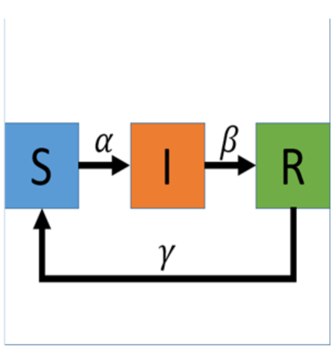
Disease Lab

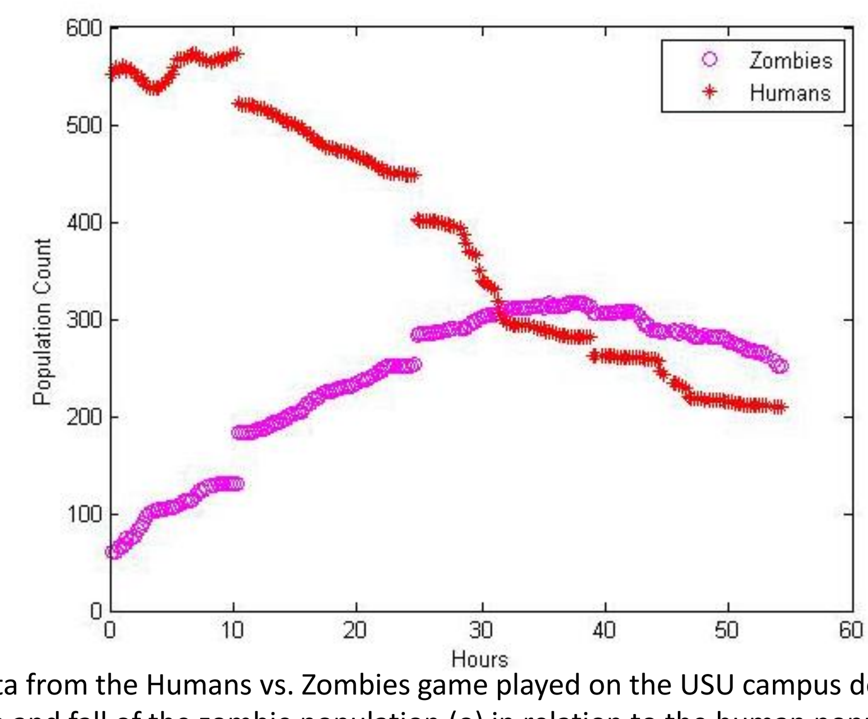
Laboratory Experiences in Mathematical Biology



Background and Extensions: At Utah State University the Disease Lab is often built around the Humans vs Zombies game of moderated tag played on campus. The game is briefly described here along with some student approaches to modeling the data.

Humans vs. Zombies

HvZ is a game of moderated tag that started at Goucher College in 2005 and is currently played on campuses worldwide. The game is played in a bounded area at specific hours of the day, e.g. on campus from 8 a.m. to 10 p.m., excluding buildings. Humans are converted into zombies by touch (tag). Humans can defend themselves by stunning zombies for 15 minutes with a Nerf dart blaster or by pelting zombies with a pair of socks rolled up into a ball. Also, a zombie dies if it does not infect a human within a 24 hour period. Additionally, humans are required to fulfill certain missions at various points during the game. These missions result in large fatalities in the human population and a corresponding increase in the zombie population. In order to track the progress of the game, zombies are required to report the ID number of each human they tag. This data drives the second portion of the Zombie Lab.



Data from the Humans vs. Zombies game played on the USU campus depicting the rise and fall of the zombie population (o) in relation to the human population (*) over time. New players were allowed to join the game during the first day (hence the increases in the human population). Additionally, the jumps in the populations were due to missions the humans were required to fulfill at various points in the game that result in many humans being turned to zombies.

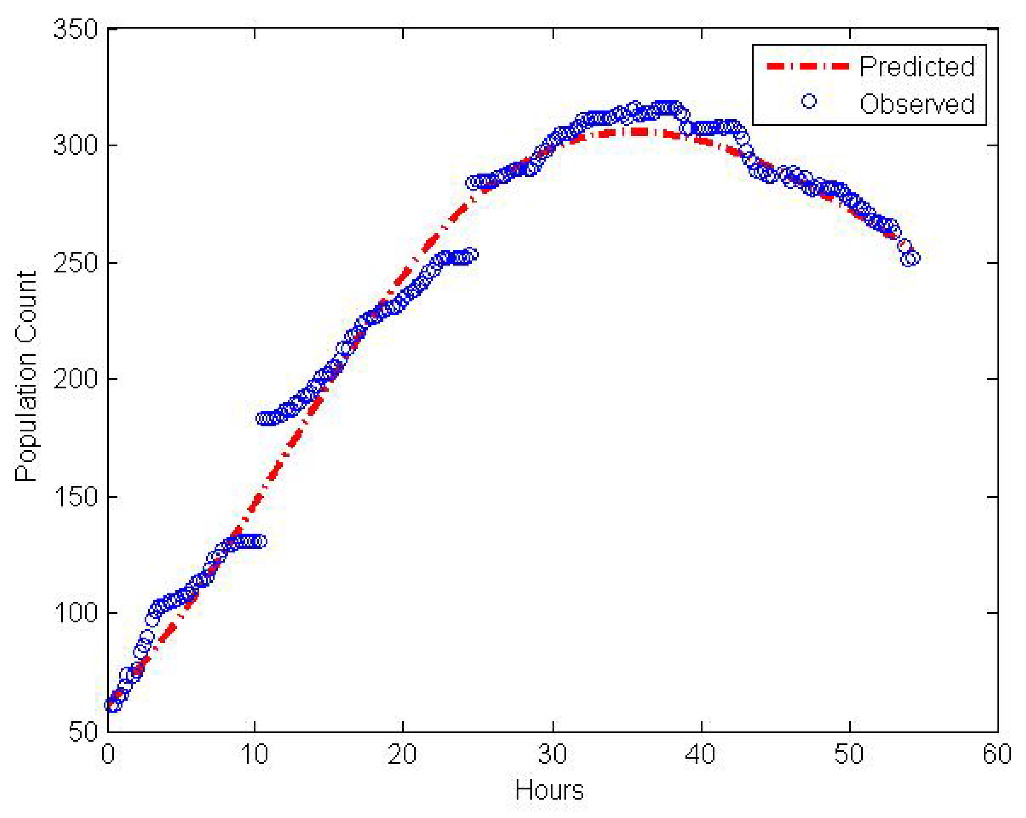
In general, students are excited to model this data since they have all played or witnessed the HvZ game on campus. Presented here are a couple of student approaches to modeling this data.

