively analyzed students’ written journals, students’ dialogue and researcher observational notes of two 11th graders during their mathematics course as they worked in groups of 3-4 students on two different mathematical tasks. Her study defined adaptive reasoning, as “the ability of building logical connections between mathematical ideas,
contents and circumstances…argumentation, justification and reasoning” (Kilpatrick et al., 2001, as cited in Corrêa, 2018, p. 456). Corrêa evidenced adaptive reasoning by examining student work that highlights students’ explanations, arguments, connections to prior knowledge, and connections between concepts. Thus, it seemed Corrêa’s adaptive reasoning definition directly informed her qualitative analysis, providing further evidence of how adaptive reasoning indicators can inform study designs and findings.

As part of a professional development study, Groth (2017) asked two teachers to analyze and reflect upon classroom data through the lens of mathematical proficiency defined by the Kilpatrick et al. document (2001). After reading the chapter on mathematical proficiency in Adding it up: Helping children learn mathematics (Kilpatrick et al., 2001), teachers developed their own indicators of adaptive reasoning, paired together, and progressively analyzed seven of their videotaped lessons. Initial lesson analysis influenced their choice of subsequent lesson plans. Teachers’ developed indicators were not presented in the article. The teachers conjectured that students who engaged in student-centered and/or hands-on activities were more likely to show evidence of adaptive reasoning.

Lepak et al. (2018) examined the strategic algebraic reasoning (SAR) of 198 ninth graders as they participated in two mathematical tasks as evidence of students’ mathematical proficiency. Designed by Lepak et al., SAR is a combination of adaptive reasoning and strategic competence. By combining the two, researchers created their own version of mathematical proficiency. Student written work evidenced adaptive reasoning in justifications, explanations, pattern recognition, and connection making. In addition,
researchers included evidence of SAR as a students’ ability to draw representative models.

**Adaptive Reasoning: Changes Due to Interventions**

A majority of studies examine changes in students’ adaptive reasoning in response to interventions, instructional methods, or programs (e.g. Kaasila et al., 2010; Samuelsson, 2010). Groves (2012), reviewed the various types of classroom practice that promote mathematical proficiency in elementary students by viewing student examples presented in a variety of studies. Establishing a list of adaptive reasoning indicators, she draws on the Australian curriculum in mathematics, Trends in International Mathematics and Science Study (TIMMS) directives, the Singapore Mathematics Framework, and the document Adding it up: Helping Children Learn Mathematics (Kilpatrick et al., 2001). Thus, Groves (2012) measured adaptive reasoning by examining students’ explanations of student thinking, students’ use of deduction, students’ justifications and conclusions, students’ ability to compare and contrast mathematical concepts, students’ ability to explain their choices, and students’ ability to justify true or false conclusions.

Mahendra et al. (2017) examined the effects of students’ problem posing and problem solving on student use of conceptual understanding and adaptive reasoning. Findings indicate that adaptive reasoning increased when seventh graders are allowed to pose their own problems. Studies that examine multiple strands of mathematical proficiency are not uncommon (Muin et al., 2018; Nussbaum & Sinatra, 2003; e.g., Groves, 2012). However, these studies oftentimes neglect to explicitly describe how adaptive reasoning is evidenced. This is the case with the Mahendra et al. (2017) study.
The lack of description raises questions to the authors evidencing of adaptive reasoning as suggested in their results.

Two quantitative studies include intuition or intuitive reasoning when measuring adaptive reasoning. Rizki et al. (2018) examined 137 seventh and eighth graders’ adaptive reasoning when engaging in activities from two different curriculum models. Researchers observe evidence of adaptive reasoning when students use informal explanation and justification, generate conjectures and determine truthfulness. Rizki, et al. (2018) include “intuitive and deductive reasoning on the basis of patterns, analogy and metaphor” (p. 1). Muin et al. (2018) distinguish intuition as a hunch or prediction, when students come to a conclusion without hesitation based on various facts.

**Connections to the Study**

This study examines adaptive reasoning through adaptive reasoning indicators by coding data similar to Awofala’s (2017) study of students’ dialogue in mathematical tasks. Studies that develop adaptive reasoning indicators are of relevance to determine indicators that operationalize adaptive reasoning in mathematical tasks. In summary, adaptive reasoning indicators for this study include: logical thought, reflection, explanation, proof, justification, pattern recognition, truthfulness determination, deductive and intuitive reasoning, analogy, metaphors, conjectures, augmentation, compare and contrast, connections making, and conclusion drawing.
Learner-Generated Drawings

Researchers utilize drawing for differing research purposes (Theron et al., 2012) such as to better understand the details of students’ thoughts (Finson, 2002) or better understand students’ emotional literacy (Wetton & McWhirter, 1998). This review focuses on research and theory of learner-generated drawings used in educational settings throughout a learning process, to better determine how drawing can evidence indicators discussed relative to adaptive reasoning. Researchers highlighted in this review, define learner-generated drawings as visual marks designed and created by the student that represent a situation or problem (Leutner & Schmeck, 2014; Van Meter & Firetto, 2013). The term learner-generated drawing also describes the strategy where students generate drawings.

The following sections: 1) review the Cognitive Model of Drawing Construction (CMDC), including self-regulated learning and the generative learning process, 2) review findings related to learner-generated drawings, and 3) make connections to the present study.

The Cognitive Model of Drawing Construction

The CMDC contends that the act of drawing influences learning by altering students’ approach to understanding an idea and in turn increases students’ success in solving problems (Van Meter & Firetto, 2013). Developed over the past forty years, this theory draws from several other theories that clarify the dynamics of how, why, and under what conditions student learning occurs.
The act of drawing requires students to engage in complicated cognitive activity. Van Meter and Firetto (2013) describe the cognitive and metacognitive dynamics of these thought processes. For instance, when students hear or see instruction, they select and organize the information into a surface representation and then a propositional representation (see Figure 2.3). Both occur internally. The *surface representation* is what students see and/or hear initially, whether written text, a visual image or sound. Through semantic processing elements are organized into a *propositional representation*, an internalized structuring of elements and relations. A visual-spatial representation is derived from integrating prior knowledge. Van Meter and Firetto call this the student’s *mental model*. Students arrange elements of the mental model into a perceptual image that then is externalized or physically drawn. Construction of knowledge occurs between the proposition representation and mental model, as prior knowledge integrates into the new representation.

**Figure 2.3**

*Cognitive Model of Drawing Construction Process*

*Note.* White boxes represent internal processes. Shaded boxes represent the external events. Black arrows represent influences of prior knowledge (adapted from Van Meter & Firetto, 2013).
Cognitive processes take place as students receive, organize, translate, and construct information into a visualized image and then construct an externalized image (Van Meter & Firetto, 2013). These processes continue throughout the creation of the drawing. Students’ cognitive processes do not necessarily progress in a linear fashion; they may progress from a surface representation to a perceptual image or from a surface representation to a mental model, depending upon the complexity of the task and a student’s prior knowledge. For instance, the progression students engage in when drawing may differ if they were asked to draw an ‘S’ shaped river compared to a fireman rescuing a cat from a twelve-foot lamppost or the digestive system of an elephant. A student’s cognitive processes, presented with familiar instructional material, might progress directly from a surface representation to a mental model; whereas presented with unfamiliar material, a student’s cognitive processes might progress to propositional representation and further organization of information.

Prior knowledge plays an important role in particular students’ cognitive paths. The more experience or prior knowledge students have of the material and how to draw the particular material determines the cognitive interaction of deriving an image. In other words, if students have drawn a giraffe several times, it’s easier for these students to draw a giraffe. Or, if you’ve taken apart a lawn mower, it’s easier to draw a model of how the mower engine functions. A student’s prior knowledge and experience is especially integral in the development of their drawing relative to their mental model, the creation of the perceptual image, and the final drawing product (Van Meter & Firetto, 2013).
Self-Regulated Learning and the Generative Learning Process

These processes and progressions exemplify how students engage in self-regulated and generative learning processes (Vosniadou, 2003; Wu & Rau, 2019). Self-regulated learning (Panadero, 2017) describes how students monitor and control their own learning processes and is an integral part of CMDC (Van Meter & Firetto, 2013). Regulation implies adaptation and is inherent in solving mathematical tasks in collaborative learning environments (Hadwin et al., 2018) and when students draw. When students are asked to “create a drawing”, they self-regulate (Schleinschok et al., 2017). They first synthesize instructional information with prior knowledge to form a mental model (Winne & Perry, 2000). With the understanding that a drawing will be the final product, the student then sets goals or standards, applies strategic operations, and integrates prior knowledge (Winne, 2018). They monitor their progress, adjusting the drawing to reach their initial goal.

The generative nature of self-regulated learning occurs when students face inconsistencies in representations within their drawing. The student receives visual feedback that creates opportunities for cognitive engagement (Hadwin et al., 2018). Cognitive dissonance occurs when students recognize gaps between their mental model and the drawing. To fill the gaps, students either return to the previous step in the CMDC process to clarify or cross reference missed information, or they engage in a generative process and change or build upon their prior knowledge (Vosniadou, 1994). Butler and Winne (1995) describe this generative cycle of self-regulated learning as internal feedback.
Generative Learning and Adaptive Reasoning

The epistemological processes that take place when visual information in a drawing is assessed and altered is not as clear within the literature. It is here, however, that theorists believe that learning takes place (Stieff, 2017). Chi (2009) concurs with this belief and explains that although dialogue incites self-explanation, drawing implicitly demands self-explanation within its construction. Prain and Tytler (2012) contend in their representational construction affordances framework (RCA) that when students construct and negotiate within the drawing process, they are making, justifying and communicating their thoughts and that the act of knowledge construction involves reasoning such as: problem solving, pattern identification, and justification. Deeper learning occurs when students create drawings rather than just copying an illustration (Edens & Potter, 2003).

Self-Regulated Learning, Generative Learning and Feedback

In the proposed study, CMDC provides details of how students interact with the drawing process and present their mental models and their learning processes. Self-regulated learning is affected by both internal and external feedback (Butler & Winne, 1995). Students experience internal feedback in the creation of their drawings. They experience external feedback through viewing their drawing product and in interaction with their environment such as interactions with fellow students. External feedback enters the CMDC process in a similar fashion as initial visual or auditory information. For example, in guided inquiry-based mathematical tasks, students engage in exploratory talk, a form of external feedback. Students receive constructive criticism that is internalized into their predetermined, self-regulated drawing goals or standards.
Generative learning occurs as students apply strategic processes to reach those goals. Internal and external feedback influences students’ decisions. The generative process of knowledge building occurs at both the individual and partner level (Prain & Tytler, 2012) when students engage in exploratory talk and adjust drawings to create representations.

**Learner-Generated Drawing Empirical Studies**

In mathematics and science education, research methods that examining learner-generated drawings generally use a pretest-treatment-posttest structure with two or more groups, one being a control group. This structure is a simple method for exploring changes or differences between groups. A few researchers, however, employ more innovative ways to determine the effectiveness of drawing as a classroom strategy, such as the examination of students’ drawings. Learning theory suggests that learner-generated drawings are an effective strategy due to the lessening of cognitive load and also the self-regulated learning dynamics that afford students generative processes to build knowledge. However, research findings indicate that learner-generated drawings do not always affect student learning in this manner. Learner-generated drawings can be time consuming, create greater cognitive load for some students, and require instructional support on strategic knowledge of drawing (Fiorella & Zhang, 2018).

The following subsections discuss (1) drawing strategies and effectiveness, (2) drawings and visualizations, (3) drawings in problem solving, and (4) drawing creation and strategic knowledge.
**Drawing as a Strategy and Effectiveness**

Research indicate that learner-generated drawings are a successful learning strategy (Schmidgall et al., 2018). Leopold and Leutner (2012) conducted two experiments to examine the effectiveness of learner-generated drawings as a strategy for notetaking. In the first experiment, students either created drawings as notes or outlined their notes in words. In the second experiment, all students outlined their notes and half created a written summary of their notes while the other half created drawing summaries. In both experiments, the drawing activity increased students’ reading comprehension, as evidenced in their ability to transfer acquired knowledge to new situations.

Csíkos et al. (2012) combined third grade students’ visual strategies with drawings in a pre-post experimental research design study. Researchers instructed two groups (experimental group and control group) of students as they completed 73 word problems in 20 lessons, which included visuals of the problems. The experimental group, consisting of 106 students, provided participants an illustration or visual of the problem and asked that they make a drawing, then solve the problem. The control group, consisting of 138 students, solved the same problems and were given the same visuals but were not asked to produce a drawing. Results showed that the experimental group experienced more achievement gains compared to the control group. Further, the experimental group experienced significant increases in their belief and attitudes when problem solving. While these groups had the same attitudes toward mathematical learning, the experimental group recognized how important the act of drawing was when problem solving.
Schwamborn et al. (2010) designed an experimental study examining 196 ninth-grade German students’ generative learning given different learner-generated drawing strategies. Participants were separated into five groups; all but the control group created drawings after reading about the chemical occurrences caused by soap and water when cleaning laundry. The first group read the text and created a drawing representing each paragraph. The second group read the text, underlined the main ideas, and created drawings to represent the main ideas. The third group read the text, visualized the information in each paragraph, then created a drawing to represent each paragraph. The fourth group read the text, underlined main ideas, visualized the main ideas, and then created a representational drawing of the main ideas. The fifth group, the control group, did not draw. In the second part of the study, all students created a drawing of the main ideas. All students participated in a series of posttests to assess transfer, retention, and drawing accuracy given a new situation.

Students that participated in any drawing strategy tested higher for transfer, retention, and drawing accuracy than students who did not draw. In addition, students with higher drawing accuracy scores also scored higher on transfer and retention. This research suggests that drawing accuracy can be a predictor of student understanding as well as integral in generative learning.

Another group of researchers (Schmidgall et al., 2018) conducted two experiments in an attempt to understand elements that contribute to the benefits of drawing as a strategy in science classes. The first experiment examined three possible mediating factors: generation, visualization, and externalization. Researchers grounded each of these factors to theoretical underpinnings of generation effect, dual coding theory,
and multimedia effect, respectively. Researchers randomly assigned 121, German undergraduates to one of four groups: (1) to draw text (visualize and generate), (2) to study multimedia (visualize only), (3) to summarize text (generate only), and (4) to study just text (no generative or visualizing). A pre-/post-/post delayed- test measured student understanding and retention. Studies evidence that drawing can be an effective strategy when cognitive load is taken into consideration. The study also shows that success in drawing comes from the internal visualization of images that occurs as a student draws rather than from the actual act of drawing.

**Drawings and Visualizations**

Research that focused specifically on visualization is important to the proposed study. *Imagining*, a construct to consider with visualization, is a type of internal visualization technique, and learner-generated drawings are a type of external visualization technique (Lin et al., 2017). Are visualization strategies just as effective as the creation of learner-generated drawings? Cooper et al. (2001) contend that learning occurs when students imagine. The following studies incorporate visualization as an alternative to drawing and/or use visualization in conjunction with drawing with varying results.

In the afore mentioned study, Schmidgall et al. (2018) suggest that students’ visualization was a primary factor that contributed to their ability to understand and retain knowledge when drawing. Researchers then conducted a second experiment to better understand how the visual and spatial elements of this factor related to students’ learning gains. Results indicated “that the benefits of drawing on learning outcomes that have
been shown in prior research stem mainly from the process of externalizing a visualization that drawing requires, rather than the actual generation of the drawing” (p. 15).

In 2009, Leutner, Leopold and Sumfleth examined students’ drawing and visualizations, or what researchers termed *mental imaging*, in science classes. Researchers used cognitive load and multimedia learning theories to frame their research. Using a 2X2 experimental design, researchers asked students to read from science textbooks and then visualize or not visualize and draw or not draw pictures representative of their reading, creating four distinct interactions between visualization and learner-generated drawings. Researchers used a pre-/post- test research design to measure students’ comprehension relative to their cognitive load and analyzed data using a general regression linear model. Initial results indicate that drawing had a negative effect on student comprehension; however, this effect was mitigated when controlling for students’ cognitive load. The act of drawing increased students’ cognitive load, which then affected their ability to comprehend text. However, visualization did not increase students’ cognitive load and results indicated an increase in their ability to comprehend text.

The Leutner et al. study (2009) is important because it evidences particular interactions between cognitive load and visualization/learner-generated drawings; results were indicative of a specific student in a given learning situation. Lin et al., (2017) conducted a similar study using a pre-/post- test experimental research design. Three groups of 21 undergraduate students read a science text and either imagined the text, created drawings of the text, or reread text. The results showed that the effects of drawing were similar when compared with students with high prior knowledge; however, results
with students with relatively low prior knowledge showed drawings were found to foster student comprehension. Known as the expertise reversal effect (Kalyuga, 2009), these findings explain that learner-generated drawings create an externalized structure that scaffolds and changes the schema of students with low prior knowledge, while creating redundancy for students with high prior knowledge.

Zhang and Linn (2011) examined the effect of drawings to enhance learning in conjunction with computer-based visualizations. In their mixed methods experimental study, 133 eighth-grade chemistry students participated in a computer-based lab experiment that contained computer generated visualizations of chemical reactions. This study, similar to Rellensmann et al. study (2017), is important because, unlike the other science-oriented studies, it utilized a task rather than student interpretation of a science text. All participants observed computer-generated visualizations of chemical reaction. One group of students created drawings of the visualizations and the other did not but were instead asked to repeatedly view lab visualizations. Pre-/post-test results showed gains in understanding for both groups. Visualizations provide both groups the opportunity to access the lab material. However, students that generated drawings were able to more fully retain details from the visuals, evidenced by increases in comprehension compared to the visualization-only group. Drawing required students to organize material, consider relevant material, and integrate prior knowledge. These findings suggest that the addition of a drawing element enhances the generation of knowledge. Students are limited by their ability to gather needed information from visualizations, producing a more superficial understanding of visualizations (Chiu & Linn, 2012). Zhang and Linn (2011) posit that drawing “provides more opportunities to
learners to recognize conflicts between their prior knowledge and new views from the visualizations” (p. 1194).

These studies examined visualizations in accompaniment to learner-generated drawings and also in comparison to learner-generated drawings. Studies present conflicting results on which strategy is more effective; however, studies concur that the combination of visualization and learner-generated drawings can be beneficial and may assist in the lessoning of students’ cognitive load.

**Drawing and Traditional Problem Solving**

As seen from the previous section, much research is presented on learner-generated drawings within the context of science education and reading scientific text. In mathematics education, learner-generated drawing research centers around problem solving and examines students use of drawings to solve problems.

The history of problem solving differs in each country; however, in the U.S. and Canada, problem solving was allocated to specific domains: mathematics, science, social studies (Sternberg & Frensch, 1991). In 1945, George Polya outlined strategies for problem solving that would guide instruction in mathematics classrooms for years to come. Polya’s first two principles, 1) understand the problem and 2) devise a plan, include two elements of drawing: “think of a drawing” and “make a drawing” (Polya, 1945). However, drawing strategies that focused on Polya’s second principle “make a drawing” oftentimes excluded the first element requirement “understand the problem.”

Lesh and Zawojewski (2007) contended that teaching students problem solving strategies void of principle number one does not necessarily lead to an increase in student
understanding. They suggest that strategic knowledge is more important than teaching
problem solving steps. In particular, Lesh and Zawojewski explain that accurate
representational ability enhances “the communication capability and conceptual
flexibility that are important to the development of solutions to many real-life problem-
solving situations” (p. 791).

**Drawing with Strategic Knowledge**

*Strategic knowledge* is the learner’s ability to understand, think about, reflect
upon and interpret problems. Rellensmann et al. (2017) investigated student strategic
knowledge and how it mediates mathematical modeling performance. Researchers
instructed 61 ninth graders from a school in Germany to complete a strategic knowledge
test. The test consisted of a word problem and a series of drawings. Students were to
select the drawing that would be most helpful in solving the problem. Following the test,
students were given eight modeling tasks and asked to create a *situational drawing*, a
*mathematical drawing*, and solve the problem. Researchers define a situational drawing
as one that depicts the storyline of the problem and a mathematical drawing as a more
abstract representation of the problem including numerical relationships. Researchers
found that students that scored higher on the strategic knowledge test created drawings of
higher accuracy (resembled task details) with correct solutions on the modeling tasks.

The study’s (Rellensmann et al., 2017) path analysis, or description of directed
dependencies of a variables, reported that both situational and mathematical drawings
were important to modeling performance for different reasons. Situational drawings help
students better understand the task (Leiss et al., 2010) and assist in their creation of a
more accurate mathematical drawing. While mathematical drawings are helpful in
solving a problem, students have more difficulty constructing mathematical drawings. There was a strong relationship between students’ ability to produce a more accurate mathematical drawing and student achievement.

Rellensmann et al. (2017) contend that future research is needed to examine educational interventions that teach students strategic knowledge and that factors which influence the efficacy of student generated drawings needs to be researched. The research implies that situational drawings are step one in Polya’s (1945) problem solving “understand the problem” and mathematical drawings are similar to step two “make a plan.”

**Connections to the Study**

Throughout the proposed study learner-generated drawings will be used by participants as a strategy to solve problems in Three-Act Mathematical Tasks. Empirical studies suggest that the creation of learner-generated drawings can enhance students’ generation of knowledge (e.g., Csikos et al., 2012; Schmidgall et al., 2018; Zhang & Linn, 2011). Research evidences that the addition of learner-generated drawings in problem solving situations enhances learning (Rellensmann et al., 2017; Zhang & Linn, 2011), especially when students are presented with strategic knowledge of drawing (Rellensmann et al., 2017). Limitations in the use of drawings in research include students’ limited knowledge of strategic drawing (Rellensmann et al., 2017), misinterpretation of drawings, and possible cognitive overload (Leutner et al., 2009). These issues were considered in the design of the study to control for potential difficulties.
Partner Dialogue

In this study, the role of partner dialogue is important in understanding student thought through the form of verbal speech and written drawings. Learner-generated drawings support student expression and student dialogue helps to explain drawings. Together, drawings and dialogue add observable insight to students’ communication of their reasoning. Researchers have studied various types of effective talk and talk pedagogies including exploratory talk, collaborative reasoning, accountable talk, thinking together, collective argumentation, and dialogic teaching (Khong et al., 2019; Phillipson & Wegerif, 2016). This review addresses one of the most researched types of classroom talk that is most closely aligned to mathematics education (Khong et al., 2019) and the present study: exploratory talk. Exploratory talk builds upon dialogic learning and the work of Vygotsky (1978) and Barnes (Barnes & Todd, 1977, 1995). Described by Mercer (1996), exploratory talk consists of a two-way dialogue between partners used in problem solving. When engaging in exploratory talk, partners share a common goal and are critical and constructive with each other’s ideas. With exploratory talk “knowledge is made more publicly accountable and reasoning is more visible” (Mercer, 1996, p. 269).

This literature review utilizes the following definition of exploratory talk (Mercer et al., 1999):

Exploratory talk is that in which partners engage critically but constructively with each other’s ideas. Statements and suggestions are sought and offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and alternative hypotheses are offered. In exploratory talk, knowledge is made publicly accountable and reasoning is visible in the talk (p. 97).
To review findings from the literature surrounding exploratory talk, the following subsections frame this discussion: 1) Dialogic learning and exploratory talk theory, 2) empirical research on exploratory talk, including the facilitation of classroom exploratory talk, adaptive reasoning and exploratory talk, coding and analysis of exploratory talk, and equity, and 3) connections to the study.

**Dialogic Learning and Exploratory Talk, Theoretical Background**

*Dialogic learning* is the learning that takes place when individuals engage in dialogue and exchange ideas that conflict or build upon each other. The theoretical underpinnings described by Vygotsky (1978) was that through students’ dialogue thought and meaning are produced. Vygotsky described the learning that occurs during dialogue as a socially constructed phenomenon where students combine thought and speech. Pertinent to this study, Vygotsky includes other forms of communication in his description of speech, such as symbols, algebraic systems, art, drawing, writing and diagrams (Vygotsky & Kozulin, 1962/2012).

Vygotsky’s contemporaries expand views of dialogue and its relationship to learning. Bakhtin, for instance, focused on the interaction of peers involved in dialogue and how meaning is situated within the context of a unique conversation (Wegerif & Mercer, 1997). Meaning is developed dependent upon the utterances of individuals and the understanding of those utterances by a student. Barnes and Todd (1977) explored three types of talk that occurred between individuals or small groups, (1) disputation, (2) cumulative, and (3) exploratory talk. They found that through exploratory talk students developed meaning from the changes in direction or the ‘exploration’ that occurs between students as they problem solve. Students sharing ideas in exploratory talk achieve more
generalizable kinds of knowledge (Mercer, 1996) situated within the dialogue. Expertise is adapted through the dynamics and interactions of the individuals comprising the group (Hatano & Inagaki, 1992). In exploratory talk, one identifies with the dialogue itself rather than with one’s own opinion or the opinion of others, which leads to an openness to new ideas and perspectives (Wegerif & Mercer, 1997).

**Exploratory Talk Empirical Studies**

Research on exploratory talk establishes that it is an effective tool for small group discussion (Khong et al., 2019; Patterson, 2018; Rutter et al., 2016). Pertinent to this study was research that discusses how to facilitate classroom exploratory talk and methods of coding and analysis of exploratory talk. The following subsections address this.

**Facilitating Classroom Exploratory Talk**

Research shows that teaching students guidelines on how to talk in an exploratory manner is beneficial to student learning (Monaghan, 2011; Murphy, 2016; Rutter et al., 2016), especially in consideration of secondary language learners (Robertson, & Graven, 2019) and marginalized learners (Moschkovitch, 2018). These guidelines are referred to as *ground rules*, or rules of partner or group dialogue engagement that promote exploratory talk. Exploratory talk research on ground rules follows a method standardized by researchers.

In 1999, Mercer et al. conducted a study on 60 nine to ten-year-old school children. The children were divided into intervention and control groups. The intervention group received training on group ground rules, the control group did not.
The ground rules included: discuss things together, make changes and alternatives explicit, think before you speak, respect and valuing other people’s opinions, share all of your ideas, and confirm group consensus. After the intervention, students worked individually and in groups on Raven’s standard and colored progressive matrices, non-verbal tests that measure abstract reasoning. Researcher video recorded the work of eight groups of students. Analysis of student exploratory talk showed that 1) intervention students increased their use of exploratory talk from pre to posttest, 2) when students used exploratory talk they were more effective at problem solving, and 3) students who were taught exploratory talk made greater gains in their individual scores on the Raven’s test. From these results, Mercer et al. found that exploratory talk assists students in problem solving and that setting ground rules in instruction is instrumental.

In a more recent study, Rabel and Wooldridge (2013) explored the benefits of exploratory talk in 39 fourth grade students using a similar method of intervention with a pre-/post-test. Rabel and Wooldridge (2013) also collected post semi-structured face-to-face group interview data. Through the examination of student dialogue, researchers compared the quality of exploratory talk and ground rule use in the intervention group with the control group that received no instruction on exploratory talk. Rabel and Wooldridge (2013) found that high-level exploratory talk was possible for the given age group when students are engaged in mathematical tasks and provided guidance in the use of exploratory talk. Researchers measured the level of exploratory talk by looking at on-task behavior combined with productive dialogue that led students to reach a solution. In addition, students of medium ability in mathematics benefited most from engaging in exploratory talk. Both same and mixed ability groups benefited. Student pairing based on
friendship improves exploratory talk (Rabel & Wooldridge, 2013). In summary, exploratory talk is beneficial to problem solving situations, instructing students on ground rules helps students to focus on productive exploratory talk, and student pairing should be considered when developing groups.

Herrlitz-Biró et al. (2013) analyzed students’ use of key words by recording exploratory talk during a discussion task and discovered that the amount of key words used in exploratory talk depends upon the level of task difficulty. In four classes of 11-12 year-old students, 26 groups of three to four students were videotaped during a discussion task. Students individually read the presented task, first forming their own ideas and solutions and then engaging in group exploratory talk. If the task level was easy for the participants, exploratory key word use was lower than if the task was challenging. Similar studies confirm that activities and tasks that challenge students subsequently promote exploratory talk (Georgius, 2013; Samuelsson et al., 2011; e.g. Dahl et al., 2018).

In connection to the proposed study, exploratory talk occurs during dialogic learning activities such as guided inquiry-based mathematical tasks. This research shows that challenging tasks and the establishment of exploratory talk ground rules promotes effective exploratory talk.

Adaptive Reasoning Indicators and Exploratory Talk

While studies report on the improvement of student reasoning within the context of exploratory talk (Rutter et al., 2016; e.g. Murphy et al., 2017), only a few studies examine students’ adaptive reasoning. Given that indicators of adaptive reasoning
overlap with elements of exploratory talk research, it is critical to consider how these concepts interact. Following are studies that combine exploratory talk and mathematical proficiency (Stevens, 2017) and exploratory talk and adaptive reasoning (Langer-Osuna & Avalos, 2015; Webb et al., 2017).

In a case study, Stevens (2017) observed four ninth grade student pairs’ dialogue over the course of 14 mathematical tasks. Stevens examined students’ dialogue to describe their exploratory talk use relative to their mathematical proficiency during an intervention. The intervention group received instruction on ground rules for exploratory talk, while the control received no guidance. Within the study, indicators of mathematical proficiency were disaggregated affording particular elements to be made more explicit, such as adaptive reasoning. The researcher examined use of adaptive reasoning indicators, “explanation of solutions, justification or explanation of procedures, reflection concerning the context of the problem, and reflective critique of another student’s solutions or explanations” (p. 101), during exploratory talk.

Results indicate that students introduced to exploratory talk exhibited increased uses of adaptive reasoning as well as other strands of mathematical proficiency. Stevens (2017) interpreted the results as student movement towards mathematical proficiency. While explanatory reasons for student movement towards proficiency were unclear, the results suggest that instruction on exploratory talk ground rules increases adaptive reasoning indicator use. The researcher found that particular student ground rules increased adaptive reasoning use. These ground rules include: (1) share your understanding, strategies, explanations and reasonings, (2) justify what you notice, (3) explain how you got your solution, and (4) share your solution paths. In addition, the
ground rule intervention seemed to help with student behavior. One participant, noted for his impulsivity, benefited from the ground rule structure through the affordance of new engagement possibilities.

Webb et al. (2017) conducted a review of a series of five studies on exploratory talk and reasoning in view of far transfer or the ability for a student to apply prior knowledge to new situations in which the context differs greatly. The collection of studies focused on the development of scientific literacy of students using exploratory talk in science and mathematics classrooms with students ages 11-14. All five studies together consisted of approximately 2,000 students. Intervention and control students took part in a pre-/post- Raven’s progressive matrices test to measure transfer as characterized by Barnett and Ceci taxonomy (2002). Teachers were trained in the use of exploratory talk and classrooms equipped with posters that reminded students of exploratory talk ground rules. Researchers examined exploratory talk by evidencing student talk with audio or video footage and comparing changes in students’ reasoning skills per the Raven’s tests. Results showed that classes that successfully implemented exploratory talk saw significant improvement in far transfer. Kilpatrick et al. (2001) explain that one indicator of adaptive reasoning is the ability for students to apply prior knowledge to new situations. Important to the proposed study, Webb et al. suggest that the use of exploratory talk improves far transfer, an indicator of adaptive reasoning.

