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ABSTRACT

Many radio pulsars exhibit glitches wherein the star’s spin rate increases fractionally by \( \sim 10^{-10} \) to \( 10^{-6} \). Glitches are ascribed to variable coupling between the neutron star crust and its superfluid interior. With the aim of distinguishing among different theoretical explanations for the glitch phenomenon, we study the response of a neutron star to two types of perturbations to the vortex array that exists in the superfluid interior: (1) thermal motion of vortices pinned to inner crust nuclei, initiated by sudden heating of the crust, (e.g., a starquake), and (2) mechanical motion of vortices (e.g., from crust cracking by superfluid stresses). Both mechanisms produce acceptable fits to glitch observations in four pulsars, with the exception of the 1989 glitch in the Crab pulsar, which is best fitted by the thermal excitation model. The two models make different predictions for the generation of internal heat and subsequent enhancement of surface emission. The mechanical glitch model predicts a negligible temperature increase. For a pure and highly conductive crust, the thermal glitch model predicts a surface temperature increase of as much as \( \sim 2 \) per cent, occurring several weeks after the glitch. If the thermal conductivity of the crust is lowered by a high concentration of impurities, however, the surface temperature increases by \( \sim 10 \) per cent about a decade after a thermal glitch. A thermal glitch in an impure crust is consistent with the surface emission limits following the 2000 January glitch in the Vela pulsar. Future surface emission measurements coordinated with radio observations will constrain glitch mechanisms and the conductivity of the crust.

Key words: dense matter – stars: evolution – stars: interiors – stars: neutron – stars: rotation.

1 INTRODUCTION

Many pulsars exhibit glitches, sudden jumps in spin rate, superimposed on the gradual spin down due to electromagnetic torque (see, e.g., Lyne, Shemar & Smith 2000). Glitches involve fractional jumps in spin rate of \( \Delta \Omega/\Omega = 10^{-10} \) to \( 10^{-6} \), with recovery to the pre-glitch spin-down rate occurring over days to months in most cases. Some pulsars show no obvious recovery, and continue to spin down faster than had the glitch not occurred. The 1989 glitch of the Crab pulsar (\( \Delta \Omega/\Omega = 7 \times 10^{-8} \)) was partially time-resolved (Lyne, Smith & Pritchard 1992). This glitch showed a quick rise on a time-scale of hours with additional spin-up taking place over approximately one day. In contrast, the Vela ‘Christmas’ glitch (\( \Delta \Omega/\Omega = 2 \times 10^{-6} \)) observed in December of 1988 (McCulloch et al. 1990) showed a very different behaviour. In this case the glitch was not time-resolved, and occurred in under two minutes. The 2000 January glitch in the Vela pulsar (\( \Delta \Omega/\Omega = 3 \times 10^{-6} \)) was similar to the Christmas glitch (Dodson, McCulloch & Costa 2000). A number of pulsars (the Crab in particular) exhibit permanent increases in spin-down rate after a glitch occurs, typically \( \Delta \Omega/\Omega = 10^{-4} \). In the Crab, these offsets produce much larger cumulative timing residuals than the glitches themselves. In addition to glitches, nearly all pulsars exhibit low-level fluctuations in their spin rate, and timing noise, believed to be of a different origin than glitches (see, e.g., D’Alessandro et al. 1995).

Glitches are thought to represent variable coupling between the stellar crust and the superfluid interior. Two questions concerning the glitch phenomena are (1) where in the star the coupling occurs, and (2) how the coupling is triggered. The rotation of the neutron superfluid interior is governed by the dynamics of vortex lines; a spin jump of the crust would result from sudden motion of vortices away from the rotation axis as a consequence of some instability. In the inner crust, vortices could pin to the solid (Anderson & Itoh 1975; Alpar 1977; Alpar, Langer & Sauls 1984c; Epstein & Baym 1988), allowing the superfluid to store angular momentum as the crust spins down under electromagnetic torque. In this picture, a giant glitch would result from the motion of \( \sim 10^{14} \) vortices over a macroscopic distance (\( \approx 1 \) m). A challenge to all glitch models has been to explain how such a global instability might occur.

