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Extensions of the Walden-Wintle Model of Charge Transport in Disordered Materials for Charge Injection with Electron Beams

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Extensions of the Walden-Wintle Model of Charge Transport in Disordered Materials for Charge Injection with Electron Beams

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USU Materials Physics Group
Charge Injection in Insulators

**Electrode Injection**

- Injection voltage, $V_{app}$
- Injection current, $I_{elec}$
- Surface Voltage, $V_s$
- Rear electrode current, $I_o$
- Conductivity,

\[
\sigma(t) = \frac{I_{elec}(t)}{F(t)} = \frac{I_{elec}(t) \cdot D}{A \cdot V_{app}(t)}
\]

**Electron Beam Injection**

- Injection energy, $eV_b$
- Injection current, $I_b$
- Electron emission current
- Surface Voltage, $V_s$
- Embedded charge distribution
- Rear electrode current, $I_o$
- Conductivity,
Electrode Injection

\[ J_{\text{elec}}(t) = J_{\text{elec}}^c(t) + J_{\text{displacement}} = \sigma(t)F(t) + \left[ \epsilon_o \frac{\partial \epsilon_r(t)}{\partial t} F(t) + \epsilon_o \epsilon_r \frac{\partial F(t)}{\partial t} \right] \]

See Lundgreen poster
The assumptions leading to the simplest model are:

1. A *parallel plate geometry* leads to a 1D model of electric transport.
2. The injected charge carriers are *electrons from a bias electrode*.
3. Injection occurs *instantaneously at surface*.

Surface voltage as a function of time,
\[
V_s(t) = \frac{\bar{J}_o \delta(t)}{\varepsilon_0 \varepsilon_r} \left[ D \left( 1 - \frac{R(E_b)}{D} \right) \right]
\]

Rear electrode current as a function of time,
\[
J_{down}(t) = \bar{J}_o \delta(t)
\]

A displacement current
\[
\nabla \times \mathbf{H} = J_c + \frac{\partial}{\partial t} (\varepsilon_0 \varepsilon_r \mathbf{F})
\]
Walden & Wintle considers general electrode injection current density as a function of applied electric field:

\[ J_{inj}(F) = \begin{cases} \frac{J_0(F) \cdot e^f(F)}{J_0} ; & \text{exponential form} \\ \frac{J_0(F)}{(F/F_o)^p} ; & \text{power law form} \end{cases} \]

\( J_0(F) \) and \( f(F) \) are arbitrary functions that vary slowly with \( F \) compared exp[f(F)].

They assume with \( 0 \leq m \equiv \frac{1}{n} \leq 1 \)

\[ \left[ \frac{dn_t}{dn} \right] = \left[ \frac{J_{inj}^c}{J_0} \right]^{(1/m)-1} \quad \text{or} \quad J_{inj}^c(t) = J_0 \left[ \frac{dn_t}{dn} \right]^{\frac{1}{n-1}} = \bar{J}_0 \left[ \frac{dn_t}{dn} \right]^{m}; \]

Total current density with displacement current density:

\[ I_{down}(t) = I_{down}^c(t) + I_{down}^{displacement} = \bar{J}_0 \left\{ \left( 1 + \frac{t}{\tau_Q} \right)^{1-m} - \frac{D^{-\frac{1}{2}} R(E_b)}{D} \right\} \cdot \left( 1 + \frac{t}{\tau_Q} \right)^{-1} \]

\[ \tau_Q = \frac{\epsilon_o \epsilon_r D m}{\bar{J}_0 \left[ D^{-\frac{1}{2}} R(E_b) \right]} \left[ \frac{\partial f(F)}{\partial F} \right]^{-1} |_{F=F(t=0)} = \frac{\epsilon_o \epsilon_r D (t-\tau_Q)}{\bar{J}_0 \left[ D^{-\frac{1}{2}} R(E_b) \right]} \frac{\partial F}{\partial t} \]

\( \tau_Q \) is characteristic onset time for the injection current density, not to be confused with a decay time.
Electron Beam Injection

- Surface Electrode Injection
- Poole-Frenkel Fowler-Nordheim Simple Exponential Constant Power Law
- Surface Injection
- Bulk Injection
- Penetrating Radiation
- Pulsed Injection
- Stepped Injection
- Periodic Injection

Injection Mode
- Positive Charging ($Y_o > 1$)
- Negative Charging ($Y_o < 1$)
- Penetrating Beam

