January 1972

The Hydraulics of Waste Stabilization Ponds

Gary Z. Watters

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THE HYDRAULICS OF WASTE STABILIZATION PONDS

Part I. The Effect of Hydraulic Flow Characteristics on Treatment Efficiency

Part II. The Effect of Wind on Mixing in Stratified and Unstratified Ponds

by

Gary Z. Watters

Utah Water Research Laboratory
College of Engineering
Utah State University
Logan, Utah 84321

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Final Report to the
Office of Water Resources Research
on Project A-008-Utah

Utah Water Research Laboratory
College of Engineering
Utah State University
Logan, Utah 84321

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PART I. THE EFFECTS OF HYDRAULIC FLOW CHARACTERISTICS ON TREATMENT EFFICIENCY
ACKNOWLEDGMENTS

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# LIST OF SYMBOLS

**Part I**

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<tr>
<td>$a$</td>
<td>thermal diffusivity</td>
</tr>
<tr>
<td>$A_m$</td>
<td>finite stage model constants</td>
</tr>
<tr>
<td>$B_m$</td>
<td>finite stage model constants</td>
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<tr>
<td>$C(\theta)$</td>
<td>C-diagram</td>
</tr>
<tr>
<td>$c$</td>
<td>concentration of tracer as a function of $\theta$ (mg/V)</td>
</tr>
<tr>
<td>$c_o$</td>
<td>mass of tracer divided by volume of pond (mg/V)</td>
</tr>
<tr>
<td>$(c/c_o)_{pk}$</td>
<td>dimensionless peak concentration</td>
</tr>
<tr>
<td>$D$</td>
<td>axial dispersion coefficient (ft$^2$/hr)</td>
</tr>
<tr>
<td>$D_{mass}$</td>
<td>mass diffusion coefficient (ft$^2$/hr)</td>
</tr>
<tr>
<td>$d$</td>
<td>diffusivity coefficient (dimensionless)</td>
</tr>
<tr>
<td>$d_o$</td>
<td>depth of pond</td>
</tr>
<tr>
<td>$E_h$</td>
<td>eddy diffusivity of heat transfer</td>
</tr>
<tr>
<td>$E_m$</td>
<td>eddy diffusivity of momentum</td>
</tr>
<tr>
<td>$F(\theta)$</td>
<td>F-diagram</td>
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<tr>
<td>$F_a$</td>
<td>perfectly mixed flow area as a fraction of total system volume</td>
</tr>
<tr>
<td>$F_b$</td>
<td>fraction of total system volume as dead flow region</td>
</tr>
<tr>
<td>$F_c$</td>
<td>fraction of total system volume as plug flow (delay time)</td>
</tr>
<tr>
<td>$F_e$</td>
<td>Froude number</td>
</tr>
<tr>
<td>$F_{\Delta}$</td>
<td>densimetric Froude number</td>
</tr>
<tr>
<td>$F_{\Delta e}$</td>
<td>densimetric Froude-Reynolds number</td>
</tr>
<tr>
<td>$f_a$</td>
<td>finite stage model constant</td>
</tr>
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\( g \) = gravity (ft/sec\(^2\))

\( K \) = first order reaction coefficient (day\(^{-1}\))

\( K_h \) = multiple of main flow interchange between live and dead flow regions

\( L \) = pond length

\( \ell \) = characteristic length

\( M \) = mass of tracer as a function of time (mg)

\( M_t \) = initial mass of tracer (mg)

\( N \) = waste material concentration as a function of time (mg/l)

\( N_0 \) = initial concentration of waste material (mg/l)

\( \overline{N}_\infty \) = waste material concentration accounting for all pollutant elements (mg/l)

\( \overline{N}_2 \) = waste material concentration at two theoretical detention times (mg/l)

\( n \) = number of basic modules in series

\( P_r \) = Prandtl number

\( Q \) = flowrate

\( Q_f \) = fictitious flowrate

\( R_e \) = Reynolds number

\( R_i \) = Richardson number

\( r_1 \) = finite stage model constant

\( r_2 \) = finite stage model constant

\( S_c \) = Schmidt number

\( t \) = time

\( \overline{t} \) = theoretical detention time

\( \overline{t}_c \) = experimental detention time
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>U</td>
<td>average fluid velocity</td>
</tr>
<tr>
<td>( U_0 )</td>
<td>reference fluid velocity</td>
</tr>
<tr>
<td>V</td>
<td>pond volume</td>
</tr>
<tr>
<td>( V_f )</td>
<td>fictitious pond volume</td>
</tr>
<tr>
<td>( V_d )</td>
<td>dead space volume</td>
</tr>
<tr>
<td>( \overline{V}_d )</td>
<td>dead space parameter</td>
</tr>
<tr>
<td>W</td>
<td>pond width</td>
</tr>
<tr>
<td>X</td>
<td>independent variable—finite stage model</td>
</tr>
<tr>
<td>x</td>
<td>horizontal coordinate</td>
</tr>
<tr>
<td>Y</td>
<td>dependent variable—finite stage model</td>
</tr>
<tr>
<td>z</td>
<td>vertical coordinate</td>
</tr>
<tr>
<td>( \theta )</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>( \theta_b )</td>
<td>dimensionless time when tracer first appears at outlet</td>
</tr>
<tr>
<td>( \theta_c )</td>
<td>dimensionless experimental mean</td>
</tr>
<tr>
<td>( \theta_{pk} )</td>
<td>dimensionless time to peak concentration of tracer</td>
</tr>
<tr>
<td>( \overline{\theta}_{pf} )</td>
<td>plug flow deviation parameter</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density</td>
</tr>
<tr>
<td>( \rho_o )</td>
<td>reference density</td>
</tr>
<tr>
<td>( \nu )</td>
<td>kinematic viscosity or diffusivity of momentum</td>
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**Part II**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>c</td>
<td>concentration of substance (wt./unit volume)</td>
</tr>
<tr>
<td>( C, C_d )</td>
<td>drag coefficient on free surface</td>
</tr>
<tr>
<td>d</td>
<td>pipe diameter (ft)</td>
</tr>
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</table>
D = diffusion coefficient (ft$^2$/sec)

D' = apparent diffusion coefficient (ft$^2$/sec)

D* = dimensionless diffusion coefficient

f = Darcy-Weisbach friction factor

g = gravitational constant (ft/sec$^2$)

H = depth of water in channel (ft or inches)

Ha = pressure head of air pressure in tunnel (ft)

H_i = depth of lower layer in density-stratified case (ft or inches)

H' = total head at bottom of channel -- H + Ha (ft)

p, P = pressure in wind tunnel or in water (lb/ft$^2$)

Rc = critical Reynolds number for complete mixing

Rh = hydraulic radius (ft)

s = concentration of substance (wt./unit volume)

S = slope of water surface

t = time (sec)

u, U = velocity of water or air (ft/sec)

um, ua = mean velocity of air flow in tunnel (ft/sec)

ur = reference velocity for air flow near inlet (ft/sec)

u* = shear velocity -- $\sqrt{\frac{\tau_s}{\rho}}$ (ft/sec)

x = distance along the channel (ft)

$\gamma$ = specific weight of water (lb/ft$^3$)

$\theta$ = critical mixing parameter of Keulegan

$\nu$ = kinematic viscosity of water (ft$^2$/sec)

$\rho$ = density of water mixture (slug/ft$^3$)
\( \tau_i \) = shear stress at interface between density layers (lb/ft\(^2\))

\( \tau_o \) = shear stress at bottom of channel (lb/ft\(^2\))

\( \tau_s \) = shear stress on surface of water (lb/ft\(^2\))
ABSTRACT

The treatment efficiency of waste stabilization ponds depends primarily on the biological factors of type of waste and organic loading. However, the biological activity in a pond is greatly influenced by the environmental conditions of temperature, wind, sunlight, and the hydraulic flow patterns. In the past little attention has been given to the hydraulic characteristics of waste stabilization ponds such as the gross flow patterns within stabilization ponds as affected by the shape of the pond or lagoon, the presence of dead spaces, the positioning of inlets and outlets and the degree of density stratification. These hydraulic flow characteristics will have an effect on the dispersion and the average detention time of the waste and on the organic (BOD) and pathogenic organism removal efficiency of the treatment process.

This research evaluated the effects of these hydraulic flow characteristics on the treatment efficiency by using certain information that can be obtained from the age distribution function of the fluid particles within a continuous flow process vessel. The age distribution function represents a history of the time of retention of the various fluid particles in the vessel and is generated by injecting a tracer into the process vessel and monitoring the outlet from the vessel. The concentration vs. time curves at the outlets, which lead to the age distribution functions for a waste or a tracer, were made dimensionless to aid in the evaluation of each experiment.
The prototype experimental data taken to establish existing flow patterns were obtained on the waste stabilization ponds of the city of Logan, Utah. Fluorometric techniques using rhodamine WT dye were used to trace the pollutant. A hydraulic model of the ponds 20 feet by 40 feet by 3 feet deep was constructed at the Utah Water Research Laboratory. This model, after verification, was used to generate data on the effects of inlet and outlet types and location, density stratification, length to width ratio, and baffling on the hydraulic flow characteristics.

The information gained from the tracer concentration vs. time curves was used in conjunction with the first order reaction equation to predict treatment efficiencies for various pond designs for determining optimum conditions.

Finally, a mathematical model of the mixing process is presented and outlet concentration vs. time curves generated by the model are compared with experimental results. This mathematical model can be used in conjunction with the first order reaction equation to predict treatment efficiencies.
INTRODUCTION AND LITERATURE REVIEW

Introduction

A waste stabilization pond can be defined as a shallow man-made basin which utilizes natural processes under partially controlled conditions for the reduction of organic matter and the destruction of pathogenic organisms in waste waters. Generally the design of the pond has been based on past experience utilizing empirical data. In the last ten years there has been an increase in interest in stabilization ponds as a method of waste treatment, basically because of the relatively low cost of construction and operation in comparison with conventional waste treatment processes. Recent design approaches have been related more to basic scientific principles rather than being based entirely on empirical data.

In stabilization ponds, the environment is beyond the control of the designer which adds to the already extreme complexity of pond behavior. To provide ponds which will be as efficient and economical as possible it is necessary to study the physical, biochemical, and environmental factors which influence pond behavior. This research was designed to basically study the physical factors with some reference to the environmental conditions that influence pond behavior.
The treatment efficiency of waste stabilization ponds depends on many factors, the most important of which are generally presumed to be the biological factors such as the type of waste and the organic loading. However, the biological activity in a pond is greatly influenced by the environmental conditions of temperature, wind, sunlight, humidity, and such physical factors as pond geometry, inlets and outlets, and the hydraulic flow patterns. In the past little attention has been given to the hydraulic characteristics of waste stabilization ponds. Specifically, little consideration has been given to the gross flow patterns within stabilization ponds which are affected by the shape of the pond or lagoon, the presence of dead spaces, the existence of density differences, and the positioning of inlets and outlets. These hydraulic flow characteristics will obviously have an effect on the dispersion of the waste as well as the average detention time for the waste and, ultimately, on the organic (BOD) and pathogenic organism removal efficiency of the treatment process.

This study considers the effects of these hydraulic flow characteristics on the treatment efficiency. The approach was to use information obtained from the age distribution functions of the fluid particles within continuous-flow process vessels or tanks. The age distribution functions represent a history of the retention of the various fluid particles in the vessel. Chemical engineers first introduced the general method of describing flow patterns by the use of age distribution
functions generated by injecting a tracer into the process vessel. A stabilization pond can be considered a biological or chemical process vessel. In addition, the biological reactions that occur in waste stabilization ponds have been found to closely follow a first order chemical reaction. Consequently, the age distribution functions can be coupled with the first order reaction equation to give a quantitative measure of treatment efficiency.

The application of a physically significant mathematical model for describing the flow patterns by use of age distribution functions is presented. This model, called the finite stage model, consists of building blocks which are composed of live and dead flow regions. The live flow section consists of plug flow and completely mixed flow regions, while the dead flow region is completely mixed and interchanges fluid slowly with the live flow region. This model was used because of its obvious physical similarity to real flow systems. The mathematical model can be derived from model studies or existing pond studies and then coupled with the first order reaction equation to give the expected treatment efficiency. After mathematical models for the model pond designs have been determined, they can be used to predict pond performance of proposed new designs.

The prototype experimental data on hydraulic circulation were taken in the waste stabilization ponds operated by the city of Logan, Utah. These ponds were also modeled in the Utah Water Research
Laboratory using a 20 foot by 40 foot tank 3 feet deep. The model was verified and then used to generate data on the effect of inlet and outlet placement and baffling on the hydraulic flow characteristics. In addition, the sensitivity of the model to Reynolds's number variation and model scale distortion was measured to provide some information on how precisely modeling laws must be satisfied to achieve good agreement between model and prototype. Additional model experiments were performed to determine the influence on pond performance of density currents in a density stratified flow.

**Literature Review**

**Review of waste stabilization ponds**

**General concept and fundamental principles**

The terms "stabilization pond," "sewage lagoon," "oxidation pond," and others are generally applied to artificially created bodies of water intended to retain waste flows containing degradable organic compounds until biological processes render them stable and hence either unobjectionable from an oxygen-demand viewpoint for discharge into natural waters or removal by percolation and evaporation. The theoretical minimum detention time of these ponds is that sufficient to permit biodegradation of organic matter and die-away of pathogenic bacteria and parasites. The theoretical maximum time is the minimum
time plus that necessary to tie up the stable products of biodegradation in algal cells.

Stabilization ponds are generally classified according to types of inflow or outflow conditions. Some of these types are raw sewage ponds, primary and secondary sewage ponds, overflow and non-overflow ponds and others. They are also classified according to the types of biological processes that occur.

The three types of biological conditions that occur in stabilization ponds are (1) aerobic conditions, (2) anaerobic conditions, and (3) combined aerobic and anaerobic conditions termed facultative.

The biological process depends on the effective use of bacteria for the degradation of organic material and the availability of green algae for oxygen-production. The bacteria break down and use up many complex organic waste materials; the algae, helped by fungi, consume the simpler degradation products.

The relative rate of production and consumption of oxygen by algae and bacteria respectively determines the nature of the pond. Whenever the rate of production of oxygen by algae is greater than the rate of consumption of oxygen by bacteria, the process is termed "aerobic." If the reverse is true, "anaerobic" conditions prevail.

Anaerobic conditions are characterized by the foul odors accompanying this biological process. This is due to the complex products of fermentation, mainly hydrogen sulfide. The gaseous product of aerobic
digestion is carbon dioxide which makes this process relatively odor free.

Most stabilization ponds are actually facultative; that is, they have combined aerobic and anaerobic zones. Aerobic conditions occur in the upper strata of the water body where greater sunlight penetration activates the photosynthesis process to greater completion than at lower depths. Anaerobic conditions occur in the lower depths because of reduced sunlight penetration which results in an oxygen deficiency. Even in shallow ponds, anaerobic conditions often exist because of the settleable organic matter at the bottom which depletes the available oxygen. Gloyna (no date given) says that the design and operation of all waste stabilization ponds must aim at an algal-bacterial balance wherein the amount of waste water discharged into the pond commensurates with the amount of available dissolved oxygen.

The fundamental principles underlying the biological operation of waste stabilization ponds is that their action depends upon the simultaneous and continuous functioning of both the right-hand and left-hand sectors of the aerobic cycle of organic growth and decay (McGauhey, 1968). The conventional system carries out only the degradation process and leaves the growth potential to be exerted in the receiving water. The significance of this principle is that the input to sewage ponds is biodegradable dead organic wastes and the output is living organic matter at a higher energy level. This brings up one very important consideration. The potential biological oxygen demand (BOD) of the effluent may be
greater than that of the influent. The living algal cells, however, are not quickly available for biodegradation because of their hardiness of life. But the underlying factor seems to be that in terms of water quality, the stabilization pond effluent may substitute an aesthetic factor for the quality factors associated with biodegradation unless the algal cells are harvested.

The following summarizes some of the most important factors in the operation of stabilization ponds.

1. Algae--The life cycle of algae influences the biology and chemistry of the aquatic environment greatly. The growth rate of algae is influenced by available light source, temperature, the presence of nutrients, pH and other environmental factors.

2. Light Energy--This factor has a direct influence on the growth of algae.

3. Temperature--This factor also, has a direct influence on algae growth as well as the hydrodynamics of stabilization ponds.

4. Bacterial Nutritional Requirements--For the most efficient reduction of organic wastes through biological oxidation adequate nutrient supplies should be provided for the bacteria. Deficiency in any of the important substances such as phosphorous, nitrogen, and sulfates, and minerals such as potassium and calcium, will result in a serious curtailment of bacterial growth and activity.

5. Algal Nutrients--These are derived from two main sources, (a) the photosynthetic process by which carbon dioxide together with ammonia and other nitrogen containing compounds produced by the
decomposition of organics and released by hydrolysis, and (b) endogenous metabolism by which algae use the products of degradation of other algae (Rich, 1963).

6. Physical Environment--The hydrodynamic and physical shape of the pond along with such parameters as depth, length, width, inlet and outlet devices and location, and the porosity of the soil have a distinct influence on the ecological performance of the pond. Hermann and Gloyna (1958) have shown that the BOD removal efficiency, for given influent loads and detention periods, is a function of pond depth. Thermal microstratification (Stahl and May, 1967) and weed growth (McKinney, 1967) have also been shown to affect BOD removal efficiency. The pond surface area has been shown to be directly related to wind agitation, reaeration, evaporation, percolation, and precipitation (Oswald, 1963).

Design consideration and criteria

The previous approaches to the design of waste stabilization ponds have been primarily empirical with such parameters as depth, retention time, physical shape of pond, and BOD reduction derived from observed practical experience. Advances in biological oxidation, photosynthetic phenomena, and algalogy make the theoretical approach both possible and feasible from the biological standpoint.

The design parameters which must be established for stabilization ponds include detention period, hydraulic loading, depth, recirculation, mixing, pond size and shape, and inlet and outlet systems. Because
there are many different methods for determining these parameters and design methods for the three types of ponds, aerobic, anaerobic, and facultative, a detailed listing of these will not be made here. But those parameters, which are all determined in essentially the same way, will be briefly mentioned.

1. Detention period (theoretical)—the volume of the pond divided by the flowrate into the pond.

2. Hydraulic loading—the depth of the pond divided by the theoretical detention time.

3. Depth.

The determination of the pond depth is somewhat arbitrary depending on the pond type. Most of the other parameters have been determined essentially from past experience with sewage ponds.

There has been at least one attempt to determine these design parameters from an experimental model and incorporate these into a design equation based on chemical reactor design methods (Thirumurthi, 1969). The goal was to try and develop design formulas based on sound scientific and mathematical principles related to chemical engineering unit operations and reactor design concepts.

It is felt by some that the future of waste stabilization ponds as a method of treatment depends on the improvement of design, and it appears that, initially, the problems of hydraulic design must be solved. Present design concepts tend to neglect the shape of the treatment pond, the
existence of dead spaces, short circuiting, density differences, and the inlet and outlet flow patterns. These hydraulic flow characteristics will have an obvious effect on molecular and turbulent diffusion as well as the detention time and, hence, on BOD removal efficiency.

Chemical reactor design

In recent years, some researchers (Oswald, 1963, and Thirumurthi, 1969) have tried to develop a design method based on sound scientific principles. The approach was to develop a method for designing algal waste stabilization ponds based on existing chemical-engineering practice for reactor design. The discussion of these approaches will be made later. First, some of the important chemical engineering terms must be defined.

1. Chemical reactor - A chemical reactor is a vessel in which a chemical reaction takes place. There are three general types of reactors, namely, the batch, the steady flow, and the unsteady flow reactors. A steady flow reactor is one in which the influent and effluent flow rates remain constant with time and as a result it is mathematically easier to work with than the unsteady flow reactor. Also, the steady flow reactor is more realistic than a batch reactor from the viewpoint of a sanitary engineer because the biological processes of stabilization pond operation are continuous rather than batch. Those two reasons point out the desirability of using steady flow reactor principles in the design of waste stabilization ponds.
2. **Types of fluid flow in a vessel** - When a fluid passes through a reactor, tank, or pond, a number of possible patterns of flow could exist depending on the entrance and exit arrangements, short circuiting, flow rate, velocity, volume of tank, and fluid properties. The two general types of flow are: (a) ideal flow (b) non-ideal flow.

   (a) **Ideal flow** - Consists of two types: plug flow (piston, slug, tabular, or non-mix flow), which is characterized by the fact that the flow of fluid through the tank is orderly with no element of fluid overtaking any other element as shown in Figure 1-a. Consequently, there is no velocity gradient and diffusion. The residence or detention time of all fluid elements is the same. **Completely mixed** (total back mix, or stirred tank) flow is characterized by a uniform composition in the tank (Figure 1-b). Any fluid element has an equal chance of being found at the outlet.

   (b) **Non-ideal flow** - An actual process vessel, like an aeration tank or a waste stabilization pond, is obviously far from ideal. In an actual situation stagnant pockets or dead space, short circuiting and dispersion will occur to create non-ideal type flows characterized as channeling, recycling, eddying, etc. Since the flow in a tank or pond is in reality non-ideal, it is obvious that the problem should be approached using non-ideal flow terms.

