Human Wildlife Interactions

Supplemental Information – A decision tool to identify population management strategies for common ravens and other avian predators

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1. THREE-STAGE LIFE HISTORY

Given a population matrix representing any 3-stage life history, where the matrix elements are denoted a_{ij} for i $= 1, 2, 3$, and j $= 1, 2, 3$, and of the form (Caswell 2001):

$$\mathrm{L} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \qquad \text{(S-1)}$$

then the characteristic equation takes the form (Beyer 1978):

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0, \qquad \text{(S-2)}$$

where the superparameters (Hanley & Dennis 2019; Shields *et al.* 2019*a*, *b*) are:

$$P = (-a_{11} - a_{22} - a_{33}), \qquad \text{(S-3)}$$

$$Q = (a_{11}a_{22} + a_{22}a_{33} + a_{11}a_{33} - a_{32}a_{23} - a_{21}a_{12} - a_{13}a_{31}), \qquad \text{(S-4)}$$

and

$$R = (a_{11}a_{32}a_{23} + a_{12}a_{21}a_{33} + a_{31}a_{13}a_{22} - a_{11}a_{22}a_{33} - a_{21}a_{13}a_{32} - a_{31}a_{12}a_{23}). \qquad \text{(S-5)}$$

2. DENSITY INDEPENDENT LIFE HISTORY OF RAVENS

Given a population matrix model representing the three-stage life history of the common raven (*Corvus corax*) and vital rates of the form:

$$L = \begin{bmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad \text{(S-6)}$$

then the characteristic equation is (Beyer 1978; Hanley & Dennis 2019; Shields et al. 2019):

$$\lambda^3 + P\lambda^2 + R = 0, \quad \text{(S-7)}$$

where,

$$P = (-a_{33}), \quad \text{(S-8)}$$

$$Q = 0, \quad \text{(S-9)}$$

and

$$R = (-a_{13}a_{21}a_{32}). \quad \text{(S-10)}$$

2.1 Symbolic reductions in one, two, or three vital rates

We will show that in a density independent life history, the reduction of:

(Case 1) a_{13} (fertility),
(Case 2) a_{21} (hatchling survival),
(Case 3) a_{32} (non-breeder survival),
(Case 4) a_{33} (breeder survival),

the reduction of dual combinations of:

(Case 5) a_{13} (fertility) and a_{21} (hatchling survival),
(Case 6) a_{13} (fertility) and a_{32} (non-breeder survival),
(Case 7) a_{13} (fertility) and a_{33} (breeder survival),
(Case 8) a_{21} (hatchling survival) and a_{32} (non-breeder survival),
(Case 9) a_{21} (hatchling survival) and a_{33} (breeder survival),
(Case 10) a_{32} (non-breeder survival) and a_{33} (breeder survival),

or the reduction of triple combination of:

(Case 11) a_{13} (fertility), a_{21} (hatchling survival), and a_{32} (non-breeder survival),
(Case 12) a_{13} (fertility), a_{21} (hatchling survival), and a_{33} (non-breeder survival),
(Case 13) a_{13} (fertility), a_{32} (non-breeder survival), and a_{33} (breeder survival),
(Case 14) a_{21} (hatchling survival), a_{32} (non-breeder survival), and a_{33} (breeder survival),

will reduce the long-term growth rate (λ) of the overall population.

<u>Case 1</u> – Reduction in fertility.

Please see Shields *et al.* (2019*a*, *b*).

<u>Case 2</u> – Reduction in hatchling survival.

We reduce a_{21} by some number such that the R in eqs. S-7 - S-10 becomes R'. Due to the negative sign, a reduction a_{21} means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. By substitution, the characteristic equation:

$$\lambda^3 + P\lambda^2 + R = 0 \quad \text{(S-11)}$$

becomes:

$$\lambda^3 + P\lambda^2 + R'\varepsilon = 0. \quad \text{(S-12)}$$

Dividing both sides by ε gives:

$$\frac{\lambda^3}{\varepsilon} + \frac{P\lambda^2}{\varepsilon} + \frac{R\prime\varepsilon}{\varepsilon} = 0, \quad \text{(S-13)}$$

which reduces to:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon})^3} + \frac{P\lambda^2}{(\sqrt[2]{\varepsilon})^2} + R' = 0, \quad \text{(S-14)}$$

and

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon})})^3 + P(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + R' = 0. \quad \text{(S-15)}$$

Since P has not changed, and $\varepsilon > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$, any reduction in hatchling survival will reduce the growth rate of the entire system. The value of hatchling survival that must be achieved to halt growth ($\lambda = 1$) is thus:

$$\frac{1 - a_{33}}{a_{13}a_{32}} = a_{21}. \quad \text{(S-16)}$$

Case 3 – Reduction in non-breeder survival.

We now reduce a_{32} by some number such that the R in eqs. S-7 - S-10 becomes R'. Due to the negative sign, a reduction a_{32} means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. By substitution, the characteristic equation:

$$\lambda^3 + P\lambda^2 + R = 0 \quad \text{(S-17)}$$

becomes:

$$\lambda^3 + P\lambda^2 + R'\varepsilon = 0. \quad \text{(S-18)}$$

Dividing both sides by ε gives:

$$\frac{\lambda^3}{\varepsilon} + \frac{P\lambda^2}{\varepsilon} + \frac{R\prime\varepsilon}{\varepsilon} = 0, \quad \text{(S-19)}$$

which reduces to:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon})^3} + \frac{P\lambda^2}{(\sqrt[2]{\varepsilon})^2} + R' = 0, \quad \text{(S-20)}$$

and

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon})})^3 + P(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + R' = 0. \quad \text{(S-21)}$$

Since P has not changed, and $\varepsilon > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$, any reduction in non-breeder survival will reduce the growth rate of the entire system. The value of non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is thus:

$$\frac{1 - a_{33}}{a_{13}a_{21}} = a_{32}. \quad \text{(S-22)}$$

Case 4 – Reduction in breeder survival.

We now reduce a_{33} by some number such that the P in eqs. S-7 - S-10 becomes P'. Due to the negative sign, a reduction a_{33} means that $P' > P$. Equivalently, $P'\varepsilon = P$ or $P' = \frac{P}{\varepsilon}$ provided $\varepsilon > 1$. By substitution, the characteristic equation:

$$\lambda^3 + P\lambda^2 + R = 0 \quad \text{(S-23)}$$

becomes:

$$\lambda^3 + P'\varepsilon\lambda^2 + R = 0. \quad \text{(S-24)}$$

Dividing both sides by ε gives:

$$\frac{\lambda^3}{\varepsilon} + P'\lambda^2 + \frac{R}{\varepsilon} = 0, \quad \text{(S-25)}$$

which is equivalent to:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon})^3} + P'\lambda^2 + \frac{R}{\varepsilon} = 0, \quad \text{(S-26)}$$

and

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon})})^3 + P'\lambda^2 + (\frac{1}{\varepsilon})R = 0. \quad \text{(S-27)}$$

Since $\frac{\lambda}{(\sqrt[3]{\varepsilon})} < \lambda$ and $(\frac{1}{\varepsilon})R$ <R, any reduction in breeder survival will reduce the growth rate of the entire system. The value of breeder survival that must be achieved to halt growth ($\lambda = 1$) is thus:

$$1 - a_{13}a_{21}a_{32} = a_{33}. \quad \text{(S-28)}$$

<u>Case 5</u> – Reduction in fertility and hatchling survival.

We reduce a_{13} (fertility) and a_{21} (hatchling survival), each by an arbitrary amount such that the R in eqs. S-7 - S-10 becomes R'. Due to the negative sign in eqs. S-7 - S-10 , a reduction a_{13} or a_{21} (or both) means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Substituting this result into eqs. S-7 - S-10 gives:

$$\lambda^3 + P\lambda^2 + R'\varepsilon = 0. \quad \text{(S-29)}$$

Dividing by ε gives:

$$\frac{\lambda^3}{\varepsilon} + \frac{P\lambda^2}{\varepsilon} + \frac{R'\varepsilon}{\varepsilon} = 0, \quad \text{(S-30)}$$

which simplifies to:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon})^3} + \frac{P\lambda^2}{(\sqrt[2]{\varepsilon})^2} + R' = 0, \quad \text{(S-31)}$$

and finally,

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon})})^3 + P(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + R' = 0. \quad \text{(S-32)}$$

Since P has not changed, and $\varepsilon > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$, a combination of strategies that produce R' will reduce the growth rate of the entire system. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is equivalent to:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0, \quad \text{(S-33)}$$

which rearranges into:

$$\frac{1 - a_{33}}{a_{32}} = a_{13}a_{21}. \quad \text{(S-34)}$$

<u>Case 6</u> – Reduction in fertility and non-breeder survival.

We now reduce a_{13} (fertility) and a_{32} (non-breeder survival) each by an arbitrary amount. The reduction produces R'. Due to the negative sign in eqs. S-7-S10, a reduction in a_{13} or a_{32} (or both) means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Substituting this result into eqs. S-7 - S-10 gives:

$$\lambda^3 + P\lambda^2 + R = 0, \quad \text{(S-35)}$$

$$\lambda^3 + P\lambda^2 + R'\varepsilon = 0, \quad \text{(S-36)}$$

$$\frac{\lambda^3}{\varepsilon} + \frac{P\lambda^2}{\varepsilon} + \frac{R'\varepsilon}{\varepsilon} = 0, \quad \text{(S-37)}$$

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon})^3} + \frac{P\lambda^2}{(\sqrt[2]{\varepsilon})^2} + R' = 0, \quad \text{(S-38)}$$

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon})})^3 + P(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + R' = 0. \quad \text{(S-39)}$$

Since P has not changed, and $\varepsilon > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-40)}$$

Rearranging gives:

$$\frac{1 - a_{33}}{a_{21}} = a_{13}a_{32}. \quad \text{(S-41)}$$

<u>Case 7</u> – Reduction in fertility and breeder survival.

We now reduce a_{13} (fertility) and a_{33} (breeder survival), each by an arbitrary amount. Due to the negative sign, a reduction in a_{13} means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Also due to the negative sign, a reduction in a_{33} means that $P' > P$. Equivalently, $P'\varepsilon' = P$ provided $\varepsilon' > 1$. Substituting these results into eqs. S-7 - S-10 gives:

$$\lambda^3 + P'\varepsilon'\lambda^2 + R'\varepsilon = 0. \quad \text{(S-42)}$$

Dividing by $\varepsilon'$and ε gives:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\varepsilon'\lambda^2}{\varepsilon\varepsilon'} + \frac{R'\varepsilon}{\varepsilon\varepsilon'} = 0, \quad \text{(S-43)}$$

which simplifies to:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\lambda^2}{\varepsilon} + \frac{R'}{\varepsilon'} = 0, \quad \text{(S-44)}$$

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon\varepsilon'})^3} + \frac{P'\lambda^2}{(\sqrt[2]{\varepsilon})^2} + \frac{R'}{\varepsilon'} = 0, \quad \text{(S-45)}$$

which collapses to:

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})})^3 + P'(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + \frac{R'}{\varepsilon'} = 0. \quad \text{(S-46)}$$

Since P' has not changed, $\frac{R'}{\varepsilon'} = \frac{\frac{R}{\varepsilon}}{\varepsilon'} = \frac{R}{\varepsilon\varepsilon'} < R$, and $\varepsilon, \varepsilon' > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-47)}$$

Fixing a_{33} at A, we have the result:

$$\frac{1 - A}{a_{21}a_{32}} = a_{13}. \quad \text{(S-48)}$$

<u>Case 8</u> – Reduction in hatching and non-breeder survival.

We now reduce a_{21} (hatchling survival) and a_{32} (non-breeder survival), each by an arbitrary amount. The reduction produces R'. Due to the negative sign in eqs. S-7-S10, a reduction in a_{21} or a_{32} (or both) means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Substituting this result into eqs. S-7 - S-10 gives:

$$\lambda^3 + P\lambda^2 + R = 0, \quad \text{(S-49)}$$

$$\lambda^3 + P\lambda^2 + R'\varepsilon = 0, \quad \text{(S-50)}$$

$$\frac{\lambda^3}{\varepsilon} + \frac{P\lambda^2}{\varepsilon} + \frac{R'\varepsilon}{\varepsilon} = 0, \quad \text{(S-51)}$$

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon})^3} + \frac{P\lambda^2}{(\sqrt[2]{\varepsilon})^2} + R' = 0, \quad \text{(S-52)}$$

$$\left(\frac{\lambda}{(\sqrt[3]{\varepsilon})}\right)^3 + P\left(\frac{\lambda}{(\sqrt[2]{\varepsilon})}\right)^2 + R' = 0. \quad \text{(S-53)}$$

Since P has not changed, and $\varepsilon > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-54)}$$

Rearranging gives:

$$\frac{1 - a_{33}}{a_{13}} = a_{21}a_{32}. \quad \text{(S-55)}$$

<u>Case 9</u> – Reduction in hatchling and breeder survival.

We now reduce a_{21} (hatchling survival) and a_{33} (breeder survival), each by an arbitrary amount. Due to the negative sign, a reduction in a_{21} means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Also due to the negative sign, a reduction in a_{33} means that $P' > P$. Equivalently, $P'\varepsilon' = P$ provided $\varepsilon' > 1$. Substituting these results into eqs. S-7 - S-10 gives:

$$\lambda^3 + P'\varepsilon'\lambda^2 + R'\varepsilon = 0. \quad \text{(S-56)}$$

Dividing by ε' and ε gives:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\varepsilon'\lambda^2}{\varepsilon\varepsilon'} + \frac{R'\varepsilon}{\varepsilon\varepsilon'} = 0, \quad \text{(S-57)}$$

which simplifies to:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\lambda^2}{\varepsilon} + \frac{R'}{\varepsilon'} = 0. \quad \text{(S-58)}$$

Further reduction gives:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon\varepsilon'})^3} + \frac{P'\lambda^2}{(\sqrt[2]{\varepsilon})^2} + \frac{R'}{\varepsilon'} = 0, \quad \text{(S-59)}$$

which collapses to:

$$\left(\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})}\right)^3 + P'\left(\frac{\lambda}{(\sqrt[2]{\varepsilon})}\right)^2 + \frac{R'}{\varepsilon'} = 0. \quad \text{(S-60)}$$

Since P' has not changed, $\frac{R'}{\varepsilon'} = \frac{\frac{R}{\varepsilon}}{\varepsilon'} = \frac{R}{\varepsilon\varepsilon'} < R$, and $\varepsilon, \varepsilon' > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-61)}$$

Fixing a_{33} at A gives the result:

$$\frac{1 - A}{a_{13}a_{32}} = a_{21}. \quad \text{(S-62)}$$

<u>Case 10</u> – Reduction in non-breeder and breeder survival.

We now reduce a_{32} (non-breeder survival) and a_{33} (breeder survival), each by an arbitrary amount. Due to the negative sign, a reduction in a_{32} means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Also due to the negative sign, a reduction in a_{33} means that $P' > P$. Equivalently, $P'\varepsilon' = P$ provided $\varepsilon' > 1$. Substituting these results into eqs. S-7 - S-10 gives:

$$\lambda^3 + P'\varepsilon'\lambda^2 + R'\varepsilon = 0. \quad \text{(S-63)}$$

Dividing by ε' and ε gives:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\varepsilon'\lambda^2}{\varepsilon\varepsilon'} + \frac{R'\varepsilon}{\varepsilon\varepsilon'} = 0, \quad \text{(S-64)}$$

which simplifies to:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\lambda^2}{\varepsilon} + \frac{R'}{\varepsilon'} = 0. \quad \text{(S-65)}$$

Further reduction gives:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon\varepsilon'})^3} + \frac{P'\lambda^2}{(\sqrt[2]{\varepsilon})^2} + \frac{R'}{\varepsilon'} = 0, \quad \text{(S-66)}$$

which collapses to:

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})})^3 + P'(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + \frac{R'}{\varepsilon'} = 0. \quad \text{(S-67)}$$

Since P' has not changed, $\frac{R'}{\varepsilon'} = \frac{\frac{R}{\varepsilon}}{\varepsilon'} = \frac{R}{\varepsilon\varepsilon'} < R$, and $\varepsilon, \varepsilon' > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-68)}$$

Fixing a_{33} at A gives:

$$\frac{1 - A}{a_{13}a_{21}} = a_{32}. \quad \text{(S-69)}$$

<u>Case 11</u> – Reduction in fertility, hatchling survival, and non-breeder survival.

