Preliminary Design of a Very-Low-Thrust Geostationary Transfer Orbit to Sun-Synchronous Orbit Small Satellite Transfer

18th Annual AIAA/USU Conference on Small Satellites

Session VII: 12th Annual Frank J. Redd Student Competition

(SSC04-VII-1)

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August 11, 2004 11:00 a.m.

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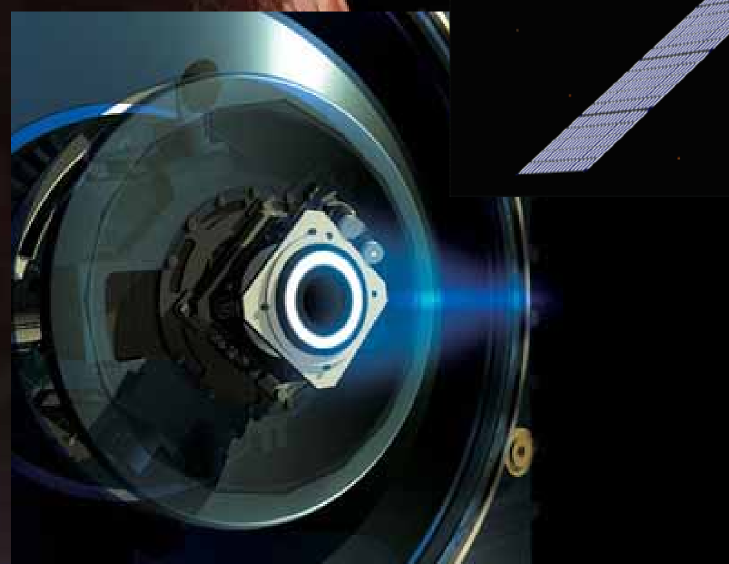
Overview

- Introduction
- Trajectory Optimization
- Problem Formulation
- Results
- Conclusions

Courtesy NASA/JPL-Caltech



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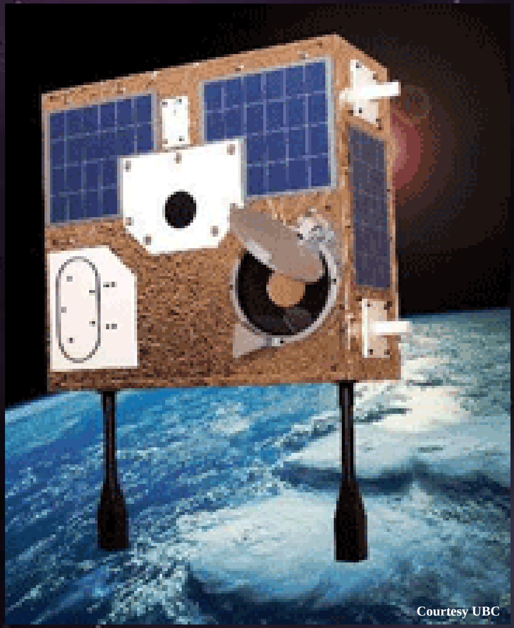
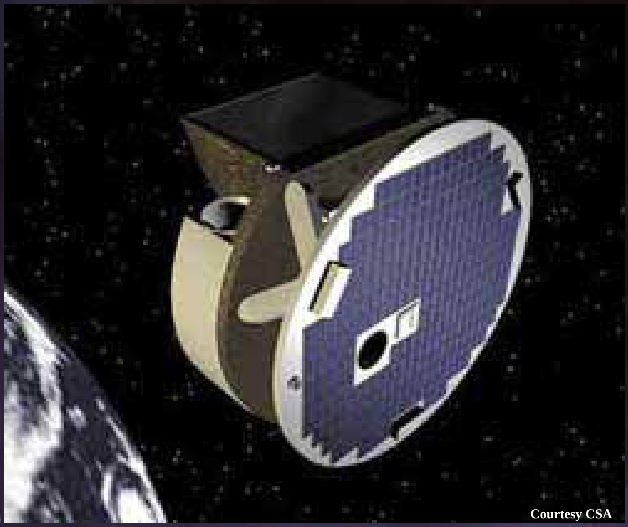
Introduction