As the crust spins down, and a velocity difference between the
solid and the superfluid develops, vortices creep through the crust at a rate that is highly sensitive to temperature (Alpar 1977; Alpar, Cheng & Pines 1989; Link & Epstein 1991; Chau & Cheng 1993a,b; Link, Epstein & Baym 1993). Based on an idea by Greenstein (1979a,b), Link & Epstein (1996) have proposed a thermal glitch mechanism in which a temperature perturbation causes a large increase in the vortex creep rate; in consequence, the superfluid quickly loses angular momentum and delivers a spin-up torque to the crust. A candidate mechanism for providing the required heat is a starquake arising from relaxation of crustal stress as the star spins down (thermal fluctuations on a global scale are negligible). Starquakes could deposit heat as much as $\sim 10^{32}$ erg in the crust (Baym & Pines 1971; Cheng et al. 1992). Ruderman (1991) has proposed a different model in which vortices strongly pinned to the inner crust solid stress the crust to the point of fracture, resulting in outward motion of vortices with plates of matter to which they are pinned. In the core, pinning may occur between the vortices and flux tubes associated with the superconducting proton fluid (Sauls 1988; Srinivasan et al. 1990; Chau, Cheng & Ding 1992), allowing the core superfluid, or a portion of it, to store angular momentum. Ruderman, Zhu & Chen (1998) have proposed a core-driven glitch mechanism in which the expanding vortex array of the core forces the magnetic flux into the highly conductive crust, stresssing it to fracture. In this model, crust cracking allows the core vortex array to suddenly expand outward, spinning down a portion of the core superfluid and spinning up the crust. Carter, Langlois & Sedrakian (2000) have suggested that centrifugal buoyancy forces are the origin of pressure gradients sufficient to crack the crust, allowing outward vortex motion. Other proposed glitch mechanisms include catastrophic unpinning of vortices in the crust through a global hydrodynamic instability (Cheng et al. 1988; Alpar & Pines 1993; Mochizuki & Izuyama 1995), although it is unclear in such models what the source of the instability would be, and vortex motion at the crust–core boundary due to proton flux tube annihilation there (Sedrakian & Cordes 1999). In any of these crust or core-driven glitch models, dissipation that accompanies outward vortex motion generates heat that might produce detectable emission as the heat arrives at the stellar surface.

Quantitative calculations are necessary to distinguish between different models for the glitch phenomenon. The thermal glitch model of Link & Epstein (1996) produced good qualitative fits to glitch observations in the Crab and Vela pulsars. This paper is an extension of that work, with more realistic physical inputs and detailed modelling of the timing data. We include non-linear thermal diffusion, which has the effect of slowing the glitch spin-up. We also include the effects of superfluid heating due to differential rotation between the superfluid and the crust (Greenstein 1975; Harding, Geyer & Greenstein 1978; Alpar et al. 1987; Shibazaki & Lamb 1989; Van Riper 1991; Van Riper, Epstein & Miller 1991; Umeda et al. 1993; Van Riper, Link & Epstein 1995; Larson & Link 1999), and study the propagation of heat to the stellar surface. We consider two types of rearrangement of the superfluid vortices: (1) thermal excitation of vortices over their pinning barriers (a thermal glitch), and (2) mechanical motion of vortices (a mechanical glitch). The first case models the response to sudden heating of the crust, e.g., from a starquake. The second case models catastrophic unpinning events, vortex motion as a result of crust cracking due to superfluid stresses, or core-driven glitches involving vortex motion near the crust–core boundary. In addition to simulations of the rotational dynamics, we predict the characteristics of the emerging thermal wave which could, in some cases, be visible from the surface of the neutron star weeks to years after a glitch occurs.

This paper is organized as follows. In Section 2 we provide an overview of the physical setting, and discuss the treatment of the coupled rotational and thermal dynamics we use in our simulations of pulsar glitches. In Section 3 we discuss the details of our numerical models. In Section 4 we present our simulations of the spin-up process and the emergence of the thermal wave at the stellar surface. We compare our simulations with spin observations of four pulsars and surface emission data following a recent glitch in the Vela pulsar. In Section 5 we conclude with discussion.

## 2 Input Physics

A neutron star consists of an atmosphere, an outer and inner crust, and the core. The inner crust begins at a density of $\rho_9 \approx 2.8 \times 10^{14} \text{ g cm}^{-3}$ and extends to approximately nuclear saturation density, $\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3}$. In this region, a neutron-rich solid coexists with a neutron superfluid, protons and relativistically degenerate electrons. The inner crust solid dissolves near nuclear density (Lorenz, Ravenhall & Pethick 1993; Pethick, Ravenhall & Lorenz 1995). Most of the mass of the core is expected to reside in superfluid neutrons and superconducting protons, with electrons and muons also present.

Glitch models that rely on the superfluid interior as an angular momentum reservoir require a metastability of the vortex state to sustain differential rotation between the solid and liquid components of the star. In the inner crust, the metastability could arise through the pinning of vortices to the solid. The details of pinning are uncertain. The form of the vortex–nucleus interaction potential is not well known due, in part, to uncertainties in the nucleon–nucleon interactions and the structure of the vortex core. Preliminary calculations of the vortex–nucleus interaction gave energies of $\sim 0.5–3\text{ MeV}$ (Alpar et al. 1984b) and $\sim 1–10\text{ MeV}$ (Epstein & Baym 1988). Pizzochero, Viverit & Broglia (1997) refined these calculations, and found interaction energies on the order of several MeV per nucleus near nuclear density. Jones (1997, 1998a, 1999) has emphasized that the self-energy of a vortex line leads to a large degree of cancellation of the nuclear forces on a vortex line. He assumes that the basic pinned state in a polycrystalline solid consists of segments pinned along crystal planes separated by unpinned kinks, and argues that pinning of vortices (in this situation) is of insufficient strength to account for the giant glitches of pulsars. Hirasawa & Shibazaki (2001) find that such a pinned state is stable as long as the vortex–nucleus interaction is stronger than $\sim 4\text{ MeV per nucleus}$. However, Jones (1998b, 2001) has recently shown that the inner-crust solid might not be a regular lattice at all, but an amorphous solid. Link & Cutler (2002) argue that, regardless of the symmetries of the solid (if any), a vortex will pin randomly to nuclei, similar to flux pinning in Type II superconductors. In this picture, a vortex threading a solid will pin to the extent that it can bend to intersect the occasional nucleus.