3. **Stimulus-response methods of characterizing flow** - A tracer or stimulus is applied at the inlet to a tank or vessel and its response is measured as a function of time at the outlet.
Figure 1. Ideal hydraulic flow patterns.
4. **Open and closed vessels** - A closed vessel is one for which the fluid moves in and out by bulk flow alone. Plug flow exists in the entering and leaving streams. An open vessel is one where neither the entering nor the leaving fluid streams satisfy the plug flow requirements of the closed vessel. A waste stabilization pond is considered to be a closed vessel.

5. **Mean residence time of fluid (theoretical)** - The theoretical detention time, $\bar{t}$, is defined as the volume of the vessel divided by the flowrate through the vessel.

6. **Reduced or dimensionless time** - The dimensionless time $\theta$, is defined as time, $t$, divided by the theoretical detention time, $\bar{t}$.

Age distribution functions

Danckwerts (1953) first introduced the general idea of age distribution functions. These functions give information about the fluid that resides a certain time in a closed vessel. This type of treatment does not give information about point-to-point changes of the variables and thus does not yield complete information about the behavior of the fluid in the vessel. But it does give some good information about the general behavior.

Some of the most useful age distribution functions follow:

**F-diagrams.** If the incoming fluid flowing into a vessel is suddenly changed from a white to a red color, then $F(\theta)$ is the fraction of red material that occurs at the outlet as a function of dimensionless time. A
plot of \( F(\theta) \) vs. \( \theta \) is called the F-diagram. \(^1\) Figure 2 shows F-diagrams for some representative types of flow functions.

Since there is always some longitudinal mixing with Newtonian fluids, due to viscous effects and molecular or eddy diffusion, the perfect piston or plug flow (curve A, Figure 2) will never occur. Curve B in Figure 2 shows a representative F-diagram for flow with some longitudinal mixing. Curve C is a diagram for complete mixing given by the equation

\[
F(\theta) = 1 - e^{-\theta} 
\]

Curve D is the F-diagram where there is considerable "dead water." The term "dead water" or "stagnant pockets" means that some fraction of the fluid is trapped in corner eddies, and spends much more than the average length of time in the vessel.

The shape of an F-diagram depends on the relative times taken by various portions of the fluid to flow through the vessel or in other words, on the distribution of particle residence times.

C-diagrams. The same information as that obtained with an F-diagram can be obtained with a C-diagram. If a known amount, \( M_T \), of tracer is instantaneously injected into the inlet, then \( C(\theta) \) is the dimensionless concentration of that tracer at the outlet as a function of dimensionless time. C-diagrams are dimensionless plots similar to F-diagrams where

\(^1\)The F-diagram, in reality, is a dimensionless mass diagram.
Figure 2. The F-diagram.

Figure 3. The C-diagram.
\[ C(\theta) = \frac{c}{c_0}, \quad c \text{ is the tracer concentration as a function of time}, \quad c_0 = \frac{M_T}{V} \]

and \( \theta \) is defined the same as for F-diagrams. Figure 3 shows some typical C-diagrams for the systems whose F-diagrams are given in Figure 2.

The relationship between F-diagram and C-diagrams can easily be shown as:

\[ C = \frac{c}{c_0} = \frac{dF}{d\theta} \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad (2) \]

Equation (3) represents the total fraction of tracer material that has left the vessel at the exit having ages between 0 and \( \theta \).

\[ \int_{0}^{\theta} \frac{c}{c_0} \, d\theta = F(\theta) \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad (3) \]

Also the total area under every C-diagram is equal to unity.

Previous design approaches—biological reactors

If a mass balance is determined for a one-dimensional flow through a chemical reactor with a first order chemical reaction occurring, the following differential equation results:

\[ D \frac{\partial^2 N}{\partial x^2} - U \frac{\partial N}{\partial x} - KN = 0 \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad (4) \]

in which

\[ D = \text{axial dispersion coefficient} \]
\[ K = \text{first order reaction coefficient} \]
\[ N = \text{reactant concentration} \]
\[ U_o = \text{fluid velocity} \]
\[ x = \text{coordinate in direction of flow} \]

This equation has been solved analytically by Wehner and Wilhelm (1958).

For any kind of entrance and exit conditions the solution is:

\[
\frac{\bar{N}_\infty}{N_0} = \frac{4a e^{1/2d}}{(1+a)^2} \frac{e^{a/2d}}{-e^{-(a-a)/2d}} \quad \ldots \ldots (5)
\]

\[
\bar{N}_\infty = \text{effluent concentration of reactant}
\]
\[
N_0 = \text{influent concentration of reactant}
\]
\[
a = 1 + 4K t_c d
\]
\[
d = \text{diffusivity coefficient}
\]
\[
t_c = \text{actual detention time}
\]

The term \( d \) in Equation (5) characterizes the non-ideal flow in a pond

\[
d = \frac{D}{U_o} \ell \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)
\]

in which

\[
D = \text{axial dispersion coefficient}
\]
\[
U_o = \text{fluid velocity}
\]
\[
\ell = \text{a characteristic length or size}
\]

The coefficient, \( d \), varies from zero for plug flow to infinity for completely mixed flow. For non-ideal flow situations \( d \) will assume a value between zero and infinity. This equation is a conversion equation and
when solved represents the performance of a reactor. The equation is based on the concept of a dispersion model which represents the resident time distribution of fluid particles $c(t)$ with $d$ the important variable that characterizes the non-ideal nature of the flow.

Thirumurthi and Nashashibi (1967) have used Equation (5) in their research. They have stated that short-circuiting in tanks, exit and entrance hydraulic devices and other hydraulic mixing characteristics can be represented by the value of $d$. Also, temperature, influent waste qualities, nutrient deficiencies, organic load and other biological factors can be accounted for by the value of $K$. The hydraulic load is represented by the value of the actual (mean) detention time, $\bar{t}_c$.

Thirumurthi and Nashashibi simulated a waste stabilization pond with a glass rectangular tank (8" x 10" x 18"), with fluorescent lights and a synthetic chemical sewage. They determined values of $K$, $d$, and $\bar{t}_c$ from their experiments. There was no direct tie with existing ponds to verify their work. However, they felt, from the results of their research, that the application of chemical engineering reactor design principles fits the complex biological reactions reasonably well.

Murphy and Timpany (1967) and Murphy and Boyko (1970) have also used a similar approach in their research to develop a design procedure and a method of analyzing aeration and spiral-flow aeration tanks.
Modeling of stratified flow

In designing a model to simulate existing conditions in waste stabilization ponds, the following types of situations can occur:

1. Inflow fluid more dense than ambient fluid (summer time flows).
2. Inflow fluid of same density as ambient fluid.
3. Inflow less dense than ambient fluid.

In reviewing past work performed on stratified flows and density currents caused from either temperature gradients or variations in composition (i.e. varying salt concentrations) use has been made of the densimetric Froude number as a modeling criteria in most all cases.

Stefan and Schiebe (1970) in their model studies of heated water flow into impoundments used an inlet Reynolds number and a densimetric Froude number with additional dimensionless heat transfer terms. The densimetric Froude number is defined as

$$ F_\Delta = \frac{U_o}{\sqrt{\Delta \rho / \rho g l}} $$

in which

- $F_\Delta$ = densimetric Froude number
- $U_o$ = reference fluid velocity
- $\Delta \rho / \rho$ = degree of stratification-density difference divided by a reference density
- $g$ = gravity
- $l$ = characteristic length
Barr (1963b) used a densimetric Froude number as a criterion for similarity. He noted that sometimes comparisons of model and prototype behavior are made on the basis of the Richardson number $R_i$, although the Richardson number was originally intended as a criterion for the degree of stability of turbulence in flows of varying stratification. $R_i$ given by Prandtl is:

$$ R_i = \frac{-g}{\rho} \frac{d\rho}{dz} \sqrt{\left( \frac{du}{dz} \right)^2} $$

in which

- $R_i$ = Richardson number
- $u$ = fluid velocity
- $z$ = vertical coordinate

Barr defines $R_i$ for simulation criterion as:

$$ R_i = \frac{-g}{\rho} \frac{\Delta \rho}{\ell} \left( \frac{u}{\ell} \right)^2 = -F_\Delta^{-2} $$

Barr derives the densimetric Froude number in two ways: (1) by expressing the ratios of the inertial and gravitational forces for model and prototype. (2) By energy concepts (kinetic energy available for mixing equals work done against the density gradient).

Barr stated that if the model scale must be distorted, i.e., the height ratio between model and prototype different than the length ratio, the discharge should be increased to maintain turbulent flow and allow the viscous forces to be neglected. If the discharge is increased, the
relative densities should be adjusted to maintain Richardson or densimetric Froude similarity.

This adjustment is:

\[
\left( \frac{\Delta \rho}{\rho} \right)_m = M^2 \left( \frac{\Delta \rho}{\rho} \right)_p
\]

in which the subscript \( m \) refers to the model and \( p \) refers to prototype,

\[ M = \frac{Q_m}{Q_m \text{ (computed)}} \]

and

\[ Q_m = \text{ the actual flowrate in the model} \]
\[ Q_m \text{ (computed)} = \text{ the flowrate from model prototype relationships for the model for Reynolds numbers} \]

Barr also suggested that exaggeration of the horizontal length scale is necessary for correct simulation of the rate of spread of the stratified layer.

In a later paper Barr (1963a) discussed modeling parameters for simulation of salinity intrusion. He combines \( F_\Delta \) and \( R_e \) to give the densimetric Froude-Reynolds number criterion for model simulation.

\[
\overline{F_\Delta R_e} = \left( \frac{\Delta \rho}{\rho} \right) \frac{1}{2} \frac{d_o^{3/2}}{\nu}
\]

In discussing the application of \( \overline{F_\Delta R_e} \) criterion, Barr stated that a model study of internal movements resulting from small density differences would be expected to correspond closely with the prototype.
in all respects, if the prototype $\frac{F_{Re}}{\Delta e}$ number could be obtained in the model, providing that the model boundary was geometrically similar to that of the prototype and two main limitations were met.

These two limitations are:

1. $(\Delta \rho / \rho)_m$ can be considerably greater than $(\Delta \rho / \rho)_p$ only if the ratio of the densities of the two liquids do not differ significantly between model and prototype. This is because of the fixed relation between the gravity and inertia forces.

2. The introduction of a second liquid means that $\nu$ is a characteristic kinematic viscosity and

\[
\frac{\nu_a \nu_b}{\nu_a \nu_b} \quad \text{must equal} \quad \frac{\nu_a \nu_b}{\nu_a \nu_b}
\]

Either $\nu_a$ or $\nu_b$ could be characteristic. Barr also stated that even equality of $\frac{F_{Re}}{\Delta e}$ numbers may not be necessary if the flow in both model and prototype were in the fully developed turbulence region.

Other authors have used similar criterion for modeling density current phenomena. A few of these are:

1. Fietz and Wood (1967) used both local and orifice or inlet Richardson and Reynolds numbers derived from dimensional analysis for their experimental three-dimensional density current study.

2. Wood (1967) used a densimetric Froude or Richardson number in his two-dimensional density current study.

3. In discussing the similarity of flow turbulence, Chen (1965) in a paper on the simulation of flow in cooling reservoirs, stated that the
model should be designed under Richardson's criterion:

\[ R_{i}^{r} = \left( \frac{V}{\sqrt{\Delta \rho / \rho}} \right)_{r} = 1 \]  \hspace{1cm} (12)

He also stated that similarity in the general flow pattern includes (1) the similarity of flow in the inflow channel and the flow into the reservoir, (2) the similarity in interchange between kinetic energy and potential energy of flow, (3) the similarity in buoyancy, and (4) the similarity in the tendency of forming a density current. The criteria thus required were:

\[ F_{r}^{r} = 1 \]  \hspace{1cm} (13a)

\[ R_{i}^{r} = 1 \]  \hspace{1cm} (13b)

\[ R_{e_{m}}^{r} > R_{e_{cr}}^{r} \]  \hspace{1cm} (13c)

\( R_{e_{cr}} \) denotes some critical value. In the case of \( (\Delta \rho / \rho) \) unequal in model and prototype, Equations (13a) and (13c) cannot be satisfied simultaneously; so Chen stated that it is better to give up \( F_{r} \) number as its scale effect is generally limited to the local inlet area and the similarity of the general flow pattern of the whole reservoir is not greatly affected. The third criterion for similarity is that the Reynolds number in the model should be greater than some critical value based on the flow pattern. Chen suggested dropping the length term and using the ratio \( Q/v \) to replace the Reynolds number. Therefore, the similarity rule becomes: \( (Q/v)_{m} > (Q/v)_{cr} \). Discharges above \( (Q/v)_{cr} \) will result
in the same general flow pattern. Chen suggested that \((Q/v)_{cr}\) be estimated from past experiments and checked in the model study. He also pointed out that \((Q/v)_r\) is governed by the model size, water depth, reservoir, topography, relative positions of the inlet and outlets, etc.

This chapter has consisted of a review of the general concepts and fundamental principles of waste stabilization ponds. Also, chemical reactor designing principles and their application to biological reactors such as sewage lagoons or aeration tanks have been discussed. Finally, the basic criteria for modeling stratified flow was presented.

The next chapter presents the important theoretical concepts that were an integral part of this study.
CHAPTER 2
THEORETICAL DEVELOPMENT

C-Diagram Parameters

To be able to account exactly for nonideal flow requires knowledge of the complete flow pattern of the fluid within the tank or pond. In obtaining this knowledge, practical difficulties arise since it would require the complete velocity distribution picture of the fluid within the vessel. Therefore, an alternate approach was used requiring knowledge only of how long different elements of the fluid remain in the vessel. This partial information is rather easy to obtain experimentally and interpret. Although it will not completely define the nonideal flow pattern within the pond, this approach yields information which is sufficient in many cases to allow a satisfactory accounting of the actual existing flow pattern and to determine the conversion of waste in the pond.

The experimental technique used for finding this desired distribution of residence times of fluid in the pond is a stimulus-response technique using tracer material in the flowing fluid. The tracer is injected in the inlet at a known concentration and the response or concentration at the outlet is measured as a function of time.

The treatment will be limited to steady-state flow with one entering and leaving stream. From the concentration vs. time curve at the outlet,
C and F diagrams will be determined.

Several parameters can be defined from the C-diagram and F-diagram which give some measure as to the extent and effectiveness of mixing in the vessel. These parameters will be used to check quantitatively the effectiveness of various factors such as inlet and outlet positions, geometrical, and fluid properties of the pond. The C-diagrams will also be used to determine the expected conversion of the reactant for selected experiments.

Mean residence time

The mean residence time $\overline{\theta}_c$ is a measure of the average time the tracer slug spends in the vessel.

$$
\overline{\theta}_c(\theta_o) = \frac{\int_{\theta_o}^{C} \frac{c}{c_o} \theta d\theta}{\int_{\theta_o}^{C} \frac{c}{c_o} d\theta}
$$

(14)

The value of $\overline{\theta}_c(\theta_o)$ is the distance from the origin to the centroid of that portion of the C-diagram between the origin and $\theta_o$. The value of $\theta_o$ is arbitrarily taken as 2 because after two detention times, $\frac{c}{c_o}$ on the C-diagram is generally small and data taken beyond this point are near the limit of readability of the tracer sensing instruments. Furthermore, if all parameters are constructed using data for $0 < \theta < 2$ then the comparison of the results will be meaningful. The equation for mean residence time then becomes
Dead space

In any flow vessel there are generally regions where mixing is less active than desirable. Generally, this occurs in corners of the vessel. These regions of poor mixing will be called dead spaces if the fluid moving through these spaces takes 5 to 10 times as long to pass through the vessel as does the main flow. An indication of the amount of dead space in a flow vessel is indicated on the C-diagram by a long tail on the C-curve.

If the flow through the vessel has a minimum of dead space then the mean residence time $\bar{t}_c$ will approach the detention time and $\bar{\theta}_c$ will approach 1.0. If there are substantial dead water regions in the flow then a large portion of the tracer will leave the vessel before $\theta = 1$. This will shift the centroid of the C-diagram toward the origin and make $\bar{\theta}_c < 1.0$.

To define the dead space a fictitious vessel will be described which will have no dead space and consequently will be smaller than the actual vessel. Its volume will be designated as $V_f$. This fictitious vessel will pass only the tracer which passed through the actual vessel up to $\theta = 2$ and this amount will be considered the total amount of tracer for the fictitious vessel. The flowrate through the fictitious vessel is $Q_f$ where

$$\bar{\theta}_c = \frac{\int_0^2 \frac{c}{c_0} \theta \, d\theta}{\int_0^2 \frac{c}{c_0} \, d\theta} \quad \ldots \quad (15)$$
\[ Q_f = Q[F]_\theta = 2 \]

\( Q_f \) may be considered that portion of the original flow rate which was moving fluid through the vessel at an acceptable rate.

The mean residence time of the fictitious vessel will be equal to that of the actual vessel and the detention time \( \bar{t}_f \) in the fictitious vessel will equal the mean residence time \( \bar{t}_c \). This means that if

\[ \bar{\theta}_c = \frac{\bar{t}_c}{\bar{t}_f} \]  

(16)

and

\[ \bar{t}_f = \frac{V_f}{Q_f} \]  

(18)

then

\[ \bar{\theta}_c = \frac{V_f \bar{\theta}}{VQ_f} \]  

(19)

Now calling the dead space

\[ V_d = V - V_f \]  

(20)

one obtains

\[ \frac{V_d}{V} = 1 - \bar{\theta}_c \{F\}_\theta = 2 \]  

(21)

If \( V_d/V \) is defined as the dead space parameter \( \bar{V}_d \) then

\[ V_d = 1 - \bar{\theta}_c \{F\}_\theta = 2 \]  

(22)
Deviation from plug flow

It is recognized that the ideal situation for flow through a waste stabilization pond is typified by plug flow. If the incoming waste mixes vertically and horizontally just enough to obtain good treatment and then moves through the pond as a slug, the optimum condition, called plug flow, has been realized. That is, all the fluid will have remained in the pond long enough for the desired degree of treatment and none will have remained longer than necessary. This condition leads to a pond of minimum volume for a given waste load and hence, minimum cost.

The deviation of a given flow from plug flow can, therefore, be considered a measure of pond efficiency. It is the waste that leaves the pond before one detention time that contributes to poor pond efficiency. Furthermore, it is not only the amount of waste leaving too soon but the amount of time it lacks of remaining in the pond one detention time that is important. Consequently, the deviation from plug flow parameter \( \overline{\theta}_{pf} \) will be defined, as shown in Figure 4, as the distance from line \( \theta = 1 \) to the centroid of the area under the C-curve from \( 0 \leq \theta \leq 1 \). The equation used to calculate \( \overline{\theta}_{pf} \) is

\[
\overline{\theta}_{pf} = \frac{\int_0^1 (1 - \theta) \frac{c}{c_0} \, d\theta}{\int_0^1 \frac{c}{c_0} \, d\theta} \quad \ldots \quad (23)
\]
Figure 4. The plug flow deviation parameter.
Computer program N-1 in the Appendix was developed to calculate the C-diagram, F-diagram, and the other important parameters discussed in this section. Each experiment was evaluated with these parameters to determine the most efficient design from a hydraulic standpoint. For best hydraulic performance that will result in maximum biological performance, the dimensionless mean residence time $\bar{\theta}_c$ should approach one. As $\bar{\theta}_c$ increases for given hydraulic conditions, the biological conversion would increase because the average detention time of the fluid particles would be greater.

Dead space has an obvious effect on conversion. Quantitatively, as dead space parameter $\bar{V}_d$ increases, the effective flow area decreases which results in a decreased detention time of the waste water and poorer biological performance.

The plug flow deviation parameter, $\bar{\theta}_{pf}$ will decrease in value as the hydraulic efficiency increases. Since maximum hydraulic efficiency and maximum conversion occurs when $\bar{\theta}_c = 1.0$ the same will hold true when $\bar{\theta}_{pf} = 0.0$.

The best flow situation that would result in an optimal detention time, and a minimum value for dead space and for the plug flow deviation parameter, would be plug or one-dimensional flow. However, in an actual pond it is important that the incoming flow mixes well at the outset to effectuate intimate contact of the waste material with the microorganisms in the pond. It is felt that for maximum conversion,
the incoming waste water should be diffused in such a way as to effectuate complete vertical and lateral mixing and then travel as a slug toward the outlet. This type of flow would utilize the complete cross-sectional area of the pond which would result in a minimum value for \( \overline{V_d} \) and \( \overline{\theta_{pf}} \) and an optimum value for \( \overline{\theta_c} \). This flow situation would then give maximum conversion for the given geometrical dimensions and hydraulic characteristics of the pond.