Injection Barrier Type
- Poole-Frenkel Fowler-Nordheim Simple Exponential Constant Power Law

Injected Charge/Energy
- (Spatial Domain)
- (Time Domain)
Electron Penetration Depth

Electron Beam Injection

- Injection energy, $eV_b$
- Injection current, $I_b$
- Electron emission current
- Surface Voltage, $V_s$
- Embedded charge distribution
- Rear electrode current, $I_o$
- Conductivity,
Electron Beam Injection and Electron Yield

Charge dependant electron yields are the key:

A simple model for surface voltage (or time) dependence of the yield for negative charging for $E_b > E_2$, based on a charging capacitor was proposed by Thomson:

$$[1 - Y(t; E_b + q_e V_s)] = [1 - Y(E_b + q_e V_s)]e^{-(Q(t)/\tau_Q)}$$

for $0 \geq q_e V_s(t) \geq (E_2 - E_b)$

$\tau_Q$ is a decay constant for the exponential approach of the yield to unity, as charge $Q(t)$ is accumulated with elapsed time and $E_2$ is the crossover energy.

Disperssion current

$$\frac{dF_s(t)}{dt} = -\frac{q_e d_1}{d\epsilon_0 \epsilon_r} \frac{d \ln F_s(t)}{d t} \frac{dn_e(t)}{dn(t)}$$

Walden assumption

$$\frac{dn_e(t)}{dn(t)} = \left( \frac{F_{inj}(t)}{F_0} \right)^{n-1}$$

Solving for time dependence

$$t = -\frac{\epsilon_0 \epsilon_r d}{(d - R) F_0} \int_{F_{inj}(0)}^{F_{inj}(t)} \left( \frac{F_{inj}(F)}{F_0} \right)^{-n} dF_s$$

Final result

$$J_S(t) = J_b \left( Y_o - 1 \right) \left( 1 + \frac{t}{\tau_{onset}} \right)^{-m} + J_{sat} \rightarrow J_b \left( \sigma_{yield} - 1 \right) + J_{sat}$$

with

$$\tau_{onset} = \left[ \frac{n_6 \epsilon_o \epsilon_r d}{J_0 [1 - Y_o(\epsilon_b)] (d - \frac{1}{2} R)} \right] F_{c6}$$
Electron Beam Injection and Electron Yield

Charge dependant electron yields are the key:

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$$[1 - Y(t; E_b + q_e V_s)] = [1 - Y(E_b + q_e V_s)] e^{-(Q(t)/\tau_Q)}$$

for $0 \leq q_e V_s(t) \leq (E_2 - E_b)$

$\tau_Q$ is a decay constant for the exponential approach of the yield to unity, as charge $Q(t)$ is accumulated with elapsed time and $E_2$ is the crossover energy.

Expression that describes the re-attraction of SE’s to the surface of a positively charged sample:

$$\sigma(E_o; V_s) = \frac{k}{6E_o} [h(eV_s; \chi) - h(50eV; \chi)] + 1,$$

where

$$h(\alpha; \chi) \equiv \frac{3\alpha + \chi}{(\alpha + \chi)^3},$$

$k$ is a material dependant proportionality constant, $\chi$ is the insulator electron affinity, and $\alpha$ is an arbitrary energy at which $h$ is evaluated.

$$J_{inj}(t) = J_o \left[1 - Y_o(\varepsilon_b) \left\{ \frac{\chi^2 [3F_s(d - \frac{1}{2}R)q_e + \chi]}{[3F_s(d - \frac{1}{2}R)q_e + \chi]^3} \right\} \right]$$
Combining all the pieces

\[
\frac{\delta_i(E_o; Q_i)}{\delta_o(E_o)} = \frac{\int_{0}^{50 \text{eV}} \frac{dN(E; E_o)}{dE} dE}{eV_s(Q_i)}
\]

Physics based model for yield SE recapture as a function of incident fluence

\[
\delta(eV_s) = \left(\sigma_o(E_o) - 1\right) \left(1 - \frac{\lambda_S}{2 \cdot d}\right) \left(\frac{h(\varepsilon_s)}{h(50 \text{eV})} - 1\right) - \left[\eta_o \left(1 - \frac{\lambda_S}{2 \cdot d}\right) - \left(1 + \frac{R}{2 \cdot d}\right)\right]
\]