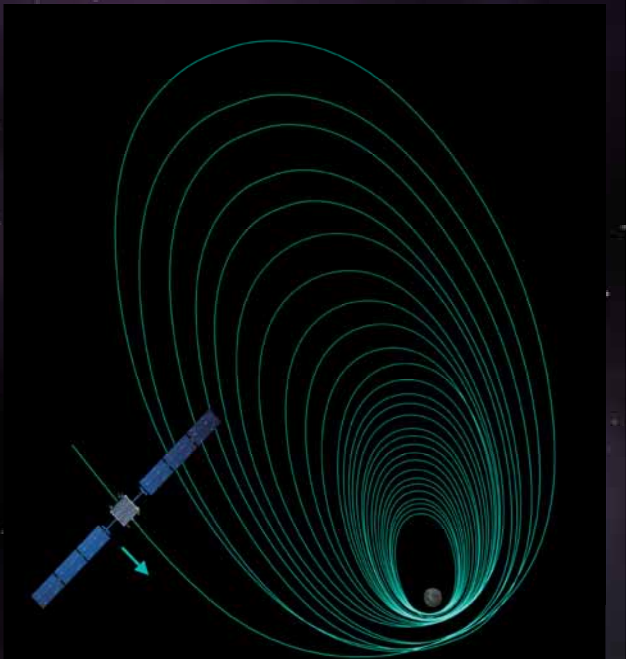
- As small satellites become more complex they will generally have more stringent mission requirements
  - Secondary launch opportunities to operational orbit may be unavailable
  - Dedicated launch opportunities expensive
  - What do we do?



- Allow satellite to transfer into operational orbit from available secondary launch opportunity
- May require a large amount of propellant for conventional thrusters
- Use low-thrust propulsion to reduce fuel requirements

Introduction

- Assessing the feasibility of going from a geostationary transfer orbit (GTO) to a sun-synchronous orbit (SSO) with low-thrust engines
- Different problem than conventional orbital transfers which employ impulsive burns
- Low-thrust engines are firing for long periods of time and slowly change the orbital elements
- Benefit of low-thrust engines:
  - High specific impulse (I_{sp})
  - Consume less fuel



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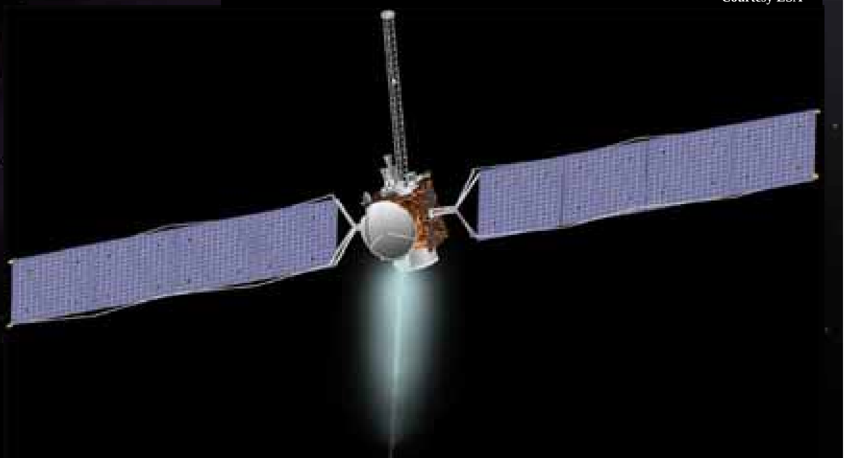