Langer-Osuna and Avalos (2015) videotaped 27 fourth graders as they used exploratory talk during 71 minutes of a mathematics discussion. The study examined the use of exploratory talk in the mathematics classroom. Students received instruction on ground rules for exploratory talk. Researchers coded discussion for turning points, places
where one line of talk encouraged a different path in conversation, and indicators of exploratory talk. The indicators of exploratory talk include: explaining, elaborating on or requesting an explanation, pointing out obvious errors, defending reasoning, offering alternatives, and agreeing or disagreeing with reasoning. Results showed a variety of talk. While some students engaged in exploratory talk and ground rules, others focused on developing a correct answer without engaging in ground rules. In addition, social skills of students enable or disable exploratory talk. For example, one student who lacked the ability to express confidence in her reasoning was overlooked by other members of her group, even when her comments contained productive insights. The study results suggest that not all student reasoning is explicit to talk.

Adaptive reasoning examined in these studies characterize adaptive reasoning through indicators such as: explanation, elaboration, reasoning defense, error observation, alternatives considered, argument or agreement over reasoning, and transfer. In addition, Stevens (2017) evidences that adaptive reasoning occurs when students use particular exploratory talk ground rules that reinforce student reasoning, explanation, and justification.

**Coding and Analysis of Exploratory Talk**

Analysis and coding of dialogic talk such as exploratory talk is established in literature. Many of the coding indicators of exploratory talk are similar to adaptive reasoning indicators. Hence, exploratory talk coding and analysis procedures in literature provides insight into methods to operationalize adaptive reasoning indicators for the proposed study.
In 2016, Hennessy et al. developed an extensive coding system called Scheme for Educational Dialogue Analysis (SEDA) by analyzing different studies that coded dialogue during whole classroom, large and small group dialogue. SEDA, developed for the purpose of coding dialogic classroom discussion across educational disciplines, is a compilation of common key words and examples of key word used in dialogic conversations (Hennessy & Rojas-Drummond, 2016). SEDA coding for exploratory talk, largely derived from the contributions of Neil Mercer (2019), breaks down coding into clusters or acts of student interactions. For example, clusters include the following: invite elaboration or reasoning, make explicit reasoning, express or invite ideas, build on ideas, position and coordinate ideas, connect and reflect or guide the direction of dialogue or an activity. Guiding notes when researchers use SEDA recommend 1) coding during student dialogue, 2) counting codes once if the same speaker reiterates the code in the same line of argument, 3) counting a repeated code more than once if it initiates a new line of thinking, 4) coding off task discussion with a U (un-coded), 5) including other forms of communication other than verbal, 6) coding not be dependent upon responses of other students, 7) accepting less sophisticated evidence of proof or argument in younger students, 8) considering prior and subsequent conversation when coding, 9) considering segmentation of data as complex as forms of embedded, interrupted, and overlapping dialogue, and 10) determining hierarchical ordering of clusters, segmentation of transcript, and categories as not mutually exclusive and grounded in the study’s conceptual framework. Once data is coded, the researcher determines analysis.

Lefstein et al. (2015) recommend the use of sequence analysis. In sequence analysis, the researcher considers dialogue in relevance groupings in the context of the
dialogue to discern meaning. When sequences are embedded, the researcher works with lag sequential analysis, where chains of dialogue are considered in context. Re-analysis of these same data using different groupings adds to the accuracy of results. In their study, Lefstein et al. video recorded 73 literary lessons in seven upper primary grades. Dialogue was coded into uninterrupted sequences, noting different pauses in dialogue as separators in sequences. Data was also coded per the discourse function such as if a student intended to explain, direct, question, respond or give feedback. Simple feedback was noted over directive talk and summarized into categories: recap, review group work or the introduction of a new task. In the proposed study, the researcher considered sequencing analysis in coding data.

**Gender and Exploratory Talk**

Studies on the effects of gender in exploratory talk support the idea that exploratory talk is more equitable with mixed-gender groups when ground rules are established prior to engaging in mathematical tasks. In a systematic review, Howe and Abedin (2013) examined 40 years of classroom dialogue across 225 studies and found relatively few studies that agree upon student gender grouping during classroom dialogue.

In a quasi-experimental mixed-methods study (T’Sas, 2018), 187 primary schools students ages 11-12 participated in 24 math and science activities over a 12 week period. The researcher examined the effects of establishing ground rules during exploratory talk of students in groups of three of varying gender. To measure exploratory talk the researcher counted interactions between male and female participants and found that
without an intervention that established ground rules, male student dominated the talk. However, in the groups that received interventions on how to talk within a group using ground rules, females participated at an equivalent rate as male students. The proposed study included mixed gender grouping of students. In addition exploratory talk was presented in the pre-study instruction.

**Connections to the Study**

The review of the literature shows that exploratory talk closely aligns with student indicators of adaptive reasoning. Guidelines that promote successful exploratory talk assist in the promotion of students use of adaptive reasoning. For example, the establishment of ground rules in the pre-implementation phase of the proposed study is designed to ensure successful exploratory talk. One ground rule of exploratory talk is for students to listen and be open to group suggestions (Rutter et al., 2016). This rule creates a perpetual openness to alternative perspectives (Wegerif & Mercer, 1997) which is also an indicator of adaptive reasoning (Kilpatrick et al., 2001). Because of the close alignment of exploratory talk and adaptive reasoning, coding and analysis connect to the proposed study methods.

The researcher found no studies on drawings and exploratory talk; although, learner-generated drawings may constitute written expression of exploratory talk (Brooks, 2006, 2009; Stevens, 2017). Learner-generated drawings were included in the coding of partner dialogue. Additionally, student pairing were considered because of its importance to successful exploratory talk in inquiry learning (Rabel & Wooldridge, 2013).
Guided Inquiry-Based Mathematical Tasks

This study takes place within Three-Act Math Tasks, a guided inquiry-based learning strategy that engages students in active learning. The following describes 1) active learning theory and mathematical tasks, 2) empirical studies, and 3) connections to the study.

Active Learning Theory and Mathematical Tasks

Guided inquiry-based mathematical tasks utilize active student learning. The Russian psychologist Leont’ev (1974), influenced by his colleagues Vygotsky and Rubenstein, developed active learning theory. In active learning, students participate actively in the learning process and construct knowledge while engaging in higher-order thinking and reasoning (Bonwell & Eison, 1991). Tasks that evoke active learning, an integral strategy for middle school students (Edwards, 2015), are generally teacher-guided and student-centered. Active learning draws from constructivist ideas (Haak et al., 2011) and increases student performance in mathematics (Freeman, et al., 2014). Learning occurs when new information challenges prior understandings. Cognitive dissonance occurs and prior knowledge is adjusted.

Mathematical Tasks Framework

Kilpatrick et al. (2001) contend that successful use of adaptive reasoning within a task requires three essential conditions. The task needs to be understandable and motivating, students need to have a sufficient knowledge base, and students need to be familiar and comfortable with the content. The mathematical tasks framework, created by
Smith and Stein (2011), outlines guidelines to create effective tasks with the idea that successful tasks create opportunities for successful student learning. These guidelines are well studied and supported with empirical research and include cognitive challenge, familiar content, teacher support of student task ownership, and management of cognitive load (Stein et al., 2009). The framework assists a teacher in development of tasks to fit the cognitive needs and interests of students given differing learning situations (Stein et al., 2009). The mathematical tasks framework involves three elements: (a) a goal or product, (b) a set of resources or *givens* in the situation and (c) the opportunity for discovery of operations that can be applied to the resources to advance to the goal (Doyle & Carter, 1984). A mathematical task is devoted to the development of a particular mathematical idea and involves a complex problem or multiple smaller yet related problems. Tasks generally take twenty to thirty minutes or less to complete and are selected or designed by the teacher, implemented by students, and summarized by the students and the teacher (Stein & Smith, 1998).

**Mathematical Tasks and Depth of Knowledge Levels**

Guided inquiry-based mathematical tasks afford students opportunities to engage in various levels of thinking. The researcher uses depth of knowledge (DOK) in the study’s design as a measure of student cognitive engagement. Litster’s (2019) findings show that students engaged in mathematical tasks may not participate at the task designed level of cognitive demanding. Student dialogue and peer interaction can change the cognitive demand of a task depending upon student knowledge and access to content. Since mathematical tasks vary in difficulty ranges of DOK, DOK level test data will be
analyzed by categorizing data into the three levels by labeling them as below, at, and above level 2. DOK levels described by Webb (1997, 2002) is a framework for teachers and practitioners to determine cognitive level of student assessment. The levels range from one to four. Task problems of DOK Level 1 can be solved using basic recall or reproduction of facts. Level 2 problems require skills and concept knowledge. Level 3 DOK problems require strategic thinking to solve and Level 4 DOK occurs when students extend thinking into other problems or situations. While mathematical tasks may be designed to engage students in all four DOK levels, students adapt tasks to allow for access or efficiency (Litster, 2019). For example a student unable to understand a task may engage differently than a student who comprehends the task meaning. Or a student that has solved a similar task may simply apply prior knowledge of solution rather than engage in more extended thinking.

The proposed study analyzed DOK level test data based on three intervals of Webb’s (1997, 2002) DOK levels: (1) below DOK level 2, (2) at DOK level 2 and (3) above DOK level 2 (Litster, 2019). These intervals were used in DOK level tests to compare student use of adaptive reasoning indicators during mathematical tasks. Students engaged in below level 2 utilize previous knowledge in a passive manner, recalling facts and procedural skills as a rote response or systemized steps. Students are not adapting their reasoning. Students engaged in level 2 move beyond habitual responses to basic reasoning. This includes classifying and organizing data, comparing data and determining solution strategies. Students may be utilizing adaptive reasoning indicators. In level 3 and above, levels 3 and 4 are combined. Both DOK levels 3 and 4 require more complex levels of reasoning. In level 4, student reasoning involves a high level of complexity.
exemplified across time and into new situations. In level’s 3 and 4, students explain and justify their reasoning which evidence indicators of adaptive reasoning. See further explanation in Chapter 3.

**Three-Act Math Tasks**

The Three-Act Math Tasks, popularized by Dan Meyer’s TED talk in 2010, is a pedagogical technique used in middle school mathematics classrooms (Redmond-Sanogo et al., 2018). Three-Act Math Tasks, also called active mathematical storytelling, engages students in a mathematical scenario presented in multimedia and then affords students the opportunity to actively contribute to the construction of the tale (Redmond-Sanogo et al., 2018). Tasks are structured in a guided inquiry task framework (Kuhlthau et al., 2012), and consist of 1) an open, a hook to catch the students’ attention and create interest, 2) an immersion, time where students research or are given enough information to investigate the interest, 3) an exploratory time, a period to gather more information and engage in problem solving, and 4) a share out, or view possible solutions time. In the Three-Act Math Tasks framework, guided inquiry-based tasks are scaffolded so that students discover and construct their knowledge by engaging in exploratory talk and thinking from multiple perspectives (Stein et al., 1996).

The first act presents a question of inquiry of low cognitive load for the student, a simple scenario generally tied to a social situation that invokes inquiry presented in photo or video format. For example, Meyer’s cola task (Meyer, 2017) begins with a photo of a swimming pool being filled with cola. The teacher asks the students what they notice and wonder. Students suggest questions: How many bottles of cola would it take to fill the
pool? or How expensive is it to fill the pool with cola? or Why would anybody put cola in a pool? The teacher chooses the question that best guides students towards the designed learning goal. The teacher then asks students to estimate an answer that is too high and one that is too low, narrowing in on a range of possible solutions. In the cola task, the teacher might ask students to guess how many liters of cola are in the pool as she holds up a liter of cola.

During the second act, the students decide on what information and resources are needed to solve the problem. The students partner up to solve the problem. The teacher monitors student conversations. For example, students might ask for the volume of the pool and the teacher might provide dimensions such as height and diameter. Solutions may be shared with the whole class.

The final act reveals one possible solution, again usually presented in video or photo form. In the cola task, a video is shown of the pool being filled, sped up time-lapsed, with a clicker count of each liter that enters the pool followed by a young man running and jumping into the pool. Students compare and verify partner solutions, concluding the task.

Mathematical Tasks Empirical Studies

Current research on Three-Act Math Tasks is limited to articles of practice. This section reviews three articles that are about classes of differing age levels, young primary, primary, and middle school, reporting on strategies for implementation and implications for learning. Each peer-reviewed article followed the Three-Act Math Tasks format presented in the previous section. All three articles were authored in affiliation
with college or university faculty. In addition, articles on gender and mathematical tasks are included in the empirical studies.

**Three-Act Math Tasks**

Lomax et al. (2017) studied three K-2nd-grade classes as they progressed through mathematical tasks. In the article, one task is presented in vignette form followed by the authors’ findings. Using the Three-Act Math Tasks structure, teachers could easily create a lesson related to their present class content and change the lesson on the fly, depending upon student responses. Three distinctions were noted for which Three-Act Math Tasks differed from other mathematics activities: 1) tasks leveraged student knowledge, empowering students to create their own knowledge, 2) tasks increased engagement by providing multiple entry points allowing all of the students to participate in some way, 3) student work engaged students in mathematical modeling as described by the Common Core’s eight Standards of Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). This article concurs with research on guided inquiry-based mathematical tasks (Kuhlthau et al., 2015). In order to create a successful task, the classroom instruction should be teacher guided and student centered. Teachers should not rush in to help if students struggle; they should allow students to make sense of their own mathematical thinking. When students’ thoughts are not mathematical, a teacher should guide them towards mathematical thinking with unrevealing questioning. When working with younger students, a teacher should break the task into smaller manageable pieces.
Flynn (2017), a professional learning provider of primary schools describes a vignette of 21 fourth graders engaged in Three-Act Math Tasks. By taking on the role of the teacher, Flynn gives an educators view of the inner workings of administering a Three-Act Math Tasks. Flynn used pre-made, online tasks downloaded from the internet. He notes that tasks designers oftentimes test their task ideas on 101 questions (www.101qs.com) by uploading task key questions, usually in the form of a photo or video. Site viewers type in the first question that comes to their mind, creating a bank of questions for the designer to consider. Flynn (2017) found that the bulk of the teacher workload during a task is on: interacting with the students, considering student ownership, and the maintaining a classroom open to suggestions and questioning.

Hallman-Thrasher et al. (2018) present a vignette of a combined 6th, 7th, and 8th-grade class as they progressed through a Three-Act Math Task called “Distance from the Camera” (Pearce, 2015). Students watch a video of a man on a merry-go-round from a fixed camera viewpoint and then create the distance-per-time graph of the man as he circles on the merry-go-round away from and towards the camera three turns and then steps off the ride. Hallman-Thrasher et al. found that the presentation of a task in video form with additional support materials helps students reason and justify. The majority of the class time was spent on student work in small and whole class discussion, with an even split of time spent between student-to-student and teacher-to-student dialogue. Further, findings note the change in teacher and student roles, suggesting that students generated and validated ideas and teachers focused on listening, questioning, and assisting in the connection of student ideas. Conclusions explain the need for cognitively
demanding tasks that challenge students in order to engage students in the learning process.

**Gender and Mathematical Tasks**

Research indicates that a student’s gender is not a significant factor in student achievement in mathematical tasks in the U.S. (Friedman, 1989; Ghasemi & Burley, 2019). Student attitudes towards inquiry-based instruction do not differ with gender variation however if gender stereotypes go unchecked in the classroom girls may be less likely to fully participate in inquiry learning (Riegle-Crumb et al., 2019). In addition, if the context of the task can make a difference in gender achievement. In a quantitative study of 523 school students ages 5-11, Zohar and Gershikov (2008) found that when students are presented with male stereotypical context in a mathematical task, male students outperform female students and when female stereotypical context is used a mixed result occurs. However, when students engage in gender-neutral contexts female students outperform male students.

Research of mixed-gender student pairing in studies derived mixed results (e.g. Kanevsky, 2015; White & Pea, 2011; Zhan et al., 2015). A variety of factors effect mixed-gender groupings during learning instances such as mathematical tasks. This study, therefore, attempted to group students in an equal amount of mixed groupings and took care to present tasks in a non-stereotypical manner.

**Connections to the Study**

The design of the study viewed adaptive reasoning indicators of students engaged in Three-Act Math Tasks. These inquiry-based tasks promoted active, student-centered
learning in a problem-solving environment that promotes mathematical proficiency (Samuelsson, 2010) and student use of adaptive reasoning (Stylianou, 2013). Elements presented within these tasks created an environment conducive to adaptive reasoning. They included authentic and non-routine problems (Muin et al., 2018) and challenging tasks that engaged students and encouraged dialogic inquiry (Kumpulainen & Lipponen, 2010).

**Gaps in the Literature and Conclusion**

Adaptive reasoning is an essential element of mathematical proficiency (Kilpatrick et al., 2001). While it is clear that students’ reason as they engage in mathematics, few empirical studies show how, and under what conditions, students adapt their reasoning. Do fifth graders adapt their reason differently than ninth graders? Do students use different types of adaptive reasoning when working whole class or in small groups? This study proposed to explore and capture adaptive reasoning use during one instance, of seventh graders as they progressed and draw through guided inquiry-based mathematical tasks. Three-Act Math Tasks were chosen to stage the research in the classroom, partly due to their popularity amongst math teachers and the free and easy access of resources online, but mainly because of implementation elements. Articles on Three-Act Math Tasks are limited to mainly practitioners reports or professional sharing of ideas; however, these reports show that Three-Act Math Tasks are classroom manageable and easy to implement, can be completed in a typical class period, engage students through video representation of a real-world situation, and because of their
structure promote the use of exploratory talk and create an atmosphere rich in adaptive reasoning.

From the empirical studies, guidelines on how to conduct a study on adaptive reasoning can be deduced. Indicators of adaptive reasoning have been defined by researchers, mainly derived from the document *Adding it up: Helping children learn mathematics* (Kilpatrick et al., 2001). Summarizing the empirical studies, adaptive reasoning indicators included in this study are: 1) relationship or connections between concepts and situations, 2) posed disagreements, arguments and/or alternative approaches, 3) justification, explanation of reasonings or proofs, 4) integration and transfer of prior knowledge, 5) pattern recognition, and 6) legitimacy determined or correctness or appropriateness of strategies. Given the variety of indicators present in the empirical studies, this study allows for the possible emergence of additional indicators.

Adaptive reasoning is difficult to observe, occurring solely within the learner’s head. Important to this study is access to observable student adaptive reasoning. In the empirical studies, researchers derive information on adaptive reasoning through various sources such as written tests, student interviews, dialogue analysis and teacher and researcher observation. A comprehensive method of observing adaptive reasoning is not presented in the body of the literature; however, there are strengths in the variety of results to inform further study.

For effective mathematical tasks, researchers need to consider the mathematical task framework by using engaging and cognitively challenging tasks, leveled in consideration of students’ prior knowledge and contextual understanding, with guided
teacher support that promotes student ownership. Empirical research strongly supports the use of mathematical tasks in this manner to engage students in inquiry-based learning.

Exploratory talk and learner-generated drawings were divided into two sections in this literature review because empirical research naturally falls within one or the other category. However exploratory talk and learner-generated drawings are two sides of the same coin in that they are communication mediums that afford researcher observation. Once again, the literature guides the presentation of both.

For effective exploratory talk, researchers need to consider student groupings based on trust, situating talk within an engaging and challenging task, and establishing talk ground rules. Empirical research strongly supports these considerations. In addition, the literature provides much guidance of how to interpret and code exploratory talk in inquiry-based learning situations primarily through work of Hennessy and Rojas-Drummond (2016) and the development of SEDA.

For effective learner-generated drawings, the literature shows that researchers need to consider student strategic knowledge of drawing in mathematics and possible student fear of drawing. In addition, students need individual time to develop drawings independent of peer interaction. Literature supports these considerations. However, while literature exists on interpretation of learner-generated drawings (e.g. Weber et al., 2011), none exists in light of interpreting adaptive reasoning in mathematical drawings. Of the existing literature, interpretation of drawings is a significant concern as the interpretation passes through the lens of the researcher in a different way than the interpretation and coding of dialogue (Literat, 2013). In this study, the researcher included a conceptual framework that describes the researcher’s lens to clarify any interpretations.
CHAPTER 3

METHODS

While problem solving has long been studied by educational researchers, new types of instructional structures that integrate problem solving, such as Three-Act Math Tasks, are emerging in mathematics classrooms to provide a guided inquiry-based learning experience for students. In this study, the researcher investigated the adaptive reasoning of students as they drew, discussed, and solved Three-Act Math Tasks to better understand how students adapt their reasoning and the relationship between adaptive reasoning and mathematical proficiency. Mathematical proficiency is the expertise necessary for a student to learn mathematics successfully. Adaptive reasoning, one aspect of mathematical proficiency, occurs when a student uses logic to explain, justify, and extend their thoughts and actions to solve something unknown. As a student solves problems, the adaptations of his or her thoughts are an integral part of the student’s mathematical proficiency (Kilpatrick et al., 2001; Schoenfeld, 2007). Students engaged in learner-generated drawings which helped students break down multi-step tasks into more manageable chunks (Eisner, 2002), in order to more easily construct and communicate their reasoning.

This chapter explains the research design, setting and participants, data sources, procedure, data collection and analysis.
Research Design

This study used an exploratory sequential mixed methods design (Creswell & Plano Clark, 2018) to observe 18 students as they discussed and created learner-generated drawings in partnered pairs during three inquiry-based tasks carried out over a two month period. Due to the COVID-19 pandemic, student interactions were restricted to an online format. Observations took place through the use of synchronous task discussions recorded on the school’s learning management system called Schoology. Observations included recorded/posted student drawings and voice explanations of drawings and student chat, and is herein called task discussion. The exploratory sequential mixed methods design consisted of two distinct phases as described by Creswell and Plano Clark, (2018): qualitative followed by quantitative. In the qualitative data analysis, the researcher collected qualitative data including task discussion data, depth of knowledge (DOK) level test data, and researcher field notes of discussion tasks or field notes. To qualitatively analyze the data, task discussion data was transcribed and divided into stanzas of meaning and structurally coded per adaptive reasoning indicators. Structural codes were developed by the researcher based on descriptions of adaptive reasoning descriptions in the document Adding it up: Helping children learn mathematics (Kilpatrick et al., 2001) and discussions coding in the Scheme for Educational Dialogue Analysis (Hennessy & Rojas-Drummond, 2016). Codes were grouped into themes to address the research questions and examined as per what adaptive reasoning indicators seventh graders use to adapt their reasoning during the mathematical tasks. In addition, DOK level test data was coded per depth of knowledge level utilized by participants and
included in the analysis. The quantitative data analysis utilized quantitized task
discussions and DOK level test data to statistically examine adaptive reasoning indicators
to address the research question.

In the study’s design, the researcher structurally coded and analyzed qualitative task
discussion and DOK level test data to address research questions 1 and 2. The DOK level
test data provided information about student thinking and was categorized as qualitative
data rather than quantitative data. The qualitative data was then quantitized and analyzed
using descriptive statistics and epistemic network analysis (ENA; Shaffer, 2017) to
address research question 3. The rationale for the use of an exploratory sequential
approach is that the qualitative data informs the quantitative data analysis in order to
provide a more general understanding to the phenomenon (Creswell & Plano Clark,
2018; Tashakkori & Teddlie, 1998). In addition, the advantage of using a mixed methods
design is that the researcher could address multiple research questions via multiple
methods and combine results into a meta inference (Teddle & Tashakkori, 2009), as
shown in Figure 3.1.

Figure 3.1

Data Collection and Analysis Overview
Research Questions

This study observed student adaptive reasoning in order to answer the following research questions.

1. What adaptive reasoning indicators do 7th grade students’ evidence throughout their discussions and solutions during guided inquiry-based mathematical tasks?

2. What adaptive reasoning indicators do 7th grade students’ evidence throughout their drawings and solutions during guided inquiry-based mathematical tasks.

3. How does adaptive reasoning indicator use relate to 7th grade students’ engagement with level 2 and 3 depth of knowledge tasks?

Setting and Participants

Setting

The study took place in a public middle school in the mountain west region, in two classrooms of approximately 30 students each. Due to COVID-19 restrictions, students wore masks covering their nose and mouth and communicated with each other online using Chromebooks.

Participants

Participants included 18 seventh-grade students, ages 12-13, enrolled in a public, rural middle school located in the midwestern United States (see Table 3.1). Twenty-two students were initially recruited with an attrition rate of 4 students. The 18 remaining students consisted of eight partner sets, and two additional individuals from two different
classes of two participating teachers. The demographic characteristics of the classrooms include 50% female, 18% free and reduced lunch, 13% special education, 8% minority and 1% English as a second language. The percentage of students achieving proficiency in mathematics and reading/language arts for the school was 58% and 59% respectively in 2018-19.

Partner sets consisted of three female-female pairs, four male-male pairs and one female-male pair with two additional individual participants (males).

Table 3.1

Participant Demographics

<table>
<thead>
<tr>
<th>Classroom</th>
<th>N</th>
<th>Male</th>
<th>Female</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. O</td>
<td>10</td>
<td>7</td>
<td>3</td>
<td>55.6</td>
</tr>
<tr>
<td>Ms. Z</td>
<td>8</td>
<td>4</td>
<td>4</td>
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<tr>
<td>Total</td>
<td>18</td>
<td>11</td>
<td>7</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Number of Participants

The number of participants (p) needed to produce explanatory power for quantitative analysis using ENA was calculated using the binomial theorem with the number of adaptive reasoning codes (n) where n \( \binom{n}{2} \) or \( \binom{n}{2} = \frac{n!}{2!(n-2)!} \). The number of necessary participants was determined by the inequality \( 3p > \binom{n}{2} \). This study used six adaptive reasoning codes and two possible emergent codes for \( n = 8 \). In this case \( n \binom{n}{2} \) equals 27 for a participation requirement of \( p = 10 \) (Eagan, personal communication, January 9, 2020). The formulation of necessary participants
was confirmed by Brendan Eagan, the co-chairman of the International Conference of Quantitative Ethnography on ENA at the University of Madison, Wisconsin. The ideal study’s participant number was increased to 18 plus four alternates, to allow for possible attrition. An ideal of 18 students was proffered; however, a population of 10 was acceptable to conduct the study.

**Age of Participants**

Research indicates that a majority of adaptive reasoning skills begin to develop around the age of twelve (Piaget & Inhelder, 1958/2013; Sternberg & Rifkin, 1979). This study explored adaptive reasoning indicators of students age 12 and 13.

**Sex of Participants**

To control for possible differences in sex, partners consisted of varying sex. The results of the pilot project suggested that adaptive reasoning variance evidenced itself differently when students were partnered in same sex pairings vs. different sex pairings. In the pilot, male/female partner’s interaction resulted in the male partner dominating the discussion. The study did not examine gender specifically; however, it used same and mixed gender partnering including, one male-female pair, three female-female pairs, and five male-male pairs.

**Sampling**

Mathematics achievement was not considered in the selection of partner teams, as students enrolled in middle level mathematics courses are already performing relatively similar to their peers. Partnering of students in this study was determined through teacher
recommendation, which was based on the following criteria when solving mathematics tasks: 1) student could produce recognizable drawings, 2) student could converse with another student, 3) student was socially compatible with their partner, 4) student regularly attended class, 5) student was adept in working with technology. To ensure these criteria were met, the researcher reviewed the protocol with the teacher. All students that met the criteria were asked to participate in the study.

**Consent and Assent**

A Qualtrics recruitment letter, including parent consent and student assent form links, was sent via email to parents of all students enrolled in participating classrooms. The principle investigator initiated the email by sending it to the participating school administration. The school’s administrator then sent it to the participating teachers, who then forwarded it to parents of all students in two of the participating teacher’s classes (see Appendix A). Eighteen participants and four alternate students were asked to participate with the intent of maintaining the male to female ratio of 1:1. However, within the pool of students with initial consent, there were not enough participants for this ratio of partner pairing. The researcher and teacher conferred and decide on pairing based on a male to female ratio of consenting participants, see Table 1.

Parents were informed of the pre-implementation lesson, DOK level test and math task dates and encouraged to notify the researcher if his or her child may be absent. If absences occurred during the task dates, student pairs were allowed to complete tasks at a different time than the rest of the class. One student left the school prior to the DOK level test. Her task data was included in analysis and DOK level test was not included.
Data Sources

Data sources collected include: (1) 108 online task discussion threads, (2) task discussion field notes, and (3) 18 depth of knowledge (DOK) level tests. These sources are described in more detail below.

Task Discussion Threads

In the first unit, online task discussion threads, were recorded on the school’s established learning management system Schoology (schoology.com). The task discussion thread format engaged partners concurrently and allowed for partner dialogue, during COVID-19 pandemic restrictions. Three units of analysis were derived from students interacting through their computers online as the pairs progressed through three tasks. The units include: 1) student chat/text component, 2) uploaded video recordings of learner-generated drawings and 3) students’ audio descriptions of their drawings. Lesh and Lehrer (2000) recommend combining recordings with other methods of data collection in order to provide a comprehensive view of the phenomena. Recording technology allowed for the observation of students’ work in task discussions while field notes helped to gain a better picture of partner online interactions. Recorded data, student drawings and student chat were important in establishing use of adaptive reasoning indicators as students progress through the stages of the task. The task discussion threads saved automatically and were used to transcribe data. Transcriptions were coded and then placed into safe university storage (e.g., Box file system).
Field Notes

Field notes were taken of the task discussions by the researcher after each completed partner task (see Researcher Field Notes Protocol in Appendix B). Collection included the observation of student partner’s drawings, recorded interactions and chat. In this way multiple data sources were used to describe how each student used adaptive reasoning indicators during the mathematical tasks (Creswell & Plano Clark, 2018). Field notes were summarized into memos for each task and summarized per partner pair.

Depth of Knowledge Level Test

A DOK level test was used to establish a mathematical proficiency baseline for each participant. The DOK level test, designed by NextLesson Incorporated (NextLesson, n.d.), was a short pre-coded test using Webb’s Depth of Knowledge (DOK) degree or complexity of knowledge levels (1997, 2002). The test was chosen for this study because it offered a variety of DOK measurement opportunities including questions of variety of depth of knowledge DOK levels. As described in the literature review, DOK is a problem difficulty measurement. The DOK level test administered was divided into two DOK levels, level 2 and level 3. Validity of the test’s DOK levels and scoring rubric was confirmed through researcher Dr. Kristy Litster of Utah State University who completed a doctoral thesis on DOK level designation in mathematical tasks.

Procedures and Data Collection

This study’s procedures (see Figure 3.2) consisted of three phases: 1) preliminary study set-up, 2) pre-implementation and 3) implementation. Data collection occurred in
phase 3 implementation. The following section details the procedures and data collection at each phase.