Link & Cutler show that vortex tension sets the average distance of a neutron to be $f_\rho \approx 2 \times 10^{16} \text{ dyne cm}^{-1}$. For a vortex–nucleus interaction potential of range $r_c \approx 10^{-16} \text{ fm}$, the effective pinning force per nucleus is then $\sim f_{\rho} \nu r_c \approx 40\text{ keV}$, which is sufficient to account for giant glitches. This estimate should also hold for a regular lattice, as long as the vortex does not closely follow one of the lattice basis vectors. Given the uncertain nature of vortex pinning, we take the effective pinning energy per nucleus to
be a free parameter. Our simulations fit the spin data for effective pinning strengths of $\approx 20 - 500 \text{ keV/nucleus}$. 

2.1 Rotational dynamics

The total angular momentum $J$ of the star is that of the effective crust (the crust and all components strongly coupled to it) plus the angular momentum of the superfluid,

$$J_{\text{tot}}(t) = I_c \Omega_c(t) + \int \text{d}l_s \Omega_s(r, t) = J_0 - N_{\text{ext}} t,$$

(1)

where $I_c$ is the moment of inertia of the effective crust, $I_s$ is the superfluid moment of inertia, $I = I_c + I_s$ is the total moment of inertia, $\Omega_c$ is the angular velocity of the effective crust, and $\Omega_s$ is the angular velocity of the superfluid. The initial angular momentum of the star is $J_0$. The star slows down an external torque, $N_{\text{ext}} = I \dot{\Omega}_c(t)$. In rotational equilibrium the effective crust and the superfluid would spin down at the same rate, $\dot{\Omega}_c(t)$. The stellar core is thought to couple to the crust on time-scales of less than a minute (Alpar et al. 1984c; Abney, Epstein & Olinto 1996). We therefore take the effective crust to include the mass of the core plus the crust, and the superfluid to exist between neutron drip ($\rho \approx 4.3 \times 10^{11} \text{ g cm}^{-3}$) and nuclear density.

We assume a geometry in which the angular velocity of the superfluid and crust are aligned with the external torque, as this is the state of lowest rotational energy for given angular momentum. The rotation rate of the inner crust superfluid is determined by the arrangement of the vortex lines which thread it. The equation of motion for the superfluid is (see, e.g., Alpar et al. 1981 and Link et al. 1993)

$$\dot{\Omega}_s(r, t) = -v_{cr} \left( \frac{2}{r_p} + \frac{\partial}{\partial r} \right) \Omega_s(r, t),$$

(2)

where $r_p$ is the distance from the rotation axis, and $v_{cr}$ is the radial component of the average vortex velocity. If vortex pinning is relatively effective, as we assume, vortices can move slowly with respect to the rotation axis through thermal excitations or quantum tunnelling in a process of vortex creep.

The average velocity of vortex creep is determined by the vortex–nucleus interaction, the vortex core structure, the characteristic energy of excitations on a pinned vortex, and the velocity difference between a pinned vortex and the superfluid flowing past it. Link & Epstein (1991) and Link et al. (1993) account for quantum effects and the vortex self-energy, and obtain a creep velocity of the general form (see equation [6.9] in Link et al. 1993),

$$v_{cr} = v_0 \exp(-E_c/T_{\text{eff}}),$$

(3)

where $E_c$ is the activation energy for a pinned vortex segment to unpin. The effective temperature is $T_{\text{eff}} = T_q \coth \frac{T}{T_q}$, where $T$ is the thermodynamic temperature, and $T_q$ is the crossover temperature which determines the transition from vortex motion through thermal activation to that by quantum tunnelling (Link et al. 1993). The crossover temperature depends on the ground-state excitation energy of a pinned vortex. In our simulations the stellar temperatures are much greater than the crossover temperature, so that thermal activation (classical creep) is the dominant creep mechanism. In this limit $T_{\text{eff}}$ reduces to the temperature $T$. The multiplicative factor $v_0$ is comparable to the radial component of the velocity of an unpinned vortex line; we take its value to be $10^6 \text{ cm s}^{-1}$ (Link & Epstein 1991; Epstein & Baym 1992; Link et al. 1993).