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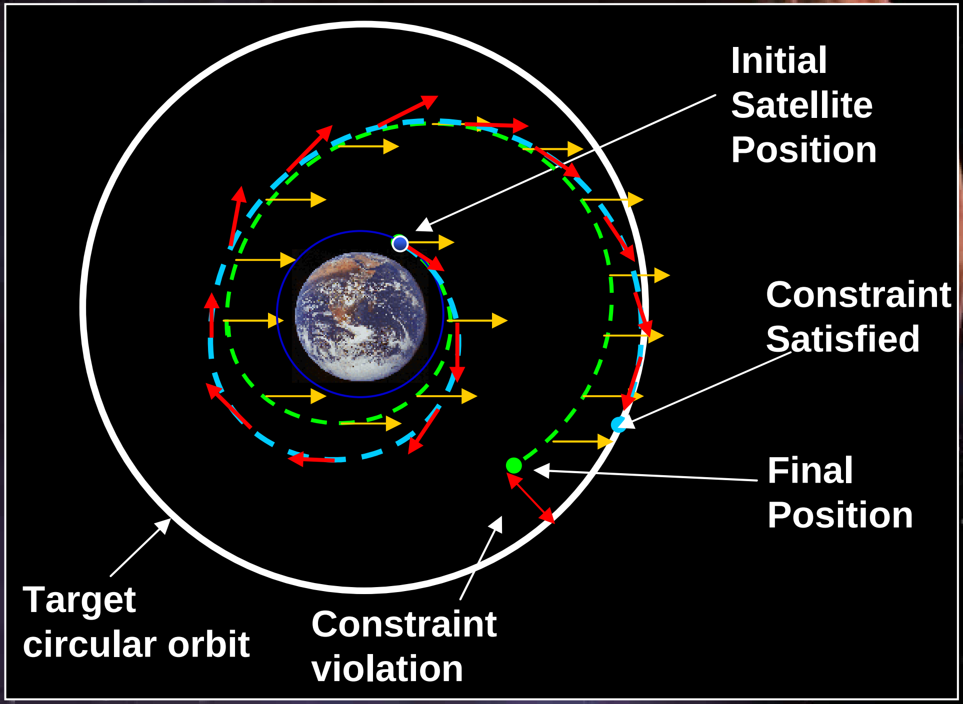
Trajectory Optimization

- Direct single-shooting approach is used
- An initial guess for the thrust profile is made
- Equations to final time of motion are numerically integrated from initial



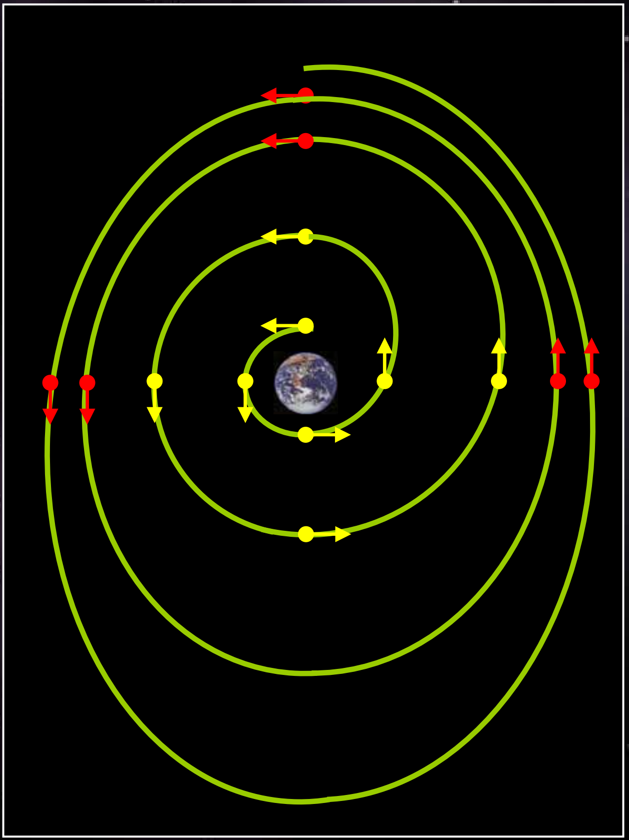
- Constraints are typically placed on the satellite's state at the final time
- Generic optimization programs are used to adjust the thrust profile
- NPSOL is used to solve the nonlinearly constrained optimization problems