- Analytic solution for SE yield as \(V_s\) changes with \(J_{in}\)
- Walden/Wintle model modified for electron beam injection gives:
  - \(V_s\) in terms of \(J_{in}\)
  - \(J_{rear}\) in terms of \(J_{in}\)

\[
V_s = \frac{Q_o (\sigma - 1)d - \sigma Q_o \lambda_{SE} + Q_o R}{\varepsilon_o \varepsilon_r A_o} + \frac{2 \varepsilon_o \varepsilon_r A_o}{2 \varepsilon_o \varepsilon_r A_o}
\]

\[
\sigma(E_o Q) = \eta(E_o) + \delta(E_o Q)
\]

Decay curve data
## Summary of Walden-Wintle Results

<table>
<thead>
<tr>
<th>Injection Barrier Type</th>
<th>Injection Current, $J_{in}(F)$</th>
<th>Time Constant, $\tau_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Surface Electrode Injection</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poole-Frenkel (Schottky/thermionic emission)</td>
<td>$C_1 F_x e^{(\phi_s + \phi_d) / d}$</td>
<td>$\left[ \frac{n_1 \varepsilon_0 \varepsilon_{r} d}{J_0 (d - R)} \right] 2 \left( \frac{V_x}{d} \right)^{1/2} F_{ci}^{1/2}$</td>
</tr>
<tr>
<td>Fowler-Nordheim (Tunneling-type field emission)</td>
<td>$C_2 F_x^2 e^{-\left(\frac{\phi_{ci}}{F_x}\right)}$</td>
<td>$\left[ \frac{n_2 \varepsilon_0 \varepsilon_{r} d}{J_0 (d - R)} \right] \left( \frac{V_x}{d} \right)^2 F_{ci}^{-1}$</td>
</tr>
<tr>
<td>Simple exponential</td>
<td>$C_3 e^{(\phi_s / F_x)}$</td>
<td>$\left[ \frac{n_3 \varepsilon_0 \varepsilon_{r} d}{J_0 (d - R)} \right] F_{ci}$</td>
</tr>
<tr>
<td>Constant (Exponential with $F_x \to \infty$)</td>
<td>$C_4$</td>
<td>$\tau_o \to \infty$</td>
</tr>
<tr>
<td>Power law $p \gg 1$</td>
<td>$J_0 (F_x / F_{ci})^p$</td>
<td>$\left[ \frac{n_5 \varepsilon_0 \varepsilon_{r} d}{J_0 (d - R)} \right] 1 \left( \frac{V_x}{d} \right)^p (d)$</td>
</tr>
<tr>
<td><strong>Electron Beam Injection</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Penetrating beam</td>
<td>0</td>
<td>$\tau_o \to \infty$</td>
</tr>
<tr>
<td>Positive Charging (Total Yield &lt; 1)</td>
<td>$J_0 \left[ 1 - Y_{o}(E_b) \right] e^{-(\phi_s / F_{ci})}$</td>
<td>$\left[ \frac{n_6 \varepsilon_0 \varepsilon_{r} d}{J_0 [1 - Y_{o}(E_b)](d - \frac{1}{2}R)} \right] F_{ci}$</td>
</tr>
<tr>
<td>Negative Charging (Total Yield &lt; 1)</td>
<td>$J_0 \left{ 1 - Y_{o}(E_b) \left[ \frac{X^2 [3F_x(d - \frac{1}{2}R)q_x + X]}{[3F_x(d - \frac{1}{2}R)q_x + X]^3} \right] \right}$</td>
<td><strong>Numerical solution required</strong></td>
</tr>
</tbody>
</table>
Band Theory of (Crystalline) Conductors, Insulators and Semiconductors

- **Conductor**: Partially filled bands
- **Insulator**: Completely filled bands
- **Semiconductor**: Insulators at finite $T$

**Groups of Atoms**
- Conduction
- Band gap
- Valence

**Number of atoms**

**Atom Density of States**

**Lattice Density of States**
General form of conductivity in HDIM with explicit time dependence

\[ \sigma(t) = \sigma_{\text{DC}} \left[ 1 + \frac{\sigma_{\text{AC}}(v)}{\sigma_{\text{DC}}} + \frac{\sigma_{\text{pol}}}{\sigma_{\text{DC}}} e^{-\frac{t}{\tau_{\text{pol}}}} + \frac{\sigma_{\text{diffusion}}}{\sigma_{\text{DC}}} t^{-1} + \frac{\sigma_{\text{dispersive}}}{\sigma_{\text{DC}}} t^{-(1-\alpha)} + \frac{\sigma_{\text{transit}}}{\sigma_{\text{DC}}} t^{-(1+\alpha)} + \frac{\sigma_{\text{RIC}}}{\sigma_{\text{DC}}} \right] \]

