THE DIFFERENTIALGEOMETRY SOFTWARE PROJECT

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Tutorial Series

Non-Isomorphic Real Simple Lie Algebras of the Same Complex Type and Character

Synopsis

- Complex simple Lie algebras are classified by their root types -- these are labeled as A_r, B_r, C_r, D_r, E_6, E_7, E_8, F_4, G_2. The type of a real simple Lie algebra $\mathfrak{g}$ is the root type of the associated complex algebra. The character δ of a real simple Lie algebra is the signature of its Killing form.

- For many root types, the character is sufficient to uniquely classify the corresponding real Lie algebras. Indeed in [1] (page 26) Cartan writes, *"Les groupes reéls d'ordre r qui correspondent à une même type complexe d'ordre r se classent en général complêtement d'aprés leur caractère."* However, as Helgason [2] (page 537) writes, one should not take this statement to be literally true – there are a few cases where the character does not suffice to distinguish all possible real forms.

- In Sections 1 and 2 we will show that the 2 real non-isomorphic Lie algebras *so*(18)* and *so(12, 6)* (with isomorphic complexifications *so(18)*) have the same character $\delta = -9$.

- A more complete understanding of the isomorphism problem for real forms of a given complex type is provided by the following theorem (Helgason [2], page 517). We recall that if $\mathfrak{g} = \mathfrak{t} \oplus \mathfrak{p}$ is the Cartan decomposition of a real simple Lie algebra, then the Killing form is negative-definite on $\mathfrak{t}$ and positive-definite on $\mathfrak{p}$. Its follows that $\delta = \dim \mathfrak{p} - \dim \mathfrak{t}$.

Theorem 1. Let $\mathfrak{g}_1$ and $\mathfrak{g}_2$ be two real simple Lie algebras of the same complex type and with Cartan decompositions $\mathfrak{g}_1 = \mathfrak{t}_1 \oplus \mathfrak{p}_1$ and $\mathfrak{g}_2 = \mathfrak{t}_2 \oplus \mathfrak{p}_2$. Then $\mathfrak{g}_1$ and $\mathfrak{g}_2$ are isomorphic as real Lie algebras if and only if the reductive Lie algebras $\mathfrak{t}_1$ and $\mathfrak{t}_2$ are isomorphic.

- In Section 3 we show that the compact part of the Cartan decomposition for so*(18)* is $\mathfrak{t} = u(1) \oplus su(9)$.
- In Section 4 we show that the compact part of the Cartan decomposition for *so(12, 6)* is

$\mathfrak{k} = su(4) \oplus so(12) = so(6) \oplus so(12)$.

1. The Real Lie Algebras of Type A_r, B_r, C_r, D_r, $4 \leq r \leq 9$

Here are the characters for all the classical Lie algebras of rank $4 \leq r \leq 9$. We see that *so*(18)* and *so(12, 6)* are the <u>*only*</u> classical Lie algebras of rank less than or equal to 9 which have the same type and character.

Type A:

A4 δ	*sl(5)* 4	*su(4, 1)* -24	*su(3, 1)* -8	*su(2, 2)* 0				
A5 δ	*sl(6)* 5	*su(6)* -35	*su(5, 1)* -15	*su(4, 2)* -3	*su(3, 3)* 1	*su*(6)* -7		
A6 δ	*sl(7)* 6	*su(7)* -48	*su(6, 1)* -24	*su(5, 2)* -8	*su(4, 3)* 0			
A7 δ	*sl(8)* 7	*su(8)* -63	*su(7, 1)* -35	*su(6, 2)* -15	*su(5, 3)* -3	*su(4, 4)* 1	*su*(8)* -9	
A8 δ	*sl(9)* 8	*su(9)* -80	*su(8, 1)* -48	*su(7, 2)* -24	*su(6, 3)* -8	*su*(5, 4)* 0		

