Space For The Next Generation

DRIFT RECOVERY AND STATION KEEPING RESULTS FOR THE HISTORIC CANX-4/CANX-5 FORMATION FLYING MISSION

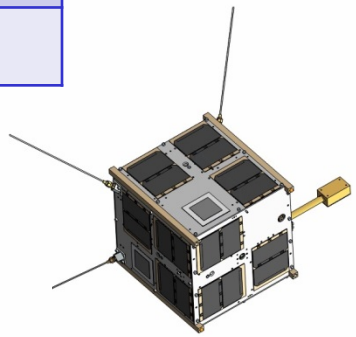
23rd Annual Frank J. Redd Student Competition

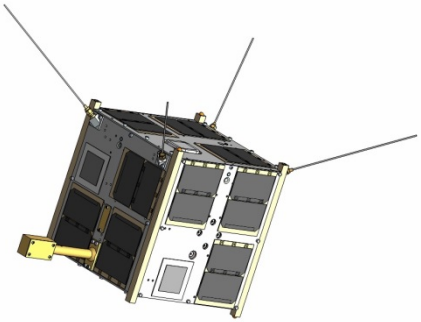
Josh Newman

CanX-4&5 Mission Overview

Number of spacecraft	2
Wet mass	6.5 kg
Size	20x20x20 cm
I_{sp}	40 s
Total ΔV	15 m/s

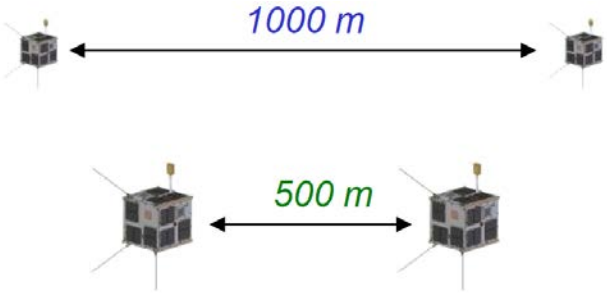
- Payloads:
  - CNAPS cold-gas thruster
  - GPS receiver
  - Inter-Satellite Link (ISL)





Formation Flying

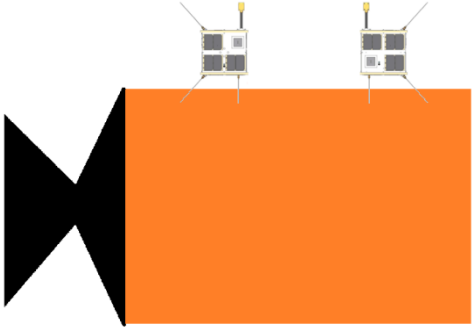
- Two Formation Types
  1. Along-Track Orbit

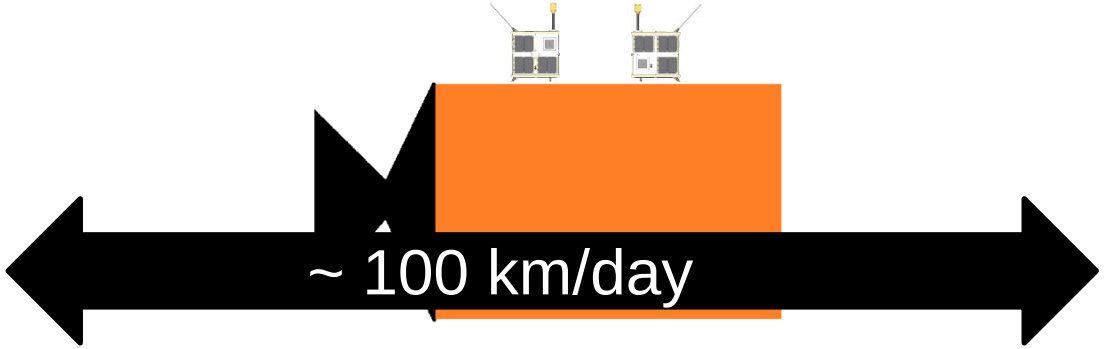


  2. Projected Circular Orbit

Chapter I

THE CHALLENGE




~ 100 km/day

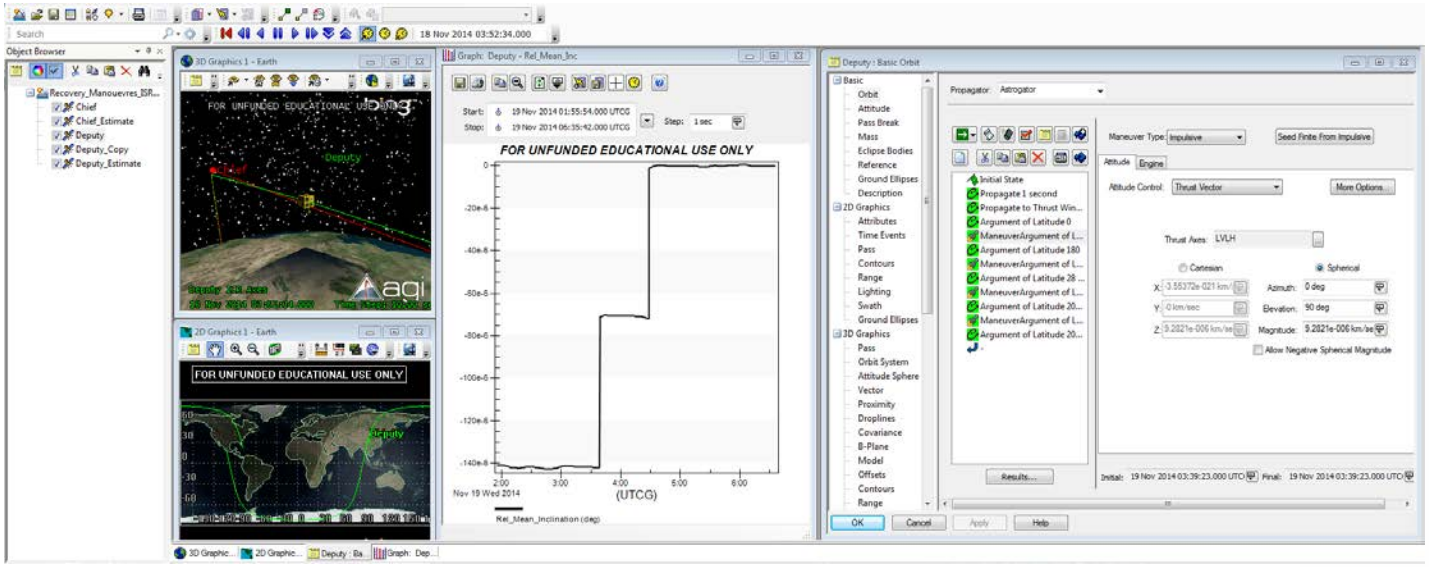
Drift Recovery Context

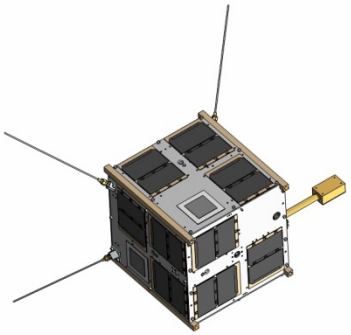
- Bring the spacecraft close
  - (a few km)
- Reduce relative motion
  - (a few cm/s)
- **Fuel efficiently**
  - (a few m/s)
- Easily extendable to include station keeping
  - (< a few headaches)

Chapter II

FORMULATING A SOLUTION

Drift Recovery and Station Keeping Overview





Mission architecture

- Chief/Deputy
- Chief – passive, is the reference trajectory by definition
- Deputy – active, manoeuvres to create, maintain, and leave formations
- State: $s_d = s_c + \delta s$

Motion Model

Gauss' Variational Equations

$$da = \frac{2a^2}{\sqrt{\mu a(1-e^2)}}\left[e\sin\theta\,\Delta V_R + (1+e\cos\theta)\Delta V_{AT}\right]$$

$$de = \sqrt{\frac{a(1-e^2)}{\mu}}\left[\sin\theta\,\Delta V_R + \frac{2\cos\theta + e(1+\cos^2\theta)}{(1+e\cos\theta)}\Delta V_{AT}\right]$$

$$di = \sqrt{\frac{a(1-e^2)}{\mu}}\frac{\cos(\omega+\theta)}{1+e\cos\theta}\Delta V_Z$$

$$d\Omega = \sqrt{\frac{a(1-e^2)}{\mu}}\frac{\sin(\omega+\theta)}{\sin i\,(1+e\cos\theta)}\Delta V_Z$$

$$d\omega = \sqrt{\frac{a(1-e^2)}{\mu}}\left[\frac{-\cos\theta}{e}\Delta V_R + \frac{(2+e\cos\theta)\sin\theta}{e(1+e\cos\theta)}\Delta V_{AT} - \frac{\sin(\omega+\theta)}{\tan i\,(1+e\cos\theta)}\Delta V_Z\right]$$

Drifting Elements

RAAN

Eccentricity

Argument of Perigee

Mean Anomaly

- Drifts due to non-spherical shape of the Earth.
- Makes the chief a moving, spiraling, oscillating target

Game Plan

1. Arrest the spacecraft drift
2. Create the opposite drift, back together
3. Phase the periodic motions
4. At the target range, stop relative motion

Trajectory Optimization

- Most important and most complicated part of the drift recovery sequence

Minimize:

$$\Delta V_{cost} = \Delta V_{Trajectory} + \Delta V_{Ecc/ArgP} + \Delta V_{Stop} + \Delta V_{Time}$$

Trajectory Optimization

$$\boldsymbol{\Delta V_{cost}} = \Delta V_{Trajectory} + \Delta V_{Ecc/ArgP} + \Delta V_{Stop} + \Delta V_{Time}$$

The fuel cost, measured in m/s, of a specific trajectory. This is the cost we want to minimize

Trajectory Optimization

$$\Delta V_{cost} = \boldsymbol{\Delta V_{Trajectory}} + \Delta V_{Ecc/ArgP} + \Delta V_{Stop} + \Delta V_{Time}$$

The fuel cost to go from the initial state, with the spacecraft moving apart, to one where the spacecraft are moving back together

Primary goal: change *a*, *i*,

Secondary goal: correct *e* , ω

Trajectory Optimization

$$\Delta V_{cost} = \Delta V_{Trajectory} + \boldsymbol{\Delta V_{Ecc/ArgP}} + \Delta V_{Stop} + \Delta V_{Time}$$

The fuel cost to correct the eccentricity and argument of perigee.

Changes: e, ω

Trajectory Optimization

$$\Delta V_{cost} = \Delta V_{Trajectory} + \Delta V_{Ecc/ArgP} + \boldsymbol{\Delta V_{Stop}} + \Delta V_{Time}$$

The fuel cost to bring the relative semi-major axis and inclination to zero when the spacecraft rendezvous

Changes: *a*, *i*

Trajectory Optimization

$$\Delta V_{cost} = \Delta V_{Trajectory} + \Delta V_{Ecc/ArgP} + \Delta V_{Stop} + \boldsymbol{\Delta V_{Time}}$$

The value that is applied to time, in m/s.