Mathematical Approach

Conversion equations

In a waste stabilization pond, the major concern is the extent of reaction of the waste material during its stay within the pond. For a reaction with a rate that is linear with concentration \( (r_c = \frac{dN}{dt} = -KN) \), the extent of reaction can be predicted solely from knowledge of the length of time each reactant element has spent in the reactor. The exact nature of the surrounding elements is of little importance. The distribution of residence times (C-diagram) gives information on how long various elements of fluid spend in a reactor, but not on the detailed exchange of matter within and between the elements. Because of this, the distribution of residence times yields sufficient information for the prediction of the average concentration in the reactor effluent. A waste stabilization pond is a biological reactor that closely corresponds to a first order (linear) chemical reactor, thereby supporting the use of chemical reactor design principles.
To determine the conversion of a chemical or biological reactor using the distribution of residence times, the following equation (Levenspiel and Bischoff, 1963) can be used:

\[
\text{Mean concentration of reactant leaving the reactor unreacted} = \sum \text{Concentration of reactant remaining in an element of age between } t \text{ and } t + dt \times \text{Fraction of exit stream which consists of elements of age between } t \text{ and } t + dt
\]  

\[\ldots \ldots \ldots \ldots \ldots \ldots \ldots \] (24a)

This equation says that for a steady flow if a sample was taken of the entire exit stream at some time and the extent of conversion of each waste or reactant element was determined and summed for all the elements in the exit stream or sample, the resulting quantity would be the mean concentration of the waste or reactant leaving the reactor unreacted. This same information can be obtained in a more convenient way by the following equation:

\[
\overline{N}_\infty = \int_0^\infty N \left( \frac{c}{c_0} \right) \, d\theta \ldots \ldots \ldots \ldots \ldots \ldots \ldots \] (24b)

in which

- \( N \) = concentration of reactant as a function of time
- \( \overline{N}_\infty \) = mean concentration of reactant leaving the reactor in an unreacted state
- \( \frac{c}{c_0} \) = residence time distribution of fluid particles
Equation 24b assumes a slug injection of reactant or pollutant and thereby requires a summation over time to account for the conversion of all the pollutant elements injected into the reactor. Referring to the first order reaction equation

\[
\frac{dN}{dt} = -KN
\]  

(25)

in which \( K \) = first order reaction coefficient. When \( t = 0 \), \( N = N_0 \) with \( N_0 \) = initial concentration of reactant. Integration of Equation 25 yields

\[
N = N_0 e^{-Kt\theta}
\]  

(26)

which is the well known equation that gives the remaining BOD of a waste as a function of \( N_0 \), \( K \), and \( t \). Incorporating Equation 26 into Equation 24 gives

\[
\frac{N_\infty}{N_0} = \int_0^\infty e^{-Kt\theta} \left( \frac{c}{c_0} \right) \, d\theta
\]  

(27)

Equation 27 is the general equation for the conversion of a reactant in a chemical or biological reactor. The reliability of this equation for accurately determining the fraction of unreacted material leaving a reactor is dependent on (1) how closely the reactant or waste material follows a first order reaction, (2) the value for the first order reaction coefficient, and (3) the appropriate expression for the residence time distribution of fluid particles in the reactor.
In using experimentally determined \( c/c_0 \) functions, the data are generally unreliable beyond \( \theta = 2.0 \). In determining the extent of reaction of a waste, Equation 27 can be modified to

\[
\frac{N_2}{N_0} = \int_0^2 e^{-Kt\theta} \left( \frac{c}{c_0} \right) d\theta \quad \ldots \ldots \ldots \quad (28)
\]

where experimental \( c/c_0 \) functions are used.

Equation 28 will give the fraction of material that has not undergone reaction at \( \theta = 2.0 \). To determine the extent of reaction or treatment efficiency, as a percent, \( \frac{N_2}{N_0} \) should be subtracted from one and the resulting quantity multiplied by 100.

Values for \( \frac{N_2}{N_0} \) and expected treatment efficiencies have been determined for a number of actual pond designs. These results are found in Chapter 5.

**Mathematical models for age distribution functions**

**Ideal flow models**

In using either Equation 27 or 28 it would be desirable to express the age distribution function \( c/c_0 \) as a mathematical function. For ideal fluid flow situations this has been done.

For completely mixed flow in a single reactor, the age distribution function is:

\[
\frac{c}{c_0} = e^{-\theta} \quad \ldots \ldots \ldots \quad (29)
\]
when incorporating this expression into Equation 29 the solution is:

\[
\frac{N_\infty}{N_o} = \frac{1}{1 + Kt}
\]  

(30)

For a series of equal-sized completely mixed flow reactors, the age distribution function is:

\[
\frac{c}{c_o} = \frac{1}{(j-1)!} \theta^{j-1} e^{-\theta}
\]  

(31)

where \( j \) = number of reactors. And the conversion equation is:

\[
\frac{N_\infty}{N_o} = \frac{1}{(1 + Kt)^j}
\]  

(32)

For plug flow, the conversion equation is

\[
\frac{N_\infty}{N_o} = e^{-Kt}
\]  

(33)

Equations 30 and 33 have been used quite extensively in the design of various wastewater treatment facilities.

Non-ideal flow models

In actual flow situations, the ideal flow conditions of plug, or completely mixed flow, are never obtained. It would therefore be desirable to develop an appropriate equation to describe the residence time distribution for use in Equation 27 or 28 which would make it possible to predict organic waste or other reactant conversion.
There have been a number of models developed and these are classified as (1) dispersed plug flow models, (2) tanks in series or mixing cell models, and (3) combined models.

Equation (5) in Chapter 1 is a mathematical expression of the dispersed plug flow situation. This equation has been used to some extent in proposed design methods for wastewater treatment facilities. Figure 5 from Levenspiel (1962) is a graphical representation of Equation (5). The ratio of reactor volume needed with dispersion to the plug-flow volume \((V/V_p)\) is plotted against the fraction of reactant remaining at the outlet, with \(d\) (dispersion effect) as the varying parameter.

From this plot, it can be seen for a given degree of treatment or percentage reactant remaining, that as \(d\) increases, the actual volume of the reactor increases in relation to the volume of a plug flow reactor to obtain the same degree of treatment. In other words this means that the best flow situation would be one which most nearly approximates a plug flow model. A design of a waste stabilization pond where its residence time distribution approaches \(c/c_o\) for a plug flow model, (i.e. \(\bar{t}_c \rightarrow \bar{t}_p\) and \(\bar{t}_p \rightarrow 0.0\) and dead space is a minimum) maximum conversion of organic matter would result.

It has been found that the only mathematical model that can adequately characterize or represent the age distribution functions in existing sewage lagoons is a combined model. A combined model is different than dispersed plug flow or mixing cell models in that it
Figure 5. Graphical representation of Equation 5—dispersed plug flow model (Levenspiel, 1962).
consists of interconnected flow regions with various modes of flow between and around these regions. A combined model consists of plug flow regions, completely mixed flow regions, dispersed plug-flow regions, and dead water regions.

The next section describes the combined model that was used in this research to represent the age distribution functions of waste stabilization ponds.

The finite stage or combined model

A combined model called a finite stage model has been developed by Hovorka (1961) and Adler et al. (1963). This model combines networks of perfectly mixed and plug flow stages. It is flexible and permits characterization of such major flow characteristics as partial mixing in the lateral and longitudinal directions, relative dead flow regions, and short circuiting. The basic module which is composed of a plug flow unit, a dead water unit, and a backmix (completely mixed) unit is shown in Figure 6. The dead water region is viewed to be in backmix flow and to be interchanging fluid slowly with the active completely mixed flow region.

To characterize a non-ideal flow situation, the basic module may be repeated any integral number of times, \( n \). Figure 7a shows a typical flow situation and Figure 7b gives the finite stage model for this case.
Figure 6. The basic module of the finite stage model.

Figure 7. Finite stage model for a given flow situation.
Figure 7a shows that the flow system may be divided into two types of regions:

1. A live flow region shown as $a_1$, $a_2$, $c_1$, and $c_2$, where the bulk of the fluid flow occurs.

2. A series of dead flow regions shown as $b_1$ and $b_2$, which have a small amount of fluid interchange with the live flow region.

Each live-flow region has limited mixing in the longitudinal direction where the dead-flow region is well-mixed, but has no direct means of mixing with other dead regions.

Any physical system with any degree of longitudinal mixing may be approximated by a model containing a combination of live and dead-flow regions. This is the basis of the finite stage model developed by Hovorka.

The finite stage model requires four parameters:

$F_a = \text{fraction of the total system volume represented by perfectly mixed units in the live-flow region.}$

$F_b = \text{fraction of the total system volume contained in dead-flow regions.}$

$F_c = \text{fraction of the total system volume contained in plug-flow elements. It also represents the delay time or time when tracer first appears at the vessel exit.}$

$K_H = \text{the fraction of the main flow which is interchanging between live and dead-flow regions.}$
\[ n = \text{the number of basic modules in series.} \]

Physically, the parameters of the model must be subject to the following restrictions:

\[ 0 \leq F_a \leq 1 \]
\[ 0 \leq F_b < 1 \]
\[ 0 \leq F_c \leq 1 \]
\[ 0 < F_a + F_b \leq 1 \]
\[ 0 \leq K_H \]
\[ n = 1, 2, 3 \ldots \text{(Integral values only)} \]

\[ F_a + F_b + F_c = 1 \]

The live-flow region can approximate any level of longitudinal mixing from one extreme of no mixing to the other extreme of perfect mixing depending upon the relative size of \( F_a \) and \( F_c \). Varying the value of \( K_H \) can also cause various levels of mixing. The value of \( n \) effects the longitudinal mixing length, small \( n \) corresponding to long mixing lengths. The value of \( K_H \) also effects the amount of inter-change between dead-flow and live-flow regions.

Parameter evaluation and model solution

Finite stage models and their parameters have definite physical significance, and it would be desirable to predict these parameters from a knowledge of the flow rate, fluid properties, and geometry within the system. At the present there have not been ways developed to predict these parameters. The best alternative method is to use
experimental residence time distribution curves from existing systems or from smaller physical models.

The following is the procedure developed by Hovorka for determining the model parameters. It is based on fitting an analytic equation to the residence time distribution curve determined from tracer data. Because the tail of the curves is generally unreliable, the procedure is designed to rely basically on the front part of the concentration versus time curves.

The exact fit is assured at two points, the minimum residence time $F_c$, where the tracer first appears, and the point at which the tracer concentration is a maximum.

The transformation equations to convert the curve in Figure 8a to the curve in Figure 8b are:

$$Y = \frac{c}{c_0} (1 - F_c) \quad \cdot \cdot \cdot \quad (34a)$$

$$X = (\Theta - F_c)/(1 - F_c) \quad \cdot \cdot \cdot \quad (34b)$$

The initial steps of the procedure are illustrated in Figure 8. The curve in Figure 8a is the normalized C-diagram for the concentration versus time data of the injected slug of tracer. The dead or delay time, equal to $F_c$, is removed using the transformation equations shown with Figure 8, thus reducing the data to $Y$ vs. $X$ coordinates shown as the curve in Figure 8b. A residence time equation without dead
Figure 8. Transformation of C-diagram to X and Y coordinates.

\[ Y = \frac{c}{c_o}(1-F_C) \]
\[ X = \frac{(\theta - F_C)}{(1-F_C)} \]
time is then fitted to the data. Equation (35) was determined by Hovorka by writing material balance total differential equations about the modules and solving these equations by Laplace transform techniques. The derivation of this equation is shown in Appendix C of Hovorka's dissertation.

\[ Y = n \sum_{m=1}^{n} \left[ A_{nm} (nX)^{m-1} e^{nr_1X} + B_{nm} (nX)^{m-1} e^{nr_2X} \right] \]  (35)

The A's, B's, \( r_1 \), and \( r_2 \) are constants which are functions of the model parameters \( K_H \), \( n \), \( F_a \), and \( F_b \). These algebraic functions and the expansion of Equation 35 for \( n = 1, 2, 3, 4, \) and 5 are listed in Appendix C of Hovorka's dissertation.

Hovorka prepared special charts and tables to permit selection of \( K_H \) and \( f_a \), where \( f_a = F_a / (1-F_c) \), for each permissible value of \( n \) in order to force Equation 35 to have the correct maximum points.

The following procedure illustrates the method.

1. The peak time and value of the peak concentration are transformed to \( X_{pk} \) and \( Y_{pk} \) by the transformation Equations 34a and 34b.

2. A graph is consulted which gives a value of \( f_a \) and \( K_H \) for \( X_{pk} \) and \( Y_{pk} \) for each \( n \).

3. The best \( n \) is chosen so as to minimize the average squared deviation between the \( Y \) values of the data and of Equation 35.
4. Having a value of \( n, K, F_a, \) and \( F_b, \) the coefficients of Equation 35 can be determined resulting in the transformed finite stage model.

5. The values of \( X \) and \( Y \) of the model equation are then transformed back to conform to the original C-diagram by the following equations:

\[
\frac{c}{c_o} = X(1-F_c) + F_c \\
\theta = Y/(1-F_c)
\]  

(36a)  
(36b)

In summary, an approximate fit over the entire residence time distribution curve for a flow system such as a waste stabilization pond may be made with two pieces of information. This information includes:

1. The \((c, \theta)\) coordinates where the tracer first appears.
2. The \((c, \theta)\) coordinates at the point where the tracer concentration is a maximum.

The value of this model is readily recognizable. If a residence time distribution curve could be specified by use of a finite stage model for a proposed waste stabilization pond, then the extent of reaction or treatment efficiency could be determined by Equations 27 or 28 provided a reliable value of the first order reaction coefficient is known.
Purpose of Model Study

In general, there are three different types of flow conditions that exist in waste stabilization ponds which must be considered when designing a model as mentioned in the previous chapter.

The flow conditions that generally occur in the Logan Pond System (with some exceptions) are these three conditions. The differences in densities are due, at least in the Logan Pond System, to temperature variations in the lagoons as well as to temperature variations in the incoming flow. To correctly model these types of flow conditions, the model probably should use temperature as a means of creating different density conditions in the pond and incoming fluid. To do this the model would have to be insulated and the room temperature and humidity controlled. For a model of the size used in this study, this was not practical. Instead, a commercial salt (NaCl) was used to obtain the required density differences needed to simulate the prototype pond flows.

The question arises as to whether this technique is valid to simulate a basically convective and conductive heat and mass transfer phenomena. An inspection of the actual flow conditions in the prototype stabilization pond will reveal that the flows are essentially turbulent in nature where the eddy diffusivity is the most important mechanism of either heat or mass transfer. From turbulent flow characteristics, it is known that $\nu \ll E_m$ and $a \ll E_h$.
in which

\[ \nu = \text{kinematic viscosity or diffusivity of momentum} \]
\[ E_m = \text{eddy diffusivity of momentum} \]
\[ a = \text{thermal diffusivity} \]
\[ E_h = \text{eddy diffusivity of heat transfer}. \]

This means that heat transfer due to conduction is negligible in comparison to convective heat transfer. The Prandtl number for water is \( Pr = \frac{\nu}{a} = 1.5 \), which means that \( a < \nu \).

Since \( \nu << E_m \), \( a << E_h \), and from \( Pr = 1.5 \), \( a < \nu \). Now the question becomes, can the molecular diffusion in the model (using salinity to create density stratification) be neglected when compared with convective mass transfer?

For a binary mixture of a low concentration of salt, the Schmidt number, which is the ratio of the diffusivity of momentum to the molecular mass diffusion coefficient is:

\[ Sc = \frac{\nu}{D_{mass}} \approx 750 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37) \]

so, \( D_{mass} << \nu << (E_h \text{ or } E_m) \). Therefore the importance of mass diffusion by conduction is even less than heat transfer by conduction.

Since heat transfer by conduction can be neglected in the prototype, mass diffusion by conduction can also be neglected in the model. It appears that the use of salinity will work well in simulating density stratification in the model.
Another problem arises in using sodium chloride or any other salt in the model to simulate density stratification. In the prototype waste stabilization pond, the major mode of heat transfer to the ambient fluid is not from heat conduction or convection from the incoming fluid, but from solar radiation heat transfer. The heat transfer from radiation depends upon many factors of which the most important is the environmental conditions. During the summer months, the net effect of solar radiation is to increase the temperature of the ambient fluid thereby reducing the density.

In modeling the flow conditions during this period of time with salinity, the net effect is to increase the density of the model pond by salt accumulation thereby reducing the density difference between the incoming flow and the ambient fluid. During the summer months' operation of the prototype ponds, the density difference actually increases. Even though this effect is wrong for correct modeling, it is assumed that it does not have a great effect on the overall flow conditions. A mathematical check was made to determine what influence an increase in salinity of the ambient fluid would have on the density difference between incoming salt water and the ambient fluid. In a conservative sense it was assumed that the pond was completely mixed with the incoming salt solution which would result in greater salinity build up and a more rapid decrease in the density difference. The change in density difference for the hypothetical flow situation was about 30 percent more than it should
have been when compared with the prototype pond data for a 30 day period of time.

In summary, the overall purpose of this research on waste stabilization ponds was to determine the existing flow patterns in the prototype ponds and to determine sound scientific design principles and methods to better design lagoons, from a hydraulic standpoint, to increase the effectiveness of this method of waste treatment. The method of analyzing pond behavior was to conduct tracer studies. Since this was expensive and time consuming, a hydraulic model was designed and used to aid in this research.

The purposes of the hydraulic model study were to:

1. Verify existing pond behavior.
2. Determine effects of geometry on the flow behavior.
3. Determine effects of the flowrate and subsequent Reynolds number on pond behavior.
4. Determine effects of different densities on the flow patterns and subsequent mixing.
5. Determine effects of inlet and outlet placement and design on the flow patterns.

There are other factors which are important and have an influence on the hydraulic characteristics of stabilization ponds such as, temperature, humidity, evaporation, precipitation, and wind. These environmental conditions were ignored. Wind is probably the most important of
these environmental conditions and the wind effects are being studied in a model at the present time by another Ph.D. candidate.

The flow patterns in waste stabilization ponds are complex and difficult—probably impossible to determine mathematically since the flow is really three-dimensional in nature. The three-dimensional velocity field must be determined before the three-dimensional dispersion equation can be solved. Consequently, hydraulic model studies to determine the general flow behavior are the most practical approach in studying the flow patterns.
CHAPTER 3
LOGAN POND SYSTEM STUDIES

Introduction

The purpose of this phase of the research was to determine the characteristics of the circulation patterns in waste stabilization ponds and to collect data on several of the existing ponds in the Logan Pond System (Figures 9 and 10). These data are compared with pond model data to ascertain the extent and influence of short circuiting, dead or stagnant areas, density difference effects, and to determine the influence of geometry, inlet and outlet types and locations on the flow patterns and hydraulic efficiency of the ponds.

The circulation patterns were determined by introducing a dye into the inlet flow and determining how rapidly and efficiently the flow disperses to all areas of the pond. By sampling at the exit, time of travel through the pond was established to determine if the requirements on the time of retention are being met. In addition, the sampling procedures were evaluated so that any corrective measures necessary could be taken.

Experimental Preparations

Experiments were performed on ponds A-1, A-2, D, and E, shown in Figure 9 as a plan of the Logan Pond System. Figure 9 also shows
Figure 9. The Logan pond system.
Figure 10. Photograph of Logan pond system.

Figure 11. Photograph of pontoon boat and sampling equipment used on the Logan City ponds.
the points where samples were taken in addition to the outlet samples of each pond. These sample points were established by using concrete anchors and small buoys. The grid size was chosen to give a good representation of the pond geometry, yet small enough (15 grid points) to allow the samples to be taken in a reasonable length of time.

The tracing dye, rhodamine WT, was ordered from DuPont in the form of a 20 percent solution. This dye was chosen because it is relatively nonabsorbent and inexpensive. A Turner Model 110 fluorometer, was calibrated to measure the dye concentrations. To avoid temperature sensitivity problems, it was decided that all dye samples from the pond would be brought to calibration temperature before readings were taken.

During the summer of 1969 a pontoon boat, Figure 11, was constructed along with the equipment necessary to obtain samples of the pond at a specified depth and at any location on the pond.

Experimental Procedure

The dye slug was introduced into a selected pond through the submerged culvert which supplies each pond. This was accomplished by inserting into the culvert a long plastic tube connected to a 30-gallon tank containing the dye-water solution. Thorough flushing of the tank and tube with water insured that the complete sample entered the pond.

At varying time intervals, depending on the dye movement, samples were taken at each of the grid points, as well as at the outlet of the pond. In some cases samples were taken of the top, middle, and bottom at
each of the grid points in addition to the outlet sample. The temperatures of the samples were measured periodically to determine if stratification was present. The sampling procedure for the entire pond took about 60 minutes to complete if samples were taken at the top, middle, and bottom of the pond. Any variation in this experimental procedure with a particular experiment is noted in describing the results of that experiment. Also the dye cloud positions were observed and noted as a function of time.

Results of Experiments Performed

Experiment No. 1

Dye was injected into the inlet of Pond E at 4:30 p.m., November 23, 1968, and the first samples were taken at 4:15 p.m., November 24, 24 hours later. Instead of the dye being concentrated near the upstream end of the pond, it was found only in the downstream one-third of the pond. The sample taken at the exit showed a strong dye concentration. Since the retention time for the pond was nearly 6 days, it was apparent that something was wrong drastically.

At the end of 48 hours, the second and third set of sample stations from the inlet towards the south (Figure 9) exhibited fluorescence. However, the concentration was only about one-half that at the fourth and fifth set of sample stations from the inlet.

At the end of 72 hours the concentration throughout the pond was essentially uniform, indicating that uniform dispersion had taken
place. But, by this time, about one-half of the dye had already left the pond.

The results show that serious short-circuiting of the pond is occurring. However, since samples were not taken the first 24 hours of operation, it was not known exactly how the dye stream progressed through the pond, therefore the results are inconclusive.

Experiment No. 2

In February Pond E was again injected with dye. The ponds were mostly ice-covered at this time so all that could be done was inject dye at the pond inlet and collect samples at the pond exit. There was a small ice-free area at both of these locations. The dye was injected into the inlet at 8:00 a.m., February 15, 1969.

After dye injections, the dye stream was observed to form into a flow streaming toward the exit. Four hours after injection a visible dye stream was flowing out of the exit, and it was coming straight from the inlet. This was good visible proof of serious short-circuiting.