A9 δ	*sl(10)* 9	*su(10)* -99	*su(9, 1)* -63	*su(8, 2)* -35	*su(7, 3)* -15	*su(6, 4)* -3	*su(5, 5)* 1	*su*(10)* -11

Type B:

B4 δ	*so(9)* -36	*so(8, 1)* -20	*so(7, 2)* --8	*so(6, 3)* 0	*so(5, 4)* 4				
B5 δ	*so(11)* -55	*so(10, 1)* --35	*so(9, 2)* -19	*so(8, 3)* -7	*so(7, 4)* 1	*so(6, 5)* 5			
B6 δ	*so(13)* -78	*so(12, 1)* -54	*so(11, 2)* -34	*so(10, 3)* -18	*so(9, 4)* -6	*so(8, 5)* 2	*so(7, 6)* 6		
B7 δ	*so(15)* -105	*so(14, 1)* -77	*so(13, 2)* -53	*so(12, 3)* -33	*so(11, 4)* -17	*so(10, 5)* -5	*so(9, 6)* 3	*so(8, 7)* 7	
B8 δ	*so(17)* -13	*so(16, 1)* -104	*so(15, 2)* -76	*so(14, 3)* -52	*so(13, 4)* -32	*so(12, 5)* -16	*so(11, 6)* -4	*so(10, 7)* 4	*so(9, 8)* 8

	6									
B9 δ	*so (19)* -171	*so (18, 1)* -135	*so (17, 2)* -103	*so (16, 3)* -75	*so (15, 4)* -51	*so (14, 5)* -31	*so (13, 6)* -15	*so (12, 7)* -3	*so (11, 8)* 5	*so (10, 9)* 9

Type C:

C4 δ	*sp (8)* -36	*sp(6, 2)* -12	*sp(4, 4)* -4	*sp (8, R)* 4		
C5 δ	*sp (10)* -55	*sp(8, 2)* -23	*sp(6, 4)* -7	*sp (10, R)* 5		
C6 δ	*sp (12)* -78	*sp (10, 2)* --38	*sp(8, 4)* -14	*sp (6, 6)* -6	*sp (12, R)* 6	
C7 δ	*sp (14)* -105	*sp (12, 2)* -57	*sp (10, 4)* -25	*sp (8, 6)* -9	*sp (14, R)* 7	
C8 δ	*sp (16)* -136	*sp (14, 2)* -80	*sp (12, 4)* -40	*sp (10, 6)* -16	*sp(8, 8)* -8	*sp (16, R)* 8
C9	*sp (18)*	*sp (16,*	*sp (14,*	*sp (12,*	*sp (10,*	*sp (18,*

δ	-171	2) - `107	4) -59	6) -27	8) -11	R) 9

Type D:

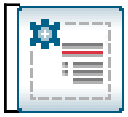
D4 δ	*so (8)* -28	*so (7,1)* -14	*so (6, 2)* -4	*so (5, 3)* 2	*so (4, 4)* 4	*so* (8) -4*					
D5 δ	*so (10)* -45	*so (9,1)* -27	*so (8, 2)* -13	*so (7, 3)* -3	*so (6, 4)* 3	*so (5, 5)* 5	*so* (10)* -5				
D6 δ	*so (12)* -66	*so (11, 1)* -44	*so (10, 2)* -26	*so (9, 3)* -12	*so (8, 4)* -2	*so (7, 5)* 4	*so (6, 6)* 6	*so* * -6	(12)		
D7 δ	*so (14)* -91	*so (13, 1)* -65	*so (12, 2)* -43	*so (11, 3)* -25	*so (10, 4)* -11	*so (9, 5)* -1	*so (8, 6)* 5	*so (7, 7)* 7	*so* (14)* -7		
D8 δ	*so (16)* -120	*so (15, 1)* -90	*so (14, 2)* -64	*so (13, 3)* -42	*so (12, 4)* -24	*so (11, 5)* -10	*so (10, 6)* 0	*so (9, 7)* 6	*so(8, 8)* 8	*so* * -8	(16)
D9 δ	*so (18*	*so (17,*	*so (16,*	*so (15,*	*so (14,*	*so (13,*	*so (12,*	*so (11,*	*so (10,*	*so (9,9)*	*so* (18)*

