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Reexamination of Rothermel's Fire Spread Equations in No-wind and No-slope Conditions

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RESEARCH SUMMARY
This paper reformulates the empirical fire spread equations of Rothermel, based on a tenfold increase in the data base of laboratory test fires. The experiment was performed at zero windspeed, with zero slope and uniform fuel particle size. Major revisions include the propagating flux ratio, the reaction velocity, and the moisture damping coefficient. These parameters remain functions of the observable fuel bed variables. Also, we redefine or reinterpret many of the fuels' descriptive parameters such as the net loading, the heat of combustion, fuel bed bulk density, flame residence time, and heat of ignition. As before, spread rate is based on an energy balance ratio of power source and heat sink terms.

We use Frandsen's (1973) formulation for effective bulk density to account for that fraction of the fuel bed that must be raised to ignition temperature. The enthalpic load of ignition also includes the energy required for complete pyrolysis of the fuel as reported earlier.

We also use Susott's (1984) method of determining the "flaming" heat of combustion, thus separating the gaseous and carbonaceous mass fractions of pyrolysis. This provides that the heat of combustion will account for many of the physiochemical effects of minerals, and so on, on the production rate of the gaseous fuel. Thus the reaction intensity is of the flames alone and specifically excludes energy derived from burning char whether in the flaming zone or not. It is still measured as an energy density within the combustion zone.

In this new formulation, moisture damping is completely divorced from fire extinction. Fire extinction is properly handled by a probability function acting on the reaction velocity. Moisture damping is a simple, negative exponential function of the fuel moisture, wherein each fuel sample has a characteristic moisture content. In the experiment, the characteristic fuel moisture was dependent only on the size of the fuel particles; however, it is further shown that it will vary greatly with the physiology of different fuels (bark covered, live or fresh-fallen foliage, decayed wood, and so on). The formula for reaction velocity begins with Anderson's 8d (1985) rule-of-thumb, which provides that the burning residence time in minutes is equal to eight times the fuel particle thickness in inches. The burning rate is limited by the free path for radiation or heat flux to propagate through the fuel bed, and further restricted by limiting the effect of excessive fuel loading or bed depth. A probability function shuts the fire down at very light fuel loads or high fuel moisture content as previously reported. The concept of optimum packing ratio is abandoned.

Following Rothermel, the propagating flux is some fraction of the reaction intensity, empirically formulated as the propagating flux ratio. It is an elementary function of the free optical path through the fuel bed.

Procedure is outlined whereby the manager in the field may estimate the probabilities that his real fire behavior will depart from the prediction.
Reexamination of Rothermel's Fire Spread Equations in No-wind and No-slope Conditions

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INTRODUCTION

Fire remains a complex, little-understood phenomenon; however, wildfire behavior prediction is an increasingly important requirement of land managers. Fire intelligence is needed in a broad range of applications, from real-time fire behavior predictions for immediate tactical use, to long-range planning, budgeting, and resource allocation. Forest and range fires are extremely complex interactions of physical, chemical, and thermodynamic processes for which satisfactory scientific theories do not exist or are far from complete. Fire researchers and model builders have therefore made heroic efforts to develop semiempirical mathematical models that anticipate management needs. Even then the mathematical models are commonly stretched beyond their capabilities by enterprising managers who will improvise tools that might solve an immediate problem.

Because a precise theory is not available and because the range of applicability is increasing in breadth and the information becoming more important and valuable, empirical mathematical fire models must be as dependable, reliable, accurate, and robust as the current state of technology can make them. This is the purpose of the research reported here—to enhance the versatility and accuracy of existing fire behavior models. Basically, this is a reworking of the Rothermel (1972) fire model equations. The primary objective of the experiment was to reexamine the fuel moisture damping effects and the marginal limits of combustion near extinction. The reformulation of the fire spread equations here should be viewed as a secondary result of the experiment where the new equations become hypotheses that require further testing.