**Figure 3.2**

*Procedures and Data Collection*

**Phase 1. Preliminary set up**

| Teacher Researcher Meeting | Discuss student pairing, identify tasks and study details | 45 Min. |

**Phase 2. Pre Implementation**

| Initial Student Instruction | Pre-teach drawing and discussion strategies | 30 Min. |

**Phase 3. Implementation**

<table>
<thead>
<tr>
<th>3.a Three-Act Math Task</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Task Immersion</td>
<td>15 Min.</td>
</tr>
<tr>
<td>Stage 1: Story Drawing</td>
<td></td>
</tr>
<tr>
<td>Stage 2: Discuss and Revise</td>
<td></td>
</tr>
<tr>
<td>Information Gathering</td>
<td>15 Min.</td>
</tr>
<tr>
<td>Stage 3: Math Drawing</td>
<td></td>
</tr>
<tr>
<td>Discuss and Revise</td>
<td></td>
</tr>
<tr>
<td>Task Resolution</td>
<td>5 Min.</td>
</tr>
<tr>
<td>Stage 4: Discuss and Revise</td>
<td></td>
</tr>
</tbody>
</table>

**Data Collection**

| 3.b. DOK level test | 15 Min | DOK level test scores |

<table>
<thead>
<tr>
<th>Online Discussions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chat</td>
<td></td>
</tr>
<tr>
<td>Voice</td>
<td></td>
</tr>
<tr>
<td>Recordings</td>
<td></td>
</tr>
<tr>
<td>Drawings</td>
<td></td>
</tr>
<tr>
<td>Field notes</td>
<td></td>
</tr>
</tbody>
</table>

**Phase 1. Preliminary Set-Up**

Prior to the study, the researcher obtained necessary permission from the school district, participating teacher, and school. Once obtained, the researcher met with the participating teachers to select and schedule different aspects of the implementation stage, including: the DOK level test and the selection of three mathematical tasks from MidSchool Math Tasks (MidSchool Math, n.d.) that aligned with the classroom
curriculum and fell within the study period. The researcher shared pre-implementation and implementation lesson plans and materials with the participating teacher (see Appendix C and D). For the selection of the 18 participants, the two participating teachers emailed student participants from one of their classes. Consent/assent email was administered as indicated in the Consent and Assent section listed above, based on the criteria listed in the participants and setting section. The Qualtrics consent/assent was followed up with emailed letters of participation to inform parents that their child was chosen as a either participant or alternate participant in the case of attrition.

No data was collected in phase 1.

Phase 2. Pre-Implementation

Prior to the study implementation, students engaged in an independent twenty-minute lesson on how to create strategic drawings (Rellensmann et al., 2017), establish exploratory talk ground rules (Mercer et al., 1999), and how to implement the draw-pair-share strategy (see Appendix C. Pre-Implementation Lesson Plan). Due to restrictions of the COVID-19 pandemic, students worked online through the learning management system Schoology.

Strategic Drawings

The pre-implementation lesson instructed students to first generate a drawing: to begin by making a situational drawing and then add in mathematical drawing elements. In the situational drawing, students illustrated the storyline of the task. The lesson referred to this type of drawing as the “story” drawing. In the mathematical drawing elements, referred to as the “math” parts, students added mathematical references needed
to solve the task. The distinction between the two types of drawings afforded students a strategic drawing skill or a method in which to approach the drawing element of the task. Studies show that students with strategic drawing knowledge create drawings more representative of the task (Rellensmann et al., 2017). Strategic drawing skills also fit the Three-Act Math Tasks format and are presented in stage 1 (Story drawing), and stage 3 (Math drawing) of the implementation phase 3a of the study (see Figure 3.2).

**Exploratory Ground Rules**

The pre-implementation lesson instructed students on how to use exploratory talk: to talk together, share what they know, to make suggestions and demonstrate their reasoning, to listen and consider suggestions, to make decisions together and reach an agreement. As seen in the literature review, establishing exploratory talk ground rules assists students to conduct discussions in inquiry situations (Barnes, 2008; Wells & Ball, 2008).

**Draw-Pair-Share**

Draw-pair-share is a collaborative learning strategy created by the researcher for this study mimicking Frank Lyman’s (Good & Lavigne, 2017) think-pair-share. In the strategy, the teacher instructs students to first “draw” without talking to their partner, then they “pair” and discuss and “share” their drawing and thoughts with their partner. Students participated in draw-pair-share using the online task discussion threads in Schoology.
Phase 3. Implementation

Phase 3 is broken down into two parts, (a) implementation of the tasks through online task discussion threads and (b) the administration of a paper-pencil DOK level test.

Phase 3a. Implementation Three-Act Math Tasks

Students participated in three mathematical tasks on three separate days, over two months. Each task lasted approximately 35 minutes (Appendix D. Implementation Detailed Lesson Plan and Task Example). Within the mathematics community, math tasks are designed to be short enough to complete in a class period so that teachers can plan a lesson and include a task within a manageable time frame (Meyer, 2011). This study adhered to this guiding timespan. Tasks were chosen based on their ability to fit into the classroom’s current unit of study and in agreement with the classroom teacher. Although Three-Act Math Tasks are divided into three acts, or sections, this study breaks the data collection aspect into four stages, in order to better analyze student adaptive reasoning use and provide space for data collection of learner-generated drawings (see Figure 3.2). Classroom conditions and the four stages of the task are described in more detail below.

Classroom Conditions. Students, with masks covering their nose and mouth, sat at their assigned desks, separated three feet from one another. To adhere to COVID-19 restrictions student engagement was limited to online. Teacher directives and task videos were posted in discussion threads on the school’s learning management system, Schoology, through a series of four discussion threads, representing each of these four
stages; see the screenshot in Figure 3.3. As the classroom of students communicated with their partner through Schoology discussions threads using video and chat, teachers tended to administrative details, such as addressing student pairing changes due to absences, resolving technological issues, and orchestrating classroom management. Masked teachers, avoiding close proximity to students, also maintained a distance from student desks as students synchronously navigated from one task discussion thread to the next, recording their drawings and voice descriptions using either the webcam of their Chromebook, propped into a cardboard stand, or a plug-and-play document camera.

**Figure 3.3**

*Example of Task Discussion Stages of Partner Dialogue*

---

**Stage 1-2. Task Immersion.** The teacher, following the lesson plan protocol (see Appendix D), directed students to the Schoology task discussion thread. Task discussions in Schoology were made visible per partnered students. Only partners could see each other’s task discussion threads. The stage 1 task discussion thread began with instructions on how to proceed through each task discussion thread. The first thread prompted the partners to independently watch a three-minute immersion video and then create a story
drawing. In stage 1 of the data collection, students drew for approximately three minutes under a document camera. The recordings captured students independent drawings and their explanations of the drawings. The student then posted the video to the task discussion thread to be read by their partner.

In stage 2, a dialogue box prompted students to view and comment on their partners recording. Students responded to each other’s work via video or text. Stage 2 provided an opportunity for students to revise their thinking. Strong evidence shows that this type of dialogue promotes reflective thinking (Zittoun & Gillespie, 2015). Students were asked to discuss what information they would need to complete the task.

**Stage 3. Information Gathering.** In stage 3 students were presented with formatted evidence or clues called artifacts. Students viewed the artifacts and revised their drawings into math drawings. The transition to math drawings within the context of dialogue allowed for a space to brainstorm and engage in partner discussion. During this phase, students were prompted to collaborate, discuss their drawings, and explain their thought processes within the task discussion thread. Student gains during partner discussion have been found to explain conceptual change in student thinking (Larraín, 2017). In stage 3, data collection of student dialogue and drawings highlighted student thinking as well as changes in their thinking. During this phase partners were encouraged to interact and discuss drawings in order to capture thought interaction. This stage lasted approximately 15 minutes.

**Stage 4. Task Resolution.** Students began stage 4 by viewing a resolution video of one possible task solution. Students were asked to compare the video solution to their own solution and were encouraged to adjust their drawings to best represent their own
thinking. This stage of the data collection provided an opportunity for students to reflect upon their work and compare solutions. When students are able to experience problem solving in more than one way, their experiences foster cognitive flexibility (Schukajlow et al., 2015). This completed the task discussion thread of phase 3a.

Once the task discussion thread was finalized, in phase 3a, the researcher reviewed the thread and recorded field notes (see Appendix B), monitoring student engagement, partner interactions, and student distractions.

**Phase 3b. Depth of Knowledge Level Test**

Student participants took a 30-minute DOL level test (see Appendix E) in phase 3b. The test was administered by the teacher five to ten days after task implementation (phase 3a). The teacher shared the completed tests with the researcher for scoring using the score coding rubric (see Appendix E). Tests were graded by the researcher, digitally copied and returned to the teacher.

At the end of the data collection, the researcher had a total of 108 task discussion threads (nine partnered pairs progressing through four task discussions in three tasks), 27 researcher field notes (one per partnered pair over three tasks), and 17 DOK level tests.

**Data Analysis**

Data collected in the phase 3a task discussions and field notes, and phase 3b DOL level test were analyzed in the following manner. Field notes (27) were open coded (Saldaña, 2016), summarized into analytic memos and then then written into narratives per partnered pair. Task discussion data was transcribed, structurally coded and analyzed
using cross-descriptive analysis (Saldaña, 2016). DOK level test results were analyzed using descriptive statistics. Task discussions coding and DOK level test results were combined, quantitized (Chi, 1997; Saldaña, 2016) and analyzed using an epistemic network analysis (ENA; Shaffer, 2017), an analysis tool used in order to model conversations through comparisons of reoccurring coded dialogue.

The following sections describe 1) qualitative data analysis, 2) mixing and quantitizing of data, 3) quantitative data analysis with ENA, and 4) combined analysis interpretation.

**Qualitative Data Analysis**

Qualitative data analysis of DOK level test data, task discussions and researcher field notes were independently analyzed, then mixed. The initial qualitative data was looked at using the different units of analysis in phase 3. The first analysis examined all 18 students individually per the 108 task discussion data and 17 DOK level test data. The second analysis examined the 18 students in pairs per the researcher field notes.

**DOK Level Tests**

The purpose of DOK level tests analysis was to determine a baseline DOK proficiency measure, a working DOK level, per participant to explore the possible relationship between adaptive reasoning use and mathematical proficiency levels. The DOK level test data was considered qualitative data because, as Creswell and Plano Clark (2018) describes, it provides information about the participants, describing thinking capabilities. The DOK level test consisted of three DOK level 2 and 3 questions. The researcher choose these levels because of their likely occurrence during mathematical
tasks, as described in Chapter 2. Students who successfully solved at least two of the three questions at each level were considered to be proficient at working within that DOK level. Students who did not achieve at least a two out of three of either DOK level 2 or 3 questions, were considered working below DOK level 2 proficiency. The DOK level tests were coded as working at a one (below level 2), two (level 2) or a three (level 3 and above); the resultant being a DOK level determination, which represented a student’s working DOK proficiency. Using descriptive statistics, DOK level test scores were compared to uses of adaptive reasoning indicators. mean, and standard deviation.

**Researcher Field Notes**

Field notes were open coded (Saldaña, 2016), considering inductively particular aspects of student language and partner, peer and teacher interactions including gestures, eye contact, and dialogue directives. Memos were developed from field notes and developed into narratives of task discussion occurrences (see Appendix B, Researcher Field Notes Protocol). These narratives were used to understand common task implementation and interaction of participants, partners, and teacher throughout the administration of the task as well as differences that might explain potential outcomes.

**Task Discussions: Chat, Dialogue and Drawings**

The researcher transcribed task discussion threads of chat dialogue and drawings into 1827 stanzas with a transcript for each partner. Because students may adapt tasks to allow for access or efficiency (Litster, 2019), Dr. Kristy Litster, an independent expert on DOK levels, reviewed the participation of four students as they progressed through the three tasks to determine if tasks were accessible for 7th grade students. Litster considered
both the DOK level participation of individual participants and partnered pairs. This
process, *task DOK assurance*, controlled for the modality of the tasks and granted
assurance that students had the capability of working at or reaching the higher levels of
DOK within the tasks (see Table 3.2).

**Table 3.2**

*Task DOK Assurance, DOK Performance per Task and Task Stages*

<table>
<thead>
<tr>
<th>Student/Partners</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
<th>DOK Level Achieved</th>
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</thead>
<tbody>
<tr>
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<tr>
<td>Maya</td>
<td>DOK1</td>
<td>DOK1</td>
<td>DOK3</td>
<td>DOK2</td>
<td>3</td>
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<td>Haddy</td>
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<td>DOK2</td>
<td>DOK3</td>
<td>DOK1</td>
<td>3</td>
</tr>
<tr>
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<td>DOK2</td>
<td>DOK3</td>
<td>DOK2</td>
<td>3</td>
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<td>DOK2</td>
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<td>DOK2</td>
<td>DOK2</td>
<td>2</td>
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<tr>
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<td>DOK2</td>
<td>DOK3</td>
<td>DOK1</td>
<td>3</td>
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<tr>
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<td>DOK2</td>
<td>DOK3</td>
<td>DOK2</td>
<td>3</td>
</tr>
<tr>
<td><strong>Doggy Diet Task</strong></td>
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<td></td>
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</tr>
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<td>Maya</td>
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<td>DOK2</td>
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<td><strong>Cosmo</strong></td>
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<td>DOK1</td>
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<tr>
<td><strong>Cosmo</strong></td>
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<td>DOK1</td>
<td>DOK1</td>
<td>2</td>
</tr>
<tr>
<td>Combined</td>
<td>DOK2</td>
<td>DOK2</td>
<td>DOK2</td>
<td>DOK1</td>
<td>2</td>
</tr>
</tbody>
</table>

*Note. DOK = depth of knowledge level*
To begin transcription, the researcher first viewed the drawings of each student with the audio portion of the recordings turned off. Screenshots of the initial drawings and subsequent changes to the drawings were made and inserted in temporal order of each partnered pair’s task discussion into a digital spreadsheet document. Descriptive text was added by the researcher to each screenshot, noting changes from the previous screenshot. For example, a description might read for Participant #1 Drawing Screenshot #8: “The fraction one-fifth was removed from the right wall of the house and changed to one-fourth.” These descriptions, called drawing descriptions, of student partners were combined temporally into one digital document. Next, student audio and task discussion chat of both partners were transcribed and inserted into the same document, also in temporal order, creating an overall transcript of partner dialogue and drawings, called a task transcript. This was repeated for all nine partners over all three tasks.

Task transcripts were then segmented into data stanzas (1,827 in total) or short lines of dialogue per associated meanings that discuss a single related concept (Gee, 1999; Shaffer et al., 2016) in a spreadsheet and then line-by-line coded (Saldaña, 2016). For example, see Figure 3.4 Code Charting Sheet.
Figure 3.4

Code Charting Sheet: Candlelight Dinner Task

<table>
<thead>
<tr>
<th>Line</th>
<th>Stage</th>
<th>Task Type</th>
<th>Gender</th>
<th>Case</th>
<th>Participant</th>
<th>Task</th>
<th>Task Check</th>
<th>Drawing Screen Cast</th>
<th>Lines - Chat, Dialogue or Drawing Description</th>
<th>Field Notes</th>
<th>Teacher</th>
<th>Adaptive Reasoning Indicator Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>817</td>
<td>3</td>
<td>CD</td>
<td>1</td>
<td>1</td>
<td>HM</td>
<td>Miyo</td>
<td>1</td>
<td>0</td>
<td>Student marks each section of the candle with a dot - counting. Student draws four briefs above the horizontal line from the man to the woman/house. Student shades in half of the first brief.</td>
<td>0 0 0 0 1 0</td>
<td>KCC PR</td>
<td></td>
</tr>
<tr>
<td>818</td>
<td>3</td>
<td>CD</td>
<td>1</td>
<td>1</td>
<td>HM</td>
<td>Miyo</td>
<td>1</td>
<td>2</td>
<td>I think there going to get done, like, at the same time because they have one, two, three, four, five, six, seven. One, two, three, four, five, six, seven, so if they keep going at this rate, they will finish at the same time.</td>
<td>0 1 0 1 0 0</td>
<td>KCC PR</td>
<td></td>
</tr>
<tr>
<td>819</td>
<td>3</td>
<td>CD</td>
<td>1</td>
<td>1</td>
<td>HM</td>
<td>Miyo</td>
<td>1</td>
<td>1</td>
<td>Disagreement with your ideas because like as the candle will be done at the same time that he gets done with his brief but he still has to drive home from his work so he won't get home in time.</td>
<td>0 1 0 0 0 0</td>
<td>KCC PR</td>
<td></td>
</tr>
</tbody>
</table>

Structural coding was used for the line-by-line coding, also known as code charting (Saldaña, 2016), and applied to individual stanzas identifying applicable adaptive reasoning indicators. In the spreadsheet, a one (1) was entered to denote that the adaptive reasoning indicator is present and a zero (0) for not present. If more than one indicator is present, stanzas were coded with multiple indicators. The adaptive reasoning indicator codes included in this study were: 1) relationship and connections, 2) alternatives proposed, 3) justification, 4) prior knowledge, 5) pattern recognition and 6) legitimacy determined (Baroody, 2003; Hatano & Inagaki, 1986; Kilpatrick et al., 2001).

Emerging codes were also considered. For further definitions of adaptive reasoning indicators, coding protocol, and examples of how they occur within partner dialogue, see
the table developed by the researcher from selections from the Scheme for Educational Dialogue Analysis tool (SEDA) from Cambridge University (Hennessy et al., 2016) in Appendix F. Additional spreadsheet descriptors included: stanza line number, task stage, task, participant identifier, student sex, partner sex, partner name, data origin (dialogue, chat or drawing), lines, field notes, and teacher.

In order to establish inter rater reliability (IRR), a portion of the stanza data was coded by an independent researcher, and compared to the researcher’s coding as per the protocol outlined in appendix F. The researcher was a mathematics education doctoral student researcher that had completed courses in qualitative research and coding. He was uniquely qualified due to his continued understanding of the study and his positionality suited within the context of the study. A Kappa rating between 80 and 100 percent agreement was achieved to meet this standard.

Next, the data was analyzed by summarizing structurally coded occurrences of adaptive reasoning indicators to address research questions 1 and 2: What adaptive reasoning indicators do 7th grade students’ evidence throughout their discussions and solutions during guided inquiry-based mathematical tasks? and What adaptive reasoning indicators do 7th grade students’ evidence throughout their drawings and solutions during guided inquiry-based mathematical tasks. Specifically, this was obtained by looking at what adaptive reasoning indicators are used by which students, at what stage in the task, and from what modality (drawing or discussion).
Mixing and Quantitizing of Data

This study quantitized the data in order to get a better understanding of the relationships between the uses of adaptive reasoning indicators, or which indicators are used in conjunction with another indicator. Quantitizing data (Sandelowski et al., 2009; Wooldridge et al., 2018) is the process of transforming coded qualitative data into quantitative data (Creswell & Plano Clark, 2018; Tashakkori & Teddlie, 1998), where qualitative data present experiences in words, quantitative data presents experiences in terms of numbers. When using video, the general steps for quantitizing data include 1) entering or transcribing the data, 2) analysis of coding and coding the data, 3) representing codes with dichotomous variables, and then finally 4) analyzing codes according to the work of the researcher (Chi, 1997). This study followed this procedure using structural coding in step 2 of the analysis and ENA to complete step four using visual models of analysis. Qualitative data including task discussion threads and student working DOK level proficiency scores were combined, or mixed, and then quantitized.

Quantitative Data Analysis with Epistemic Network Analysis

Integrated data, guided by the qualitative results, were analyzed using ENA to address research question 3: How do adaptive reasoning indicators relate to 7th grade students’ engagement with DOK level 2 and 3 tasks? To understand how students adapted their reasoning the researcher developed and analyzed ENA plots for combinations of students working with in tasks: individually, in relation to partner interaction, as a whole group, within tasks and task stages, between the modalities of drawing and discussion, and per student performance on the DOK level test. Because
ENA is a relatively new technique, this section is divided into two parts, the first part describes how ENA works followed by how ENA was utilized within the study.

**How Epistemic Network Analysis Works**

ENA builds on epistemic frame theory (Safoutin et al., 2000; Shaffer, 2006; Shaffer, 2017) and the concept that learning occurs within a community, culture or practice, also in line with the work of Lave and Wenger’s *communities of practice* (1991). By observing shared discourse, described by Gee (1999) as Big-D discourse, which includes spoken words and other communicatory elements such as listening, writing, interacting, feelings, and drawing artifacts, researchers establish a better understanding of this learning. ENA is a mathematical method that quantifies an *epistemic frame*, or the description of how codes are related to one another, and is particularly helpful in understanding complex interactions in discourse. In this study, the epistemic frame described how adaptive reasoning indicators relate to one another (see Figure 3.5). Observing adaptive reasoning is a complex process (see Chapter 2, adaptive reasoning) and the choice of ENA proved valuable to view adaptive reasoning of students as they discussed and drew during guided inquiry-based mathematical tasks.

Within ENA coded dialogue, lines were grouped into *windows*. A window included continuous lines of dialogue. Within the window, the relation of code co-occurrences formed an *epistemic network*, which was then moved progressively, through the dialogue (Siebert-Evenstone et al., 2017). Epistemic networks were summarized and plotted on a graph. The nodes represented calculated centroids for each epistemic network. The values of the centroids were bound by the strength of the connections.
between the nodes. In addressing this study, nodes represented the adaptive reasoning indicator codes. Lines, or edges, between nodes were formed by the relative repeated use of codes within windows of dialogue. Thicker, more saturated lines between nodes indicated a stronger connection or reoccurring combinations of codes within a window. In addition to network graphs, ENA allowed for the generation of subtractive networks, or networks that compared differences in relationships. ENA is further described in the following section and the quantitative section in Chapter 4.

**Epistemic Network Analysis Within the Study**

The researcher determined patterns of adaptive reasoning indicators per 1) tasks and stages of the tasks, 2) drawing and discussion modalities, and 3) a student’s working DOK level proficiency scores. Different combinations of coded descriptors were created to discover possible network patterns. Because the data collected did not have a normal distribution, nonparametric Mann-Whitney U tests were conducted to determine statistical significance between descriptors. Nonparametric tests are often used in educational research due to assumption violations such as the non-normal data seen in this study and were therefore appropriate in the determination of significance in this study.

In question 1 and 2, What adaptive reasoning indicators do seventh-grade student’s evidence in their discussion and drawings when engaged in inquiry-based mathematical tasks?, ENA comparisons were developed per student and partners use of adaptive reasoning within tasks, task stages, and drawings and dialogue. This process addressed what, when, and how students use adaptive reasoning in guided inquiry-based
tasks. To answer research question 3, How do 7th grade students adapt their reasoning within DOK2 and 3 level inquiry-based tasks, student working DOK proficiencies were compared to patterns that arose during the analysis of adaptive reasoning indicators in questions one and two. Subtractive networks were used to compare relationships between adaptive reasoning indicator use, modalities, and student working DOK level proficiencies. For example, if the indicator justification was predominantly used in conjunction with prior knowledge during a task, a students working DOK level proficiency was compared to examine possible connections or patterns.

Underlying processes described and interpreted using ENA were checked against original data to close the interpretive loop (Shaffer, 2017). Closing the interpretive loop (see Figure 3.5) is a validation method, key to ENA. This process involves taking sample snippets of original data and tracing it through ENA to the final interpretation.

**Figure 3.5**

*Closing the Interpretive Loop, Epistemic Network Analysis*

![Diagram of ENA process]

**Combined Interpretation**

Inferences were made from the qualitative and quantitative analysis and combined into a meta-inference to generate conclusions (Teddlie & Tashakkori, 2009). The
researcher combined quantitative results and qualitative findings in a joint interpretation (Creswell & Plano Clark, 2018), summarizing graph results and related qualitative findings. The researcher examined similarities and discrepancies. In the case of discrepancies, the researcher re-evaluated existing data to close the interpretive loop.

**Limitations**

This study was conducted during the COVID-19 pandemic. While the study took place in a traditional classrooms setting, students were restricted to online communications. Partner and teacher interaction may have been different without these restrictions. Because of COVID-19 restrictions students were to maintain a 3 foot distance from one another. In addition, teachers were to maintain a 3 foot distance, disallowing the teacher a view of student computers, see Classroom Conditions under Phase 3 section for a more detailed description. In this way, students were dependent upon online instruction and communicating remotely with their partner via a Chromebook. This may have limited organic exploratory talk, shifting conversation to a more explanatory nature. Additionally, communication took more time and was limited due to the process of creating drawings and discussions through recordings and online chat.

**Ethical Considerations**

IRB approval was obtained from Utah State University prior to commencement of study, including COVID-19 restrictions. Parents/guardians of participants were sent
appropriate consent forms and letter of information (see Appendix A) which indicated the goals and steps involved in the study and participants’ rights to privacy and their ability to leave the study at any time. Parents/guardians and participants were also notified of the nature and inclusion of video recording. Anonymity of participants were kept confidential via alternate name coding of individual identities. Case participants were not compensated in any manner.
CHAPTER 4

RESULTS

The purpose of this study was to examine what adaptive reasoning indicators seventh grade students evidenced through their drawings and discussions during guided inquiry-based mathematical tasks in the classroom. The study used both quantitative and qualitative analysis to answer the research questions. The three research questions guiding this study includes: (1) What adaptive reasoning indicators do 7th grade students’ evidence throughout their discussions and solutions during guided inquiry-based mathematical tasks? (2) What adaptive reasoning indicators do 7th grade students’ evidence throughout their drawings and solutions during guided inquiry-based mathematical tasks. (3) How do adaptive reasoning indicators relate to 7th grade students’ engagement with level 2 and 3 depth of knowledge tasks?

The findings presented in the following sections are based on data collection of eighteen 7th graders, from two different classrooms, ages 12-13 from a public intermediate school. Results of this exploratory sequential design are organized into the following four sections, 1) qualitative results, 2) quantitizing of the qualitative results, 3) quantitative results and 4) combined interpretation of qualitative and quantitative results.

Qualitative Results

Qualitative results were derived from structurally coded task discussions and drawings, researcher field note narratives, and DOK level tests results. Descriptive statistics were used to summarize the qualitative data and answer the first two research
questions. The results are discussed from analyses of (1) task discussion threads (2) field note narratives, and (3) the DOK level test scores in the three sections following.

**Task Discussion Thread Results**

*Task discussion threads* were how students communicated their discussions and drawings with partners during the three tasks online on their school’s learning management system, Schoology. Discussion threads include student chat and audio/visual recordings. The audio/video recordings, called *recordings*, include student drawings and student audio explanations of their drawings. In this study, chat and recorded audio are referred to as *discussions*. Drawn elements from the recordings are referred to as *drawings*.

Six structural codes identifying adaptive reasoning indicators were used to analyze students discussions and drawings. Results are described using the following abbreviated descriptors: *relationships and connections*, *alternates pursued*, *justifications*, *prior knowledge*, *pattern recognition*, and *legitimacy determined*, for more details on code descriptions see Appendix F. Emergent coding of these data resulted in additional indicators of adaptive reasoning, that were not used by students consistently enough to substantiate additional coding categories. Inter-coder reliability of four students’ (two partners’) interactions through three tasks discussions (291 stanzas) was conducted with a mathematics education doctoral student. Inter-coding reliability produced a Cohen’s kappa value of 0.61 which suggests a good strength of agreement above 80% (Mahmud, 2010).

To assure that students engaged in tasks at DOK 2 and 3 levels, a task DOK
assurance analysis was conducted with four students progressing through the three tasks. Results concluded that the students reached a DOK level of 2 or higher 100% of the time in both individual levels and partner levels. Additionally, students reached a level 3 or higher 58.3% of the time per individual levels and 83.3% per partner levels (see Table 3.2).

In all three tasks, the researcher identified 820 incidences of adaptive reasoning (see Table 4.1). About two-thirds (538) of these were identified within discussions and the remaining third (282) within drawings. These results may be explained by the fact that students were asked to draw in two of the four stages of the task.