- \( \sigma_{\text{DC}} \equiv q \tau_{\text{ne}} \mu_e \) dark current or drift conduction—very long time scale equilibrium conductivity.
- \( \sigma_{\text{AC}}(v) \equiv \sum_i \left[ (\varepsilon_r(v) - \varepsilon_r^0) \varepsilon_0 \frac{1}{1+(v/v_i)^2} \right] \)
  frequency-dependent AC conduction—dielectric response to a periodic applied electric field
- \( \sigma_{\text{pol}}(t) \equiv \left[ (\varepsilon_r^\infty - \varepsilon_r^0) \varepsilon_0 / \tau_{\text{pol}} \right] \cdot e^{-\frac{t}{\tau_{\text{pol}}}} \) long time exponentially decaying conduction due to polarization
- \( \sigma_{\text{diffusion}}(t) \equiv \sigma_{\text{diffusion}}^0 \cdot t^{-1} \)
  diffusion-like conductivity from gradient of space charge spatial distribution.
- \( \sigma_{\text{dispersive}}(t) \equiv \begin{cases} \sigma_{\text{dispersive}}^0 \cdot t^{-(1-\alpha)} ; & (\text{for } t < \tau_{\text{transit}}) \\ \sigma_{\text{transit}}(t) \equiv \sigma_{\text{transit}}^0 \cdot t^{-(1+\alpha)} ; & (\text{for } t > \tau_{\text{transit}}) \end{cases} \) broadening of spatial distribution of space charge through coupling with energy distribution of trap states.
- \( \sigma_{\text{RIC}}(t; \dot{D}, \tau_{\text{RIC}}^1, \tau_{\text{RIC}}^2) \equiv \sigma_{\text{RIC}}^0(\dot{D}(t)) \left( 1 - e^{-\tau_{\text{RIC}}^1/(t-t_{\text{on}})} \right) \left( 1 + (t - t_{\text{off}}) / \tau_{\text{RIC}}^2 \right)^{-1} \)
  radiation induced conductivity term resulting from energy deposition within the material.

Refer to (Wintle, 1983), (Dennison et al., 2009), and (Sim, 2012)
Other Displacement in Conductivity in HDIM

Dark Current

Polarization

Diffusion

Dispersive/Transit

RIC

Current Density

Pre-Transit

Post-Transit

5/2/2012
Extension of the Models of Surface Voltage of HDIM

\[
\mathbf{F}(z, t) = \begin{cases} 
0 \\
\mathbf{F}_{up}(z, t) = -\frac{V_{bias}(t) - \Sigma_{d}(t)}{z_{d}(t) - z_{ext}} \mathbf{\hat{z}} & ; 0 < z < z_{d}(t) \\
\mathbf{F}_{down}(z, t) = -\frac{V_{bias}(t) + \Sigma_{d}(t)}{D} \mathbf{\hat{z}} & ; z_{d}(t) \leq z \leq D \\
0 & ; z > D
\end{cases}
\]

\[
V(z, t) = \begin{cases} 
V_{bias}(t) + V_{ext} \\
V_{bias}(t) + V_{ext} - \frac{V_{ext} + \Sigma_{d}(t)}{\varepsilon_{o}\varepsilon_{r}} \frac{(z - z_{ext})}{z_{d}(t) - z_{ext}} & ; z \leq z_{ext} \\
V_{bias}(t) + V_{ext} \left[ \frac{D}{z_{d}(t) - z_{ext}} + \frac{\Sigma_{d}(t)}{\varepsilon_{o}\varepsilon_{r}} \frac{z_{ext}}{z_{d}(t) - z_{ext}} \right] & ; 0 < z < z_{ext} \\
V_{bias}(t) \left[ \frac{D}{z_{d}(t) - z_{ext}} - \frac{\Sigma_{d}(t)}{\varepsilon_{o}\varepsilon_{r}} \right] & ; z = 0 \\
V_{bias}(t) \left[ \frac{D - z_{d}(t)}{D} \right] - \frac{\Sigma_{d}(t)}{\varepsilon_{o}\varepsilon_{r}} \left[ D - z_{d}(t) \right] & ; 0 \leq z < z_{d}(t) \\
V_{bias}(t) \left[ \frac{D - z}{D} \right] - \frac{\Sigma_{d}(t)}{\varepsilon_{o}\varepsilon_{r}} \left[ D - z \right] & ; z_{d}(t) \leq z \leq D \\
0 & ; z > D
\end{cases}
\]
Conclusions