The equations introduced here are based on a tenfold increase in the data base of laboratory test fires. This experiment was expressly designed to cover the range of all the physical (geometric) variables, fuel moisture content, and fuel loadings that might be encountered in surface fuel beds in the field. The revised equations are expressed as functions of the fuel bed packing ratio, β, the fuel particle surface area-to-volume ratio, ω, the fuel bed depth, δ, and the fuel moisture content, Mf. The experiment was performed at zero wind speed, with zero slope, and uniform fuel particle size. Fuel particles ranged from 0.6 cm to 1.27 cm in thickness; however, each fuel bed was constructed of a single, unmixed size class. The tabulated experimental data are available from the Intermountain Fire Sciences Laboratory (IFSL), Missoula.

We used heuristic reasoning and statistical "because it works" types of argument to reach "conclusions" and to generate empirical equations to describe events. For example, our procedure for forming the expectation value for spread rate as the product of other primary "expectation values" is not vigorously legitimate because those expectation values were not derived independently. Thus, the exposition here cannot be labeled as good physical theory; knowledge of wildfire phenomena has not yet reached that state.

New empirical formulas were constructed through judicious use of modified multiple linear regression—analysis of variance techniques with constraints and transformation of variables imposed to conform to preconceived notions of physical principles. We examined most possible combinations of the independent fuel bed variables and selected the functions of these variables to "reasonable" formulations that might be expected from any future physical theory or that could be easily revised by further analysis. That is, the rigorous statistical curve-to-data fitting is here tempered by arbitrarily limiting the independent variables to those that "ought to have" a proper physical effect and by incorporating much of the extensive practical knowledge of several colleagues whose experience with field applications has given guidance to how fire models should behave. The chosen combinations of independent variables are about the most efficient descriptors of the dependent fire parameters that make good physical sense.

Similarly, most of the numerical coefficients are truncated to show only meaningful digits. In some cases, the experimental statistics indicated that more significant digits were appropriate, but the practical physics overruled, for example, an exponent was evaluated to be 0.486 ± 0.011, but in the formulation, 0.5 (square root) makes more sense. The data would confirm that the two numbers were not really different at some reasonable significance level; so, the square root is used for practical reasons. Note that this problem would be superfluous if we had theoretically hypothesized a square root function before the experiment because the data would have confirmed it. We are in the philosophical position of using this publication to form a new hypothesized fire spread model that must be confirmed or rejected by further testing and validation. This research is not really different at some reasonable significance level; so, the square root is used for practical reasons.

We have used metric measure exclusively, and dimensionless ratios are expressed as fractions.

DISCUSSION OF EXPERIMENTAL TEST METHODS AND RESULTS

Rothermel's model is the most widely applicable and robust of the current wildland fire spread models. The reader should be familiar with the Rothermel formulation. We have found no reason or justification in the extended data set to change Rothermel's basic formulation, in which spread rate is based on an energy balance ratio of power source and heat sink terms:

\[ R = \frac{\text{Power density of propagating flux}}{\text{Preheating energy of fuel}} \]

\[ R = \frac{I_{p}}{\text{w}_{b} Q_{rr}} \]  

(1)

where \( I_{p} \) is the fire propagating intensity provided by the heat source, \( Q_{rr} \) is the heat sink energy required to heat, dry, and pyrolyze the fuel, and \( w_{b} \) is the fuel bed bulk density.

We reformulate the empirical equations by fitting the curves to the expanded data set including: the propagating flux ratio \( \xi \), the reaction velocity, \( \Gamma \), and the moisture-damping coefficient, \( \text{w}_{\text{m}} \), meters remain functions of the observable fuel bed variables. Also, we redefine or reinterpret many of the fuels' descriptive parameters such as \( \text{w}_{\text{m}} \), \( \text{h}_{\text{m}} \), \( \text{w}_{\text{w}} \), and \( Q_{\text{r}} \) which plays an equivalent role to the formerly used \( Q_{\text{rr}} \).