	) -15 3	*1)* -119	*2)* -89	*3)* -63	*4)* -41	*5)* -23	*6)* -9	*7)* 1	*8)* 7	9	-9

Details

Here are some simple programs used to generate the characters for the above tables.

```
alg > with(DifferentialGeometry): with(LieAlgebras):
```



For example, the command Adelta(r) calculates the characters for the real forms of A_r , namely, *sl(r+1), su(r+1), su(r,1), ...* :

```
alg > Adelta(5);
```

$$5, [5, -35, -15, -3, 1, -7] \quad (2.1.1)$$

```
alg > Bdelta(5);
```

$$5, [55, -35, -19, -7, 1, 5] \quad (2.1.2)$$

2. Simple roots and Satake Diagrams for so*(18) and so(12, 6)

In this section we initialize the Lie algebras *so*(18)* and *so(12, 6)* and calculate their properties. We check that the characters of each are equal to -9. From the simple roots we are easily able to confirm that these Lie algebras are not isomorphic.

```
> with(DifferentialGeometry): with(LieAlgebras):
```

Part 1 - so*(18):

Retrieve the structure equations for *so*(18)* and initialize the algebra.

```
> LD1 := SimpleLieAlgebraData("so*(18)", alg1):
> DGsetup(LD1, [e], [o]):
```

Calculate the properties of *so*(18)* (this will take several minutes to complete).

```
alg1 > Properties1 := SimpleLieAlgebraProperties(alg1):
```

Here is the Cartan decomposition.

```
alg1 > T1, P1 := Properties1:-CartanDecomposition;
```

T1, *P1* := [*e1*, *e2*, *e3*, *e4*, *e5*, *e6*, *e7*, *e8*, *e9*, *e10*, *e11*, *e12*, *e13*, *e14*, *e15*, *e16*, *e17*, *e18*, *e19*, *e20*, *e21*, *e22*, *e23*, *e24*, *e25*, *e26*, *e27*, *e28*, *e29*, *e30*, *e31*, *e32*, *e33*, *e34*, *e35*, *e36*, *e73*, *e74*, *e75*, *e76*, *e77*, *e78*, *e79*, *e80*, *e81*, *e82*, *e83*, *e84*, *e85*, *e86*, *e87*, *e88*, *e89*, *e90*, *e91*, *e92*, *e93*, *e94*, *e95*, *e96*, *e97*, *e98*, *e99*, *e100*, *e101*, *e102*, *e103*, *e104*, *e105*, *e106*, *e107*, *e108*, *e109*, *e110*, *e111*, *e112*, *e113*, *e114*, *e115*, *e116*, *e117*], [*e37*, *e38*, *e39*, *e40*, *e41*, *e42*, *e43*, *e44*, *e45*, *e46*, *e47*, *e48*, *e49*, *e50*, *e51*, *e52*, *e53*, *e54*, *e55*, *e56*, *e57*, *e58*, *e59*, *e60*, *e61*, *e62*, *e63*, *e64*, *e65*, *e66*, *e67*, *e68*, *e69*, *e70*, *e71*, *e72*, *e118*, *e119*, *e120*, *e121*, *e122*, *e123*, *e124*, *e125*, *e126*, *e127*, *e128*, *e129*, *e130*, *e131*, *e132*, *e133*, *e134*, *e135*, *e136*, *e137*, *e138*, *e139*, *e140*, *e141*, *e142*, *e143*, *e144*, *e145*, *e146*, *e147*, *e148*, *e149*, *e150*, *e151*, *e152*, *e153*] **(3.1**