We now reduce a_{13} (fertility), a_{21} (hatching survival), and a_{32} (non-breeder survival), each by an arbitrary amount. The reduction produces *R*'. Due to the negative sign in eqs. S-7 - S-10 and the reduction in one, two, or all three vital rates (a_{13}, a_{21}, or a_{32}), then $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Substitution of this result into eqs. S-7 - S-10 gives:

$$\lambda^3 + P\lambda^2 + R = 0, \quad \text{(S-70)}$$

$$\lambda^3 + P\lambda^2 + R'\varepsilon = 0, \quad \text{(S-71)}$$

$$\frac{\lambda^3}{\varepsilon} + \frac{P\lambda^2}{\varepsilon} + \frac{R'\varepsilon}{\varepsilon} = 0, \quad \text{(S-72)}$$

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon})^3} + \frac{P\lambda^2}{(\sqrt[2]{\varepsilon})^2} + R' = 0, \quad \text{(S-73)}$$

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon})})^3 + P(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + R' = 0. \quad \text{(S-74)}$$

Since P has not changed, and $\varepsilon > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-75)}$$

Rearranging gives:

$$1 - a_{33} = a_{13}a_{21}a_{32} \quad \text{(S-76)}$$

<u>Case 12</u> – Reduction in fertility, hatchling survival, and breeder survival.

We now reduce a_{13} (fertility), a_{21} (hatchling survival), and a_{33} (breeder survival), each by an arbitrary amount. Due to the negative sign, a reduction in a_{13} or a_{21} (or both) means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Also due to the negative sign, a reduction in a_{33} means that $P' > P$. Equivalently, $P'\varepsilon' = P$ provided $\varepsilon' > 1$. Substituting these results into eqs. S-7 - S-10 gives:

$$\lambda^3 + P'\varepsilon'\lambda^2 + R'\varepsilon = 0. \qquad \text{(S-77)}$$

Dividing by ε' and ε gives:

$$\frac{\lambda^3}{\varepsilon\varepsilon\prime} + \frac{P\prime\varepsilon\prime\lambda^2}{\varepsilon\varepsilon\prime} + \frac{R\prime\varepsilon}{\varepsilon\varepsilon\prime} = 0, \qquad \text{(S-78)}$$

which simplifies to:

$$\frac{\lambda^3}{\varepsilon\varepsilon\prime} + \frac{P\prime\lambda^2}{\varepsilon} + \frac{R\prime}{\varepsilon\prime} = 0. \qquad \text{(S-79)}$$

Further reduction gives:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon\varepsilon\prime})^3} + \frac{P\prime\lambda^2}{(\sqrt[2]{\varepsilon})^2} + \frac{R\prime}{\varepsilon\prime} = 0, \qquad \text{(S-80)}$$

which collapses to:

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon\prime})})^3 + P'(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + \frac{R\prime}{\varepsilon\prime} = 0. \qquad \text{(S-81)}$$

Since P' has not changed, $\frac{R\prime}{\varepsilon\prime} = \frac{\frac{R}{\varepsilon}}{\varepsilon\prime} = \frac{R}{\varepsilon\varepsilon\prime} < R$, and $\varepsilon, \varepsilon' > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon\prime})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \qquad \text{(S-82)}$$

Fixing a_{33} at A gives:

$$\frac{1-A}{a_{32}} = a_{13}a_{21}. \qquad \text{(S-83)}$$

<u>Case 13</u> – Reduction in fertility, non-breeder survival, and breeder survival.

We now reduce a_{13} (fertility), a_{32} (non-breeder survival), and a_{33} (breeder survival), each by an arbitrary amount. Due to the negative sign, a reduction in a_{13} or a_{32} (or both) means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Also due to the negative sign, a reduction in a_{33} means that $P' > P$. Equivalently, $P'\varepsilon' = P$ provided $\varepsilon' > 1$. Substituting these results into eqs. S-7 - S-10 gives:

$$\lambda^3 + P'\varepsilon'\lambda^2 + R'\varepsilon = 0. \qquad \text{(S-84)}$$

Dividing by $\varepsilon'$and ε gives:

$$\frac{\lambda^3}{\varepsilon\varepsilon\prime} + \frac{P\prime\varepsilon\prime\lambda^2}{\varepsilon\varepsilon\prime} + \frac{R\prime\varepsilon}{\varepsilon\varepsilon\prime} = 0, \qquad \text{(S-85)}$$

which simplifies to:

$$\frac{\lambda^3}{\varepsilon\varepsilon\prime} + \frac{P\prime\lambda^2}{\varepsilon} + \frac{R\prime}{\varepsilon\prime} = 0. \qquad \text{(S-86)}$$

Further reduction gives:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon\varepsilon\prime})^3} + \frac{P\prime\lambda^2}{(\sqrt[2]{\varepsilon})^2} + \frac{R\prime}{\varepsilon\prime} = 0, \qquad \text{(S-87)}$$

which collapses to:

$$(\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon\prime})})^3 + P'(\frac{\lambda}{(\sqrt[2]{\varepsilon})})^2 + \frac{R\prime}{\varepsilon\prime} = 0. \qquad \text{(S-88)}$$

Since P' has not changed, $\frac{R'}{\varepsilon'} = \frac{\frac{R}{\varepsilon}}{\varepsilon'} = \frac{R}{\varepsilon\varepsilon'} < R$, and $\varepsilon, \varepsilon' > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-89)}$$

Fixing a_{33} at A gives:

$$\frac{1-A}{a_{21}} = a_{13}a_{32}. \quad \text{(S-90)}$$

<u>Case 14</u> – Reduction in hatchling survival, non-breeder survival, and breeder survival.

We now reduce a_{21} (hatchling), a_{32} (non-breeder survival), and a_{33} (breeder survival), each by an arbitrary amount. Due to the negative sign, a reduction in a_{21} or a_{32} (or both) means that $R' > R$. Equivalently, $R'\varepsilon = R$ provided $\varepsilon > 1$. Also due to the negative sign, a reduction in a_{33} means that $P' > P$. Equivalently, $P'\varepsilon' = P$ provided $\varepsilon' > 1$. Substituting these results into eqs. S-7 - S-10 gives:

$$\lambda^3 + P'\varepsilon'\lambda^2 + R'\varepsilon = 0. \quad \text{(S-91)}$$

Dividing by ε' and ε gives:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\varepsilon'\lambda^2}{\varepsilon\varepsilon'} + \frac{R'\varepsilon}{\varepsilon\varepsilon'} = 0, \quad \text{(S-92)}$$

which simplifies to:

$$\frac{\lambda^3}{\varepsilon\varepsilon'} + \frac{P'\lambda^2}{\varepsilon} + \frac{R'}{\varepsilon'} = 0. \quad \text{(S-93)}$$

Further reduction gives:

$$\frac{\lambda^3}{(\sqrt[3]{\varepsilon\varepsilon'})^3} + \frac{P'\lambda^2}{(\sqrt[2]{\varepsilon})^2} + \frac{R'}{\varepsilon'} = 0, \quad \text{(S-94)}$$

which collapses to:

$$\left(\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})}\right)^3 + P'\left(\frac{\lambda}{(\sqrt[2]{\varepsilon})}\right)^2 + \frac{R'}{\varepsilon'} = 0. \quad \text{(S-95)}$$

Since P' has not changed, $\frac{R'}{\varepsilon'} = \frac{\frac{R}{\varepsilon}}{\varepsilon'} = \frac{R}{\varepsilon\varepsilon'} < R$, and $\varepsilon, \varepsilon' > 1$, $\frac{\lambda}{(\sqrt[3]{\varepsilon\varepsilon'})} < \lambda$ and $\frac{\lambda}{(\sqrt[2]{\varepsilon})} < \lambda$. The combinatorial value that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-96)}$$

Fixing a_{33} at A gives:

$$\frac{1-A}{a_{13}} = a_{21}a_{32}. \quad \text{(S-97)}$$

2.2 Symbolic reductions in the proportions of one, two, or three vital rates

Now suppose we wish to calculate the reduction in symbolic *proportions* when one, two, or three vital rates are altered to produce a growth rate equal to one ($\lambda = 1$).

<u>Case 1</u> – Reduction in fertility.

Please see Shields *et al.* (2019*a*, *b*).

<u>Case 2</u> – Reduction in hatchling survival.

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings that survive. Hatchling survival can therefore be broken up into those hatchlings that are killed (X), and those hatchlings that are not killed:

$$Xa_{21} + (1 - X)\, a_{21}. \quad \text{(S-98)}$$

Surviving hatchlings reduces eq. S-98 to:

$$(1 - X)\, a_{21}. \quad \text{(S-99)}$$

Substitution of eq. S-99 into eqs. S-7 - S-10 gives:

$$1 - \ a_{33} - a_{13}(a_{21} - Xa_{21})a_{32} = 0. \quad \text{(S-102)}$$

Rearranging, and solving for X, we have:

$$1 - \ a_{33} - a_{13}a_{21}a_{32} + Xa_{13}a_{21}a_{32} = 0, \quad \text{(S-103)}$$

which reduces to:

$$X = \frac{1 - a_{33} - a_{13}a_{21}a_{32}}{-a_{13}a_{21}a_{32}}. \quad \text{(S-104)}$$

Equation S-104 specifies the proportion of hatchlings that must be killed (X) to reduce the growth rate to 1.

<u>Case 3</u> – Reduction in non-breeder survival.

Let X be the proportion of non-breeders killed. Then $1 - X$ is the proportion of non-breeders that survive. Non-breeder survival can therefore be broken up into those birds that are killed (X), and those that are not killed:

$$Xa_{32} + (1 - X)\, a_{32}. \quad \text{(S-105)}$$

Surviving birds reduces eq. S-105 to:

$$(1 - X)\, a_{32}. \quad \text{(S-106)}$$

Substitution of eq. S-99 into eqs. S-7 - S-10 gives:

$$1 - \ a_{33} - a_{13}a_{21}(a_{32} - Xa_{32}) = 0. \quad \text{(S-107)}$$

Rearranging, and solving for X, we have:

$$1 - \ a_{33} - a_{13}a_{21}a_{32} + Xa_{13}a_{21}a_{32} = 0, \quad \text{(S-108)}$$

which reduces to:

$$X = \frac{1 - a_{33} - a_{13}a_{21}a_{32}}{-a_{13}a_{21}a_{32}}. \quad \text{(S-109)}$$

Equation S-109 specifies the proportion of non-breeders that must be killed (X) to reduce the growth rate to 1.

<u>Case 4</u> – Reduction in breeder survival.

Let X be the proportion of breeders killed. Then $1 - X$ is the proportion of breeders that survive. Breeder survival can therefore be broken up into those birds that are killed (X), and those that are not killed:

$$Xa_{33} + (1 - X)\, a_{33}. \quad \text{(S-110)}$$

Surviving birds reduces eq. S-110 to:

$$(1 - X)\, a_{33}. \quad \text{(S-111)}$$

Substitution of eq. S-111 into eqs. S-7 - S-10 gives:

$1 - a_{33} + Xa_{33} - a_{13}a_{21}a_{32} = 0.$ (S-112)

Rearranging, and solving for X, we have:

$X = \frac{a_{13}a_{21}a_{32} - 1 + a_{33}}{a_{33}}.$ (S-113)

Equation S-113 specifies the proportion of non-breeders that must be killed (X) to reduce the growth rate to 1.

Case 5 – Reduction in fertility and hatchling survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs that hatch. Fertility can therefore be broken up into those eggs that are killed (X), and those that are not killed:

$Xa_{13} + (1 - X)\,a_{13}.$ (S-114)

Surviving eggs reduces eq. S-114 to:

$(1 - X)\,a_{13}.$ (S-115)

Survival can also be broken up into those hatchlings that are killed (Y), and those that are not killed:

$Ya_{21} + (1 - Y)\,a_{21}.$ (S-116)

Surviving hatchlings reduces eq. S-116 to:

$(1 - Y)\,a_{21}.$ (S-117)

Substitution of eq. S-115 and eq. S-1171 into eqs. S-7 - S-10 gives:

$1 - a_{33} - (a_{13} - Xa_{13})(a_{21} - Ya_{21})a_{32} = 0.$ (S-118)

Rearranging, and solving for X and Y, we have:

$1 - a_{33} - (a_{13} - Xa_{13})(a_{21}a_{32} - Ya_{21}a_{32}) = 0.$ (S-119)

$1 - a_{33} - (a_{13}a_{21}a_{32} - a_{13}Ya_{21}a_{32} - Xa_{13}a_{21}a_{32} + Xa_{13}Ya_{21}a_{32}) = 0,$ (S-120)

which reduces to:

$1 - a_{33} - a_{13}a_{21}a_{32} + a_{13}a_{21}a_{32}Y + a_{13}a_{21}a_{32}X - a_{13}a_{21}a_{32}XY = 0.$ (S-121)

Equation S-121 specifies the proportion of hatchlings that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

Case 6 – Reduction in fertility and non-breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs that hatch. Fertility can be broken up into those eggs that are killed (X), and those that are not killed:

$Xa_{13} + (1 - X)\,a_{13}.$ (S-122)

Surviving eggs reduces eq. S-122 to:

$(1 - X)\,a_{13}.$ (S-123)

Survival of non-breeders can also be broken up into those birds that are killed (Y), and those that are not killed:

$Ya_{32} + (1 - Y)\,a_{32}.$ (S-124)

Surviving non-breeders reduces eq. S-124 to:

$(1 - Y)\,a_{32}.$ (S-125)

Substitution of eq. S-123 and eq. S-125 into eqs. S-7 - S-10 gives:

$1 - a_{33} - (a_{13} - Xa_{13})a_{21}(a_{32} - Ya_{32}) = 0.$ (S-126)

Rearranging, and solving for X and Y, we have:

$1 - a_{33} - (a_{13} - Xa_{13})(a_{21}a_{32} - Ya_{21}a_{32}) = 0.$ (S-127)

$1 - a_{33} - (a_{13}a_{21}a_{32} - a_{13}Ya_{21}a_{32} - Xa_{13}a_{21}a_{32} + Xa_{13}Ya_{21}a_{32}) = 0,$ (S-128) which reduces to:

$$1 - a_{33} - a_{13}a_{21}a_{32} + a_{13}Ya_{21}a_{32} + Xa_{13}a_{21}a_{32} - Xa_{13}Ya_{21}a_{32} = 0. \quad \text{(S-129)}$$

Equation S-129 specifies the proportion of non-breeders that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

<u>Case 7</u> – Reduction in fertility and breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs that hatched. Fertility can be broken up into those eggs that are killed (X), and those eggs that are not killed:

$$Xa_{13} + (1 - X)\,a_{13}. \quad \text{(S-130)}$$

Surviving eggs reduces eq. S-130 to:

$$(1 - X)\,a_{13}. \quad \text{(S-131)}$$

Survival of breeders can also be broken up into those birds that are killed (Y), and those that are not killed:

$$Ya_{33} + (1 - Y)\,a_{33}. \quad \text{(S-132)}$$

Surviving breeders reduces eq. S-132 to:

$$(1 - Y)\,a_{33}. \quad \text{(S-133)}$$

Substitution of eq. S-131 and eq. S-133 into eqs. S-7 - S-10 gives:

$$1 - (a_{33} - Ya_{33}) - (a_{13} - Xa_{13})a_{21}a_{32} = 0. \quad \text{(S-134)}$$

Rearranging, and solving for X and Y, we have:

$$1 - a_{33} + Ya_{33} - a_{13}a_{21}a_{32} + Xa_{13}a_{21}a_{32} = 0. \quad \text{(S-135)}$$

Equation S-135 specifies the proportion of breeders that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

<u>Case 8</u> – Reduction in hatchling survival and non-breeder survival.

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings that survive. Hatchling survival can be broken up into those birds that are killed (X), and those birds that are not killed:

$$Xa_{21} + (1 - X)\,a_{21}. \quad \text{(S-136)}$$

Surviving hatchlings reduces eq. S-136 to:

$$(1 - X)\,a_{21}. \quad \text{(S-137)}$$

Survival of non-breeders can also be broken up into those birds that are killed (Y), and those that are not killed:

$$Ya_{32} + (1 - Y)\,a_{32}. \quad \text{(S-138)}$$

Surviving non-breeders reduces eq. S-138 to:

$$(1 - Y)\,a_{32}. \quad \text{(S-139)}$$

Substitution of eq. S-137 and eq. S-139 into eqs. S-7 - S-10 gives:

$$1 - a_{33} - a_{13}(1 - X)\,a_{21}(1 - Y)\,a_{32} = 0. \quad \text{(S-140)}$$

Rearranging, and solving for X and Y, we have:

$$\frac{1 - a_{33}}{a_{13}a_{21}a_{32}} = (1 - X)\,(1 - Y). \quad \text{(S-141)}$$

Equation S-141 specifies the proportion of hatchlings that must be killed (X) alongside the proportion of non-breeders (Y) must be killed to reduce the growth rate to 1.

Case 9 – Reduction in hatchling survival and breeder survival.