Variable List

- **Fuel bed descriptors**: 
  1. Fuel ovendry loading, \( \omega_{b} \) kg/m²
  2. Fuel particle size: ratio of surface area-to-volume, \( \text{w}_{\text{s}} \) km²/kg
  3. Fuel bed depth, δ cm
  4. Fuel moisture content, \( M_{f} \), fraction ovendry weight
  5. Wind speed, \( U \), m/min
  6. Slope, tan(θ), was invariant, zero, for this experiment for completeness, one should include the ambient air temperature

The measured thermochemical properties of the fuels include:

- 7. Heat of combustion of pyrolyzate gases, \( h_{\text{p}} \), kJ/kg (\( h_{\text{p}} \) is measured as the combustion energy of the pyrolyzate gases per unit initial mass of the solid dry fuel. The total heat of combustion, \( h_{\text{p}} \), was measured, compared, and rejected in the analysis of variance procedure. The mineral content was measured, but found unnecessary when the burning rates and intensities are based on the heat of combustion, \( h_{\text{p}} \), of the vaporized fraction of fuel).
  8. Fuel particle ovendry density, \( \rho_{p} \), kg/m³
  9. Heat for pyrolysis, \( Q_{\text{r}} \), kJ/kg
  10. A characteristic fuel moisture content, \( M_{\text{m}} \), fraction ovendry weight, is used to characterize moisture damping, replacing the moisture of extinction, \( M_{e} \), used in Rothermel's 1972 model.

In addition, we have a few new numeric parameters in the equations below that may vary with the thermochemical character of a different fuel type. Specific comment is made where appropriate.

- The dependent fuel variables are also the input variables for the revised spread equations and are intended to be comparable to Rothermel's (1972, page 27) list of input parameters that definitively describe the fuel bed and burning conditions. For
The onset of flaming combustion occurs when the woody forest fuels reach about 320 °C. Even though the entire fuel particle is not raised to ignition temperature uniformly, but only the surface of the fuel particle must be ignited, we have found that the ignition enthalpy load of the fuel mass must include the energy required to complete pyrolyzation of the fuel. Susott (1982b, 1984) has shown that pyrolyzation, that is, flammable gas production, is essentially complete very near 400 °C. The energy of fuel ignition, $Q_e$, is newly defined as:

$$Q_e = Q_M + M_M$$

(3)

where

$$Q_M = \int_0^{400} \left( \frac{dQ}{dT} \right) dT$$

and the heat to vaporize the fuel moisture is

$$M_M = 4.18(100 - T_{mb}) = 540 \text{ kJ/kg}$$

and where Susott's procedure provides values of $dQ/dT$ to 400 °C and above, where Dunlap's specific heat equation (as used by Frandsen and Rothermel) fails dramatically. Note that we have just circumvented a conflict in semantics: $Q_e$ is no longer the heat load of "preignition" and the term "heat of pyrolysis" has an explicit meaning elsewhere, so we will use "heat of ignition" for the total heat load. $Q_e$. We hope this terminology will be less confusing to those concerned.

### Reaction Intensity

The concept of propagating flux that drives the fire is significantly changed from Rothermel's original description. We propose that the energy released from only the flaming area and due to burning of the pyrolyzates is the major source of power density that drives the fire. The propagating intensity is still calculated as a fraction, $I_p$, of reaction intensity, $I_0$, but here $I_0$ is really the reaction intensity of the flames and specifically excludes energy derived from the burning char whether in the flaming zone or not. It is still measured as an energy release rate per unit area within the combustion zone. Susott (1982a) and Wilson (1985, 1987) argue that the flaming pyrolyzates are the primary energy source for propagation of the fire front, and Susott has developed techniques to measure the gaseous heat of combustion, $h_g$. Because Susott's method determines the "flaming" heat of combustion per unit of total (initial) fuel mass, thus separating the gaseous and carbonaceous fractions, we have the additional advantage that $h_g$ accounts for many of the physical-chemical effects (minerals, etc.) on the mass fraction and on the production rate of the gaseous fuel. For example, the effects of minerals on volatiles release creates a pyrolyzate flow with a lower yield of pyrolyzate gas and more char are reflected in the lower measured $h_g$. Although they vary widely in woody fuels, the pyrolyzate gas account for about 60 percent of the total combustion energy and about 80 or 90 percent of the total fuel mass loss. While we have settled on the gaseous heat, $h_g$ for use in modeling the reaction intensity for the fire spread equations, this does not preclude using the total heat of combustion, $h_m$, for many other applications. For example, it may be inappropriate in "fire effects" work to use $h_m$ to calculate the total heat released to the site which includes the burnout of the residual char fraction. Thus the reaction intensity is defined:

$$I_p = \frac{h_m}{h_g} \frac{dw}{dt}$$

(4)

The other terms in the reaction intensity that must be experimentally measured are the mass loss rate per unit area of the combustion zone and the reaction or residence time. Here, again, our method differs from that of Rothermel (1972). In this experiment, we measured the mass loss rate of the entire combustion zone ($h_m$ is measured relatively to total load) and the width and depth of the burning—flaming area. Thus a consistent new calculation of flashing reaction intensity follows as the average heat release rate per unit area of the flaming zone. The (flaming) reaction time is calculated directly by dividing the flame zone depth by the spreading velocity.