Typically, the fuel leak rate is used.

Testing

- Thousands of simulations, varying:
  - Deployment geometry
  - Attitude control
  - Navigational errors
  - Delays in commissioning
- Tested on spacecraft hardware simulator
- Algorithms robust against initial conditions and control errors
- Total confidence in mission success

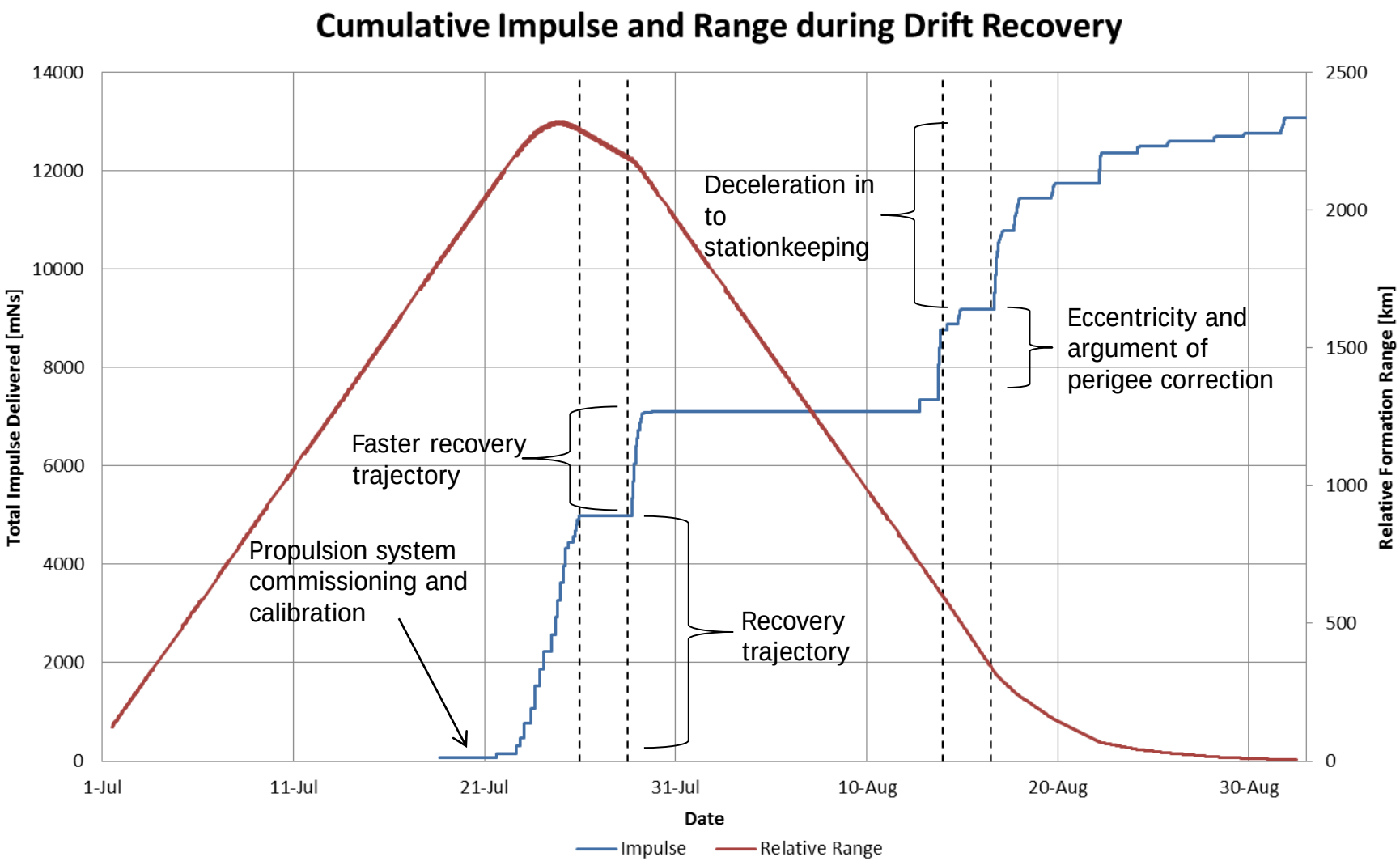
Chapter III

ON-ORBIT RESULTS

Launch

- Launched 30 June 2014 aboard PSLV-C23
- 650 km, circular, sun-synchronous
- Ejected from separate deployment systems

PSLV C-23 lifting off from Satish Dhawan Space Centre


Cumulative Impulse and Range during Drift Recovery
Total Impulse Delivered [mNs]
Relative Formation Range [km]
Date
14000
12000
10000
8000
6000
4000
2000
0
2500
2000
1500
1000
500
0
1-Jul
11-Jul
21-Jul
31-Jul
10-Aug
20-Aug
30-Aug
Deceleration in to stationkeeping
Eccentricity and argument of perigee correction
Faster recovery trajectory
Propulsion system commissioning and calibration
Recovery trajectory
Impulse
Relative Range

Drift Recovery Summary

- 18 July to 2 September 2014
- Maximum separation: 2320 km
- Final separation: 2.95 km

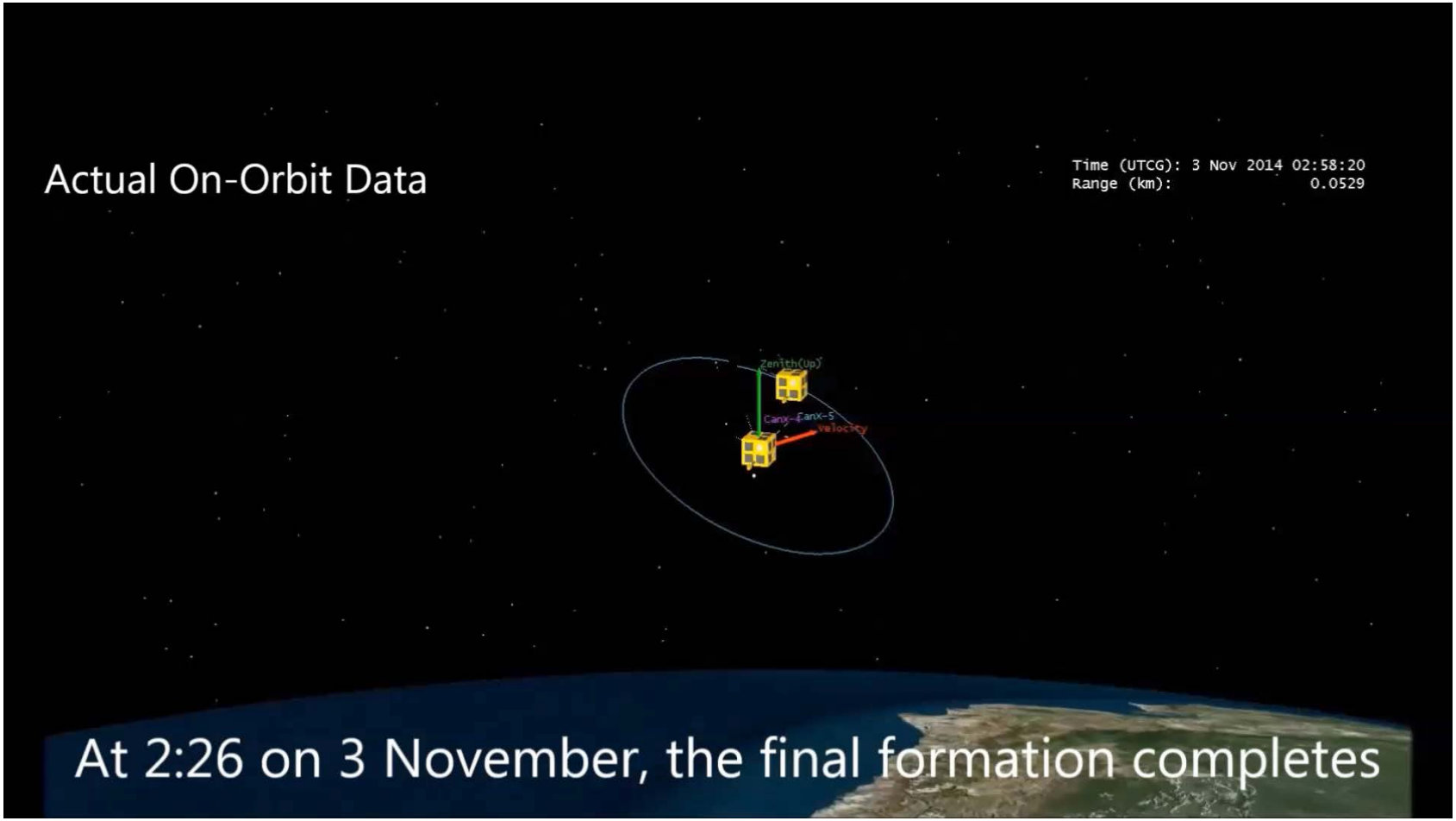
- 102 individual manoeuvres commanded
- ΔV: 2.03 m/s

- Another 1st on the nanosatellite scale

Station Keeping Summary

- 02 September to 19 November 2014
- 59 manoeuvres
- 0.81 m/s

Passive Collision Avoidance



Where we are and What's Next?

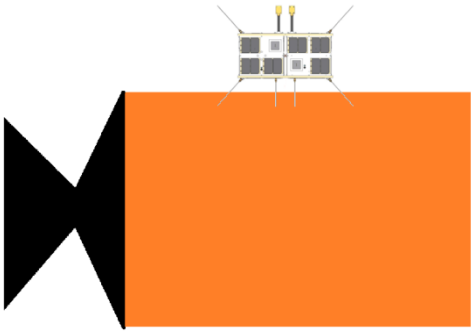
- Primary mission objectives completed in November 2014
- Currently in safe-hold mode, 125 km apart.
- Hopeful to acquire funding to perform further testing, but nothing final yet

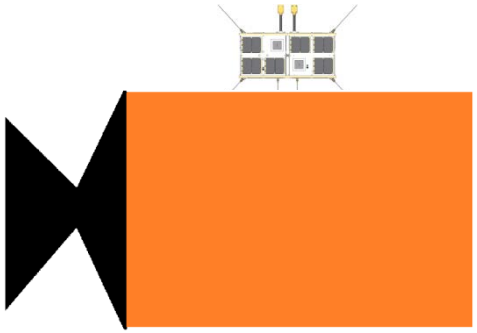
Thanks!

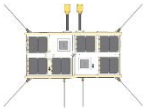
ANY QUESTIONS?

Special Thanks

- My family
- Grant Bonin, Niels Roth, Scott Armitage, Ben Risi
- Brad Cotten, Thomas Sears, Jamie Fine, John Chung
- SFL Community
- Dr. Robert E. Zee







Orbital Mechanics 101

Keplerian two-body equation of motion:

Gravitational Parameter

Acceleration: $$\ddot{\vec{\boldsymbol{r}}} = -\frac{\mu \vec{\boldsymbol{r}}}{\left|\vec{\boldsymbol{r}}\right|^3}$$

Position: $$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$$

Eccentricity

Velocity $$\dot{r} = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)}$$

Semi-Major Axis

Where a is semi-major axis, e is eccentricity, θ is true anomaly, and μ is the gravitational parameter of the primary body (Earth)

Drifting Elements

Mean Anomaly

- Drifts secularly as a function of semi-major axis, due to non-spherical shape of the Earth.
  - Two spacecraft in similar orbits but different semi-major axes will drift apart without bound.
  - Two spacecraft in similar orbits but separated along-track can be brought together over time by changing their relative semi-major axes.