What this our extend model yields:

- Modified Walden-Wintle formalism to include electron beam injection
- Yields general predictions for rear electrode current, surface voltage, and conductivity vs time
- Key are models of the electron yield as a function of charge build up
- Can now model electrode injection AND surface, non-penetrating and penetrating particle injection

More Generally:

- Model ties various methods and allows cross checks of materials properties
- Key to understanding all this is the spatial and energy density of trap states occupied in highly disordered insulating materials
Parallel Plate Models
Finite injection, no dissipation

The assumptions leading to the simplest model are:

1. A *parallel plate geometry* leads to a 1D model of electric transport.
2. The injected charge carriers are *electrons from a bias electrode*.
3. Injection occurs *instantaneously at surface for finite time and space*.

Surface voltage as a function of time,

\[ V_s(t) = \frac{J_o}{\varepsilon_o \varepsilon_r} \left[ D \left( 1 - \frac{R(E_b)}{D} \right) \right] x \]

Rear electrode current as a function of time,

\[ J_{down}(t) = J_o x \]

A displacement current

\[ \vec{V} \times \vec{H} = I_c + \frac{\partial}{\partial t} (\varepsilon_o \varepsilon_r F) \]
Electron Beam Injection and Emission, no dissipation

Surface voltage as a function of time,

\[ V_s(t) = \frac{I_0}{\varepsilon_0 \varepsilon_r} \left[ D \left( 1 - \frac{R(E_B)}{D} \right) \right] x \]

Rear electrode current as a function of time,

\[ J_{down}(t) = I_0 x \]

A displacement current

\[ \vec{\nabla} \times \vec{H} = J_c + \frac{\partial}{\partial t} (\varepsilon_0 \varepsilon_r \vec{F}) \]
The assumptions leading to the simplest model for this are:

1. A **parallel plate geometry** leads to a 1D model of electric transport.
2. The incident (or injected) charge carriers are **electron**.
3. The magnitude of the injected current density is approximated as the time-averaged incident beam current
   \[
   \overline{J_0} \equiv \frac{\int_0^{t_{dep}} J_{\text{inj}}(t) \, dt}{\int_0^{t_{dep}} dt} \approx J_0 \left[ \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} \right]
   \]
4. All charge is initially deposited at **single penetration depth**, \( R \)
   \[
   \Sigma(z, t) = \int_0^t J_{\text{inj}}(t') \, dt' \quad \delta[z - R]
   \]
5. Charge deposited in the region \( 0 < x < R \) quickly redistributes to a uniform volume charge distribution
   \[
   \int_0^t \frac{J_{\text{inj}}(t')}{q_e R \left( E_b - q_e V_s(t') \right)} \Theta[z - R \left( E_b - q_e V_s(t') \right)] \, dt'
   \]
6. **There is no electron emission**; that is, the total electron yield \( Y = 0 \).
7. **There is no charge dissipation**. The dielectric acts as a perfect charge integrator.
What is different about e beam deposition?

- Electron emission modifies current and voltage
- Extra current term for emitted charged particles
- Energy-depended emission
- Reattraction of secondary electrons to positive surface
- Different injection barrier in RIC region
- RIC
Electron Beam Injection and Emission, no dissipation

Surface voltage as a function of time,

\[ V_s(t) = V_{\text{nd}}^\text{inj}(t) \left( \frac{t}{\tau_o} \right) \]

