Online Dynamic Modeling and Localization for Small-Spacecraft Proximity Operations

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Small-Spacecraft Proximity Operations

- Missions:
  - Docking
  - Inspection
  - repair of target vehicle
- Constraints:
  - Mass
  - Power
  - Fuel

Short list of related missions:
- XSS-10, XSS-11 *(AF)*
- SNAP-1 *(SSTL)*
- MiTEx *(DARPA)*
- SHPERES *(MIT, NASA)*
- AERCam, Mini-AERCam *(NASA)*
- Orbital Express *(DARPA)*
- MEPSI *(DARPA, AF)*
- BX-1 *(China)*
- PARADIGM *(TAMU, UT, NASA)*

Bandit Mission

Proximity Operations
- **Bandit**

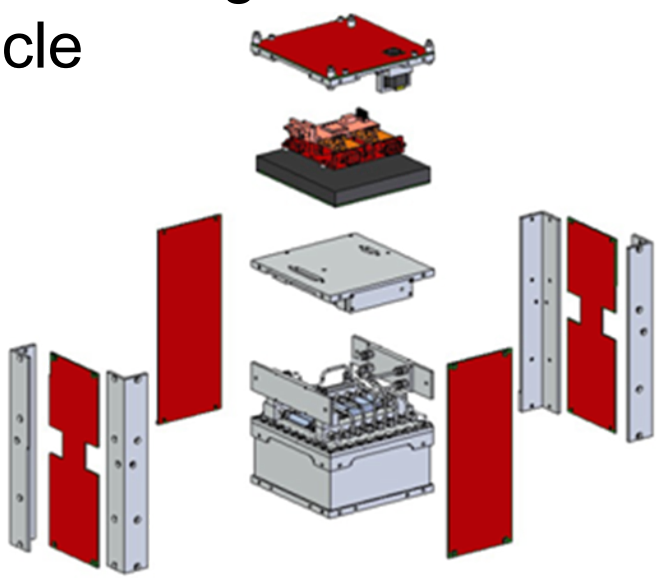
Localization
- UKF
- Results

Dynamic Modeling
- BLR
- GPR
- Results

Conclusion

- 3-kg Service Vehicle
  - Repeatable docking
  - Station keeping
  - Blended autonomous control
- Decouples orbital functions with larger host vehicle

- Cold-gas propulsion
- 8 thrusters round the center
- ARM-32 processor
- Sensors:
  - 3 roll-rate gyros
  - 3-axis MEMS accelerometer
  - Camera for image-based navigation

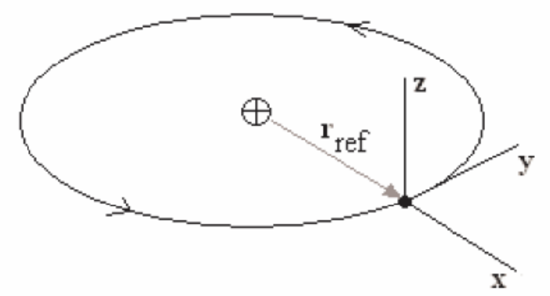


Localization and Pose Estimation

- Problem:
  - COTS unfiltered noisy sensors
  - Delayed image navigation
  - Thrust impulse is close to the accelerometer's sensor noise
  - Non-linear thrust characteristics

- Clohessy-Wiltshire Relative Coordinate Frame
  - Used for relative orbital dynamics

$$X = \begin{bmatrix} \vec{x} \\ \dot{\vec{x}} \\ \vec{q} \\ \dot{\vec{q}} \end{bmatrix} \quad \vec{u} = \begin{bmatrix} \vec{\alpha} \\ \vec{\omega} \end{bmatrix} \quad \vec{X} \in \mathbb{R}^{12}, \vec{u} \in \mathbb{R}^{6}$$



Unscented Kalman Filter

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- Allows for non-linear state transition and observation functions
  - Unscented transform sigma points
- Transition function includes INS integration and CW orbital propagation
- Image Navigation used for observation updates
- Error quaternion constructed, δq, by rotational error vector, ϕ, in sigma points:

$$\delta q = \begin{bmatrix} \frac{\sin(|\phi|)/2}{|\phi|}\phi \\ \cos(|\phi|/2) \end{bmatrix}$$