Table 4.1

<table>
<thead>
<tr>
<th>Description</th>
<th>Adaptive Reasoning Indicators</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RC</td>
<td>J</td>
</tr>
<tr>
<td>Count</td>
<td>187</td>
<td>194</td>
</tr>
<tr>
<td>% of Discussion</td>
<td>34.7%</td>
<td>36.0%</td>
</tr>
<tr>
<td>Count</td>
<td>138</td>
<td>75</td>
</tr>
<tr>
<td>% of Drawings</td>
<td>48.9%</td>
<td>26.6%</td>
</tr>
<tr>
<td>Total Count</td>
<td>325</td>
<td>269</td>
</tr>
<tr>
<td>% of Total.</td>
<td>39.6%</td>
<td>32.8%</td>
</tr>
</tbody>
</table>

Note. RC=relationships and connections, AP=alternates pursued, J=justifications, PK=prior knowledge, PR=pattern recognition, LD=legitimacy determined

Table 4.2 shows the number of adaptive reasoning indicators student used per category, and the proportional difference between the number of adaptive reasoning indicators divided by the standard deviation over all three tasks. Students adapted their reasoning across all tasks on average 45.6 times with a standard deviation (SD) of 16.8, 29.9 times
in discussions (SD 11.8), and 15.7 times in drawings (SD 5.7), see Table 4.3, or on average approximately a third of the incidences of adaptive reasoning occurred in drawings while two-thirds occurred in discussions.
### Table 4.2

**Student Use of Adaptive Reasoning Indicators for All Three Tasks**

<table>
<thead>
<tr>
<th>Name</th>
<th>RC</th>
<th>AP</th>
<th>J</th>
<th>PK</th>
<th>PR</th>
<th>LD</th>
<th>Total</th>
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<tbody>
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<td>N</td>
<td>SD</td>
<td>N</td>
<td>SD</td>
<td>N</td>
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<tr>
<td>Aisha</td>
<td>22</td>
<td>0.6</td>
<td>10</td>
<td>1.8</td>
<td>15</td>
<td>0.0</td>
<td>0</td>
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<tr>
<td>Harlee</td>
<td>19</td>
<td>0.1</td>
<td>10</td>
<td>1.8</td>
<td>21</td>
<td>0.9</td>
<td>1</td>
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<tr>
<td>Total</td>
<td>41</td>
<td>20</td>
<td>36</td>
<td>1</td>
<td>17</td>
<td>1</td>
<td>6</td>
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</table>

<table>
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<th>SD</th>
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<th>SD</th>
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<th>SD</th>
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<tbody>
<tr>
<td>Max</td>
<td>26</td>
<td>1.2</td>
<td>7</td>
<td>0.9</td>
<td>32</td>
<td>2.5</td>
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<tr>
<td>Ryan</td>
<td>35</td>
<td>2.5</td>
<td>4</td>
<td>-0.1</td>
<td>20</td>
<td>0.7</td>
<td>2</td>
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<tr>
<td>Total</td>
<td>61</td>
<td>11</td>
<td>52</td>
<td>6</td>
<td>4</td>
<td>8</td>
<td>142</td>
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<tr>
<td>Jacob</td>
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<td>1.0</td>
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<td>-0.7</td>
<td>27</td>
<td>1.8</td>
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<tr>
<td>Jason</td>
<td>18</td>
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<td>6</td>
<td>0.6</td>
<td>13</td>
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<tr>
<td>Total</td>
<td>43</td>
<td>8</td>
<td>40</td>
<td>7</td>
<td>4</td>
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<td>105</td>
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<td>Brad</td>
<td>15</td>
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<td>2</td>
<td>-0.7</td>
<td>13</td>
<td>-0.3</td>
<td>2</td>
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<tr>
<td>Kevin</td>
<td>13</td>
<td>-0.8</td>
<td>3</td>
<td>-0.4</td>
<td>12</td>
<td>-0.4</td>
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<tr>
<td>Total</td>
<td>28</td>
<td>5</td>
<td>25</td>
<td>4</td>
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<td>2</td>
<td>64</td>
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<td>Mike</td>
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<td>-0.7</td>
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<td>-0.9</td>
<td>2</td>
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<td>Sean</td>
<td>12</td>
<td>-0.9</td>
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<td>-0.7</td>
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<td>-0.7</td>
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<tr>
<td>Total</td>
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<td>19</td>
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<td>0</td>
<td>7</td>
<td>59</td>
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<tbody>
<tr>
<td>Sadie</td>
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<td>3</td>
<td>-0.4</td>
<td>12</td>
<td>-0.4</td>
<td>2</td>
</tr>
<tr>
<td>Kate</td>
<td>11</td>
<td>-1.0</td>
<td>4</td>
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<tr>
<td>Total</td>
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<td>2</td>
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<tr>
<td>Maya</td>
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<td>0.4</td>
<td>19</td>
<td>-0.6</td>
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</tr>
<tr>
<td>Total</td>
<td>43</td>
<td>4</td>
<td>27</td>
<td>5</td>
<td>14</td>
<td>11</td>
<td>104</td>
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<table>
<thead>
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<th>SD</th>
<th>N</th>
<th>SD</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Cam</td>
<td>16</td>
<td>-0.3</td>
<td>11</td>
<td>2.1</td>
<td>16</td>
<td>0.2</td>
<td>3</td>
</tr>
<tr>
<td>Bodhi</td>
<td>18</td>
<td>0.0</td>
<td>4</td>
<td>-0.1</td>
<td>17</td>
<td>0.3</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>325</td>
<td>75</td>
<td>269</td>
<td>44</td>
<td>44</td>
<td>52</td>
<td>55</td>
</tr>
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</table>

<table>
<thead>
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<th>SD</th>
<th>N</th>
<th>SD</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>18.1</td>
<td>4.2</td>
<td>15</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>45.6</td>
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<tr>
<td>Stand.</td>
<td>6.74</td>
<td>3.3</td>
<td>6.8</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>16.8</td>
</tr>
</tbody>
</table>

**Note.** RC=relationships and connections, AP=alternates pursued J=justifications, PK=prior knowledge, PR=pattern recognition, LD=legitimacy determined. N=Number of adaptive reasoning indicators Used, SD=(N-mean)/standard deviation.
The number of times a student adapted their reasoning varied across students and student partners. For example, at the individual level, Max displayed the most adaptive reasoning indicators at a total count of 75, while Cosmo demonstrated the least at 18. Within student partners, Max and Ryan displayed the most indicators, 142 while Cosmo and Silvia demonstrated the least number of indicators, 35. All students’ individual adaptive reasoning use within each indicator category fell within 2.5 standard deviations from the mean, see Table 4.2 for details. Each student adapted their reasoning more frequently within discussions than drawings, apart from Kevin who demonstrated an equal amount in both modalities (see Table 4.2).
Table 4.3

Adaptive Reasoning Indicator use per Student Categorized by Modality Source

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>AP</th>
<th>J</th>
<th>PK</th>
<th>PR</th>
<th>LD</th>
<th>Total</th>
<th>Draw</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aisha</td>
<td>10</td>
<td>12</td>
<td>3</td>
<td>7</td>
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<td>Harlee</td>
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<td>Jason</td>
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<td>2</td>
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<td>0</td>
<td>3</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Silvia</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
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<td>Brad</td>
<td>6</td>
<td>9</td>
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<td>1</td>
<td>4</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Kevin</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Mike</td>
<td>7</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Sean</td>
<td>5</td>
<td>7</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>Kate</td>
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<td>6</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>0</td>
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<td>1</td>
</tr>
<tr>
<td>Sadie</td>
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<td>12</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Haddy</td>
<td>9</td>
<td>12</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Maya</td>
<td>10</td>
<td>12</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>13</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
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<td>Cameron</td>
<td>7.0</td>
<td>9</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>11</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Bodhi</td>
<td>7</td>
<td>11</td>
<td>0</td>
<td>4</td>
<td>5</td>
<td>12</td>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Mean: 7.7 10.4 1.2 3.0 4.2 10.8 0.8 1.7 1.4 1.4 0.4 2.6 15.7 29.9 35%
Std. Dev: 2.6 4.2 1.2 2.3 2.4 5.0 0.8 1.4 1.7 1.5 0.5 1.7 5.7 11.8 6%

Note. RC=relationships and connections, AP=alternates pursued J=justifications, PK=prior knowledge, PR=pattern recognition, LD=legitimacy determined, Draw=drawn, Dis=discussion

In this study, the researcher divided the six adaptive reasoning indicators into two categories of overarching thematic codes based on the predominant student use of two indicators: (a) primary indicators, consisting the two indicators most used by students: 
relationships and connections, and justifications, and (b) secondary indicators consisting of the four lesser used indicators: alternates pursued, prior knowledge, pattern
recognition, and legitimacy determined. Results of the adaptive reasoning found in the primary and secondary indicators are described in more detail, followed by sections on how students adapted their reasoning per the different tasks and task stages.

**Primary Indicators: Relationship and Connections, and Justifications in Discussion Threads**

*Relationships and connections* occur when students recognize one occurrence that affects another or there is a connection or comparison of elements. *Justifications* occur when students explain, clarify or discuss occurrences that follow logical steps. Oftentimes a *justification* may be embedded in an argument or counter argument, see Appendix F. The majority (72.4%) of adaptive reasoning indicator use was seen within the two primary indicators: *relationships and connections*, and *justifications*. Every student utilized these two indicators more than the other four indicators to varying degrees. *Relationships and connections* was used 325 times or 39.6% of the overall indicator count, 187 times within discussions and 138 times within drawings. *Justifications* occurred less often than *relationships and connections*, 269 times or 32.8% of overall indicator use, 194 were recognized in discussions and 75 in drawings. Examples of these indicators used within the two modalities of discussion and drawing follows.

**Discussions Modality.** To better understand the two most utilized indicators, percentages of use per the modalities of drawing and discussion were analyzed, see Table 4.1. Of the total indicators identified within discussions, participants’ used *relationships and connections* (34.7%) about the same percentage of times as *justifications* (36.0%). Student use of *relationships and connections* can be seen in Table 4.4. In the Ghost
Island task, Kevin recognized that the hypotenuses of the two overlapping triangles in his drawing form a “straighter” line pointing to the island. He connected this idea with the direction of the boat when he said it’s “headed to the island.” By talking about his work in this way, Kevin demonstrated the indicator relationships and connections because he connected the concepts. Aisha demonstrated the use of justifications in the Ghost Island task. She explained thought processes using steps to prove her point. She uses phrases like “from here to here”, “I’m going to do”, “So then I’m going to do”, “which is” denoting steps and concluded with the idea that the distance from the ship to the big island is 45 leagues.

**Table 4.4**

*Coded Discussions: Relationships and Connections and Justifications Excerpt*

<table>
<thead>
<tr>
<th>Indicator, Task, Stage</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relationships and Connections Ghost Island, Stage 3</td>
<td>Kevin: And this is where we are headed, and it is straighter. From the drawing, it looks like we are headed to the island.</td>
</tr>
<tr>
<td>Justifications Ghost Island, Stage 3</td>
<td>Aisha: From here to here is 35. So, this was from here to here. So, I'm going to do 32 plus 48. So, I got 80 for that one. So, then I'm going to do 80 minus 35, which is 45. So, I think from here, from this ship to the big island is 45 leagues.</td>
</tr>
</tbody>
</table>

**Drawing Modality.** Students evidenced adaptive reasoning in drawings through sketched elements such as arrows, lines, check marks, circling, measurement lines, variables, equal signs, crossing out, erasing, written descriptions, pictorial representations, talk bubbles, graphs, repeated drawing of images or cross-outs, proximity of objects to one another, action indicators and alterations to drawings. To indicate relationships and connections in drawings, students used arrows, various symbols that
connect elements or crossed out items in a one-to-one corresponding pattern. For example, in the excerpt in Table 4.5, Silvia demonstrated *relationships and connections* by drawing two right triangles and then showed the relationship between the different legs of the triangles by including fractions in her drawing. *Justifications* within a drawing consisted of multiple elements drawn in steps that concluded in a point. For example, in Harlee’s drawing in Table 4.5, she first drew the initial island and then added the triangle relationship to the ship. Next, she added another triangle connecting the ship to the treasure island. She denoted the outcome of the ship journey with arrows, one leading the ship to the island and the other showing that the ship may be off course and go out to sea.

Of the two most utilized indicators, students demonstrated *relationships and connections* (48.9%) more often than *justifications* (26.6%) in drawings (see Table 4.1). The difference in student use was due to the nature of the indicators. Justification is a lengthier process involving multiple steps to demonstrate reasoning. Students demonstrated *justifications* more readily through verbalization. Whereas the illustration of *relationships and connections* is a one-step process that students accessed more readily in drawings.
Table 4.5

*Coded Drawings: Relationships and Connections and Justifications Excerpt*

<table>
<thead>
<tr>
<th>Indicator, Task, Stage</th>
<th>Drawing</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relationships and Connections</td>
<td></td>
<td>Silvia: And so, if you make, simplify this, the smaller triangle becomes 14 twenty-firsts and the second one becomes 32 forty-eighths.</td>
</tr>
<tr>
<td>Ghost Island, Stage 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Justifications</td>
<td></td>
<td>Harlee: All we have to do is, this is a triangle and this part’s 21 leagues, and this other part is 14 leagues. So, I think that that would make this part 21 leagues and this part 14. So, I think that they could do that in order to get there.</td>
</tr>
<tr>
<td>Ghost Island, Stage 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Secondary Indicators: Alternates Pursued, Prior Knowledge, Pattern Recognition and Legitimacy Determined in Discussion Threads*

The other four adaptive reasoning indicator uses were utilized by students far less than *relationships and connections* and *justifications*, 27.5% of the total adaptive reasoning indicators when combined. Students used *alternates pursued* (9.1%) more often than *prior knowledge* (5.4%), *pattern recognition* (6.3%) or *legitimacy determined* (6.7%), see Table 4.1. When students used the indicator *alternates pursued*, they offered an acknowledgement or suggestion of a change of position or viewpoint. Oftentimes this occurred in conjunction with a challenge connected with the redirection of thought. *Prior knowledge* was represented as a past tense reference linked to a present situation, oftentimes indicated in a formula or process. Students show *pattern recognition* when
they noticed a repetition or regularity and demonstrate legitimacy determined when making sense of a solution, such as too big, too small, or just right. Oftentimes legitimacy determined was expressed in the form of a question or statement remarking on a found absurdity.

**Discussion Modality.** In each indicator category, students adapted their reasoning a majority of the time in discussions rather than in drawings except for pattern recognition. This suggests that these categories are easier to discuss or are more easily observed in discussion than in drawings. Over all discussions, students accessed these four indicators in the following percentages: alternates pursued (10%), prior knowledge (5.6%), pattern recognition (4.8%) and legitimacy determined (8.7%), see table 4.1.

In Table 4.6, excerpts from the data demonstrate use of the secondary indicators. Haley shows alternates pursued when she said, “If we think about it a little more”. She suggested another way to think about the problem. Kevin, in the Doggy Diet task, used prior knowledge of proportions when he said “I’ll just set up a proportion for this. Cameron demonstrated use of pattern in the Candlelight Dinner tasks as he noticed the pattern of repeated halves. And Harlee noted that “This one makes more sense” describing that her second approach of the problem was more on track, showing a determination of legitimacy.
Table 4.6

Coded Discussions: Alternates Pursued, Prior Knowledge, Pattern Recognition and Legitimacy Determined Excerpt

<table>
<thead>
<tr>
<th>Indicator, Task, Stage</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternates Pursued Candlelight Dinner, Stage 4</td>
<td>Haley: If we think about it a little more, you have 4 briefs that the husband has to fill out, and for every 1/2 of the briefs he fills out, 1/8 of the candle will burn.</td>
</tr>
<tr>
<td>Prior Knowledge Doggy Diet, Stage 3</td>
<td>Kevin: I'll just set up a proportion for this. So, two grams for one pounds...</td>
</tr>
<tr>
<td>Pattern Recognition Candlelight Dinner, Stage 3</td>
<td>Cameron: So, and he's halfway through this one, so he has to get through. If you break these into halves, so 1/2, 1/2, 1/2, 1/2, so he's finished with one half of it, so he has seven more halves to go.</td>
</tr>
<tr>
<td>Legitimacy Determined Candlelight Dinner, Stage 3</td>
<td>Harlee: I rerecorded it. I understand like way more now that I thought about it more. I thought that was how much the candle melted when he finished. This one makes more sense.</td>
</tr>
</tbody>
</table>

**Drawing Modality.** In student drawings, the following percentages of adaptive reasoning were observed: *alternates pursued* (7.4%), *prior knowledge* (5.0%), *pattern recognition* (9.2%) and *legitimacy determined* (2.8%), see Table 4.1. Excerpts of these uses are shown in Table 4.7. *Pattern recognition* was the one indicator observed the same number of times in drawing as in discussion, demonstrating that students were equally likely to express the category in either modality.

In stage 3 of the Candlelight Dinner task, Max wrote an equation showing that one-eighth a candle equals one-half a briefing. He showed a total of four briefs in the upper right corner. Max determined that the man in the task won’t make it home in time. He then pursued an alternate approach to the problem by determining that the man in the task needed to finish the briefings faster, opening a new train of thought. Jason
demonstrated *prior knowledge* by drawing a foot long candle and then breaking it down into 12 inches. Aisha showed a pattern in her drawing by repeatedly shading in one-eighth of the candle and then one-half of a brief in the Candlelight Dinner task. In her discussion she said “we are just going to keep on like this” indicating repetition in her discussion also. Aisha in stage 4 of Doggy Diet, calculated incorrectly that the dog needs to be fed 2 grams of food a day. Verbally she checked the legitimacy of here solution by saying “Which I knew wasn’t a lot.” In her drawing she shows that 2 grams equals one pound, illustrating *legitimacy determined*, see Table 4.7.
Table 4.7

Coded Drawings: Alternates Pursued, Prior Knowledge, Pattern Recognition and Legitimacy Determined Excerpt

<table>
<thead>
<tr>
<th>Indicator, Task, Stage</th>
<th>Drawing</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternates Pursued</td>
<td></td>
<td>Max: So that means he would have briefings, all of them, right as the candle burns out, so that means he won't make it. But in order to make it, he would have to go faster on the briefings. So, the briefings would have to be faster.</td>
</tr>
<tr>
<td>Candlelight Dinner, Stage 3</td>
<td></td>
<td>Jason: So that's the candle burning one foot tall. It is one inch each minute. So, one foot is 12 inches. So, he has 12 minutes to get home.</td>
</tr>
<tr>
<td>Prior Knowledge</td>
<td></td>
<td>Aisha: By the time he's done with the first half of his first meeting, the candle is 1/8 of the way melted. So, what we are going to do is, we are just going to keep on like this until we fill in all of the meetings and see if he is able to get home in time for his anniversary dinner with his wife.</td>
</tr>
<tr>
<td>Pattern Recognition,</td>
<td></td>
<td>Aisha: I thought that on the graph it was saying that every day he was eating two grams of food. Which I knew wasn't a lot. But that was the only thing I could come up with.</td>
</tr>
<tr>
<td>Candlelight Dinner, Stage 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pattern Recognition,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Candlelight Dinner, Stage 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Legitimacy Determined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doggy Diet, Stage 4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Per Task Adaptive Reasoning Use in Discussion Threads

Students adapted their reasoning more in the Candlelight Dinner task, 45.1% of the total adaptive reasoning use, than the two other tasks Doggy Diet (27.4%) and Ghost
Island (27.4%), see Table 4.8. The three tasks were different in content. Students solved the Candlelight Dinner task using pattern recognition or proportional reasoning, the Doggy Diet task through graphing skills, multiplicative reasoning or unit rate strategies, and the Ghost Island task through of similar triangles or proportions. Students successfully solved the Candlelight Dinner tasks more often than the other two tasks. In all three tasks, students utilized drawings and discussions in the same two to one ratio as the overall task ratio, demonstrating that the task itself did not mediate the amount of adaptive reasoning that took place between the different modalities.

Regardless of the task, students adapted their reasoning of the primary indicators similarly, however different tasks mediated the use of the other four indicators, see Table 4.8. Notable are the lower percentage use of prior knowledge and legitimacy determined and higher percentage use of pattern recognition in the Candlelight Dinner task, a decrease percentage use of pattern recognition in the Doggy Diet task, and in increase percentage use of prior knowledge and decrease percentage use of alternates pursued and justification in the Ghost Island task.
Table 4.8

Adaptive Reasoning Use Per Task Per Modality

<table>
<thead>
<tr>
<th>Mode</th>
<th>Adaptive Reasoning Indicator</th>
<th>Total</th>
<th>Draw-Disc%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RC</td>
<td>AP</td>
<td>J</td>
</tr>
<tr>
<td>Candlelight Dinner Task</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discussion</td>
<td>87</td>
<td>28</td>
<td>98</td>
</tr>
<tr>
<td>Drawing</td>
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<td>11</td>
<td>37</td>
</tr>
<tr>
<td>Total</td>
<td>138</td>
<td>39</td>
<td>135</td>
</tr>
<tr>
<td>% of task</td>
<td>37.2%</td>
<td>10.5%</td>
<td>36.5%</td>
</tr>
<tr>
<td>Doggy Diet Task</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discussion</td>
<td>51</td>
<td>17</td>
<td>50</td>
</tr>
<tr>
<td>Drawing</td>
<td>42</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>Total</td>
<td>93</td>
<td>25</td>
<td>74</td>
</tr>
<tr>
<td>% of task</td>
<td>41.3%</td>
<td>11.1%</td>
<td>32.9%</td>
</tr>
<tr>
<td>Ghost Island Task</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discussion</td>
<td>49</td>
<td>9</td>
<td>46</td>
</tr>
<tr>
<td>Drawing</td>
<td>45</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>Total</td>
<td>94</td>
<td>11</td>
<td>60</td>
</tr>
<tr>
<td>% of task</td>
<td>41.8%</td>
<td>4.9%</td>
<td>26.7%</td>
</tr>
</tbody>
</table>

Note. RC=relationships and connections, AP=alternates pursued J=justifications, PK=prior knowledge, PR=pattern recognition, LD=legitimacy determined, AR=adaptive reasoning. Draw-Disc% indicates percent of discussion and drawing per task.

Per Task Stages Adaptive Reasoning in Discussion Threads

Examination of adaptive reasoning use in task stages helped the researcher gain a better understanding of adaptive reasoning use within the stages of the tasks. For example, in stage 1 of the Ghost Island task, Jacob and Jason were tasked to determine if a ship leaving the Isle of Verde will reach the Isle of Fantasma. The two students created story drawings, depicting a boat leaving the Isle of Verde approaching the Isle of Fantasma. The two boys adapt their reasoning nine times, four times within discussions and five times in their drawings. Over all three tasks, in stage 1, the 18 students adapted
their reasoning 126 times, 78 times within discussions and 48 times within drawings.

Along with stage 3, stage 1 was where students were asked specifically to make drawings. Adaptive reasoning resembled stage 3, with more *justification* and less *relationships and connections*. Of the secondary indicators, students used more *prior knowledge* and less *pattern recognition* in stage 1, see Table 4.9.
Table 4.9

*Percent Difference of Indicator Use per Stage*

<table>
<thead>
<tr>
<th>Mode</th>
<th>Adaptive Reasoning Indicators</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RC</td>
<td>AP</td>
</tr>
<tr>
<td>Discussion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discussion</td>
<td>29</td>
<td>5</td>
</tr>
<tr>
<td>Discussion</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>52</td>
<td>8</td>
</tr>
<tr>
<td>%Difference</td>
<td>+1.7%</td>
<td>-2.8%</td>
</tr>
<tr>
<td>Drawing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drawing</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>29</td>
<td>2</td>
</tr>
<tr>
<td>% Difference</td>
<td>-7.4%</td>
<td>-6.9%</td>
</tr>
<tr>
<td>Stage 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discussion</td>
<td>109</td>
<td>19</td>
</tr>
<tr>
<td>Discussion</td>
<td>97</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>206</td>
<td>28</td>
</tr>
<tr>
<td>% Difference</td>
<td>+5.0%</td>
<td>-3.0%</td>
</tr>
<tr>
<td>Stage 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discussion</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>Discussion</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>%Difference</td>
<td>-12.8%</td>
<td>+17.0%</td>
</tr>
<tr>
<td>Task Totals</td>
<td>325</td>
<td>75</td>
</tr>
<tr>
<td>Percent of Total</td>
<td>39.6%</td>
<td>9.1%</td>
</tr>
</tbody>
</table>

*Note.* RC=relationships and connections, AP=alternates pursued J=justifications, PK=prior knowledge, PR=pattern recognition, LD=legitimacy determined, AR=adaptive reasoning.

*Note.* Bottom row of data represents the percent of totals adaptive reasoning indicator use in all stages. Differences from the percent of total is calculated within each stage and shown as % difference.

In stage 2, Jason and Jacob discussed what additional information they needed to solve the problem. For this they utilized discussion rather than drawings. In a chat box Jacob says to Jason, “They need to know the wind speed/direction and which way they
are traveling/facing. They also need to know how much their ship is affected by the wind…” The boys adapted their reasoning four times, two of which were coded as *justifications*. *Justifications* was the dominate indicator used in stage 2, representing 58.9% of the adaptive reasoning. Many students, in creating their list of needed information, presented the list as a series of logical arguments, projecting how to solve the problem. These arguments were coded as *justifications*. In stage 2, adaptive reasoning occurred the least (90 times) and most students did not create drawings, see Table 4.9.

Once Jacob and Jason established their list of needs, they continued to stage 3 where they were given two artifacts containing sextant readings. The first artifact shows a right triangle between both islands and the second shows a triangle between Isle of Verde and the ships current location. Jacob and Jason created drawings, describing their thoughts of possible solutions. Jacob expressed the idea that the triangles have similar angles so the hypotenuses must be pointing in the same direction and “and as long as they keep going in the same direction, then they will eventually reach the island.” Jason used the legs of the triangle to form a proportion and develops the slope of the hypotenuses but does not see the connection to how the slopes determine a similar route to Isle of Fantasma. The boys, chatted back and forth, over possible ways to solve the problem, adapting their reasoning ten times. Jacob said, “The triangles have the same angles, so if they travel in the same direction then they will eventually reach the island.” Jason challenged Jacob’s statement, “they don’t necessarily; one could be 46 and the other 44. I’m pretty sure they are going northeast.”

The bulk of all adaptive reasoning predominantly occurred in stage 3 (462 incidences). In fact, 206 of the adaptive reasoning incidences occurred in the category of
finding *relationships and connections* and 103 in *justifications*. The modality of the indicator use in this stage resembled overall task use; two-thirds of the adaptive reasoning indicators were observed in discussion and one-third within drawings. For example in stage 3, Jacob and Jason finally agreed that the ship would reach the Isle of Fantasma, however their reasons justifying their results were unclear.

After watching the resolution video in stage 4, both boys agreed with the video solution without commenting on why they agreed, and no adaptive reasoning indicators occurred. In stage 4, the total category count for the 18 students across all tasks was 126, similar to the count in stage 1, see Table 4.9. It was here, however, that the researcher found the biggest variety of use of adaptive reasoning categories. Most notably, *relationships and connections* and *justifications* decreased, and *alternates pursued* and *legitimacy determined* increased. The response to stage 4 differed between student pairs. Most pairs compared their work to the resolution video, considering alternate solutions or determining legitimacy of their solutions and the video solution. To a lesser degree, some pairs reworked their solutions, demonstrating *relationships and connections*, and *justifications*. Students were not asked specifically to create a drawing in this stage, however many did, resulting in 44 of the indicators being identified within drawings.

In addition to the adaptive reasoning counts of the task discussion threads, researcher field notes helped in the observation of student adaptive reasoning within the tasks.

**Field Note Results**

This section reports on the analysis of field notes. Research field notes were open
coded (Saldaña, 2016), summarized into analytic memos and written up as narratives from the task discussion threads (see Appendix B, Researcher field notes protocol). Open coding considered inductive aspects of peer interaction such as gestures, conversation directives, engagement, and behavior. Themes were created from the coded data to understand common task implementation and interactions of students, partners, and teachers through task administration that might explain potential outcomes. The themes are organized as follows: 1) Peer leads and student autonomy, 2) student challenges and agreements, 3) gestures, emotions, and expressions, 4) task completion, time in tasks and task stages.

**Peer Leads and Student Autonomy, Field Notes**

In the first theme, common instances, such as task administration and classroom conditions, mediate the way in which students adapted their reasoning. Partners worked online, separate from other members of the class and teacher due to COVID-19 pandemic restrictions. Limited exposure to classroom teachers gave way to student autonomy such as peer leads. *Peer leads* are students that assumed the role of the teacher (Stein et al., 2009). For example, in the Candlelight Dinner and Doggy Diet tasks, Cameron demonstrated on-task behavior, while his partner Bodhi was off-task, demonstrating little adaptive reasoning. Cameron persistently stayed on-task, undeterred by his partner’s actions, becoming the peer lead. In both tasks, Bodhi returned to on-task behavior, following Cameron’s lead and increasing adaptive reasoning. Additionally, when Cameron and Bodhi worked through the Doggy Diet task, lesson protocol was questioned by Bodhi. Rather than seeking teacher help, Cameron assumed the peer lead and copy-
pasted lesson instructions into the discussion chat box.

Lack of a teacher interaction or peer leads or instances where students did not take charge, also effected adaptive reasoning. In another example, Brad and Keven demonstrated off-task behavior in the Ghost Island task. Reduced engagement lessened their use of adaptive reasoning indicators. Lacking teacher or peer lead interaction, the guidance to move to on-task did not take place. In another case, Ryan and Max drew conclusions that were incorrect in the Doggy Diet task. Unchecked, by teacher intervention, they continued with incorrect assumptions. Still another example, Cosmo did not contribute to the task discussion thread. He instead waited for his partner Silvia to come up with solutions. Cosmo was not held accountable and did not demonstrate adaptive reasoning. Lack of teacher or peer lead interaction allowed Cosmo’s actions to go unchecked.

**Student Challenges and Agreements, Field Notes**

In the second theme, a common narrative developed around students who challenged or agreed with their partner's reasoning or task processes. While challenges were generally expressed verbally, in discussions, the conversation could center around a drawing. Challenges or agreements were generally coded as *legitimacy determined*, and typically accompanied a student search for *alternates pursued* or the use of *prior knowledge*. For example, in stage 3 of the task, Haddy demonstrated *legitimacy determined* in the Candlelight Dinner task when she texted: “I disagree with your video because like ya (yes) the candle will be done at the same time that he gets done with his briefs but he still has to drive home from his work, so he won't get home in time.” Maya
chatted back, “yeah, that’s what I was thinking, lol,” agreeing with Haddy. Following this interaction, the partners work together to figure out a new solution to the problem demonstrating *alternates pursued*. In the same task, Ryan explained in his recording “So the second the candle goes out, he's done with his work, that means he doesn't have enough time to go home. Even if he lived this close to the building (student draws a small circle next to an office building). He wouldn't have time to go home...” Ryan used his prior understanding (*prior knowledge*) that ‘work’ is completed at a workplace, separate from a home. Ryan also demonstrated *legitimacy determined* by determining that the husband (he) won’t get home in time.

Ultimately, students who challenged their partner’s drawing elements generally demonstrated a greater understanding of the problem. For example, when a student criticized their partner for not attending to detail, the partner responded by adding detail and increasing adaptive reasoning indicator use. In stage 1 of the Candlelight Dinner task Mike chatted to Sean “Nice drawing but you could improve a few things, you could talk more during the video, and you missed the whole phone talk conversation part.” This challenged Sean consider other aspects of the task, to add detail and describe his reasoning. Challenges such as this increased use of adaptive reasoning. Agreements had a similar effect. In addition, if the student accompanied their challenge with a logical, step-by-step description of the discrepancy in their thinking process, it was coded as a justifications.

Agreements between partners were coded as *legitimacy determined* if the argument was used as a verification process, however, they were not coded if the agreement was used as a compliment. For example, in the Doggy Diet task, Maya chatted
to Haddy, “I agree on your video, when I saw the first one, I thought that yes if you feed him 2 grams a day, he would be skinny really fast, but then I saw your second video and I got the same answer as you.” In this example, Maya gave reasons for her agreement and demonstrated a legitimacy determined in recognizing that if the dog was fed two grams a day, he would get skinny. She also made a connection with her partner’s explanation by saying she got the same answer. Her comment also furthers Haddy’s thinking about her own solution. Similarly, in the Candlelight Dinner task, Bodhi asked his partner, “Do you agree that the husband will be sleeping outside?” to verify that his solution was correct, to determine legitimacy. Occasionally, however, agreements were delivered out of courtesy, to be agreeable or supportive or to speed movement through a task. These types of agreements were not coded as students adapting their reasoning.

**Gestures, Emotions, and Expressions, Field Notes**

In the third theme, general trends of note that emerged in the field notes that helped to categorize adaptive reasoning in discussions and drawings include 1) gestures, hand movements or vocal sounds. 2) emotional engagement, seen in student drawn and vocal expressions, and 3) elaborations, changes in the amount a student spoke. Gestures, often seen when students created drawings, contributed to evidence of adaptive reasoning indicator use, such as the repeated touching or tapping of pencil on drawing elements, fist pounding, opening, and closing of the hand and emotional reactions such as sighing or humming. Touching or tapping usually indicated a recognition of a pattern or a connection between the touched objects. Extensive tapping on discrete drawn elements indicated justifications. Sighing was usually paired with a student either not
understanding the problem or the realization that their solution method was incorrect and categorized as a *legitimacy determined*. Similarly, fist pounding in frustration or humming contently also accompanied determinations of legitimacy as students expressed a feeling of being on or off track with their processes. When students turned over their paper abruptly and began writing on the opposite side, it was usually in concert with an alternative pursuit, an act of starting a new thought process.