We see that the character of *so*(18)* is -9.

```
alg1 > nops(T1), nops(P1);
```

81, 72 **(3.2**

Here are the simple roots for *so*(18)*.

```
alg1 > SimRts1 := Properties1:-SimpleRoots;
```

$$SimRts1 := \left[\begin{bmatrix}0\\0\\0\\0\\2\,I\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\0\\-I\\-I\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\0\\2\,I\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\0\\0\\-I\\-I\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\0\\0\\2\,I\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\-1\\0\\0\\-I\\-I\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\2\,I\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\-I\\-I\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\-I\\I\end{bmatrix}\right] \qquad \textbf{(3.3}$$

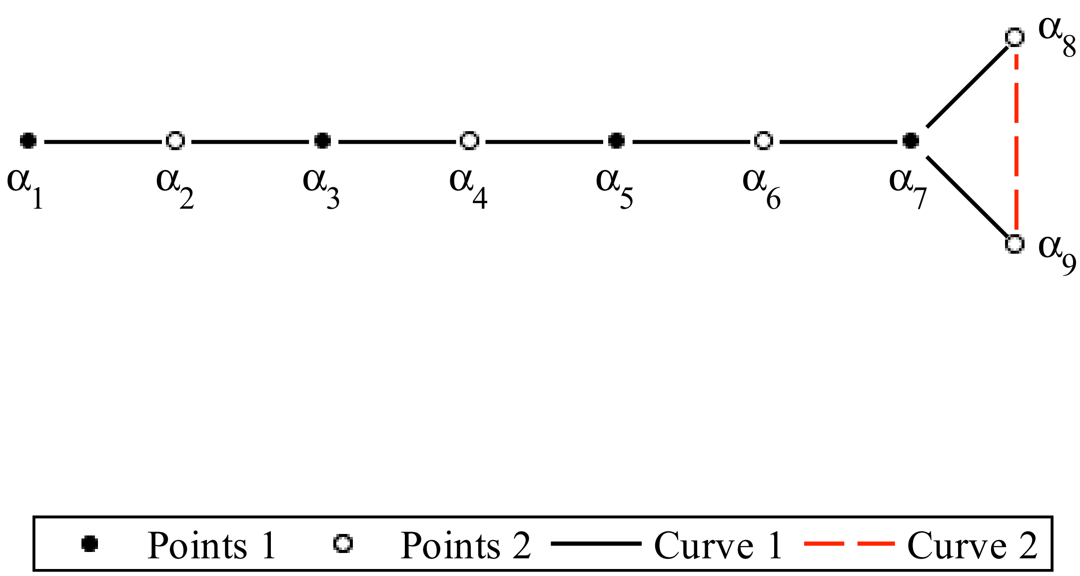
We see that there are 4 pure imaginary roots so there are 4 black dots in the SatakeDiagram. The Satake associate of root 8 is root 9.

```
alg1 > SatakeAssociate(SimRts1[8], SimRts1);
```

$$\begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\-I\\I\end{bmatrix} \qquad \textbf{(3.4}$$

This 2 facts are coded in the Satake diagram for *so*(18)*.

```
alg1 > SatakeDiagram("so*(18)");
```



Now we turn to the second Lie algebra.

Part 2 - so(12, 6):

Retrieve the structure equations for *so(12, 6)* and initialize the algebra.

```
> LD2 := SimpleLieAlgebraData("so(12, 6)", alg2):
> DGsetup(LD2, [f], [o]):
```

Calculate the properties of *so(12, 6)* (this will take several minutes to complete).

```
alg2 > Properties2 := SimpleLieAlgebraProperties(alg2):
```