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The mechanics and energetics of unpinning are affected by the vortex self-energy, or tension, $\hat{T}$. For a vortex line with a sinusoidal perturbation of wavenumber $k$, the tension takes the form (Fetter 1967; see Appendix A in Link & Epstein 1991)

$$\hat{T} = \frac{\rho \kappa^2}{4\pi} \Lambda_{\varphi},$$

(4)

where $\Lambda = (0.116 - \ln k \xi)$, and $\xi$ is the vortex coherence length. The circulation associated with each vortex line is $\kappa = h/2m_n$, where $m_n$ is the mass of a neutron and $h$ is Planck’s constant. Typically, $2 \leq \Lambda \leq 10$ in the inner crust (Link & Epstein 1991). The relative importance of tension is determined by the value of the stiffness parameter $\tau = \hat{T}_{\tau}/\hat{T}_{\varphi}$, where $\hat{T}_{\varphi}$ is the maximum attractive force between a nucleus and a vortex, $r_o$ is the range of the pinning potential, and $I$ is the internuclear spacing (Link & Epstein 1991). We take $F_{\varphi} = U_{\text{cr}}/r_o$, where $U_{\text{cr}}$ is the effective pinning energy per nucleus. In terms of fiducial values, the stiffness $\tau$ is

$$\tau = 100 \left( \frac{\rho}{10^{19} \text{ g cm}^{-3}} \right) \left( \frac{U_{\text{cr}}}{100 \text{ keV}} \right)^{-1} \left( \frac{r_o}{10 \text{ fm}} \right)^2 \left( \frac{I}{50 \text{ fm}} \right)^{-1}.$$

(5)

Let the angular velocity lag between the superfluid and the crust be $\omega = \Omega_c - \Omega_s$. The critical angular velocity difference above which the Magnus force prevents vortex pinning is $\omega_c = F_{\varphi}/r_np$. The number of pinning sites involved in an unpinning event is determined by the value of $\tau$. When $\tau > 1$, many pinning bonds must be broken for a vortex segment to unpin. Exact expressions for the activation energy in this limit can be found in Link & Epstein (1991). In the limiting cases of $\omega \ll \omega_c$ and $\omega = \omega_c$, $E_c$ is (equations [B.12] and [5.1] in Link & Epstein 1991)

$$E_c = \left\{ \begin{array}{ll} 2.2U_{\text{cr}} \sqrt{\frac{\omega}{\omega_c}} & \omega \ll \omega_c \\ 5.1U_{\text{cr}} \sqrt{1 - \omega_c \sqrt{\omega}} & \omega = \omega_c \end{array} \right.$$

(6)

In our simulations, we use the exact expressions for $E_c$ given in Link & Epstein (1991).

2.2 Thermal dynamics

Changes in the local temperature affect the vortex creep rate and hence the rotation rate of the star. A temperature enhancement generates a thermal wave which propagates through the star according to the thermal diffusion equation

$$c_v \frac{\partial T}{\partial t} = \nabla \cdot (\kappa T \nabla T) + h_{\text{friction}} - \Lambda_{\varphi},$$

(7)

Here $c_v$ is the specific heat, $\kappa$ is the thermal conductivity, $h_{\text{friction}}$ is the heating rate per unit volume from friction between the superfluid crust, and $\Lambda_{\varphi}$ is the cooling rate through neutrino emission. The quantities $c_v$, $\kappa$, and $\Lambda_{\varphi}$ are functions of density and temperature. The integrated heating rate due to superfluid friction (Alpar et al. 1984a; Shibazaki & Lamb 1989; Umeda et al. 1993; Van Riper et al. 1995) is

$$H_{\text{friction}} = \int \text{d}l_s \omega |\dot{\Omega}_s|.$$

(8)

Relevant cooling mechanisms include neutrino cooling via the modified URCA process, neutron–neutron and neutron–proton bremsstrahlung in the core (Friman & Maxwell 1979), and electron
bremsstrahlung (Itoh et al. 1984) in the crust. The neutrino cooling rates are

\[ \Lambda_{\nu}^{URCA} = 1.8 \times 10^{13} m_n^{3 \times 3} \left( \frac{\rho}{\rho_0} \right)^{2/3} T_9^{3/2} \text{ erg cm}^{-3} \text{ s}^{-1} \]  

\[ \Lambda_{\nu}^{\alpha} = 4.4 \times 10^{10} m_n^{8/4} \left( \frac{\rho}{\rho_0} \right)^{1/3} T_9 \text{ erg cm}^{-3} \text{ s}^{-1} \]  

\[ \Lambda_{\nu}^{p} = 5.0 \times 10^{11} m_n^{2/2} \left( \frac{\rho}{\rho_0} \right)^{2/3} T_9 \text{ erg cm}^{-3} \text{ s}^{-1} \]  

\[ \Lambda_{\nu}^{\text{brems}} = 1.6 \times 10^{20} \left( \frac{Z^2}{A} \right) \left( \frac{\rho}{\rho_0} \right) T_9 \text{ erg cm}^{-3} \text{ s}^{-1}, \]  

where \( m_n^* \) is the ratio of the effective mass to the bare mass of the neutron, and similarly for the proton. We take \( m_n^* = m_p^* = 0.8 \) in our calculations (Bäckman, Källman & Sjöberg 1973). Our values for \( A \) (the ion mass number) and \( Z \) (the ion proton number) are from Lattimer et al. (1985), and \( T_9 \) is the internal temperature in units of \( 10^9 \) K.