Adaptive reasoning was often accompanied with student emotional engagement and drawn action elements. For example, Kate and Sadie depicted characters in the Candlelight Dinner task with expressions of surprise, anger, frustration, and happiness (see Table 4.10). The girls’ voices intonated with each voiced expression. While the girls' expressions themselves were not indicators of adaptive reasoning, their enthusiasm prompted engagement in the task and accompanied their use of adaptive reasoning. In the same task, Bodhi demonstrated *legitimacy determined* by drawing a door slamming to represent his solution, that the man in the task would have to sleep outside with the dog. The student erased and redrew the door in a ‘more closed’ position three times to emphasize his point that his solution was correct.
**Table 4.10**

*Emotional Engagement, Candlelight Dinner Task*

<table>
<thead>
<tr>
<th>Student Drawing</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Sadie's Drawing" /></td>
<td>Sadie: draws an angry woman on the phone talking to a concerned man at work. The angry woman points to a candle. An expression bubble emerging from her head. A dog bone is drawn within the bubble, meaning that if the man is not home in time for dinner, he will be sleeping outside with their dog. An upset man sits at his desk frowning with an exclamation point above his head.</td>
</tr>
<tr>
<td><img src="image" alt="Kate's Drawing" /></td>
<td>Kate: draws an upset man in an office talking on the phone with his wife at home. When describing the man and woman, Kate used phrases that focus on emotions. She described the man as very depressed and the woman as sad and “looking like she is about to kill him.” Kate repeats that the man is depressed, and the woman is sad three times in her description as she drew. After her discovery of the solution to the problem, and a happy ending, Kate erased and redrew the woman with a smiley face.</td>
</tr>
<tr>
<td><img src="image" alt="Bodhi's Drawing" /></td>
<td>Bodhi: draws upset people and adds a door slamming. Student erases the door and draws it closed a bit more each time. Student turns over paper and draws the slamming door again. Bodhi says, “And I’m also going to make this door slamming shut, because DRAMA!”</td>
</tr>
</tbody>
</table>

Elaboration, or the amount of talking required to convey meaning, oftentimes had a modifying effect on student’s adaptive reasoning. For example, Jacob greatly detailed his story drawing in stage 1 of the Candlelight Dinner task, in many instances
anticipating the solution intricacies before acquiring needed information to solve the problem (see Table 4.11). His partner, overwhelmed, did not engage in the conversation nor did he request clarification, but instead moved on to the next task discussion thread. Jacob’s excessive talk curbed his partner’s interaction. Lack of elaboration may also indicate less adaptive reasoning use or a shift to the use of drawings instead of discussion. Oftentimes when students found a task to be too simple, they simplified their drawings or discussions, demonstrating less adaptive reasoning. Kevin did not talk as much as his partner. He did, however, adapt his reasoning within drawings (50%) at a higher percent than the other students.

**Table 4.11**

*Excessive Description, Candlelight Dinner Task*

<table>
<thead>
<tr>
<th>Student</th>
<th>Drawing and Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jacob (drawing)</td>
<td><img src="image.png" alt="Image" /></td>
</tr>
<tr>
<td>Jacob (drawing description)</td>
<td>And so, we’re going to say Y. That’s Y and so then if we want to know how long the candle will be burning for, we need to do x times y. And then if we get x times y then that is how long it was burning for. And if we have to know how many minutes it is going to take to get home, as well. And so, we will have ‘A’ is what we used for basically how many cases he has to do, how many folders. And ‘B’ would be how long it takes him to do each folder. And then we also need to know how far away, and that will give us, the A times B will give us, how long it will take him to finish all of his work. And then the distance, we need to know the distance, how far away he is from his house. And so, we are using ‘C’ for that. So, we have the equation here is C plus A times B. And that's going to give us how long he is going to take him to get home. So, if Y times X is greater than C plus A times B, if that is true, then he didn't make it in time. But if it's not true, if this doesn't work. Then it would be C plus A times B is greater than Y times X then he would hopefully make it in time.</td>
</tr>
<tr>
<td>Jason (chat)</td>
<td>yes, it was good now let's move on geeeeee...</td>
</tr>
</tbody>
</table>
Some students felt as though they lacked drawing skills and rather than draw, relied on more elaborate descriptions. In the Candlelight Dinner task Kate said to Sadie “I'm not a very good artist, so stick figures it is. So, this is the lady at home, at her table and she has the candle...” Katie continued to describe the situation using expressions that described the task. In all three tasks Haddy did not like her drawings and claimed that each task was difficult. Instead, she engaged in extensive on task talk, oftentimes writing words instead of using pictorial representations in her drawings. Her partner, Maya completed detailed pictorial drawings demonstrating understanding each problem. In each task, including the final task which proved difficult for both students, Maya drew more and exhibited more adaptive reasoning indicators, however both students completed the tasks and evidenced adaptive reasoning.

Task Completion, Time in Tasks and Task Stages, Field Notes

In the fourth theme from the analysis of the field notes, the researcher found that students completed the tasks and stages at different rates, with varying success rates. They successfully completed the Candlelight Dinner task considerably more often than the Doggy Diet or Ghost Island task. In addition, the Candlelight Dinner task took the most time for students to complete. As students progressed through the tasks, the routine of task administration became more familiar and following tasks, Doggy Diet and Ghost Island were completed more quickly. In addition, students spent most of their time in stages 1 and 3 of the task in which they were prompted to produce a drawing. The least amount of time was spent in stage 2, when students were asked to create a list of needed information to solve the task.
Themes developed from the analysis of the field notes help in the coding of and understanding of student adaptive reasoning use in when students challenged or agreed with one another, used gestures, emotions and expressions, worked autonomously or when taking on the role of peer lead. These themes including the understanding of task completion time and student success combined with the analysis of the depth of knowledge level tests, described in the next section, added helpful information to the combined interpretation.

**Depth of Knowledge Level Test Results**

The final section under qualitative results reports on the DOK level tests. Following the three tasks, students completed a DOK level test. The test was adapted from NextLesson (n.d.). DOK question levels were confirmed by independent researcher, Kristy Litster, and administered to 17 of the 18 students. One student moved before the test was provided. The test consisted of eight DOK level 2 and 3 questions and were graded as per the grading rubric in Appendix E, DOK Level Test. Students that did not pass DOK level 2 or 3 questions were categorized as working at DOK 1 or below. Designated students working at the DOK 2 level, completed the DOK level 2 questions correctly but did not sufficiently complete the DOK level 3 test questions. And students categorized as working at the DOK level 3 or above consisted of students who completed both level 2 and 3 test questions sufficiently.

In level 2 of the DOK level test, the researcher was looking for students to ‘calculate’, ‘compare’ and ‘explain’ correct solutions. For example, in Figure 4.1 on question number one, the student correctly calculated ‘Wins to Games Played Ratio’ and
‘Winning Percentage’. He then compared scores to determine that Aaron Rodgers is the best all-time quarterback. He explained his rationale, stating “because his win percentage is higher.” To achieve a level 3 the student also correctly answered questions five through eight as per the rubric. In the DOK 3 level, students needed to rank the players and decide which rankings were more important than the others and explain their reasoning. In the example the student compared the different rankings and explained that the Competition and TD and INT rates are the most important rankings to consider because the rankings are not skewed by other players' performances.
Figure 4.1

Depth of Knowledge Test Level 2 and 3 Example Questions

### Level 2

**Passing Pros Green Bay Packers**

Who is the all-time best quarterback for the Packers? You are a sports columnist for an online blog. You are writing an article comparing Aaron Rodgers and Brett Favre. Which quarterback is best? Use data to justify your opinion.

One way to compare performance is to look at winning percentages. Use the ratio of wins to games played to determine the winning percentage for each player. Round to the nearest tenth. The first one is done for you.

<table>
<thead>
<tr>
<th>Name</th>
<th>Win-Loss Ratio</th>
<th>Wins to Games Played Ratio</th>
<th>Winning Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td>80:39</td>
<td>160/253</td>
<td>67.2%</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>160:93</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Who has the highest win per games played ratio? Which quarterback appears to be the best all-time quarterback for the Packers? Explain your reasoning.

Aaron Rodgers has a higher win per games played ratio. Aaron Rodgers appears to be the best quarterback because he wins a higher percentage of games.

You could rank the quarterbacks by completion percentages.

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>Passes Completed</th>
<th>Passes Attempted</th>
<th>Completion percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td>2,633</td>
<td>4,047</td>
<td>65.1%</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>5,377</td>
<td>8,754</td>
<td>61.4%</td>
</tr>
</tbody>
</table>

### Level 3

You have looked at four different statistics to determine which quarterback is the best. Now compare those statistics.

5. Use your calculations from questions #1-4 and rank both players as (1) for highest and (2) for lower.
6. Add up the rankings to find the total rank.

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>Winning percentages rank</th>
<th>Completion percentage rank</th>
<th>Number of Wins per year rank</th>
<th>TD to INT Rate rank</th>
<th>Combined total Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

7. Which quarterback has the best combined rank total?

Aaron Rodgers

8. Are some statistics more important than others? Who is the best quarterback? Explain your reasoning.

I think that the completion percentage and the TD to INT rates are the most important because that is the most important thing that the QB does. It also is not skewed by other players. In conclusion, Aaron Rodgers is the better player.

**Note.** Test adapted from NextLesson (n.d.), nextlesson.org
Fourteen of the seventeen students (82.4%) performed at a working DOK level of 3 or higher. These students demonstrated more abstract thinking including planning or demonstration of evidence in their answers. Two students (11.8%) performed at level 2, indicating multi-step thinking involving comparing, organizing, summarizing, predicting, or estimating. One student (5.9%) performed at a working DOK level 1 or lower demonstrating simple procedures solutions.

Patterns were discovered in adaptive reasoning indicator use per student working DOK levels, see Table 4.12. Results showed that as student working DOK level increased there was a decrease in the student use of alternates pursued and an increased use of prior knowledge. This suggests that students of higher working DOK levels accessed prior knowledge rather than utilizing a guess-and-check strategy of trying alternate approaches. In addition, pattern recognition was utilized more by the student working at DOK1 than the students working at DOK2 or DOK3.

Table 4.12

<table>
<thead>
<tr>
<th>DOK Level</th>
<th>N</th>
<th>RC</th>
<th>AP</th>
<th>J</th>
<th>PK</th>
<th>PR</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOK&lt;=1</td>
<td>1</td>
<td>37.9%</td>
<td>17.2%</td>
<td>25.9%</td>
<td>0%</td>
<td>12.1%</td>
<td>6.9%</td>
</tr>
<tr>
<td>DOK=2</td>
<td>2</td>
<td>42.1%</td>
<td>10.5%</td>
<td>40.4%</td>
<td>3.5%</td>
<td>0%</td>
<td>3.5%</td>
</tr>
<tr>
<td>DOK&gt;=3</td>
<td>14</td>
<td>40.5%</td>
<td>7.6%</td>
<td>32.7%</td>
<td>6.4%</td>
<td>5.5%</td>
<td>7.3%</td>
</tr>
</tbody>
</table>

*Note*. RC=relationships and connections, AP=alternates pursued J=justifications, PK=prior knowledge, PR=pattern recognition, LD=legitimacy determined, AR=adaptive reasoning. DOK=Depth of Knowledge

The qualitative results were summarized into qualitative findings to guide quantitative analysis. Summarized conclusions are presented in the next section.
Quantitizing of the Qualitative Results for Quantitative Analysis

Qualitative data were then quantitized. To better understand this process, this section describes the interface between the qualitative and quantitative results of the study’s exploratory sequential design (Creswell & Plano Clark, 2018). The qualitative results explained what adaptive reasoning indicators 7th grade students evidenced as they solved mathematical tasks in their drawings and discussions. Building upon these results, qualitative data were quantitized using ENA (Wooldridge et al., 2018), to further understand how students adapted their reasoning. ENA is a quantitative ethnographic technique (Shaffer, 2017; Shaffer & Ruis, 2017; Shaffer et al., 2016) used to model the structure of connections quantifying temporal co-occurrence of adaptive reasoning indicator use within conversations and drawings, producing a weighted network of co-occurrences. Using the ENA tool, researchers can critically analyze and compare networks both visually and statistically, making it a useful technique for modeling how students adapt their reasoning.

This section summarizes the elements of the qualitative results, explaining the determined direction of how the quantitative data were analyzed and then discusses the process of quantitizing the data using ENA.

Qualitative Results: Instigation of Quantitative Directives

Trends and patterns found in qualitative results are organized by (a) student and partner overall use, use in different tasks, and within stages of the tasks, (b) adaptive reasoning indicator use within the different modalities of discussion and drawing, and (c) adaptive reasoning indicator use per DOK level test results. The following sections and
subsections are delineated by qualitative findings and quantitative analysis directives, describing qualitative findings that initiated quantitative directions of analysis.

**Student Adaptive Reasoning, Overall Use by Students and Partners**

**Qualitative Findings.** The qualitative analysis revealed a use of six adaptive reasoning indicators with no significant emergent indicators. Students adapted their reasoning 820 times, predominantly utilizing the two primary indicators: *relationships and connections* (39.6%) and *justifications* (32.8%). Students also used secondary indicators: *alternates pursued* (9.1%), *prior knowledge* (5.4%), *pattern recognition* (6.3%) or *legitimacy determined* (6.7%). Students adapted their reasoning in this general pattern of percentages; however, differences were seen in individual use within the number of incidences observed and categories of incidences. Similar patterns of adaptive reasoning use were evidenced between partners, including number of times students adapted their reasoning, the type of indicator and whether the indicator was used within drawing or discussion, demonstrating the mediation of partner interaction.

Students solved tasks using the primary indicators and to a varying degree the secondary indicators. Tasks were of different mathematical content and were solved by students using different processes with assorted success. Students demonstrated 45.1% of adaptive reasoning indicator use in the first task, *Candlelight Dinner*, and 27.4% in each of the two other tasks *Doggy Diet* (13.7%) and *Ghost Island* (13.7%).

Within the task stages, students spent most of their time in stage 1 and 3, with stage 3 accounting for over half of the adaptive reasoning occurrences (56.3%). The least amount of student time was spent in stage 2, with the least amount of adaptive reasoning
indicator uses identified. Use of adaptive reasoning indicators varied between stages. Stages 1 and 2 represent high use of justifications. Especially in stage 2 where justifications occurred with an increase in prior knowledge and alternates pursued. And in stage 4, there was a higher use of alternates pursued, legitimacy determined and a much lower use of justifications within student adaptive reasoning.

Quantitative Analysis Directives. Connections between indicators were explored to better understand how students adapted their overall adaptive reasoning indicator use, individual student use, and partner use by creating ENA network models and differences graphs. Analysis included a focused look at the two primary indicators, relationships and connections and justifications, and how these indicators related to the other four secondary indicators. Variation in student use of adaptive reasoning instigated further analyses of student and partner patterns of adaptive reasoning. Differences in task and task stages were also analyzed to more fully understand how students adapted their reasoning within varying tasks and the immersion, information gathering, and resolution phases of the task.

Adaptive Reasoning in Drawing and Discussion Modalities

Qualitative Findings. Two-thirds of the total adaptive reasoning indicators were identified within student discussions in the following percentages per discussion total indicator use: relationships and connections 34.7%, justifications 36.0%, alternates pursued 10.0%, legitimacy determined 8.7%, prior knowledge 5.7%, pattern recognition 4.8%, see Table 4.1. The remaining third were identified in student drawings in the following percentages per drawing total indicator use: relationships and connections
48.9%, justifications 26.6%, alternates pursued 7.4%, legitimacy determined 2.8%, prior knowledge 5.0%, pattern recognition 9.2%, see Table 4.1. The percent of adaptive reasoning that occurred in discussions was higher in justifications, alternates pursued, and legitimacy determined. In drawings the percent of adaptive reasoning was higher in relationships and connections and pattern recognition, demonstrating differences in student use of indicators within modalities. Specifically, when looking at the two primary indicators, percentage use differed within drawing, relationships and connections was higher than justifications by 23%, whereas in discussions the primary indicators were seen in similar percentages.

Individual students adapted their reasoning generally two-thirds of the time during discussions and one-third during drawings, except for Kevin who spent an equal amount of time reasoning in both modalities. Students drew common symbols, elaboration, expressions, gestures that represented adaptive reasoning.

**Quantitative Analysis Directives.** Quantitative analysis was further used to scrutinize differences in drawing and discussion modalities over all tasks, between tasks and within task stages. Use between categories of indicators was explored to better understand the higher use of justifications, alternates pursued and legitimacy determined within discussions and pattern recognition within drawings. Individual student comparisons were conducted to better understand and identify outliers, such as Kevin.

**Adaptive Reasoning and Depth of Knowledge Levels**

**Qualitative Findings.** DOK level test results showed that students that achieved varying DOK levels adapted their reasoning differently. The use of alternates pursued
decreased and prior knowledge increased as working DOK level increased. In addition, pattern recognition is utilized more by students working at DOK1 than students working at DOK2 and DOK3.

**Quantitative Analysis Directives.** Quantitative analysis was used to better understand the differences in student uses of alternates pursued, prior knowledge, and pattern recognition between students working at DOK1 and DOK3. In addition, quantitative analysis was used to observe adaptive reasoning of students working at differing DOK levels in different tasks and within task stages.

**Quantitizing the Qualitative Data with Epistemic Network Analysis**

In this study, ENA was applied (Shaffer, 2017; Shaffer & Ruis, 2017; Shaffer et al., 2016) to these data using the ENA Web Tool, version 1.7.0 (Marquart et al., 2018) to model the structure of connections amongst codes in the task discussion threads. The qualitative coded data sheet was entered into the ENA Web Tool and turned into network graphs or network models; images consisting of a summary of the temporal connections of adaptive reasoning incidences, wherein nodes correspond to the adaptive reasoning indicator codes and edges reflect the relative frequency of co-occurrences, or the relative repeated connections between two codes. Task discussion threads were analyzed with an ENA algorithm called infinite stanzas, a technique that constructs the network model by comparing all lines that precede the current line in temporal context (Siebert-Evenstone et al., 2017). Infinite stanza windows were chosen due to the shorter components of dialogue in task discussion threads, the segmentation of tasks and task stages, and the recordings within the stages. The resulting networks are aggregated visualizations of all
the associations between indicators, modeling the use of adaptive reasoning within each conversation and drawing. ENA is described in more detail in the quantitative results section.

In addition to ENA network models, differences graphs were created to compare networks. These graphs are calculated by subtracting the weight of each connection in one network from the corresponding connections in another comparative model. Criteria of statistical significance was placed on the comparison between groups, determining that patterns found within the graphs were systematic and of value to analyze (Shaffer & Serlin, 2004). Due to the study’s small sample sizes and non-symmetrical data distribution, nonparametric Mann-Whitney U tests were used to determine statistically significance between group centroids. Once significance between centroids is established, comparisons between networks and student adaptive reasoning indicators are possible. Nonparametric tests are oftentimes used in educational research instead of parametric tests because assumptions are frequently violated, such as normality. Because of this, Mann-Whitney U tests, based on rank order, were chosen as the most appropriate test for this study and performed to show statistical significance of comparisons between student adaptive reasoning 1) within different tasks, 2) between the different modalities of drawing and discussion, and 3) between student working DOK levels. Reporting of the Mann-Whitney U test include the median of each group (Mdn), the alpha level (significance level), the p-value, the effect size (r), and the U-value (the smaller of the differences equations between the two groups’ rank totals). When the p-value < the alpha level the centroids are considered statistically significant.

Connections found between different adaptive reasoning categories in ENA
graphs were subjected to *closing of the interpretive loop*, a process of going back to the original data that created the connections. This process affords a second view at instances of adaptive reasoning occurrences within the context how the two codes connect or how students utilize the different categories of adaptive reasoning in concert.

**Quantitative Results**

Quantitized results, inferential statistical analysis of the modeled data in ENA networks, and comparison graphs were used to address research question 3. ENA networks and differences graphs were constructed comparing each individual to the group, each partnered pair to the group, and overall group between the different modalities. Additionally, these comparisons were also viewed in relation to the different modalities of drawing and discussion, student working DOK levels, tasks and task stages.

Significant results and results related to the qualitative findings are reported in this section and a summary is presented in Table 3.13. Candlelight Dinner and Ghost Island task, all stages of the tasks, discussion and drawing modalities, and differences between students that tested at levels of DOK1 and 3 were compared, as seen in the green highlighted boxes. In the yellow boxes, The two tasks were compared by modality and students’ working DOK levels by tasks, task stages and modality. In Table 4.13, column and row heading list comparison categories. Green and yellow boxes indicate statistically significant events and white boxes indicate differences graphs that were not statistically significant.
### Table 4.13

**Differences Graphs Constructed Between Component Categories**

<table>
<thead>
<tr>
<th>Tasks</th>
<th>Task Stages 1-4</th>
<th>Modality Discussion</th>
<th>DOK Test DOK1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tasks</strong> Ghost Island</td>
<td>Two Tasks</td>
<td>GI and CD Tasks Per Modality</td>
<td>GI and CD Tasks per DOK1&amp;3</td>
</tr>
<tr>
<td><strong>Task Stages 1-4</strong></td>
<td>All 4 stages</td>
<td>Stages per Modality (NSS)</td>
<td>Stages Per DOK1&amp;3</td>
</tr>
<tr>
<td><strong>Modality Drawing</strong></td>
<td></td>
<td>Modalities</td>
<td>Modality per DOK1&amp;3</td>
</tr>
<tr>
<td><strong>DOK Test DOK3</strong></td>
<td></td>
<td></td>
<td>DOK Test Scores</td>
</tr>
</tbody>
</table>

*Note. Statistically significant – green and yellow boxes, not statistically significant (NSS) white boxes. GI = Ghost Island task, CD = Candlelight Dinner task, DOK = Depth of Knowledge*

The quantitative results, guided by the qualitative findings, consist of ENA network models and differences graphs used to address research question 3, How do adaptive reasoning indicators relate to 7th grade students’ engagement with level 2 and 3 depth of knowledge tasks? The results were organized into three sections that look at student adaptive reasoning across all tasks, within tasks, and within task stages of: 1) student, group, and partner use, 2) use per the modalities of drawing and discussion, and 3) use per student working DOK levels.

**Student, Group, and Partner Adaptive Reasoning**

This section begins with how students adapt their reasoning individually across all three tasks. Next, 18 student adaptive reasoning networks are combined into one network
to better generalize students adaptive reasoning and to identify how students adapted their reasoning compared to the group. Partner adaptive reasoning is then explored in order to determine mediation effects. And finally, group adaptive reasoning is compared between the three different tasks and the four task stages, to better understand how students adaptive reasoning under different task circumstances and stages.

*Student Adaptive Reasoning Across All Tasks*

Results show that students adapt their reasoning differently. Figure 4.2 is an ENA network representing the student Max’s adaptive reasoning across the three tasks. In the top left corner of Figure 4.2 is the title of the unit that this epistemic network is modeling, ‘Max’. The network includes longitudinal data across which the relationships between adaptive reasoning indicators are modeled. In the bottom left of the figure, units and conversations denotes that Max is a participant and that the data is nested within the conversations in the tasks and interactions between partners. The six adaptive reasoning indicators are presented as dark grey dots of varying sizes called *nodes*. The nodes represent associations between the six adaptive reasoning indicators. The positions of the six nodes on the grid does not directly inform about adaptive reasoning, but instead are placed in these positions based on an ENA optimization routine that provides a visual map to interpret the relationships between adaptive reasoning indicators. The optimization routine is a process in which ENA rotates the network in high dimensional space and positions the network along the X and Y axes so that networks of groups or individual students can be compared. The axes themselves do not provide direct
information on adaptive reasoning. For more information on ENA see Shaffer et al. (2016).

Thickness of the lines connecting nodes or edges are determined by the number of relative indicators of adaptive reasoning incidences identified between the two indicators, temporally, in each span of dialogue or student drawings. The thickness of the edge is an indicator of the relative frequencies of co-occurrences (RFC), or the occurrences of adaptive reasoning use, relative to the entire network. Max adapted his reasoning using the two primary indicators together, see Figure 4.2. The edge between these two indicators justifications and relationships and connections (RFC 0.60) is seen by the size of two nodes and the thickness of the edges between the indicators. The closeness of these two nodes to each other and the origin of the graph indicates that these two indicators were also used together by other students. The RFC’s are not correlations, they are numerical strengths weighted so that comparisons can be made between nodes or indicators of adaptive reasoning.

The blue dot labeled ‘Max’ indicates a centroid, or the arithmetic mean of the edge weights of the network connections, see Figure 4.2. It is the center of mass of Max’s network, in that it summarizes the connections of his adaptive reasoning use into a single point. Centroids are influenced by the position of the nodes and the strength between nodes. Max’s centroid is positioned near the origin of the graph, indicated by a the red square; meaning that Max adapted his reasoning with similar connectivity, or in a similar pattern as the average use of all students in the study. Bolder edges display connections and are considered as viable results, while light edges or weak connections between nodes, such as from prior knowledge to pattern recognition, did not provide enough
statistical evidence quantitatively to be considered viable and are generally discussed as lacking connection or having no connection, see Figure 4.2.

Max’s network connections link *prior knowledge, pattern recognition, legitimacy determined*, and *alternates pursued* to the two primary indicators. This means that Max adapted his reasoning mostly by finding relationships and connections and justifying. In this process, he used his prior knowledge, recognized patterns, determined legitimacy, and to a lesser extent, pursued alternative approaches in conjunction with the primary indicators. Rarely did Max use prior knowledge to pursue alternatives, or recognize a pattern or determine the legitimacy of his processes, denoted by the light edges between these nodes.
Figure 4.2

*Max’s Adaptive Reasoning ENA Network Model with Relative Frequencies of Co-Occurrences*

![Diagram](image)

*Note.* The blue dot = centroid of the network. The red square on the origin represents the center of the network for all students.

To investigate Max’s use of the primary indicators, all incidences of adaptive reasoning in the transcribed and coded data that occurred between the two primary nodes were examined. This process of examining connections of the original coded data to the network graph is called *closing the interpretive loop*. For example, the edge strength between the two primary indicators used by Max in stage 3 of the Ghost Island task was traced back to the following comment:

*Max:* *And then from here* like that it was 32 and 48. *And so we would need to compare [Relationship and Connection] these.* So how many times does 14 go
into 32? *Like you need to compare them. And yeah, compare them and see if they are going on course. Because if we compare them, and this doesn't line up with this, then we could be going off course.* [Justification]

The green text illustrates Max’s use of the adaptive reasoning indicator *justification* and the red text illustrates his use of *relationships and connections*. Max demonstrates the indicator *relationships and connections* when he “compares” elements in his drawing. He demonstrates *justification* by showing steps of reasoning that could lead him to a conclusion. Max draws triangles and labels the legs of the triangles, showing the comparison, and demonstrating the indicator relationships and connections. However, he does not demonstrate *justification* in the drawing. In this case, Max exemplifies a generalizable event; students demonstrating both primary indicators within the modality of discussion and only one indicator when drawing. As seen in the qualitative results, the indicator *justification* is not used as readily within drawings as is the indicator *relationships and connections*.

Student *baseline networks* were analyzed in consideration of how each students adapted their reasoning. The variety of connectivity seen in the resulting student networks in Figure 4.3 indicate uniqueness in adaptive reasoning with generalizable patterns. Connectivity patterns include strong edges between the two primary indicators with varied connections to the other four secondary indicators. Dense networks that signify more connections, such as Cameron’s network, while less dense networks demonstrate fewer connections, for example Cosmo’s network.

ENA networks show temporal connections between indicators visually representing how students adapt their reasoning. For example, in Figure 4.3, Jason’s
network shows strong connections from *alternates pursued* to the primary indicators. This means that Jason examined different ways to solve the task, using *alternates pursued*. He discovered relationships and connections and justified his pursuits. Ryan, however, used pattern recognition in conjunction the primary indicators. The strength from *pattern recognition* to *relationships and connections* is much stronger than from *pattern recognition* to *justifications*. Ryan recognized patterns as he found relationships and connections. He also, but to a lesser degree, justified when recognizing patterns.
Figure 4.3

ENA Networks for Each Student Over All Tasks
Students’ adaptive reasoning can be compared using differences graphs. In Figure 4.4 on the left side, Harlee’s and Max’s networks models look different. Max’s centroid, as discussed earlier, is near the center of his network graph, whereas Harlee’s centroid is in the top left corner of her network graph, centered between *alternates pursued* to *pattern recognition*. In Harlee’s network, the edges from *prior knowledge* to the primary indicators are thinner and the edges from *alternates pursued* to the two primary indicators are thicker than in Max’s network. Comparisons are more clearly displayed in differences graph in Figure 4.4, right side. The red edge from *alternates pursued* to *justifications* (RFC 0.25) and the blue edge from *prior knowledge* to *justifications* (RFC 0.30) show how Harlee more often pursued other alternatives and justified her processes and Max accessed prior knowledge and justified. Harlee used a guess-and-check strategy, while Max used prior knowledge to justify and adapt his reasoning.

Similarities of adaptive reasoning use can also be viewed in the ENA graphs. For example, in Figure 4.4, the absence of an edge from *pattern recognition* to *relationships and connections* indicates that these two indicators were used in similar occurrences between Max and Harlee. However, since the difference graph eliminates co-occurrences, it is not possible to understand how much use occurred without looking at the baseline network models of each student. Viewing the network models and differences graph together tells a more complete story of how Max and Harlee adapted their reasoning. To further interpret the results, Max and Harlee adapted their reasoning with strong connections from *pattern recognition* to *relationships and connections*, at about the same number of relative occurrences.
**Figure 4.4**

Left: ENA Networks for Max and Harlee, Right: ENA Differences Graph Comparing Max VS. Harlee, with Relative Frequencies of Co-Occurrences

**Harlee’s Network**

**Max’s Network**

**Max (Blue) VS. Harlee (Red)**

**Group Adaptive Reasoning Across All Tasks**

Results of student adaptive reasoning use is summarized in Figure 4.5. This group network model takes into consideration all 18 students’ 820 adaptive reasoning incidences throughout the three tasks. Colored dots represent student centroids in relation to the overall group network. Dots of the same color indicate partners. Bodhi and Cameron did not have consistent partners and are indicated by black dots.

The connectivity of the group network, see Figure 4.5, shows that students adapted their reasoning by using the two primary indicators in conjunction, justifications and relationships and connections (RFC 0.63). This relationship between the two indicators formed a center hub of student adaptive reasoning in which the secondary
indicators, connect like spokes to the two primary indicators. *Prior knowledge, alternates pursued, legitimacy determined, and pattern recognition,* were mainly used in conjunction with the primary indicators with relative frequencies of co-occurrences ranging from 0.16 to 0.25. Weaker connections were considered statistically insignificant in this model, such as the edge from *alternates pursued* to *legitimacy determined.*

**Figure 4.5**

*Mean ENA Group Network of Adaptive Reasoning Indicators with Relative Frequencies of Co-Occurrences*

Positioning of centroids on the group network reveal how students adapt their reasoning as reflected within the group, see Figure 4.5. For example, Sadie and Max are located close to the origin of the graph, showing that they adapted their reasoning in a
similar manner to each other and the group network connectivity. Students farther from
the origin, such as Brad, Mike, Harlee and Jason, are outliers. They adapted their
reasoning apart from the rest of the students. Brad’s centroid, located in the lower right of
the graph, showed that he adapted his reasoning using mainly indicators below the
horizontal line and to the right of the vertical line, mainly prior knowledge, justifications,
relationships and connections and legitimacy determined. Jason, with a centroid situated
to the far left, solved problems by pursuing alternatives, using prior knowledge to justify
relationships and connections. Mike, to the far right; utilized more relationships and
connections and legitimacy determination.