Threshold Model

This group conjectured that the rate of zombies dying or simply quitting the game would grow with time due to the increasingly scant supply of humans to feed on as the game progressed as well as zombified students simply quitting when the action died down a bit. Additionally, they supposed that there is a critical zombie threshold population, P, that if crossed would cause a collapse of the zombie population. In order to accommodate these two hypotheses the students' model took the following form:

$$\frac{dZ}{dt} = -r\left(1-\frac{Z}{P}\right)\left(1-\frac{Z}{K}\right)Z - stZ,$$

where r is the intrinsic growth rate, K is the carrying capacity and s describes the increasing rate at which zombies die or leave the game.



Plot of students' Threshold Model fitted to the HvZ zombie population data using least squares. Students estimated the intrinsic growth rate at 0.122 and the death acceleration term $s \approx 0.0014$. The model is based on the hypothesis that once the zombie population has been reduced below a critical threshold it would naturally collapse to zero; students did not notice that their threshold, $P \approx 840$, was very similar to the corresponding $K \approx 845$.

The Answer's a Parabola, Right? Model

Some students have had experience fitting polynomial curves to data, and when confronted with the HvZ data immediately want to fit a parabola. Most of these students are initially straightforward with their intentions, suggesting models of the form

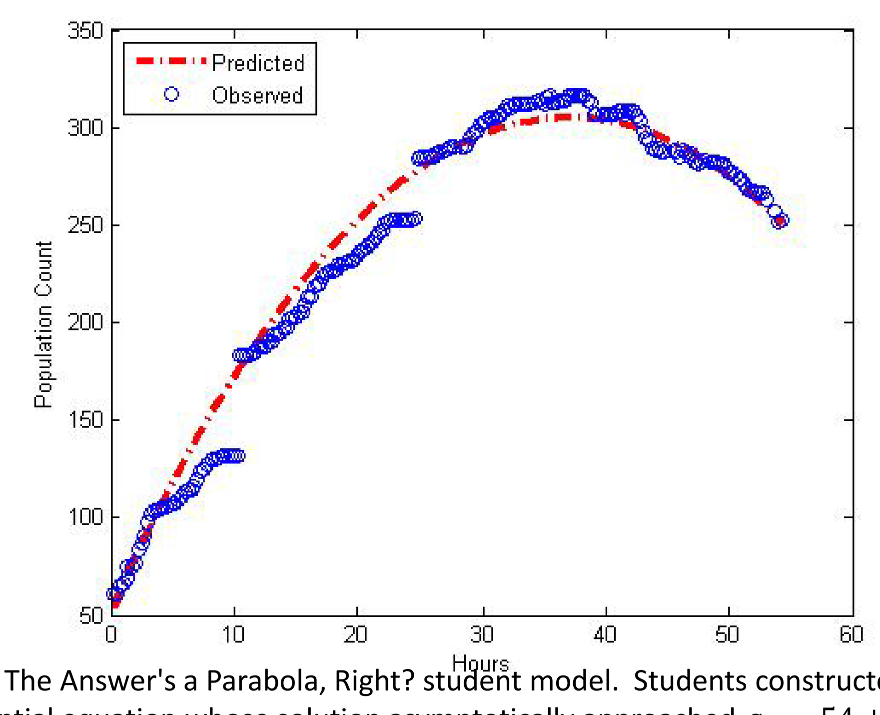
$$\frac{dZ}{dt} = At + B$$

where A represents the population's rate of acceleration and B is the growth rate. Many realize their approach is off target when asked to either describe the relationships between the variables they used to create the model or outline the physical concepts upon which the model was derived. They come to understand the models are supposed to be mechanistic and not strictly empirical models used to drive a curve through points. However, others remain determined that the parabola is the right answer, but it simply needs to be dressed up more.

In one class, the lab occurred shortly after the students had learned the method of integrating factors. In one homework assignment students were asked to "…construct a first order linear differential equation whose solutions have the required behavior as $t \to \infty$". The students were then assigned a variety of functions their solutions should approach. Inevitably, some students noted they could simply extend their homework experience to the HvZ scenario and produced the model

$$\frac{dZ}{dt} + Z(t) = \frac{dg}{dt} + g(t)$$

where $g(t) = At^2 + Bt + C$, the parabola the students want to use to model the data. The students proceeded to solve the differential equation using the method of integrating factors to get $Z(t) = At^2 + Bt + C + De^{-t}$, a function that approaches $g(t) = At^2 + Bt + C$ asymptotically.



Plot of The Answer's a Parabola, Right? student model. Students constructed a differential equation whose solution asymptotically approached $g = 54 + 13.55t - 0.18t^2$, the quadratic fit.

So, while every student model will not be mechanistic, the students were using mathematics to explain and describe data fluently which is seldom seen in a typical differential equations classroom.