We take the surface of the star to cool through blackbody radiation,

\[ L_{bb} = 4 \pi \sigma R_\odot T_\odot^4, \]  

where \( R_\odot \) and \( T_\odot \) are the radius and surface temperature seen by a distant observer. These quantities are related to their values at the surface through the redshift \( e^{-\Phi} = (1 - 2 GM/Rc^2)^{-1/2} \) as

\[ T_{s,\infty} = e^{\Phi} T_s, \quad R_{s,\infty} = e^{-\Phi} R. \]  

The specific heat of the star is due predominantly to degenerate electrons (Glen & Sutherland 1980) with significant contributions from the ions at lower densities (Van Riper 1991; see Chong & Cheng 1994 for corrections). The thermal conductivity is a function of density and temperature. We use the results of Itoh et al. (1983, 1984) and Mitake, Ichimaru & Itoh (1984) for a pure crust.

Impurities may arise in the crust as a result of the cooling history of the star. Early in the star’s thermal evolution (\( T = 10^{10} \) K), lattice crystallization is expected to occur more quickly than beta equilibration processes (Flowers & Ruderman 1977; Jones 1999). Consequently, nuclei with different nuclear charge (impurities) from the dominant nuclei are likely to be formed. The concentration of impurities lowers the mean-free-path of the electrons, reducing the thermal conductivity in the crust. The electron–impurity thermal conductivity \( \kappa_{TQ} \) is (see, e.g., Ziman 1972, equation 7.92)

\[ \kappa_{TQ} = \frac{\pi^2 n_e k_B^2 T}{3m_e^*} \tau_{Q}, \]  

where \( n_e \) is the density of electrons, \( m_e^* \) is the effective mass of the electron, \( k_B \) is the Boltzmann constant, and \( \tau_{Q} \) is the electron–impurity relaxation time. For a high concentration of impurities, the impurity relaxation time can be approximated by the electron–ion relaxation time (Yakovlev & Urpin 1980):

\[ \tau_{ci} = \frac{P_V^2 \gamma_{PE}}{4 \pi Z^2 e^4 n_e} \Lambda_{ci}^{-1}, \]  

where \( P_V \) and \( \gamma_{PE} \) are the momentum and velocity of an electron at the Fermi surface, and \( \Lambda_{ci} = \inf[2(2\pi Z^3)^{1/2}(1.5 + 3F_2^{1/2})]^{-1} \). The ion density is \( n_{ei} \), and \( T \) is the lattice order parameter. To obtain a lower limit on the thermal conductivity for an impure crust, we calculate the conductivity due to electron–ion scattering, treating the ions as if they were liquefied (see discussion in Brown 2000).

We obtain the liquid-state thermal conductivity numerically, using the results of Itoh et al. (1983) and Mitake et al. (1984). We also use the liquid-state neutrino emissivity (Haensel, Kaminker & Yakovlev 1996) in this case.

3 MODELS

We consider the transfer of angular momentum from the superfluid to the crust through two mechanisms: (1) a deposition of thermal energy which liberates pinned vortex lines from their pinning barriers (a thermal glitch), and (2) mechanical motion of vortices (a mechanical glitch). The first case would arise from the heat deposition associated with a starquake. The second case applies if crust cracking occurs through direct vortex forces as a result of strong pinning (Ruderman 1991), or through magnetic stresses arising from the forcing of the field through the crust by core vortices (Ruderman et al. 1998). Mechanism (2) would also describe a catastrophic unpinning event (Cheng et al. 1988; Alpar & Pines 1993; Mochizuki & Iyusaya 1995). In both models, the vortices are allowed to creep as described in Section 2.1. We initiate vortex motion in both models at a density of \( 1.5 \times 10^{14} \text{ g cm}^{-3} \), representative of the densest regions of the inner crust, where the contribution to the moment of inertia of pinned superfluid is largest. Since this density is near nuclear density, where the solid is expected to dissolve, our treatment also models core-driven glitches when the vortex motion occurs near the crust–core boundary (Ruderman et al. 1998).

Sudden vortex motion generates heat. For a glitch which conserves angular momentum, the heating associated with the vortex motion \( E_{\text{motion}} \) is determined by the change in rotational energy. Angular momentum conservation gives

\[ I_c \Delta \Omega_c + \int dl_i \Delta \Omega_i = 0. \]  

The heat liberated is

\[ E_{\text{motion}} = \Delta \left( \frac{1}{2} I_s \Omega_s^2 + \frac{1}{2} \int dl_i \omega_i^2 \right) = \int dl_i \omega_i(r) \Delta \Omega_i(r). \]  

Here \( \omega_i(r) \) is the angular velocity lag before the glitch, and \( \Delta \Omega_i(r) \) is the change in the superfluid angular velocity due to vortex motion.