Similarities between student adaptive reasoning can be seen in the clustering of
the centroids. Harlee, Kate and Aisha are all positioned between alternates pursued and
legitimacy determined and adapted their reasoning using these two indicators. Mike,
Sean, Haddy, Ryan and Maya all adapted their reasoning by determining the legitimacy
of their processes. The centroids of Cameron, Sadie, Jason, Jacob, Kevin and Bodhi lie
between alternates pursued and prior knowledge, demonstrating adaptive reasoning from
these two indicators to the primary indicators. Silvia and Brad show strong connections to
prior knowledge and the two primary indicators, and Cosmo adapted his reasoning using
mostly the two primary indicators.

**Partner Adaptive Reasoning Across All Tasks**

Similarities were seen in connectivity between partner’s networks indicating a
mediating effect of adaptive reasoning indicator use due to partner interaction. Student
baseline networks in Figure 4.3 are color coded per partners to visualize similarities in
connectivity. To better understand these similarities, student centroids were mapped using color coded dots, in Figure 4.6, on an overlayed of the group network. The *partner mean* is plotted as a solid square. This solid square is also described as an outlier box, which surrounds the mean, indicating the normal range of the mean, with a dotted rectangle. Cameron and Bodhi did not have consistent partners and were excluded from model.

Partner outlier boxes are fairly specific, with some outlier boxes overlapping. Students with centroids located close to their partner’s, such as Aisha and Harlee, Max and Ryan, Haddy and Maya, and Mike and Sean indicate similar network connectivity. When working together, these students varied less in how they adapted their reasoning. Partners that are farther apart, such as Kate and Sadie, Jason and Jacob, Brad and Kevin, and Cosmo and Silvia showed more variability. For example, partners Haddy and Maya (light blue rectangle) were similar along the X-axis or *horizontally*, but were different along the Y-axis or *vertically*. Maya demonstrated strong connections between the two primary indicators and Haddy demonstrated stronger connections from *legitimacy determined* and *pattern recognition* to the primary indicators.

Proximities of partner centroid’s demonstrates partner mediation in adapting their reasoning. Distinct concentrations of partner adaptive reasoning, as indicated by outlier boxes, show connections that the partners share. Three partners were selected from the results to demonstrate partner interaction, Kate and Sadie (grey rectangle), Ryan and Max (aqua rectangle), and Kevin and Brad (green rectangle). Kate and Sadie were chosen because of the vertical difference between their centroids. Max and Ryan have centroids that are horizontally different. And Kevin and Brad’s centroids are both horizontally and
vertically different. In addition, Kate and Sadie’s outlier box overlaps Max and Ryan’s, but not Kevin and Brad’s outlier box.

**Figure 4.6**

*ENA All Student Network Overlay Across Partner Centroids and Outlier Boxes*

Closeness of centroids show similarities and distance between centroids show individuality of student adaptive reasoning. In Figure 4.6, Sadie and Kate share adaptive reasoning indicators that are horizontally similar above the X-axis. Meaning they adapt their reasoning similarly when determining legitimacy, seeking alternate approaches, and
recognizing pattern. However, they differ vertically. Sadie shows stronger connections to prior knowledge. She utilizes her prior knowledge more often when solving tasks than Kate, see Figure 4.7. Kevin and Brad’s adaptive reasoning outlier box is shared at the far bottom of the network in Figure 4.6. They adapt their reasoning using the two primary indicators, with strong connections from prior knowledge to the primary indicators. Brad however determined legitimacy in conjunction with the primary indicators, while Kevin followed alternates pursued in conjunction with the primary indicators to solve the task, see Figure 4.7. Ryan and Max, a pair with fairly close centroids differing horizontally, adapted their reasoning in a similar manner, as seen in Figure 4.7 left, in the lighter edge weights. Max adapted his reasoning more using indicators located on left side of the graph including justification, pattern recognition, prior knowledge, and to a lesser extent, alternates pursued. Max determined legitimacy when he found relationships or connections when solving the task.

Figure 4.7

ENA Partners Differences Graphs: Sadie and Kate, Kevin and Brad, Ryan and Max
Differences between partner pairs can be compared because partners mediate each other’s adaptive reasoning creating similarities in partners. Brad and Kevin are compared to Sadie and Kate and also Ryan and Max in the differences graph in Figure 4.8. In the left graph, Brad and Kevin show a stronger connectivity from prior knowledge to relationships and connections (RFC 0.47) and from prior knowledge to justifications (RFC 0.23). From the student networks in Figure 4.3, both Brad and Kevin contributed to these connections of reasoning as seen in their network graphs in Figure 4.8. Kate and Sadie, by comparison, showed connectivity from alternates pursued to relationships and connections (RFC 0.14), pattern recognition to relationships and connections (RFC 0.16), pattern recognition to justifications (RFC 0.14), and relationships and connections to justifications (RFC 0.15). The stronger connections from alternates pursued to relationship and connections and pattern recognition to relationships and connections is largely attributed to Kate. Whereas the stronger connections from relationships and connections is seen in Sadie’s network.

In the graph on the right in Figure 4.8, Brad and Kevin show stronger connections from prior knowledge to relationships and connections (RFC 0.48) and prior knowledge to justifications (RFC 0.28). Mike and Sean show connectivity from legitimacy determined to relationships and connections (RFC 0.38) and also legitimacy determined to justifications (RFC 0.26), demonstrating the differences between the partnered pairs. Brad and Kevin adapted their reasoning in a similar fashion as Mike and Sean in other connections such as between the two primary indicators, alternates pursued and pattern recognition to the two primary indicators.
Figure 4.8

Partner differences graphs, Left: Brad and Kevin VS. Kate and Sadie, Right: Brad and Kevin VS. Mike and Sean

Brad and Kevin (BK, Green) VS Kate and Sadie (KS, Grey)  
Brad and Kevin (BK, Green) VS. Mike and Sean (MS, Orange)  

Group Adaptive Reasoning: Within Tasks

Results show that students may adapt their reasoning differently based on the particular task. Nonparametric Mann-Whitney U tests were used between the three tasks to determine degrees of statistical significance of adaptive reasoning use. The tests showed differences between Candlelight Dinner and the other two tasks, but no significant difference between the Doggy Diet and Ghost Island task. To exemplify the differences in adaptive reasoning between tasks, an ENA differences graph between the Candlelight Dinner task and the Ghost Island task is provided using an ENA means rotation, or a representation of the two variables constructed into a differences graph that shows the maximum difference between their groups, see Figure 4.9. Centroids are shown with confidence intervals, dotted rectangles around the centroid. Confidence
intervals that do not overlap generally show statistical significance. In this figure, variance was maximized using a Mann-Whitney U test along the X-axis, showing that the centroid of adaptive reasoning used in the Candlelight Dinner task (Mdn=0) was statistically significantly different at the alpha=0.05 level from the centroid of the Ghost Island task (Mdn=0), U=1528.0, p=0, r=0.38. This indicates that the use of adaptive reasoning in the Candlelight Dinner task was significantly different at a medium to large effect size.

Qualitative findings showed that students adapted their reasoning more times within the Candlelight Dinner task, which was administered first, solved through proportional reasoning or pattern recognition, and was more successfully solved by students. Ghost Island was administered last, solved using proportional reasoning or similar triangles and successfully solved the least by students. In the ENA networks and differences graph in Figure 4.9, the Candlelight Dinner task illustrates strong connections between the primary indicators (RFC 0.16), predominately solving the task by finding relationships and connections and justifying. In addition, stronger connections from *alternates pursued* to the primary indicators (RFC 0.05) and *pattern recognition* to *justifications* (RFC 0.06) suggested that more guess-and-check strategies and identification of patterns were used in the Candlelight Dinner task. In Ghost Island, students focused more on prior knowledge and determining the legitimacy of their solutions, as can be seen in the stronger connections from *legitimacy determined* to *relationships and connections* (RFC 0.06) and from *prior knowledge* to the primary indicators (RFC 0.05).
Of interest is the red edge from *justifications to relationships and connections* (RFC 0.16) in the differences graph, Figure 4.9, left. This indicates that students justified and found relationships and connections more readily in the Candlelight Dinner task over the Ghost Island task. These results demonstrate how adaptive reasoning differed, however reasons for these differences are less clear. Knowing that students solved the Candlelight Dinner task more successfully than the Ghost Island task may help explain differences based on the task’s mathematical content or task challenge level.

**Figure 4.9**

*Left: ENA Differences Graph Between Candlelight Dinner Task (Red) and Ghost Island Task (Green) With Mean Confidence Intervals, Right: ENA Network Models per Task*

*Note. GI = Ghost Island Task, CD = Candlelight Dinner Task*
**Group Adaptive Reasoning: Within Task Stages**

Results of adaptive reasoning indicator use during task phases shows that students adapt their reasoning differently within the immersion stages 1 and 2, the information gathering and solving stage 3 and the resolution stage 4.

Qualitative findings indicated more than half of the student adaptive reasoning indicator use occurred in stage 3. In the ENA network in Figure 4.10 bottom left, stage 3’s dense network of recurring connections, closely resembles student overall adaptive reasoning indicator connectivity, i.e., a strong connection between the primary indicators with connections to all other indicators. In stage 1, students adapted their reasoning in a similar fashion to stage 3, quantitative results showed no statistical differences between adaptive reasoning indicator use between the two stages.

In stage 2, an immersion stage, students made lists of needed information and rarely created a drawing. Qualitative results demonstrated a higher use of *justifications*, which can be seen quantitatively in the larger *justifications* node in Figure 4.10. Indicator connectivity was strong between the primary indicators, however, other connections were limited to the connection from *justifications* to *prior knowledge* and *alternates pursued*. In other words, when students developed their lists of needed information to solve the task, they primarily justified relationships and connections, and used prior knowledge and considered alternatives when justifying.

In the resolution stage 4, when students were presented with a solution, the network evidences a connectivity pattern distinctly different from the overall task. Rather than connections between the primary indicators extending the secondary indicators; there is a shift to strong connections of all adaptive reasoning indicators to *alternates*...
pursued, except for pattern recognition. The heavy use of alternates pursued is primarily due to students' consideration of different task solutions, see Figure 4.10.

**Figure 4.10**

*Mean ENA Networks of Adaptive Reasoning per Stage*
**Drawing and Discussion, Adaptive Reasoning in Modalities**

Drawing and discussion modalities were explored through ENA network models and differences graphs and examined to further address the research question of how students adapt their reasoning. Statistical differences between modalities were found across all tasks and between tasks, however not between tasks stages. Because the ENA network models of each modality closely resembled each other, a differences graph was constructed comparing the two using a means rotation. Along the X axis, a Mann-Whitney U test showed that the drawing group centroid (Mdn=0) was statistically significantly different at the alpha=0.05 level from the discussion group centroid (Mdn=0), U=14718.00, p=0.00, r=-0.17, see Figures 4.11 and 4.12.

**Drawing and Discussion Modalities Across All Tasks**

Across all three tasks stronger connectivity is seen in the drawing modality from *pattern recognition* to *relationships and connections* (RFC 0.03) as compared to discussion, see Figure 4.11. However, when students discussed, a denser network occurred from *legitimacy determined* to *justifications* (RFC 0.03). This suggests that students demonstrated pattern recognition, and recognized relationships and connections at a relatively higher rate when they drew than when they discussed; and that they determined legitimacy and justified, they were more readily in discussions rather than drawings.
Figure 4.11

ENA Difference Graph Between Discussion (Blue) and Drawing (Red) Modalities with Confidence Intervals

Note. The model had co-registration correlations of 0.93 (Pearson) and 0.97 (Spearman) for the first dimension and co-registration correlations of 0.95 (Pearson) and 0.93 (Spearman) for the second. These measures indicate that there is a strong goodness of fit between the visualization and the original model. Dotted lines represent confidence levels of each group. Disc.=discussion

Drawing and Discussion Modalities Within Tasks

Student use of adaptive reasoning indicators varied between discussion and drawing modalities in different tasks. Looking at both tasks, a higher variety of adaptive reasoning indicator use is seen in drawing in the Candlelight Dinner task. In the Ghost Island task, variety both drawing and discussion are more equally seen. In the Candlelight Dinner task, stronger connection are seen from pattern recognition to the primary
indicators. This signifies that students used *justifications* and *pattern recognition* (RFC 0.08) as well as made *relationships and connections* and *recognized patterns* (RFC 0.10) more in their drawings than in their discussions, see Figure 4.12. In the Ghost Island task weaker connections are seen in the use of drawing in most all indicators over discussion, except for an increase in relative connection between the primary indicators (RFC 0.10). Within discussions in the Ghost Island task, strong connections are seen from prior *knowledge* to *justifications* (RFC 0.05) and from *legitimacy determined* to *justifications* (RFC 0.03).

**Figure 4.12**

*ENA Difference Graphs, Discussion and Drawing in Candlelight Dinner to Ghost Island*

*Note.* The model had co-registration correlations of 0.93 (Pearson) and 0.97 (Spearman) for the first dimension and co-registration correlations of 0.95 (Pearson) and 0.93 (Spearman) for the second. These measures indicate that there is a strong goodness of fit between visualization and the original model.
**Depth of Knowledge Levels, Levels 1 and 3 Adaptive Reasoning**

Patterns were seen within students’ adaptive reasoning based on students’ working DOK levels over all tasks, within tasks and within task stages. Qualitative findings showed that students working at increased DOK levels demonstrated a decrease in use of the adaptive reasoning indicator *alternates pursued* and an increase use of *prior knowledge*. In addition *pattern recognition* was utilized more by students working at DOK1 than DOK2 or DOK3. To gain a better understanding of the qualitative results, differences graphs between students working at DOK1 and DOK3 were created, see Figures 4.13 - 4.15. These two working DOK levels were chosen for analysis because the difference between students working at DOK2 and DOK3 proved statistically non-significant and both qualitative findings and statistical confidence intervals suggest that students working at DOK2 and DOK3 adapt their reasoning in a similar fashion. In addition, the smaller confidence interval of the group working at DOK3 did not overlap with the group working at DOK1 due to the larger number of students working at DOK3.

To accommodate the smaller group number if students working at DOK1, a means rotation across the X-axis was performed to maximize statistical power, creating the difference graphs in Figures 4.13 – 4.15. Along the X axis, a Mann-Whitney U test showed that the centroid showing students working at DOK1 (Mdn=-0.03) was statistically significantly different at the alpha=0.05 level from centroid showing students working at DOK3 (Mdn=-0.03) $U=1237.50$, $p=0.01$, $r=0.33$. Connections seen within each graph are described in the following sections.
Depth of Knowledge Levels Across All Tasks

Results show that students working at DOK3 exhibited stronger connections from prior knowledge to relationships and connections (RFC 0.05) and justifications (RFC 0.04) with a stronger edge from relationships and connections to justifications (RFC 0.07), see Figure 4.13. Suggesting students working at DOK3 used prior knowledge more often in relation to the primary indicators. The student working at DOK1 exhibited stronger connections from alternates pursued to justifications (RFC 0.13) and, to a lesser extent, from alternates pursued to relationships and connections (RFC 0.06), meaning that the student working at DOK1 adapted her reasoning by pursuing alternative processes, a guess-and-check type strategy, and justifying her different approaches.
**Figure 4.13**

*ENA Differences Graph Between Working Test Level DOK1 (Red) to DOK3 (Purple), Across All Tasks*

![ENA Differences Graph](image)

*Note.* The model had co-registration correlations of 0.87 (Pearson) and 0.9 (Spearman) for the first dimension and co-registration correlations of 0.93 (Pearson) and 0.91 (Spearman) for the second. These measures indicate that there is a strong goodness of fit between the visualization and the original model. DOK = Depth of Knowledge.

**Depth of Knowledge Levels Within Tasks**

In the Candlelight Dinner task, the student working at DOK1 demonstrated a variety of indicator connections, especially from **pattern recognition** to the **alternates pursued** (RFC 0.14), discovering **relationships and connections** (RFC 0.08), and **justifications** (RFC 0.11), see Figure 4.14. Whereas the students working at DOK3 showed strong connection differences solely from **justifications** to **relationships and connections** (RFC 0.27). Conversely, in the Ghost Island task, the student working at
DOK1 utilized a limited amount of adaptive reasoning indicators in her attempt to solve the task, using the primary indicators more so than the students working at DOK3 (RFC 0.08). In the Ghost Island task, the students working at DOK3 adapted their reasoning in a more diverse network than the student working at DOK1. Prior knowledge was utilized more with primary indicators, prior knowledge to relationships and connections (RFC 0.14) and prior knowledge to justifications (RFC 0.13). Additionally, the students working at DOK3 determined the legitimacy of found relationships and connections (RFC 0.09).

**Figure 4.14**

ENA Difference Graphs, Left: Candlelight Dinner Task, DOK1 and DOK3 Working Level, Right: Ghost Island for DOK1 and DOK3 Working Level

**Note.** Model co-registration first dimension correlations 0.96 (Pearson) and 0.94 (Spearman), and second co-registration correlations of 0.97 (Pearson) and 0.95 (Spearman). Strong goodness of fit between visualization and the original model. **Note.** 1.CD = Mean at DOK1 in the Candlelight Dinner task, 3.CD = Mean of students working at DOK3 in the Candlelight Dinner task, 1.GI = Mean of DOK1 in the Ghost Island, 3.GI = Mean of DOK3 in the Ghost Island task.
Depth of Knowledge Level Within Tasks Stages

The differences graphs in Figure 4.15 help to clarify how students of differing DOK working levels adapt their reasoning within stages of the task. In stage 3, where most of the adaptive reasoning indicator use occurred, the student working at DOK1 exhibited stronger connections from alternates pursued to pattern recognition (RFC 0.10), and pattern recognition to legitimacy determined (RFC 0.10). Students working at DOK3 students stronger connections from alternates pursued to the primary indicators (RFC 0.10 and 0.06), and prior knowledge to the primary indicators (RFC 0.11 and 0.13). The student working at DOK1 considered alternate approaches as she recognized patterns, where students working at DOK3 looked at relationships and connections in conjunction with considering alternates. Determining the legitimacy of the patterns was key to the student working at DOK1, whereas prior knowledge came into play with students working at DOK3.

The student working at DOK1 exhibited similar adaptive reasoning indicator use in stages 1, 2 and 4 of the graph from the students working at DOK3. Particularly noticeable are the strong connections from alternates pursued to justifications in stages 1 and 2 (RFC 0.16 and 0.30), and a strong connection from alternates pursued to relationships and connections (RFC 0.64), in stage 4. More than the students working at DOK3, the student working at DOK1 primarily utilized a guess-and-check strategy throughout the entire task except for stage 3, where she adapted her reasoning mainly using the two primary indicators and to a lesser extent legitimacy determined, alternates pursued, and pattern recognition.
In stage 4, the students working at DOK3 adapted their reasoning by checking out alternate solutions and determining the legitimacy of their solutions. They were more likely to solve the tasks in stage 3, requiring little adaptation to their reasoning in stage 4. Whereas the student working at DOK1 also used *alternates pursued*, however predominantly in conjunction with *relationships and connections*. Oftentimes the student working at DOK1 calculated incorrect solutions in stage 3 and continued to search for a solution in stage 4.
Figure 4.15

*Top Row, Difference Graphs of DOK1 (Red) VS. DOK3 (Purple) Working Level of Student’s Adaptive Reasoning Per Stages*

**Stage 1**

**Stage 2**

**Stage 3**

**Stage 4**

*Note.* The model has a co-registration correlations of 0.88 (Pearson) and 0.88 (Spearman) for the first dimension and a co-registration correlation of 0.93 (Pearson) and 0.88 (Spearman) for the second dimension.
**Combined Interpretation**

To address the research questions, this study examined what adaptive reasoning indicators 7th graders evidenced through their 1) discussions and 2) drawings during inquiry-based mathematical tasks and 3) how students adapted their reasoning within the context of DOK 2 and 3 level mathematical tasks. Qualitative findings address the first two research questions about what adaptive reasoning indicators students evidence, distinguishing between the different modalities of drawing and discussion. Quantitative findings address how students adapted their reasoning by showing which adaptive reasoning indicators students used temporally. ENA models and differences graphs show connections between indicators for comparisons between tasks, task stages, modalities and student DOK working levels. The combined interpretation describes the ‘what’ and ‘how’ of adaptive reasoning across tasks, within tasks, and within task stages of (a) students, partners and group, (b) use within the modalities of drawing and discussion, and (c) use by DOK working levels.

**Student, Partners, and Group Adaptive Reasoning**

This section describes student, partner and group adaptive reasoning organized by tasks and task stages. Qualitative results, show that the 18 students adapted their reasoning using all six of the adaptive reasoning indicators: 1) *relationships and connections*, 2) *justifications*, 3) *alternates pursued*, 4) *prior knowledge*, 5) *pattern recognition*, and 6) *legitimacy determined*. Students adapted their reasoning primarily using *relationships and connections* and *justifications* and secondarily using *alternates pursued, prior knowledge, pattern recognition, and legitimacy determined*. Student
adaptive reasoning was identified within drawings and discussions accompanied by gestures, emotions, and expressions. Notably, individual student adaptive reasoning resembled the group network. Differences illustrated the unique ways in which each student adapted their reasoning. Connectivity patterns within partners suggest that partners are a mediating factor when students adapt their reasoning. Incidences in task discussion threads between partners, when students challenge one another or when a student assumes peer lead and other interactions, may indicate moments of similar adaptive reasoning indicator use, explaining this connectivity.

Quantitative results also show how students adapted their reasoning through the connectivity patterns in the ENA networks. The two indicators, relationships and connections and justifications, were used primarily together. In other words, students for the most part adapted their reasoning by finding relationships and connections of elements within the task and justifying these findings. In addition to these two indicators, students utilized the other four indicators to a lesser extent, however mostly in conjunction with the two primary indicators.

**Group Adaptive Reasoning Within Tasks**

Differences in how students adapted their reasoning between tasks is evidenced in both the qualitative and quantitative results. Of the three tasks, qualitative findings show that students used adaptive reasoning indicators more in the Candlelight Dinner task than the other two tasks. In quantitative results, the ENA networks show quantitative findings showed a strong effect size in the differences of adaptive reasoning indicator use between the Candlelight Dinner and Ghost Island tasks. In both tasks, students adapted their
reasoning using the primary indicators, however, in the Candlelight dinner, students adapted their reasoning more through the pursuit of alternatives and recognition of patterns. In Ghost Island, students utilized more prior knowledge and determined legitimacy to validated solutions or processes.

**Group Adaptive Reasoning Within Task Stages**

Common themes of what adaptive reasoning indicators students use were found in the different stages of the tasks. Qualitative findings show students used all six adaptive reasoning indicators and spent more time in stages 1 and 3, when students were asked to create a story drawing and a math drawing. In stage 2, students used mostly *prior knowledge* and *alternates pursued* as they created their needs lists and in stage 4 or the resolution stage, qualitative results shows that students utilized the primary indicators less and *alternates pursued* and *legitimacy determined* more.

Quantitative results help to delineate how students used the indicators. No statistical differences were found in adaptive reasoning use between stages 1 and 3, however the dense ENA networks show similarities to the overall group network. In stage 2, students adapted their reasoning by discussing and drawing *relationships and connections* and *justifications*. Unique to this stage, students use of the secondary indicators was limited to *alternates pursued* and *prior knowledge* in relation to *justification*. Students thought through how to solve the problem using the two primary indicators, however, to create the list of needed information for stage 2, students justified using prior knowledge and justified alternate ways to solve the problem.

In the resolution stage 4 of the task, qualitative results showed a decrease in the
primary indicators and an increase in *alternates pursued*. Adaptive reasoning indicator use composition changed drastically from the other stages. Quantitative results help to clarify these changes. Students were less concerned with finding connections, relationships, and justifying their results. Instead, when presented with the resolution video, students considered alternate approaches, using prior knowledge and determining the legitimacy of the different approaches.

**Drawing and Discussion Modalities**

Differences in adaptive reasoning indicators use between the modalities of drawing and discussion were noticed overall. Qualitative results showed one-third of all incidences of adaptive reasoning use occurred within drawings and two-thirds occurred within discussions. Student modality results may be explained by the use of drawings in only two of the three stages of the tasks. In discussions, students were more likely to adapt their reasoning using *justifications, legitimacy determined* and *alternates pursued*. In drawings students were more likely to adapt their reasoning using *relationships and connections* and *pattern recognition*. *Prior knowledge* was demonstrated similarly in drawings and discussions.

Quantitative ENA network graphs showed similarities in adaptive reasoning indicator use between the two modalities, while ENA differences graphs demonstrated differences. In discussions, the increased use of the indicator *legitimacy determined* in the qualitative results is seen in connections from *legitimacy determined* to *alternates pursued* and *justifications*. As students determined legitimacy, they considered alternate approaches and also justified their determinations. In addition, stronger connections in
the ENA discussion network were seen between *justifications* and all of the other indicators, confirming that when a student justifies, they are more likely to be discussing. While in drawings, qualitative results showed an increased use of the indicators *relationships and connections* and *pattern recognition*. Quantitative ENA drawing networks clarify these findings, showing that *pattern recognition* was used almost exclusively with the two primary indicators. *Relationships and connections* show strong connections to all of the other indicators, confirming that when a student discovers relationships and connections, they are more likely to do so in the form of a drawing.

**Drawing and Discussions Within Tasks**

Relationships between drawing and discussion, such as whether students altered their use of adaptive reasoning between modalities within different tasks and task stages, were further explored in the quantitative analysis. Between tasks, differences in students’ adaptive reasoning between modalities differed between the Candlelight Dinner task and the Doggy Diet and Ghost Island tasks. However, statistically significant differences were not found between the Doggy Diet and Ghost Island tasks. This suggests that students may draw and discuss using different adaptive reasoning indicators in different tasks. The comparison of the Candlelight Dinner and the Ghost Island tasks in the quantitative results exemplifies the differences.

In the Candlelight Dinner task, stronger connections from *pattern recognition* to *justifications* exist when students drew than when they discussed. In the Ghost Island task, students used *prior knowledge* with *justifications*, when students discussing more so than drawing. In addition, when drawing in the Ghost Island task, students adapted their
reasoning between the primary indicators more than through discussions.

**Students’ Working Depth of Knowledge Levels**

Findings suggest that students working at different DOK levels used different adaptive reasoning indicators when solving inquiry-based tasks. Qualitative results show progressive differences between students working at DOK1, 2 and 3 levels; as DOK working levels increased, *prior knowledge* increased while *alternates pursued* decreased. Indicating that students of increasingly higher DOK working levels utilized prior knowledge rather than pursuing alternative strategies. Quantitative analysis shows that the student working at DOK1 utilized *alternates pursued* in connection with *justifications*. She used guess-and-check strategies and justified alternative approaches more often than the students working at DOK3. The students working at DOK3 used *prior knowledge* more when adapting their reasoning especially in connection to the primary indicators. The use of prior knowledge was integrated into DOK3 working students’ adaptive reasoning strategies when finding relationships and connections and justifying processes more than the student working at DOK1.

**Depth of Knowledge levels Within Tasks**

Students of varying DOK working levels used different adaptive reasoning indicators within the tasks. In Candlelight Dinner, the student working at DOK1 adapted her reasoning differently from students working at DOK3 by demonstrating strong connections from *pattern recognition* to a variety of adaptive reasoning indicators. She pursued alternate solutions, justified and discovered relationships in the tasks, all in connection with pattern recognition. In the Ghost Island task, she used solely the primary
indicators. Conversely, the students working at DOK3 utilized mostly primary indicators in the Candlelight Dinner task while using all the indicators, especially prior knowledge and legitimacy determined in the Ghost Island task. While a pattern is recognized within these differences between students working at DOK1 and DOK3, the reason for these differences is unclear.

**Depth of Knowledge Level Within Task Stages**

The student working at DOK1 consistently used the primary indicators with alternates pursued in all four stages. Signifying that students primarily found relationships and connections and justified their reasoning using a guess-and-check type strategies. By comparison, the students working at DOK3 reasoned with mostly the primary indicators in the first two stages, using all adaptive reasoning indicators in the third stage, especially prior knowledge. And in the final stage 4, the students working at DOK3 determined the legitimacy of their solutions and considered alternates pursuits in solutions. In this final stage, the students working at DOK3 compared their solutions to the presented video solution and finalized the task, as opposed to the student working at DOK1 who did not consistently find a correct solution and continued to try to understand and re-work the problem.

To summarize, a combined interpretation of student adaptive reasoning was developed from qualitative and quantitative findings, demonstrating what adaptive reasoning indicators students used in discussions and drawings and how they developed their reasoning within level 2 and 3 mathematical tasks. Transcribed and coded task discussion threads and student drawings were interpreted qualitatively, quantitized and
observed quantitatively through ENA network and differences graphs. One-third of the adaptive reasoning indicators were identified within drawings and two-thirds within the modality of discussion. Students individual adaptive reasoning were unique, but also resembled their partners baseline networks and the overall group network, demonstrating partner mediation as well as similar group use of adaptive reasoning indicators. Students primarily utilized the two indicators, *relationships and connections*, and *justifications*, regardless of modality; varying use within the different modalities; with an equal amount of use of the two indicators within discussion and a more prominent use of *relationships and connections* in student drawings. Students utilized these two indicators in concert with one another, demonstrating a strong bond between these two indicator’s use by 7th grade students. Patterns in ENA network graphs show that use of the four lesser used indicators were utilized by students in connection with the primary indicators, demonstrating a strong connection between the primary indicators in relationship to the secondary indicators. Of the secondary indicators, students demonstrated *pattern recognition* in drawings while student used *legitimacy determined* more within discussion.