- For given separation and relative semi-major axis, the time to intersection can be found.

Drifting Elements

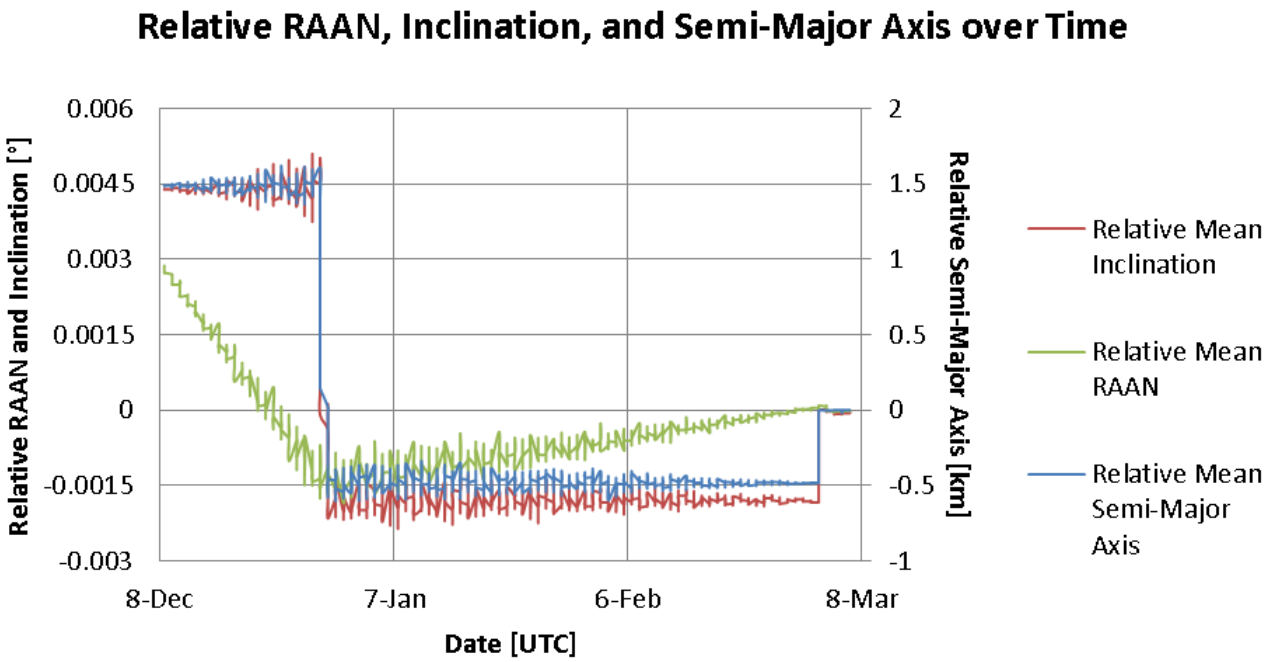
RAAN

- Drifts secularly as a function of semi-major axis and inclination.

$$\frac{d\Omega}{dt} = -\frac{3}{2}\frac{J_2 R_{\oplus}^2}{(1-e^2)^2}\sqrt{\frac{\mu}{a^7}}\cos i$$

- Non-zero relative Ω manifests as a periodic cross-track motion.
- Can be fixed instantaneously, or via relatively small changes in a,i over time.

Drifting Elements



Drifting Elements

Argument of Perigee and Eccentricity

- For moderately low values of eccentricity, ω varies secularly:

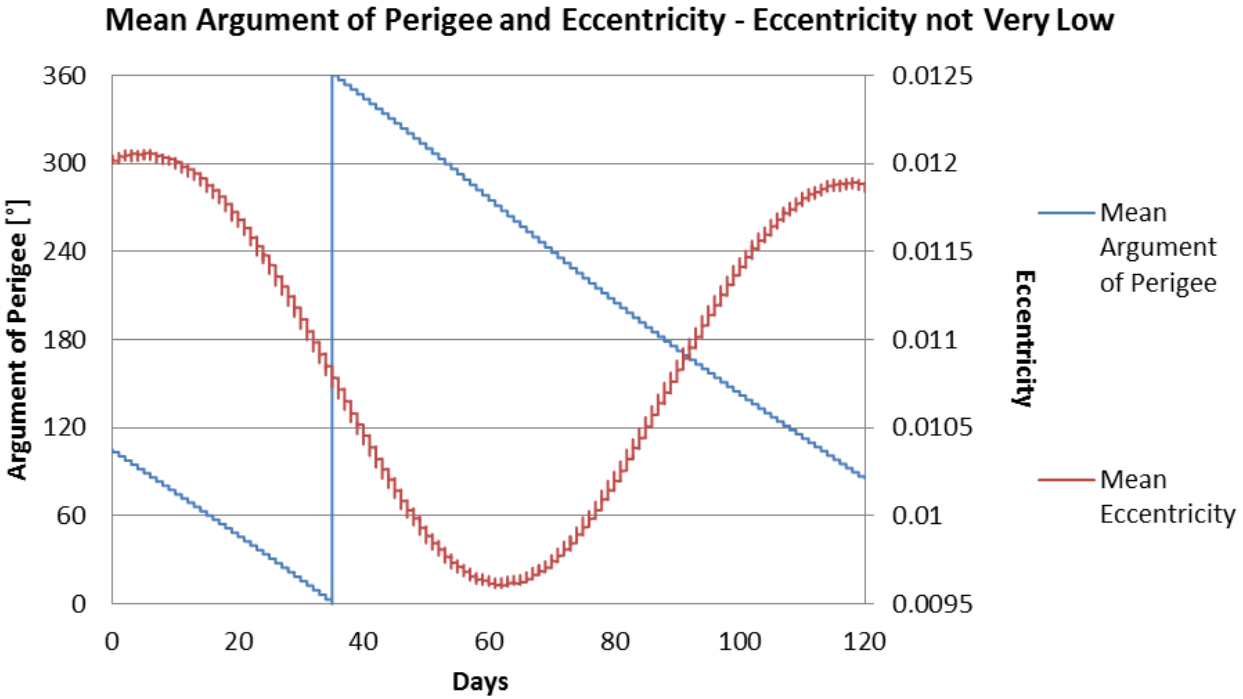
$$\dot{\omega} = \frac{1}{2} G\eta(1 - 5\cos^2 i)$$