Algorithm *Unscented Kalman Filter*

Input: $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$

1. $\mu^a_{t-1} = \left(\mu^\top_{t-1} \quad 0^\top_{m\times 1} \quad 0^\top_{p\times 1}\right)^\top$
2. $\Sigma^a_{t-1} = \begin{pmatrix} \Sigma_{t-a} & 0 & 0 \\ 0 & M_t & 0 \\ 0 & 0 & Q_t \end{pmatrix}$
3. $\chi^a_{t-1} = (\mu^a_{t-1}, \; \mu^a_{t-1} + \gamma\sqrt{\Sigma^a_{t-1}}, \; \mu^a_{t-1} - \gamma\sqrt{\Sigma^a_{t-1}})$
4. $\bar{\chi}^x_t = g(u_t + \chi^u_t, \chi^x_{t-1})$
5. $\bar{\mu}_t = \sum_{i=0}^{2L} w_i^{(m)} \bar{\chi}^x_{i,t}$
6. $\bar{\Sigma}_t = \sum_{i=0}^{2L} w_i^{(c)} (\bar{\chi}^x_{i,t} - \bar{\mu}_t)(\bar{\chi}^x_{i,t} - \bar{\mu}_t)^\top$
7. $\bar{Z}_t = h(\bar{\chi}^x_t) + \chi^z_t$
8. $\hat{z}_t = \sum_{i=0}^{2L} w_i^{(m)} \bar{Z}_{i,t}$
9. $S_t = \sum_{i=0}^{2L} w_i^{(c)} (\bar{Z}_{i,t} - \hat{z}_t)(\bar{Z}_{i,t} - \hat{z}_t)^\top$
10. $\Sigma^{x,z}_t = \sum_{i=0}^{2L} w_i^{(c)} (\bar{\chi}^x_{i,t} - \bar{\mu}_t)(\bar{Z}_{i,t} - \hat{z}_t)^\top$
11. $K_t = \Sigma^{x,z}_t S_t^{-1}$
12. $\mu_t = \bar{\mu}_t + K_t(z_t - \hat{z}_t)$
13. $\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^\top$
14. $p_{z_t} = det(2\pi S_t)^{-\frac{1}{2}} exp\{-\frac{1}{2}(z_t - \hat{z}_t)^\top S_t^{-1}(z_t - \hat{z}_t)\}$
15. **return** μ_t, Σ_t, p_{z_t}

Table 1: Unscented Kalman Filter

UKF Results

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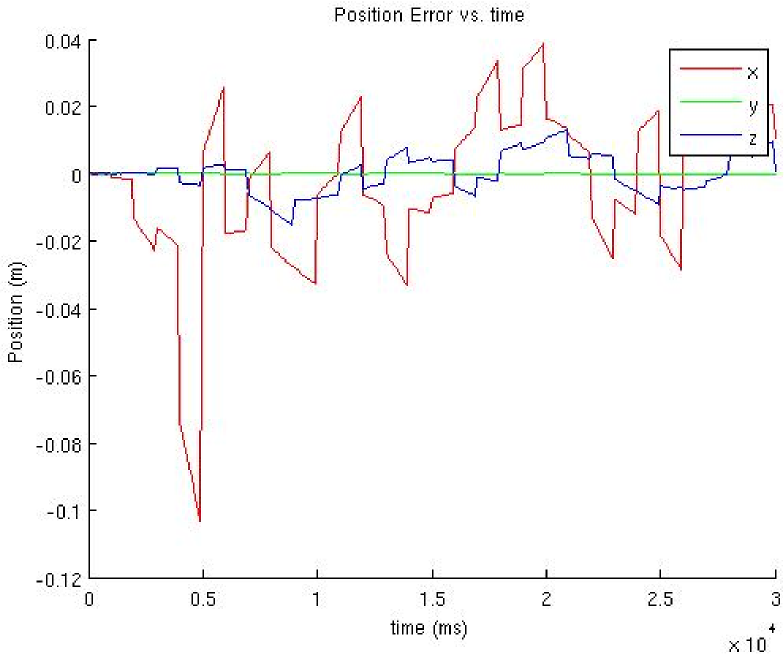
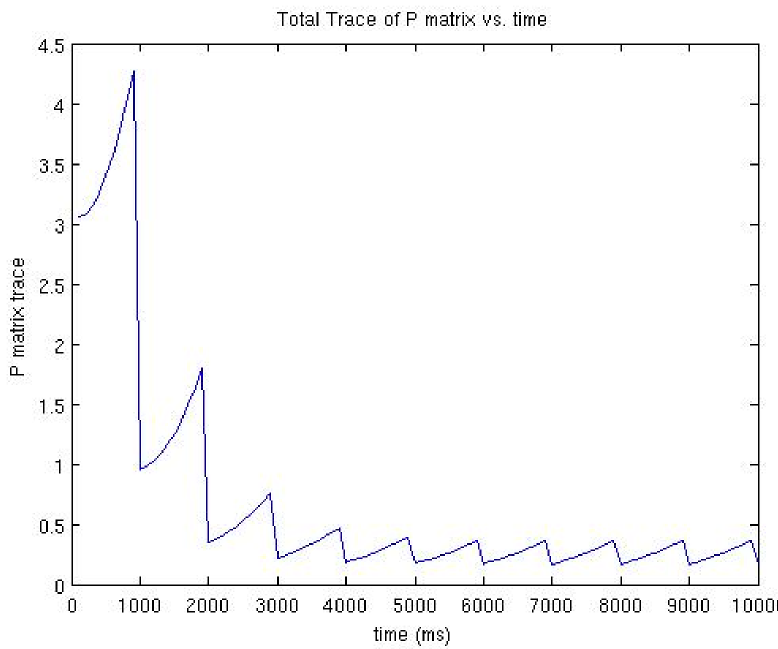
Position Errors during Autonomous Docking