(no dissipation) \hspace{1cm} (4.13)

with

\[
V_{\text{nd}}^\text{inj}(t) = \begin{cases} 
V_{\text{ne}}^\text{inj} = \frac{I_o}{\sigma_o} \left[ D \left( 1 - \frac{R(E_b)}{D} \right) \right] \\
V_{\text{se}}^\text{inj} = \frac{I_o}{\sigma_o} \left[ 1 - Y(E_b) \right] \left[ D \left( 1 - \frac{R(E_b)}{D} \right) \right] \\
V_{\text{de}}^\text{inj}(t) = \frac{I_o}{\sigma_o} \left[ 1 - Y(E_b) \right] \left( \frac{\tau_Q}{t} \right) \left[ D \left( 1 - \frac{R(E_b)}{D} \right) \right] \left[ 1 - e^{-\left( t/\tau_Q \right)} \right] \\
V_{\text{ge}}^\text{inj}(t) = \frac{I_o}{\sigma_o} \left[ D \left( 1 - \frac{R(E_b)}{D} \right) \right] \left\{ \int_0^t \{ 1 - Y [E_b - q_e V_s(t')] \} \, dt' \right\} 
\end{cases}
\]

(no emission) \hspace{1cm} (static emission) \hspace{1cm} (dynamic emission) \hspace{1cm} (general emission)

Total current density with displacement current density:

\[
J_{\text{down}}(t) = J_{\text{down}}^c(t) + J_{\text{down}}^{\text{displacement}} = \bar{J}_0 \left\{ \left( \frac{1 + t/\tau_Q}{\tau_Q} \right)^{-m} - \left( \frac{D - \frac{1}{2} R(E_b)}{D} \right) \cdot \left( 1 + \frac{t}{\tau_Q} \right)^{-1} \right\}
\]
Electron Beam Injection and Emission, with dissipation

Surface voltage as a function of time,

\[ V_s(t) = \frac{I_o}{\varepsilon_0\varepsilon_r} \left[ D \left( 1 - \frac{R(E_b)}{D} \right) \right] \]

Rear electrode current as a function of time,

\[ J_{down}(t) = I_o \]

A displacement current

\[ \vec{\nabla} \times \vec{H} = J_c + \frac{\partial}{\partial t} (\varepsilon_0\varepsilon_r F) \]
Localized states for HDIM Conductivity Theory
Disorder introduces localized states in the gap.

A quantum mechanical model of the spatial and energy distribution of the electron states.

DELocalized in real space

\[ |\psi(r)|^2 \]

Localized in momentum space

\[ |\psi(q)|^2 \]

Details of the HDIM conductivity mechanisms follow from descriptions of the spatial and energetic distributions of the trap density of state in the band gap of the HDIM. Theses model predict the time, T, Q, F, dose and dose rate dependence of the mechanisms and relate them to transition probabilities.

\[
N_{ex} = \dot{D} \rho / \varepsilon_0 \varepsilon_r
\]

(a) Excitation by thermal or external source:  
(b) Trapping

\[
\int_{\varepsilon_F}^{\varepsilon_C} \alpha_{te}(\varepsilon) N_c n_t(\varepsilon, t) d\varepsilon f(\varepsilon)
\]

(c) De-trapping due to thermal excitation

\[
\alpha_{tr} (\varepsilon) n_e (t) [n_e (t) + n_t (\varepsilon, t)]
\]

(d) Recombination

\[
\alpha_{th} (\varepsilon) n_t (\varepsilon, t) n_h (t) P_{tr} (\varepsilon)
\]

(e) Low temperatures

(f) Mid band recombination
Experimental Test
Electron Emission

Electron emission uses incident (charged, energetic) electrons injected with a pulsed or continuous electron beam and measures conducted electrons, emitted electrons and stored charge.
Absolute Electron Yields

Hemispherical Grid Retarding Field Analyzer Electron Emission Detector

- Works with incident:
  - 20 eV to 30 keV electrons
  - ~100 eV to 5 keV ions
  - ~0.5 eV to 7 eV photons

- Precision absolute yield
  - ~1-2% accuracy with conductors
  - ~2-5% accuracy with insulators
  - measures all currents
  - in situ absolute calibration

- low energy e⁻ and UV charge neutralization
- in situ surface voltage probe

- multiple sample stage
- ~100 K < T < 400 K

Fig. 2. Hemispherical Grid Retarding Field Analyzer (HGRFA). (a) Photograph of sample stage and HGRFA detector (side view). (b) Cross section of HGRFA. (c) Photograph of sample stage showing sample and cooling reservoir. (d) Side view of the mounting of the stepper motor. (e) Isometric view of the HGRFA detailing the flood gun, optical ports, and wire harness.
Low Charge Capabilities

\[ J_{\text{inj}}(t) = J_o \left( 1 - Y_o(\varepsilon_b) \right) \left[ \frac{\chi^2}{3F_s \left( d - \frac{1}{2}R \right) q_e + \chi} \right] \]
SVP Charging and Discharging