In addition, patterns of adaptive reasoning were identified within tasks and task stages. Students adapted their reasoning differently given different tasks. While consistently utilizing primary indicators throughout all three tasks, student use of the secondary indicators varied in different tasks. Differences in adaptive reasoning use between students of varying DOK working levels, showed similar results; all students utilized the primary indicators consistently, with varying patterns of secondary indicator use depending upon DOK working level. Students adapted their reasoning differently...
within the stages of the tasks. In stage 1, when students were presented with the problem and created a story drawing, students utilized the adaptive reasoning indicator prior knowledge at an increased rate and pattern recognition at a decrease rate compared to the group network. In stage 2, when students were gathering information needed to solve the problem, students adapted their reasoning the least, utilizing mostly justifications. Students adapted their reasoning most often in stage 3, with strong connections of adaptive reasoning use representative of the group network. During this stage students were actively working on solving the problem and creating a math drawing. In stage 4, a recognizable decrease in the primary indicators was seen as students focused more on the adaptive reasoning indicator alternates pursued when presented with a solution to the task.
CHAPTER 5

DISCUSSION AND CONCLUSION

This exploratory mixed methods study examined adaptive reasoning evidenced through drawings and discussions of 18 seventh graders paired in three guided inquiry-based mathematical tasks. The study examined what adaptive reasoning indicators 7th grade students’ evidence in their drawings and discussions and how these indicators were evidenced in inquiry-based mathematical tasks of depth of knowledge (DOK) levels 2 and 3. Descriptive statistics and epistemic network analysis (ENA) were used to elucidate: individual and partner student use of adaptive reasoning, adaptive reasoning use within the modalities of discussion and drawing, and adaptive reasoning use distinguished by student DOK working levels. Adaptive reasoning, an important precept of mathematical proficiency (Kilpatrick et al., 2001), is central to this study's conceptual framework which contends that adaptive reasoning is not yet fully understood in mathematics education but can be evidenced through student discussions and drawings in mathematical tasks.

A discussion and conclusions of the study are offered in terms of 1) a summary of the findings outlined by the research questions followed by sections that discuss 2) future research suggestions, 3) implications for researchers and educators, and 4) a conclusion.

Summary of Findings

Adaptive reasoning is the ability for a student to ‘think logically about the relationships between concepts and situations’ (Kilpatrick et al., 2001). The findings of
this study help to characterize the nature of adaptive reasoning, applicable to this particular set of students, by identifying what adaptive reasoning indicators students use and how they use those indicators when solving inquiry-based tasks. For example, the students in the study primarily used the two indicators, relationships and connections and justification, demonstrating a varied weighted distribution of indicator use when adapting their reasoning. These findings suggest that these students ‘think logically’ through the indicator justification; and they are thinking ‘about the relationships and concepts between situations’ when they are using the indicator relationships and connections. By drawing on these two primary indicators, students in this study may have found more opportunities in the inquiry-based tasks to reflect on the logic of their reasoning and the reasoning of others. Also, by making connections between mathematical ideas, students in this study were also given more opportunities to adapt their reasoning. In addition, the students utilized the secondary indicators: alternates pursued, legitimacy determined, prior knowledge, and pattern recognition to a lesser extent.

The summary of findings are presented per research question within the context of the literature. The first two questions summarize student adaptive reasoning within the two modalities by looking at of the primary and then secondary indicators. The third question discussing how students adapted their reasoning within tasks, task stages, per working DOK test level.

**Research Question 1: Adaptive Reasoning Used in Discussions**

In consideration of the first research question, *What adaptive reasoning indicators do 7th grade students evidence in their discussions and solutions as they progress*
Through inquiry-based mathematical tasks?, adaptive reasoning was identified in discussions when students engaged in exploratory talk; when they challenged each other, proposed new ideas, and described their reasoning. During the mathematical tasks, students engaged in a type of discussion called exploratory talk which afforded opportunities to adapt their reasoning (Barnes, 2008). Exploratory talk is a type of partner discussion where students engage critically and constructively with each other such as during in inquiry-based tasks (Barnes, 1976/1992). In exploratory talk, partners are active listeners; they challenge each other, argue, disagree, and extend each other’s reasonings as they seek a common solution (Mercer & Hodgkinson, 2008). While research shows that students use a variety of strategies when engaged in exploratory talk (Hennessy & Rojas-Drummond, 2016; T’Sas, 2018), this study focused on those strategies where students adapted their reasoning within the scope of inquiry-based tasks.

Two-thirds of the present study’s adaptive reasoning indicators occurred in student discussions and solutions. This higher use of adaptive reasoning within the modality of discussions may be explained by the fact that students were instructed to draw in two of the three stages in the tasks whereas discussion occurred in all four stages. Within discussions, students adapted their reasoning using the two primary indicators: justifications (36%) and relationships and connections (34.7%). The indicator justifications was seen in discussions when students argued, clarified their thoughts, presented a series of conjectures or explanations, and compared their solutions using multi-step explanations of a process or solution. The indicator relationships and connections was identified when students used if-then, when-then statements and used words such as “different,” “similar,” “together.” Different from justifications,
relationships and connections were generally seen as a single-step transaction; where students recognized a connection between occurrences. The primary indicators were utilized by students predominantly within the first three stages of the task, when students engaged in solving the task rather than when students compared solutions in the final fourth stage of the task. Students engaged in exploratory talk as they explained their thinking and proposed new ideas (Mercer & Hodgkinson, 2008).

Prior research helps to explain why students utilized the primary indicators. The strategy of justifying is important in the development of middle schooler’s learning of proof making (Francisco & Maher, 2005), creating generalizations and thinking algebraically (Bieda et al., 2006) using mathematical reasoning (Mata-Pereira & da Ponte, 2017) and in the building of mathematical knowledge (Stylianou, 2013). Engagement in exploratory talk, as when students are adapting their reasoning in inquiry-based tasks and co-construct understandings (Kruger, 1993), supports a student’s ability to justify and create proofs (Blanton & Stylianou, 2014). Additionally, relationships and connections are fundamental to aspects of student mathematical thinking (National Council of Teachers of Mathematics, 2000), adaptive reasoning (Kilpatrick et al., 2001), and a pivotal focus of instruction in 7th grade core standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Students utilized the secondary indicators less than the primary indicators, with a slightly higher use of alternates pursued (10.0%) and legitimacy determined (8.7%), than prior knowledge (5.6%) and pattern recognition (4.8%). While few studies in the literature address these indicators in relation to adaptive reasoning, studies on exploratory talk (Barnes, 1976/192; Hennessy & Rojas-Drummond, 2016; Wells & Ball, 2008;
Winne, 2018) generally concur with the present study’s findings on student use of these indicators. Students determined the legitimacy of their solutions using words such as “too big,” “just right,” and “it can’t be,” oftentimes pointing out an absurdity. Alternates pursued were recognized when students changed paths, usually after a disagreement or a challenge presented by their partner. Students proposed new ideas and described their change in thinking using words like “instead of” and “rather.” Prior knowledge was most often identified when students referred to a concept that was previously learned, primarily when students recited a formula, mathematical rule or use words like “remember when” and “before when.” Patterns recognition was identified when students repeated a phrase or described their thinking by expressing a notice of reoccurrence.

Students used words like “again” and “over and over.”

Students use of secondary indicators often occurred in connection with the primary indicators. For example, a common theme was seen when partners argued or challenged each other, processes often seen in exploratory talk (Mercer & Hodgkinson, 2008). Typically, one partner would begin an argument utilizing the secondary indicator legitimacy determined by identifying an incorrect process or solution and questioning its legitimacy. If the described argument was accompanied with a step-by-step description of the contention, the adaptive reasoning indicator justifications was identified. Generally following the dispute, partners would consider a new path of reasoning, demonstrating the adaptive reasoning indicator alternates pursued. Students oftentimes utilized justifications in conjunction with alternates pursued, prior knowledge and legitimacy determined. When students justified their reasoning, they were more likely to use prior knowledge, determine the legitimacy of their solutions and change their approach to the
task. This pattern was predominantly seen in student discussions rather than within student drawings.

**Research Question 2: Adaptive Reasoning use in Drawings**

Addressing research question two *What adaptive reasoning indicators do 7th grade students’ evidence throughout their drawings and solutions during guided inquiry-based mathematical tasks?*, adaptive reasoning was identified in student written gestures, erasures, scribble-outs, and connecting arrows in their drawings. Learner-generated drawings, an effective learning strategy (Csikos et al., 2012; Leopold & Leutner, 2012; Schmidgall et al., 2018), are drawings created by learners to convey a story or process (Van Meter et al., 2006). Students created situational drawings in stage 1 of the task and mathematical drawings stage 3 of the task. Situational drawings are drawings that described the task situation and mathematical drawings described mathematical aspects of the task (Rellensmann et al., 2017). The literature shows that as students create drawings, they adapt their reasoning and evolve their mathematical thinking through the cognitive processes involved in learner-generated drawings (Van Meter & Firetto, 2013).

Adaptive reasoning was identified less within drawings than discussions; one-third of the adaptive reasoning indicators were identified in drawings. Additionally, students utilized adaptive reasoning indicators in different percentages per modality. *Relationships and connections* and *justifications* represented the primary indicators as seen with discussions; however, *relationships and connections* (48.9%) were seen considerably more than *justifications* (26.6%) in drawings. According to multi-representational learning literature, these differences per modalities were to be expected
The use of different modalities affords students different representational views or lenses in which to problem solve (Tripathi, 2008). Developmental stages of 7th grade students and their drawing capabilities may also come into play (Lowenfeld, 1947/1987; Piaget, 1964). Research shows that 7th graders are leaving the concrete operational stage and entering the formal operational stage of cognitive development. This age students are beginning to think more abstractly and reason about hypothetical problems (Piaget, 1964). Additionally, these students are at the age of reason or the pseudo-naturalistic stage of their artistic abilities; where students pay close attention to the relationships between drawings and realism, focusing on the relationships between objects and proportionality (Lowenfeld, 1947/1987).

Common themes were seen when students adapted their reasoning using the primary indicators. Student represented relationships and connections in drawing with directional arrows, connector lines or circled paired elements in student drawings. Justifications were evidenced when students paired drawing elements into steps or added to the drawings in a step-like manner. Consistent with other studies (James et al., 2016), justifications steps were usually expressed using arrows, connector lines and ordered placement of objects within the drawings.

With the secondary indicators, a higher percentage use of pattern recognition (9.2%), a similar use of prior knowledge (5.0%), and reduced use of alternates pursued (7.4%) and legitimacy determined (2.8%) was noticed compared to discussions. The researcher identified alternates pursued when students scribbled out or erased substantial elements of their drawings, and then redrew or added on substantial elements. This was similar to a study involving children’s drawings of representational change (Karmiloff-
Smith, 1990). Prior knowledge was recognized when students wrote down a formula or built upon the drawing with a formerly known concept or relationship. Legitimacy determined was recognized when students scribbled out a solution or element. Hand gestures and sighing oftentimes accompanied drawn elements signifying legitimacy determined. Pattern recognition was represented with repeated lines, shading, symbols, gestures or images. A study (Yoon et al., 2021), of students drawing in science class, found similar patterns, showing connections with repeated images of increasing object size.

As with discussions, interactions between indicator use was seen within drawings. For example, a common theme occurred when students drew patterns and then made a connection between the pattern and another aspect of their drawing. Students commonly utilized the two indicators relationships and connections and pattern recognition in conjunction; suggesting a connection between these two indicators.

This process of using representations as an adaptable tool in problems solving, affords students situations in which to use varying mathematical strategies to co-create meaning (Stylianou, 2011; Stylianou, 2020).

**Research Question 3: How Student Adaptive Reasoning Relates to Tasks**

Research shows that reasoning mechanisms are rarely distinct and separate acts (Ellis et al., 2007). In consideration of research question three, how adaptive reasoning indicators relate to 7th grade students’ engagement with level 2 and 3 depth of knowledge tasks, ENA network and differences graphs were used to take a closer look at the interactions between adaptive reasoning use temporally, within tasks and task stages.
Inquiry-based mathematical tasks are teacher chosen, student lead, engaging tasks where students work together to solve a story-based mathematics problem. This study utilized Three-Act Math tasks, a guided inquiry-based task framework where students solve story-based problems. In these types of tasks, students discover and construct their knowledge by engaging in exploratory talk and thinking from multiple perspectives (Stein et al., 1996). Inquiry-based tasks (Brodie, 2010; Stein et al., 2009), exploratory talk (Barnes, 2008), and learner-generated drawings (Van Meter & Firetto, 2013) prompt student-centered learning and adaptive reasoning use (Samuelsson, 2010; Stylianou, 2013).

Prominent findings of the present study show that: 1) students primarily adapted their reasoning utilizing the primary indicators, relationships and connections and justifications, 2) the primary indicators were utilized in conjunction with one another, and 3) secondary indicators were used in distinct connection with the primary indicators and rarely seen in connection with one another. 4) Additionally, students adapted their reasoning uniquely, with similarities seen between partners’ ENA networks, indicating that partner interaction mediated adaptive reasoning, as seen in studies on exploratory talk, where students construct meaning from one another’s thought processes (Cervetti et al., 2014).

This study found that the students used different adaptive reasoning indicators given different tasks. This further clarified how different adaptive reasoning strategies were used by the students within tasks and task stages. For example, in the Candlelight Dinner task an increased use of alternates pursuits and pattern recognition was seen in connection with the primary indicators. In comparison, in the Ghost Island task, an
increased use of prior knowledge and legitimacy determined was seen in conjunction with the primary indicators. While the present study identifies the occurrence of these differences per task, the reasons why students adapted their reasoning differently was not explored within the scope of this study. Initial indications suggest that when a task corresponded with a student’s ability level, the student utilized more adaptive reasoning indicators.

Additional evidence of how students adapted their reasoning differently within the tasks was seen through the observation of ENA differences graphs between the modalities of discussion and drawing. An increased use of pattern recognition in the Candlelight Dinner was seen in conjunction with the two primary indicators in drawings, however not within discussions. In the Ghost Island tasks the connections between pattern recognition were primarily seen in conjunction with relationships and connections and not within justifications. This split demonstrates how the primary indicators differed between modalities and tasks. Additionally, students primarily adapted their reasoning using pattern recognition in drawings in conjunction with relationships and connections; in discussions students were more likely to use pattern recognition with justifications. However, within the two tasks, uses varied as seen in the overall higher use of pattern recognition in the Candlelight Dinner task.

The three tasks in the study, designed using the guided inquiry-task framework (Kuhlthau et al., 2012; MidSchool Math, n.d.), consisted of immersion, exploratory and resolution stages. The literature explains that active learning (Bonwell & Eison, 1991) in tasks invoke reasoning (Stein et al., 1996), however little is said about how students adapt their reasoning, especially in the different stages of the task. In the present study, findings
clarified how students adapt their reasoning within the stages. In the immersion and exploratory stages (see Figure 4.10), in which students were asked to create drawings, students adapted their reasoning in a similar pattern to the overall network use (see Figure 4.5). When students were asked to list needed information to solve the task, students adapted their reasoning differently, using the primary indicators in conjunction with prior knowledge and alternates pursued. Students considered different ways to solve the problem, relying more on prior knowledge, during this stage. In the resolutions stage, a shift in the pattern of adaptive reasoning also occurred. Fewer connections from the secondary indicators to the primary indicators are witnessed with a change in emphasis to all the indicators connecting to alternates pursued, except for pattern recognition. In this stage, as students compared solutions, they focused on alternate solutions or processes.

This study contributes information on how the students, working at different DOK levels adapted their reasoning. Overall task results show that the students of increasingly higher DOK working levels demonstrated an increased use of prior knowledge and a decreased use of alternates pursued. The students working at lower DOK levels used more guess-and-check strategies, oftentimes, recognizing a pattern, testing a strategy, and justifying the strategy without determining the legitimacy of their processes or solutions. The students working at DOK3 also pursued different alternatives, however to a lesser extent than the student working at DOK1. The students working at DOK3 utilized alternates pursued in conjunction with prior knowledge and legitimacy determined, with stronger connections between alternates pursued and the primary indicators. The differences in adaptive reasoning use per DOK working level, as seen in the quantitative and qualitative results, may demonstrate the level of abstraction required for these
students to move from one working level to the next or from DOK1 to DOK3. Students begin by using guess and check, pattern recognition reasoning strategies and then move to determining the legitimacy and the use of prior knowledge.

Within the individual tasks, a similar pattern occurred when students of varying DOK working levels adapted their reasoning (see Figure 4.14). The student working at DOK1, in the Candlelight Dinner task, exhibited a broader variety of adaptive strategies, whereas students working at DOK3 solved this task utilizing only the primary indicators. The Ghost Island task however was more challenging for students and students working at DOK3 exhibited a greater variety of adaptive reasoning indicators, specifically an increased use of prior knowledge and determination of legitimacy. The student working at DOK1, challenged beyond her skill level, used only the primary indicators in the Ghost Island task, and failed to solve the task. While it is unclear that this pattern is specifically related to task challenge, other studies have found that difficulty levels can influence student reasoning (Litster, 2019; Stein et al., 2009) and it is interesting to note specific indicator uses within different tasks through the lens of adaptive reasoning.

Within the task stages, the student working at DOK1 demonstrated a dominant use of *alternates pursued* in all stages, whereas students working at DOK3 showed more varied use of adaptive reasoning between stages (see Figure 4.15). In the first two stages, students working at DOK3 used mostly the primary indicators. In stage 3, they utilized all indicators and in stage 4 they used primarily *alternates pursued* and *legitimacy determined*. These results show how the student working at DOK1 used guess and check strategies, whereas the students working at DOK3 evidenced a more complex use of adaptive strategies.
Further, students adapted their reasoning uniquely, but within discernable patterns when compared to their partners, classmates, and within the tasks, meaning student adaptive reasoning changed due to the circumstances and the learning environment. This implies that teachers could utilize adaptive reasoning as a baseline assessment instrument to detect student growth in adaptive reasoning, personalize learning towards adaptive reasoning and design tasks to suit instructional needs of individual students and the class. For example, Kevin adapted his reasoning utilizing drawing half of the time rather than the 2 to 1 discussion to drawing ratio demonstrated by the rest of the students. Kevin’s teacher could pair Kevin with a student that struggled with adapting her reasoning when drawing, but excelled in verbalizing her adaptive reasoning, to create an environment in which both students might experience growth.

**Future Research**

Given the importance of understanding mathematical reasoning within tasks (Brodie, 2010), the following research is recommended. The first recommendation is the repetition of the study with students interacting face-to-face. Exploratory talk is grounded in student interaction. However, due to COVID-19 restrictions, this study was conducted with partner interaction occurring through computer discussion threads. Because of this, student interaction may have taken on a more explanatory nature, particularly when students created videos and justified the nature of their drawings through discussion. Repetition of the study with students working face-to-face would help to identify how students adapt their reasoning under different communication situations.
Secondly, a repetition of the study in different grade bands or within other classroom activities is suggested. This study provides robust information on how 7th grade students adapted their reasoning; however, it does not capture the scope of adaptive reasoning within different grade bands. Repetition of the study in different grade bands would paint a more comprehensive picture of how students adapt their reasoning as they progress in their mathematical thinking in terms of adaptive reasoning. In addition, repetition of the study with students engaging in other inquiry-type activities in the classroom would also expand understanding of how students adapt their reasoning in different contexts.

The third recommendation is to conduct a similar study that investigates adaptive reasoning in terms of student engagement or challenge level. The present study clearly identified differences in student adaptive reasoning used per the different mathematical tasks. However, it is unclear why these differences occurred. Alan Schoenfeld (2019) stated that observing adaptive reasoning is easier when the task is challenging. In this study, the student working at DOK1 adapted her reasoning using more indicators in the Candlelight Dinner tasks and less indicators in the Ghost Island task. The reverse was true of the students working at DOK3. It may be that students utilize different adaptive reasoning strategies based on the engagement or the level of task challenge. Additional exploration is needed to better understand this dynamic and determine why certain indicators were used by students of working DOK levels in different tasks.

Finally, the use of ENA is recommended in further studies involving student interaction within the mathematics classroom. In the present study, ENA enabled the construction of visual analysis networks which showed comprehensive profiles of
student, partner and overall group adaptive reasoning within different contexts. A relatively new analysis, ENA served as a valuable tool to better understand student complex thinking in discussions and drawings. The use of ENA could be expanded to identify relationships between other types of mathematical reasoning (Kollosche, 2021) or assess elements of mathematical proficiency.

Use of ENA or similar analysis methods are the future of classroom real-time assessments. Recent research, focused on how to assess students engaged in problem solving is becoming more prominent. An example of this is Whole Class Think Alouds, where student partners collaborate to problem solve while the teacher walks around, collecting assessment data on how students problem solve (Hicks & Bostic, 2021). Presently it is easier to assess student calculation skills than problem solving skills (Nortvedt & Buchholtz, 2018), however this thought is being challenged by innovative research such as Problem Solving Measures (PSM) which investigates new ways to measure problem solving performances (Bostic et al., 2022). Ramifications from PSM research may inform student assessment tool development, which can be used during task discussions and while student draw when problem solving. The intersection of these studies would have the potential to improve student performance in problem solving.

For instance, studies that take on these intersections might include real-time data collection of partner dialogue. By collecting data in real-time educators and scholars could use speech-to-text technology, which could be analyzed instantaneously using user selected codes representing elements of student performance and generate student reports similar to the ENA network and differences graphs used in this study. Implications for such advancements could include instantaneous assessment of classroom performance as
the primary method of grading, eliminating the need for standardized assessments, and reflecting student growth through performance-based standards of mathematical proficiency. Time previously spent on testing could be applied to inquiry-based learning methods, shifting the classroom learning environment from traditional to more inquiry-based instruction.

**Implications for Educators and Researchers**

This study provides educators insight on how students might adapt their reasoning, an integral component of mathematical proficiency (Kilpatrick et al., 2001), in inquiry-based math tasks. Understanding students’ thinking is a key factor in a mathematics teachers’ ability to conduct group discussions and improve teaching in tasks (Stein et al., 2008). Long-term implications, that include technological advances in monitoring and assessing mentioned in the previous section, could include a better understanding of patterns of reasoning and the ability to instruct students in how they adapt their reasoning. While the small sample size of this study prevents the results from being generalizable; leveraging new information from this study enhances an educator’s ability to provide strategic instruction to increase student learning. In such, these findings are preliminary and should be followed up with more research to confirm findings.

**Educators**

Findings indicate that students adapted their reasoning in recognizable patterns in mathematical tasks. With an understanding of these patterns, educators can better design learning experiences based on the personal needs of seventh graders. Personalized
learning is a beneficial learning strategy in which teachers customize lessons to fit students’ strengths, needs and skills and interests (Pane et al., 2017). Based on the information gained in this study, a seventh grade teacher might choose task instruction that emphasizes the development of adaptive reasoning skills. Knowing that students working at DOK3 utilized prior knowledge and legitimacy determined to successfully navigate through a task and that the student working at DOK1 failed to use these strategies, a teacher might personalize DOK working students’ learning experiences. The teacher might include a ‘too-high, too-low’ guessing strategy (Meyer, 2010) to increase student legitimacy determined skills. She might have students create and share a K-W-L chart: What do I know?, What do I wish to know?, What have I learned? (Miller & Veatch, 2011; Ningsih & Retnowati, 2020), to develop skills in accessing prior knowledge. Since this study showed that these two strategies were more readily accessed through discussion, the task design might include partner interaction in discussion. Another strategy might include purposeful pairing.

Purposeful partner pairing is recommended as a teacher tool to personalize learning and to strategically expose a student to new adaptive reasoning strategies. The results of this study showed that partners mediated student use of adaptive reasoning. This is salient during inquiry-based math tasks in which learning is student-centered and peer interaction is important (Liljedahl, 2016; Stein et al., 2009). This was especially evidenced during these COVID-19 teaching situations in which teacher interaction was limited and oftentimes replaced by peer leads. Purposefully pairing students of different DOK levels would expose learners to different strategies of adaptive reasoning.
A main contribution of this study was the clarification of how these students adapt their reasoning differently within tasks and tasks stages. Inquiry-based tasks are designed to ensure active student thinking by making tasks accessible to all students and challenging students to work to their highest potential (Stein et al., 2009), a ‘low-floor, high-ceiling’ type of activity. Teachers should consider adaptive reasoning indicator use when choosing or designing inquiry-based tasks. For example, a teacher might strategically embed modalities of discussion or drawing into task lesson plans to facilitate student use of certain adaptive reasoning indicators. In light of how students adapted their reasoning differently between tasks in this study, a teacher might consider how DOK working levels might relate to the task difficulty level, in the creation of task and the adaptive reasoning they would like to leverage.

**Researchers**

This study contributes to research by identifying the adaptive reasoning indicators within the different representation modalities of drawing and discussion. It is unclear how students engage with different representations when problem solving (Stylianou, 2020); however this study identifies adaptive reasoning used within student drawing and discussions by analyzing these students adaptive reasoning temporally within inquiry-based tasks, tasks stages, and per student DOK working level. Whin these modalities, the study reports on student thinking within the scope of pre-selected and emergent codes of adaptive reasoning. And while new codes were not found, new ways in which these codes were evidenced were identified. Identified patterns of adaptive reasoning used within the group, by partners, and individuals contributes to current research on student thinking.
within the different representations and in inquiry-based tasks in terms of how students learn.

This study adds to the literature on middle schoolers’ thinking strategies, specifically on how 7th graders use justification when problem solving. In elementary grades, students are beginning to use justification within the context of relationships and connections, typically seen when students are asked to ‘compare and contrast’ (Widjaja et al., 2021). The development of justification is also seen as an essential skill for students learning to generalize and engage in functional reasoning in 8th grade (Vale et al., 2017). The research on the progression of student justification skills use through the middle school years is incomplete. The present study, helps to fill this gap by identifying how the presented students justified in seventh grade. In addition, this study examines other strategies of adaptive reasoning in this grade band.

ENA network models and differences graphs were used in this study as an analysis tool to create a dynamic view of complex student thinking. Methodologically, the use of ENA lent the ability to observe chosen aspects of student dialogue and drawings at different student DOK working levels to better understand temporal connections of student adaptive reasoning. Liljedahl, (2016) in the book *Posing and Solving Mathematical Problems* stated that “It is not possible to measure how much a student is thinking during any activity, or how that thinking is individual or predicated on and with the other members of his or her group” (p. 398). In his studies, he tries to capture student thinking during different aspects of problem solving.

The process of using qualitative observation quantitized into ENA models, as in this study, redefines what is possible in terms of measurement of student thinking. This
study also adds to the body of literature on ENA through the unique exploration of student reasoning within learner-generated drawings. Coded interpretation of drawings through written transcriptions is a new and relatively uncharted method in ENA.

**Conclusion**

Adaptive reasoning, or the ability for a student to think logically to solve a problem strategically, is one of five strands of mathematical proficiency as outlined by the Mathematics Learning Study Committee (Kilpatrick et al., 2001). Adaptive reasoning has been researched in terms of student performance and mathematical proficiency in relation to different curricula (Awofala, 2017; Mahendra et al., 2017; Rizki et al., 2018). This study progresses research and informs educators on how these students adapted their reasoning within inquiry-based Three Act Math tasks, a relatively new and popular method to teach mathematical concepts. While there are books describing how teachers create and administer inquiry-based tasks (Liljedahl, 2020; Stein et al., 2009), there is little information on how students actively adapt their reasoning within a task. Reasoning behavior found in this study’s findings helps researchers and educators begin to understand how adaptive reasoning relates to different types of mathematical proficiencies and provides insight into how these students adapt their reasoning in hopes of creating better learning opportunities for others engaged in mathematical tasks and to increase student mathematical proficiency.
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APPENDICES
Appendix A. Parent Consent and Student Assent
Figure A.1

Parent Consent

Relationships Between Middle School Students’ Adaptive Reasoning when Creating Learner-Generated Drawings and Partner Talk During Inquiry-Based Mathematical Tasks

Introduction
Your child is invited to participate in a research study conducted by Angela Frabasilo, a doctoral student in School of Teacher Education and Leadership department (TEAL) at Utah State University. This study is approved by the Washington County School District. The purpose of this study is to use engaging methods of instruction to examine and measure changes in students’ adaptive reasoning. Your child’s participation is entirely voluntary. Research studies such as this help us learn more about how students learn so that instruction methods can be improved. By participating, your family will be assisting this research endeavor.

This form includes detailed information on the research to help you decide whether to participate. Please read it carefully and ask any questions you have before you agree to your child’s participation.

 Procedures
Your child’s participation will involve taking part in three scheduled learning tasks in their mathematics classroom. In the tasks, paired students will be presented with a real-life mathematical problems that they are to solve. All students in the class will be participating in the same learning tasks that will take place over three different days in October through December. Your child will be engaging in the same curriculum, lessons and activities as the rest of his/her classmates. Each task will take approximately 30 minutes with students working in pairs. The task will be presented on a Schoology discussion thread. If in the event that your child’s classroom is receiving remote instruction, then the tasks will take place during their remote (online) class. Your child and their partner will share drawings and communicate through the computer through this discussion thread. All students in the class will be participating in the lesson in this same manner so your child will not miss out on any class instruction or time. The three tasks will be graded as classroom assignments for all students in the class. The difference between study participants with parental consent is that their work will be collected for research purposes.

If you agree to your child’s participation, the researchers will collect video recordings of their hands and drawings as submitted by your child. The recording will including audio recordings of the dialogue that occurs during the task.

In addition, your child will take a pre and posttests that last approximately 15 minutes. The pre and posttest will be taken by your child’s entire class. All of the students tests will be graded but not entered as part of their grades, unless specified by the student. We anticipate that 10 to 18 students will participate in this research study at Sunrise Ridge Intermediate School.

In the event that school conditions restrict student interaction during COVID-19 social distancing regulations will be maintained. During in-class distancing limitations or in a remote learning situation students will take the pre and posttest on their learning management system (Schoology). Tasks will be conducted in Schoology on an online discussion thread utilizing Schoology’s video

(Figure continues)
Figure A.1

Parent Consent

recording and texting capabilities. All interactions between students would be conducted through a computer. If instruction is remote, students will be provided necessary equipment such as a Chromebook, a document camera, and Wi-Fi connect if it is not available in your home.

Risks
This is a minimal risk research study. That means that the risks of participating are no more likely or serious than those your child encounters in everyday activities.

Benefits
Although you or your child will not directly benefit from this study, it has been designed so that your child may learn more about their thoughts on the math that they participate in during class by creating drawings.

Confidentiality
The researchers will make every effort to ensure that the information provided as part of this study remains confidential. Your child’s identity will not be revealed in any publications, presentations, or reports resulting from this research study. While we will ask all participants to keep the information they hear in this study confidential, we cannot guarantee that everyone will do so.

We will collect student information such as their names through the video of student’s drawings, including audio recordings of student partners discussing their thoughts as they draw and work through math tasks. This data will be stored online. Online activities always carry a risk of a data breach, but we will use systems and processes that minimize breach opportunities. Videos and student dialogue will be securely recorded and stored in a restricted-access folder on Box.com, an encrypted, cloud-based storage system through Utah State University. Recordings and this form will be kept for three years after the study is complete, and then they will be destroyed. In addition, student pre, posttest and drawings will also be kept in a secure location, locked within the researcher’s office. Pre, posttest and drawings data identifying information will be removed and replaced with pseudonyms within 48 hours by the student researcher. The data will then be digitally scanned and uploaded into the secure box.com mentioned above. Once uploaded, pre, posttest and drawings will be returned to your child’s teacher.