Problem Formulation
Satellite Equations of Motion

- State representation:
  - Cartesian:
    - Simple equations of motion
    - Equations change quickly with time
  - Equinoctial:
    - Complex equations of motion
    - Equations change slowly with time
  - Equinoctial equations take 1.31 times longer to compute (for equivalent integration steps)
  - Equinoctial equations require 40 times less integration steps for similar accuracy

Problem Formulation

- Very low thrust levels give rise to extremely large optimization problems
- A multiple-orbit thrust parameterization strategy was developed to accommodate the large number of orbit revolutions
- Thrust is parameterized for a single orbit and the thrust parameterization is repeated for several orbits
- Reduces the optimality of the solution, but produces feasible near-optimal solutions to large optimization problems



Problem Formulation
Satellite Model

- Small satellite parameters:
  - Initial Mass: 100 kg
  - Low-thrust engines (two versions considered):
    - Moderately-high I_{sp} (800 s)
    - High I_{sp} (3500 s)
    - Max thrust: 25 mN
  - Assumed a variable power thruster model with fixed I_{sp}
    - Thrust: $T = 2\eta P / \left(g_{sl} I_{sp} \right)$
    - Propellant mass flow rate: $\dot{m} = \dfrac{T}{g_{sl} I_{sp}}$

Problem Statement

- Small satellite launched into a GTO with an operational SSO
  - 3 minimum-fuel case studies
  - 1400 orbit revolutions
  - 40 orbit repeat cycle for thrust parameterization
- Initial orbit (GTO):

a (km)	e	i (deg)	Ω (deg)	ω (deg)	θ (deg)
24,370.0	0.73	28.6	0.0	0.0	0.0

- Target orbit (SSO):

a (km)	e	i (deg)	Ω (deg)	ω (deg)	θ (deg)
7,178.0	0.0	98.6	---	---	---

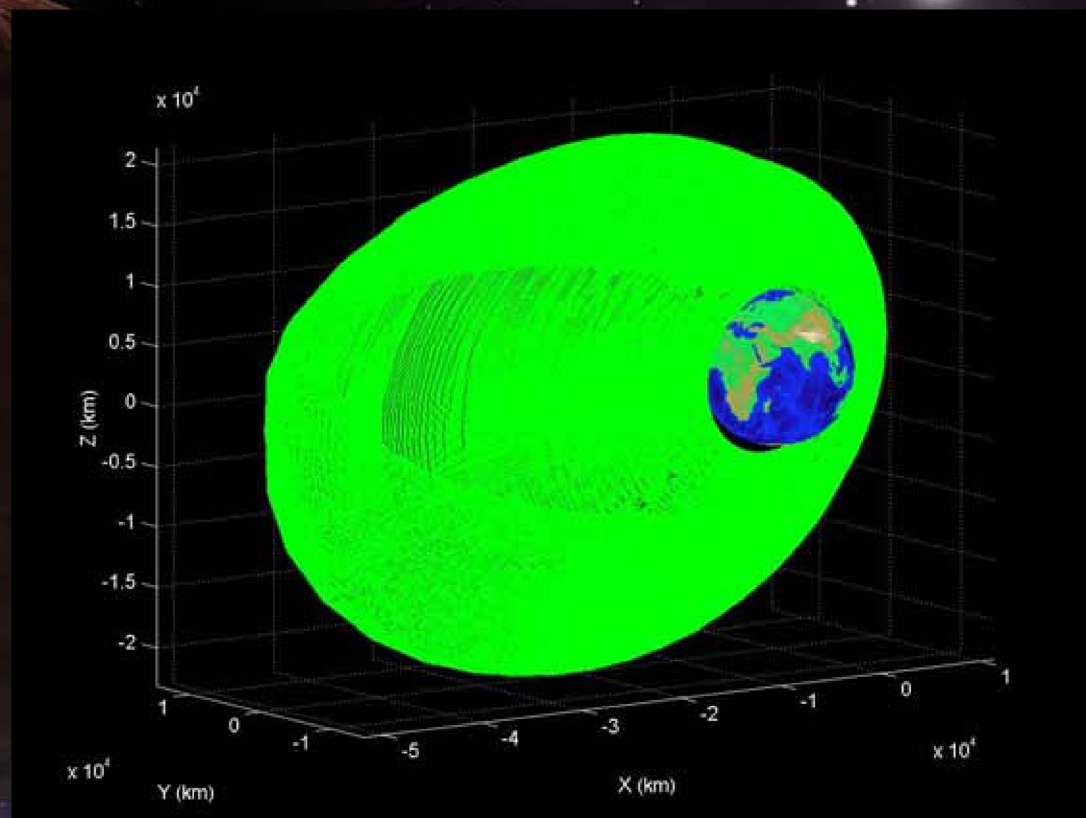
Case 1 – Moderately-High I_{sp} Transfer