Note that Rothermel's formulation is consistent in the same sense: he directly measured the mass loss rate per unit area of the combustion zone, that when multiplied by the total heat of combustion, gives a reaction intensity equal to the total heat release rate per unit area of combustion zone. The reaction time (and hence the combustion zone area) was then determined by the duration of constant mass loss in the combustion zone. The difference between the two methods is that Rothermel's combustion zone was defined by the area with attached flaming. Then the reaction intensity is:

$$I_p = \frac{h_m}{h_g} \frac{dw}{dt}$$

(5)

Recall that Rothermel used the net loading, $w_r$, to provide for less than 100 percent burning efficiency, and which fraction of the total load, $w_{rb}$, is to be accounted for in the reaction velocity, $\Gamma$, along with the reaction time, $\tau$. In the present experiment, the residual load after complete burnout never exceeded 0.005 fraction of the initial load, so by Rothermel's method, the net and total loads were equivalent in the experiment. Also, in field application, fuel modellers are well versed in selecting "that portion of the fuel bed that will contribute to propagating the fire." Our concept here is that all inefficiencies in flame production will be accounted for by Susott's technique for $h_g$ and that the reaction velocity should concern only the time dependence of the burning rate function. We wish to be explicit that the carbonaceous char in many fires was almost completely consumed in the flaming zone, thus contributing to the total mass loss rate of the flaming zone. In many other fires there was considerable glowing/ burning char left after passage of the flaming front. Due to the nature of our fuels, this residual char was quickly consumed by glowing combustion, generally leaving no residual char. This trailing edge of glowing char obviously had very little effect on the propagation of the fire but in several cases it provided a significant amount of the mass loss and total heat released.

So, we have experimentally measurable reaction intensities and hence by equation 5, the reaction velocity is:

$$\Gamma = \frac{1}{w_{rb}} \frac{dw}{dt} \text{ min}^{-1}$$

(6)
The third assumption is as follows:

2. All fuel moisture effects on reaction velocity (and intensity) are contained in a separable and independent moisture damping coefficient, \( \eta_w \), that is defined as the ratio of reaction intensity with fuel moisture, \( I(M, f) \), to that without moisture, \( I(0) \).

Each experimental fuel bed configuration was burned at several fuel moisture contents, so that we have experimental measurements of \( \eta_w \).

Thus, we speculate that thermally thin, fine fuels should have very little moisture damping, \( \eta_w \approx 1 \), and otherwise, "moisture damping" may imply the ability of a fuel to hold and carry moisture into the combustion zone where it may have an additional cooling or gaseous dilution effect.

2. A second reason for using the experimental method of equation 10 is that there is much uncertainty and ambiguity in defining and measuring the "size of the combustion zone," which area is divided into the mass loss rate for calculation of reaction intensity (and inherently, the mass loss rate has a larger experimental measurement error than does spread rate). Also, the effect of the enthalpic moisture is obscured in the ratio of mass loss rate over combustion zone area.

If, indeed, some fires do generally travel in an "overdamp," moisture-free, combustion zone, then the reaction intensity must be independent of the initial fuel moisture, that is, the observed effect of increasing fuel moisture would be to slow the spread rate and decrease the size of the combustion zone without, however, changing the rate of mass loss per unit area within that zone.

3. In the above mathematical construction, the two methods of measuring \( \eta_w \) are equivalent, by the spread rate method, we measure a more dynamic variable with greater precision and explicitly exclude the enthalpic moisture effect from moisture damping.