The highest concentration of dye passed through the exit some 6-10 hours after injection (Figure 12). This indicates an average velocity of the dye slug of about 0.05 feet per second through the pond.

In this test the dye load was twice that of test No. 1 to insure more accurate readings. This change, coupled with more frequent samples, has substantiated the tentative conclusions of test No. 1.
Figure 12. Concentration vs. time curve at outlet for Experiment 2 - Pond E.
Experiment No. 3

On October 23, 1969, 12 liters of rhodamine WT 20 percent solution were placed into the transfer structure between ponds "C" and "D." Concentrated slugs of dye appeared on the surface in an erratic, random pattern.

Top, mid-depth, and bottom samples were collected at each grid point at various time intervals, varying from 2 to 12 hours, and the dye concentration determined. Also, samples were collected at the outlet of the pond at various time intervals. The dye cloud moved rapidly through the pond as detailed below.

By 12:00 noon, October 23, the dye cloud had spread across the north end of the pond. The movement of the visible dye cloud is shown in Figure 13. By 6:00 p.m., 8 hours after release, the dye had traveled more than half way through the pond. The exact time when the dye first began to leave the pond could not be visually determined because of darkness. Fluorescence was not detected in the samples taken at the outlet at 10:00 p.m., October 23, but readings were obtained on outlet samples taken at 6:00 a.m., October 24.

Various curves were obtained from the data collected. Figure 14 shows the concentration (mg/L) of dye vs. the time at the outlet. The peak concentration at the outlet was attained 28 hours after the dye had been released into the pond. The dye concentration dropped off quickly, then steadily decreased to only trace amounts. A more informative
Figure 13. Dye clouds observed from boat at various time (hrs) after dye injection experiment.
Figure 14. Concentration vs. time curve at outlet for Experiment 3 - Pond D.
picture is presented by the mass diagram in Figure 15. One-half of
the dye placed in the pond had reached the outlet 120 hours after the dye
had been released. The retention time for this flow rate is about 250
hours, so it is apparent that the waste water is passing through too
quickly. Samples were last taken 340 hours after release and 88 percent
of the dye had left the pond. Figure 16 shows the average concentration
of all grid points in the ponds. The concentration at the bottom was
initially lower than that at the top, due to the lower velocities along the
bottom. The pond became fairly well mixed 33 hours after the dye had
been released. By this time about 11 percent of the dye had left the
pond. After this time the dye was quite evenly dispersed throughout the
pond. Most of the fluorometer readings indicated that the concentration
of the dye was decreasing fairly evenly throughout the pond. Eventually
a region of clear water developed at the north end of the pond and the
dyed fluid slowly moved toward the outlet.

Calculations based on the average flow rate indicated a design
velocity of .0026 fps. However, based on the time for the dye to reach
the outlet, the velocity was .035 fps which is an order of magnitude
greater. The maximum flow velocity was higher than the average velocity
by a factor of approximately 1.5 to 2.0.

There seemed to exist a certain amount of vertical stratification
as the dye cloud moved through the pond. Low readings were obtained
on the bottom, but roughly equal readings were obtained on the surface
Figure 15. Mass curve for Experiment 3 - Pond D.

\[ M_t = 2.4 \times 10^6 \text{ mg} \]
Figure 16. Average concentration of all grid points vs. time curves for the top, middle, and bottom of Pond D - Experiment 3.
Figure 16. Average concentration of all grid points vs. time curves for the top, middle, and bottom of Pond D - Experiment 3.
and at the 4 foot depth. Temperature measurements indicated the bottom was 1°C cooler than the top of the pond. After the edge of the dye cloud had passed, nearly equal readings were obtained throughout the depth. Thirty-three hours after the release the dye had become evenly dispersed. The pond decreased in fluorescence evenly until about 110 hours after release.

Experiment No. 4

On November 20, 1969, at 7:00 a.m., 12 liters of rhodamine dye were injected into the 30" pipe that runs into pond "D." Shortly after all the dye had been placed in the pipe, it appeared at the outlet coming to the surface as slugs of dye. As time went on, it spread out and moved laterally across the north end of the pond towards the west bank. The dye first appeared at the outlet between 30-33 hours after the dye was introduced into the pond.

At 10:00 a.m., 1:00 p.m., and 4:00 p.m. on November 20 aerial photographs were taken of the pond to record the dye movement. At 2:00 p.m. on November 21, 1969, aerial photographs were again taken of the dye movement in the pond. These photographs were reduced to sketches, and Figure 17 shows the position of the dye cloud at each time photographs were taken. For this experiment, samples were taken only at the outlet and not at the sample points throughout the pond. The concentration versus time curve for this data is shown in Figure 18.
Figure 17. Dye clouds as observed from the air at various times after dye injection for Experiment 4 - Pond D.
Figure 18. Concentration vs. time curve at outlet for Experiment 4 - Pond D.
Figure 19 shows the accumulated mass curve versus the time following the introduction of dye. This curve represents the quantity of dye that has left the pond as a function of time.

At 2:30 p.m. on November 20, 1969, a longitudinal traverse was made through the front of the advancing dye cloud. Samples were taken at 6 different points as shown in Figure 20 to determine concentration profile of the dye as a function of depth. Figure 20 shows the concentration profiles at the 6 sample points.

In the front portion of the dye cloud the concentration profile was similar to the actual velocity profile as measured with drogues. After moving into the dye cloud, the concentration became more uniform with depth because of the vertical convective and conductive mass diffusion of the dye from the region of greater mass concentration near the surface to the regions of lower concentration near the bottom of the pond.

Four drogues were placed on the I-3 side of I-2 (Figure 20) to determine the approximate velocity profile by time displacement. It was assumed that the bulk fluid movement would be in the direction of the outlet. It was surprising to find that the movement was not toward the outlet as was thought, but was circular in motion and moving in the general direction of H-2. The flow near the west bank of the pond was in the general direction of the outlet while on the east bank it was in the direction of the inlet. These visual observations indicate a counter-clockwise circular flow pattern apparently caused by the jet type inflow.
Figure 19. Mass vs. time curve for Experiment 4 - Pond D.

\[ M_t = 2.4 \times 10^3 \text{ mg} \]
Figure 20. Dye concentration profiles as a function of depth for Experiment 4 - Pond D.
from the 30" diameter inlet pipe.

This experiment strengthened the conclusions of the previous experiments, which indicated "short-circuiting" in the waste stabilization ponds. The aerial photographs, referred to above, show the gross movement of the fluid directly towards the outlet. There is some lateral mixing, but as shown in the pictures, some areas of the pond were void of high concentrations of dye. The concentration versus time curves, Figures 14 and 18 indicate a circular flow pattern for Experiments 3 and 4. The series of mild peaks shown in Figure 14 is a result of a quantity of tracer that is continually being circulated and diffused so that its concentration is decreased. But each time around, a quantity of this tracer leaves the pond at the outlet which results in the peaks shown in Figure 14. Figure 14 indicates a more serious problem than does Figure 18. The peak concentration was higher for Experiment No. 3 than for Experiment No. 4. This can be explained by the Reynolds number variation of the inlet flow pattern. The Reynolds number in Experiment No. 3 was about twice that of Experiment No. 4. This is due mainly to the increased flowrate, which causes a strong circulation pattern to be developed. This results in a more serious short circuiting problem as is evidenced by the higher concentration of dye at the outlet, as is shown in Figure 14.
Experiment No. 5

On May 19, 1970, at 2:00 p.m., 44 liters of rhodamine WT 20 percent solution were placed in the inlet channel just ahead of the Parshall flume that measures the total wastewater flow before it enters ponds A-1 and A-2. After passing through the Parshall flume, the wastewater flow enters a splitter box that equally divides the flow into the primary ponds A-1 and A-2. It was assumed for the purposes of this experiment that the flow was equally divided (22 liters) into each pond.

Samples at the outlets of both ponds were collected at varying time intervals. Also, samples were collected at the grid points of pond A-1 as shown in Figure 9 at various time intervals.

It is not known when the dye first appeared at the outlets of both ponds, but 7 hours after the dye placement, the dye had been leaving pond A-1 for some time as the concentration had already reached its peak. Six days after the dye was first introduced into the ponds, the sampling was discontinued due to an unexpected change in the flow conditions caused by an operational change made by Logan City personnel. At this time, the dye concentration of the fluid leaving pond A-2 had not reached its peak. In brief, there was a serious short circuiting problem in A-1 but it wasn't nearly so severe in A-2.

The probable reasons for this different flow behavior are: on the day that the dye was placed into the pond, special note was made of a strong wind blowing from the southwest to the northeast during the whole
day. By referring to Figure 9 of the Logan pond system, the wind movement in relation to the layout of the inlets and outlets at ponds A-1 and A-2 can be seen.

It is felt that this consistently strong wind from the southwest caused a vertical circulation pattern to be developed in the ponds with the surface fluid being dragged along by the wind in a northeasterly direction and the fluid in the lower depths of the pond moved in a southwesterly direction. Since the waste water entering the ponds was of a higher density, due to the lower temperature than the receiving fluid, it tended to stay near the bottom and was carried directly to the outlet of pond A-1 by the bulk fluid movement in the lower depths. This resulted in a large quantity of dye being carried through the pond via the short circuit route. Pond A-2 did not reflect this short circuit problem because of the placement of the outlet (Figure 9). The dye, due to this vertical circulation pattern, tended to collect in the southwest corner of the pond and since the outlet was located in the northwest corner, the problem of short circuiting was much less severe in pond A-2.

This shortened experiment did point out the need for considering outlet and inlet placement in relation to prevailing winds in order to prevent serious short circuiting and subsequent loss of hydraulic efficiency of waste stabilization ponds.
Experiment No. 6

On August 5, 1970, 19.57 liters of rhodamine WT 20 percent solution were placed into the inlet pipes of primary pond A-1. On the same date and time 6 liters of the same dye were placed into the inlet pipes of primary pond A-2.

Samples at the outlets of both ponds were collected at varying time intervals. Also, samples were taken at the grid points shown in Figure 9 at various time intervals. Temperature readings were taken at both inlets and outlets of the two ponds to determine the existing density difference between the ponds and the influent to the ponds.

Dye first appeared at the outlet of pond A-1 somewhere between 5 and 24 hours after dye placement in pond A-2. Dye didn't appear until 28 hours after dye placement in pond A-2. The peak concentration in both ponds A-1 and A-2 occurred 56 hours after dye placement. Figure 21 shows the normalized concentration vs. time curves for these two ponds. The quantities $c_o$ and $t$ are defined as:

$$c_o = \frac{\text{mass of dye placed in pond}}{\text{volume of pond}}$$

$$t = \frac{\text{volume of pond}}{\text{flow rate}}$$

Samples taken at the grid points of pond A-1 indicated the location or movement of the dye. The data revealed that the dye seemed to stay near the bottom especially near the diffuser. This is as expected since
Figure 21. Normalized concentration vs. time curves at outlets, uncorrected for Experiment 6 - Ponds A-1 and A-2.
the density of the influent was greater than in the pond. After 72 hours the dye concentration was essentially uniform.

Figure 23 is the mass diagram for ponds A-1 and A-2. At the time (72 hours) that the dye became thoroughly mixed in pond A-1, about 6 percent of the dye had left the pond. After one theoretical detention time, 37 percent of the dye had left the pond. This percentage, when added to the percentage of dye left in the pond, should have been equal to the amount of dye placed in the pond. For pond A-1 only about 54 percent of the total quantity of dye was accounted for. For pond A-2, after one theoretical detention time, 77 percent of the dye was accounted for. The data for the experiments on ponds A-1 and A-2 were corrected for this loss of dye by reducing the quantity of dye placed in the ponds by the amount that was determined to be lost. The corrected curves for ponds A-1 and A-2 are shown in Figures 22 and 23. This is an approximation, but it was felt that the results are more realistic when compared with this reduced mass of injected tracer than with the amount originally placed in the ponds.

In an attempt to locate the missing dye, samples were taken of the sludge on the pond bottom after the dye concentration had reached zero at the outlet. Samples of water were also taken just above the bottom at several locations throughout the pond. The sludge samples were mixed thoroughly and the solids centrifuged out. The water samples near the bottom indicated a zero concentration of dye. The dye concentration of the fluid remaining from the centrifuged sludge samples indicated an
Figure 22. Normalized concentration vs. time curves at outlets, corrected for Experiment 6 - Ponds A-1 and A-2
Figure 23. Normalized mass diagrams for Experiment 6, corrected and uncorrected for Ponds A-1 and A-2.
average of 0.001 mg/L for three samples. This means that the dye was either in the sludge or in the wastewater located in the pond bottom depressions. Another test was conducted to determine if the dye had been adsorbed on the sludge. Samples of sludge were taken at several locations throughout the pond and the solids centrifuged out. The solids were then mixed with water containing a known concentration of dye. After two weeks the concentration was measured and compared with the original concentration. The samples indicated no change in the dye concentration. Apparently there was little or no adsorption of the dye on the organic matter in the pond. The only explanation for the loss of dye must be that a certain quantity of dye, mixed with the more dense liquid entering the pond became entrapped in small depressions or pockets in the boundary layer on the bottom of the pond and thus was a lost quantity.

There did exist in pond A-1 a definite vertical dye stratification as the dye moved through the pond. High readings were generally found near the bottom. This was because the fluid entering the pond was more dense than the pond fluid itself, and because of the vertical temperature profile that existed in the pond. Temperatures varied from 1°C to 10°C cooler at the bottom than at the top. After 72 hours the dye concentration in the pond became essentially uniform.
Experiment No. 7

On November 17, 1970, 9.475 liters of rhodamine WT 20 percent dye were placed into the transfer pipes between ponds D and E. The temperatures at the inlet and outlet of the pond were essentially constant throughout the test run.

This experiment was performed to get better performance data for the particular type of inlet and outlet facilities that exist in pond E. The inlet is two 30" pipes and the outlet is one 36" pipe located in plan as shown in Figure 9. The inlet and outlet pipes are on the bottom of the pond. Samples were taken hourly at the outlet for 24 hours, then at varying time intervals after that.

The dye first appeared 5 hours after placement into the pond. The peak concentration was reached 11 hours after the start of the experiment. Two curves were obtained from the data collected. Figure 24 is the normalized concentration vs. time curve at the outlet and Figure 25 is the normalized mass curve. The dye seemed to stay near the west bank after entering the pond, and it occupied essentially the west half until reaching the outlet. The tracer was carried in a counter-clockwise direction to the east bank. Figure 24 shows the typical peaks that would be expected due to the counter-clockwise circulation patterns developed in the pond.

The ponds in the Logan pond system that were studied all experienced short circuiting to some degree depending upon the environmental
Figure 24. Normalized concentration vs. time curve at outlet for Experiment 7 - Pond E.
Figure 25. Normalized mass curve for Experiment 7 - Pond E.
conditions, the hydraulic characteristics, and the geometry of the pond.

Reliable data were difficult to obtain in the field studies because of (1) inaccurate flowrate measurements, (2) the location of the sampling points, and (3) temperature variations.

Tied closely to flowrate measurements is the unknown quantity of seepage and evaporation and its effect on the flowrate out of each pond. Flowrate measurements were only made at the inlet of ponds A-1 and A-2 and at the outlet of pond E. It was assumed that the flowrate out of pond A-1 and A-2 was equal to the flowrate into these ponds. Similar assumptions were made on ponds D and E. This, of course, neglects seepage and evaporation losses. The effect of the location of the sampling point at the outlet on the reliability of the results is shown in Figure 18. Temperature was constantly changing which made it difficult to obtain representative measurements. Temperature was particularly important in Experiments 5 and 6.

Considering all of the possibilities for experimental error and its influence on the data and other factors that affect tracer experiments, it is felt that the results reported herein do give reliable estimates of pond behavior, and show the influence of certain design factors on the flow patterns through waste stabilization ponds.
CHAPTER 4

EXPERIMENTAL PROCEDURES-MODEL

Description of Experimental Equipment

Experimental apparatus

The hydraulic model was constructed with plywood coated on the inside with fiberglass and resin. The facility is shown schematically in Figure 26 and photographically in Figure 27. The tank is 40 feet long, 20 feet wide, and 3.5 feet deep. The facility was designed so that the tracer concentration at the outlet could be continuously monitored with a Turner Fluorometer. The effluent of the model was carried by a 2 1/2" diameter pipe. Samples of the outflow were pumped at a constant rate through the fluorometer from a sample tap in the pipe. The fluorometer dial reading was recorded with a strip chart recorder. The flowrate at the outlet was measured with a Venturi meter and controlled with a gate valve. Figure 28 shows the outlet apparatus and tracer concentration measuring equipment.

The inflow apparatus was designed so that the flowrate could be continuously recorded and the salt water flow could be adjusted to give the required density for the density experiments. The inflow apparatus is shown in Figure 29. The apparatus consisted of two tanks, one for fresh water, and the other for a salt water solution. The fresh water
Figure 26. Experimental apparatus and model pond.
Figure 27. Photograph of hydraulic model.

Figure 28. Photograph of outflow apparatus and tracer concentration measuring equipment.
Figure 29. Photograph of inflow apparatus.

Figure 30. Photograph of diffusers used in experimental studies.
was pumped to a constant level tank and then mixed with a metered flow
of salt water to give the required density when making density experi-
ments. A booster pump was then used to give the required flowrate,
which was carried by a 2 inch diameter pipe.

The model pond was equipped with a motorized carriage that spanned
the width to enable samples to be taken at any location in the pond.

An extra wall was constructed along with baffles and a number of
inlet and outlet devices. The inlet devices were constructed so as to
break up or diffuse the incoming flow to nullify the effects of jets on the
flow patterns. This was accomplished by diffusers made out of plastic
pipe and graded gravel. The outflow was mixed so as to give good
representative sample before being pumped through the fluorometer.

**Diffusers used in experiments**

Three types of diffusers were used in the experiments. Diffuser
A in Figure 30 was designed and constructed to represent the inflow
behavior of Pond A-1 in the Logan pond system. It was the same length
as the diffuser in Pond A-1 (reduced by the length ratio between model
and prototype) that being 8 feet long. Diffuser B in Figure 30 was 2
feet long and was constructed like diffuser A. Diffuser C was a vertical
diffuser 2 feet in height which was designed to break up and distribute
the flow uniformly around the diffuser. Diffuser C was constructed by
drilling a large number of holes around the two foot length of plastic
pipe and then placing the pipe in a cylindrical screen which was 2 inches
greater in diameter than the pipe. This annular space was then filled with graded gravel.

Experimental Method and Design

Test procedure

1. Tank was filled to desired depth with river water and allowed to become quiescent to enable the eddies and currents to die out.

2. Inlet and outlet flowrate was adjusted to desired quantity.

3. The fluorometer was turned on.

4. Saltwater was injected at desired rate to give desired density of incoming flow. (For constant density experiments, this step was omitted.)

5. The model was operated for a certain time to allow for the flow to stabilize.

6. A known quantity of rhodamine WT 20 percent dye was injected into the inlet essentially as a slug. The recorder on the fluorometer was started at this same time.

7. The experiment was terminated when two theoretical detention times were reached.

8. The temperature of the fluid at the outlet was measured at different times during the experiment.
Data analysis and design of experiments

The raw data from the strip charts were extracted at equal intervals of time and the concentrations of the tracer were determined from temperature dependent calibration charts. The data were processed using computer program N-1 to calculate dimensionless C and F-diagrams and other pertinent parameters. The computer program used a simple numerical method for making the required calculations.

The following are the different groups of experiments that were performed.

1. Identically designed experiments to determine reproducibility.

These experiments all had the following dimensions and hydraulic characteristics.

\[
\begin{align*}
\text{Length} & = 40 \text{ feet} \\
\text{Width} & = 20 \text{ feet} \\
\text{Depth} & = 1.5 \text{ feet} \\
Q & = 80 \text{ gpm}
\end{align*}
\]

The 8 foot diffuser was centered in the model 8 feet from the end wall. The outlet consisted of one 2\(\frac{1}{2}\) inch diameter pipe located one foot from the floor of the model and one foot from the side wall.

2. Reynolds number variations - The model was designed the same as number 1 above except that \(Q\) was varied for the purpose of determining the effect of the Reynolds number on the C-diagram and pertinent parameters.
3. Depth variations - The model was designed the same as number 1 above except the depth was varied for the purpose of determining the effect of depth on the flow patterns.

4. Outlet and inlet variations - The model was designed as number 1 above except the inlet and outlet devices were varied as well as $Q$ in some instances for the purpose of evaluating inlets and outlets in terms of the hydraulic efficiency.

5. Length to width ratio variations - The 8 foot diffuser was placed against the end wall centered between the two side walls of the model pond. The length was varied for most of the experiments with the proper placement of an extra wall with the width held constant. The depth was 1.5 feet and $Q = 80$ gpm generally. The purpose here was to determine the effect of the $L/W$ ratio on the hydraulic efficiency.

6. Baffling - The inlet diffuser was placed on the side wall at one end of the pond and baffles were placed in the pond in such a fashion as to cause the flow to meander on its way to the pond exit. The spacing and length of the baffles were varied to determine the effect on hydraulic efficiency. Over and under baffles were also used to force the flow to mix vertically.