$$\eta = \sqrt{1 - e^2}$$

$$G = -\frac{3}{2} J_2 \left(\frac{R_\oplus}{a\eta^2}\right)^2$$

- e oscillates about a mean point

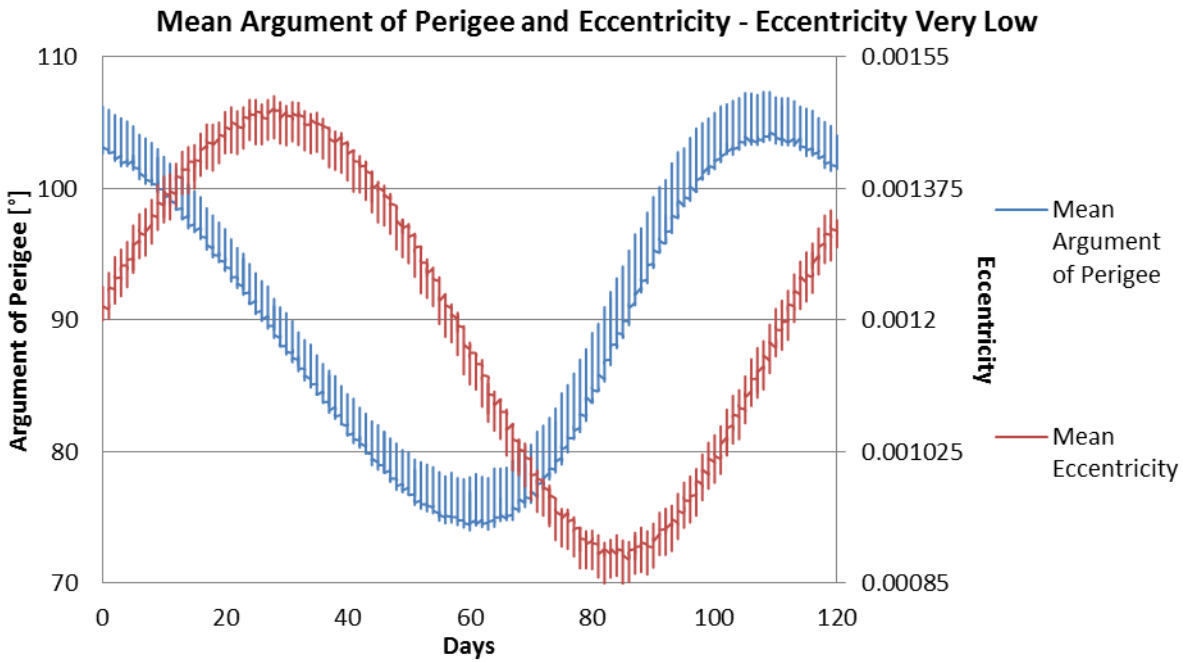
Drifting Elements



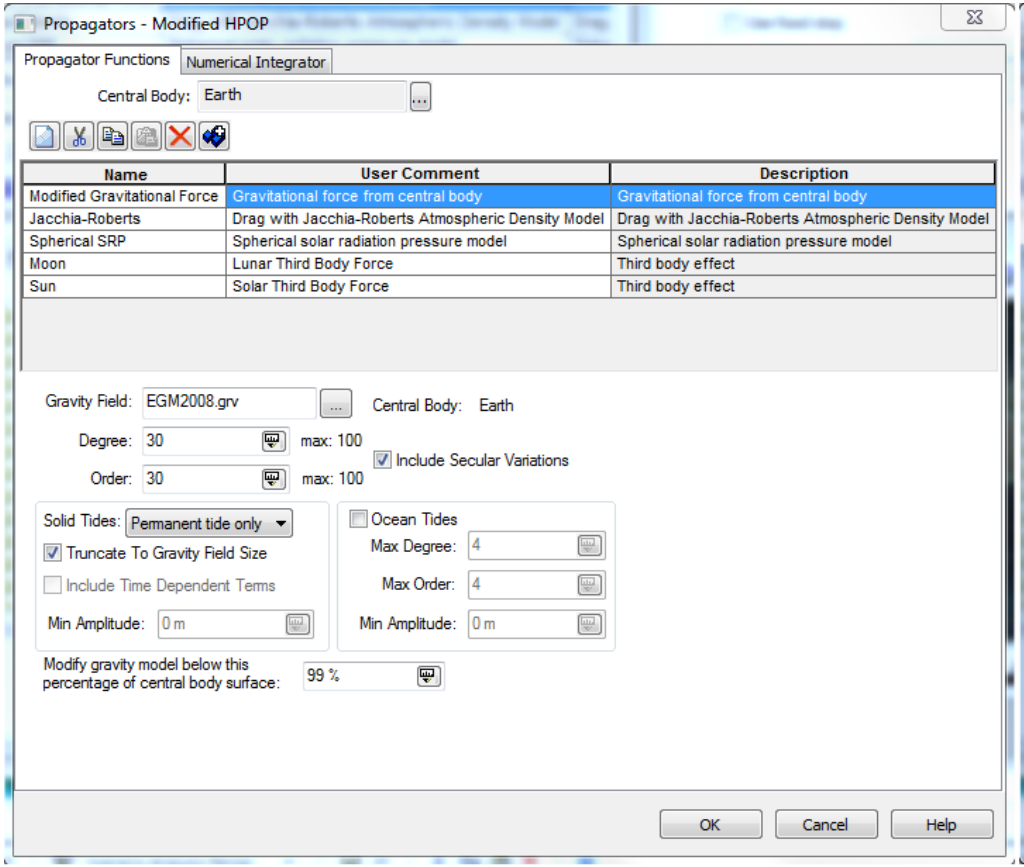
Drifting Elements

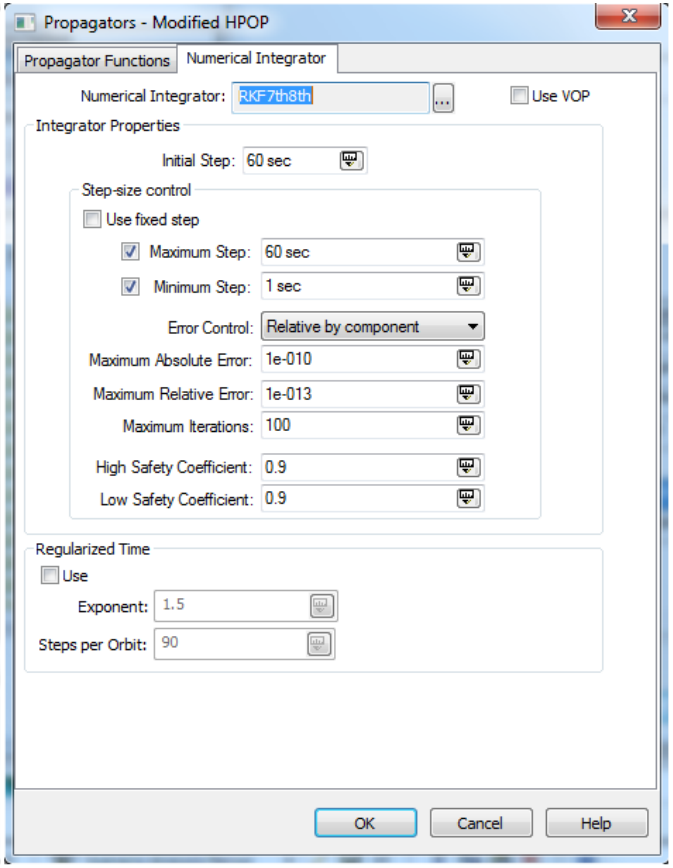
Argument of Perigee and Eccentricity

- For very low values of eccentricity, ω oscillates about 90°

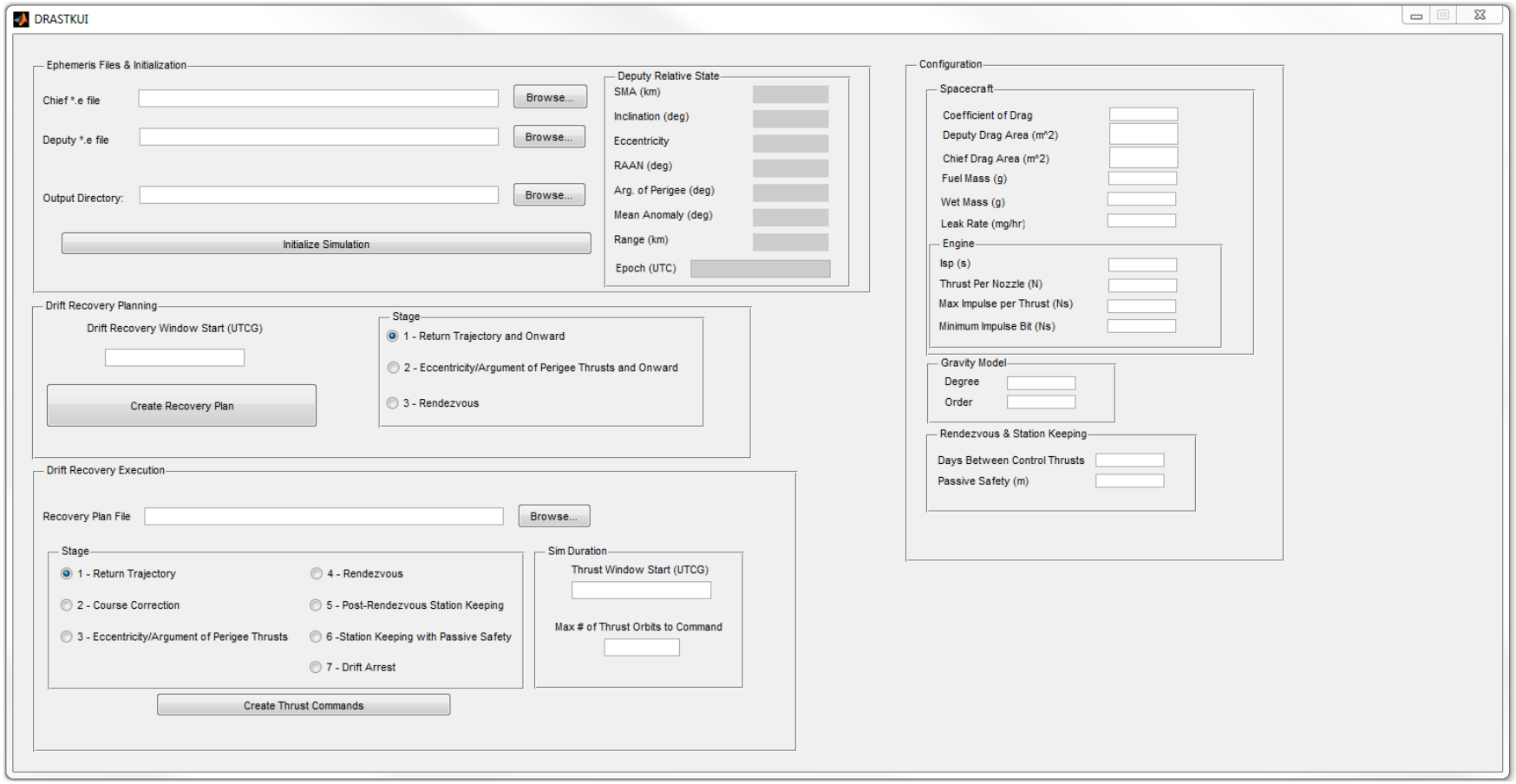


Propagator Configuration





DRASTK GUI


DRASTKUI
Ephemeris Files & Initialization
Chief *.e file
Browse...
Deputy *.e file
Browse...
Output Directory:
Browse...
Initialize Simulation
Deputy Relative State
SMA (km)
Inclination (deg)
Eccentricity
RAAN (deg)
Arg. of Perigee (deg)
Mean Anomaly (deg)
Range (km)
Epoch (UTC)
Configuration
Spacecraft
Coefficient of Drag
Deputy Drag Area (m^2)
Chief Drag Area (m^2)
Fuel Mass (g)
Wet Mass (g)
Leak Rate (mg/hr)
Engine
Isp (s)
Thrust Per Nozzle (N)
Max Impulse per Thrust (Ns)
Minimum Impulse Bit (Ns)
Gravity Model
Degree
Order
Rendezvous & Station Keeping
Days Between Control Thrusts
Passive Safety (m)
Drift Recovery Planning
Drift Recovery Window Start (UTCG)
Create Recovery Plan
Stage
1 - Return Trajectory and Onward
2 - Eccentricity/Argument of Perigee Thrusts and Onward
3 - Rendezvous
Drift Recovery Execution
Recovery Plan File
Browse...
Stage
1 - Return Trajectory
2 - Course Correction
3 - Eccentricity/Argument of Perigee Thrusts
4 - Rendezvous
5 - Post-Rendezvous Station Keeping
6 -Station Keeping with Passive Safety
7 - Drift Arrest
Sim Duration
Thrust Window Start (UTCG)
Max # of Thrust Orbits to Command
Create Thrust Commands

DRASTK Plan File

Drift Recovery Plan	
Date Created (local):	7/6/2014 20:27
Date of Return Trajectory Burn:	27 Jul 2014 17:25:45.000 UTCG
Target Relative SMA (km):	0.384
Target Relative Inc (deg):	0.001382573
Estimated DV (m/s):	0.764614191
Date of Ecc/ArgP Correction Burn:	5 Aug 2014 07:39:56.000 UTCG
Estimated DV (m/s):	0.305721221
Date to Begin Rendezvous Burn:	18 Sep 2014 17:35:21.602 UTCG
Relative range at Start of Rendezvous burn	167.3401463
Days Taken to Rendezvous	27.98105785
Rendezvous Time Constant	0.134047209
Estimated DV (m/s):	0.325985503
Date of Drift Recovery Completion:	19 Oct 2014 17:08:04.000 UTCG
Total DV Spent on Manoeuvres (m/s)	1.396320915
Total DV Lost to Leakage (m/s)	0.629735117
Total DV spent on Drift Recovery (Leakage and Manoeuvres) (m/s)	2.026056032
Ecc/ArgPFlag	0
Initial Relative SMA	-0.756099255
Initial Relative Inc	-0.002140855

Trajectory Optimization

$$\Delta V_{cost} = \boldsymbol{\Delta V_{Trajectory}} + \Delta V_{Ecc/ArgP} + \Delta V_{Stop} + \Delta V_{Time}$$

The fuel cost to go from the initial state, with the spacecraft moving apart, to one where the spacecraft are moving back together

Change *a*, to reverse secular along-track motion

$$\frac{d\Omega}{dt} = -\frac{3}{2}\frac{J_2 R_{\oplus}^2}{(1-e^2)^2}\sqrt{\frac{\mu}{a^7}}\cos i$$

Solution: change *i* at the same time

Trajectory Optimization

$$\Delta V_{cost} = \Delta V_{Trajectory} + \boldsymbol{\Delta V_{Ecc/ArgP}} + \Delta V_{Stop} + \Delta V_{Time}$$

$$da = \frac{2a^2}{\sqrt{\mu a(1-e^2)}}\left[e\sin\theta\,\Delta V_R + (1+e\cos\theta)\Delta V_{AT}\right]$$