Here is the Cartan decomposition.

```
alg2 > T2, P2 := Properties2:-CartanDecomposition;
```

$$T2, P2 := [f2 - f7, f3 - f13, f4 - f19, f5 - f25, f6 - f31, f9 - f14, f10 - f20, f11 - f26, f12 - f32, f16 - f21, f17 - f27, f18 - f33, f23 - f28, f24 - f34, f30 - f35, f37 + f52, f38 + f53, f39 + f54, f40 + f55, f41 + f56, f42 + f57, f43 + f58, f44 + f59, f45 + f60, f46 + f61, f47 + f62, f48 + f63, f49 + f64, f50 + f65, f51 + f66, f67 + f103, f68 + f104, f69 + f105, f70 + f106, f71 + f107, f72 + f108, f73 + f109, f74 + f110, f75 + f111, f76 + f112, f77 + f113, f78 + f114, f79 + f115, f80 + f116, f81 + f117, f82 + f118, f83 + f119, f84 + f120, f85 + f121, f86 + f122, f87 + f123, f88 + f124, f89 + f125, f90 + f126, f91 + f127, f92 + f128, f93 + f129, f94 + f130, f95 + f131, f96 + f132, f97 + f133, f98 + f134, f99 + f135, f100 + f136, f101 + f137, f102 + f138, f139, f140, f141, f142, f143, f144, f145, f146, f147, f148, f149, f150, f151, f152, f153], [f1, f2 + f7, f3 + f13, f4 + f19, f5 + f25, f6 + f31, f8, f9 + f14, f10 + f20, f11 + f26, f12 + f32, f15, f16 + f21, f17 + f27, f18 + f33, f22, f23 + f28, f24 + f34, f29, f30 + f35, f36, f37 - f52, f38 - f53, f39 - f54, f40 - f55, f41 - f56, f42 - f57, f43 - f58, f44 - f59, f45 - f60, f46 - f61, f47 - f62, f48 - f63, f49 - f64, f50 - f65, f51 - f66, f67 - f103, f68 - f104, f69 - f105, f70 - f106, f71 - f107, f72 - f108, f73 - f109, f74 - f110, f75 - f111, f76 - f112, f77 - f113, f78 - f114, f79 - f115, f80 - f116, f81 - f117, f82 - f118, f83 - f119, f84 - f120, f85 - f121, f86 - f122, f87 - f123, f88 - f124, f89 - f125, f90 - f126, f91 - f127, f92 - f128, f93 - f129, f94 - f130, f95 - f131, f96 - f132, f97 - f133, f98 - f134, f99 - f135, f100 - f136, f101 - f137, f102 - f138] \quad (3.5$$

We see that the character of *so(12, 6)* is -9, which is the same as *so*(18)*.

```
alg2 > nops(T2), nops(P2);
```

$$81, 72 \quad (3.6$$

Here are the simple roots for *so(12, 6)*.

```
alg2 > SimRts2 := Properties2:-SimpleRoots;
```

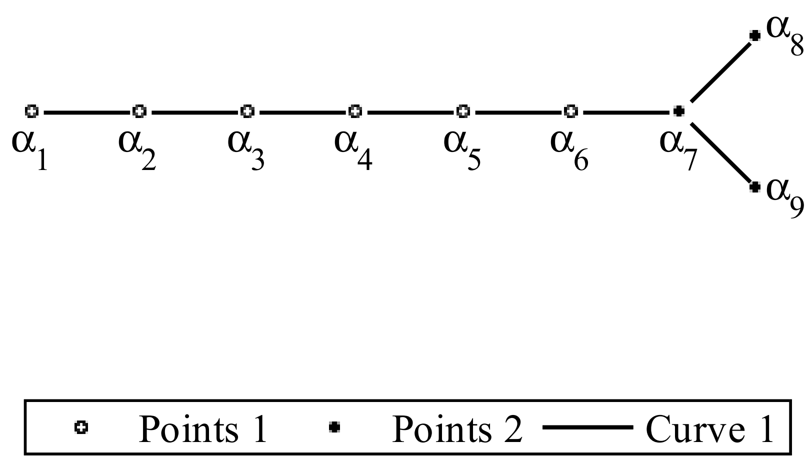
(3.7

$$SimRts2 := \left[\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -I \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I \\ -I \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I \\ -I \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I \\ I \end{bmatrix} \right] \qquad \textbf{(3.7}$$

We see that there are 3 pure imaginary roots (the last 3 simple roots) so there are 3 black dots in the Satake diagram. *This suffices to show that the Lie algebras so*(18) and so(12, 6) are not isomorphic*.