I_{sp} (s)	Transfer Time (days)	# of Orbits	m_i (kg)	m_f (kg)	m_p (kg)
800	363.5	1400	100.0	47.9	52.1

- Low arrival mass at operational orbit
- Propellant consumption could be reduced by longer transfer time or higher I_{sp}

Results - Case 1
Moderately-High I_{sp} Transfer

- 1400 orbit revolutions
- Maximum apogee over 50,000 km

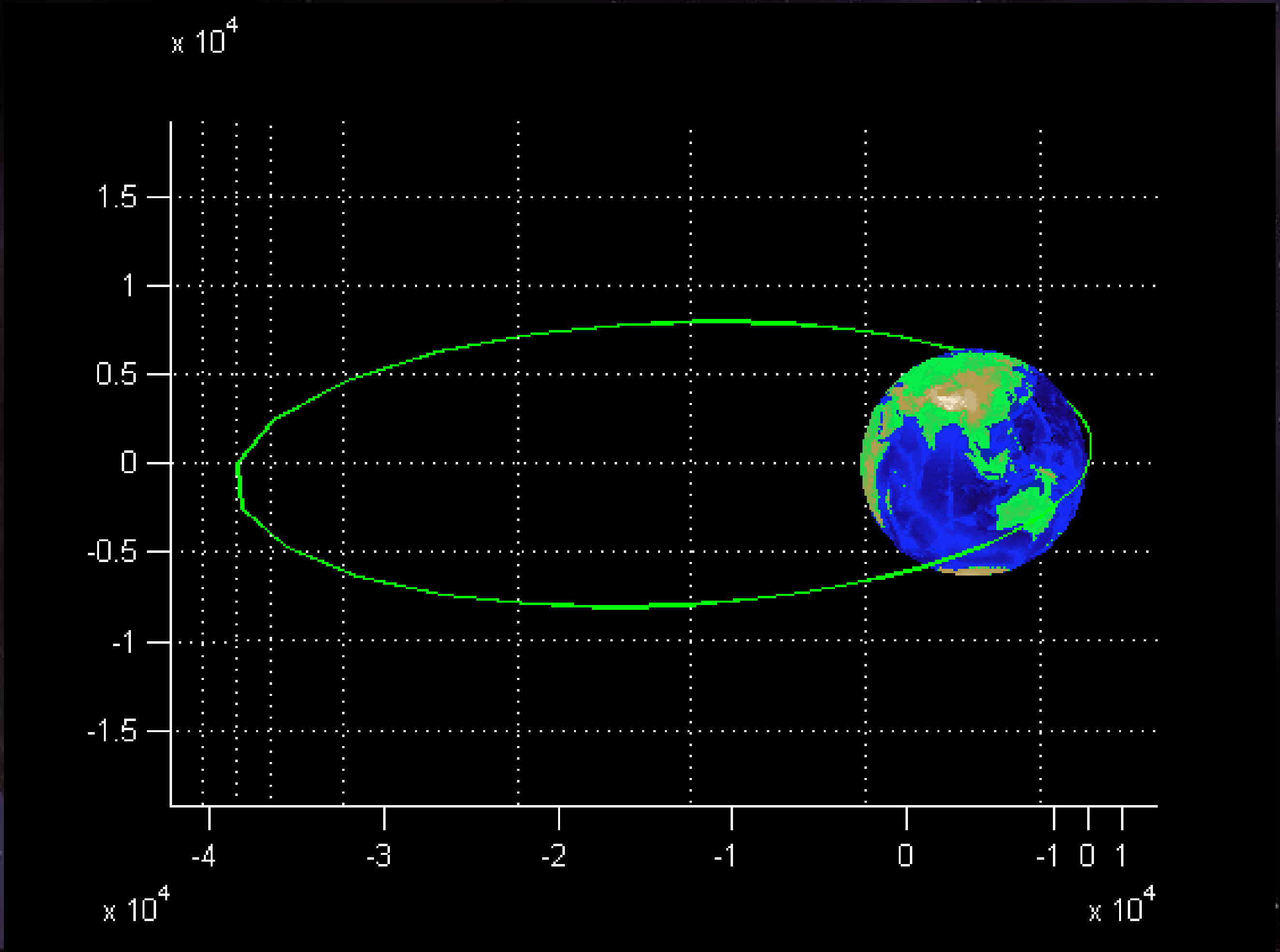


Results (Case 2)
High I_{sp} Transfer

I_{sp} (s)	Transfer Time (days)	# of Orbits	m_i (kg)	m_f (kg)	m_p (kg)
3500	357.7	1400	100.0	81.2	18.8

- Low fuel requirements to obtain SSO