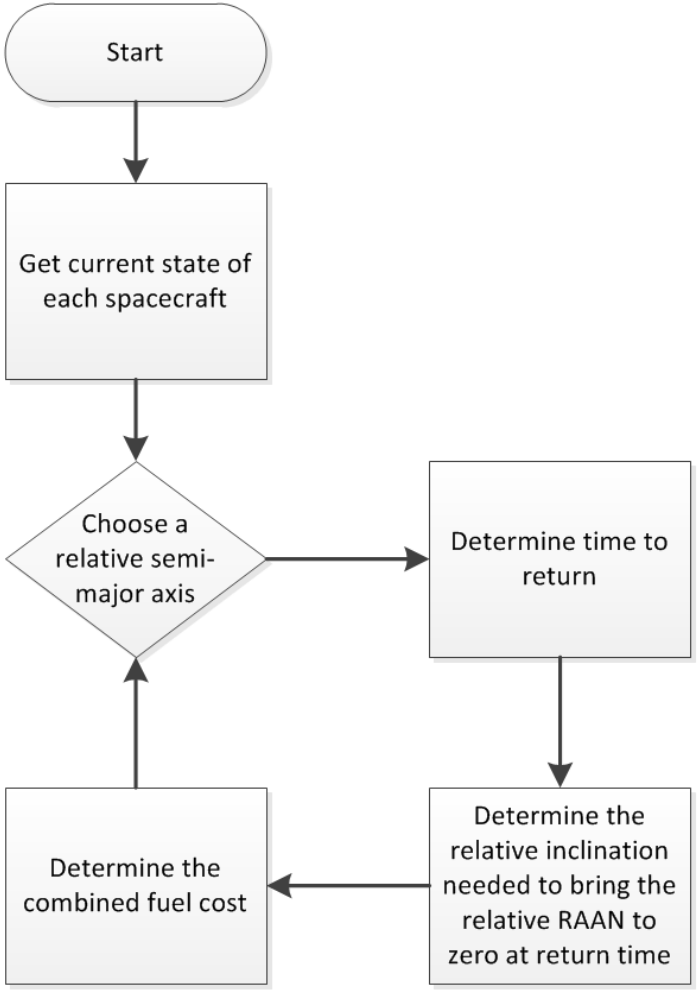
$$de = \sqrt{\frac{a(1-e^2)}{\mu}}\left[\sin\theta\,\Delta V_R + \frac{2\cos\theta + e(1+\cos^2\theta)}{(1+e\cos\theta)}\Delta V_{AT}\right]$$

$$di = \sqrt{\frac{a(1-e^2)}{\mu}}\frac{\cos(\omega+\theta)}{1+e\cos\theta}\Delta V_Z$$

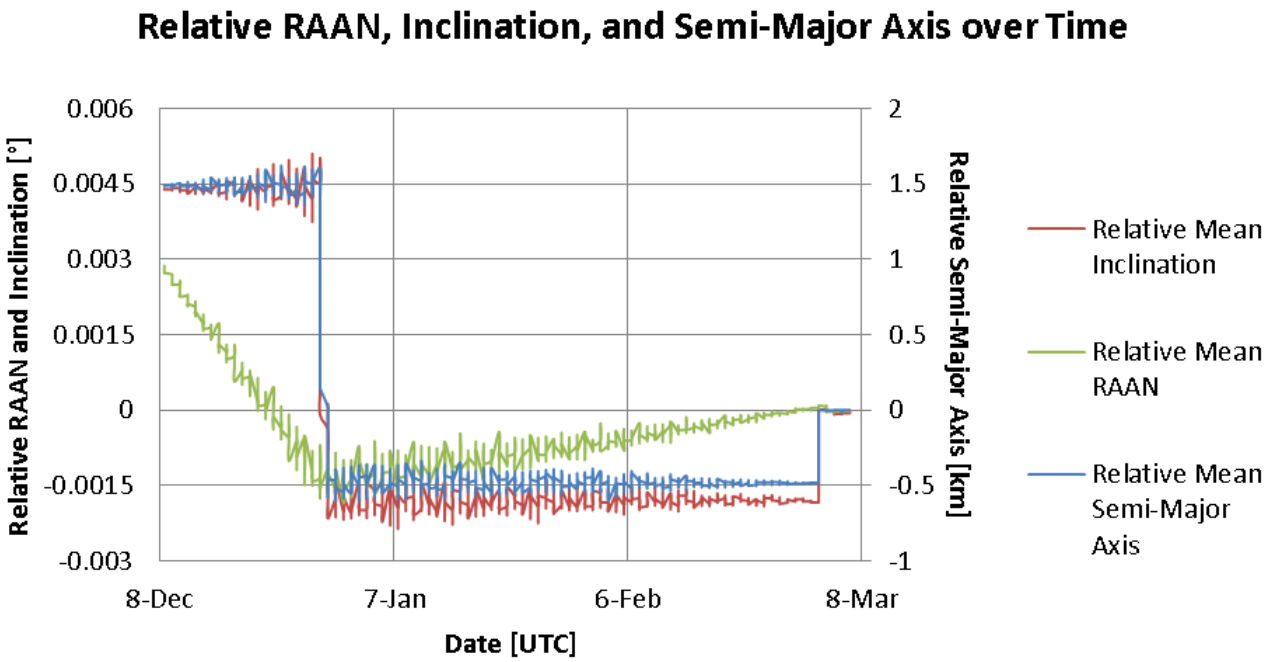
$$d\Omega = \sqrt{\frac{a(1-e^2)}{\mu}}\frac{\sin(\omega+\theta)}{\sin i\,(1+e\cos\theta)}\Delta V_Z$$

$$d\omega = \sqrt{\frac{a(1-e^2)}{\mu}}\left[\frac{-\cos\theta}{e}\Delta V_R + \frac{(2+e\cos\theta)\sin\theta}{e(1+e\cos\theta)}\Delta V_{AT} - \frac{\sin(\omega+\theta)}{\tan i\,(1+e\cos\theta)}\Delta V_Z\right]$$

Trajectory Optimization



Trajectory Optimization



Trajectory Optimization

$$\Delta V_{Ecc/ArgP} = \Delta V_{Ecc/ArgP_0} + S_{EccArgP} \Delta V_{AT,SMA}$$

where

$$S_{Ecc} = -\left| \sin\left(2\omega \frac{\pi}{\frac{\pi}{2} + A} + \frac{\pi}{2} + A \right) \right|$$

For CanX-4&5, $A \sim 55°$

Ecc/ArgP Correction

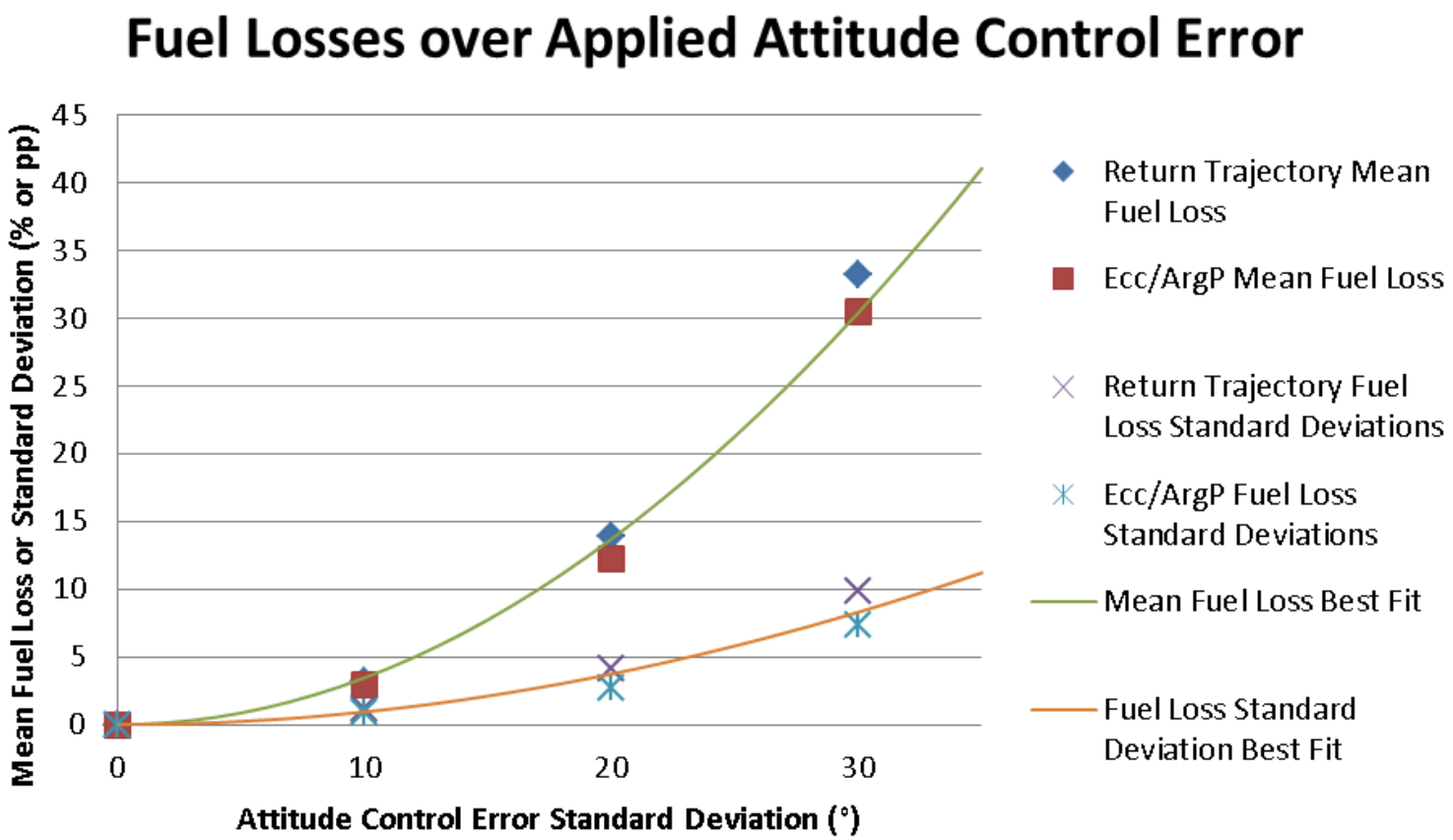
$$A = \sqrt{\frac{a(1-e^2)}{\mu}} \begin{bmatrix} 2\cos\theta & \sin\theta) \\ \frac{2\sin\theta}{e} & -\frac{\cos\theta}{e} \end{bmatrix}$$