If vortices unpin in a mechanical glitch, vortex drag will limit the spin-up time-scale to \( \sim 100 \) rotation periods (Epstein & Baym 1992) or \( \sim 1 \) min in the slowest rotating pulsar that we consider. If vortex motion occurs through motion of crust material with little unpinning of vortices, the spin-up time-scale is approximately the distance that the vortices move divided by the transverse sound speed in the crust, \( t \sim \Delta r/c_s = 10^{-4} \text{ s} \), a small fraction of a rotation period. Since these time-scales are shorter than any time-resolved glitch observations, we approximate the spin-up in a mechanical glitch as an infinitely fast transfer of angular momentum from the superfluid to the crust. This produces a step-like initial increase in the spin rate of the crust.

3.1 Geometry

A complete treatment of the thermal and rotational dynamics described above would require a multidimensional analysis. The energy deposition due to a starquake could occur in a localized region (Link, Franco & Epstein 1998; Franco, Link & Epstein 2000); the subsequent thermal diffusion probably lacks any simple
symmetry, making the vortex dynamics complicated. Moreover, the heat dissipated by a moving vortex depends on position along the line, further complicating the dynamics. These effects make the coupled thermal and rotational dynamics a difficult two- or three-dimensional problem. We adopt a one-dimensional (radial) treatment as a first step in modelling thermal and rotational changes, as we now describe.

The vortices of the rotating superfluid would align themselves with the rotation axis of the solid if there were no pinning. The number of vortex lines present in the superfluid is

$$N = \frac{2\pi R^2 \Omega_c}{\kappa} = \frac{10^{16}}{P}.$$  

where $P$ is the spin period in seconds. A bundle of $\Delta N$ vortex lines has $\Delta N$ times the circulation, and therefore $(\Delta N)^2$ times the tension of a single line (equation 4), and effectively resists bending. Consequently, a bundle of vortices tends to remain straight and aligned with the rotation axis, even when forces vary along the bundle. We therefore treat the vortex array as infinitely stiff over the dimensions of the crust, and average the vortex creep velocity along a vortex. In this approximation, the vortices are always aligned with the rotation axis of the crust. We take the vortex distribution to be axisymmetric, and follow changes in the superfluid rotation rate as a function of the distance from the rotation axis.

We follow thermal changes in the star by solving the thermal diffusion equation (equation 7) with spherical symmetry. Though a crude approximation, this treatment of the thermal evolution captures the essence of the dynamics, while conserving energy. We account for the frictional heat generation, which has axisymmetry, in the following way. The heat generated (equation 8, supplemented by equation 18 for a mechanical glitch) is integrated over each cylindrical shell in the inner crust. The total heat liberated is then divided by the volume of the inner crust, and included as a source term in equation (7) in the spherical treatment of the thermal diffusion. Because the heat is distributed with spherical symmetry throughout the crust, this approach underestimates the temperature increase in some parts of the star. We begin the stellar core at a density of $10^5 \, \text{g cm}^{-3}$, and average the vortex creep velocity along a vortex. In this approximation, the vortices are always aligned with the rotation axis of the crust. We take the vortex distribution to be axisymmetric, and follow changes in the superfluid rotation rate as a function of the distance from the rotation axis.

As a result, the external torque temporarily acts on a smaller moment of inertia, and the star spins down at a greater rate than before the glitch. In older (presumably colder) stars, which have a lower specific heat, the heat deposition causes a larger increase in temperature, which generates a faster glitch than the same energy deposition would in a younger star. Our results depend on the location of the energy deposition. Less energy is required when the deposition is at lower density to produce a similar spin rate.

The thermal glitches we examine require typical energy depositions of $\Delta E = 10^{42} \, \text{erg}$ (see Table 2). Most of the energy is lost to neutrinos, with approximately 0.1 per cent being radiated.

### Table 1. Physical parameters for the four pulsars studied in our glitch simulations.

| Pulsar | $t_{\text{age}}$ | $\Omega_c$ | $|\Omega_c|$ | $T_{\text{int}}$ | $T_{\text{crust}}$ |
|--------|-----------------|-----------|--------------|-----------------|-------------------|
| 0531+21 | $1.2 \times 10^3$ | 189 | $2.4 \times 10^{-9}$ | $8.6 \times 10^7$ | $7.9 \times 10^7$ |
| 0833−45 | $1.1 \times 10^4$ | 70.4 | $1.0 \times 10^{-10}$ | $7.3 \times 10^7$ | $7.2 \times 10^7$ |
| 1822−09 | $2.3 \times 10^5$ | 8.2 | $5.6 \times 10^{-13}$ | $7.2 \times 10^7$ | $7.1 \times 10^7$ |
| 0355+54 | $1.2 \times 10^6$ | 40.2 | $1.1 \times 10^{-12}$ | $6.8 \times 10^7$ | $6.9 \times 10^7$ |

ultimately as photons. The typical increase in surface temperature is \( \sim 2-10 \) per cent, with a corresponding increase in luminosity of \( \sim 8-50 \) per cent.