Uncertainty



Largest Source of Error is Accelerometer

Online Dynamic Modeling

- Bandit's dynamics change
  - Moments shift due to propellant usage
- Thruster performance change:
  - Temperature
  - Misalignment and un-modeled fluid dynamics
  - Fluid dynamic losses
  - System degradation

Total Force and torque is additive:

$$\vec{T}_{thrusters} = \sum_{i=1}^{8} \lambda_i \vec{t_i}$$

$$\vec{F}_{thrusters} = \sum_{i=1}^{8} \lambda_i \vec{f_i}$$

$$\lambda_i \in \{0, 1\}$$

We can linearize around current ω and *v*:

$$\Delta \vec{v} = \frac{1}{m} \vec{F}_{thrusters} + \Delta \vec{v}_{orbital}$$

$$\Delta \vec{\omega} = \mathbf{J}^{-1} \vec{\omega} \times (\mathbf{J} \vec{\omega}) \delta_t + \mathbf{J}^{-1} \vec{T}_{thrusters} \delta_t$$

Bayes Linear Regression

- Online linear regression for Gaussian mapping:

$$y_t = \theta^\top x_t + \epsilon_t$$

- Given a new data point, we can update the natural Gaussian parameterization:

$$p(\vec{y}|\vec{x},\vec{\theta})p(\vec{\theta}) \propto e^{\frac{-(y-\theta^\top x)^2}{2\sigma^2}} e^{\frac{-1}{2}\theta^\top} = e^{\frac{-1}{2}\theta^\top(P-\frac{xx^\top}{\sigma^2})\theta+(J+y\frac{yx^\top}{\sigma^2})^\top\theta}$$

Update Steps:

$$P \leftarrow P + \frac{x_t x_t^\top}{\sigma^2}$$

$$J \leftarrow J + \frac{y_t x_t^\top}{\sigma^2}$$

- Moment parameterization:

$$\mu = P^{-1}J$$

$$\Sigma = P^{-1}$$

$$y = (P^{-1}J)^\top x$$

Gaussian Process Regression

- Gaussian Kernels capture non-linearities in the system
- Sliding window allows for time varying dynamics
- GPR defines a probability distribution over functions, with inference taking place directly in the space of functions

$$\mu_{y_t} = K(x_t, \mathbf{X})K(\mathbf{X},\mathbf{X})^{-1}\mathbf{Y}$$

$$\Sigma_{y_t} = K(x_t, x_t) - K(x_t, \mathbf{X})K(\mathbf{X},\mathbf{X})^{-1}K(\mathbf{X}, x_t)$$