$$C = \begin{bmatrix} \Delta e \\ \Delta\omega \end{bmatrix}$$

$$\Delta V_{Ecc/ArgP} = |A^{-1}C|$$

Find θ such that $\Delta V_{Ecc/ArgP}$ is minimized

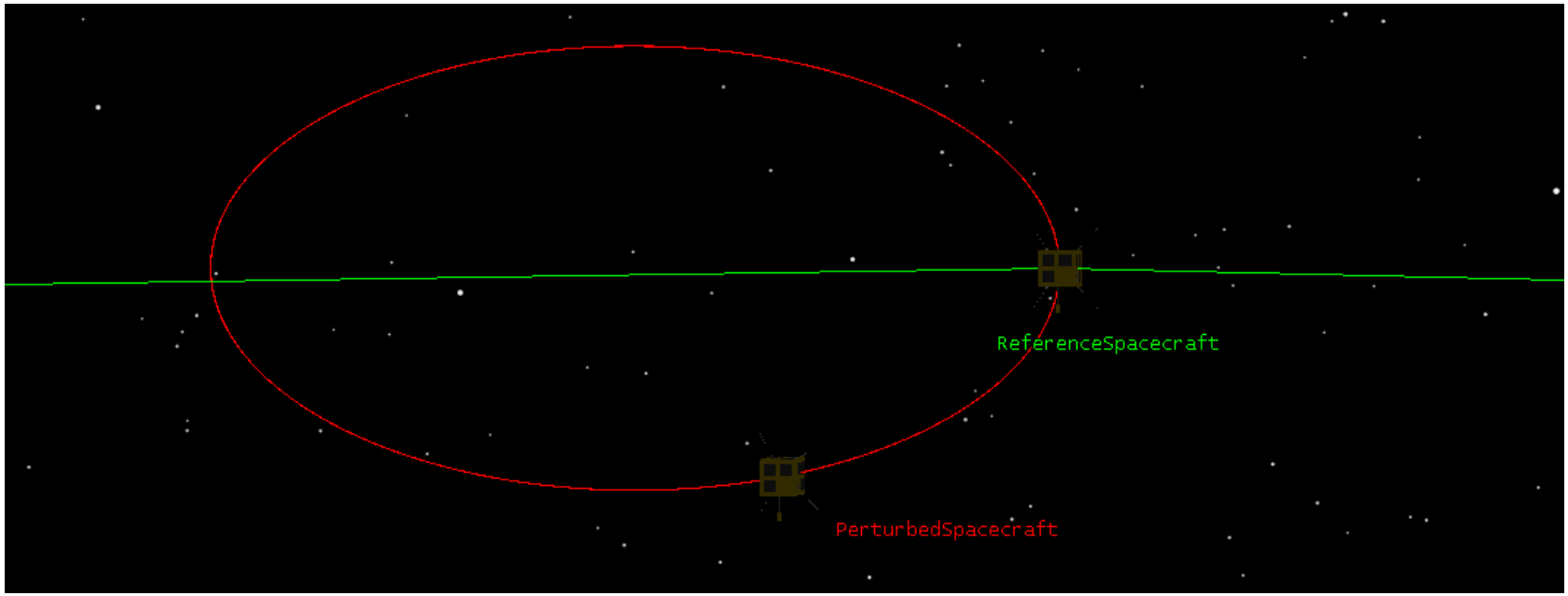
Attitude Errors



Navigational Errors

Gain	Position Error St. Dev. (m)	Absolute Velocity Error St. Dev. (mm/s)	Relative Velocity Error St. Dev. (mm/s)	Mean Fuel Error (%)	St. Dev. (pp)
0	0.000	0.000	0.000	0.00	0.00
15	0.178		1.903	5.02	3.33
25	0.289		3.100	17.83	11.97
35	0.419		4.489	34.06	23.70
85	0.993	1.063		0.37	0.53
200	2.281	2.443		1.09	1.21
300	3.285	3.519		3.14	2.98

Passive Collision Avoidance



An unsafe relative orbit. If left uncorrected, these spacecraft will collide.

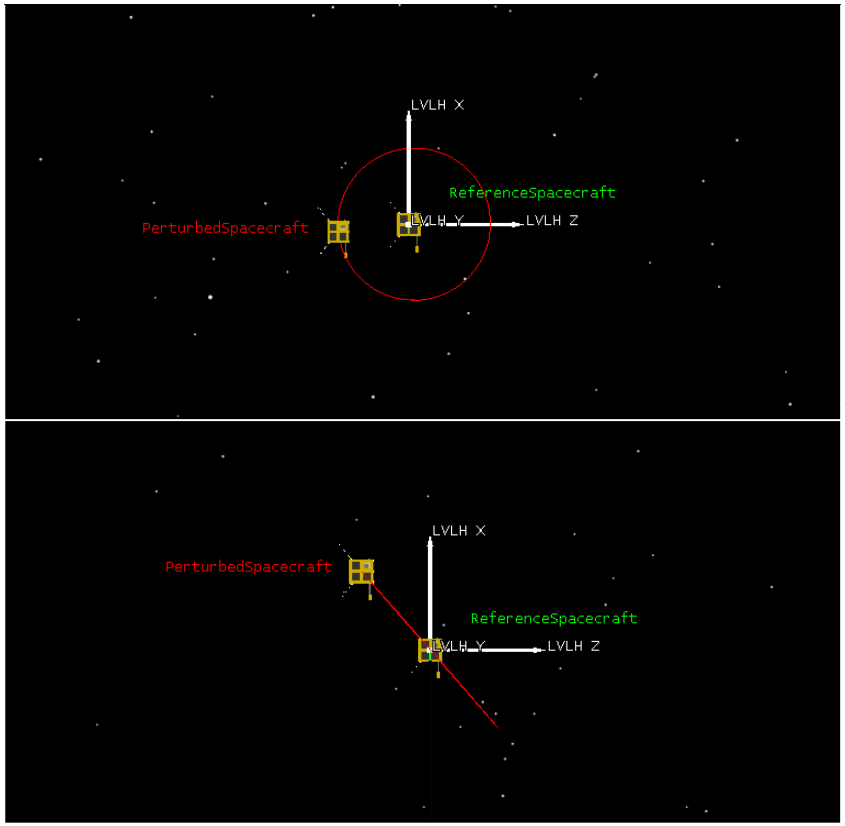
Combined errors

- Using 10 degree mean attitude error, absolute navigation error gain of 85 and relative navigation error of 15
- Expected fuel loss error of 5.75%, standard deviation of 2.80 pp
- 3σ fuel loss error is expected to be 14.15%
  - Within budget

Station Keeping

- Between autonomous formation flying experiments
- Keeps spacecraft safely separated
- Navigational errors, perturbations will cause the spacecraft to drift, possibly on a collision course
- Desire a passive state that will keep the spacecraft a safe minimum distance apart.

Passive Collision Avoidance



Passively safe (top) and unsafe (bottom) relative motion, as viewed from along the reference spacecraft's velocity vector.

Passive Collision Avoidance

1. Identify uncorrected radial, cross-track motions
2. Identify desired radial, cross-track motions
3. Find the intersections of the initial and desired states
4. Choose the intersection with the lowest relative velocity
5. Perform thrust

Passive Collision Avoidance

Perturbations - RAAN

- Recall:

$$\frac{d\Omega}{dt} = -\frac{3}{2}\frac{J_2 R_{\oplus}^2}{(1-e^2)^2}\sqrt{\frac{\mu}{a^7}}\cos i$$

- Can cause cross-track motion to change over time, possibly creating a dangerous state

Passive Collision Avoidance

Perturbations - RAAN

- For a desired relative semi-major axis, an relative inclination can be found to negate the procession change

$$\Delta i_p = \cos^{-1}\left(\cos i_r \sqrt{\left(\frac{a_r + \Delta a_p}{a_r}\right)^7}\right) - i_r$$

$$\sqrt{\Delta i_p{}^2 + \left(\Delta \Omega_p \sin i_r\right)^2} = \sqrt{2}\frac{A_{safety}}{a_r}$$

$\Delta i_p, \Delta\Omega_p \rightarrow z(0), \dot{z}(0)$

Passive Collision Avoidance

Perturbations – Argument of Perigee

- To first order:

$$\frac{d\omega}{dt} = -\frac{3}{2}\frac{J_2 R_{\oplus}^2}{(1-e^2)^2}\sqrt{\frac{\mu}{a^7}}\left(2 - \frac{5}{2}\sin^2 i\right)$$

- For CanX-4&5, -3.19 degrees/day
- Therefore, radial motion doesn't have a period of 1 orbit, but actually 3.5 seconds shorter.
- Takes 112.8 days to complete one cycle

Passive Collision Avoidance

Perturbations – Argument of Perigee

- Therefore, if desired state is left for 56.4 days, at one point the cross-track and radial motions will line up. If the along-track distance crosses zero at that time, a collision will occur.

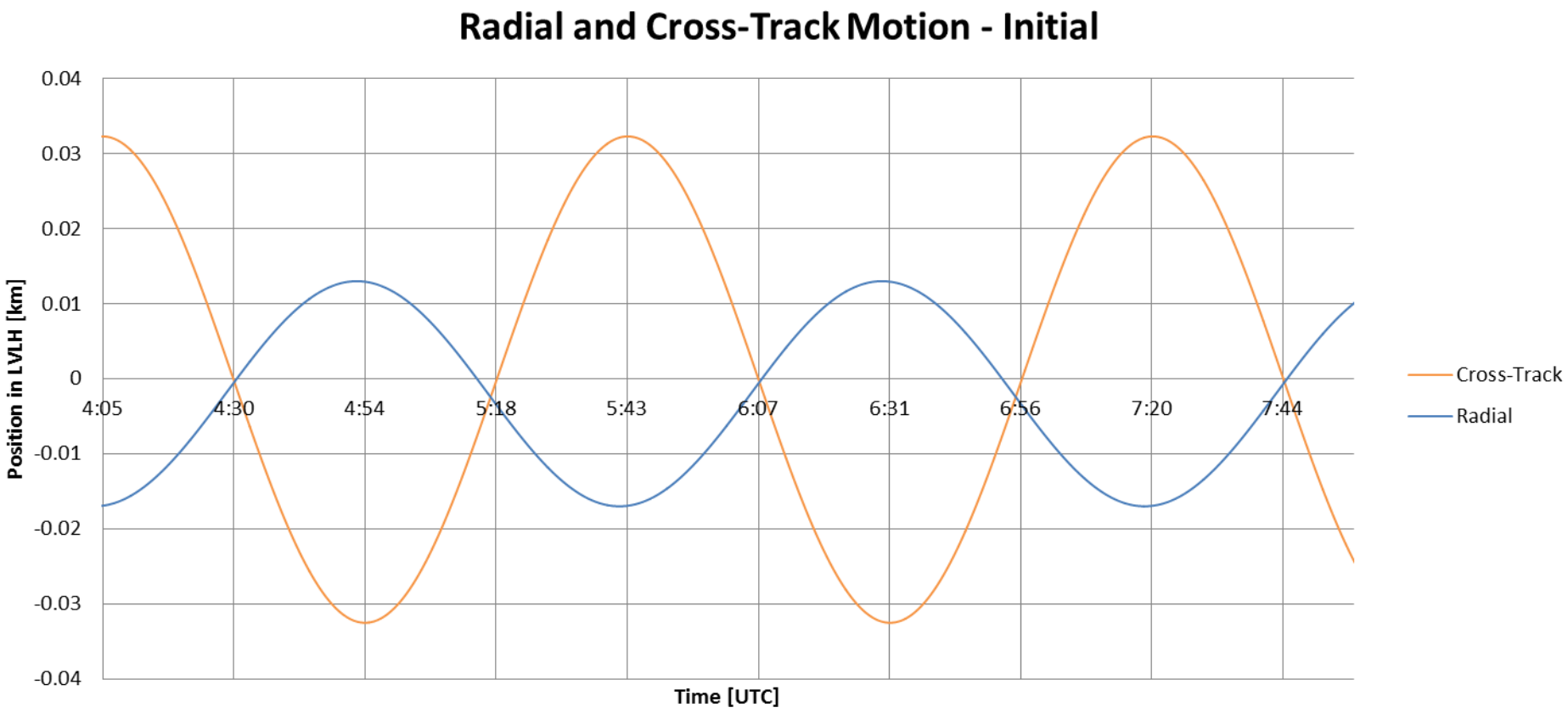
Passive Collision Avoidance

1. Identify uncorrected radial, cross-track motions

Time [UTC]	x [km]	y [km]	z [km]	v_x [km/s]	v_y [km/s]	v_z [km/s]
10/7/2014 4:00:00 AM	-0.016419	0.265132	0.029962	-4.432 E-06	8.1179 E-05	1.3002 E-05