7. Verification experiments for comparing with existing ponds - These model experiments were designed according to the proper model laws and selected geometrical ratios. The purpose of these experiments was to verify existing pond behavior.
8. Density stratified flow experiments - The model was designed as the experiments in number 1 except the pond was either filled with a saltwater of predetermined density or the inflow was saltwater of appropriate density to give the desired densimetric Froude number. The desired density of the inflow to the model pond was obtained by mixing saltwater, containing a high concentration of salt, with the incoming fresh water in the proper quantities to give the inflow the desired density. The highly concentrated saltwater was pumped from the small supply tank, shown in Figure 29, through a constant level tank and into the suction side of the booster pump, located immediately in front of the flowmeter. An orifice meter and valve were used to control the saltwater flowrate.

The purpose of the density stratified flow experiments was to determine the influence of density differences, quantitatively measured by the densimetric Froude number, on the C-diagrams and other pertinent parameters.
CHAPTER 5

PRESENTATION AND DISCUSSION OF RESULTS

Verification of the Hydraulic Model

Experimental data from Experiments 4, 5, and 6 of Chapter 3 for ponds D, A-I, and E, respectively, were used to compare with the model experiments for verification purposes. The results of the model experiments performed to verify existing pond behavior were divided into two main groups: (1) unstratified flow (Reynolds number criterion) and (2) density stratified flow (densimetric Froude number criterion).

Plain flow (constant density)

Hydraulic model experiments were performed to verify the experimental tracer data of ponds D and E of the Logan pond system. The model parameters were determined using Reynolds number criterion since the prototype data for ponds D and E indicated constant density flows.

Figure 31 gives the dimensionless C-diagrams for the prototype data, P-2 and two model runs, M-12 and M-4. Table 1 gives the model and prototype hydraulic parameters, the values for the dimensionless mean residence time, plug flow deviation parameter, dead space, and the modeling parameters.
$\left( \frac{c}{c_0} \right)_{pk}$ of M-12 = 2.72

Figure 31. C-diagram for Experiments P-2, M-12, and M-4.
Table 1. Model and prototype pond data and experimental results for constant density experiments.

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Depth (ft)</th>
<th>Width (ft)</th>
<th>Length (ft)</th>
<th>Q (gpm)</th>
<th>Re No.</th>
<th>T (min)</th>
<th>$\theta_c$ @ 2.0</th>
<th>$\theta_b$</th>
<th>$\theta_{pk}$ (c/c_o)_{pk}</th>
<th>$\theta_{pf}$</th>
<th>$\bar{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-2(pond D)</td>
<td>7.5</td>
<td>870</td>
<td>1980</td>
<td>4039</td>
<td>617</td>
<td>398.8 hr.</td>
<td>.652</td>
<td>.080</td>
<td>.183</td>
<td>1.94</td>
<td>.509</td>
</tr>
<tr>
<td>M-12</td>
<td>1.5</td>
<td>20</td>
<td>40</td>
<td>76</td>
<td>613</td>
<td>118.1</td>
<td>.625</td>
<td>.103</td>
<td>.159</td>
<td>2.72</td>
<td>.573</td>
</tr>
<tr>
<td>M-4</td>
<td>1.5</td>
<td>20</td>
<td>40</td>
<td>80</td>
<td>606</td>
<td>112.2</td>
<td>.632</td>
<td>.083</td>
<td>.176</td>
<td>1.75</td>
<td>.555</td>
</tr>
<tr>
<td>P-5(pond E)</td>
<td>7.75</td>
<td>870</td>
<td>1421</td>
<td>6790</td>
<td>1040</td>
<td>175.9 hr.</td>
<td>.599</td>
<td>.026</td>
<td>.063</td>
<td>3.42</td>
<td>.646</td>
</tr>
<tr>
<td>M-6</td>
<td>1.5</td>
<td>20</td>
<td>32.5</td>
<td>80</td>
<td>594</td>
<td>91.2</td>
<td>.560</td>
<td>.101</td>
<td>.165</td>
<td>3.67</td>
<td>.630</td>
</tr>
</tbody>
</table>

Modeling Parameters (Experiments P-2, M-12 and M-4)

\[
\begin{align*}
(d_o)_{r} &= 1/5^{(a)} \\
R_e &= \frac{v_{r}(d_o)_{r}}{v_r} = \frac{Q_r}{L_r} = 1.0 \\
L_r &= 1/46 \\
v_r &= .915 \\
\tau_r &= \frac{L_r d_r}{v_r} = 1/210
\end{align*}
\]

Modeling Parameters (Experiments P-5 and M-6)

\[
\begin{align*}
(d_o)_{r} &= 1/5.17 \\
Q_r &= L_r v_r = 1/45.7^{(b)} \\
L_r &= 1/43.5 \\
\tau_r &= \frac{L_r^2}{Q_r} \\
(d_o)_{r} &= 1/125.4
\end{align*}
\]

(a) $(d_o)_{r}$ means the depth of the model divided by the depth of the prototype.

(b) $Q_r$ was not satisfied for M-6.
The main difficulty in modeling pond D was that of duplicating the inlet flow patterns. The difference between model Experiment M-12 and M-4 was that of inlet design. It was found by trial and error that the inlet jet in the model had to be greatly diffused in order to approximate the correct peak concentration of the tracer, the peak time, and the time when the tracer first appeared at the outlet. The C-diagrams for Experiments M-12 and M-4 show the sensitivity of the inlet design in the modeling of pond D. Experiment M-4 appears to be fairly close to P-2 but the inlet design was somewhat arbitrary in that the inlet flow was diffused in such a way as to nullify the jet. This influence of the inlet indicates a need to be able to model the spread of prototype jet flows. An improvement in inlet design would undoubtedly lead to better duplication of the prototype by hydraulic model studies.

For Experiment M-4 the 2 foot long diffuser described in Chapter 4 was used. It was laid on the bottom of the model pond in the appropriate location to satisfy geometrical similarity.

Figure 31 for Experiments P-2, M-12, and M-4 shows the typical humps in the C-diagram that result from the gross circulation patterns that are developed by the inlet flow jet. Visual observation of the model experiments verified the general counter-clockwise circulation patterns around the pond that were observed in Experiments 3 and 4 of Chapter 3. It was found that the degree of short circuiting with inlets, such as exist in pond D, is affected by the Reynolds number and also the model inlet design.
The M-4 represents a good model representation of pond D. The C-diagrams for P-2 and M-4 are quite similar and the values of the dimensionless mean, dimensionless beginning time and time to peak, peak concentration, and the plug flow deviation parameter are all within 10 percent of each other. Dead space differs by 14 percent for P-2 and M-4. This difference is felt to be within the realm of experimental reproducibility.

In verifying Experiment 4 on pond E the same problem of modeling the inlet flow pattern was encountered. Pond E has two 30 inch inlet pipes that lay on the bottom of the pond discharging the flow south from the northwest corner of the pond (see Figure 4).

The method of modeling the inflow was the same as for pond D except that the 2 foot diffuser was placed in the proper location for geometrical similarity of pond E.

Figure 32 gives the C-diagrams for Experiment 7, P-5, on pond E and the verification Experiment, M-6. Table 1 gives the dimensions, hydraulic data, and the pertinent parameters determined from the experimental results of P-5 and M-6. The dimensionless mean, \( \bar{c}_{pf} \) and peak concentration are fairly close in numerical value. The time to peak and beginning time of the tracer is off considerably along with the dead space quantity.

In modeling ponds D and E, the main problem seems to be that of inlet design. The dimensionless concentration versus time curves are
Figure 32. C-diagram for Experiments for P-5 and M-6.
very sensitive, especially the front portion, to inlet effects. It has been observed that undiffused inlet flows such as exist in ponds D and E, create pond-wide circulation patterns which have a direct influence on the degree of short circuiting. This same type of behavior took place in the model studies to varying degrees depending on the type of inlet used. It was found that the more the incoming flow was diffused, the closer the model could be made to resemble the actual pond results. The difference that exists in the results for P-5 and M-6 is attributed mainly to that of inlet design. The effect of decreasing the flowrate ratio between model and prototype for pond E (i.e. \( Q_m < Q_{m} \) computed from Reynolds number criterion) probably had an effect also on the results of Experiment M-6 in relation to P-5.

Density stratified flow

The model experiment to verify Experiment 6 on primary pond A-1 was designed according to the densimetric Froude number criterion. If the model experiment had been designed according to the densimetric Froude-Reynolds number criterion, very large quantities of salt would have been needed to give the proper density ratio between model and prototype. Therefore a lower flowrate was chosen but still resulting turbulent flow, so as to reduce the amount of salt needed.

Figure 33 gives the C-diagrams for Experiment P-3 in pond A-1, and the model verification Experiment, M-7. Table 2 gives the pertinent data and experimental results for these two experiments where the
Figure 33. C-diagrams for Experiments P-3 and M-7.
Table 2. Model and prototype pond data and experimental results for stratified flow experiments.

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Depth (ft)</th>
<th>Width (ft)</th>
<th>Length (ft)</th>
<th>Q (gpm)</th>
<th>Re</th>
<th>t (min)</th>
<th>$\bar{\theta}_c$</th>
<th>$\theta_b$</th>
<th>$\theta_{pk}$ (c/c_o)_{pk}</th>
<th>$\bar{\theta}_{pf}$</th>
<th>$\bar{V}_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-3 (pond A-1)</td>
<td>5.5</td>
<td>1421</td>
<td>2912</td>
<td>4765</td>
<td>690</td>
<td>595.4 hr.</td>
<td>.586</td>
<td>.017</td>
<td>.092</td>
<td>1.78</td>
<td>.596</td>
</tr>
<tr>
<td>M-7</td>
<td>2.5</td>
<td>20</td>
<td>40</td>
<td>50</td>
<td>362</td>
<td>299.2</td>
<td>.706</td>
<td>.086</td>
<td>.108</td>
<td>1.73</td>
<td>.443</td>
</tr>
</tbody>
</table>

Modeling Parameters

$$F_\Delta = \frac{V}{\sqrt{g_d_o \left(\frac{\Delta \rho}{\rho}\right)}}$$

$$\left(\frac{\Delta \rho}{\rho}\right)_r = \frac{Q_r^2}{L_r^2 d_o^3} = 8.38$$

$$t_r = \frac{L_r^2 d_o}{Q_r} = 1/130$$

$$l = \frac{V_r}{\sqrt{g_r d_o \left(\frac{\Delta \rho}{\rho}\right)_r}} = \frac{Q_r}{L_r d_o^{3/2} \sqrt{g_r \left(\frac{\Delta \rho}{\rho}\right)_r}}$$

$$L_r = 1/75$$

$$(d_o)_r = 1/2.2$$

$$Q_r = 1/95.3$$
inflow was more dense than the receiving fluid. It is easy to recognize from Figure 33 and the results in Table 2 that the model did not fit the prototype experimental data very well. There are a number of possible reasons for this lack of fit of model and prototype experiments. Some of these are:

1. Reynolds number equality was not followed. With the large model that was used in the model experiments, a reduction in the flow-rate was a necessity due to the large amounts of salt that would have been needed to give the proper density differences. It is not known exactly what effect this had except that the Reynolds number should have been greater which would have resulted in a higher turbulence level of the flow.

In visually observing Experiment M-7, streaks of tracer were noticed during the experiment. These streaks of concentrated tracer show up at the outlet concentration versus time curve as peaks. The obvious lack of mixing which did occur in the pond A-1 after a certain time is probably due to low turbulence level of the model experiment, particularly near the bottom where the tracer was highest in concentration.

2. The data on pond A-1 were corrected due to the loss of tracer that occurred during Experiment 6 (Chapter 3). Therefore it is felt that the model data are probably as reliable as the prototype data.

3. Another possible reason is attributed to the approximate modeling of density currents and stratified flow situations with salt water to
give the necessary density difference. But this was necessary, as it was impossible for the water to be heated sufficiently to give the required density difference to satisfy the densimetric Froude number criterion.

4. The last reason might be due to errors in flowrate measurement, measurement of depth and temperature, fluorometer calibration curves, variations in inlet flow patterns and others which are involved in experimental data taking.

In summarizing, it is concluded that existing pond behavior can be verified and simulated in the lab for situations involving plain or constant density flows as long as the inlet flow pattern can be closely approximated. (Proper simulation of prototype inlet design.)

It is not known how well density-stratified flows in stabilization ponds can be modeled. To be able to determine the validity of the modeling laws, better data must be taken on existing ponds to be used to compare with the hydraulic model. The prototype data, on density stratified flows, taken in this study were not adequate for a proper accounting of the modeling laws. The main reasons for the lack of reliable data for density stratified flows in this study were due to two main factors: (1) The large size of the primary ponds (A-1 and A-2 of Figure 9) which posed problems for obtaining data due to the very long theoretical detention times, and (2) the environmental and hydraulic changes that occurred. These two factors actually had an effect on the experiments that were performed. (Refer to Experiments 5 and 6, Chapter 3.)
Hydraulic Model Study Results

Plain flow (constant density)

Experimental reproducibility

A number of experiments were performed on the hydraulic model to determine the extent of the reproducibility of the results to obtain an idea of the reliability of the model data. For this series of experiments, including the experiments on depth and Reynolds number variations, the model was designed as outlined in Chapter 5.

Figures 34, 35, and 36 show the C-diagrams for the experiments performed with the hydraulic and geometric conditions given in Table 3, groups A, A-1, and A-2 respectively. Table 3 also gives the values for the important parameters that were computed for each experiment. The C-diagrams for these sets of experiments, exhibit quite a bit of variability particularly between \( \theta = 0.0 \) to \( \theta = 1.0 \). The parameter variation tabulated in Table 3 is not as extreme as the C-diagrams seem to indicate. If the lack of exact reproducibility is assumed to be of the order of 10 percent, then the variability about the dimensionless mean, \( \bar{\theta}_c \), plug flow deviation parameter, \( \bar{\theta}_{pf} \), and dead space, \( \bar{V}_d \), are within the realm of experimental reproducibility. It obviously takes more pronounced variations in the C-diagram to greatly effect these parameters.

There are a number of factors that have an influence on reproducibility and are important to consider in determining the reliability of the
Figure 34. C-diagrams for Experiments D-53, D-54, and D-56.
Figure 35. C-diagrams for Experiments R-21, R-23, and R-24.

\[
\frac{c}{c_0}_{pk} \text{ of R-23} = 2.93
\]
Figure 36. C-diagrams for Experiments O-6, O-61, and O-62.
Table 3. Model pond data and experimental results for constant density experiments.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Depth (ft)</th>
<th>Q (gpm)</th>
<th>Re No.</th>
<th>$\overline{t}$ (min)</th>
<th>$\overline{\theta}_c$</th>
<th>$\overline{\theta}_pf$</th>
<th>$\overline{V}_d$</th>
<th>$d_0/W$</th>
<th>$F$ $\theta=2.0$</th>
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<tr>
<td><strong>Group A</strong></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>D-53</td>
<td>1.5</td>
<td>80</td>
<td>632</td>
<td>112.2</td>
<td>.620</td>
<td>.494</td>
<td>.450</td>
<td>.075</td>
<td>.886</td>
</tr>
<tr>
<td>D-54</td>
<td>1.5</td>
<td>80</td>
<td>594</td>
<td>112.2</td>
<td>.638</td>
<td>.536</td>
<td>.479</td>
<td>.075</td>
<td>.816</td>
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<tr>
<td>D-56</td>
<td>1.5</td>
<td>80</td>
<td>564</td>
<td>112.2</td>
<td>.622</td>
<td>.545</td>
<td>.578</td>
<td>.075</td>
<td>.679</td>
</tr>
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<td><strong>Group A-1</strong></td>
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</tr>
<tr>
<td>R-21</td>
<td>1.5</td>
<td>70</td>
<td>619</td>
<td>128.2</td>
<td>.603</td>
<td>.527</td>
<td>.464</td>
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<td>.888</td>
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<td>R-23</td>
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<td>.574</td>
<td>.562</td>
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<td>619</td>
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<td>.514</td>
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<td>.856</td>
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<tr>
<td><strong>Group A-2</strong></td>
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<td>0-6</td>
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<td>662</td>
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<td>687</td>
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<td>.558</td>
<td>.490</td>
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<td>.746</td>
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<tr>
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<td>.621</td>
<td>.673</td>
<td>.075</td>
<td>.632</td>
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<td>D-56</td>
<td>1.5</td>
<td>80</td>
<td>564</td>
<td>112.2</td>
<td>.622</td>
<td>.545</td>
<td>.578</td>
<td>.075</td>
<td>.679</td>
</tr>
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<td><strong>Group B</strong></td>
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<tr>
<td>D-12</td>
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<td>80</td>
<td>632</td>
<td>224.3</td>
<td>.429</td>
<td>.603</td>
<td>.710</td>
<td>.150</td>
<td>.677</td>
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<tr>
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<td>632</td>
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<td>.565</td>
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<td>.100</td>
<td>.896</td>
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<td>.528</td>
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<td>.818</td>
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<td>D-62</td>
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<td>632</td>
<td>93.5</td>
<td>.566</td>
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<td>.604</td>
<td>.0625</td>
<td>.700</td>
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<td><strong>Group C</strong></td>
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<td>840</td>
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<td>395</td>
<td>179.5</td>
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<td>.814</td>
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<tr>
<td>D-54</td>
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<td>80</td>
<td>594</td>
<td>112.2</td>
<td>.638</td>
<td>.536</td>
<td>.479</td>
<td>--</td>
<td>.816</td>
</tr>
</tbody>
</table>
data. Following is a list of the most important ones:

1. Residual eddies or currents resulting from filling the model pond prior to each experiment.

2. Variations in flow patterns that depend, to a considerable degree, on the length of time the model was run for stabilization purposes before experiment was started.

3. Errors in flow measurement at both the inlet and outlet of the model pond.

4. Temperature variations of fluid during the experiment. The temperature changes affect density changes of the fluid that could result in density currents. Also, the concentration readings obtained with the fluorometer are dependent upon the temperature and temperature fluctuations affect the fluorometer readings.

5. Errors in the calibration curves for the fluorometer.

6. Errors in geometrical measurements, particularly depth measurements.

7. Errors due to random fluid behavior in inlet and outlet flows and random large scale turbulence of flow within the pond.

Figure 37 gives the C-diagrams for two separate experiments that were performed to obtain an idea of how factor (2) could affect the results of the experiments performed. The results are tabulated as group A-3 of Table 3. A simple explanation of the differences that exist can be given as follows: In order for the eventual flow patterns to stabilize,
Figure 37. C-diagrams for Experiments D-55 and D-56.
sufficient time must pass for the moving fluid particles to exert shearing stresses to effectuate movement of the adjacent fluid particles and they in turn exert shearing stress on other adjacent fluid particles until all the fluid within the pond is moving as it eventually would after a long period of time. Experiments D-55 and D-56 were not started until the fluid within the pond had stood overnight to allow for the currents and eddies, resulting from the filling procedure, to die out. The difference between these two experiments was that experiment D-55 was run at the proper flowrate for 20 minutes before injecting the tracer into the pond while D-56 was operated for over $2\frac{1}{2}$ hours before dye injection. The resulting C-diagrams verify the explanation above. Experiment D-55 had a higher peak concentration and a greater mass of dye leaving the pond at the beginning than did D-56, indicating a more serious short circuiting problem. This is because insufficient time had elapsed during the stabilization phase to effectuate total fluid movement within the pond which resulted in a reduced area of flow and subsequently higher concentrations of tracer leaving the pond.

In evaluating the importance of changes in pond geometry or the hydraulic characteristics of the ponds and their effects on the hydraulic efficiency, all experimental variability must be considered and taken into account.
Effect of depth

A number of experiments were performed to evaluate the effect of changing the depth with all other variables held constant. Group A-1 and Group B of Table 3 are the results of these experiments. Figure 38 shows the plot of these parameters versus the depth to width ratio $d_o/W$, for each experiment. From the plots, as the $d_o/W$ ratio increases, i.e. as the depth increases, $\bar{V_d}$ increases, $\bar{\theta}_{pf}$ increases, and $\bar{\theta}_c$ decreases, which all indicate less hydraulic efficiency and subsequently less biological conversion of organic matter. There is considerable scatter but the data plotted in Figure 38 suggest a trend probably exists.

Effect of Reynolds number

Groups A, A-1, and C of Table 3 contain the results of the experiments conducted to show the effect of Reynolds number. The Reynolds number was defined and used as

$$Re = \frac{V d_o}{\nu} = \frac{Q}{W \nu}$$

in which

$\Omega$ = flowrate
$W$ = width of pond
$d_o$ = depth of pond
$\nu$ = kinematic viscosity

Figure 39 shows $\bar{V_d}$, $\bar{\theta}_{pf}$, and $\bar{\theta}_c$ plotted against the Reynolds number. When considering the possible variability just due to experimental error,
Figure 38. Plot of $\overline{\theta_c}$, $\overline{\theta_{pf}}$, and $\overline{V_d}$ vs. $d_o/W$ for D-series experiments.
Figure 39. Plot of $\bar{\theta}_c$, $\bar{\theta}_{pf}$, and $V_d$ vs. $R_e$ for R-series experiments.
Figure 39 shows essentially no relationship for these parameters as a function of Reynolds number. Dead space and $\bar{\theta}_{pf}$ show a slight decrease as $Re$ increases which means greater efficiency of operation. At higher Reynolds numbers, this would be expected due to greater turbulence and larger diffusivity coefficients which result in greater mixing and interchange of the fluid particles.

Effects of inlets and outlets

Two types of outlet configurations were tried along with a number of inlet variations. Figure 40 shows the inlet and outlet configurations that were used in this study. D and E of Table 4 give the results of these experiments.