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings that survive. Hatchling survival can be broken up into those birds that are killed (X), and those birds that are not killed:

$$Xa_{21} + (1 - X)\, a_{21}. \quad \text{(S-142)}$$

Surviving hatchlings reduces eq. S-142 to:

$$(1 - X)\, a_{21}. \quad \text{(S-143)}$$

Survival of breeders can also be broken up into those birds that are killed (Y), and those that are not killed:

$$Ya_{33} + (1 - Y)\, a_{33}. \quad \text{(S-144)}$$

Surviving breeders reduces eq. S-144 to:

$$(1 - Y)\, a_{33}. \quad \text{(S-145)}$$

Substitution of eq. S-143 and eq. S-145 into eqs. S-7 - S-10 gives:

$$1 - (1 - Y)\, a_{33} - a_{13}(1 - X)\, a_{21}a_{32} = 0. \quad \text{(S-146)}$$

Equation S-146 specifies the proportion of hatchlings (X) that must be killed alongside the proportion of breeders that must be killed (Y) to reduce the growth rate to 1.

Case 10 – Reduction in non-breeder survival and breeder survival.

Let X be the proportion of non-breeders killed. Then $1 - X$ is the proportion of non-breeders that survive. Non-breeder survival can be broken up into those birds that are killed (X), and those birds that are not killed:

$$Xa_{32} + (1 - X)\, a_{32}. \quad \text{(S-147)}$$

Surviving non-breeders reduces eq. S-147 to:

$$(1 - X)\, a_{32}. \quad \text{(S-148)}$$

Survival of breeders can also be broken up into those birds that are killed (Y), and those that are not killed:

$$Ya_{33} + (1 - Y)\, a_{33}. \quad \text{(S-149)}$$

Surviving breeders reduces eq. S-149 to:

$$(1 - Y)\, a_{33}. \quad \text{(S-150)}$$

Substitution of eq. S-148 and eq. S-150 into eqs. S-7 - S-10 gives:

$$1 - (1 - Y)\, a_{33} - a_{13}a_{21}(1 - X)\, a_{32} = 0. \quad \text{(S-151)}$$

Equation S-151 specifies the proportion of non-breeders (X) that must be killed alonside the proportin of breeders that must be killed (Y) to reduce the growth rate to 1.

Case 11 – Reduction in fertility, hatchling survival, and non-breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs that survive. Fertility can be broken up into those eggs that are killed (X), and those eggs that are not killed ($1 - X$):

$$Xa_{13} + (1 - X)\, a_{13}. \quad \text{(S-152)}$$

Surviving eggs reduces eq. S-152 to:

$$(1 - X)\, a_{13}. \quad \text{(S-153)}$$

Survival of hatchlings can also be broken up into those birds that are killed (Y), and those that are not killed ($1 - Y$):

$$Ya_{21} + (1 - Y)\, a_{21}. \quad \text{(S-154)}$$

Surviving hatchlings reduces eq. S-154 to:
$(1 - Y)\ a_{21}$. (S-155)
Survival of non-breeders can also be broken up into those birds that are killed (Z), and those that are not killed $(1 - Z)$:
$Za_{32} + (1 - Z)\ a_{32}$. (S-156)
Surviving non-breeders reduces eq. S-156 to:
$(1 - Z)\ a_{32}$. (S-157)
Substitution of eq. S-153, S155 and eq. S-157 into eqs. S-7 - S-10 gives:
$1 - \ a_{33} - \ (1 - X)\ a_{13}(1 - Y)a_{21}\ (1 - Z)a_{32} = 0.$ (S-158)
Equation S-158 specifies the proportion of eggs (X), proportion of hatchlings (Y), and proportion of non-breeders (Z) that must be killed to reduce the growth rate to 1.

<u>Case 12</u> – Reduction in fertility, hatchling survival, and breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs that survive. Fertility can be broken up into those eggs that are killed (X), and those eggs that are not killed $(1 - X)$:
$Xa_{13} + (1 - X)\ a_{13}$. (S-159)
Surviving eggs reduces eq. S-159 to:
$(1 - X)\ a_{13}$. (S-160)
Survival of hatchlings can also be broken up into those birds that are killed (Y), and those that are not killed $(1 - Y)$:
$Ya_{21} + (1 - Y)\ a_{21}$. (S-161)
Surviving hatchlings reduces eq. S-161 to:
$(1 - Y)\ a_{21}$. (S-162)
Survival of breeders can also be broken up into those birds that are killed (Z), and those that are not killed $(1 - Z)$:
$Za_{33} + (1 - Z)\ a_{33}$. (S-163)
Surviving non-breeders reduces eq. S-163 to:
$(1 - Z)\ a_{33}$. (S-164)
Substitution of eq. S-160, S162 and eq. S-164 into eqs. S-7 - S-10 gives:
$1 - \ (1 - Z)a_{33} - \ (1 - X)\ a_{13}(1 - Y)a_{21}\ a_{32} = 0.$ (S-165)
Equation S-165 specifies the proportion of eggs (X), proportion of hatchlings (Y), and proportion of breeders (Z) that must be killed to reduce the growth rate to 1.

<u>Case 13 –</u> Reduction in fertility, non-breeder survival, and breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs that survive. Fertility can be broken up into those eggs that are killed (X), and those eggs that are not killed $(1 - X)$:
$Xa_{13} + (1 - X)\ a_{13}$. (S-166)
Surviving eggs reduces eq. S-166 to:
$(1 - X)\ a_{13}$. (S-167)
Survival of non-breeders can also be broken up into those birds that are killed (Y), and those that are not killed $(1 - Y)$:
$Ya_{32} + (1 - Y)\ a_{32}$. (S-168)
Surviving non-breeder reduces eq. S-168 to:

$(1 - Y)\ a_{32}$. (S-169)

Survival of breeders can also be broken up into those birds that are killed (Z), and those that are not killed ($1 - Z$):

$Za_{33} + (1 - Z)\ a_{33}$. (S-170)

Surviving non-breeders reduces eq. S-170 to:

$(1 - Z)\ a_{33}$. (S-171)

Substitution of eq. S-167, S169 and eq. S-171 into eqs. S-7 - S-10 gives:

$1 - (1 - Z)a_{33} - (1 - X)\ a_{13}a_{21}(1 - Y)\ a_{32} = 0$. (S-172)

Equation S-172 specifies the proportion of eggs (X), proportion of non-breeders (Y), and proportion of breeders (Z) that must be killed to reduce the growth rate to 1.

<u>Case 14</u> – Reduction in hatchling survival, non-breeder survival, and breeder survival.

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings that survive. Hatchling survival can be broken up into those birds that are killed (X), and those birds that are not killed ($1 - X$):

$Xa_{21} + (1 - X)\ a_{21}$. (S-173)

Surviving birds reduces eq. S-173 to:

$(1 - X)\ a_{21}$. (S-174)

Survival of non-breeders can also be broken up into those birds that are killed (Y), and those that are not killed ($1 - Y$):

$Ya_{32} + (1 - Y)\ a_{32}$. (S-175)

Surviving non-breeders reduces eq. S-175 to:

$(1 - Y)\ a_{32}$. (S-176)

Survival of breeders can also be broken up into those birds that are killed (Z), and those that are not killed ($1 - Z$):

$Za_{33} + (1 - Z)\ a_{33}$. (S-177)

Surviving breeders reduces eq. S-177 to:

$(1 - Z)\ a_{33}$. (S-178)

Substitution of eq. S-174, S176 and eq. S-178 into eqs. S-7 - S-10 gives:

$1 - (1 - Z)a_{33} - a_{13}(1 - X)\ a_{21}(1 - Y)\ a_{32} = 0$. (S-179)

Equation S-179 specifies the proportion of hatchlings (X), proportion of non-breeders (Y), and proportions of breedesr (Z) that must be killed to reduce the growth rate to 1.

3. DENSITY DEPENDENT LIFE HISTORY OF RAVENS

Given a population matrix representing the three-stage life history of the common raven, where the vital rates are of the form:

$$\mathrm{L} = \begin{bmatrix} 0 & 0 & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad \text{(S-180)}$$

then the characteristic equation is (Beyer 1978; Hanley & Dennis 2019; Shields et al. 2019),

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0, \quad \text{(S-181)}$$

where

$$P = (-a_{22} - a_{33}), \quad \text{(S-182)}$$

$$Q = (a_{22}a_{33} - a_{32}a_{23}), \quad \text{(S-183)}$$

and

$$R = (-a_{13}a_{21}a_{32}). \quad \text{(S-184)}$$

3.1 Symbolic proportion reductions in one, two or three vital rates

We will show that in a density dependent life history, the proportional reduction of

(Case 1) a_{13} (fertility),
(Case 2) a_{21} (hatchling survival),
(Case 3) a_{22} (non-breeder survival without transition),
(Case 4) a_{23} (breeder survival with demotion to non-breeder stage),
(Case 5) a_{32} (non-breeder survival with transition to breeder stage),
(Case 6) a_{33} (breeder survival),

the duel proportional reduction of a combination of:

(Case 7) a_{13} (fertility) and a_{21} (hatchling survival),
(Case 8) a_{13} (fertility) and a_{22} (non-breeder survival without transition),
(Case 9) a_{13} (fertility) and a_{23} (breeder survival with demotion),
(Case 10) a_{13} (fertility) and a_{32} (non-breeder survival with transition),
(Case 11) a_{13} (fertility) and a_{33} (breeder survival),
(Case 12) a_{21} (hatchling survival) and a_{22} (non-breeder survival without transition),
(Case 13) a_{21} (hatchling survival) and a_{23} (breeder survival with demotion),
(Case 14) a_{21} (hatchling survival) and a_{32} (non-breeder survival with transition),
(Case 15) a_{21} (hatchling survival) and a_{33} (breeder survival),
(Case 16) a_{22} (non-breeder survival without transition) and a_{23} (breeder survival with demotion),
(Case 17) a_{22} (non-breeder survival without transition) and a_{32} (non-breeder survival with transition),
(Case 18) a_{22} (non-breeder survival without transition) and a_{33} (breeder survival),
(Case 19) a_{23} (breeder survival with demotion) and a_{32} (non-breeder survival with transition),
(Case 20) a_{23} (breeder survival with demotion) and a_{33} (breeder survival),
(Case 21) a_{32} (non-breeder survival with transition) and a_{33} (breeder survival),

or the triple proportional reduction of any combination of:

(Case 22) a_{13} (fertility), a_{21} (hatchling survival), and a_{22} (non-breeder survival without transition),

(Case 23) a_{13} (fertility), a_{21} (hatchling survival), and a_{23} (breeder survival with demotion),

(Case 24) a_{13} (fertility), a_{21} (hatchling survival), and a_{32} (non-breeder survival with transition),

(Case 25) a_{13} (fertility), a_{21} (hatchling survival), and a_{33} (breeder survival),

(Case 26) a_{13} (fertility), a_{22} (non-breeder survival without transition), and a_{23} (breeder survival with demotion),

(Case 27) a_{13} (fertility), a_{22} (non-breeder survival without transition), and a_{32} (non-breeder survival with transition),

(Case 28) a_{13} (fertility), a_{22} (non-breeder survival without transition), and a_{33} (breeder survival),

(Case 29) a_{13} (fertility), a_{23} (breeder survival with demotion), and a_{32} (non-breeder survival with transition),

(Case 30) a_{13} (fertility), a_{23} (breeder survival with demotion), and a_{33} (breeder survival),

(Case 31) a_{13} (fertility), a_{32} (non-breeder survival), and a_{33} (breeder survival),

(Case 32) a_{21} (hatchling survival), a_{22} (non-breeder survival without transition), and a_{23} (breeder survival with demotion),

(Case 33) a_{21} (hatchling survival), a_{22} (non-breeder survival without transition), and a_{32} (non-breeder survival with transition),

(Case 34) a_{21} (hatchling survival), a_{22} (non-breeder survival without transition), and a_{33} (breeder survival),

(Case 35) a_{21} (hatchling survival), a_{23} (breeder survival with demotion) and a_{32} (non-breeder survival with transition),

(Case 36) a_{21} (hatchling survival), a_{23} (breeder survival with demotion) and a_{33} (breeder survival),

(Case 37) a_{21} (hatchling survival), a_{32} (non-breeder survival with transition), and a_{33} (breeder survival),

(Case 38) a_{22} (non-breeder survival without transition), a_{23} (breeder survival with demotion), and a_{32} (non-breeder survival with transition),

(Case 39) a_{22} (non-breeder survival without transition), a_{23} (breeder survival with demotion), and a_{33} (breeder survival),

(Case 40) a_{22} (non-breeder survival without transition), a_{32} (non-breeder survival with transition), and a_{33} (breeder survival),

(Case 41) a_{23} (breeder survival without demotion), a_{32} (non-breeder survival), and a_{33} (breeder survival),

will have varying effects on the growth rate.

<u>Case 1</u> – Reduction in fertility.

Please see Shields *et al.* (2019*a*, *b*).

<u>Case 2</u> – Reduction in hatchling survival.

The reduction in hatchling survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-185)}$$

Rearranging gives and isolating the term for hatchling survival gives:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0.$$

$$a_{21} = \frac{1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23}}{a_{13}a_{32}}. \quad \text{(S-186)}$$

<u>Case 3</u> – Reduction in non-breeder survival (without transition).

The reduction in non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-187)}$$

Rearranging and isolating the term for non-breeder survival (without transition) gives:

$$a_{22} = \frac{1 - a_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32}}{(1 - a_{33})}. \quad \text{(S-188)}$$

<u>Case 4</u> – Reduction in breeder survival (with demotion).

The reduction in breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-189)}$$

Rearranging and isolating the term for breeder survival (with demotion) gives:

$$a_{23} = \frac{1 - a_{22} - a_{33} + a_{22}a_{33} - a_{13}a_{21}a_{32}}{a_{32}}. \quad \text{(S-190)}$$

<u>Case 5</u> – Reduction in non-breeder survival (with transition).

The reduction in non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-191)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{(a_{23} + a_{13}a_{21})}. \quad \text{(S-192)}$$

<u>Case 6</u> – Reduction in breeder survival.

The reduction in breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-193)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1 - a_{22} - a_{32}a_{23} - a_{13}a_{21}a_{32}}{(1 - a_{22})}. \quad \text{(S-194)}$$

<u>Case 7</u> – Reduction in fertility and hatchling survival.

Let the reduction of fertility (a_{13}) be represented as A. Then the reduction of hatchling survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - Aa_{21}a_{32} = 0. \quad \text{(S-195)}$$

Rearranging and isolating the term for hatchling survival gives:

$a_{21} = \frac{1-a_{22}-a_{33}+a_{22}a_{33}-a_{32}a_{23}}{Aa_{32}}.$ (S-196)

Case 8 – Reduction in fertility and non-breeder survival (without transition).
Let the reduction of a_{22} be represented as A. Then the reduction of fertility that must be achieved to halt growth ($\lambda = 1$) is:
$1 - A - a_{33} + Aa_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0.$ (S-197)
Rearranging and isolating the term for fertility gives:
$a_{13} = \frac{1-A-a_{33}+Aa_{33}-a_{32}a_{23}}{a_{21}a_{32}}.$ (S-198)

Case 9 – Reduction in fertility and breeder survival (with demotion).
Let the reduction of a_{23} be represented as A. Then the reduction of fertility that must be achieved to halt growth ($\lambda = 1$) is:
$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}A - a_{13}a_{21}a_{32} = 0.$ (S-199)
Rearranging and isolating the term for fertility gives:
$a_{13} = \frac{1-a_{22}-a_{33}+a_{22}a_{33}-a_{32}A}{a_{21}a_{32}}.$ (S-200)

Case 10 – Reduction in fertility and non-breeder survival (with transition).
Let the reduction of a_{32} be represented as A. Then the level of fertility that must be achieved to halt growth ($\lambda = 1$) is:
$1 - a_{22} - a_{33} + a_{22}a_{33} - Aa_{23} - a_{13}a_{21}A = 0.$ (S-201)
Rearranging and isolating the term for fertility gives:
$a_{13} = \frac{1-a_{22}-a_{33}+a_{22}a_{33}-Aa_{23}}{a_{21}A}.$ (S-202)

Case 11 – Reduction in fertility and breeder survival.
Let the reduction of a_{33} be represented as A. Then the reduction of fertility that must be achieved to halt growth ($\lambda = 1$) is:
$1 - A - a_{22} + Aa_{22} - a_{32}a_{23} - a_{21}a_{13}a_{32} = 0.$ (S-203)
Rearranging and isolating the term for fertility gives:
$a_{13} = \frac{1-A-a_{22}+Aa_{22}-a_{32}a_{23}}{a_{21}a_{32}}.$ (S-204)

Case 12 – Reduction in hatchling survival and non-breeder survival (without transition).
Let the reduction of a_{21} be represented as A. Then the reduction in non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:
$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}Aa_{32} = 0.$ (S-205)
Rearranging and isolating the term for non-breeder survival (without transition) gives:

$$a_{22} = \frac{1-a_{33} - a_{32}a_{23} - a_{13}Aa_{32}}{(1-a_{33}).}. \quad \text{(S-206)}$$

Case 13 – Reduction in hatchling survival and breeder survival (with demotion).