In this new formulation of moisture damping, the most significant improvement is to completely divorce moisture damping from fire extinction. Each fuel sample must still have a characteristic moisture content, but extinction is properly handled by the probability function, \( P_x \), in the reaction velocity, \( \Gamma \), discussed later. So the new moisture damping equation is very much simplified:

\[
\eta_w = \exp(-M/M_a)
\]

The experimental curves are shown in figure 1a, a, b, c, d, e. In figure 1a, moisture damping (that moisture effect in excess of \( Q(M, f) \) effect) is shown to be negligibly small in the lightly and moderately packed (\( \beta < 0.04 \)) fine fuels in support of the observations above. Figure 1a shows the moisture damping (in excess of \( Q(M) \)) for the very compact fuel beds, \( \beta > 0.04 \), where \( M_a = 0.50 \).

Figure 1b shows a definite moisture-damping effect in \( \frac{3}{8} \)-inch-thick sticks with characteristic \( M_a = 0.49 \). This relatively large \( M_a \) indicates this fuel is easily dried and thermally thin. Figures 1c, d, and e show the progressively greater moisture damping in successively larger fuel particles.

Figure 1a—Experimental scatter and lack of definitive trend for moisture damping in light-to-moderate loads of fine fuels (excess).

Figure 1a—Moisture damping effect in the more densely packed—heavier loaded fine fuel beds, \( \beta = 0.04 \). The curve is drawn with characteristic \( M_a = 0.50 \).

Figure 1b—Moisture damping for \( n = 25.21/cm \) (\( \frac{3}{8} \)-inch sticks). Here \( M_a = 0.49 \), indicating that it takes a considerable amount of water to dampen fire in the smaller (thermally thin) fuels.
Figure 2 shows the dependence of $M_e$ on fuel particle size for all of the experimental fuels. Equation 11 derives from a $dI/dM = - (1/M) dM/dM_e$ type of model. Note that the characteristic fuel moisture, $M_e$, is shown in figure 2 as functions of the size of fuel particles; the experimental fuels were otherwise very similar, all being sound cured cellulose heartwoods—of different species, however. An empirical equation, $M_e = f(\alpha)$, could be constructed from the data in figure 2, but would be very misleading. Such an equation would only satisfy pragmatic needs for such a predictive equation by model builders of fire behavior systems; it is a strongly held opinion of the author that values of $M_e$ will vary greatly with the physiology of different fuels (bark covered, live or fresh-fallen foliage, decayed wood, etc.). This opinion is based primarily on private communications with Hal Anderson and extrapolating his moisture response observations (Anderson 1985) into a forced drying-fire environment. Hypothetically, we might expect that a fuel that has a long ambient moisture response time will also more easily retain moisture by forced drying, thus exhibiting a lower $M_e$ (smaller values of $M_e$ will produce a stronger response in $\eta_d$ due to change in fuel moisture content, $M_f$). In figure 3a, b, c, we use equations 10 and 11 and replot Anderson's original data from which Rothermel's original moisture damping coefficient was derived. Note the relatively strong $M_e$'s for the natural pine needles when compared to cellulose pine heartwood, with nearly the same surface area-to-volume ratio.
A future project should compare Anderson's ambient moisture response times with these $M_1$. An attempt should be made to relate these $M_1$ to diffusion coefficients and other physical characteristics of the fuel particles.

**Reaction Velocity**

Note that Rothermel's model of reaction velocity was characterized by an optimum packing ratio which, in turn, depended on the fuel particle size. The equations predicted maximum reaction velocity when the packing ratio was at its "optimum" value. Then the reaction velocity slowed as packing ratio moved away from optimum in either direction, but was independent of fuel bed depth and hence the total amount of fuel. The reaction velocity as defined here has a similar character but we wish to order a request for $B_0$. In the base case, the equivalent effect is dependent on the total amount of fuel mass and/or fuel surface area in the fuel bed. Indeed, as the fuel bed is fluffed up to lighter packing (at constant load, meaning greater depth) the reaction velocity will increase as fires are shown to burn faster and hotter as the fuel bed is opened up. But instead of reaching a maximum reaction rate and then slowing down as a full contiguous fire front, the fuel becomes sparse and the fire front breaks up and ultimately goes out as determined by the function $P_E$ (see fig. 4a).