- GTO transfer to SSO is a feasible concept with high I_{sp} engines
- Other considerations:
  - Radiation
  - Launch costs and availability

Results (Case 2)
High I_{sp} Transfer



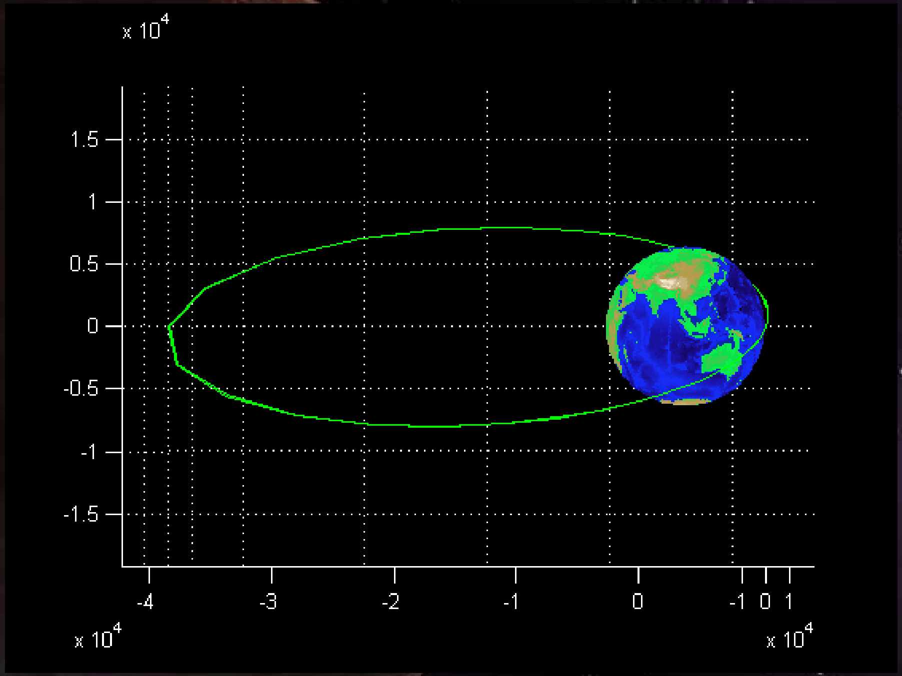
Results (Case 3)
High I_{sp} Transfer with Node Change

- May want to define a specific right ascension of the ascending node (Ω) for operational orbit
  - Target Ω was assumed to be 60 degrees

I_{sp} (s)	Transfer Time (days)	# of Orbits	m_i (kg)	m_f (kg)	m_p (kg)
3500	541.9	1400	100.0	78.2	21.8