Algorithm *Gaussian Process Regression*

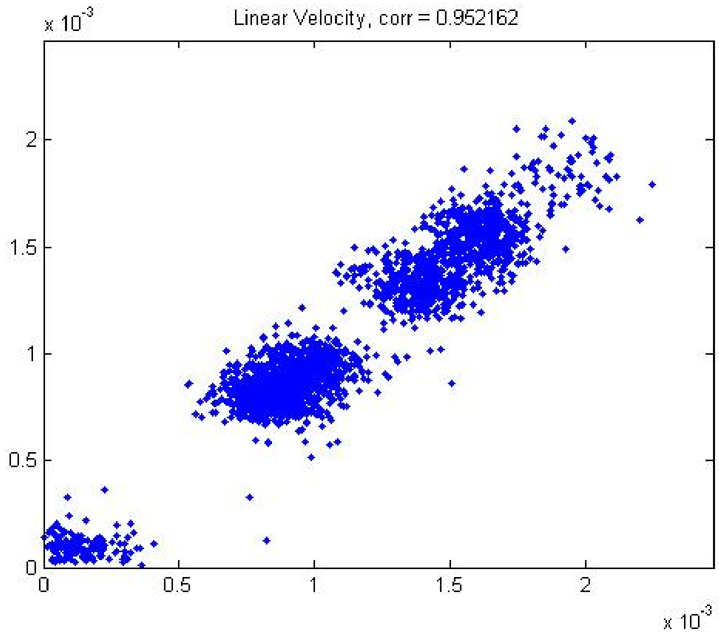
Input: X (inputs), y (target), k (covariance function), σ_n^2 (noise level), x_* (test input)

1. $L = cholesky(K + \sigma_n^2 I)$
2. $\alpha = L^\top \; (L \; y)$
3. $\hat{f}_* = k_*^\top \alpha$
4. $v = L \; k_*$
5. $V[f_*] = k(x_*, x_*) - v^\top v$
6. $\log p(y|X) = -\frac{1}{2} y^\top \alpha - \Sigma_i \log L_{ii} - \frac{n}{2} \log 2\pi$
7. **return** $\hat{f}_*$ (mean), $V[f_*]$ (variance), $\log p(y|X)$ (log marginal likelihood)

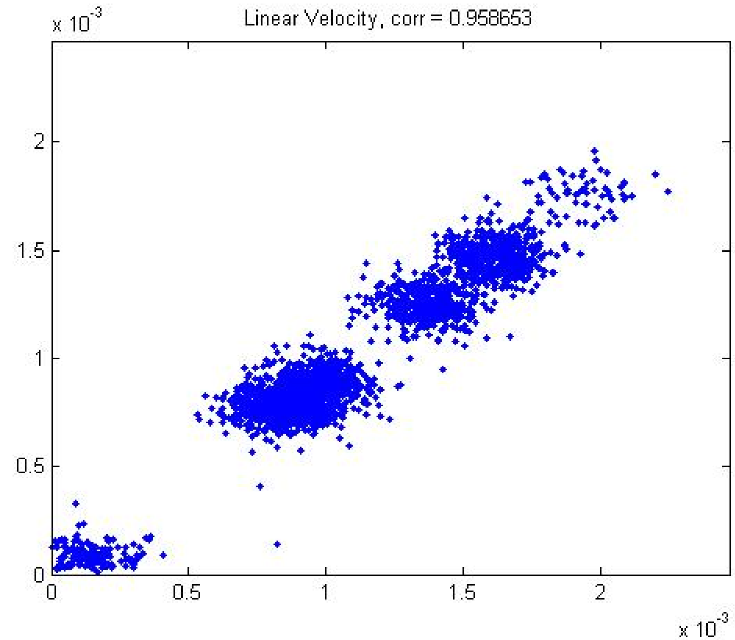
Table 2: Gaussian Process Regression Algorithm

Online Dynamic Modeling Results

GP



BLR



	Velocity Correlation		Time Complexity
	Linear	Angular	
BLR	0.959	0.925	$O(d^3)$, d = 8
GP	0.952	0.881	$O(n^3)$, n ~ 100

Conclusion

- Localization via Unscented Kalman Filter
  - Captures nonlinear dynamics
  - Uses COTS sensors and image navigation
  - Handles quaternion rotation and orbital dynamics

- Online dynamic modeling via Bayes Linear Regression
  - Handles noisy thrusters and inputs
  - Good time complexity
  - Sliding window captures time-varying dynamic changes

- These techniques will allow small spacecrafts, with constrained sensors and actuators, to autonomously navigate despite noisy and poorly observable states

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