Let the reduction of a_{21} be represented as A. Then the reduction in breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}Aa_{32} = 0. \quad \text{(S-207)}$$

Rearranging and isolating the term for non-breeder survival (without transition) gives:

$$a_{23} = \frac{1-a_{22}-a_{33}+a_{22}a_{33} - a_{13}Aa_{32}}{a_{32}}. \quad \text{(S-208)}$$

Case 14 – Reduction in hatchling survival and non-breeder survival (with transition).

Let the reduction of a_{21} be represented as A. Then the reduction in non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}Aa_{32} = 0. \quad \text{(S-209)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1-a_{22}-a_{33}+a_{22}a_{33}}{(a_{23}+a_{13}A)}. \quad \text{(S-210)}$$

Case 15 – Reduction in hatchling survival and breeder survival.

Let the reduction of a_{33} be represented as A. Then the reduction of hatchling survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - A + a_{22}A - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-211)}$$

Rearranging and isolating the term for hatchling survival gives:

$$a_{21} = \frac{1-a_{22}-A+a_{22}A - a_{32}a_{23}}{a_{13}a_{32}}. \quad \text{(S-212)}$$

Case 16 – Reduction in non-breeder survival (without transition) and reduction in breeder survival (with demotion).

Let the reduction of a_{22} be represented as A. Then the reduction of breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - A - a_{33} + Aa_{33} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-213)}$$

Rearranging and isolating the term for breeder survival (with demotion) gives:

$$a_{23} = \frac{1-A-a_{33}+Aa_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32}}{a_{32}}. \quad \text{(S-214)}$$

Case 17 – Reduction in non-breeder survival (without transition) and non-breeder survival (with transition).

Let the reduction of a_{22} be represented as A. Then the reduction in non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - A - a_{33} + Aa_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-215)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1-A-a_{33}+Aa_{33}}{(a_{23}+a_{13}a_{21})}. \quad \text{(S-216)}$$

Case 18 – Reduction in non-breeder survival (without transition) and breeder survival.

Let the reduction of a_{22} be represented as A. Then the reduction of breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - A - a_{33} + Aa_{33} - a_{32}a_{23} - a_{13}a_{21}a_{32} = 0. \quad \text{(S-217)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1-A - a_{32}a_{23} - a_{13}a_{21}a_{32}}{(1-A)}. \quad \text{(S-218)}$$

Case 19 – Reduction in breeder survival (with demotion) and non-breeder survival (with transition).

Let the reduction of a_{23} be represented as A. Then the reduction of non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}A - a_{13}a_{21}a_{32} = 0. \quad \text{(S-219)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1-a_{22}-a_{33}+a_{22}a_{33}}{(A+a_{13}a_{21}).}. \quad \text{(S-220)}$$

Case 20 – Reduction in breeder survival (with demotion) and breeder survival.

Let the reduction of a_{23} be represented as A. Then the reduction of breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}A - a_{13}a_{21}a_{32} = 0. \quad \text{(S-221)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1-a_{22} - a_{32}A - a_{13}a_{21}a_{32}}{(1-a_{22})}. \quad \text{(S-222)}$$

Case 21 – Reduction in non-breeder survival (with transition) and breeder survival.

Let the reduction of a_{32} be represented as A. Then the reduction of breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - Aa_{23} - a_{13}a_{21}A = 0. \quad \text{(S-223)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1-a_{22} - Aa_{23} - a_{13}a_{21}A}{(1-a_{22})}. \quad \text{(S-224)}$$

<u>Case 22</u> – Reduction in fertility, hatchling survival, and non-breeder survival (without transition).

Let the reduction of a_{13} be represented as A. Let the reduction in hatchling survival be represented by B. Then the reduction of non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - ABa_{32} = 0. \quad \text{(S-225)}$$

Rearranging and isolating the term for non-breeder survival (without transition) gives:

$$a_{22} = \frac{1 - a_{33} - a_{32}a_{23} - ABa_{32}}{(1 - a_{33})}. \quad \text{(S-226)}$$

<u>Case 23</u> – Reduction in fertility, hatchling survival, and breeder survival (with demotion).

Let the reduction of a_{13} be represented as A. Let the reduction in hatchling survival be represented by B. Then the reduction of breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - ABa_{32} = 0. \quad \text{(S-227)}$$

Rearranging and isolating the term for breeder survival (with demotion) gives:

$$a_{23} = \frac{1 - a_{22} - a_{33} + a_{22}a_{33} - ABa_{32}}{a_{32}}. \quad \text{(S-228)}$$

<u>Case 24</u> – Reduction in fertility, hatchling survival, and non-breeder survival (with transition).

Let the reduction of a_{13} be represented as A. Let the reduction in hatchling survival be represented by B. Then the reduction of non-breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - ABa_{32} = 0. \quad \text{(S-229)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{(a_{23} + AB)}. \quad \text{(S-230)}$$

<u>Case 25</u> – Reduction in fertility, hatchling survival, and breeder survival.

Let the reduction of a_{13} be represented as A. Let the reduction in hatchling survival be represented by B. Then the reduction of breeder survival that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - ABa_{32} = 0. \quad \text{(S-231)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1 - a_{22} - a_{32}a_{23} - ABa_{32}}{(1 - a_{22})}. \quad \text{(S-232)}$$

<u>Case 26</u> – Reduction in fertility, non-breeder survival (without transition) and breeder survival (with demotion).

Let the reduction of a_{13} be represented as A. Let the reduction in a_{22} be represented by B. Then the reduction of a_{23} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - B - a_{33} + Ba_{33} - a_{32}a_{23} - Aa_{21}a_{32} = 0. \quad \text{(S-233)}$$

Rearranging and isolating the term for breeder survival (with demotion) gives:

$a_{23} = \frac{1-B-a_{33}+Ba_{33} - Aa_{21}a_{32}}{a_{32}}$. (S-234)

<u>Case 27</u> – Reduction in fertility, non-breeder survival (without transition) and non-breeder survival (with transition).

Let the reduction of a_{13} be represented as A. Let the reduction in a_{22} be represented by B. Then the reduction of a_{32} that must be achieved to halt growth ($\lambda = 1$) is:

$1 - B - a_{33} + Ba_{33} - a_{32}a_{23} - Aa_{21}a_{32} = 0$. (S-235)

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$a_{32} = \frac{1-B-a_{33}+Ba_{33}}{(a_{23}+Aa_{21})}$. (S-236)

<u>Case 28</u> – Reduction in fertility, non-breeder survival (without transition) and breeder survival.

Let the reduction of a_{13} be represented as A. Let the reduction in a_{22} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$1 - B - a_{33} + Ba_{33} - a_{32}a_{23} - Aa_{21}a_{32} = 0$. (S-237)

Rearranging and isolating the term for breeder survival gives:

$a_{33} = \frac{1-B - a_{32}a_{23} - Aa_{21}a_{32}}{(1-B)}$. (S-238)

<u>Case 29</u> – Reduction in fertility, breeder survival (with demotion) and non-breeder survival (with transition).

Let the reduction of a_{13} be represented as A. Let the reduction in a_{23} be represented by B. Then the reduction of a_{32} that must be achieved to halt growth ($\lambda = 1$) is:

$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}B - Aa_{21}a_{32} = 0$. (S-239)

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$a_{32} = \frac{1-a_{22}-a_{33}+a_{22}a_{33}}{(B+Aa_{21})}$. (S-240)

<u>Case 30</u> – Reduction in fertility, breeder survival (with demotion) and breeder survival.

Let the reduction of a_{13} be represented as A. Let the reduction in a_{23} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}B - Aa_{21}a_{32} = 0$. (S-241)

Rearranging and isolating the term for breeding survival gives:

$a_{33} = \frac{1-a_{22} - a_{32}B - Aa_{21}a_{32}}{(1-a_{22})}$. (S-242)

<u>Case 31</u> – Reduction fertility, non-breeder survival (with transition) and breeder survival.

Let the reduction of a_{13} be represented as A. Let the reduction in a_{32} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - Ba_{23} - Aa_{21}B = 0. \qquad \text{(S-243)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1 - a_{22} - Ba_{23} - Aa_{21}B}{(1 - a_{22})}. \qquad \text{(S-244)}$$

<u>Case 32</u> – Reduction in hatchling survival, non-breeder survival (without transition), and breeder survival (with demotion).

Let the reduction of a_{21} be represented as A. Let the reduction in a_{22} be represented by B. Then the reduction of a_{23} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - B - a_{33} + Ba_{33} - a_{32}a_{23} - a_{13}Aa_{32} = 0. \qquad \text{(S-245)}$$

Rearranging and isolating the term for breeder survival (with demotion) gives:

$$a_{23} = \frac{1 - B - a_{33} + Ba_{33} - a_{13}Aa_{32}}{a_{32}}. \qquad \text{(S-246)}$$

<u>Case 33</u> – Reduction in hatchling survival, non-breeder survival (without transition), and non-breeder survival (with transition).

Let the reduction of a_{21} be represented as A. Let the reduction in a_{22} be represented by B. Then the reduction of a_{32} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - B - a_{33} + Ba_{33} - a_{32}a_{23} - a_{13}Aa_{32} = 0. \qquad \text{(S-247)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1 - B - a_{33} + Ba_{33}}{(a_{23} + a_{13}A)}. \qquad \text{(S-248)}$$

<u>Case 34</u> – Reduction in hatchling survival, non-breeder survival (without transition) and breeder survival.

Let the reduction of a_{21} be represented as A. Let the reduction in a_{22} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - B - a_{33} + Ba_{33} - a_{32}a_{23} - a_{13}Aa_{32} = 0. \qquad \text{(S-249)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1 - B - a_{32}a_{23} - a_{13}Aa_{32}}{(1 - B)}. \qquad \text{(S-250)}$$

<u>Case 35</u> – Reduction in hatchling survival, breeder survival (with demotion) and non-breeder survival (with transition).

Let the reduction of a_{21} be represented as A. Let the reduction in a_{23} be represented by B. Then the reduction of a_{32} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}B - a_{13}Aa_{32} = 0. \qquad \text{(S-251)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{(B + a_{13}A)}. \qquad \text{(S-252)}$$

<u>Case 36</u> – Reduction hatchling survival, breeder survival (with demotion), and breeder survival.

Let the reduction of a_{21} be represented as A. Let the reduction in a_{23} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}B - a_{13}Aa_{32} = 0. \quad \text{(S-253)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1 - a_{22} - a_{32}B - a_{13}Aa_{32}}{(1 - a_{22})}. \quad \text{(S-254)}$$

<u>Case 37</u> – Reduction in hatchling survival, non-breeder survival (with transition) and breeder survival.

Let the reduction of a_{21} be represented as A. Let the reduction in a_{32} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - Ba_{23} - a_{13}AB = 0. \quad \text{(S-255)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1 - a_{22} - Ba_{23} - a_{13}AB}{(1 - a_{22})}. \quad \text{(S-256)}$$

<u>Case 38</u> – Reduction in reduction in non-breeder survival (without transition), breeder survival (with demotion), and non-breeder survival (with transition).

Let the reduction of a_{22} be represented as A. Let the reduction in a_{23} be represented by B. Then the reduction of a_{32} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - A - a_{33} + Aa_{33} - a_{32}B - a_{13}a_{21}a_{32} = 0. \quad \text{(S-257)}$$

Rearranging and isolating the term for non-breeder survival (with transition) gives:

$$a_{32} = \frac{1 - A - a_{33} + Aa_{33}}{(B + a_{13}a_{21})}. \quad \text{(S-258)}$$

<u>Case 39</u> – Reduction in reduction in non-breeder survival (without transition), breeder survival (with demotion), and breeder survival.

Let the reduction of a_{22} be represented as A. Let the reduction in a_{23} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - A - a_{33} + Aa_{33} - a_{32}B - a_{13}a_{21}a_{32} = 0. \quad \text{(S-259)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1 - A - a_{32}B - a_{13}a_{21}a_{32}}{(1 - A)}. \quad \text{(S-260)}$$

<u>Case 40</u> – Reduction in reduction in non-breeder survival (without transition), non-breeder survival (with transition), and breeder survival.

Let the reduction of a_{22} be represented as A. Let the reduction in a_{32} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - A - a_{33} + Aa_{33} - Ba_{23} - a_{13}a_{21}B = 0. \quad \text{(S-261)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1-A - Ba_{23} - a_{13}a_{21}B}{(1-A).}. \quad \text{(S-262)}$$

Case 41 - Reduction in breeder survival (with demotion), non-breeder survival (with transition), and breeder survival.

Let the reduction of a_{23} be represented as A. Let the reduction in a_{32} be represented by B. Then the reduction of a_{33} that must be achieved to halt growth ($\lambda = 1$) is:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - BA - a_{13}a_{21}B = 0. \quad \text{(S-263)}$$

Rearranging and isolating the term for breeder survival gives:

$$a_{33} = \frac{1-a_{22} - BA - a_{13}a_{21}B}{(1-a_{22})}. \quad \text{(S-264)}$$

3.2 Symbolic reductions in proportions of one, two, or three vital rates

Now suppose we wish to calculate the symbolic reduction in proportion of one, two, or three vital rates necessary to drive the growth rate to one ($\lambda = 1$).

Case 1 – Reduction in fertility.

Please see Shields *et al.* (2019*a*, *b*).

Case 2 – Reduction in hatchling survival.

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings not killed. Hatchlings can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{21} + (1 - X)\, a_{21}. \quad \text{(S-265)}$$

Surviving hatchings reduces eq. S-265 to:

$$(1 - X)\, a_{21}. \quad \text{(S-266)}$$

Substitution of eq. S-266 into the characteristic equations gives:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - a_{13}\,(1 - X)\, a_{21}a_{32} = 0. \quad \text{(S-267)}$$

Rearranging, and solving for X gives the solutions for X that will reduce the growth rate to one:

$$1 - \left(\frac{1-a_{22}-a_{33}+a_{22}a_{33} - a_{32}a_{23}}{a_{13}a_{21}a_{32}}\right) = X. \quad \text{(S-268)}$$

Equation S-268 specifies the proportion of hatchlings that must be killed (X) to reduce the growth rate to 1.

Case 3 – Reduction in non-breeder survival (without transition).

Let X be the proportion of hatchlings non-breeders killed. Then $1 - X$ is the proportion of non-breeders not killed. Non-breeders can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{22} + (1 - X)\, a_{22}. \quad \text{(S-269)}$$

Surviving non-breeders reduces eq. S-269 to:

$$(1 - X)\, a_{22}. \quad \text{(S-270)}$$

Substitution of eq. S-266 into the characteristic equations gives:

$1-(1-X)\,a_{22}-a_{33}+(1-X)\,a_{22}a_{33}-a_{32}a_{23}-a_{13}\,a_{21}a_{32}=0.$ (S-271)

Rearranging, and solving for X gives the solutions for X that will reduce the growth rate to one:

$1-(\frac{1-a_{33}-a_{32}a_{23}-a_{13}\,a_{21}a_{32}}{(a_{22}-a_{22}a_{33})})=X.$ (S-272)

Equation S-272 specifies the proportion of non-breeders (without transition) that must be killed (X) to reduce the growth rate to 1.

<u>Case 4</u> – Reduction in breeder survival (with demotion).

Let X be the proportion of breeders that are killed. Then $1-X$ is the proportion of breeders not killed. Breeders can be broken up into those that are killed (X), and those that are not killed:

$Xa_{23}+(1-X)\,a_{23}.$ (S-273)

Surviving breeders reduces eq. S-273 to:

$(1-X)\,a_{23}.$ (S-274)

Substitution of eq. S-274 into the characteristic equations gives:

$1-a_{22}-a_{33}+a_{22}a_{33}-a_{32}(1-X)\,a_{23}-a_{13}\,a_{21}a_{32}=0.$ (S-275)

Rearranging, and solving for X gives the solutions for X that will reduce the growth rate to one:

$1-\left(\frac{1-a_{22}-a_{33}+a_{22}a_{33}-a_{13}\,a_{21}a_{32}}{a_{32}a_{23}}\right)=X.$ (S-276)

Equation S-276 specifies the proportion of breeders (with demotion) that must be killed (X) to reduce the growth rate to 1.

<u>Case 5</u> – Reduction in non-breeder survival (with transition).