Figure 41 illustrates the C-diagrams for the experiments of group D. From this figure and from Table 4 it appears that O-5 is the best design for maximum hydraulic efficiency. It has the highest dimensionless mean retention time and the lowest values for $\bar{\theta}_{pf}$ and dead space which indicate maximum efficiency. Figure 42 shows the C-diagrams for the experiments of Group E of Table 4. From these results and in visually evaluating the C-diagrams of Figure 42, it is not readily evident which is best, but probably I-5 and M-4 would give maximum hydraulic efficiency.

Length to width ratio effects

Figure 43 shows how the model was modified to determine the effects of $L/W$ ratio changes on the hydraulic efficiency of ponds. The
Figure 40. Inlet and outlet configurations employed.
Table 4. Model pond data and experimental results for constant density experiments.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Depth (ft)</th>
<th>Q (gpm)</th>
<th>Re No.</th>
<th>( \bar{t} ) (min)</th>
<th>( \bar{\theta}_c )</th>
<th>( \bar{\theta}_{pf} )</th>
<th>( \bar{\nu}_d )</th>
<th>L/W</th>
<th>F</th>
<th>( \Theta = 2.0 )</th>
</tr>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-1</td>
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<td>72.0</td>
<td>561</td>
<td>145.4</td>
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<td>.534</td>
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<td>.844</td>
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</tr>
<tr>
<td>0-2</td>
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<td>72.0</td>
<td>561</td>
<td>145.4</td>
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<td>.599</td>
<td>.614</td>
<td>2.0</td>
<td>.744</td>
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</tr>
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<td>.572</td>
<td>.574</td>
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<td>.778</td>
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<td>.814</td>
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</tr>
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<td>.668</td>
<td>.570</td>
<td>2.0</td>
<td>.944</td>
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<tr>
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<td>80</td>
<td>606</td>
<td>112.2</td>
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<td>.576</td>
<td>.493</td>
<td>2.0</td>
<td>.791</td>
<td></td>
</tr>
<tr>
<td>M-4</td>
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<td>80</td>
<td>606</td>
<td>112.2</td>
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<td>.555</td>
<td>.479</td>
<td>2.0</td>
<td>.824</td>
<td></td>
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<tr>
<td>I-5</td>
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<td>632</td>
<td>112.2</td>
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<td>.552</td>
<td>.449</td>
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<td>.887</td>
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</tr>
<tr>
<td>Group F</td>
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<td></td>
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<tr>
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<td>.508</td>
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<td>586</td>
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<td>.613</td>
<td>.569</td>
<td>.625</td>
<td>.769</td>
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</tr>
<tr>
<td>G-4</td>
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<td>80</td>
<td>578</td>
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<td>.616</td>
<td>.571</td>
<td>.750</td>
<td>.740</td>
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<td>80</td>
<td>578</td>
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<td>.656</td>
<td>.579</td>
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</tr>
<tr>
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<td>632</td>
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<td>.503</td>
<td>.545</td>
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Figure 41. C-diagrams for Experiments O-1, O-2, O-3 and O-5.
Figure 42. C-diagrams for Experiments M-3, M-4, O-4 and I-5.

\[
\left( \frac{c}{c_0} \right)_{pk} \text{ of O-4} = 10.5
\]
G-series utilized a movable wall as shown in Figure 43. The results of these experiments are tabulated in Table 4 as Group F. Figure 44 gives the experimentally determined C-diagrams for these experiments. In referring to the C-diagrams and the tabulated results for Experiments G-2, G-3, G-4, and G-5, an increase in the hydraulic efficiency occurs for an increase in the L/W ratio. As the L/W ratio increases, the conditions for plug flow are closer to being satisfied and as an end result, the efficiency of the treatment increases.

Baffling effects

There were 17 tests performed using different arrangements of baffles. These tests are divided into 3 series. The first series utilized 12 tests with horizontal baffling. The tests of the first series are divided into 3 groups, each group having a different baffle length. The dimensionless
Figure 44. C-diagrams for experiments with different length to width ratios.
parameter used to express baffle length is the length of the baffles $WB$ divided by the width of the pond $B$. $WB/B$ for each of the three groups is 0.50, 0.70, and 0.90, respectively. (See Figure 45.)

For each group four different baffle spacings were used. The spacing of baffles is defined by a dimensionless parameter obtained by dividing spacing of the baffles $X$ by the length of the pond $L$. Since the length of the pond is constant, this parameter is related directly to the baffle spacing. The four spacings used, expressed as $X/L$, are: 1/3, 1/5, 1/7, and 1/9, respectively. These spacings were achieved by using 2, 4, 6, and 8 baffles, respectively.

Each experiment is numbered to represent the particular baffling scheme. The first digit of the number represents $L/X$ followed by the decimal form of $WB/B$. An experiment numbered 3.5 denotes that $L/X = 3$ and $WB/B = 0.50$. Figure 46 shows C-diagrams for the group of tests showing $WB/B = 0.50$. Figure 47 and Figure 48 show C-diagrams of groups of tests with $WB/B = 0.70$ and 0.90, respectively.

Figure 45. Model pond showing baffle configuration.
Figure 46. C-diagrams for experiments 3.5, 5.5, 7.5, and 9.5.
Figure 47. C-diagrams for experiments 3.7, 5.7, 7.7, and 9.7.
Figure 48. C-diagrams for experiments 3.9, 5.9, 7.9, and 9.9.
The three parameters, WB/B, X/L, and $\bar{\theta}_{pf}$ are plotted in Figure 49. With $\bar{\theta}_{pf}$ as the ordinate and X/L as the abscissa, a curve for each value of WB/B is plotted. Table 5 shows some selected parameters for each experiment with explanations of the symbols below the table. Two important points should be noted when examining the plots in Figure 49. First, the reverse curvature of the curve for WB/B = .50 indicates the best performance with X/L at approximately 1/7. That is $\bar{\theta}_{pf}$ decreases as X/L became smaller, until X/L became equal to 1/7, then efficiency begins to decrease again. This trend can be explained with the help of Figure 50 which shows a drawing of the pond with the baffles in place. It was observed during the tests that, as more baffles were added to the pond they developed a pattern of short circuiting, as shown in Figure 50. When X/L became less than 1/7, this pattern became strong enough to dominate the flow pattern of the pond. This short circuiting occurred through the center of the pond around the end of each baffle, thereby increasing $\bar{\theta}_{pf}$. This short circuiting did not occur with larger values of WB/B because the flow was more directly confined. The longer baffles forced the flow out toward the side of the pond.

The second unexpected characteristic shown in Figure 49 is that the curve for WB/B = .70 is below the curve for WB/B = .90. This unexpected characteristic can be explained with the help of Figure 51. With WB/B = .90 the baffles extend 90 percent across the width of the
Figure 49. Plot of $\bar{\theta}_{pf}$ versus $X/L$ versus WB/B.
Table 5. Baffled pond experimental results.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\bar{\theta}_c$</th>
<th>$\bar{\theta}_pf$</th>
<th>$\theta_b$</th>
<th>$\theta_{pk}$</th>
<th>F</th>
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<td>.267</td>
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<td>.850</td>
</tr>
<tr>
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<td>.668</td>
<td>.630</td>
</tr>
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<td>.314</td>
<td>.144</td>
<td>.798</td>
<td>.866</td>
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<td>.251</td>
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<td>.794</td>
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<td>.467</td>
<td>.827</td>
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<td>.782</td>
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<td>.884</td>
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<td>5.9</td>
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<td>.283</td>
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<td>.954</td>
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<td>7.9</td>
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<td>.417</td>
<td>.803</td>
<td>.985</td>
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<tr>
<td>9.9</td>
<td>.947</td>
<td>.188</td>
<td>.442</td>
<td>.812</td>
<td>.952</td>
</tr>
<tr>
<td>No baffles</td>
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<td>.550</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>.234</td>
<td>.610</td>
<td>.828</td>
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<td>.802</td>
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<td>OU-3</td>
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<td>.234</td>
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<td>C-3</td>
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<td>.250</td>
<td>.359</td>
<td>.677</td>
<td>.870</td>
</tr>
</tbody>
</table>

Explanation of Symbols

$\bar{\theta}_c$: Dimensionless mean residence time
$\bar{\theta}_pf$: Deviation from plug flow
$\theta_b$: Dimensionless time for dye to reach the outlet
$\theta_{pk}$: Dimensionless time for dye concentration to reach a maximum
F: The F-distribution parameter
Figure 50. Experiment No. 9.5 showing short circuiting between baffles. Showing model pond in plan view.

Figure 51. Experiment No. 5.9 showing currents created by baffles extended .90 across the width of the pond. Showing model pond in plan view.
pond which creates a very narrow channel for the water to flow around the end of the baffles, and thus increases the velocity. As seen in Figure 51, as the dye goes around the first baffle the velocity is increased and as it enters the area between the first and second baffles the increased velocity carries the dye along the edge of the second baffle. The velocity remains almost constant and the width of the dye path was observed to remain almost constant. This pattern continues around each baffle until it reaches the outlet. A small portion of the dye remains between each of the baffles and the rest is carried to the outlet in a comparably shorter time than with $WB/B = .70$. This flow pattern increases $\theta_{pf}$ and therefore decreases the hydraulic efficiency of the pond.

The second series of tests were 4 tests designed to evaluate the effect of vertical baffling on the hydraulic efficiency. The baffle configuration is shown on Figure 52. The results of this series of tests is tabulated in Table 5. It is interesting to note that comparing OU-1 with OU-3 and OU-2 with OU-4 leads to the conclusion that 4 baffles are better than 6. The cause of this unexpected result has already been observed and discussed previously for horizontal baffle schemes. Table 5 also indicates that, in general, the horizontal baffling schemes seem to be more efficient for a comparable number of baffles.

The third series of tests was just one test. Its purpose was to determine if longitudinal baffling was more efficient than transverse
<table>
<thead>
<tr>
<th>Experiment</th>
<th>No. of Baffles</th>
<th>$y$-ft</th>
<th>$d_0$-ft</th>
<th>$y/d_0$</th>
</tr>
</thead>
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<tr>
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<td>.50</td>
<td>1.50</td>
<td>.333</td>
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<tr>
<td>OU-4</td>
<td>6</td>
<td>.915</td>
<td>1.915</td>
<td>.477</td>
</tr>
</tbody>
</table>

Figure 52. Vertical baffle configuration.

Figure 53. Longitudinal baffle configuration.
baffling. The baffle configuration is shown on Figure 53 and the experimental results are listed in Table 5 identified as C-3. The width of the flow passage for this scheme is 6.7 feet which suggests it would compare fairly with schemes 5.7 and 7.7 of Table 5 because they have channel widths of 8 feet and 5.7 feet, respectively. Both schemes 5.7 and 7.7 have about a 6 foot spacing between the end of the baffle and the wall. Comparison of these two schemes shows that the longitudinal baffling scheme is midway between scheme 5.7 and 7.7 in terms of efficiency. This result strongly suggests that both longitudinal and transverse baffling give essentially the same results.

Discussion of plain flow experiments

It was mentioned previously that it apparently takes significant changes in geometry or in the hydraulic characteristics to greatly affect the parameters $\bar{\theta}_c$, $\bar{\theta}_p$, and $\bar{V}_d$. In reviewing the results of the model experiments and also the existing pond Experiments P-1, P-2, and P-5 for plain flow tabulated in Tables 3 and 4, it is recognized that there is not a great deal of variability between the experiments. The values of $\bar{\theta}_c$ for most of the experiments varied between $\bar{\theta}_c = .50$ to $\bar{\theta}_c = .65$, for $\bar{\theta}_p$ between $\bar{\theta}_p = .50$ to $\bar{\theta}_p = .65$ and for dead space between $\bar{V}_d = .45$ to $\bar{V}_d = .60$. For one type of design in comparison with
another there might be an increase in $\bar{\theta}_c$ of 30 percent and a decrease of $\bar{\theta}_{pf}$ and $\bar{V}_d$ of 30 percent. This would probably not result in a straight increase of 30 percent in treatment efficiency as the efficiency would be determined by Equation 28 with $c/c_0$ for both pond designs.

A dramatic exception to this conclusion appears in the baffled pond experiments. As shown in Figure 49, $\bar{\theta}_{pf}$ values dropped from about .50 to .60 for unbaffled ponds to as low as .10 to .20 for properly baffled ponds. This result is logical, for as baffles are placed in the ponds the length of travel of each fluid particle is increased and the width of flow is decreased thereby guiding the flow so that plug flow conditions are more closely approached.

The conditions for plug flow are desirable for maximum treatment efficiency, provided sufficient mixing occurs between the inflow and the microorganisms in the pond to cause treatment to occur. Since an intimate contact of degradable organic matter and the microorganisms is necessary, the desirable flow situation would be where the inflow laterally and vertically mixes with the ambient fluid then flows as a slug towards the outlet. This would result in minimum values for $\bar{\theta}_{pf}$ and $\bar{V}_d$ and a maximum value for $\bar{\theta}_c$ which in turn would increase
the treatment efficiency for a given pond design.

**Stratified flow (variable density)**

The model was designed as was described in Chapter 4. Prior to the placement of dye for each experiment, the flowrate into and out of the model was set at the proper value for a certain length of time to allow for the flow to stabilize. The stabilizing time for the stratified flow experiments was considerably less than the stabilizing time for the constant density experiments. The reduced time was necessary due to the problem of salt buildup or washout that would occur for long periods of model operation prior to dye injection. The stabilizing time was arbitrarily chosen between 15 and 30 minutes depending on the flowrate.

Table 6 gives a list of the density stratified flow experiments with the experimental results that were performed. The density of the inflow of group A was greater than the density of the pond fluid while the reverse was true for the experiments listed in group B. Figure 54 illustrates some typical C-diagrams for two of the density stratified flow experiments. Also shown in this figure is the constant density Experiment D-55. In all three of these experiments the flow was allowed to stabilize for about 20 minutes before dye injection. These curves show that in
Table 6. Model pond data and experimental results for stratified flow experiments.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Depth (ft)</th>
<th>Q (gpm)</th>
<th>Re</th>
<th>$\tilde{t}_c$ (min)</th>
<th>$\tilde{v}_b$</th>
<th>$\tilde{v}_p$</th>
<th>$\tilde{v}_f$</th>
<th>$\tilde{v}_D$</th>
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<td></td>
<td></td>
</tr>
<tr>
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<td>.086</td>
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<tr>
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<tr>
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<td>112.2</td>
<td>.540</td>
<td>.060</td>
<td>.097</td>
<td>.623</td>
<td>.586</td>
</tr>
<tr>
<td>S-5</td>
<td>1.5</td>
<td>80</td>
<td>598</td>
<td>112.2</td>
<td>.497</td>
<td>.057</td>
<td>.072</td>
<td>.655</td>
<td>.627</td>
</tr>
</tbody>
</table>
Figure 54. C-diagrams for Experiments S-2 and S-4 (density stratified flow) and D-55.
general when the density of the influent is greater than in the receiving body, the values of the time to peak \((\theta_{pk})\), beginning time, \((\theta_b)\), and \(c/c_o\) are greater than the corresponding values when \(\rho_{in} < \rho_{pond}\). The reasons for these differences are given as follows: When \(\rho_{in} > \rho_{pond}\), the inflow tracer or waste material flows along the bottom of the pond due to its higher density. The movement through the pond is slower due to the lower velocities that occur near the pond bottom due to the influence of the bottom boundary of the pond. This causes the tracer to take a longer time to reach the outlet. The efficiency of mixing in the lower depths of the pond is less due to reduced turbulence level and subsequently the tracer or waste material is higher in concentration \((c/c_o)_{pk}\) when it leaves the pond. The reverse is true when \(\rho_{in} < \rho_{pond}\). The tracer is concentrated in the higher velocity region of the pond, with respect to depth, and it thus reaches the outlet in a shorter time interval. In this region of flow there is greater mixing efficiency due to the higher turbulence level which exhibits itself in greater turbulent diffusion of the tracer or waste material. The greater turbulent diffusion may explain the lower peak concentration at the outlet. When \(\rho_{in} = \rho_{pond}\), the C-diagram and the values of \(\theta_c\), \(\theta_{pf}\), \(\theta_b\), \(\theta_{pk}\), and \(V_d\) should be somewhere between
the values of these same parameters for the two types of density-stratified flows. Referring to the results of Experiment D-55 recorded in group A-3 of Table 3 and the results of the density-stratified flow experiments recorded in Table 6 and shown in Figure 54, this seems to be true.

Figures 55 and 56 are photographs of the dye tracer as it flows out of the diffuser for \( \rho_{in} < \rho_{pond} \) and \( \rho_{in} > \rho_{pond} \) respectively. These photographs visually show the tracer-flowing near the bottom or near the surface of the pond for the two cases of density-stratified flow. Figures 57 and 58 are photographs taken from the side and above the diffuser respectively when \( \rho_{in} > \rho_{pond} \).

The effect of the densimetric Froude number on the results is shown in Table 6. The important parameters listed in Table 6 are shown in Figures 59, 60, and 61 plotted versus the densimetric Froude number, \( F_{\Delta} \). Figure 59 shows \( \bar{\theta}_{pf} \) and \( \theta_{b} \) for both density difference situations, plotted against \( F_{\Delta} \). The curves for \( \bar{\theta}_{pf} \) vs. \( F_{\Delta} \) for \( \rho_{in} > \rho_{pond} \) are close to bracketing the values of \( \bar{\theta}_{pf} \) for Experiments D-55 and D-56 when \( F_{\Delta} = \infty \) (\( \rho_{in} = \rho_{pond} \)). Figure 60 shows similar curves for \( \bar{\theta}_{c} \) and \( \theta_{pk} \) plotted against \( F_{\Delta} \). These curves also bracket the values of \( \bar{\theta}_{c} \) for Experiments D-55 and D-56 when \( F_{\Delta} = \infty \). Figure 61 shows \( \bar{V}_{d} \) plotted against \( F_{\Delta} \) with the two curves bracketing \( \bar{V}_{d} \) for Experiments D-55 and D-56 when \( F_{\Delta} = \infty \). It is not known exactly why the curves for \( \rho_{in} > \rho_{pond} \) and \( \rho_{in} < \rho_{pond} \) for \( \bar{\theta}_{c} \), \( \bar{\theta}_{pf} \), and \( \bar{V}_{d} \) cross but if these
Figure 55. Photography of tracer discharging from diffuser for density stratified flow $\rho_{in} < \rho_{pond}$.

Figure 56. Photograph of tracer discharging from diffuser for density stratified flow $\rho_{in} > \rho_{pond}$. 
Figure 57. Photograph of tracer discharging from diffuser for density stratified flow $\rho_{\text{in}} > \rho_{\text{pond}}$.

Figure 58. Photograph of tracer discharging from diffuser for density stratified flow $\rho_{\text{in}} > \rho_{\text{pond}}$. 
Figure 59. Plots of $\bar{\theta}_{pf}$ and $\theta_{pk}$ vs. $F_\Delta$ for $\rho_{in} > \rho_{pond}$ and $\rho_{in} < \rho_{pond}$.
Figure 60. Plots of $\bar{\theta}_c$ and $\theta_b$ vs. $F_\Delta$ for $\rho_{in} > \rho_{pond}$ and $\rho_{in} < \rho_{pond}$. 

- $\rho_{in} > \rho_{pond}$
- $\rho_{in} < \rho_{pond}$

Range of $\theta_c$ when $F_\Delta = \infty$

Range of $\theta_b$ when $F_\Delta = \infty$

Dimensionless tracer delay time, $\theta_{b, c}$, and mean $\bar{\theta}_c$.
Figure 61. Plots of $V_d$ vs. $F$ for $\rho_{in} > \rho_{pond}$ and $\rho_{in} < \rho_{pond}$. 

Densimetric Froude number, $F \Delta \times 10^2$ 

Range of $V_d$ when $F \Delta = \infty$ 

$\bullet$ $\rho_{in} > \rho_{pond}$ 

$\circ$ $\rho_{in} < \rho_{pond}$
curves are extended to infinity they should be bracketed by Experiments D-55 and D-56. Experiments D-55 and D-56 were run to obtain an idea of the influence that the stabilizing time would have upon the results. This has been discussed before under the section on experimental reproducibility. The values of $F_\Delta$ greater than 0.015 represent stratified flows of low density differences. It is felt that when $F_\Delta$ is greater than 0.015 that there is little change in the parameter values or in other words that there is a constant relationship between $F_\Delta$ and the parameter values. If this is the case then the difference between the values of $\bar{\theta}_c$, $\bar{\theta}_p$, and $\bar{V}_d$ when $F_\Delta > 0.015$ for $\rho_{in} > \rho_{pond}$ and $\rho_{in} < \rho_{pond}$ might be due to differences because of lack of reproducibility in the model.

More experiments with a greater range of densimetric Froude numbers should have been made to verify this explanation. Also, more experiments should have been performed to obtain a better idea of the effect of stabilization time on the results. Due to the lack of time and the large amounts of salt required to make the density experiments, this was not done in this research.

In summarizing, it is felt that the results contained herein for different degrees of stratified flow are good considering the variability due to lack of experimental reproducibility and the approximations that were made. There appears to be no choice but to accept the results with certain approximations when using salt (mass transfer) as a method of creating density stratified flows. The effect of both the use of salt to
create density differences and the short stabilization time is to probably shift the curves of $\bar{\theta}_c$, $\bar{\theta}_{pf}$, and $\bar{\nu}_d$ vs. $F_\Delta$ either up or down depending on the flow situation but it is felt that the general relationship with $F_\Delta$ is valid. The existing ponds in the Logan pond system have values of $F_\Delta$ that are within the range of the values of $F_\Delta$ that were used in these experiments and therefore the results should apply to existing ponds.