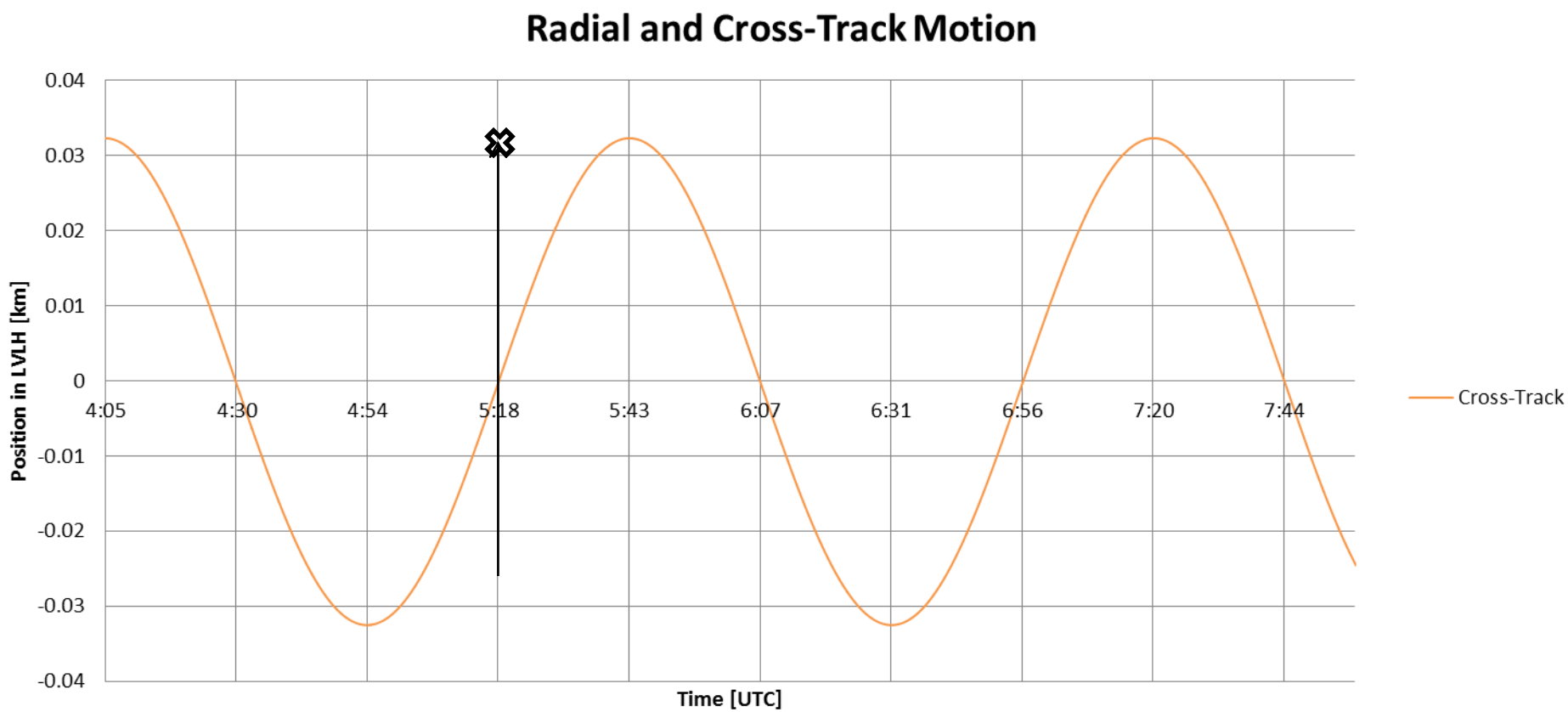
$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} 4-3\cos nt & 0 & 0 & n^{-1}\sin nt & 2n^{-1}(1-\cos nt) & 0 \\ 6(\sin nt - nt) & 1 & 0 & -2n^{-1}(1-\cos nt) & n^{-1}(4\sin nt - 3nt) & 0 \\ 0 & 0 & \cos nt & 0 & 0 & n^{-1}\sin nt \\ 3n\sin nt & 0 & 0 & \cos nt & 2\sin nt & 0 \\ -6n(1-\cos nt) & 0 & 0 & -2\sin nt & 4\cos nt - 3 & 0 \\ 0 & 0 & -n\sin nt & 0 & 0 & \cos nt \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{bmatrix}$$