- Uses pulsed non-penetrating electron beam injection with no bias electrode injection.
- Fits to exclude AC, polarization, transit and RIC conduction.

\[
\sigma(t) = \sigma_0 \left\{ 1 + \left[ \frac{\sigma_0^{\text{diffusion}}}{\sigma_0} \right] t^{1} + \left[ \frac{\sigma_0^{\text{dispersive}}}{\sigma_0} \right] t^{(1-\alpha)} \right\}
\]

Charging

\[
V_s(t) = \frac{q_e n_{tr}^{\text{max}}}{\varepsilon_o \varepsilon_r} \left[ R(E_b) D \left( 1 - \frac{R(E_b)}{2 D} \right) \right] \left[ \frac{t_0}{t} \right] \left[ 1 - e^{-\left( \frac{1}{t_0} \right) \left[ 1 + \left( \frac{t}{\sigma_0} \right) \left[ 1 + \frac{\sigma_0^{\text{diffusion}}}{\sigma_0} (t) + \frac{\sigma_0^{\text{dispersive}}}{\sigma_0} (t-(1-\alpha)) \right] \right]^{-1} \right]^{-1}
\]

Discharge

\[
V(t) = V_o e^{-t \sigma(t)/\varepsilon_o \varepsilon_r}
\approx V_o \left[ 1 - \left( \frac{\sigma_0}{\varepsilon_o \varepsilon_r} t \right) \left[ 1 + \left( \frac{\sigma_0^{\text{diffusion}}}{\sigma_0} \right] t^{1} + \left[ \frac{\sigma_0^{\text{dispersive}}}{\sigma_0} \right] t^{(1-\alpha)} \right] \right]
\]
Constant Voltage Conductivity

Constant Voltage Chamber configurations inject a continuous charge via a biased surface electrode with no electron beam injection.

- **Pre-Transit**
- **Diffusion**
- **Dark Current**
- **Polarization**
Electrostatic Discharge

Chamber configurations is the same as the constant voltage conductivity chamber, but with high field effects. It inject a continuous charge via a biased surface electrode with no electron beam injection.
RIC chamber uses a combination of charge injected by a biased surface electrode with simultaneous injection by a pulsed penetrating electron.
Photoyield is a variant of electron emission with incident (uncharged, energetic) photons and emitted and conducted electrons.

Cathodoluminescence is a variant of electron emission with incident (charged, energetic) electrons, conducted electrons, and emitted photons.
Supplemental Slides

HDIM Conductivity Theory
Conductivity in HDIM

Dark Current

Polarization

Diffusion

Dispersive/Transit

RIC

Current Density

Pre-Transit

Post-Transit

5/2/2012
An Extended Microscopic Model

Complete set of dynamic transport equations

\[ J = q_e n_e(z, t) \mu_e F(z, t) + q_e D \frac{dn_{tot}(z,t)}{dz} \quad \{\text{Sum of electron drift and diffusion current densities } J_i\} \]

\[ \frac{\partial}{\partial z} F(z, t) = q_e n_{tot} / \epsilon_0 \epsilon_r \quad \{\text{1D Gauss’s Law}\} \]

\[ \frac{\partial n_{tot}(z,t)}{\partial t} - \mu_e \frac{\partial}{\partial z} [n_e(z, t)F(z, t)] - q_e D \frac{\partial^2 n_e(z,t)}{\partial z^2} = N_{ex} - \alpha_{er} n_e(z, t)n_{tot}(z, t) + \alpha_{et} n_e(t)[N_t(z) - n_t(z, t)] \quad \{\text{1D Continuity equation with drift, diffusion and source terms}\} \]

\[ \frac{dn_h(z,t)}{dt} = N_{ex} - \alpha_{er} n_e(z, t)n_h(z, t) \quad \{\text{1D hole continuity equation with Generation and recombination terms}\} \]

\[ \frac{dn_t(z,\varepsilon,t)}{dt} = \alpha_{et} n_e(z, t)[N_t(z, \varepsilon) - n_t(z, \varepsilon, t)] - \alpha_{te} N_e \exp \left[-\frac{\varepsilon}{kT}\right] n_t(z, \varepsilon, t) \quad \{\text{1D trapping continuity equation for electrons}\} \]