It is unlikely, but possible, that Utah State University or state or federal officials may require us to share the information from the study to ensure that the research was conducted safely and appropriately. We will only share your child’s information if law or policy requires us to do so. If the researchers learn about suspected abuse or neglect of a vulnerable individual, state law requires that the researchers report this suspicion to the authorities. If the researchers learn about suspected abuse/neglect of a vulnerable individual, state law requires that the researchers report this suspicion to the authorities.

Voluntary Participation & Withdrawal
Your child’s participation in this research is completely voluntary. If you agree to participate now and change your mind later, you may withdraw your child at any time by contacting the principal investigator. If you choose to withdraw your child after we have already collected information, the data collected will be destroyed. Your child’s grade or class participation will not be affected in any

(Figure continues)
Figure A.1

Parent Consent

way. The researchers may choose to end your child’s participation in this research study if he or she does not choose to draw and discuss their drawings with a partner or are absent regularly.

IRB Review
The Institutional Review Board (IRB) for the protection of human research participants at Utah State University has reviewed and approved this study. If you have questions about the research study itself, please contact the Principal Investigator at 435-797-1097 or beth.macdonald@usu.edu. If you have questions about your rights or would simply like to speak with someone other than the research team about questions or concerns, please contact the IRB Director at (435) 797-0567 or irb@usu.edu.

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(435) 773-1311; angie.frabasilio@washk12.org

Informed Consent
By signing below, you agree to participate in this study. You indicate that you understand the risks and benefits of participation, and that you know what you will be asked to do. You also agree that you have asked any questions you might have, and are clear on how to stop your participation in the study if you choose to do so. Please be sure to retain a copy of this form for your records.

Participant’s Signature  Participant’s Name Printed  Date
Figure A.2

Student Assent

We are doing a research study about student reasoning in math tasks. Research studies help us learn more about people. If you would like to be a part of this research study, you and a partner will solve math tasks with the rest of your class through a discussion thread on Schoolology. The discussion thread will be shared with us so that we can better understand how you are solving the task. You will also take part in two short, 15 minute, math assessments.

When the researchers do things like take videos, some other things could happen. For example, video technology might fail and we might have to repeat the task in a later session. We will do everything we can to prevent those things from happening, but there is still a chance, so we want you to know that first.

Not everyone who is a part of research studies receives a something good from it. In this study, nothing directly good will happen to you, but you will help us learn more about people like you. Also, we will tell other people about what we learned from doing this study with you and the five to seven other people who are in the study, but we won’t tell anyone your name or that you were in the study.

If this sounds like something you would like to do, we will ask you to say that you understand what we talked about, and that you do want to participate. You do not have to be in this study if you do not want to be. If you decide to stop after we begin videotaping the task that’s okay, too. No one will be upset if you don’t want to do this, or change your mind later.

You can ask any questions you have, now or later. Your parents know about this research study, and they have said you can participate, if you want.

If you would like to be in this study, please sign your name and write the date.

_________________________  _______________________
Name                        Date
Appendix B. Researcher Field Notes Protocol
Researcher will observe students’ discussions threads following each Three-Act Math Tasks, protocol adapted from Shekhar et al. (2015). Data collection includes three notes per task for a total of 9 fields notes.

Task Name: _____________________________________________________________

Date of recording: ___________________________ Class Period: _________________

Class size (# students participating): __________

1. Field notes: (for each partnered pair).

<table>
<thead>
<tr>
<th>Time Begin:</th>
<th>Time End:</th>
<th>Task Stage(s):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partner a</td>
<td></td>
<td>Partner b</td>
</tr>
</tbody>
</table>

Partner interaction: gestures and directing conversation.

Drawings: gestures, side doodles, erasures

2. Instructor introduction of and response during active learning (circle)

<table>
<thead>
<tr>
<th>Follows lesson protocol</th>
<th>Explains expectations clearly</th>
<th>Answers questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gives verbal feedback</td>
<td>Solicits student responses</td>
<td>Encourages students</td>
</tr>
</tbody>
</table>

Uses strategies to reduce student resistance to task

3. Classroom response during active learning

<table>
<thead>
<tr>
<th>Classroom Engagement: Above 80%</th>
<th>Between 80-50%</th>
<th>Between 20-50%</th>
<th>Below 20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-task behavior: Above 80%</td>
<td>Between 80-50%</td>
<td>Between 20-50%</td>
<td>Below 20%</td>
</tr>
</tbody>
</table>

Additional comments:
Teacher instructs students to open Schoology, take out a blank piece of paper and invites students to begin working through the discussion thread presented in Schoology.

**Figure C.1**

*Pre-Implementation Lesson Plan*

---

**Creating drawings in math tasks**

*Story drawings and Math drawings*

- Have students seated with partners, their paper and pencils.
- Teacher: We are going to create drawings as we complete some tasks in the classroom. Today I’ll be showing you a way to create drawings to solve math tasks. We will begin with a task about pirates.

**Talking norms: Respecting your partner**

1. Talk together to think about what to do
2. Share what we know with each other
3. Ask everyone to say what they think
4. Listen carefully to each other and consider what we hear
5. Give reasons for what we say
6. Pay attention and try to think of good ideas
7. Make decisions together
8. Reach an agreement

---

**Ghost Island**

Aboard the ship Isosceles, Captain Mary Read and sailing master Bonny Anne are headed to find the rumored treasure of Ghost Island. They have departed from Isla Vista, setting a course of 32 leagues North and 48 leagues East in hopes of finding the hidden treasure.

*Is the ship on course to hit the island?*

**Story Drawing**

**Math Drawing**

Teacher: These are two drawings. Two drawings made about the Ghost Island task. The one on the left helps to tell the story of what is happening. See how the ship is heading towards Ghost Island. The drawing on the right is different. It shows the numbers important to solving the problem. This is what we call a math drawing. It shows the math used to solve the problem.

---

For your next task, you will be asked to first make a story drawing and then add numbers to turn it into a math drawing. See the story drawing. Can you tell the story? Click on the drawing. Now can you see how the artist added the math story?

(Figure continues)
Figure C.1

Pre-Implementation Lesson Plan

Let's give it a try, using Draw-Pair-Share

Draw-Pair-Share is similar to
Think-Pair-Share except you will be drawing
when you think.

Teacher: Let's give this a try. We will be using Draw-Pair-Share as we draw.
Draw-Pair-Share is similar to Think-Pair-Share where you first draw without talking to
your partner. Then when I say Pair, you and your partner talk to each other and
discuss your drawings. Explain to your partner how your drawing represents the story
of the problem. Next I will call on one person to share their thoughts on what should
be in a story drawing.

You want to know the height of a
tree so you can buy a ladder to put a
birdhouse on top. The tree is too tall
to measure. The tree casts a
shadow. At the same time your body
casts a shadow. How tall is the tree?

Create a Story Drawing

Read the passage.
Teacher: On your paper, create a story drawing. Remember that the drawing does
not have to be fancy, it just needs to tell the story.
(Give students time to talk)
Teacher: Now Pair with your partner. Look at each other’s drawings and discuss what
you’ve drawn. Make sure you both talk about your drawings.
(Give students time to talk)
Teacher: Who would like to share their thoughts about what they drew? Or
__________ (student’s name) Please share what your partner put in his/her drawing.
Choose one person to share.

You want to know the height of a tree.
The tree casts a 38-foot shadow. At the same time you, who are 63 inches tall,
cast a shadow measuring 42 inches.
How tall is the tree?

Add to your drawing - make a Math Drawing

Now look at the additional information and add to your drawing to make a Math
Drawing. You may erase parts of your first drawing if you wish. You may work with
your partner, but remember to create your own math drawing that shows the numbers
involved. Show your work and answer on the drawing also.
Appendix D. Implementation Detailed Lesson Plan and Task Example
Candlelight dinner task and video (MidSchool Math, n.d.)

Figure D.1

Candlelight Dinner Lesson Plan

Ratios & Proportional Relationships

7.R.P.A.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

DETAILED LESSON PLAN: CANDLELIGHT DINNER

Will Attorney Matherstone make it home before the candle burns out?

Being able to recognize that a complex fraction is simply one fraction divided by another may help simplify this concept for students. During Candlelight Dinner, Attorney Matherstone tells his wife, Grace, that he will again be late, finishing briefs at the office. Having planned dinner for their anniversary, she is discouraged. She lights a new candle and informs him that if he makes it home before it burns out, they can eat a nice dinner together. If not, he will be sleeping outside with their dog, Roger. The data provided are a picture of the number of briefs he needs to finish, as well as a text message exchange, displaying how much work he has finished and how much of the candle has burned.

Lesson Plan Overview

Lesson Length: 4 Days

Prerequisite Standards

- 6.R.P.A.2

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.

  On Day 1 during the Immersion and Data & Computation phases, students will intuitively make sense of this situation and will approach the solution in their own way. They will then see how a complex fraction can lead them to the same solution.

Vocabulary

- Complex fraction: A fraction where the numerator, denominator, or both, contain a fraction.
- Ratio: Comparison of two quantities (a and b). Can be written with a colon (a:b), in fraction form (a/b), and in words (a to b).
- Unit rate: An item per one unit of another item; a rate with a denominator of one.

Cluster Connection

Cluster Heading: Analyze proportional relationships and use them to solve real-world and mathematical problems.

- Direct Connection: In Candlelight Dinner, students will explore the relationship between the number of briefs read by Attorney Matherstone and the amount of candle burned, resulting in a complex fraction and ultimately a unit rate.
- Cross-Cluster Connection: This activity connects 7.R.P.A to 6.R.P.A as students are extending their knowledge of ratios and rates to include complex fractions.

Common Misconceptions

- Students may not divide fractions correctly.
- Students may not realize there is more than one way to set up and interpret a ratio or rate.

English Language Learners: Provide a word wall with the words Ratio, Rate, and Unit Rate (from Grade 6) displayed and defined for students to reference.
Figure D.2

Candlelight Dinner Task

1. Immersion
   Allow 12 minutes

Materials
- Candlelight Dinner immersion video
- Blank Paper and Pencils

Procedure
1. Play the Immersion video to the whole class.

Immersion Video

Husband from work calls wife. Phone rings.

Wife answers call.
Wife: Happy anniversary, sweet heart.
Husband (in a sad voice): Hi, honey.
Wife (angry): Oh James. Don’t tell me.
Husband: I know honey, but Stevens kept me in meetings all afternoon and you know I’ve got a huge day in court tomorrow, four cases. I just have to review all of the briefs and then I will be right home.
Wife: Well I can’t say I’m exactly shocked.
Husband: You know the only place I want to be right now is home with you.
Wife: You work so hard, honey. And I know how important tomorrow is to you but tonight is special. I have an idea that might give you a little motivation.
Husband: Oh?
Wife: I have a brand new candle that I’m going to light; and if it’s still burning by the time you get home, we’ll have a lovely anniversary dinner together, but if it goes out…

(Figure continues)
Figure D.2

Candlelight Dinner Task

Husband: Yes?  Wife: The door will be locked and you can sleep outside with Roger.

Wife hangs up and lights the candle. Husband begins reading briefs.

Video ends.

2. Restate the question and keep it visible: *Will attorney Matherstone make it home before the candle burns out?*
4. Ask students: “What is the story line and what do we need to know?”

**Draw-Pair-Share Protocol**
Discussion thread prompts students to think individually about what they know and think about what they need to know and make a drawing (2-3 min). Students make a drawing. Students pair with their partner and discuss their drawings (3 min) through the discussion thread. Finally, the teacher may show student work to the whole class by sharing their partner strategies and drawings on Schoology. (3 min).

2  Data & Computation

Allow 10 minutes

**Materials**
- Copies of *Candlelight Dinner Data Artifact*, one per student

**Procedure**
1. Distribute the *Data Artifact* to each student.

*Data Artifact*

(Figure continues)
2. Invite students to work with their partner and add to their drawings to create a math drawing.
3. Observe students at work (avoid confirmation of the solution). Look for opportunities to clarify vocabulary, identify student strategies or work samples to be shared, and ask students questions to further their thinking.

Questions may vary depending on the method of simulation chosen by students.

- What information are we given?
- How many briefs does Attorney Matherstone have to read?
- According to the text, how many briefs has Attorney Matherstone read?
- According to the text, how much of the candle has burned?
- Assuming they both continue to happen at the same rate, can you represent this relationship visually?
- What other information about the briefs and the candle burning can you get from the text message?

4. After determining students are nearing ‘sufficient’ progress, either consider using an additional teaching protocol, or ask students to provide a thumbs up/middle/down to indicate readiness to see the Resolution video. You may grant students additional work time, if necessary.

3 Resolution

Allow 7 minutes

Materials
- Candlelight Dinner Resolution Video

Procedure
1. Play Resolution video to the whole class, and have the students mentally compare their solutions as they watch.

(Figure continues)
Figure D.2

Candlelight Dinner Task

Video begins with a shot of the Matheson’s house in time-lapse photography of the afternoon passing to evening. The husband enters the house quietly.

Wife: You made it!
Wife: Now I am impressed. How? You had four briefs to read, those things are epic.

Husband: Well when I got your text, I realized that in the time it took me to read half a brief, one-eighth of the candle had burned. So I did a little math. I wanted to figure out how many briefs I could read in the time it took for the entire candle to burn. I realized that I was looking for the unit rate. Which I was able to find by setting up a ratio and dividing.

Husband: One-half divided by one-eighth is the same as one-half multiplied by eight over one.

Husband: I got eight over one, which reduced to four over one. Which meant that in the time it took for the entire candle to burn, I could read all four briefs.

(Figure continues)
Figure D.2

Candlelight Dinner Task

**Wife:** So you had enough time to read all four briefs. But you still needed more time to make it home.

**Husband:** Exactly. So I decided to read the first three at the office and read the last one on my walk home. And I even picked a few flowers along the way.

**Wife:** Oh, honey! Thank you. This is the best anniversary gift ever, us. Husband and wife blow out the candle. Video ends.

2. After the video, prompt students with the following questions:
   - What did you do that was the same?
   - What was different?
   - What strategy do you think was more efficient? Why?

It is not always necessary for students to respond. The questions can simply be used to cue thinking prior to instruction.

3. Invite students to work with their partner to add to and complete their math drawings.

4. Instruct students to write their names on the back of their drawings. Students turn in drawings, teacher passes out posttest. Allow 10 minutes for posttest. Collect posttest.
Appendix E. Depth of Knowledge Level Test
Depth of Knowledge (DOK) Level Test was adapted from NextLessons (n.d).

**Figure E.1**

*Depth of Knowledge Level Test*

<table>
<thead>
<tr>
<th>DOK Level Test</th>
<th>Problems</th>
<th>Below DOK2</th>
<th>Problems</th>
<th>At DOK2</th>
<th>Problems</th>
<th>DOK3 and above</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#1-4</td>
<td>#1-4</td>
<td>#5-8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Level 2**

**Passing Pros Green Bay Packers**

Name: ________________

Who is the all-time best quarterback for the Packers? You are a sports columnist for an online blog. You are writing an article comparing Aaron Rogers and Brett Favre. Which quarterback is best? Use data to justify your opinion.

One way to compare performance is to look at winning percentages. Use the ratio of wins to games played to determine the winning percentage for each player. Round to the nearest tenth. The first one is done for you.

<table>
<thead>
<tr>
<th>Name</th>
<th>Win-Loss Ratio</th>
<th>Wins to Games Played Ratio</th>
<th>Winning Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td>80:39</td>
<td>80/119</td>
<td>67.2%</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>160:33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Who has the highest win per games played ratio? Which quarterback appears to be the best all-time quarterback for the Packers? Explain your reasoning.

You could rank the quarterbacks by completion percentages.

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>Passes Completed</th>
<th>Passes Attempted</th>
<th>Completion percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td>2,633</td>
<td>4,047</td>
<td>65.1%</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>5,377</td>
<td>8,754</td>
<td></td>
</tr>
</tbody>
</table>

2. Who has the highest completion percentage?

You could rank the quarterbacks by wins per year.

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>Wins</th>
<th>Years Played</th>
<th>Wins per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td>80</td>
<td>10</td>
<td>8.0</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>160</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

3. Who has the highest number of wins per year?

You could rank quarterbacks by the highest touchdown (TD) to interception (INT) ratio.

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>TD</th>
<th>INT</th>
<th>TD to INT Ratio</th>
<th>Unit Rate TD per INT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td>257</td>
<td>65</td>
<td>257:65</td>
<td>4.0</td>
</tr>
<tr>
<td>Brett Favre</td>
<td>442</td>
<td>286</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Who has the highest touchdown (TD) to interception (INT) ratio?

(Figure continues)
Figure E.1

Depth of Knowledge Level Test

**Level 3** You have looked at four different statistics to determine which quarterback is the best. Now compare those statistics.

5. Use your calculations from questions #1-4 and rank both players as (1) for highest and (2) for lower.
6. Add up the rankings to find the total rank.

<table>
<thead>
<tr>
<th>Quarterback</th>
<th>Winning percentages rank</th>
<th>Completion percentage rank</th>
<th>Number of Wins per year rank</th>
<th>TD to INT Rate rank</th>
<th>Combined total Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaron Rodgers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brett Favre</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Which quarterback has the best combined rank total?

8. Are some statistics more important than others? Who is the best quarterback? Explain your reasoning.
Appendix F. Adaptive Reasoning: Keywords, Descriptions, Coding Protocol
Table F.1

Adaptive Reasoning Indicators: Keywords, Descriptions, and Examples

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Keywords</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
</table>
| Relationship Connections and Comparisons (RCC) | if...then, compare, unlike, different, like, similar, both, together   | Recognition of one occurrence that affects another or connection between. A comparison of elements.                                                                                                          | “See, when I increase this side of the triangle, the other gets smaller.”  
Student draws an arrow from one drawing of a house to another and labels the arrow ‘larger than.’ |
| Alternatives Pursued (AP)                      | Acknowledgement or suggestion of a change of position or viewpoint. instead of, rather, more like | Viewing a different or alternate solution(s) path.  
Clarifying or challenging a misconception connected with a redirection of thought.  
Drawing a representation of a path and then erasing it for another representation that leads towards a solution.  
Changing number drawn in relation to a representation that changes the solution path, not just the situational understanding of the problem. | “Why don’t we try to divide instead.”  
“I see what you mean, I agree with you now. This is probably right not that.”  
Could we say that the line over here represents the hypotenuse instead?  
Student draws two triangles and then a formula for the Pythagorean theorem, then erases the formula and writes a proportion. |
<table>
<thead>
<tr>
<th>Indicator</th>
<th>Keywords</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Justifications (J)</td>
<td>evidence, justify, elaborate, because, clarify, explain, prove</td>
<td>Justification: An explanation, clarification or discussion of an occurrence that follows logical steps or reasonings.</td>
<td>“Wait, I'll prove it to you, you can’t change the one side of the figure without it affecting the other side.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Suggesting sources of reasoning. A series of logical arguments (Newton, 2013). Establishment of a what is seen as a truth.</td>
<td>“When I was dividing the ribbon into eight pieces, I only cut it seven times. Dividing into sections is different from cutting. The number is one less that if I were to divide.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>May include argument and counter argument. Does not include simply repeating someone else’s statement.</td>
<td>Student draws an arrow from the main representation of a geometric shape and then draws a smaller triangle noting a right angle and degrees of other angles.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>May include bringing in evidence from the inside or outside of the current context into the dialogue to support an argument, opinion, proposal, prediction or theory.</td>
<td>Student draws a grid and then skip-counts along the grid then creates a number line and skip-counts along it to verify a solution.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Drawing side explanations that discuss main elements of the drawing utilizing steps, oftentimes expressed in conjunction with dialogue and gestures towards drawing.</td>
<td></td>
</tr>
<tr>
<td>Prior Knowledge Integration (PK)</td>
<td>Past tense references, refers to, remember when, before, a formula, like, unlike</td>
<td>Use of knowledge previously attained. A linking of new information to old understandings. Transfer.</td>
<td>“Remember when we studied integers, a positive times a positive gave us a positive answer. Using exponents is sort of like that.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Linking concepts, beliefs, hypothesis, agreements, conclusions reached, learned content to a current topic or activity. Knowledge typically is derived from information gained prior to task, but may build upon learning from current task.</td>
<td>“This is like when we solved equations, except we need to use an inequality. We do the same steps.”</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Drawing a previously known formula or understanding. Building onto a drawing a formerly known image.</td>
<td>Student draws C=d(pi) next to a semi-circle.</td>
</tr>
</tbody>
</table>

*(Table continues)*
<table>
<thead>
<tr>
<th>Indicator</th>
<th>Keywords</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern Recognition (PR)</td>
<td>pattern, notice, clue, reoccurring, same, chain, similar, again, over and over, happening,</td>
<td>Noticing repetition or regularity. Iteration. Remarks on the understanding that something repeats or completes a cycle. Algebraic thinking. Drawing a repeated symbol, number, marks or interval. Quantity progression illustrated in similar quantities or steps.</td>
<td>&quot;It goes 1/2, then 1/4, then 1/8.&quot; Student draws repeated marks on three candles indicating rates of burn.</td>
</tr>
<tr>
<td>Legitimacy Determination (LD)</td>
<td>too much (little), too many (few), too big (small), can't (must) be, wrong, right, fit</td>
<td>Student concludes validity of solution. Oftentimes seen as a quantity being too large or too small to fit the situation. Determining relevancy. Recognizing an absurdity, a fit, a synchronization. Legitimacy can occur in the form of a question. The legitimization may occur in response to a previous comment. Legitimizing does not include a justification but may occur with or without an attached justification. Drawing decisions with unit analysis or conversion factors may come into play. Size of objects drawn or redrawn.</td>
<td>&quot;That can't work, this answer is way too big.&quot; &quot;That answer seems to make sense.&quot; Crossing out or erasing a number or representation of a solution.</td>
</tr>
<tr>
<td>Emergent Code (EC-)</td>
<td>Codes that emerge during the mathematical task that do not follow the structured codes.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note*. The Cam-UNAM Scheme for Educational Dialogue Analysis (SEDA: ©2016) was developed by a research team from the University of Cambridge, UK, and the National Autonomous University of Mexico, led by Sara Hennessy and Sylvia Rojas-Drummond and funded through a grant from the British Academy. The original scheme and list of co-creators are available at [http://tinyurl.com/BAdialogue](http://tinyurl.com/BAdialogue). Coding scheme was altered to fit the needs of this study (Hennessy & Rojas-Drummond, 2016).
Coding Protocol

1) only apply codes when they occur in context of the dialogue

2) coding is line-by-line, count the code once if the same speaker reiterates the code in the same line of argument

3) count a repeated code more than once if it initiates a new line of thinking

4) code off task discussion with a U (un-coded)

5) code drawings, if speaker reinforces drawing code count once for drawing and once for speaker

6) coding is not dependent upon responses of other learners

7) less sophisticated evidence of proof or argument is acceptable under justification coding

8) consider prior and subsequent conversation when coding

9) within segmentation of data considering embedded, interrupted, and overlapping dialogue and count once in the case of reiteration of speaker

10) hierarchical ordering of clusters, segmentation of transcript, and categories that are not mutually exclusive will be determined by researcher(s) as they emerge within the study’s particulars.
Angela M. Frabasilio  Curriculum Vita
PO Box 630247, Rockville, Utah 84763 - 0247 ◊ angiefrab@gmail.com

EDUCATION / CERTIFICATIONS
PhD, 2022
Utah State University
Specialization: Curriculum and Instruction
Concentration: Mathematics Teacher Education and Leadership

Masters of Education, May 2004
Southern Utah University

Bachelors of Science, Mathematics, May 1993
Southern Utah University, Magna Cum Laude
Major: Mathematics, Minors: Art, History, Utah Teaching Certification

State of Utah Teaching Certification, 1993-Current
Professional Educator’s License for the State of Utah Secondary Education, 6th - 12th Grade, Level 2
Endorsements: Mathematics Level 4, Visual Arts, Technology, History, Journalism

State of California Teaching Certification 1995-2000
Professional Educator’s License for the State of California Single Subject, PreK-12
Endorsements: Mathematics Level 4, Visual Arts, Social Sciences

State of Utah Technology Endorsement 2012
Technology endorsement, technology learning coach

RESEARCH INTERESTS
1) Learner Generated Drawings
2) STEAM and Passion Learning
3) Adaptive reasoning of learners engaged in mathematical tasks and problem solving

EMPLOYMENT HISTORY
Mathematics 7th, STEAM (Science, Technology, Engineering, Art and Mathematics), Sign Language, and the Art of Science. Responsibilities include teacher development, technology coach, website design, mathematics department chair, school leadership committee member, team leader committee member, technology committee chair.

**Librarian, Springdale Public Library.** (2009-2011). Washington County Library, Utah. Responsibilities include shelving and checking out books, organizing reading programs.

**Teacher, Zion Canyon Middle School.** (2007-2009). Washington County, Utah. All subjects, 6th and 7th grades. Responsibilities include school organization manager, events coordinator.


**Teacher, Hurricane Middle School.** (2001-2002). Washington County School District, Utah. Pre-algebra.


**Vice President, International Legislative Exchange (ILE).** (1991-1997). Rockville, Utah. ILE created exchanges for Delaware/Estonia, Michigan/Latvia, Illinois/Lithuania, Colorado/Tibetan parliament in exile. Responsibilities include organizing a nonprofit, assessing needs of visiting legislators, creating and arranging substantive meetings with constitutional scholars, land managers from the National Park Service, foreign policy experts, etc.

running the corporation: contracting and budgeting, managing bank accounts, payroll, tax filings, petty cash accounts, purchasing, management of rental properties, distributions across four state offices: California, Iowa, Philadelphia and Washington D.C.

UNIVERSITY TEACHING
Professor’s Assistant. (1994). Southern Utah University Responsibilities: Teach mathematics to undergraduates and grade papers. Mathematics 1010, Intermediate Algebra

GRANTS, SCHOLARSHIPS, AND PRIVATE FUNDING
Funded:


Principal Investigator. ($8,100). Three-Act Math Tasks in the Classroom. Lead of six teachers in a group STEM Action Center PreK-12 Classroom Grant. Grant proposes funds for the integration of mathematical tasks in the classroom using MidSchool Math resources. (2019)

Graduate Enhancement Award. ($4,000). Student Involvement and Leadership, Utah State University (2018)


Principal Investigator. ($25,000). HOMELESS BACKPACK STEAM PROJECT. Students help homeless people in their local community by designing an

**Utah Active Teacher Scholarship. ($1,000).** Utah Retired School Employees Association. (2017).


**Co-Principal Investigator. ($2,941).** Fantastic Ekphrastic -Poetry meets Art. (2013) Utah Teachers & Technology Grant, CenturyLink, 2014 Project goal: Partner art and language art students to create and discuss ekphrastic art. Principal - Julie Green, Sunrise Ridge Intermediate School.

**Principal Investigator. ($2,455).** iPads in the Mathematics Classroom. (2013) Teachers & Technology Grant, CenturyLink. Project Goal: The primary goal is to put technology in the hands of the student by providing them daily access to iPads in the mathematics classroom. Primarily this will give students a tool to create, solve and share group projects, and secondarily it is a tool to provide students extra help or extra opportunities beyond classroom instruction through online programs. Sunrise Ridge Intermediate School.


**Submitted, but not funded:**

**Principal Investigator. ($5,000).** HELPING HANDS; STEAM students aid people with disabilities. Project goal: to provide startup funds to purchase project tools and materials. Teachers & Technology Grant, CenturyLink

**Principal Investigator. ($2,500).** HELPING HANDS STEAM students aid
children with disabilities. Select 25, Select Health. Project goal: to provide startup funds to purchase project tools and materials.

**Principal Investigator - Online crowdsourcing.** ($1,245). HELPING HANDS STEAM students aid children with disabilities, Generosity by Indiegogo, Project goal: To provide startup funds to purchase project tools and materials. Website: www.generosity.com/education-fundraising/sunrise-helping-hands-adapting-for-special-needs.

**PUBLICATIONS**
**Journal Articles (Refereed)**

**PRESENTATIONS**
**NATIONAL**


**STATE**


**LEADERSHIP AND SERVICE**

**Adaptive Design Workshop.** (January 30-31, 2017). Organized a workshop on how to design adaptive furniture for children with disabilities, Sunrise Ridge Intermediate School. Guest speakers Clay Christensen and Amy Henningson, USU Assistive Technology lab, attendees: Teachers and administrators at Sunrise Ridge Intermediate School, Dixie Regional Medical Center Pediatrics Center, Washington County Special Education Department personnel, Zion National Park Resource manager, Zion National Park education specialist

**Technology learning coach.** (2015-2018). Responsible for schoolwide assistance of teachers implementing technology in their classrooms, Sunrise Ridge Intermediate School (SRIS), St. George, Utah

**Mathematics team leader.** (2013-2017). SRIS, St. George, Utah

**Technology group team leader.** (2015-2017). Responsible for schoolwide technology plan, management of computers and other technologies, SRIS, St. George, Utah

**School leadership team.** (2015-2017). Responsible for working with other leaders of the school and guiding direction of program implementation, SRIS, St. George, Utah

**MathCounts team leader.** (2014-2016). Sunrise Ridge Intermediate School, St. George, Utah

**Annual art competitions and educational weeks.** (2000-2005). Creating and organizing district-wide art competitions and art educational weeks, Hurricane High School, Hurricane, Utah

**St. George Art Festival.** (2014-2016). participate with students, SRIS, St. George, Utah

**Kayenta Street Painting Festival.** (2014-2016). participate with students, SRIS, St. George, Utah

**Day on Cedar Mountain Annual Art Camp.** (2001-2). participate with students, Southern Utah University, Cedar City Utah

**PROFESSIONAL SERVICE**

Sunrise Ridge Intermediate School, Regional Education Representative. (2015-2017). Technology PLC Southwest Educational Development Center, Cedar City, Utah
ORGANIZATIONS AND COMMUNITY INVOLVEMENT

Youth Activities of Zion (YAZ). (1998-2014). Zion Canyon, Utah
Director/Project Coordinator. Organizing summer activities for children and young adults in Zion Canyon, Utah. Service area includes the communities of Virgin, Rockville, Springdale and employees of Zion National Park. Responsibilities included creating educational summer program content for low or no cost to participants and raising, budgeting and distribution of community funds. Website: https://sites.google.com/site/yazkids/

Canyon Community Center, Board of Directors. (1999-2001). Springdale, Utah. Responsibilities included guiding inspiration for the building, organizing, staffing and maintenance of the canyon community center in the town of Springdale

Founder and incorporator of the recycling program for the communities of Virgin, Rockville and Springdale

Zion Overlook. (1994-1995). Zion Canyon, Utah
Vice President and volunteer for organization, political and environmental watchdogged observing changes that occurred on Bureau of Land Management lands and Zion National Park Lands in Zion Canyon area. Responsibilities included reading environmental assessments and advising community members on courses of action.

Zion Harvest Food Co-op. (1992-2000). Zion Canyon, Utah
Volunteer and financial manager. Responsibilities included organizing community gatherings and monthly management of food goods distribution.

TECHNOLOGY

Technology Learning Endorsement (2013). Zion Canyon, Utah
Systems Experience