Let X be the proportion of non-breeders killed. Then $1-X$ is the proportion of non-breeders not killed. Non-breeders can be broken up into those that are killed (X), and those that are not killed:

$Xa_{32}+(1-X)\,a_{32}.$ (S-277)

Surviving non-breeders reduces eq. S-277 to:

$(1-X)\,a_{32}.$ (S-278)

Substitution of eq. S-278 into the characteristic equations gives:

$1-a_{22}-a_{33}+a_{22}a_{33}-(1-X)\,a_{32}a_{23}-a_{13}\,a_{21}(1-X)\,a_{32}=0.$ (S-279)

Rearranging, and solving for X gives the solutions for X that will reduce the growth rate to one:

$1-(\frac{1-a_{22}-a_{33}+a_{22}a_{33}}{(a_{32}a_{23}+a_{13}\,a_{21}\,a_{32})})=X.$ (S-280)

Equation S-280 specifies the proportion of non-breeders (with transition) that must be killed (X) to reduce the growth rate to 1.

<u>Case 6</u> – Reduction in breeder survival.

Let X be the proportion of breeders killed. Then $1-X$ is the proportion of breeders not killed. Breeders can be broken up into those that are killed (X), and those that are not killed:

$Xa_{33}+(1-X)\,a_{33}.$ (S-281)

Surviving breeders reduces eq. S-281 to:

$(1-X)\,a_{33}.$ (S-282)

Substitution of eq. S-282 into the characteristic equations gives:

$1{-}a_{22} - (1 - X)\ a_{33} + a_{22}(1 - X)\ a_{33} - a_{32}a_{23} - a_{13}\ a_{21}a_{32} = 0.$ (S-283)
Rearranging, and solving for X gives the solutions for X that will reduce the growth rate to one:
$1 - (\frac{1-a_{22} - a_{32}a_{23} - a_{13}\ a_{21}a_{32}}{(a_{33} - a_{22}\ a_{33})}) = X.$ (S-284)
Equation S-284 specifies the proportion of breeders that must be killed (X) to reduce the growth rate to 1.

<u>Case 7</u> – Reduction in fertility and hatchling survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:
$Xa_{13} + (1 - X)\ a_{13}.$ (S-285)
Surviving eggs reduces eq. S-285 to:
$(1 - X)\ a_{13}.$ (S-286)
Survival can also be broken up into those hatchlings that are killed (Y), and those that are not killed:
$Ya_{21} + (1 - Y)\ a_{21}.$ (S-287)
Surviving hatchlings reduces eq. S-287 to:
$(1 - Y)\ a_{21}.$ (S-288)
Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:
$1{-}a_{22} - a_{33} + a_{22}a_{33} - a_{32}a_{23} - (1 - X)a_{13}(1 - Y)\ a_{21}\ a_{32} = 0.$ (S-289)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$\frac{1-a_{22}-a_{33}+a_{22}a_{33} - a_{32}a_{23}}{a_{13}a_{21}\ a_{32}} = (1 - X)(1 - Y).$ (S-290)
Equation S-290 specifies the proportion of hatchlings that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

<u>Case 8</u> – Reduction in fertility and non-breeder survival (without transition).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:
$Xa_{13} + (1 - X)\ a_{13}.$ (S-291)
Surviving eggs reduces eq. S-291 to:
$(1 - X)\ a_{13}.$ (S-292)
Survival can also be broken up into those non-breeders that are killed (Y), and those that are not killed:
$Ya_{22} + (1 - Y)\ a_{22}.$ (S-293)
Surviving non-breeders reduces eq. S-293 to:
$(1 - Y)\ a_{22}.$ (S-294)
Substitution of eq. S-292 and eq. S-294 into the characteristic equation gives:
$1{-}(1 - Y)a_{22} - a_{33} + (1 - Y)a_{22}a_{33} - a_{32}a_{23} - (1 - X)a_{13}\ a_{21}\ a_{32} = 0.$ (S-295)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$1 - a_{33} - a_{32}a_{23} = (1 - X)a_{13}\ a_{21}\ a_{32} - (1 - Y)a_{22}a_{33} + (1 - Y)a_{22}.$ (S-296)

Equation S-296 specifies the proportion of non-breeders (without transition) that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

Case 9 – Reduction in fertility and breeder survival (with demotion).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{13} + (1 - X)\, a_{13}. \qquad \text{(S-297)}$$

Surviving eggs reduces eq. S-297 to:

$$(1 - X)\, a_{13}. \qquad \text{(S-298)}$$

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$$Ya_{23} + (1 - Y)\, a_{23}. \qquad \text{(S-299)}$$

Surviving breeders reduces eq. S-287 to:

$$(1 - Y)\, a_{23}. \qquad \text{(S-300)}$$

Substitution of eq. S-298 and eq. S-300 into the characteristic equation gives:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}(1 - Y)a_{23} - (1 - X)a_{13}\, a_{21}\, a_{32} = 0. \qquad \text{(S-301)}$$

Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:

$$\frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{a_{32}} = (1 - Y)a_{23} + (1 - X)a_{13}\, a_{21}. \qquad \text{(S-302)}$$

Equation S-302 specifies the proportion of breeders (with demotion) that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

Case 10 – Reduction in fertility and non-breeder survival (with transition).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{13} + (1 - X)\, a_{13}. \qquad \text{(S-303)}$$

Surviving eggs reduces eq. S-303 to:

$$(1 - X)\, a_{13}. \qquad \text{(S-304)}$$

Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:

$$Ya_{32} + (1 - Y)\, a_{32}. \qquad \text{(S-305)}$$

Surviving non-breeders reduces eq. S-305 to:

$$(1 - Y)\, a_{32}. \qquad \text{(S-306)}$$

Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - (1 - Y)\, a_{32}a_{23} - (1 - X)a_{13}\, a_{21}\, (1 - Y)a_{32} = 0. \qquad \text{(S-307)}$$

Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:

$$\frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{a_{32}} = (1 - Y)a_{23} + (1 - X)a_{13}\, a_{21}\, (1 - Y). \qquad \text{(S-308)}$$

Equation S-308 specifies the proportion of non-breeders (with transition) that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

Case 11 – Reduction in fertility and breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$Xa_{13} + (1 - X)\, a_{13}$. (S-309)

Surviving eggs reduces eq. S-309 to:

$(1 - X)\, a_{13}$. (S-310)

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$Ya_{33} + (1 - Y)\, a_{33}$. (S-311)

Surviving breeders reduces eq. S-311 to:

$(1 - Y)\, a_{33}$. (S-312)

Substitution of eq. S-310 and eq. S-312 into the characteristic equation gives:

$1 - a_{22} - (1 - Y)a_{33} + a_{22}(1 - Y)a_{33} - a_{32}a_{23} - (1 - X)a_{13}\, a_{21}\, a_{32} = 0$. (S-313)

Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:

$1 - a_{22} - a_{32}a_{23} = (1 - Y)a_{33} - a_{22}(1 - Y)a_{33} + (1 - X)a_{13}\, a_{21}\, a_{32}$. (S-314)

Equation S-314 specifies the proportion of breeders that must be killed (Y) alongside the proportion of eggs that must be killed (X) to reduce the growth rate to 1.

Case 12 – Reduction in hatchling survival and non-breeder survival (without transition).

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings not killed. Hatchlings survival be broken up into those that are killed (X), and those that are not killed:

$Xa_{21} + (1 - X)\, a_{21}$. (S-315)

Surviving hatchlings reduces eq. S-315 to:

$(1 - X)\, a_{21}$. (S-316)

Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:

$Ya_{22} + (1 - Y)\, a_{22}$. (S-317)

Surviving non-breeders reduces eq. S-317 to:

$(1 - Y)\, a_{22}$. (S-318)

Substitution of eq. S-316 and eq. S-318 into the characteristic equation gives:

$1 - (1 - Y)a_{22} - a_{33} + (1 - Y)a_{22}a_{33} - a_{32}a_{23} - a_{13}\,(1 - X)a_{21}\, a_{32} = 0$. (S-319)

Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:

$1 - a_{33} - a_{32}a_{23} = (1 - Y)a_{22} - (1 - Y)a_{22}a_{33} + a_{13}(1 - X)\, a_{21}\, a_{32}$. (S-320)

Equation S-320 specifies the proportion of non-breeders (without transition) that must be killed (Y) alongside the proportion of hatchlings that must be killed (X) to reduce the growth rate to 1.

Case 13 – Reduction in hatchling survival and breeder survival (with demotion).

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings not killed. Hatchling survival can be broken up into those that are killed (X), and those that are not killed:

$Xa_{21} + (1 - X)\, a_{21}$. (S-321)

Surviving hatchlings reduces eq. S-321 to:

$(1-X)\ a_{21}$. (S-322)
Survival can also be broken up into breeders that are killed (Y), and those that are not killed:
$Ya_{23}+(1-Y)\ a_{23}$. (S-323)
Surviving breeders reduces eq. S-323 to:
$(1-Y)\ a_{23}$. (S-324)
Substitution of eq. S-322 and eq. S-424 into the characteristic equation gives:
$1-a_{22}-a_{33}+a_{22}a_{33}-a_{32}(1-Y)a_{23}-a_{13}(1-X)\ a_{21}\ a_{32}=0$. (S-325)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$\frac{1-a_{22}-a_{33}+a_{22}a_{33}}{a_{32}}=\ (1-Y)a_{23}+a_{13}(1-X)a_{21}$. (S-326)
Equation S-326 specifies the proportion of breeders (with demotion) that must be killed (Y) alongside the proportion of hatchlings that must be killed (X) to reduce the growth rate to 1.

<u>Case 14</u> – Reduction in hatchling survival and non-breeder survival (with transition).

Let X be the proportion of hatchlings killed. Then $1-X$ is the proportion of hatchlings not killed.
Hatchling survival can be broken up into those that are killed (X), and those that are not killed:
$Xa_{21}+(1-X)\ a_{21}$. (S-327)
Surviving hatchlings reduces eq. S-327 to:
$(1-X)\ a_{21}$. (S-328)
Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:
$Ya_{32}+(1-Y)\ a_{32}$. (S-329)
Surviving non-breeders reduces eq. S-329 to:
$(1-Y)\ a_{32}$. (S-330)
Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:
$1-a_{22}-a_{33}+a_{22}a_{33}-(1-Y)\ a_{32}a_{23}-a_{13}(1-X)\ a_{21}\ a_{32}=0$. (S-331)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$\frac{1-a_{22}-a_{33}+a_{22}a_{33}}{a_{32}}=\ (1-Y)a_{23}+a_{13}(1-X)\ a_{21}$. (S-332)
Equation S-332 specifies the proportion of non-breeders (with transition) that must be killed (Y) alongside the proportion of hatchlings that must be killed (X) to reduce the growth rate to 1.

<u>Case 15</u> – Reduction in hatchling survival and breeder survival.

Let X be the proportion of hatchlings killed. Then $1-X$ is the proportion of hatchlings not killed.
Hatchling survival can be broken up into those that are killed (X), and those that are not killed:
$Xa_{21}+(1-X)\ a_{21}$. (S-333)
Surviving hatchlings reduces eq. S-333 to:
$(1-X)\ a_{21}$. (S-334)
Survival can also be broken up into breeders that are killed (Y), and those that are not killed:
$Ya_{33}+(1-Y)\ a_{33}$. (S-335)
Surviving breeders reduces eq. S-335 to:
$(1-Y)\ a_{33}$. (S-336)

Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:
$1 - a_{22} - (1-Y)a_{33} + a_{22}(1-Y)a_{33} - a_{32}a_{23} - a_{13}(1-X)\,a_{21}\,a_{32} = 0.$ (S-337)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$1 - a_{22} - a_{32}a_{23} = (1-Y)a_{33} - a_{22}(1-Y)a_{33} + a_{13}(1-X)\,a_{21}\,a_{32}.$ (S-338)
Equation S-338 specifies the proportion of breeders that must be killed (Y) alongside the proportion of hatchlings that must be killed (X) to reduce the growth rate to 1.

<u>Case 16</u> – Reduction in non-breeding survival (without transition) and breeder survival (with demotion).

Let X be the proportion of non-breeders killed. Then $1-X$ is the proportion of non-breeders not killed. Non-breeder survival can be broken up into those that are killed (X), and those that are not killed:
$Xa_{22} + (1-X)\,a_{22}.$ (S-339)
Surviving non-breeders reduces eq. S-339 to:
$(1-X)\,a_{22}.$ (S-340)
Survival can also be broken up into those breeders that are killed (Y), and those that are not killed:
$Ya_{23} + (1-Y)\,a_{23}.$ (S-341)
Surviving breeders reduces eq. S-341 to:
$(1-Y)\,a_{23}.$ (S-342)
Substitution of eq. S-340 and eq. S-342 into the characteristic equation gives:
$1 - (1-X)a_{22} - a_{33} + (1-X)a_{22}a_{33} - a_{32}(1-Y)a_{23} - a_{13}\,a_{21}\,a_{32} = 0.$ (S-343)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$1 - a_{33} - a_{13}\,a_{21}\,a_{32} = (1-X)a_{22} - (1-X)a_{22}a_{33} + a_{32}(1-Y)a_{23}.$ (S-344)
Equation S-344 specifies the proportion of breeders (with demotion) that must be killed (Y) alongside the proportion of non-breeders (without transition) that must be killed (X) to reduce the growth rate to 1.

<u>Case 17</u> – Reduction in non-breeding survival (without transition) and non-breeder survival (with transition).

Let X be the proportion of non-breeders killed. Then $1-X$ is the proportion of non-breeders not killed. Non-breeder survival can be broken up into those that are killed (X), and those that are not killed:
$Xa_{22} + (1-X)\,a_{22}.$ (S-345)
Surviving non-breeders reduces eq. S-345 to:
$(1-X)\,a_{22}.$ (S-346)
Survival of the remaining non-breeders can be broken up into those that are killed (Y), and those that are not killed:
$Ya_{32} + (1-Y)\,a_{32}.$ (S-347)
Surviving non-breeders reduces eq. S-347 to:
$(1-Y)\,a_{32}.$ (S-348)
Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:

$1-(1-X)a_{22} - a_{33} + (1-X)a_{22}a_{33} - (1-Y)\, a_{32}a_{23} - a_{13}\, a_{21}\, (1-Y)a_{32} = 0.$ (S-349)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$1 - a_{33} = (1-X)a_{22} - (1-X)a_{22}a_{33} + (1-Y)a_{32}a_{23} + a_{13}\, a_{21}\, (1-Y)a_{32}$ (S-350)
Equation S-350 specifies the proportion of non-breeders (with transition) that must be killed (Y) alongside the proportion of non-breeders (without transition) that must be killed (X) to reduce the growth rate to 1.

<u>Case 18</u> – Reduction in non-breeding survival (without transition) and breeder survival.

Let X be the proportion of non-breeders killed. Then $1 - X$ is the proportion of non-breeders not killed. Non-breeder survival can be broken up into those that are killed (X), and those that are not killed:
$Xa_{22} + (1-X)\, a_{22}.$ (S-351)
Surviving non-breeders reduces eq. S-351 to:
$(1-X)\, a_{22}.$ (S-352)
Survival can also be broken up into breeders that are killed (Y), and those that are not killed:
$Ya_{33} + (1-Y)\, a_{33}.$ (S-353)
Surviving hatchling ravens reduces eq. S-353 to:
$(1-Y)\, a_{33}.$ (S-354)
Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:
$1-(1-X)a_{22} - (1-Y)a_{33} + (1-X)a_{22}(1-Y)a_{33} - a_{32}a_{23} - a_{13}\, a_{21}\, a_{32} = 0.$ (S-355)
Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:
$1 - a_{32}a_{23} - a_{13}\, a_{21}\, a_{32} = (1-X)a_{22} + (1-Y)a_{33} - (1-X)a_{22}(1-Y)a_{33}.$ (S-356)
Equation S-356 specifies the proportion of breeders that must be killed (Y) alongside the proportion of non-breeders (without transition) that must be killed (X) to reduce the growth rate to 1.

<u>Case 19</u> – Reduction in breeding survival (with demotion) and non-breeder survival (with transition).

Let X be the proportion of breeders killed. Then $1 - X$ is the proportion of breeders not killed. Survival of breeders can be broken up into those that are killed (X), and those that are not killed:
$Xa_{23} + (1-X)\, a_{23}.$ (S-357)
Surviving breeders reduces eq. S-357 to:
$(1-X)\, a_{23}.$ (S-358)
Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:
$Ya_{32} + (1-Y)\, a_{32}.$ (S-359)
Surviving non-breeding ravens reduces eq. S-359 to:
$(1-Y)\, a_{32}.$ (S-360)
Substitution of eq. S-358 and eq. S-360 into the characteristic equation gives:
$1-a_{22} - a_{33} + a_{22}a_{33} - (1-Y)\, a_{32}(1-X)a_{23} - a_{13}\, a_{21}\, (1-Y)a_{32} = 0.$ (S-361)

Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:

$$\frac{1-a_{22}-a_{33}+a_{22}a_{33}}{a_{32}} = (1-Y)(1-X)a_{23} + a_{13}\, a_{21}\,(1-Y). \tag{S-362}$$

Equation S-362 specifies the proportion of non-breeders (with transition) that must be killed (Y) alongside the proportion of breeders (with demotion) that must be killed (X) to reduce the growth rate to 1.