A quantum mechanical model
of the spatial and energy distribution of the electron states
Temperature Dependence of Hopping Conductivity

At high temperatures, the conductivity is proportional to a Boltzmann factor, with trap depth $\Delta H$:

$$\sigma(T) \propto \exp\left[-\frac{\Delta H}{k_B \cdot T}\right] \quad \text{or} \quad \rho(T) \propto \exp\left[\frac{\Delta H}{k_B \cdot T}\right]$$

At low temperatures, the variable-range hopping conductivity is proportional to a Mott factor:

$$\sigma(T) \propto \exp\left[-\frac{1}{k_B \cdot T^{1/4}}\right] \quad \text{or} \quad \rho(T) \propto \exp\left[\frac{1}{k_B \cdot T^{1/4}}\right]$$
E-Field Dependence of Hopping Conductivity

At low field, the conductivity is independent of E-field:

\[
\sigma_{\text{hop}}(T) \rightarrow \left[ \frac{n(T) \cdot v \cdot a^2 \cdot e^2}{k_B \cdot T} \right] \exp \left( \frac{-\Delta H}{k_B \cdot T} \right)
\]

or

\[
\rho_{\text{hop}}(T) \rightarrow \left[ \frac{n(T) \cdot v \cdot a^2 \cdot e^2}{k_B \cdot T} \right]^{-1} \exp \left( \frac{\Delta H}{k_B \cdot T} \right)
\]

At high field, the potential wells distort. Poole-Frenkle theory predicts:

\[
\rho(E;T) = \rho_o(T) \exp \left[ -\frac{\beta \cdot E^{0.5}}{k_B \cdot T} \right]
\]
Conductivity in Highly Disordered Insulation Materials

\[ \sigma(t) = \sigma_{DC} + \sigma_{\text{Polarization}}(t) + \sigma_{\text{Diffusion}}(t) + \sigma_{\text{Dispersion}}(t) + \sigma_{\text{Transit}}(t) + \sigma_{\text{RIC}}(t) \]

Polarization

\[ \sigma_{\text{Pol}}^0 e^{-t/\tau_{\text{Pol}}} \]
Drift and hopping conductivity

\[ \sigma(t) = \sigma_{DC} + \sigma_{Polarization}(t) + \sigma_{Diffusion}(t) + \sigma_{Dispersion}(t) + \sigma_{Transit}(t) + \sigma_{RIC}(t) \]

\[ \sigma_{Diff}^{-1} \]

\[ \sigma_{hop}(E, T) = \left[ \frac{2 \cdot n(T) \cdot v \cdot a \cdot e}{E} \right] \exp \left[ \frac{-\Delta H}{k_B \cdot T} \right] \sinh \left[ \frac{\varepsilon \cdot E \cdot a}{2 \cdot k_B \cdot T} \right] \]

\[ \Delta H_m \]

\[ e \varepsilon a \]

\[ \ln \sigma \]

Boltzmann

Transition

Mott

(i)

(ii)

(iii)

(iv) \( T^{-1/4} \)
Dispersive transport

\[ \sigma(t) = \sigma_{DC} + \sigma_{Polarization}(t) + \sigma_{Diffusion}(t) + \sigma_{Dispersion}(t) + \sigma_{Transit}(t) + \sigma_{RIC}(t) \]

\[ \sigma_{\text{Disp}}^\alpha t^{-(1-\alpha)} + \sigma_{\text{Trans}}^\alpha t^{-(1+\alpha)} \]

\[ I(t) \sim \begin{cases} t^{-(1-\alpha)}, & t < t_T \\ t^{-(1+\alpha)}, & t > t_T \end{cases} \]
Normal vs. Dispersive Transport

I/t Curves

Pulse Propagation

Dispersive Transport

- Photoconductivity experiment on semiconductors by Pfister and Sherr
- Amorphous Selenium
- Dispersive transport causes unique shape in log-log graph
- Hopping and trapping mechanisms responsible for dispersive transport
- Dispersive transport results from wide range of hopping times, e.g. from range of $\Delta H$ and $a$
- ‘Universality’ a result of dispersive transport
- Note transit times in ms

\[ \sigma(t) = \sigma_{DC} + \sigma_{Polarization}(t) + \sigma_{Diffusion}(t) + \sigma_{Dispersion}(t) + \sigma_{Transit}(t) + \sigma_{RIC}(t) \]