We initiate a mechanical glitch with an axisymmetric impulsive change in the superfluid angular velocity from its steady-state value centred on a density of \( 1.5 \times 10^{14} \) g cm\(^{-3}\). The shell has a Gaussian distribution in radius, with a full width of 40 m. Sudden movement of the superfluid vortices generates heat, as described in Section 3 (see equation 18); we deposit the heat suddenly in the region in which the vortices move, as described in Section 3.1. After the initial angular velocity change, we solve for the thermal and rotational responses of the star. A mechanical glitch occurs as a step corresponding to the repositioning of vortex lines with respect to their steady-state locations. Following the initial spin jump, the superfluid relaxes and eventually the lag is recovered. The thermal pulse associated with a mechanical glitch is orders of magnitude smaller than that resulting from a thermal glitch. This difference occurs because the heat deposition required to mobilize pinned vortices in a thermal glitch is much larger than the heat generated by vortex motion in a mechanical glitch. A larger change in the superfluid angular velocity is required to produce the same spin response if the vortex motion occurs at lower density, holding all other parameters constant. If the change in superfluid velocity occurs through a wider shell, a smaller peak in the Gaussian profile is needed to give the same spin response.

The parameters which best fit the data, along with the \( \chi^2/\text{dof} \), are listed in Tables 2 and 3 for the thermal and mechanical glitch models, respectively. For comparison, the \( \chi^2/\text{dof} \) of the steady-state observations is given (pre-glitch \( \chi^2/\text{dof} \)) as a measure of the inherent scatter in the data. We report values for \( U_0, r_0, \Delta E \) and \( \bar{\omega} \), where \( \Delta E \) is the energy deposited in a thermal glitch, and \( \bar{\omega} \) is the angular velocity difference between the superfluid and the crust, averaged over the superfluid moment of inertia.

### 4.1 PSR 0531+21 (Crab pulsar)

Simulations of the 1988 glitch in the Crab pulsar (Lyne, Smith & Pritchard 1992) are shown in Fig. 1. The glitch is best modelled by a thermal glitch with an energy deposition of \( 1.5 \times 10^{52} \) erg (solid line). The thermal pulse triggers transfer of angular momentum to the crust over a time-scale of minutes. As the thermal pulse dissipates, the remaining spin-up occurs over approximately one day. The glitch has a fractional increase in rotation rate of \( \Delta n/\nu = 7 \times 10^{-5} \). The mechanical glitch model is unable to simulate the gradual rise of the Crab observations. The quick spin-up is followed by slow decay of the spin increase (Fig. 1, dashed line). The thermal pulse associated with the thermal glitch peaks at the crust over a time-scale of minutes. As the thermal pulse dissipates, the remaining spin-up occurs over approximately one day. The glitch has a fractional increase in rotation rate of \( \Delta n/\nu = 7 \times 10^{-5} \). The mechanical glitch model is unable to simulate the gradual rise of the Crab observations. The quick spin-up is followed by slow decay of the spin increase (Fig. 1, dashed line). The thermal pulse associated with the thermal glitch peaks at
the stellar surface ~20 days after glitch onset and shows a maximum temperature increase of ~0.2 per cent (Fig. 2, solid line). The surface temperature increase associated with the mechanical glitch is of negligible magnitude.

4.2 PSR 0833−45 (The Vela pulsar)

The 1989 ‘Christmas’ glitch (\(\Delta\nu/\nu = 2 \times 10^{-6}\)) in the Vela pulsar (McCulloch et al. 1990) is well modelled as either a thermal or a mechanical glitch (Fig. 3); the simulations are virtually indistinguishable. However, the two models are also indistinguishable in the 2000 January glitch (Dodson, McCulloch & Costa 2000) in the Vela pulsar (Fig. 4). The thermal pulse which reaches the stellar surface differs significantly between the two mechanisms, with a thermal glitch generating a pulse approximately two orders of magnitude larger than that resulting from the mechanical glitch. In Figs 5 and 6 we show the surface temperature increase following a thermal and mechanical glitch for the 2000 January glitch in Vela. Chandra observations of thermal emission from the surface following the glitch in the Vela pulsar on 2000 January 16 limit the temperature difference to <0.2 per cent 35 days after the glitch (Helfand, Gotthelf & Halpern 2001; Pavlov et al. 2000), and <0.7 per cent 361 days after the glitch (Pavlov, private communication). These upper limits are marked in Fig. 6. Lattice impurities in the stellar crust would delay the arrival of the thermal wave at the stellar surface. The low thermal conductivity of an impure crust also prevents the thermal wave from entering the core, resulting in a larger increase in the surface temperature. We find that for a highly impure crust, the thermal pulse from a thermal glitch peaks at the surface ~16 years after the glitch, having a magnitude of ~10 per cent (Fig. 5). Lowering the thermal conductivity in this manner does not affect the spin-up, because vortex motion occurs over a relatively short time-scale during the...
initial energy deposition. The upper limits marked in Fig. 6 are inconsistent with a thermal glitch in a pure crust, but are consistent with a highly impure crust. Our simulation of an impure crust gives an upper limit on the effects that impurities could have. A smaller impurity fraction would decrease the time at which the thermal pulse peaks at the surface.