Passive Collision Avoidance



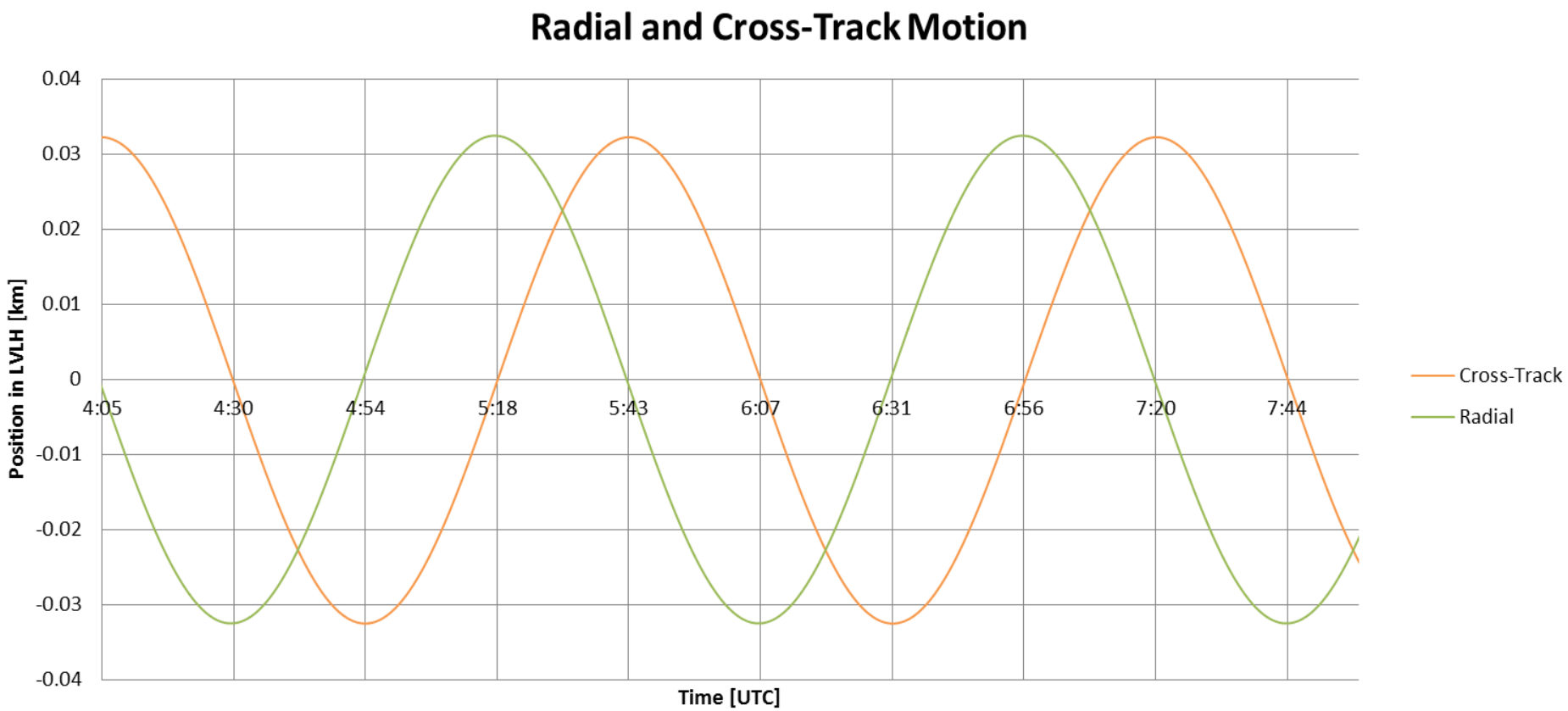
Passive Collision Avoidance

2. Identify desired radial, cross-track motions



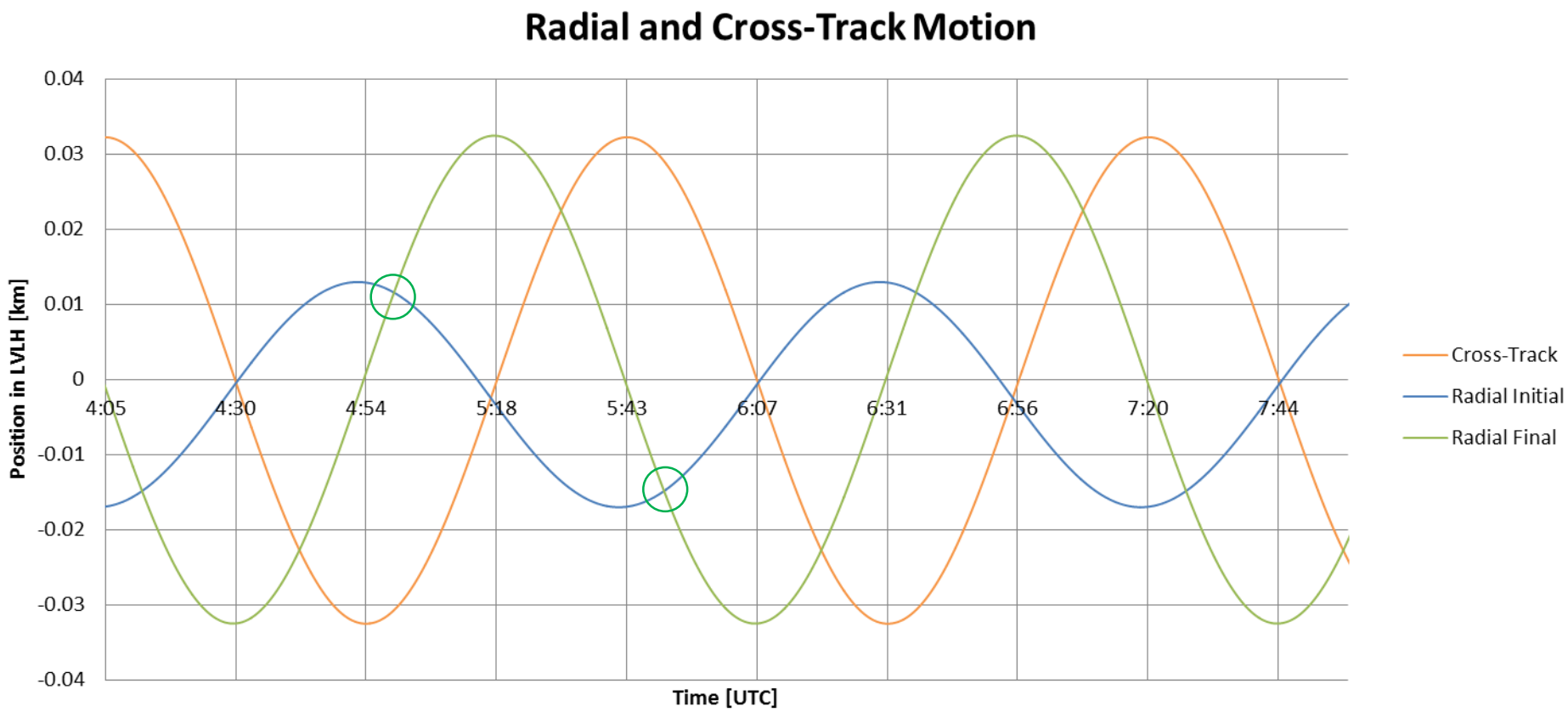
Passive Collision Avoidance

2. Identify desired radial, cross-track motions



Passive Collision Avoidance

3. Find the intersection(s) of the initial and desired state



Passive Collision Avoidance

4. Choose the intersection with the lowest relative velocity

	Intersection 1	Intersection 2
Time [UTC]	04:59:50	05:50:16
Radial displacement [m]	11.60	-14.83
Initial radial velocity [cm/s]	-0.6797	0.8338
Desired radial velocity [cm/s]	3.2598	-3.1055
Manoeuvre ΔV [cm/s]	-3.9395	3.9393

Commissioning

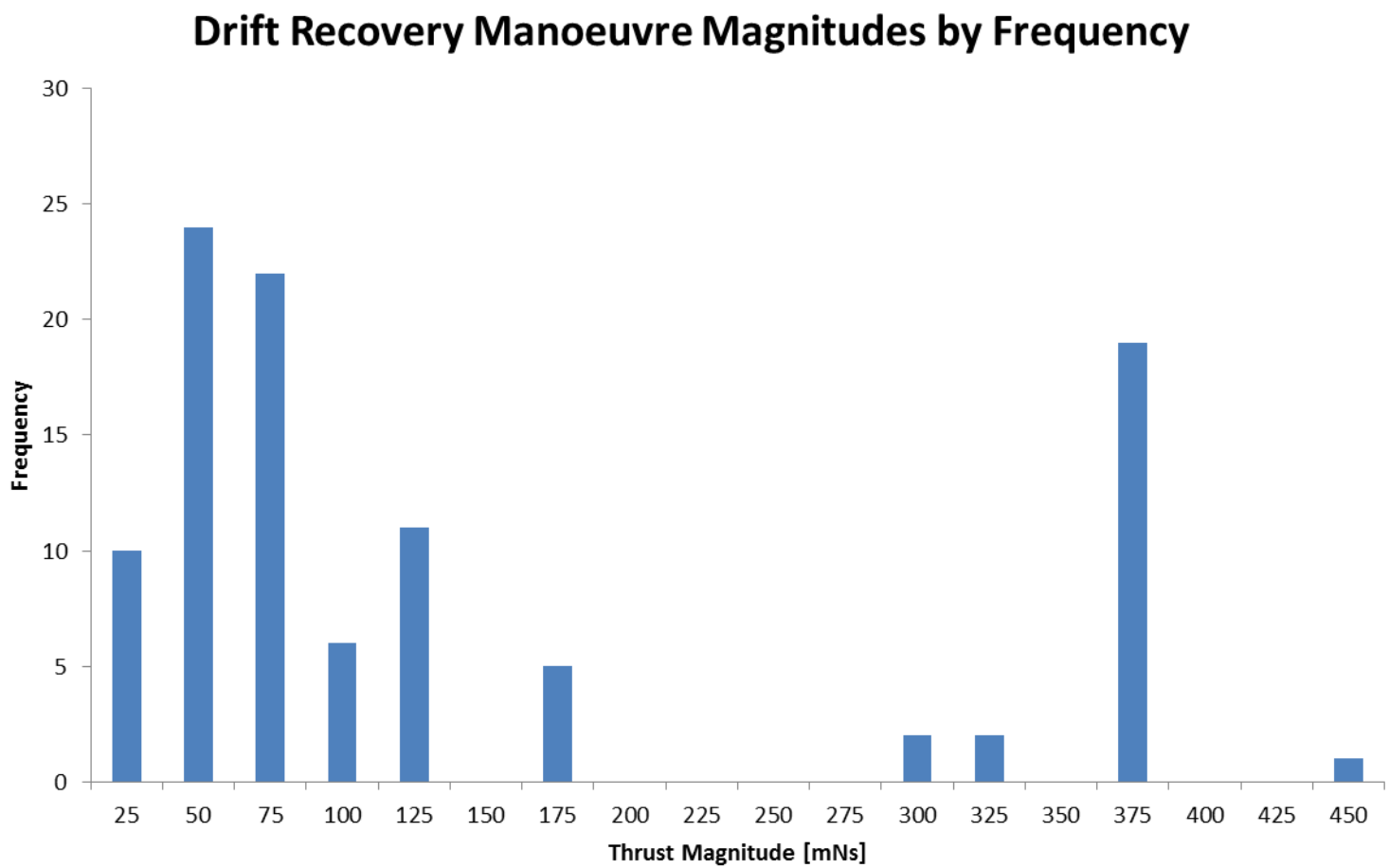
- Spacecraft both contacted within hours of launch
- Basic spacecraft health (power generation, temperatures) confirmed
- GPS and ISL systems validated within 24 hours

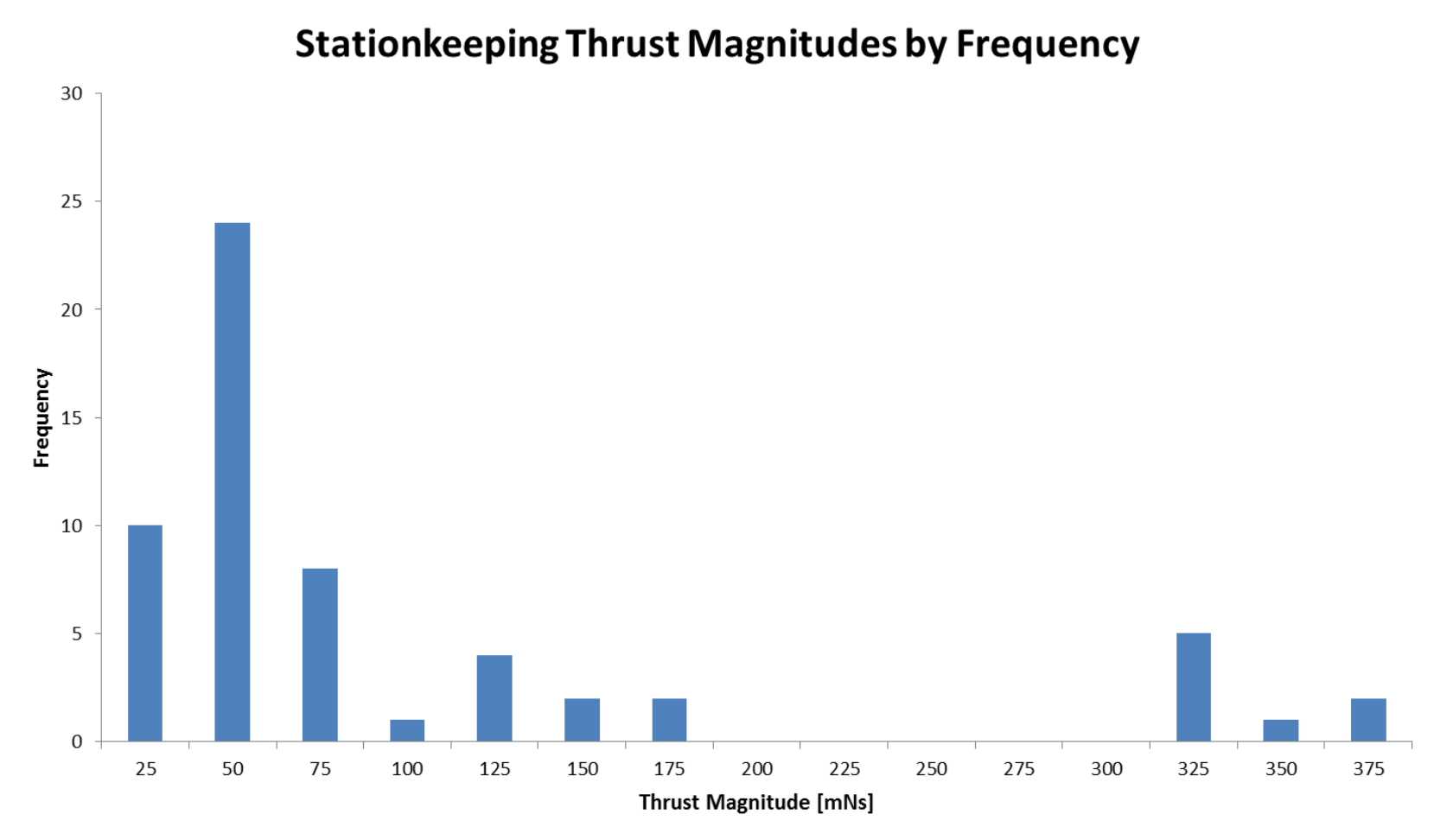
DRASTK Timeline

- Day 18 – “Dry run” zero-impulse thrust commands with inertial target attitudes executed.
- Day 19 – Following the success of the dry run, first real thrust performed
- Day 19-24 – Larger thrusts are performed, starting at 65 mNs up to the maximum 375 mNs.
- Day 28: on fuel-optimal return trajectory, 43 km/day

Drift Recovery

- Day 30: decision made to accelerate recovery (increase value of time in the cost function) to 105 km/day
- Day 45-46: Performed Ecc/ArgP correction thrusts
- Day 48: began deceleration from 320 km relative range
- Day 64: entered station keeping at 3 km relative range


Drift Recovery Manoeuvre Magnitudes by Frequency
Frequency
30
25
20
15
10
5
0
25
50
75
100
125
150
175
200
225
250
275
300
325
350
375
400
425
450
Thrust Magnitude [mNs]


Stationkeeping Thrust Magnitudes by Frequency
Frequency
30
25
20
15
10
5
0
25
50
75
100
125
150
175
200
225
250
275
300
325
350
375
Thrust Magnitude [mNs]