The fire extinction equation that provides the practical upper limit to reaction velocity is identical to the fire state (of extinction) probability function in Wilson (1985): 

$$P_E(n_1 = 1) = 1 + \exp(-2(n_1 - 0.5)/1.2)$$  

(12)

Note here, however, that the parameters, 1.2 and $n_1$ (see fig. 4a), are potential "adjustment" factors that would allow $I_c$ to work correctly for a different fuel type; a number smaller than 1.2 would "sharpen" the edge between good and bad burning, and conversely; $n_1 = 3.0$ approximately for woody cellulosic fuels; larger values of $n_1$ will move the "NO-BURN cutoff" toward heavier fuel loadings or lower fuel moisture. Note that $n_1$, given in equation 13, is given independently on fuel thermochemistry and fuel moisture content.

The extinction index equation that partitions "good burning" from "poor burning" fires is given by:

$$n_2 = \ln(\text{M}_2 h_{Q1} h_{Q2} h_{M1} + M_1 Q_i Q_j)$$  

(13)

Wilson's extinction index is described in detail in Combustion Science and Technology (1985). The index, $n_2$, was especially designed to reflect the energy balance of fire near the marginal limits of sustained burning.

An interesting speculation (without experimental foundation) is to include the wind coefficient, $A_w$, in agreement of the numerators' logarithmic function; in addition to its normal effect on spread rate (Rothermel 1972, p. 26), this use of $A_w$ also has the effect of "turning I_c" on in sufficient wind when otherwise the fuels might be too light or too wet to burn; and similarly for slope, $A_s$.

We must caution that the previous formulation of the wind coefficient is very likely to produce strange results if used here as in the 1972 fire spread model as a function to increase rate of spread. Careful examination of the "old wind model" using sketchy data has been inconclusive and net encouraging. Ralph Nelson (1988) has suggested a form of wind coefficient, $A_w = \exp(0.12\log^{1/3}w)$ that might work better.

Until a new wind experiment is performed, these formulations should be used with great caution. It is almost imperative that a new set of laboratory wind data be generated for revision and updating of Rothermel's wind model (and similarly for slope). The importance in the field of wind effects and the depth of supporting experimental data make it the next priority item in fire behavior research.

The new experimental formulation for reaction velocity, $\Gamma$, (fig. 4a and b), is:

$$\langle C > = 0.34 \cdot \exp (-0.8/3) \cdot P_E(n_1) \text{ min}^{-1}$$  

(14)

The $0.34$ is similar to Anderson's old 8d rule-of-thumb, which provides that the burning residence time, $\tau$, in minutes, is equal to 8 times the fuel particle thickness, $d$, in inches. $P_E$ provides the by dependence and $\exp(-0.8/3)$ measures the absorption cross section of fuel particles per unit area of the combustion zone.

The exponential function of $\exp(-0.8/3)$ limits $\Gamma$ by a characteristic distance that propagating flux is projected through the fuel bed to heat fuel ahead of the combustion zone. The equation components with $\exp(-0.8/3)$ arguments slow the reaction (velocity and intensity) for heavier and denser fuel loads. The $P_E$ function shuts the fire down at very light loads or high moisture, for example, and is discussed extensively in Wilson (1985). Fire modelers in the future will probably separate this probability function from the reaction velocity. Only in very slow, marginally burning fires, only very deep and very lightly packed fuel beds where flame heights were much less than the depth of the fuel bed, did the $P_E$ function contribute significantly to the burning rate. Otherwise, $P_E$ acts only to turn the fire on or off.

**Propagating Flux Ratio**

The propagating flux, $I_p$, is that portion of the fire's power density that goes to drive the fire: 

$$I_p = \frac{\text{M}_2 Q_1 Q_2}{2 \cdot \rho_{CH4}}$$  

(15)

Following Rothermel, the propagating flux must also be some fraction, $\xi$, of the reaction intensity, $I_r = \xi \cdot I_p$, such that $\xi$ may be calculated from experimental measurements by:

$$\xi = \frac{R \cdot \rho_{Q1} Q_2}{h_{\lambda} (\text{d}u/\text{d}t)}$$  

(16)

and an empirical prediction equation constructed to fit:

$$\xi = \frac{1}{(\text{d}u/\text{d}t)}$$  

(17)