The Satake diagram for *so(12, 6)* is:

```
alg1 > SatakeDiagram("so(12, 6)");
```



3. Classification of the Compact Part of the Cartan Decomposition for so*(18)

Initialize the compact part T1 of the Cartan decomposition of *so*(18)* as a Lie algebra in its own right.

(4.1

```
alg2 > LDC1 := LieAlgebraData(T1, Compact1):
alg2 > DGsetup(LDC1, [x], [o]);
```

Lie algebra: Compact1 (4.2

Since the compact part of the Cartan decomposition is always reductive but perhaps not semi-simple, we calculate the Levi decomposition. This takes several minutes to calculate so we store the result.

```
Compact1 > #Levi1 := LeviDecomposition(Compact1);
```

```
Compact1 > Levi1 := [[x37+x46+x54+x61+x67+x72+x76+x79+x81], [x1, x2, x3, x4, x5,
           x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20,
           x21, x22, x23, x24, x25, x26, x27, x28, x29, x30, x31, x32, x33, x34,
           x35, x36, x37-x81, x38, x39, x40, x41, x42, x43, x44, x45, x46-x81, x47,
```

```
x48, x49, x50, x51, x52, x53, x54-x81, x55, x56, x57, x58, x59, x60, x61
-x81, x62, x63, x64, x65, x66, x67-x81, x68, x69, x70, x71, x72-x81,
x73, x74, x75, x76-x81, x77, x78, x79-x81, x80]];
```

$Levi1 := [[x37 + x46 + x54 + x61 + x67 + x72 + x76 + x79 + x81], [x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20, x21, x22, x23, x24, x25, x26, x27, x28, x29, x30, x31, x32, x33, x34, x35, x36, x37 - x81, x38, x39, x40, x41, x42, x43, x44, x45, x46 - x81, x47, x48, x49, x50, x51, x52, x53, x54 - x81, x55, x56, x57, x58, x59, x60, x61 - x81, x62, x63, x64, x65, x66, x67 - x81, x68, x69, x70, x71, x72 - x81, x73, x74, x75, x76 - x81, x77, x78, x79 - x81, x80]]$ (4.3

We see there is 1-dimensional center.

```
Compact1 > Levi1[1];
```

$$[x37 + x46 + x54 + x61 + x67 + x72 + x76 + x79 + x81]$$ (4.4

Initialize the semi-simple part of the Levi decomposition.

```
Compact1 > LDCss1 := LieAlgebraData(Levi1[2], Compact1ss):
Compact1 > DGsetup(LDCss1, [y], [o]);
```

Lie algebra: Compact1ss (4.5

To classify the Lie algebra Compact1ss, we need a Cartan subalgebra. Because this algebra is quite large, it helps to first identify an abelian Lie algebra *A* of semi-simple (that is, diagonalizable) elements in the algebra and then look for a Cartan subalgebra containing *A*.

```
Compact1ss > Query(Adjoint(y1), "Diagonalizable");
```

true (4.6

```
Compact1ss > GNS1 := GeneralizedNullSpace([y1]);
```

$GNS1 := [y1, y16, y17, y18, y19, y20, y21, y22, y23, y24, y25, y26, y27, y28, y29, y30, y31, y32, y33, y34, y35, y36, y37 + y46, y54, y55, y56, y57, y58, y59, y60, y61, y62, y63, y64, y65, y66, y67, y68, y69, y70, y71, y72, y73, y74, y75, y76, y77, y78, y79, y80]$ (4.7

```
Compact1ss > A := [y1, y16];
```

$$A := [y1, y16]$$ (4.8

```
Compact1ss > Query(A, "Abelian");
```

true (4.9

```
Compact1ss > CSA1 := CartanSubalgebra(contains = A);
```

(4.10

$$CSA1 := [y1, y16, y27, y37 + y46, y54 + y61, y67 + y72, y76, y79] \quad (4.10)$$