- 3 kg more fuel than previous case
- Extremely long transfer time

Results (Case 3)
High I_{sp} Transfer with Node Change



Conclusions

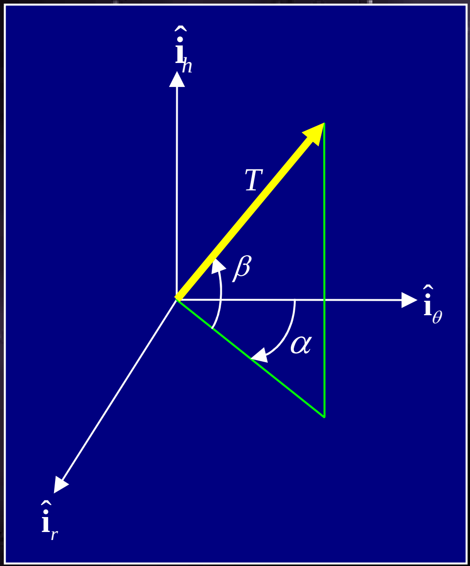
- A robust method for solving small satellite low-thrust transfer problems has been developed
- Small satellite can feasibly transfer (in terms of propellant) into SSO from a GTO secondary launch opportunity with high-Isp low-thrust engines

Questions?

Problem Formulation
Satellite Equations of Motion

- Write Equinoctial equations
- Thrust acceleration vector, **a**

$$a_r = (T/m)\sin(\alpha)\cos(\beta)$$
$$a_\theta = (T/m)\cos(\alpha)\cos(\beta)$$
$$a_h = (T/m)\sin(\beta)$$



Equinoctial Equations of Motion

$$\dot{\mathbf{y}} = \mathbf{A}(\mathbf{y})\mathbf{u} + \mathbf{b}$$

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{2p}{\xi}\sqrt{\frac{p}{\mu}} & 0 \\ \sqrt{\frac{p}{\mu}}\sin L & \sqrt{\frac{p}{\mu}}\frac{1}{\xi}\{(\xi+1)\cos L + f\} & -\sqrt{\frac{p}{\mu}}\frac{g}{\xi}\{h\sin L - k\cos L\} \\ -\sqrt{\frac{p}{\mu}}\cos L & \sqrt{\frac{p}{\mu}}\frac{1}{\xi}\{(\xi+1)\sin L + g\} & \sqrt{\frac{p}{\mu}}\frac{f}{\xi}\{h\sin L - k\cos L\} \\ 0 & 0 & \sqrt{\frac{p}{\mu}}\frac{s^2\cos L}{2\xi} \\ 0 & 0 & \sqrt{\frac{p}{\mu}}\frac{s^2\sin L}{2\xi} \\ 0 & 0 & \sqrt{\frac{p}{\mu}}\frac{1}{\xi}\{h\sin L - k\cos L\} \end{bmatrix}$$

$$\mathbf{b}^\mathsf{T} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \sqrt{\mu p}\left(\frac{\xi}{p}\right)^2 \end{bmatrix}$$

$$\mathbf{u}^\mathsf{T} = [u_r, u_\theta, u_h]$$

$$\begin{aligned} \xi &= 1 + f\cos L + g\sin L, \\ r &= \frac{p}{\xi}, \\ \alpha^2 &= h^2 - k^2, \\ \chi &= \sqrt{h^2 + k^2}, \\ s^2 &= 1 + \chi^2. \end{aligned}$$

Cartesian Equations of Motion

$$\dot{\mathbf{y}} = \mathbf{A}(\mathbf{y})\mathbf{u} + \mathbf{b}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{u}^{\mathrm{T}} = \left[u_x, u_y, u_z\right]$$

$$\mathbf{b}^{\mathrm{T}} = \begin{bmatrix} v_x & v_y & v_z & \dfrac{-\mu}{\|\mathbf{r}\|^3} r_x & \dfrac{-\mu}{\|\mathbf{r}\|^3} r_y & \dfrac{-\mu}{\|\mathbf{r}\|^3} r_z \end{bmatrix}$$