It is only fair to admit here that all of the parameters on the right side of equation 16 are measured in the experiment, except $c$. Both $c$ and $\xi$ are fractions that diminish their respective power densities so that the two intensities are equivalent to one another; that is, so that the propagating flux, $I_p$, equals the product of the supply rate, $R$, and the heat load $Q_1 = RQ_2$. It follows that the experimental measurements of $\xi$ (effective heating ratio) and $c$ (effective heat number) are interrelated. As a consequence, they are difficult to measure independently. We rely on Frandsen's (1972) experiment for the independent prediction of $<\xi> = \exp(-1/220)$. Thus, in equation 17 we are actually making an estimate of $<\xi>/c = f'(\text{d}u/\text{d}t)$, and the ratio, $c/\xi$, is simply a rotation operator that measures the scalar value of the ratio of two vectors (the horizontal propagating flux to the vertical source flux):

$$c/\xi = \frac{R \cdot \rho_{Q1} Q_2}{h_{\lambda} (\text{d}u/\text{d}t)}$$  

(18)

In any case, by accepting Frandsen's expression for $c$, a fairly simple straightforward formula for $c/\xi$ is bad.

Note that we have here an interesting experimental exercise: 

- if, in equation 13, we interpret $R = \text{d}u/\text{d}t$ 
- and $\text{d}u/\text{d}t = \rho_{CH4} (\text{d}u/\text{d}t)$ 
- and assume that

$$\frac{\text{d}u}{\text{d}t} = \frac{\text{d}t}{\delta} = \text{"flame depth"}$$

then the experimental measurement of $\xi/c$ becomes:

$$\frac{\text{d}t}{\delta} = \frac{\text{d}u}{\text{d}t}$$  

(18a)
which, facetiously, compares very well with experimental determination of the $\tau_i$ ratio by equation 18. Statistical analysis showed that the product of the fuel particles' surface-to-volume ratio times the packing ratio, $\phi$, is for the most dominant variable governing the propagating flux ratio. This product is dimensionally equivalent to an (optical) absorption coefficient of power transport where power flux density decreases with distances propagated through an absorbing (heating) medium. This formulation is adaptable to a model of propagating energy transport in which $d_1 = -\varepsilon \phi d_2$ and $k = \varepsilon_0 c d_2$, etc.

Our new equation for propagating flux ratio, $\langle R \rangle$ (Fig. 5) is:

$$\langle R \rangle = 1 - e^{-0.17 \phi}.$$  

(19)

Note that the factor, 0.17 cm, (for $\sigma$ in cm$^{-1}$) is construed as a physical skin thickness on the fuel particles that is absorbing power (being pyrolyzed) and is comparable to Frandsen's "effective heating" coefficient, 0.02 cm. Susott (private communication) has observed a charring pyrolyzing zone in woody material that consistently measures about 2 mm deep.

Furthermore, we should note that all of our experimental fuels were beds of long narrow pine needles, namely, "one dimensional," for which characteristic optical absorption distance depends on the geometric cross-section of the particle. A fuel complex of long narrow pine needles will have a very different optical extinction length than will a corresponding bed of wide (two-dimensional) oak leaves. The point is that functions that depend here on the surface area-to-volume ratio of the fuel particle might not be universally applicable to other fuel particle configurations; particularly, the equations of propagating flux ratio, reaction velocity, and moisture damping.

Rate of Spread

The predicted value of spread rate, $\langle R \rangle$, is given by substitution of the foregoing equations 19, 14, 11, 2 and 3 into equation 1 (see Fig. 6):

$$\langle R \rangle = \frac{e^x}{\langle R \rangle} = \frac{e^x}{\langle R \rangle} =$$

(20)

From Figure 6 we see that equation 20 overpredicts rate of spread in the faster burning fine dry fuels and, to a lesser degree, in the very slow, light loads of larger fuel particles. This effect is the result of combining in equation 20 the several prediction equations for $\langle R \rangle$, $\langle R \rangle$, $\langle R \rangle$, which were not derived from statistically independent data sets. Thus, for example, the same experimental statistical errors in spread rate are applicable to $\langle R \rangle$, $\langle R \rangle$, and $\langle R \rangle$; thus, in equation 20 they are applied thrice.