4.3 PSR 1822−09

Shabanova (1998) observed an extremely small glitch, with a fractional change in spin rate of only $2.0 \times 10^{-10}$, in PSR 1822−09 in September of 1994. The thermal glitch simulation in Fig. 7 (solid line) turns over slowly as the thermal wave propagates through the star, consistent with the findings of Link & Epstein (1996). Although the mechanical glitch appears to the eye to be a better fit (Fig. 7, dashed line), both models are statistically consistent with the observations, having a $\chi^2$/dof within the scatter of the data. The thermal pulse peaks at the surface of the star approximately 16 days after the onset of a thermal glitch (Fig. 8), and has a maximum value of only 0.075 per cent. There is a negligible increase in the surface temperature following a mechanical glitch.

4.4 PSR 0355+54

The largest glitch observed in any pulsar occurred in PSR 0355+54 in 1986 March with a glitch magnitude of $\Delta n / n = 4.4 \times 10^{-6}$ (Shemar & Lyne 1996). The glitch is well fitted by either glitch model; the simulations are indistinguishable in Fig. 9. A thermal glitch generates a thermal pulse at the surface, which peaks $\sim 17$ days after the glitch with a fractional temperature increase of $\sim 2$ per cent (Fig. 10, solid line). A mechanical glitch generates a thermal pulse which peaks at a magnitude of $\sim 0.02$ per cent, about 13 days after the initial spin up (Fig. 10, dashed line).

5 DISCUSSION

This work has considered two different glitch scenarios involving
the movement of superfluid vortex lines near nuclear density. The thermal glitch model of Link & Epstein (1996) is consistent with all glitches modelled in this study. The mechanical glitch model produces a spin jump that is too quick to explain the 1988 glitch in the Crab pulsar, but is consistent with glitch data in the other pulsars in our sample. However, plastic flow of the crust following a mechanical glitch, an effect we have not accounted for, might provide a slow component to the spin-up in this younger, and presumably hotter star.

At present, timing measurements of glitching pulsars are too sparse to distinguish between the predicted spin behaviour of thermal and mechanical glitches in most cases. However, a distinguishing feature between these models is the size of the surface temperature pulse which accompanies the glitch. The larger temperature increase associated with a thermal glitch lowers the thermal conductivity, slowing the pulse as it travels through the crust as compared to a mechanical glitch. In general, a thermal glitch in a pure crust exhibits a thermal pulse at the stellar surface which is much larger, and occurs a few days later, than a mechanical glitch. Impurities in the stellar crust lower the thermal conductivity and could delay the surface pulse by years. A thermal glitch in an impure crust is consistent with surface emission limits following the 2000 January glitch in the Vela pulsar (Helfand et al. 2001; Pavlov et al. 2000). However, the temperature simulation curve for an impure crust is still rising at the time of the latest upper limit (see Fig. 6), and refinement of this upper limit, as well as additional measurements, may conflict with a thermal glitch in an impure crust as well. Future glitch observations coordinated with surface emission measurements will play a key role in distinguishing between these two models. Further work is needed to fully understand the effects of crust impurities on the emerging thermal wave.

Other time-resolved observations of slow spin-ups in young pulsars, similar to the Crab glitch of 1989, would support the thermal glitch model. A mechanical glitch does not occur slowly enough to explain such behaviour, but is a viable model for glitches which spin up the star quickly. However, detection of an early thermal wave of large amplitude could rule out the mechanical glitch model in a pure crust. It is possible that both thermal and mechanical mechanisms are at work in generating pulsar glitches.

For both models we obtain effective pinning energies $U_0$ in the range $\sim 10$–$500$ keV. These values are less than calculated pinning energies per nucleus (Pizzochero et al. 1997), and may indicate the presence of vortex tension effects which act to reduce the effective pinning energy (Jones 1997, 1998, 1999; Link & Cutler 2002). The relatively low values of $U_0$ we find for the Crab pulsar could indicate that glitches originate in a different pinning region for this object than in other pulsars. The values we obtain for the range of the pinning potential $r_p$ are consistent with existing estimates (Pizzochero et al. 1997; Epstein & Baym 1988). Larson & Link (1999) obtained constraints on the average angular velocity difference $\dot{\omega}$, assuming that superfluid friction accounts for the unexpectedly high surface temperatures of some older pulsars. That work also provided upper limits on $\dot{\omega}$ in younger pulsars, by requiring that young stars be stable against a thermal-rotational instability. For stars of the same age, our values of $\dot{\omega}$ are in agreement with those results.

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REFERENCES

Anderson P. W., Itoh N., 1975, Nat, 256, 25
Dodson R. G., McCulloch P. M., Costa M. E., 2000, IAU Circ. 7347
Greenstein G., 1979b, Nat, 277, 521
Yakovlev D. G., Urpin V. A., 1980, SvA, 24, 303

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