<u>Case 20</u> – Reduction in breeding survival (with demotion) and breeder survival.

Let X be the proportion of breeders killed. Then $1-X$ is the proportion of breeders not killed. Survival of breeders can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{23} + (1-X)\,a_{23}. \tag{S-363}$$

Surviving breeders reduces eq. S-363 to:

$$(1-X)\,a_{23}. \tag{S-364}$$

Survival can also be broken up into breeders are killed (Y), and those that are not killed:

$$Ya_{33} + (1-Y)\,a_{33}. \tag{S-365}$$

Surviving breeders reduces eq. S-365 to:

$$(1-Y)\,a_{33}. \tag{S-366}$$

Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:

$$1-a_{22} - (1-Y)a_{33} + a_{22}(1-Y)a_{33} - a_{32}(1-X)a_{23} - a_{13}\,a_{21}\,a_{32} = 0. \tag{S-367}$$

Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:

$$1-a_{22} - a_{13}\,a_{21}\,a_{32} = (1-Y)a_{33} - a_{22}(1-Y)a_{33} + a_{32}(1-X)a_{23}. \tag{S-368}$$

Equation S-368 specifies the proportion of breeders that must be killed (Y) alongside the proportion of breeders (with demotion) that must be killed (X) to reduce the growth rate to 1.

<u>Case 21</u> – Reduction in non-breeding survival (with transition) and breeder survival.

Let X be the proportion of non-breeders that are killed. Then $1-X$ is the proportion of non-breeders not killed. Survival of non-breeders can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{32} + (1-X)\,a_{32}. \tag{S-369}$$

Surviving non-breeders reduces eq. S-369 to:

$$(1-X)\,a_{32}. \tag{S-370}$$

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$$Ya_{33} + (1-Y)\,a_{33}. \tag{S-371}$$

Surviving breeders reduces eq. S-371 to:

$$(1-Y)\,a_{33}. \tag{S-372}$$

Substitution of eq. S-286 and eq. S-288 into the characteristic equation gives:

$$1-a_{22} - (1-Y)a_{33} + a_{22}(1-Y)a_{33} - (1-X)a_{32}a_{23} - a_{13}\,a_{21}\,(1-X)a_{32} = 0, \tag{S-373}$$

Rearranging, and solving for X and Y gives all the combinatorial solutions for X and Y that will reduce the growth rate to one:

$$1-a_{22} = (1-Y)a_{33} - a_{22}(1-Y)a_{33} + (1-X)a_{32}a_{23} + a_{13}\,a_{21}\,(1-X)a_{32}. \tag{S-374}$$

Equation S-374 specifies the proportion of breeders that must be killed (Y) alongside the proportion of non-breeders (with transition) that must be killed (X) to reduce the growth rate to 1.

<u>Case 22</u> – Reduction in fertility, hatchling survival, and non-breeder survival (without transition).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{13} + (1 - X)\, a_{13}. \quad \text{(S-375)}$$

Surviving eggs reduces eq. S-375 to:

$$(1 - X)\, a_{13}. \quad \text{(S-376)}$$

Survival can also be broken up into those hatchlings that are killed (Y), and those that are not killed:

$$Ya_{21} + (1 - Y)\, a_{21}. \quad \text{(S-377)}$$

Surviving hatchling ravens reduces eq. S-377 to:

$$(1 - Y)\, a_{21}. \quad \text{(S-378)}$$

Survival can also be broken up into non-breeders that are killed (Z), and those that are not killed:

$$Za_{22} + (1 - Z)\, a_{22}. \quad \text{(S-379)}$$

Surviving non-breeders reduces eq. S-379 to:

$$(1 - Z)\, a_{22}. \quad \text{(S-380)}$$

Substitution of eq. S-376, S378 and eq. S-380 into the characteristic equation gives:

$$1-(1 - Z)a_{22} - a_{33} + (1 - Z)a_{22}a_{33} - a_{32}a_{23} - (1 - X)a_{13}(1 - Y)\, a_{21}\, a_{32} = 0. \quad \text{(S-381)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 - a_{33} - a_{32}a_{23} = (1 - Z)a_{22} - (1 - Z)a_{22}a_{33} + (1 - X)a_{13}(1 - Y)\, a_{21}\, a_{32}. \quad \text{(S-382)}$$

Equation S-382 specifies the proportion of eggs that must be killed (X), the proportion of hatchlings that must be killed (Y), and the proportion of non-breeders (without transition) that must be killed (Z) to reduce the growth rate to 1.

<u>Case 23</u> – Reduction in fertility, hatchling survival, and breeder survival (with demotion).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{13} + (1 - X)\, a_{13}. \quad \text{(S-383)}$$

Surviving eggs reduces eq. S-383 to:

$$(1 - X)\, a_{13}. \quad \text{(S-384)}$$

Survival can also be broken up into those hatchlings that are killed (Y), and those that are not killed:

$$Ya_{21} + (1 - Y)\, a_{21}. \quad \text{(S-385)}$$

Surviving hatchling ravens reduces eq. S-385 to:

$$(1 - Y)\, a_{21}. \quad \text{(S-386)}$$

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$$Za_{23} + (1 - Z)\, a_{23}. \quad \text{(S-387)}$$

Surviving hatchling ravens reduces eq. S-287 to:

$(1 - Z)\ a_{23}$. (S-388)

Substitution of eq. S-384, S386 and eq. S-388 into the characteristic equation gives:

$1 - a_{22} - a_{33} + a_{22}a_{33} - a_{32}(1 - Z)a_{23} - (1 - X)a_{13}(1 - Y)\ a_{21}\ a_{32} = 0.$ (S-389)

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$\frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{a_{32}} = (1 - Z)a_{23} + (1 - X)a_{13}(1 - Y)\ a_{21}.$ (S-390)

Equation S-390 specifies the proportion of eggs that must be killed (X), the proportion of hatchlings that must be killed (Y), and the proportion of non-breeders (without transition) that must be killed (Z) to reduce the growth rate to 1.

<u>Case 24</u> – Reduction in fertility, hatchling survival, and non-breeder survival (with transition).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$Xa_{13} + (1 - X)\ a_{13}.$ (S-391)

Surviving eggs reduces eq. S-391 to:

$(1 - X)\ a_{13}.$ (S-392)

Survival can also be broken up into those hatchlings that are killed (Y), and those that are not killed:

$Ya_{21} + (1 - Y)\ a_{21}.$ (S-393)

Surviving hatchling ravens reduces eq. S-393 to:

$(1 - Y)\ a_{21}.$ (S-394)

Survival can also be broken up into non-breeders that are killed (Z), and those that are not killed:

$Za_{32} + (1 - Z)\ a_{32}.$ (S-395)

Surviving non-breeders reduces eq. S-395 to:

$(1 - Z)\ a_{32}.$ (S-396)

Substitution of eq. S-392, S394 and eq. S-396 into the characteristic equation gives:

$1 - a_{22} - a_{33} + a_{22}a_{33} - (1 - Z)\ a_{32}a_{23} - (1 - X)a_{13}(1 - Y)\ a_{21}(1 - Z)\ a_{32} = 0.$ (S-397)

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$\frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{a_{32}} = (1 - Z)a_{23} + (1 - X)a_{13}(1 - Y)\ a_{21}(1 - Z).$ (S-398)

Equation S-399 specifies the proportion of eggs that must be killed (X), the proportion of hatchlings that must be killed (Y), and the proportion of non-breeders (with transition) that must be killed (Z) to reduce the growth rate to 1.

<u>Case 25</u> – Reduction in fertility, hatchling survival, and breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$Xa_{13} + (1 - X)\ a_{13}.$ (S-399)

Surviving eggs reduces eq. S-399 to:

$(1 - X)\ a_{13}.$ (S-400)

Survival can also be broken up into those hatchlings that are killed (Y), and those that are not killed:

$$Ya_{21} + (1 - Y)\, a_{21}. \quad \text{(S-401)}$$

Surviving hatchling ravens reduces eq. S-401 to:

$$(1 - Y)\, a_{21}. \quad \text{(S-402)}$$

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$$Za_{33} + (1 - Z)\, a_{33}. \quad \text{(S-403)}$$

Surviving breeders reduces eq. S-403 to:

$$(1 - Z)\, a_{33}. \quad \text{(S-404)}$$

Substitution of eq. S-400, S402 and eq. S-404 into the characteristic equation gives:

$$1 - a_{22} - (1 - Z)a_{33} + a_{22}(1 - Z)a_{33} - a_{32}a_{23} - (1 - X)a_{13}(1 - Y)\, a_{21}\, a_{32} = 0. \quad \text{(S-405)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 - a_{22} = (1 - Z)a_{33} - a_{22}(1 - Z)a_{33} + a_{32}a_{23} - (1 - X)a_{13}(1 - Y)a_{21}\, a_{32}. \quad \text{(S-406)}$$

Equation S-406 specifies the proportion of eggs that must be killed (X), the proportion of hatchlings that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

<u>Case 26</u> – Reduction in fertility, non-breeder survival (without transition), and breeder survival (with demotion).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{13} + (1 - X)\, a_{13}. \quad \text{(S-407)}$$

Surviving eggs reduces eq. S-407 to:

$$(1 - X)\, a_{13}. \quad \text{(S-408)}$$

Survival can also be broken up into those non-breeders that are killed (Y), and those that are not killed:

$$Ya_{22} + (1 - Y)\, a_{22}. \quad \text{(S-409)}$$

Surviving non-breeders reduces eq. S-409 to:

$$(1 - Y)\, a_{22}. \quad \text{(S-410)}$$

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$$Za_{23} + (1 - Z)\, a_{23}. \quad \text{(S-411)}$$

Surviving breeders reduces eq. S-411 to:

$$(1 - Z)\, a_{23}. \quad \text{(S-412)}$$

Substitution of eq. S-408, S410 and eq. S-412 into the characteristic equation gives:

$$1 - (1 - Y)a_{22} - a_{33} + (1 - Y)a_{22}a_{33} - a_{32}(1 - Z)a_{23} - (1 - X)a_{13}\, a_{21}\, a_{32} = 0. \quad \text{(S-413)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 = (1 - Y)a_{22} + a_{33} - (1 - Y)a_{22}a_{33} + a_{32}(1 - Z)a_{23} + (1 - X)a_{13}\, a_{21}\, a_{32}. \quad \text{(S-414)}$$

Equation S-414 specifies the proportion of eggs that must be killed (X), the proportion of non-breeders (without transition) that must be killed (Y), and the proportion of breeders (with demotion) that must be killed (Z) to reduce the growth rate to 1.

<u>Case 27</u> – Reduction in fertility, non-breeder survival (without transition), and non-breeder survival (with transition).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$Xa_{13} + (1 - X)\, a_{13}$. (S-415)

Surviving eggs reduces eq. S-415 to:

$(1 - X)\, a_{13}$. (S-416)

Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:

$Ya_{22} + (1 - Y)\, a_{22}$. (S-417)

Surviving non-breeders reduces eq. S-417 to:

$(1 - Y)\, a_{22}$. (S-418)

Survival can also be broken up into those non-breeders that are killed (Z), and those that are not killed:

$Za_{32} + (1 - Z)\, a_{32}$. (S-419)

Surviving hatchling ravens reduces eq. S-419 to:

$(1 - Z)\, a_{32}$. (S-420)

Substitution of eq. S-416, S418 and eq. S-420 into the characteristic equation gives:

$1-(1 - Y)a_{22} - a_{33} + (1 - Y)a_{22}a_{33}$- $(1 - Z)a_{32}a_{23} - (1 - X)a_{13}\, a_{21}(1 - Z)\, a_{32} = 0$. (S-421)

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$1 = (1 - Y)a_{22} + a_{33} - (1 - Y)a_{22}a_{33}$+$(1 - Z)a_{32}a_{23} + (1 - X)a_{13}\, a_{21}(1 - Z)\, a_{32}$. (S-422)

Equation S-422 specifies the proportion of eggs that must be killed (X), the proportion of non-breeders (without transition) that must be killed (Y), and the proportion of non-breeders (with transition) that must be killed (Z) to reduce the growth rate to 1.

<u>Case 28</u> – Reduction in fertility, non-breeder survival (without transition), and breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$Xa_{13} + (1 - X)\, a_{13}$. (S-423)

Surviving eggs reduces eq. S-423 to:

$(1 - X)\, a_{13}$. (S-424)

Survival can also be broken up into those non-breeders that are killed (Y), and those that are not killed:

$Ya_{21} + (1 - Y)\, a_{22}$. (S-425)

Surviving non-breeders reduces eq. S-425 to:

$(1 - Y)\, a_{22}$. (S-426)

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$Za_{33} + (1 - Z)\, a_{33}$. (S-427)

Surviving breeders reduces eq. S-427 to:

$(1 - Z)\, a_{33}$. (S-428)

Substitution of eq. S-424, S426 and eq. S-428 into the characteristic equation gives:

$1-(1 - Y)a_{22} - (1 - Z)a_{33} + (1 - Y)a_{22}(1 - Z)a_{33} - a_{32}a_{23} - (1 - X)a_{13}\, a_{21}\, a_{32} = 0$.

(S-429)

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 = (1-Y)a_{22} + (1-Z)a_{33} - (1-Y)a_{22}(1-Z)a_{33} + a_{32}a_{23} + (1-X)a_{13}\, a_{21}\, a_{32}. \quad \text{(S-430)}$$

Equation S-430 specifies the proportion of eggs that must be killed (X), the proportion of non-breeders (without transition) that must be killed (Y), and the proportion of the breeders that must be killed (Z) to reduce the growth rate to 1.

<u>Case 29</u> – Reduction in fertility, breeder survival (with demotion), and non-breeder survival (with transition).

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{13} + (1-X)\, a_{13}. \quad \text{(S-431)}$$

Surviving eggs reduces eq. S-431 to:

$$(1-X)\, a_{13}. \quad \text{(S-432)}$$

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$$Ya_{23} + (1-Y)\, a_{23}. \quad \text{(S-433)}$$

Surviving breeders reduces eq. S-433 to:

$$(1-Y)\, a_{23}. \quad \text{(S-434)}$$

Survival can also be broken up into non-breeders that are killed (Z), and those that are not killed:

$$Za_{32} + (1-Z)\, a_{32}. \quad \text{(S-435)}$$

Surviving non-breeders reduces eq. S-435 to:

$$(1-Z)\, a_{32}. \quad \text{(S-436)}$$

Substitution of eq. S-432, S434 and eq. S-436 into the characteristic equation gives:

$$1 - a_{22} - a_{33} + a_{22}a_{33} - (1-Z)a_{32}(1-Y)a_{23} - (1-X)a_{13}\, a_{21}\, a_{32} = 0. \quad \text{(S-437)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$\frac{1 - a_{22} - a_{33} + a_{22}a_{33}}{a_{32}} = (1-Z)(1-Y)a_{23} + (1-X)a_{13}\, a_{21}. \quad \text{(S-438)}$$

Equation S-438 specifies the proportion of eggs that must be killed (X), the proportion of breeders (with demotion) that must be killed (Y), and the proportion of the non-breeders (with transition) that must be killed (Z) to reduce the growth rate to 1.

<u>Case 30</u> – Reduction in fertility, breeder survival (with demotion), and breeder survival.

Let X be the proportion of eggs killed. Then $1 - X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{13} + (1-X)\, a_{13}. \quad \text{(S-439)}$$

Surviving eggs reduces eq. S-439 to:

$$(1-X)\, a_{13}. \quad \text{(S-440)}$$

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$$Ya_{23} + (1-Y)\, a_{23}. \quad \text{(S-441)}$$

Surviving breeders reduces eq. S-441 to:
$(1-Y)\ a_{23}$. (S-442)
Survival can also be broken up into breeders that are killed (Z), and those that are not killed:
$Za_{33}+(1-Z)\ a_{33}$. (S-443)
Surviving breeders reduces eq. S-443 to:
$(1-Z)\ a_{33}$. (S-444)
Substitution of eq. S-440, S442 and eq. S-444 into the characteristic equation gives:
$1-a_{22}-(1-Z)a_{33}+a_{22}(1-Z)a_{33}-a_{32}(1-Y)a_{23}-(1-X)a_{13}\ a_{21}\ a_{32}=0$. (S-445)
Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:
$1-a_{22}=(1-Z)a_{33}-a_{22}(1-Z)a_{33}+a_{32}(1-Y)a_{23}+(1-X)a_{13}\ a_{21}\ a_{32}$. (S-446)
Equation S-446 specifies the proportion of eggs that must be killed (X), the proportion of breeders (with demotion) that must be killed (Y), and the proportion of remaining breeders that must be killed (Z) to reduce the growth rate to 1.