**Expected Treatment Efficiency For Selected Experiments**

In the design of a waste stabilization pond, it would be desirable to be able to predict the percent conversion of the waste material to stable end products during its residence in the pond. Equation 27 or Equation 28 of Chapter 2 can be used for this purpose, providing the age distribution function, $c/c_0$, the theoretical detention time, $\bar{t}$, and the first order reaction coefficient, $K$, are known. Since all pollutant remaining in the pond longer than two detention times is ignored in computing $\bar{N}_2/N_0$, the quantity $(1 - \bar{N}_2/N_0)$ represents the fraction of waste material that has undergone reaction.

Equation 28 consists of two parts: 1) the residence time distribution of the fluid particles $c/c_0$ as a function of $\theta$ and 2) the biological first order reaction equation, $N/N_0 = e^{-K\bar{t}\theta}$ as a function of $K$, $\bar{t}$, and $\theta$. To determine the effect of the hydraulic design on the treatment efficiency, $K$ and $\bar{t}$ must be held constant. Since the extent of reaction of a waste is highly dependent on $\bar{t}$ for a given $K$, the detention time
Table 7. Treatment efficiency for selected experiments.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>t (days)</th>
<th>t</th>
<th>( \frac{N_2}{N_0} )</th>
<th>( K=0.20 )</th>
<th>( K=0.25 )</th>
<th>( K=0.30 )</th>
<th>( K=0.35 )</th>
<th>( K=0.40 )</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-53</td>
<td>5.0</td>
<td></td>
<td></td>
<td>0.504</td>
<td>0.444</td>
<td>0.393</td>
<td>0.350</td>
<td>0.312</td>
<td>69</td>
</tr>
<tr>
<td>D-54</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.466</td>
<td>0.413</td>
<td>0.369</td>
<td>0.331</td>
<td>0.298</td>
<td>71</td>
</tr>
<tr>
<td>D-56</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.395</td>
<td>0.352</td>
<td>0.316</td>
<td>0.285</td>
<td>0.258</td>
<td>74</td>
</tr>
<tr>
<td>D-12</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.455</td>
<td>0.416</td>
<td>0.381</td>
<td>0.351</td>
<td>0.323</td>
<td>68</td>
</tr>
<tr>
<td>O-6</td>
<td>5.0</td>
<td></td>
<td></td>
<td>0.442</td>
<td>0.390</td>
<td>0.345</td>
<td>0.307</td>
<td>0.273</td>
<td>73</td>
</tr>
<tr>
<td>O-61</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.446</td>
<td>0.399</td>
<td>0.358</td>
<td>0.323</td>
<td>0.293</td>
<td>71</td>
</tr>
<tr>
<td>O-62</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.410</td>
<td>0.361</td>
<td>0.320</td>
<td>0.285</td>
<td>0.255</td>
<td>74</td>
</tr>
<tr>
<td>O-2</td>
<td>5.0</td>
<td></td>
<td></td>
<td>0.495</td>
<td>0.452</td>
<td>0.415</td>
<td>0.383</td>
<td>0.355</td>
<td>64</td>
</tr>
<tr>
<td>O-3</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.477</td>
<td>0.428</td>
<td>0.386</td>
<td>0.348</td>
<td>0.316</td>
<td>68</td>
</tr>
<tr>
<td>O-4</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.618</td>
<td>0.575</td>
<td>0.539</td>
<td>0.507</td>
<td>0.479</td>
<td>52</td>
</tr>
<tr>
<td>O-5</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.463</td>
<td>0.408</td>
<td>0.361</td>
<td>0.320</td>
<td>0.285</td>
<td>71</td>
</tr>
<tr>
<td>M-4</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.480</td>
<td>0.429</td>
<td>0.386</td>
<td>0.349</td>
<td>0.318</td>
<td>68</td>
</tr>
<tr>
<td>I-5</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.486</td>
<td>0.424</td>
<td>0.372</td>
<td>0.307</td>
<td>0.289</td>
<td>71</td>
</tr>
<tr>
<td>G-2</td>
<td>5.0</td>
<td></td>
<td></td>
<td>0.547</td>
<td>0.503</td>
<td>0.464</td>
<td>0.430</td>
<td>0.401</td>
<td>60</td>
</tr>
<tr>
<td>G-3</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.517</td>
<td>0.461</td>
<td>0.418</td>
<td>0.381</td>
<td>0.349</td>
<td>65</td>
</tr>
<tr>
<td>S-2</td>
<td>5.0</td>
<td></td>
<td></td>
<td>0.453</td>
<td>0.409</td>
<td>0.372</td>
<td>0.339</td>
<td>0.310</td>
<td>69</td>
</tr>
<tr>
<td>S-3</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.495</td>
<td>0.456</td>
<td>0.422</td>
<td>0.393</td>
<td>0.368</td>
<td>63</td>
</tr>
<tr>
<td>S-4</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.418</td>
<td>0.376</td>
<td>0.341</td>
<td>0.310</td>
<td>0.284</td>
<td>72</td>
</tr>
<tr>
<td>S-6</td>
<td>&quot;</td>
<td></td>
<td></td>
<td>0.404</td>
<td>0.358</td>
<td>0.320</td>
<td>0.287</td>
<td>0.259</td>
<td>74</td>
</tr>
<tr>
<td>P-5</td>
<td>5.0</td>
<td></td>
<td></td>
<td>0.571</td>
<td>0.521</td>
<td>0.479</td>
<td>0.444</td>
<td>0.413</td>
<td>59</td>
</tr>
<tr>
<td>Plug</td>
<td>Flow</td>
<td>5.0</td>
<td></td>
<td>0.368</td>
<td>0.286</td>
<td>0.223</td>
<td>0.173</td>
<td>0.135</td>
<td>86</td>
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</table>

Table 8. Values of K for wastewaters of various concentrations.\(^a\)

<table>
<thead>
<tr>
<th>K(day(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak wastewater</td>
</tr>
<tr>
<td>Strong wastewater</td>
</tr>
<tr>
<td>Primary effluent</td>
</tr>
<tr>
<td>Secondary effluent</td>
</tr>
<tr>
<td>Tap water</td>
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</table>

\(^a\)Fair, Geyer, and Okun.
Table 9. Treatment efficiency for baffled experiments.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>θ_pf</th>
<th>% Treatment Efficiency</th>
<th>K = .30</th>
<th>K = .40</th>
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</thead>
<tbody>
<tr>
<td>3.5</td>
<td>.416</td>
<td>66.3</td>
<td>74.4</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>.309</td>
<td>74.2</td>
<td>81.9</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>.303</td>
<td>82.2</td>
<td>87.5</td>
<td></td>
</tr>
<tr>
<td>9.5</td>
<td>.314</td>
<td>74.9</td>
<td>82.4</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>.374</td>
<td>73.6</td>
<td>80.7</td>
<td></td>
</tr>
<tr>
<td>5.7</td>
<td>.286</td>
<td>73.4</td>
<td>83.0</td>
<td></td>
</tr>
<tr>
<td>7.7</td>
<td>.212</td>
<td>82.8</td>
<td>88.7</td>
<td></td>
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<tr>
<td>9.7</td>
<td>.136</td>
<td>80.9</td>
<td>87.9</td>
<td></td>
</tr>
<tr>
<td>3.9</td>
<td>.398</td>
<td>70.0</td>
<td>77.8</td>
<td></td>
</tr>
<tr>
<td>5.9</td>
<td>.296</td>
<td>73.1</td>
<td>81.5</td>
<td></td>
</tr>
<tr>
<td>7.9</td>
<td>.216</td>
<td>74.3</td>
<td>83.1</td>
<td></td>
</tr>
<tr>
<td>9.9</td>
<td>.188</td>
<td>74.4</td>
<td>83.4</td>
<td></td>
</tr>
<tr>
<td>C-3</td>
<td>.250</td>
<td>76.5</td>
<td>84.3</td>
<td></td>
</tr>
<tr>
<td>OU-1</td>
<td>.325</td>
<td>73.9</td>
<td>81.5</td>
<td></td>
</tr>
<tr>
<td>OU-2</td>
<td>.364</td>
<td>73.5</td>
<td>80.7</td>
<td></td>
</tr>
<tr>
<td>OU-3</td>
<td>.355</td>
<td>72.5</td>
<td>80.1</td>
<td></td>
</tr>
<tr>
<td>OU-4</td>
<td>.395</td>
<td>70.1</td>
<td>77.6</td>
<td></td>
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</table>
rarely reach 70 percent. This points to the desirability of using baffled ponds to effectively increase the L/W ratio.

Equation 28 can be used to determine the treatment efficiency if the experimental age distribution function is known and a proper value of K can be determined. Equation 27 can be used if a mathematical model for the age distribution function can be developed.

Application of the Finite Stage Model

The purpose of this section is to illustrate the application of the finite stage model described in Chapter 2. This section will give some models developed from the experimental data resulting from this research. With a mathematical model for $c/c_0$, then either Equation 27 or 28 could be used to determine the theoretical treatment efficiency of a proposed or existing pond.

The principles and the method discussed in Chapter 2 were applied to a few selected experiments to show the value of the application of the finite stage model for predicting the age distribution functions of waste stabilization ponds. A computer program was written to make the parameter evaluations for each experiment after which these parameters were used to generate residence time distribution data for each experiment for the purpose of comparing the actual experimental C-diagram with this derived mathematical model of the C-diagrams and the computed C-diagrams for the selected experiments. Figure 62 illustrates a
Figure 62. Experimental and theoretical C-diagrams for Experiments C-1 and C-2.

- **C-1**
  - \( n = 2 \)
  - \( K_H = 0.050 \)
  - \( F_a = 0.6534 \)
  - \( F_b = 0.0066 \)
  - \( F_c = 0.340 \)

- **C-2**
  - \( n = 2 \)
  - \( K_H = 0.30 \)
  - \( F_a = 0.3665 \)
  - \( F_b = 0.3665 \)
  - \( F_c = 0.2670 \)
relatively flat curve while Figure 66 shows the C-diagram for a steep curve. These figures illustrate the versatility of the finite stage model, and its ability to describe highly skewed distributions as well as relatively flat curves. The parameters for the model of each experiment are given with the figure for that experiment.

The finite stage models for the experimental C-diagrams were computed with the aid of a computer program. This program uses the coefficients determined by Hovorka (1961) for his derived mathematical equation. The input to the program consisted of values for $K_H$ and $f_a'$ for each integer value of $n$ and $F_c'$, the delay time. The values for $f_a'$ and $K_H$ were taken from the plots of $Y_{pk}$ vs. $X_{pk}$ for each value of $n$ in Hovorka's dissertation using the transformed values of $\bar{\theta}_{pk}$ and $(c/c_o)_{pk}$ for each set of experimental data. Also, the experimental residence time curve (C-diagram) was input data to the program to determine the value of $n$, the number of model units. The output consisted of the derived model of the residence time curve with its appropriate values of $F_a'$, $F_b'$, $F_c'$, $K_H'$, and $n$. Each of the figures illustrating the experimental and derived C-diagrams give the computed values of these parameters. This mathematical model fits actual pond data, Figure 66, as well as the data generated with the use of the model pond.

To apply or use this mathematical model to describe the age distribution function and to determine the treatment efficiency, the values of
Dimensionless concentration, $c/c_0$

Dimensionless time, $\theta$

---

Figure 63. Experimental and theoretical C-diagrams for Experiment O-5.

$n = 2$

$K_H = 0.30$

$F_a = 0.309$

$F_b = 0.5053$

$F_c = 0.185$
Figure 64. Experimental and theoretical C-diagrams for Experiment R-21.
Figure 65. Experimental and theoretical C-diagrams for Experiment S-2.
Figure 66. Experimental and theoretical C-diagrams for Experiment P-5.

--- P-5
- model

n = 2
$K_H = .780$
$F_a = .1266$
$F_b = .8474$
$F_c = .026$
\( \theta_b, \theta_{pk}, (c/c_o)_{pk} \) and the experimental residence time distribution curve must be known. For a given rectangular pond design the results of the model studies reported in this chapter could be consulted to estimate values of \( \theta_b, \theta_{pk}, (c/c_o)_{pk} \) and obtain a representative C-diagram. Mathematical models for many different pond designs could be determined from hydraulic model studies and then used to compute the expected treatment efficiency of existing or proposed waste stabilization ponds. Model studies on triangular and circular ponds with various inlet devices could also be made and mathematical expressions developed from the results for use in future pond designs.

**Discussion of Results**

The experiments that were performed both on the model and the existing ponds for verification purposes indicated a number of important pond design considerations. The experiments on ponds D and E and the model verification experiments for these two ponds revealed that inlets that create jet inflows considerably affect the flow patterns and have considerable influence on the degree of short circuiting. The typical flow pattern caused by this type of inlet (i.e. pipe laid in a corner on the bottom of a prototype pond) is a pond wide circulation current which results in short circuiting, the extent of which is dependent upon the Reynolds number. The C-diagram resulting from this type of circulation is a series of humps spaced at intervals equal to the time it takes for the
bulk fluid to circulate around the pond. The model experiments verify this behavior and indicate the degree that the resulting short circuiting is dependent on the type of inlet used.

The hydraulic model was effectively verified using Reynolds number equality for ponds D and E. The model design was somewhat arbitrary in that the inlet design was a trial and error procedure to obtain proper inlet flow patterns. It was found that the inlet flow had to be greatly diffused in order to approximate the prototype pond behavior.

The verification of the model for the density stratified flow in pond A-1 was not adequate due to the lack of data of sufficient detail or accuracy to describe occurrences in both the model and prototype.

In all of these experiments including those on the existing ponds, the front portion of the C-diagram was found to be quite sensitive to such factors as inlet types and locations, degree of stratification, Reynolds number, and environmental factors that have been ignored in this study. There are other factors that affect the shape of the C-diagram and also influence the magnitude of experimental error. One of the most important was the occurrence of residual eddies and circular currents that were developed in filling the model prior to each experiment. Particular care was taken to minimize this influence on the flow behavior by waiting a certain length of time to allow for the dissipation or die away of the currents and eddies. Then the model pond was put into operation for a length of time before each experiment was begun to allow the flow to
stabilize. The random nature of turbulence is another factor that influences the fluid flow behavior. Because of the randomness of turbulence which exhibits itself in rapidly varying velocities, it is reasonable to conclude that the gross fluid flow patterns are subsequently random in nature. This randomness of flow has a particular influence on the shape of the first portion of the dimensionless concentration versus time curves.

Other factors which influence the experimental error are (1) flow measurement, (2) fluorometer calibration, (3) variations in inlet flow, and (4) temperature measurements.

In reviewing the results of the experiments performed on existing ponds and the experiments on the model, much of the variability among experiments might be due to the random behavior of flow turbulence. The effects of changes in L/W ratio, depth, Reynolds number, densimetric Froude number, inlets and outlets, on the significant parameters \( \bar{\theta}_{pf}, \bar{\theta}_c, \) and \( \bar{V}_d \) were not as large as would be expected even though some of these changes in hydraulic or geometric conditions greatly altered the shape of the C-diagram. There were some notable exceptions, however, with experiments employing L/W ratio changes and certain inlet and outlet types producing the most dramatic changes. The values of the parameters for Reynolds number changes were essentially constant while depth changes did influence the values of these parameters as much
as 30 percent but when the treatment efficiency was considered there was only about 6 percent difference among experiments involving depth changes. The density stratified experiments did indicate a definite relationship of these parameters including $\theta_{pk}$ and $\theta_b$, with the densimetric Froude number. Certain inlet and outlet configurations were obviously poor designs for best hydraulic performance. But it was found that significant parameter change occurred as the L/W ratio became large. The L/W ratio is the most important single factor that effects dramatic changes in the important parameters used here.

The L/W ratio is also significant in the overall treatment efficiency. Significant increases in the calculated treatment efficiency were noted as the L/W ratio increased.

The finite stage model was shown to be an excellent model for representing the age distribution functions of waste water or fluid particles within a stabilization pond. It has been shown that the finite stage model for a flow system can probably be derived to represent a given flow situation with less variability than might occur due to the lack of reproducibility. In other words, this mathematical model is as reliable as the experimental data itself. The finite stage model has physical significance and is very versatile in representing highly skewed to relatively flat age distribution curves. This model can be used to predict expected treatment efficiency if the representative C-diagram is available and the mean detention time is known for the proposed pond design.
The data indicate that waste stabilization ponds should be constructed with large values of $L/W$ ratio. This will give a maximum detention time and the minimum value for dead space which will result in greater treatment of the wastewater. Large values of $L/W$ can be accomplished effectively and economically by baffling a large rectangular pond as has been shown in this study. Also, the data indicate that certain types of inlets and outlets and their placement such as Experiments O-4 and O-2, are obviously poor for maximum efficiency of pond operation and should not be used in constructing future wastewater ponds.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

Based on the experimental work conducted on existing waste stabilization ponds in the Logan pond system and on the hydraulic model, the following conclusions are made:

1. Short circuiting in existing ponds in the Logan pond system occurs to varying degrees depending on the environmental conditions, the hydraulic characteristics, and the geometry of the pond. The experiments on the Logan City ponds revealed that short circuiting can be greatly influenced by wind. The types of inlets or outlets and their location in the pond can also directly influence the degree of short circuiting, particularly if the outlet is downwind and the inlet is upwind of the prevailing winds. Density stratification also effects the flow patterns and the degree of short circuiting in wastewater ponds. Density differences between the inlet flow and the ambient fluid can cause the main flow to be in the upper or bottom half of the pond which results in more serious short circuiting because of the reduced effective volume of the pond.

2. The most important consideration in verifying the pond behavior for the unstratified flow situation is the variable behavior of the inflow jet. Proper inflow jet modeling is important if model behavior is to duplicate prototype behavior.
3. For duplicating pond behavior where stratified flow exists, no firm conclusion has been reached. More reliable prototype data must be obtained in order to fully verify the use of the densimetric Froude number as a modeling criteria. However, the preliminary results look promising.

4. Reynolds number changes had little effect on the significant parameters and on the age distribution functions as long as the flow was turbulent. This conclusion assumes that the inflow jet is well diffused.

5. The degree of depth distortion did have an influence on the values of $\bar{\theta}_c$, $\bar{\theta}_{pf}$, and $\bar{V}_d$. A change of about 30 percent was indicated by the data for $\bar{\theta}_{pf}$, but in comparing treatment efficiencies, influence of depth resulted in only about a 6 percent change.

6. Inlet and outlet types and configurations had a significant effect on the hydraulic characteristics and subsequent treatment efficiency determinations. A change of as much as 42 percent was indicated by the data for the important hydraulic parameters, while a change of 19 percent in treatment efficiency was indicated for the two extreme conditions of inlets and outlets.

7. The length-to-width ratio $L/W$ has the greatest influence on the overall efficiency of waste stabilization ponds. Large $L/W$ ratios created by baffling were the best for maximum hydraulic performance.

8. The degree of stratification had an influence on the hydraulic performance. For an increase in the degree of stratification, i.e.,
decreasing densimetric Froude numbers, the influence on the hydraulic parameters and treatment efficiency depended on the type of stratification. When $\rho_{in} > \rho_{pond}$ the hydraulic performance improved. When $\rho_{in} < \rho_{pond}$ the hydraulic efficiency decreased.

9. The biological treatment efficiency can be predicted for a wastewater pond if a reliable value of $K$, the first order reaction coefficient, and a C-diagram are available.

10. The finite stage model represents well the age distribution function of a flow system. The model derivation depends on reliable values for $\theta_b$, $\theta_{pk}$, $(c/c_o)_{pk}$, and a representative C-diagram. The finite stage model can also be used for predicting pond performance for proposed designs.

11. The most significant conclusion of the research is that the hydraulics of waste stabilization ponds are important in determining the treatment efficiency of a pond. The hydraulics should be considered in proposed pond designs to insure maximum economy of construction and operation for maximum conversion of the waste material.

The following are recommendations for extensions of this study.

1. Obtain more reliable prototype data. This could be accomplished by performing tracer experiments on smaller ponds.

2. Obtain a considerable amount of tracer study data on model ponds utilizing different baffling schemes in increasing the effective $L/W$ ratio. Also, obtain tracer data on odd shaped ponds such as (1).
triangular ponds, (2) circular ponds and others.

3. Determine the influence of the stabilizing time on the shape of the $C$-diagram for density-stratified flow experiments.

4. Obtain more data for a greater range of densimetric Froude numbers.

5. Determine mathematical equations for given pond designs from hydraulic model data and existing pond data for many different pond designs to be used for future pond designs for predicting treatment efficiency.
SELECTED REFERENCES


PART II. THE EFFECT OF WIND ON MIXING IN STRATIFIED AND UNSTRATIFIED PONDS
Environmental pollution is one of the major crises of our times and inadequate waste treatment is a prime contributor. The role of the engineer in waste treatment is the selection of an economical and technically feasible treatment process, its design and its operation. While there are many types of waste treatment processes for industrial, agricultural and domestic waste products, this work concerns waste water treatment, specifically, treatment by waste stabilization ponds.

In regions of the country where land is low-priced, the waste stabilization pond can be an economical means of waste water treatment. For the treatment to be effective, the waste must enter the pond, disperse, and remain in the pond a required number of days to undergo biological treatment processes. Any factor which affects this retention time is of interest to the engineer.