Alternatively, the physical problem might be that the particle-to-particle variation in fuel moisture or heat content, etc., may have a more pronounced effect on the spread rate in the drier fuel beds with few fuel particles (very small $\beta$). These variations among the individual particles were not measured.

PROBABILITIES

Let any one of the critical dependent variables such as spread rate, $R$, or reaction intensity, $I_R$, be represented by the parameter $Y$. Then, of course, the "real" observed value, $Y$, will be different than the predicted fire behavior, $\langle Y \rangle$, which implies that there is a probability distribution of $Y$ about $\langle Y \rangle$.

The variable $\langle Y \rangle$ is a deterministic function of the independent variables, (a, $\beta$, $M$, $f$), and it describes in one number the effects (fire behavior) that result from the conjunction of explicit values of the independent variables. By definition, it is the most likely value of $Y$ that may occur, but it is very unlikely that $Y$ will precisely equal $\langle Y \rangle$. It follows that there must exist a probability distribution of the real fire behavior when we are given the expected (predicted) fire behavior that can be calculated from measured values of $\beta$, $\beta$, $M$, $f$, etc. From Wilson (1987), we expect to be given these probability distributions so that persons that must make risks may calculate their chances of being wrong.

Now let us review the probability problem and solution. To begin, the fire manager in the field will have at hand a prediction, $\langle Y \rangle$, of predicted fire behavior that is theoretically different from the actual "real" behavior, $Y$. Also, the manager will have some imperfect knowledge, $P(Y)$, of the prior historical behavior of fire in similar circumstances.

The manager needs to know the probability distribution, $P(Y|\langle Y \rangle)$, of what the real behavior, $Y$, might be, given prior knowledge of $\langle Y \rangle$ and the expected behavior, $\langle Y \rangle$, where $\langle Y \rangle$ is estimated from the current conditions, $\sigma$, $\beta$, $M$, $f$, etc. The calculation of that probability follows from Bayes theorem (Wilson 1987):

$$P(Y|\langle Y \rangle) = \frac{P(\langle Y \rangle|Y)P(Y)}{P(\langle Y \rangle)}.$$  

(21)

where the probability distribution, $P(\langle Y \rangle|Y)$, is determined by independent experiments such as the one described here.

Figure 7 shows the error distribution, $P(\langle R \rangle/R)$ for spread rate, $\langle R \rangle$, from the experiment described here. This figure is the one to be used for probability calculations of rate of spread while using the prediction equations presented in this paper. When a more precise experiment is performed or more accurate equations are derived, then a new $P(\langle R \rangle/R)$ of Figure 7 must be presented.

These error distributions give a clue to the source of the "factor-of-two" estimate of maximum error in fire behavior predictions (Albini 1976); we note the following characteristics of these distributions:

1. The experimental observations of $\langle Y \rangle$ are found to be log-normal for all of the dependent variables. Thus, observations of $Y < \langle Y \rangle$ and $Y > \langle Y \rangle$ occur with about equal probability.

2. The standard deviation for the spread rate observation is in the approximate range of $0.6 < \sigma < 1.0$. (The common factor-of-two error would be about two standard deviations on the log-normal distribution.)

3. The experimental variation of the independent variables, $\sigma$, $\beta$, and $M$, is at the very most less than 5 percent (rms error in $\beta$ and $M$ < 1 percent; in $M$, $< 2$ percent and $< 3$ percent).

Thus, the experimental errors in observation of the independent variables do not explain all of the variance in the dependent variables. The model prediction error is most likely a result of inaccuracy in the model relationships or the effect of missing or neglecting some unknown physicochemical thermal parameter. As Albini said, we "consider models successful if the relationships predict fire behavior within a factor of two or three over a range of two or three decades."
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REFERENCES


Major revisions to Rothermel's fire spread equations include the propagating flux ratio, reaction velocity, and moisture damping coefficient. The reaction intensity is of the flames alone and specifically excludes energy derived from burning char whether or not it lies in the flaming zone. In this new formulation, moisture damping is completely divorced from fire extinction. Fire extinction is a probability function.

KEYWORDS: fire behavior, fire modeling, intensity, fuels, moisture

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