<u>Case 31</u> – Reduction in fertility, non-breeder survival (with transition), and breeder survival.

Let X be the proportion of eggs killed. Then $1-X$ is the proportion of eggs not killed. Fertility can be broken up into those that are killed (X), and those that are not killed:
$Xa_{13}+(1-X)\ a_{13}$. (S-447)
Surviving eggs reduces eq. S-447 to:
$(1-X)\ a_{13}$. (S-448)
Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:
$Ya_{32}+(1-Y)\ a_{32}$. (S-449)
Surviving non-breeders reduces eq. S-449 to:
$(1-Y)\ a_{32}$. (S-450)
Survival can also be broken up into breeders that are killed (Z), and those that are not killed:
$Za_{33}+(1-Z)\ a_{33}$. (S-451)
Surviving breeders reduces eq. S-451 to:
$(1-Z)\ a_{33}$. (S-452)
Substitution of eq. S-448, S450 and eq. S-452 into the characteristic equation gives:
$1-a_{22}-(1-Z)a_{33}+a_{22}(1-Z)a_{33}-(1-Y)\ a_{32}a_{23}-(1-X)a_{13}\ a_{21}(1-Y)\ a_{32}=0$. (S-453)
Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:
$1-a_{22}=(1-Z)a_{33}-a_{22}(1-Z)a_{33}+(1-Y)a_{32}a_{23}+(1-X)a_{13}\ a_{21}(1-Y)a_{32}$. (S-454)
Equation S-454 specifies the proportion of eggs that must be killed (X), the proportion of non-breeders (with transition) that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

<u>Case 32</u> – Reduction in hatchling survival, non-breeder survival (without transition), and breeder survival (with demotion).

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings not killed. Hatchling survival can be broken up into those that are killed (X), and those that are not killed:
$Xa_{21} + (1 - X)\, a_{21}$. (S-455)
Surviving hatchlings reduces eq. S-455 to:
$(1 - X)\, a_{21}$. (S-456)
Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:
$Ya_{22} + (1 - Y)\, a_{22}$. (S-457)
Surviving non-breeders reduces eq. S-457 to:
$(1 - Y)\, a_{22}$. (S-458)
Survival can also be broken up into breeders that are killed (Z), and those that are not killed:
$Za_{23} + (1 - Z)\, a_{23}$. (S-459)
Surviving breeders reduces eq. S-459 to:
$(1 - Z)\, a_{23}$. (S-460)
Substitution of eq. S-456, S458 and eq. S-460 into the characteristic equation gives:
$1-(1 - Y)\, a_{22} - a_{33} + (1 - Y)\, a_{22}a_{33} - a_{32}(1 - Z)a_{23} - a_{13}(1 - X)\, a_{21}a_{32} = 0.$ (S-461)
Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:
$1 - a_{33} = (1 - Y)a_{22} - (1 - Y)\, a_{22}a_{33} + a_{32}(1 - Z)a_{23} + a_{13}\, (1 - X)a_{21}a_{32}.$ (S-462)
Equation S-462 specifies the proportion of hatchlings that must be killed (X), the proportion of non-breeders (without transition) that must be killed (Y), and the proportion of breeders (with demotion) that must be killed (Z) to reduce the growth rate to 1.

Case 33 – Reduction in hatchling survival, non-breeder survival (without transition), and non-breeder survival (with transition).

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings not killed. Hatchling survival can be broken up into those that are killed (X), and those that are not killed:
$Xa_{21} + (1 - X)\, a_{21}$. (S-463)
Surviving hatchlings reduces eq. S-463 to:
$(1 - X)\, a_{21}$. (S-464)
Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:
$Ya_{22} + (1 - Y)\, a_{22}$. (S-465)
Surviving non-breeders reduces eq. S-465 to:
$(1 - Y)\, a_{22}$. (S-466)
Survival of the remaining non-breeders that are killed (Z), and those that are not killed:
$Za_{32} + (1 - Z)\, a_{32}$. (S-467)
Surviving non-breeders reduces eq. S-467 to:
$(1 - Z)\, a_{32}$. (S-468)
Substitution of eq. S-464, S466 and eq. S-468 into the characteristic equation gives:
$1-(1 - Y)a_{22} - a_{33} + (1 - Y)a_{22}a_{33} - (1 - Z)\, a_{32}a_{23} - a_{13}\, (1 - X)a_{21}(1 - Z)\, a_{32} = 0.$
(S-469)
Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:
$1 - a_{33} = (1 - Y)a_{22} - (1 - Y)a_{22}a_{33} + (1 - Z)a_{32}a_{23} + a_{13}\, (1 - X)a_{21}(1 - Z)\, a_{32}$.(S-470)

Equation S-470 specifies the proportion of hatchlings that must be killed (X), the proportion of non-breeders (without transition) that must be killed (Y), and the proportion of the non-breeders (with transition) that must be killed (Z) to reduce the growth rate to 1.

Case 34 – Reduction in hatchling survival, non-breeder survival (without transition), and breeder survival.

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings not killed. Hatchling survival can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{21} + (1 - X)\, a_{21}. \quad \text{(S-471)}$$

Surviving hatchlings reduces eq. S-471 to:

$$(1 - X)\, a_{21}. \quad \text{(S-472)}$$

Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:

$$Ya_{22} + (1 - Y)\, a_{22}. \quad \text{(S-473)}$$

Surviving non-breeders reduces eq. S-473 to:

$$(1 - Y)\, a_{22}. \quad \text{(S-474)}$$

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$$Za_{33} + (1 - Z)\, a_{33}. \quad \text{(S-475)}$$

Surviving breeders reduces eq. S-475 to:

$$(1 - Z)\, a_{33}. \quad \text{(S-476)}$$

Substitution of eq. S-472, S474 and eq. S-476 into the characteristic equation gives:

$$1-(1 - Y)a_{22} - (1 - Z)a_{33} + (1 - Y)a_{22}(1 - Z)a_{33} - a_{32}a_{23} - a_{13}\,(1 - X)a_{21}\, a_{32} = 0. \quad \text{(S-477)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 = (1 - Y)a_{22} + (1 - Z)a_{33} - (1 - Y)a_{22}(1 - Z)a_{33} + a_{32}a_{23} - a_{13}\,(1 - X)a_{21}\, a_{32}. \quad \text{(S-478)}$$

Equation S-478 specifies the proportion of hatchlings must be killed (X), the proportion of non-breeders (without transition) that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

Case 35 – Reduction in hatchling survival, breeder survival (with demotion), and non-breeder survival (with transition).

Let X be the proportion of hatchlings killed. Then $1 - X$ is the proportion of hatchlings not killed. Hatchlings can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{21} + (1 - X)\, a_{21}. \quad \text{(S-479)}$$

Surviving hatchlings reduces eq. S-479 to:

$$(1 - X)\, a_{21}. \quad \text{(S-480)}$$

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$$Ya_{23} + (1 - Y)\, a_{23}. \quad \text{(S-481)}$$

Surviving breeders reduces eq. S-481 to:

$$(1 - Y)\, a_{23}. \quad \text{(S-482)}$$

Survival can also be broken up into non-breeders that are killed (Z), and those that are not killed:

$$Za_{32} + (1 - Z)\, a_{32}. \quad \text{(S-483)}$$

Surviving non-breeders reduces eq. S-483 to:

$(1-Z)\ a_{32}$. (S-484)

Substitution of eq. S-286, S222 and eq. S-288 into the characteristic equation gives:

$1-a_{22}-a_{33}+a_{22}a_{33}-(1-Z)\ a_{32}(1-Y)a_{23}-a_{13}(1-X)\ a_{21}(1-Z)\ a_{32}=0.$ (S-485)

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$\frac{1-a_{22}-a_{33}+a_{22}a_{33}}{a_{32}}=\ (1-Z)(1-Y)a_{23}+a_{13}(1-X)a_{21}(1-Z).$ (S-486)

Equation S-486 specifies the proportion of hatchlings must be killed (X), the proportion of breeders (with demotion) that must be killed (Y), and the proportion of non-breeders (with transition) that must be killed (Z) to reduce the growth rate to 1.

Case 36 – Reduction in hatchling survival, breeder survival (with demotion), and breeder survival.

Let X be the proportion of hatchlings killed. Then $1-X$ is the proportion of hatchlings not killed. Hatchlings can be broken up into those that are killed (X), and those that are not killed:

$Xa_{21}+(1-X)\ a_{21}$. (S-487)

Surviving hatchlings reduces eq. S-487 to:

$(1-X)\ a_{21}$. (S-488)

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$Ya_{23}+(1-Y)\ a_{23}$. (S-489)

Surviving breeders reduces eq. S-489 to:

$(1-Y)\ a_{23}$. (S-490)

Survival can also be broken up into the remaining breeders that are killed (Z), and those that are not killed:

$Za_{33}+(1-Z)\ a_{33}$. (S-491)

Surviving breeders reduces eq. S-491 to:

$(1-Z)\ a_{33}$. (S-492)

Substitution of eq. S-488, S490 and eq. S-492 into the characteristic equation gives:

$1-a_{22}-(1-Z)a_{33}+a_{22}(1-Z)a_{33}-a_{32}(1-Y)a_{23}-a_{13}(1-X)\ a_{21}\ a_{32}=0.$ (S-493)

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$1-a_{22}=\ (1-Z)a_{33}-a_{22}(1-Z)a_{33}+\ a_{32}(1-Y)a_{23}+a_{13}(1-X)\ a_{21}\ a_{32}.$ (S-494)

Equation S-494 specifies the proportion of hatchlings must be killed (X), the proportion of breeders (with demotion) that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

Case 37 – Reduction in hatchling survival, non-breeder survival (with transition), and breeder survival.

Let X be the proportion of hatchlings killed. Then $1-X$ is the proportion of hatchlings not killed. Hatchlings can be broken up into those that are killed (X), and those that are not killed:

$Xa_{21}+(1-X)\ a_{21}$. (S-495)

Surviving hatchlings reduces eq. S-495 to:

$(1-X)\ a_{21}$. (S-496)

Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:
$Ya_{32} + (1 - Y)\ a_{32}$. (S-497)
Surviving non-breeders reduces eq. S-497 to:
$(1 - Y)\ a_{32}$. (S-498)
Survival can also be broken up into breeders that are killed (Z), and those that are not killed:
$Za_{33} + (1 - Z)\ a_{33}$. (S-499)
Surviving breeders reduces eq. S-499 to:
$(1 - Z)\ a_{33}$. (S-500)
Substitution of eq. S-496, S498 and eq. S-500 into the characteristic equation gives:
$1 - a_{22} - (1 - Z)a_{33} + a_{22}(1 - Z)a_{33} - (1 - Y)a_{32}a_{23} - a_{13}(1 - X)\ a_{21}\ (1 - Y)a_{32} = 0$. (S-501)
Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:
$1 - a_{22} = (1 - Z)a_{33} - a_{22}(1 - Z)a_{33} + (1 - Y)a_{32}a_{23} + a_{13}(1 - X)\ a_{21}\ (1 - Y)a_{32}$. (S-502)
Equation S-502 specifies the proportion of hatchlings must be killed (X), the proportion of non-breeders (with transition) that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

<u>Case 38</u> – Reduction in non-breeder survival (without transition), breeder survival (with demotion), and non-breeder survival (with transition).

Let X be the proportion of non-breeders killed. Then $1 - X$ is the proportion of non-breeders not killed. Non-breeder can be broken up into those that are killed (X), and those that are not killed:
$Xa_{22} + (1 - X)\ a_{22}$. (S-503)
Surviving non-breeders reduces eq. S-503 to:
$(1 - X)\ a_{22}$. (S-504)
Survival can also be broken up into breeders that are killed (Y), and those that are not killed:
$Ya_{23} + (1 - Y)\ a_{23}$. (S-505)
Surviving breeders reduces eq. S-505 to:
$(1 - Y)\ a_{23}$. (S-506)
Survival can also be broken up into non-breeders that are killed (Z), and those that are not killed:
$Za_{32} + (1 - Z)\ a_{32}$. (S-507)
Surviving non-breeders reduces eq. S-507 to:
$(1 - Z)\ a_{32}$. (S-508)
Substitution of eq. S-504, S506 and eq. S-508 into the characteristic equation gives:
$1 - (1 - X)a_{22} - a_{33} + (1 - X)a_{22}a_{33} - a_{32}(1 - Y)a_{23} - a_{13}\ a_{21}\ a_{32} = 0$. (S-509)
Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:
$1 - (1 - X)a_{22} - a_{33} + (1 - X)a_{22}a_{33} - (1 - Z)\ a_{32}(1 - Y)a_{23} - a_{13}\ a_{21}(1 - Z)\ a_{32} = 0$.
(S-510)
Equation S-510 specifies the proportion of non-breeders (without transition) that must be killed (X), the proportion of breeders (with demotion) that must be killed (Y), and the proportion of non-breeders (with transition) that must be killed (Z) to reduce the growth rate to 1.

Case 39 – Reduction in non-breeder survival (without transition), breeder survival (with demotion), and breeder survival.

Let X be the proportion of non-breeders killed. Then $1 - X$ is the proportion of non-breeders not killed. Non-breeders can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{22} + (1 - X)\, a_{22}. \quad \text{(S-511)}$$

Surviving non-breeders reduces eq. S-511 to:

$$(1 - X)\, a_{22}. \quad \text{(S-512)}$$

Survival can also be broken up into breeders that are killed (Y), and those that are not killed:

$$Ya_{23} + (1 - Y)\, a_{23}. \quad \text{(S-513)}$$

Surviving breeders reduces eq. S-513 to:

$$(1 - Y)\, a_{23}. \quad \text{(S-514)}$$

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$$Za_{33} + (1 - Z)\, a_{33}. \quad \text{(S-515)}$$

Surviving breeders reduces eq. S-515 to:

$$(1 - Z)\, a_{33}. \quad \text{(S-516)}$$

Substitution of eq. S-512, S514 and eq. S-516 into the characteristic equation gives:

$$1-(1 - X)a_{22} - (1 - Z)a_{33} + (1 - X)a_{22}(1 - Z)a_{33} - a_{32}(1 - Y)a_{23} - a_{13}\, a_{21}\, a_{32} = 0. \quad \text{(S-517)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 - a_{13}\, a_{21}\, a_{32} = (1 - X)a_{22} + (1 - Z)a_{33} - (1 - X)a_{22}(1 - Z)a_{33} + a_{32}(1 - Y)a_{23}. \quad \text{(S-518)}$$

Equation S-518 specifies the proportion of non-breeders (without transition) that must be killed (X), the proportion of breeders (with demotion) that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

Case 40 – Reduction in non-breeder survival (without transition), non-breeder survival (with transition), and breeder survival.

Let X be the proportion of non-breeders killed. Then $1 - X$ is the proportion of non-breeders not killed. Non-breeders can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{22} + (1 - X)\, a_{22}. \quad \text{(S-519)}$$

Surviving non-breeders reduces eq. S-519 to:

$$(1 - X)\, a_{22}. \quad \text{(S-520)}$$

Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:

$$Ya_{32} + (1 - Y)\, a_{32}. \quad \text{(S-521)}$$

Surviving non-breeders reduces eq. S-521 to:

$$(1 - Y)\, a_{32}. \quad \text{(S-522)}$$

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$$Za_{33} + (1 - Z)\, a_{33}. \quad \text{(S-523)}$$

Surviving breeders reduces eq. S-523 to:

$$(1 - Z)\, a_{33}. \quad \text{(S-524)}$$

Substitution of eq. S-520, S522 and eq. S-524 into the characteristic equation gives:

$$1-(1 - X)a_{22} - (1 - Z)a_{33} + (1 - X)a_{22}(1 - Z)a_{33} - (1 - Y)\, a_{32}a_{23} - a_{13}\, a_{21}(1 - Y)\, a_{32} = 0. \quad \text{(S-525)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 = (1 - X)a_{22} + (1 - Z)a_{33} - (1 - X)a_{22}(1 - Z)a_{33} + (1 - Y)a_{32}a_{23} + a_{13}\, a_{21}(1 - Y)a_{32}. \quad \text{(S-526)}$$

Equation S-526 specifies the proportion of non-breeders (without transition) that must be killed (X), the proportion of non-breeders (with transition) that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

<u>Case 41</u> – Reduction in breeder survival (with demotion), non-breeder survival (with transition), and breeder survival.