Now classify the Lie algebra.

```
Compact1ss > infolevel[ClassifyComplexSemiSimpleLieAlgebra] := 2;
```

$$infolevel_{DifferentialGeometry:-LieAlgebras:-ClassifyComplexSemiSimpleLieAlgebra} := 2 \quad (4.11)$$

```
Compact1ss > ClassifyComplexSemiSimpleLieAlgebra(CSA1);
Step 1. Find the root space decomposition
Step 2. Find the positive and simple roots
Step 3. Find the Cartan matrix
Step 4. Put the Cartan matrix in standard form
```

"A8" (4.12)

We conclude that the compact part of the Cartan decomposition for *so*(18)* is $\mathfrak{t}_1 = u(1) \oplus su(9)$.

4. Classification of the Compact Part of the Cartan Decomposition for so(12, 6)

Initialize the compart part T2 of the Cartan decomposition of *so(12, 6)*.

```
Compact1ss > LDC2 := LieAlgebraData(T2, Compact2):
alg2 > DGsetup(LDC2, [z], [o]);
```

Lie algebra: Compact2 (5.1)

This time the compact part is semi-simple and so there is no need to calculate the Levi decomposition. We need a Cartan subalgebra and, as before, it helps to first identify an abelian Lie algebra *A* of semi-simple (that is, diagonalizable) elements in the algebra.

```
Compact2 > Query(Adjoint(z1), "Diagonalizable");
```

true (5.2)

```
Compact2 > GNS2 := GeneralizedNullSpace([z1]);
```

$$GNS2 := [z1, z10, z11, z12, z13, z14, z15, z16, z25, z26, z27, z28, z29, z30, z43, z44, z45, z46, z47, z48, z49, z50, z51, z52, z53, z54, z55, z56, z57, z58, z59, z60, z61, z62, z63, z64, z65, z66, z67, z68, z69, z70, z71, z72, z73, z74, z75, z76, z77, z78, z79, z80, z81] \quad (5.3)$$

```
Compact2 > A := [z1, z10];
```

$$A := [z1, z10] \tag{5.4}$$

```
Compact2 > Query(A, "Abelian");
```

$$true \tag{5.5}$$

```
Compact2 > CSA2 := CartanSubalgebra(contains = A);
```

$$CSA2 := [z1, z10, z15, z16, z25, z30, z67, z76, z81] \tag{5.6}$$

```
Compact2 > ClassifyComplexSemiSimpleLieAlgebra(CSA2);
Step 1. Find the root space decomposition
Step 2. Find the positive and simple roots
Step 3. Find the Cartan matrix
Step 4. Put the Cartan matrix in standard form
```

$$\text{"6A3"} \tag{5.7}$$

We conclude that the compact part of the Cartan decomposition for *so(12, 6)* is $\mathfrak{t}_2 = su(4) \oplus so(12) = so(6) \oplus so(12)$.

Highlighted Commands

ClassifyComplexSemiSimpleLieAlgebra, GeneralizedNullSpace, SimpleLieAlgebraProperties, SatakeDiagram

Release Notes

- This worksheet was compiled with Maple 17 and DG release USU1, available by request from ian.anderson@usu.edu .

References

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2. S. Helgason, *Differential Geometry, Lie Groups and Symmetric Spaces*, Pure and Applied Mathematics **80**, Academic Press (1978).

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