Let X be the proportion of breeders killed. Then $1 - X$ is the proportion of breeder not killed. Breeders can be broken up into those that are killed (X), and those that are not killed:

$$Xa_{23} + (1 - X)\, a_{23}. \quad \text{(S-527)}$$

Surviving breeders reduces eq. S-527 to:

$$(1 - X)\, a_{23}. \quad \text{(S-528)}$$

Survival can also be broken up into non-breeders that are killed (Y), and those that are not killed:

$$Ya_{32} + (1 - Y)\, a_{32}. \quad \text{(S-529)}$$

Surviving non-breeders reduces eq. S-529 to:

$$(1 - Y)\, a_{32}. \quad \text{(S-530)}$$

Survival can also be broken up into breeders that are killed (Z), and those that are not killed:

$$Za_{33} + (1 - Z)\, a_{33}. \quad \text{(S-531)}$$

Surviving breeders reduces eq. S-531 to:

$$(1 - Z)\, a_{33}. \quad \text{(S-532)}$$

Substitution of eq. S-528, S530 and eq. S-532 into the characteristic equation gives:

$$1 - a_{22} - (1 - Z)a_{33} + a_{22}(1 - Z)a_{33} - (1 - Y)a_{32}(1 - X)a_{23} - a_{13}\, a_{21}\, (1 - Y)a_{32} = 0. \quad \text{(S-533)}$$

Rearranging, and solving for X, Y, and Z gives all the combinatorial solutions for X, Y, and Z that will reduce the growth rate to one:

$$1 - a_{22} = (1 - Z)a_{33} - a_{22}(1 - Z)a_{33} + (1 - Y)a_{32}(1 - X)a_{23} + a_{13}\, a_{21}\, (1 - Y)a_{32} \quad \text{(S-534)}$$

Equation S-534 specifies the proportion of breeders (with demotion) that must be killed (X), the proportion of non-breeders (with transition) that must be killed (Y), and the proportion of breeders that must be killed (Z) to reduce the growth rate to 1.

4. ONE-TIME RESET TO HISTORICAL LEVELS

4.1 Reset using a density-independent life history of ravens

The life history of ravens in a density-independent life history is:

$$\mathrm{L} = \begin{bmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}. \quad \text{(S-535)}$$

Letting t represent time (in years), H_t represent the current abundances of hatchlings, S_t the current abundance of non-breeders, and A_t the current abundances of breeders at the onset of the breeding period, then the stage-wise abundances of ravens at the onset of the breeding period in year $t + 1$ are then (Caswell 2001):

$$\begin{bmatrix} H_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} H_t \\ S_t \\ A_t \end{bmatrix}, \quad \text{(S-536)}$$

or equivalently,

$$H_{t+1} = a_{13}A_t, \quad \text{(S-537)}$$
$$S_{t+1} = a_{21}H_t, \quad \text{(S-538)}$$
$$A_{t+1} = a_{32}S_t + a_{33}A_t. \quad \text{(S-539)}$$

Suppose we want to find the one-time vital rates needed to reset abundances to desired (target) densities. Letting these one-time vital rates be denoted ${a_{13}}'$, ${a_{21}}'$, ${a_{32}}'$, and ${a_{33}}'$, and letting H_h represent the desired abundances of hatchlings, S_h the desired abundances of non-breeders, and A_h the desired abundances of breeders, the goal would be to find the vital rates that satisfy the equations:

$$H_h = {a_{13}}'A_t, \quad \text{(S-540)}$$
$$S_h = {a_{21}}'H_t, \quad \text{(S-541)}$$
$$A_h = {a_{32}}'S_t + {a_{33}}'A_t. \quad \text{(S-542)}$$

Equation S-540 suggests that the desired abundances of hatchlings can be reset from current levels to desired levels provided fertility can be reduced to:

$$a'_{13} = \frac{H_h}{A_t}. \quad \text{(S-543)}$$

Similarly, desired abundances can be reset from current abundances to desired abundances given that hatchling survival can be reduced to:

$$a'_{21} = \frac{S_h}{H_t}. \quad \text{(S-544)}$$

Finally, desired abundances may be attained from current levels given non-breeder survival can be reduced to:

$$a'_{32} = \frac{A_h - {a_{33}}'A_t}{S_t}. \quad \text{(S-545)}$$

while

$$a'_{33} = \frac{A_h - {a_{32}}'S_t}{A_t}. \quad \text{(S-546)}$$

The one-time matrix necessary to reset abundances to desired levels is then:

$$\begin{bmatrix} 0 & 0 & \frac{H_h}{A_t} \\ \frac{S_h}{H_t} & 0 & 0 \\ 0 & \frac{A_h - a_{33}{}' A_t}{S_t} & \frac{A_h - a_{32}{}' S_t}{A_t} \end{bmatrix}. \tag{S-547}$$

4.2 Reset using a density-dependent life history of ravens

The life history of ravens in a density-dependent life history is:

$$\mathrm{L} = \begin{bmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}. \tag{S-548}$$

Letting t represent time (in years), H_t represent the current abundances of hatchlings, S_t the current abundance of non-breeders, and A_t the current abundances of breeders at the onset of the breeding period, then the stage-wise abundances of ravens at the onset of the breeding period in year $t + 1$ are (Caswell 2001):

$$\begin{bmatrix} H_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} H_t \\ S_t \\ A_t \end{bmatrix}, \tag{S-549}$$

or equivalently,

$$H_{t+1} = a_{13} A_t, \tag{S-550}$$

$$S_{t+1} = a_{21} H_t + a_{22} S_t + a_{23} A_t \tag{S-551}$$

$$A_{t+1} = a_{32} S_t + a_{33} A_t. \tag{S-552}$$

Suppose we wish to find the one-time vital rates needed to reset abundances to desired densities. Letting these one time vital rates be denoted $a_{13}{}'$, $a_{21}{}'$, $a_{22}{}'$, $a_{23}{}'$, $a_{32}{}'$, and $a_{33}{}'$, and letting H_h represent the desired abundances of hatchlings, S_h the desired abundances of non-breeders, and A_h the desired abundances of breeders, the goal would be to find the vital rates that satisfy the equations:

$$H_h = a_{13}{}' A_t, \tag{S-553}$$

$$S_h = a_{21}{}' H_t + a_{22}{}' S_t + a_{23}{}' A_t, \tag{S-554}$$

$$A_h = a_{32}{}' S_t + a_{33}{}' A_t. \tag{S-555}$$

Equation S-555 suggests that the desired abundances of hatchlings can be reset to desired levels from current levels provided fertility can be reduced to:

$$a_{13}' = \frac{H_h}{A_t}. \tag{S-556}$$

Similarly, desired abundances can be reset from current abundances to desired abundances given that hatchling survival can be reduced to:

$$a_{21}' = \frac{S_h - a_{22}{}' S_t - a_{23}{}' A_t}{H_t}. \tag{S-557}$$

As for the non-breeder class, desired abundances may be attained given their survival can be reduced to:

$$a_{22}' = \frac{S_h - a_{21}{}' H_t - a_{23}{}' A_t}{S_t}. \tag{S-558}$$

$$a'_{32} = \frac{A_h - a_{33}{'}A_t}{S_t}. \tag{S-559}$$

While the breeders can be reset when:

$$a'_{23} = \frac{S_h - a_{21}{'}H_t - a_{22}{'}S_t}{A_t}. \tag{S-560}$$

$$a'_{33} = \frac{A_h - a_{32}{'}S_t}{A_t}. \tag{S-561}$$

The one-time matrix necessary to reset abundances to historical levels is then:

$$\begin{bmatrix} 0 & 0 & \frac{H_h}{A_t} \\ \frac{S_h - a_{22}{'}S_t - a_{23}{'}A_t}{H_t} & \frac{S_h - a_{21}{'}H_t - a_{23}{'}A_t}{S_t} & \frac{S_h - a_{21}{'}H_t - a_{22}{'}S_t}{A_t} \\ 0 & \frac{A_h - a_{33}{'}A_t}{S_t} & \frac{A_h - a_{32}{'}S_t}{A_t} \end{bmatrix}. \tag{S-562}$$

5. COMPREHENSIVE KILLING OF EGGS, HATCHLINGS, NON-BREEDERS, OR BREEDERS

When the life history is assumed to be closed to dispersal, the life history of ravens contains a bottleneck at the youngest ages (eggs or hatchlings) by which targeted removal will lead to eventual complete collapse of the entire population, if desired.

5.1 Comprehensive killing of eggs

Let A_t be the abundance of breeding adults at time t and H_{t+1} be the number of hatchlings at time $t+1$. Then the comprehensive killing of eggs (such that $a_{13}=0$) gives:

$$H_{t+1} = a_{13}A_t = 0A_t = 0, \quad \text{(S-563)}$$

or a complete collapse in the hatchling cohort for the next year. Projecting the equations out three years in the density independent life history gives:

$$S_{t+2} = a_{21}H_{t+1}, \quad \text{(S-564)}$$

$$A_{t+3} = a_{32}S_{t+2} + a_{33}A_{t+2}. \quad \text{(S-565)}$$

Substitution of eq. S-563 into S564, then eq. S-564 into S565 gives:

$$A_{t+3} = a_{33}A_{t+2}, \quad \text{(S-566)}$$

which means the reduction in breeders from comprehensive egg killing in one year will manifest itself in the breeding population in three-years time. Projecting the equations out three years in a density dependent life history gives:

$$S_{t+2} = a_{21}H_{t+1} + a_{22}S_{t+1} + a_{23}A_{t+1} \quad \text{(S-567)}$$

$$A_{t+3} = a_{32}S_{t+2} + a_{33}A_{t+2}. \quad \text{(S-568)}$$

Substitution further gives:

$$A_{t+3} = a_{32}(a_{22}S_{t+1} + a_{23}A_{t+1}) + a_{33}A_{t+2}. \quad \text{(S-569)}$$

Should comprehensive egg killing be continued such that existing breeders and immatures naturally die off, then the activity will lead to total collapse of population in as few years as the maximum lifetime of the raven, if desired.

5.2 Comprehensive killing of hatchlings

Let S_t be the abundance of non-breeders at time t and let H_{t+1} be the number of hatchlings at time $t+1$. Then the comprehensive culling of hatchlings (such that $a_{21}=0$) in a density dependent system gives:

$$S_{t+1} = a_{21}H_t = 0H_t = 0, \quad \text{(S-570)}$$

but in a density-dependent system gives:

$$S_{t+1} = a_{21}H_t + a_{22}S_t + a_{23}A_t = 0H_t + a_{22}S_t + a_{23}A_t = a_{22}S_t + a_{23}A_t. \quad \text{(S-571)}$$

Projection of the system gives:

$$A_{t+2} = a_{32}S_{t+1} + a_{33}A_{t+1}, \quad \text{(S-572)}$$

which reduces to:

$$A_{t+2} = a_{32}S_{t+1} + a_{33}A_{t+1} = 0 + a_{33}A_{t+1} = a_{33}A_{t+1}, \quad \text{(S-573)}$$

in a density-independent system, and:

$$A_{t+2} = a_{32}(a_{22}S_t + a_{23}A_t) + a_{33}A_{t+1}, \quad \text{(S-574)}$$

in a density-dependent system. In both cases, the continued comprehensive culling of hatchlings will cause the number of breeders to remain non-zero for only the duration of a raven lifetime, after which there will be total collapse of the population.

5.3 Comprehensive killing of non-breeders

Now let us consider the comprehensive removal of non-breeders. Suppose we cull all birds that are not hatchlings, nor breeders (such that $a_{22} = 0$ and $a_{32} = 0$). In a density independent life history, this would yield:

$$A_{t+3} = a_{32}S_{t+2} + a_{33}A_{t+2} = 0S_{t+2} + a_{33}A_{t+2} = a_{33}A_{t+2}. \quad \text{(S-575)}$$

while in a density dependent life history, this would yield:

$$S_{t+1} = a_{21}H_t + a_{22}S_t + a_{23}A_t = a_{21}H_t + 0S_t + a_{23}A_t = a_{21}H_t + a_{23}A_t, \quad \text{(S-576)}$$

and

$$A_{t+3} = a_{32}S_{t+2} + a_{33}A_{t+2} = 0S_{t+2} + a_{33}A_{t+2} = a_{33}A_{t+2}. \quad \text{(S-577)}$$

Equation S-575 shows that the abundances of breeders will remain non-zero as long as the lifetime of the existing birds. Equations S576-S577 leads to a total collapse of the breeding population in as little as one additional year.

5.4 Comprehensive killing of breeders

The life history of ravens contains an adaptation that enables the population to remain viable (or even grow) despite annual categorical devastation of their breeders. For example, in the density-independent life history, a raven progresses through their life history in a predictable manner: a bird may transition from the non-breeder stage to the breeder stage, and after having done so, will remain a breeder for the duration of their lifetime. By contrast, in a density-dependent life history, a bird that has achieved breeding status can revert into non-breeding status, where it can remain a non-breeder, or re-advance into breeding status.

Under both density independent and density dependent configurations, the abundances of breeders are the sum of two categories of birds: those individuals that were in non-breeding status (S_t), survived and transitioned to breeders (a_{32}), and those individuals that were in breeding status (A_t), survived, and remained breeders (a_{33}):

$$A_{t+1} = a_{32}S_t + a_{33}A_t. \quad \text{(S-588)}$$

Equation S-588 shows that even complete culling of breeding adults at time t (e.g., the reduction of a_{23} and a_{33} to 0) will not destroy the breeding population. In both a density dependent and density independent system, complete culling of breeders, year after year will lead to:

$$A_{t+1} = a_{32}S_t + a_{33}A_t = a_{32}S_t + 0A_t = a_{32}S_t, \quad \text{(S-589)}$$

in year one,

$$A_{t+2} = a_{32}S_{t+1} + a_{33}A_{t+1} = a_{32}S_{t+1} + 0A_{t+1} = a_{32}S_{t+1}, \quad \text{(S-590)}$$

in year two, all the way up to:

$$A_{t+n+1} = a_{32}S_{t+n} + a_{33}A_{t+n} = a_{32}S_{t+n} + 0A_{t+n} = a_{32}S_{t+n}, \quad \text{(S-591)}$$

in year n. While the culling of breeders will reduce reproductive output, comprehensive culling of breeders every single year will not necessarily collapse the population, even in a system assumed to be closed to gains from dispersal.

6. CONVERSION FROM ABUNDANCES TO DENSITIES

Coates et al. (2020) identified negative impacts to nesting sage-grouse, especially where raven density exceeded ~0.40 km^{-2}, meaning that this figure may be used as an ecological threshold.

Here we illustrate how to convert between density and abundance. Let the geographic area of a monitoring or management strata be represented by A, where A is reported in square kilometers. Then the total number of ravens (of all ages; R_h) in a monitoring or management strata under historical densities would be:

$$R_h = 0.4A. \quad \text{(S-592)}$$

We must now determine the historical number of hatchlings (H_h), immature and non-breeders (S_h), and breeders (A_h) that comprise the total historical abundances such that:

$$R_h = H_h + S_h + A_h. \quad \text{(S-593)}$$

Any number of H_h, S_h, and A_h may be used so long as the summation satisfies eq. S-593. While we have no way to know the historical structure of the population, we will show how to satisfy eq. S-593 using the stable stage distribution (SSD; Caswell 2001).

7. STRUCTURE USING THE STABLE STAGE DISTRIBUTION

Letting λ be the dominant eigenvalue (population growth rate), and the notation described above, then the symbolic expressions for the stable stage distributions (w) for a density independent system is (Hanley 2018):

$$w = \begin{bmatrix} \frac{H_h}{A_t\lambda} & \frac{H_h S_h}{A_t H_t \lambda^2} & 1 \end{bmatrix}', \quad \text{(S-594)}$$

and for a density dependent system is:

$$w = \begin{bmatrix} \frac{H_h}{A_t\lambda} & \frac{\left(\frac{H_h(S_h - a_{22}'S_t - a_{23}'A_t)}{A_t H_t} + \frac{\lambda(S_h - a_{21}'H_t - a_{22}'S_t)}{A_t}\right)}{\lambda^2 - \frac{\lambda(S_h - a_{21}'H_t - a_{23}'A_t)}{S_t}} & 1 \end{bmatrix}'. \quad \text{(S-595)}$$

Plugging these into the expression for density gives an expression that satisfies the abundances of ravens in each life stage in a density independent system:

$$0.4A = \left(\frac{H_h}{A_t\lambda}\right)H_h + \left(\frac{H_h S_h}{A_t H_t \lambda^2}\right)S_h + A_h. \quad \text{(S-596)}$$

and for the density dependent system:

$$0.4A = \left(\frac{H_h}{A_t\lambda}\right)H_h + \left(\frac{\left(\frac{H_h(S_h - a_{22}'S_t - a_{23}'A_t)}{A_t H_t} + \frac{\lambda(S_h - a_{21}'H_t - a_{22}'S_t)}{A_t}\right)}{\lambda^2 - \frac{\lambda(S_h - a_{21}'H_t - a_{23}'A_t)}{S_t}}\right)S_h + A_h. \quad \text{(S-597)}$$

Any number of H_h, S_h, and A_h may be used so long as the summation satisfies eqs. S-596 and S597.

8. LITERATURE CITED

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