One of the most important aspects of the treatment process is the mixing within the pond and the overall circulation patterns in the pond. Part I of this report has been directed to the basic circulation patterns and diffusion in a pond as affected by the shape of the pond, entrance and exit conditions, and density stratifications. One aspect which can
exert considerable influence on the hydraulic behavior, which was not considered in Part I, is wind.

The wind causes internal circulation patterns to form which can have a marked effect on the residence time of the waste in the pond. The wind can greatly enhance the mixing within the pond and it can create strong enough internal currents to breakdown density stratification in shallow ponds. All of these effects are of interest to the designer of a waste stabilization pond.

Because of the complex nature of wind-driven mixing, this work is directed to one aspect of this problem. That aspect is the dispersion and diffusion of a pollutant in a two-dimensional channel representing both constant density and density-stratified ponds. Mixing due only to internal circulation is considered. Other researchers have studied the mixing occurring beneath wind-generated waves, hence, this topic will not be investigated.

Although not a classical one-dimensional problem, the results of longitudinal dispersion are presented in terms of one-dimensional dispersion coefficients to facilitate the use of the results of the research. The wind shear which generates internal circulations strong enough to break up density stratification is also evaluated for selected density differences and density-layer thicknesses.
Review of Literature

Two distinct topics are associated with a review of the past work on this subject. One topic is that of shear stress on a water surface caused by airflow above the water. The initial studies in this area attempted to predict the rise of the water surface (wind tide) at the downwind edge of the water body. The most recent studies were to gain an understanding of the wind-water interaction and to explain the formulation of wind waves. The other topic reviewed in the literature was diffusion and mixing caused by wind forces. A considerable amount of effort has been extended developing the basic theories of diffusion and mixing but few researchers have considered the problem of wind-generated diffusion and mixing. The review of the literature will be presented for each topic.

Wind-generated shear stress

The following is a review of the pertinent research on wind shear or circulation caused by wind shear. Two significant studies were performed in 1951, one in the United States and the other in Great Britain.

In the United States G. H. Keulegan (1951) studied the wind set-up on a channel 11.3 cm by 28.5 cm in cross-section and 20 meters long. Set-up was measured by inclined manometers. Shear stress was calculated from the relationship
\[
\frac{\tau_s}{\rho g H} = \frac{1}{n} \frac{dH}{dx} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (1)
\]

in which \( \frac{dH}{dx} \) is the slope of the water surface corrected for pressure drop and \( n \) was defined as \( \frac{\tau_o}{\tau_s} + 1 \). The main relationships obtained during his study were:

\[
\tau_s \approx 0.0037 \frac{\rho_a U^2}{2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2)
\]

\[
U_s \approx 0.033 u_a \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (3)
\]

\[
S_1 = 3.3 \times 10^{-6} \frac{U^2}{gH} \quad L \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (4)
\]

in which

- \( \tau_s \) = surface shear stress
- \( U_s \) = the water surface velocity
- \( S_1 \) = the set-up

He found that Equation 4 should be modified for the condition when waves were on the water surface.

In Great Britain, Francis (1951, 1954) used a wind-water tunnel with dimensions similar to Keulegan's and concluded that

\[
\tau_s = C \rho_a U^2 \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (5)
\]

He compared the 1951 data with that of other researchers and formulated the relationship \( C = 0.0013 \) in which \( C \) is non-dimensional and \( U \) has units of m/sec.
Van Dorn (1953) measured set-up on a shallow pond 240 meters long, 60 meters wide and 2 meters deep. Wind speeds were measured at heights up to 10 meters above the water surface. He compared his data with that of Keulegan and obtained nearly the same results

\[ \tau_s = C \rho_a U^2 \]  \hspace{1cm} (6)

However, values of \( C \) in Equation 6 varied from 0.0011 to 0.0037 depending on which height above the water surface the velocity was measured. Consequently, the value of the drag coefficient \( C \) was shown to be a function of the velocity profile.

Hay (1955) measured air velocities above the sea at five heights up to 8 meters. He applied the equations from boundary layer theory for flow over smooth boundaries and rough boundaries. His results indicated that the airflow above the water varied logarithmically with height when the temperature gradient was small. He obtained the values of \( C = \frac{\tau_s}{\rho_a U^2} \) which varied linearly with wind speed. The results of his studies concluded \( \tau_s = 0.00166 \rho_a U^3 \) which gives a \( C \)-value slightly higher than that obtained by Francis (1954).

Munk (1955) compared results from the shear stress measurements based on set-up to the results from geostrophic flow of the wind near the surface. This was an attempt to bring the two schools of thought together by proposing that the shear stress equation have the following form \( \tau_s = C_1 U^2 + C_2 U^3 \). To try and implement this idea,
the data of Van Dorn (1953) was fit by method of least squares in this way:

\[ \tau_s = C' \rho \left( \frac{g}{V} \right)^{-1/3} U^3 \quad \ldots \quad \ldots \quad (7) \]

in which \( C' = 0.68 \times 10^{-6} \). The factor \( (gV)^{-1/3} \) was introduced from dimensional considerations (see Jefferey (1925), Keulegan (1951) for details). A sheltering model was proposed much like Jefferey's (1925) model. The portion of shear stress governed by the \( U^3 \) law was related to the high frequency wave spectrum. Newman's (1953) wave spectrum was used to make the comparisons. Problems concerning this method of evaluating shear stress are: 1) At what elevation should the wind be measured? 2) What is the effect of lapse rate? 3) Can the coefficient for form drag be evaluated? In view of these unanswered questions, the hypothesis of Munk was not followed by many researchers.

Keulegan and Brame (1960) in a contract for the Corps of Engineers studied the problem of mixing, shear stress, and wave growth in a density-stratified wind-water tunnel. The tunnel used for this research was the same one Keulegan built for his 1951 study. One of the main objectives of this study was to determine the rate of mixing (diffusion) of a lower heavier layer with the upper layers of the flow. Results of this study were hampered by the time required to obtain the samples. Also, the mean value of density was assumed to be the density measured at the centerline of the channel midway between the interface and the free
surface. This assumption may have influenced the results of the experiment, because the diffusion is a function of the turbulent intensity, as well as the water velocity. Thus the amount of mixing would be influenced by distance from the free surface. Final expressions for the density change due to diffusion were

\[ \Delta \rho' = \Delta \rho e^{-K \left( \frac{\xi}{H_0} \right)} \]  

in which \( \xi_0 = f \left( \frac{V}{\sqrt{\rho g H_0}} \right) \) and \( K = 1.42 \left( \frac{Vt}{H_0} \right)^{-1/3} \).

Other facets of his work were the influence of the density stratification on set-up, wave growth, and the disposition of the interface. A relationship of set-up to shear stress was developed and data taken to obtain a new equation for surface shear

\[ \tau_s = 2.5 \times 10^{-3} \rho_a U^2 \]  

This equation was based on

\[ \frac{\tau_s}{\rho g H} = \frac{dH_a}{dx} + \frac{dH}{dx} + \frac{\Delta \rho}{\rho} \frac{dH_i}{dx} \frac{H_i}{H} \]  

in which \( H_i \) is the height of interface above the channel bottom and \( \frac{dH_a}{dx} \) is the gradient of the air pressure in the channel with pressure expressed in height of an equivalent water column. Surface velocities obtained for the flows of this study had the same relationship given by Equation 3 as Keulegan obtained in 1951. A theoretical explanation developed for the velocity patterns within each layer was limited because
it assumed laminar flow. The general form of the velocity profiles have been observed although few velocity profile measurements have ever been made on stratified flows.

Knapp (1962) investigated wind shear with a modified flume 3 feet by 3 feet and 42 feet long. The depth of water in the flume was 1 foot. Shear stress on the water surface was calculated from set-up measurements. Also, the effect of shear stress on internal velocity profiles was observed in the channel. Surface velocities were in close agreement to the results of Keulegan (1951).

Masch (1963) verified the relationship between water surface velocity and the mean wind velocity in a 100-foot long wind-wave channel. A 4-foot by 4-foot cross-section was used to determine the effect of wind and wave action on horizontal and vertical dispersion. Some difficulty was encountered in controlling the variables that affect the diffusion coefficient. A relationship was established between surface current and wind velocity $U_s = 0.027 u$ which was comparable with Equation 3 but with a lower coefficient. In addition the air velocity was related to the diffusion coefficient by the equation

$$D_y = 1.8 \times 10^{-6} [U^2 C]^{1.2}$$

in which $C = 5.12 \times$ wave period. The results did not support the 4/3 relationship of Richardson or Kolomogoroff between diffusion coefficient, rate of energy dissipation and length to the 4/3 power. The problems
uncovered by this study indicated that further research into the mechanism of wind-wave generation was essential to properly interpret diffusion in terms of wave characteristics.

Fitzgerald (1963, 1964) measured the set-up on a short wind-water tunnel and related it to surface velocity and shearing stress. Surface measurements were made at the ends of a 6-foot long channel. Data were also obtained by measuring the velocity profile in the airflow above the water surface. From the airflow data a value of shear stress was determined from the logarithmic velocity profile equation. Values obtained by both methods were similar to those of Keulegan (1951). A drag coefficient $C$ was found to be nearly constant at $2.24 \times 10^{-3}$ until wind velocity reached 412 m/sec; then $C$ tended to increase as a function of wind speed. The validity of the results may be doubted because of the short length of the apparatus which would have required super-sensitive instrumentation to get accurate measurements.

Baines and Knapp (1965) reported the results of Knapp's (1962) work and additional research. An argument based on dimensional analysis demonstrated that the shear velocity is the basic parameter for dynamic similarity in the internal turbulent flow

$$\frac{U}{U_*} = f \left( \frac{U_* H}{\nu}, \frac{H}{H_0} \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (12)$$
Equation 12 is similar to the functional form of the boundary layer solution for shear on a rigid wall. Near the wall viscous stresses dominate and the shear stress is a function of Reynolds number, whereas, away from the wall viscous forces are minimal. Consequently, the stresses are a function of distance from the wall for large Reynolds numbers. One problem experienced was that measured current profiles indicated that continuity was not satisfied by integrating the velocity profile over the depth. This problem may have occurred because the flow may have been unsteady, as was pointed out in the paper, or the fact that the current velocity apparatus did not measure the vertical velocity vector in the channel.

Bye (1965) developed a theoretical equation based on the similarity theory of turbulence to give an internal velocity profile. The profile was very similar to those obtained from the experimental results of Francis (1951) throughout most of the water depth. Near the bottom of the channel similarity theory does not predict the return to zero velocity. Assumptions made to obtain a solution require that there exist a finite velocity at the boundary. Since the shear stress of the water surface must be known to evaluate the velocity profile, application of this method is limited. Approximations to the true value of the shear stress were calculated either from set-up measurements or by fitting a logarithmic profile to the airflow.
Plate and Goodwin (1965) studied the influence of wind on an open channel flow. Air velocity profiles above the water surface were fit by log-velocity relationships. Shear stress on the water surface was given by the relationship $\tau_s = C\rho U^2$ where $C = 0.000225 U$. The shear stress computed by this equation is slightly higher than that obtained by Keulegan (1951).

Further work by Plate and Hidy (1966) with the same experimental facility obtained the following relationship between shear stress and the air velocity

$$\tau_s = 6.67 \times 10^3 \rho U$$  \hspace{1cm} (13)$$

in which $U$ is in meters/sec. This was based on the calculations of shear stress from the equation

$$\tau_s = \rho g H \left[ S + \frac{1}{\rho g} \frac{d P_a}{dx} \right]$$  \hspace{1cm} (14)$$

in which $S$ is the slope of the water surface and $P_a$ is the air pressure above the water. Their study was an effort to explain the mechanism of wave generation by wind.

With the exception of the work by Hidy and Plate most of the previous researchers have not attempted to explain the mechanism of energy exchange from the airflow to the water. Beginning in the early sixties several investigations attempted to explain this mechanism.

Shermdin and Hsu (1967) used a 3-foot wide, 6-foot high and 115-foot long channel to examine the mechanism of energy transfer to the water
from wind. The theory, proposed by Miles, was compared with the velocity and pressure measurements. These variables and others were measured with respect to a moving frame of reference. The moving reference was established so that the measurements were obtained at a fixed distance above the wavy surface. The velocity profile measurements were fit with a logarithmic velocity profile. The data exhibited a systematic deviation from the best squares fit of the logarithmic profile.

Liggett and Hajitheodorou (1969) discuss the results of the latter's thesis, Hajitheodorou (1967). Basically, suitable assumptions were made so that a finite-difference solution could be obtained for the stream function. From this solution velocity equations in the x, y, and z directions and the pressure equations could be solved. Some of the main assumptions were laminar flow (implied from shear stress relationships), constant shear stress, small Rossby number (Rossby number is the ratio of inertia forces to Coriolis forces), shallow lake, hydrostatic pressure distribution, negligible horizontal diffusion of momentum, and constant eddy viscosity and Coriolis parameter. Some of these assumptions may lead to erroneous results, most likely the laminar flow, constant shear stress, and constant eddy viscosity assumptions. However, without these assumptions the mathematics are not tractable. No attempt was made to derive the mechanism by which the wind imparts momentum to the water surface because of the uncertainty of the energy transfer process. The results were plots of the velocity field at various depths. Some
circulation features that have been observed in lakes were demonstrated in the model. Output was generally of a qualitative nature rather than being quantitative.

Liggett (1969) used the same approximations to develop a solution for the unsteady equations of motion. The solutions of these equations were much more difficult than in the steady case and a significant amount of numerical computation was involved to obtain a solution. For details of the solution the reader is referred to the paper. Liggett obtained results from the program that approximated the response of a rectangular lake approximately the size of Lake Michigan. A nearly steady state condition was reached with a period of oscillation equal to 17.7 hours, about 12 hours after the wind started. Output was similar to that described previously. The main advantage of a computer solution is that the effect of different variables on the solution can be determined by changing the value of one variable at a time, then comparing that solution with previous solutions. The results may be only qualitative, but they may provide good insight into some of the effects caused by changing various parameters.

Only two studies have investigated the effect of wind on a stratified system. The first was the previously discussed work of Keulegan and Brame (1960) and a computer model. The latter was done by Lee and Liggett (1970) who extended the original work of Liggett and others to include the case of a density-stratified lake. The equations solved are similar to those used in the unstratified case, except that the layers must be solved separately.
and coupled at the density discontinuity. Assumptions used in the earlier studies were required to obtain a solution. One of the interesting characteristics exhibited in the solution was jets along the coastal areas parallel to the wind stress. Results of the computations are rather tentative because of the assumptions required to obtain a solution.

In another computer model, Plate (1970) solved for relationships between the airflow and the resulting shear on the water by dividing the flow into four regions. Theory and experiment yielded the same results when the drift current remained laminar. However, the theory does not predict well for the case of turbulent flow in the water nor when waves exist on the water surface. That is, theory and experiment gave the same results because the theory is pertinent to the region where it was applied. But the solution fails to give correct values where waves are present and the flow is turbulent, because the theory is not applicable.

Shemdin and Lai (1970) investigated experimentally the structure of turbulent shear flow above propagating waves. Mean velocity profiles and the turbulent velocity fluctuations were measured with a hot-film anemometer. Power spectrum and crosspower spectrum analyses were performed on the air-velocity data. Also spectra of the water surface displacements were obtained. They presented the mean velocity profiles over the water surface as logarithmic velocity profiles.

The sensors above the water were fixed in space. Separate log profiles were obtained for each velocity setting of the wind tunnel. These
data were compared with the data obtained from measurements of the current in the water. Shear velocity was obtained by assuming the velocity profile in the water followed a logarithmic profile. Figure 1 shows the relationship of shear stress to air velocity based on both airflow and water velocity data. The drag coefficient was given as

\[ C_d = \frac{\tau_s}{\rho_a U_s} \frac{U^2}{\omega} = 2.45 \times 10^{-3} \quad . \quad . \quad . \quad (15) \]

This value compares very well to the work of Keulegan (1951) and Fitzgerald (1963).

Liu and Perez (1971) report a study of wind-induced circulation based on the equations of Liggett. The effect of the Coriolis forces was eliminated because the pond was so small that they are negligible. Results were graphic displays of the velocity field and velocity profiles which approximate those of Keulegan (1951) for the laminar case. As with the previous numerical solutions only approximate solutions were obtained.

**Diffusion and mixing**

Studies that relate to wind-driven dispersion and mixing are scarce. However, the general topic of diffusion, dispersion and mixing has numerous studies reported. Several good surveys of previous research will be cited on the topics of diffusion and dispersion in lieu of an extensive citation of individual research papers. Then, studies pertinent to wind related mixing and dispersion will be discussed.
Figure 1. Surface stress vs. $U_\infty$ determined from wind and current profiles. $\tau_s = 2.45 \times 10^{-3} \rho_w U_\infty^2$. 
Harleman, Jordan and Lin (1959) reported results of an investigation of diffusion in stratified fluid in a homogeneous turbulent field. The beginning sections are an excellent review of both classical diffusion theories and turbulent diffusion theories. Only some of the more basic theories will be described here. The reader is referred to the report for further details. They first discuss the development of Fick's first and second laws of molecular diffusion which are

\[ N = -D \frac{\partial s}{\partial x} \quad \ldots \quad (16) \]

and

\[ \frac{\partial s}{\partial t} = D \frac{\partial^2 s}{\partial x^2} \quad \ldots \quad (17) \]

in which \( D \) is the molecular diffusion coefficient and \( s \) is the concentration of a substance. Solutions for various boundary conditions are in Carslaw and Jaeger (1959) and will not be explained here.

The type of mixing occurring in turbulent flow depends strongly on the turbulence phenomena and is referred to as turbulent diffusion. The pioneering work of Taylor, Kampe de Feriet and Frenkiel verified that the eddy diffusivity of turbulent flow is not necessarily a function of turbulent intensity alone. However, only the small region around the diffusing source that has a radius of the same order as the eddy scale of turbulence is governed by their equations. They conclude when distance is much larger than the average eddy size, the assumption of constant
diffusion coefficient would be valid for the theories of turbulent diffusion. Thus the general equation for diffusion with convection was shown to be

$$\frac{\partial c}{\partial t} + \nabla (u \cdot c) = \text{div} (D \cdot \nabla c) \quad \ldots \ldots \ldots \ldots \quad (18)$$

in which $c$ is the concentration of a substance and $u$ is the velocity. Equation 18 may be reduced to the following if an analogy between molecular and turbulent diffusion is made. Substituting the temporal mean values $\overline{c}$ for $c$, $\overline{u}$ for $u$, etc., for isotropic turbulence

$$\frac{\partial \overline{c}}{\partial t} + \overline{u} \cdot \nabla \overline{c} = D \nabla^2 \overline{c} \quad \ldots \ldots \ldots \ldots \quad (19)$$

Equation 19 further reduces to

$$\frac{\partial \overline{c}}{\partial t} + u \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2} \quad \ldots \ldots \ldots \ldots \quad (20)$$

for one-dimensional flow. Further, they referenced Taylor's solution of the diffusion equation which was based on the log-velocity profile and Reynolds analogy. The analogy assumes that transfer of heat, mass and momentum by turbulence is analogous.

It was shown that

$$D = 1.785 \, d \, U \sqrt{f} \quad \text{for pipes} \quad \ldots \ldots \ldots \ldots \quad (21)$$

$$D = 7.14 \, R_h \, U \sqrt{f} \quad \text{for rivers and channels} \quad \ldots \ldots \quad (22)$$
in which

\[ d = \text{pipe diameter} \]
\[ R_h = \text{hydraulic radius} \]
\[ U = \text{mean flow velocity} \]
\[ f = \text{friction factor (Darcy-Weisbach)} \]

They reference Taylor's solution (1954) for a finite dosage of a conservative substance introduced at \( x = 0 \)

\[ c = \frac{M}{A \sqrt{4 \pi Dt}} e^{-\frac{(x - Ut)^2}{4Dt}} \ldots \ldots \ldots \ldots \ldots \quad (23) \]

in which \( M = \text{weight of the dosage of the substance} \).

Several other studies were referenced but are not directly applicable to the present work. Various methods for determining diffusion coefficients in homogeneous turbulence were discussed.

Typical of the application of the one-dimensional diffusion equation is the work of Krenkel and Orlob (1962) who presented results of several tests to evaluate the reaeration and turbulent diffusion in a stream.

Several schemes to evaluate the reaeration and gas absorption into the water were explored by the authors. Relationships between the reaeration coefficient and turbulent diffusion were established. The techniques used to compute the diffusion coefficients were used also for the writer's research. Krenkel and Orlob used the one-dimensional diffusion equation (Equation 23) developed by Harleman, Jordan, and Lin. By taking the
log of both sides the authors demonstrated that \( \frac{1}{4D} \) is the slope of the plot of \( \log_e (ct^{1/2}) \) versus \( (x - ut)^2/t \). The advantage of this method over other methods is that the exact concentration of dye does not need to be known.

Diachishin (1963) reviews various formulations of the diffusion equation and suggests several methods to determine the diffusion coefficient. Although methods for obtaining diffusion coefficients in more than one dimension are presented, only those for one-dimensional flow will be discussed here. The governing partial differential equation (Equation 20) has been shown above. Solutions are for a point source in a fluid infinite extent. Note the similarity between this solution and that of Krenkel and Orlob (1962)

\[
c = \frac{K}{t^{1/2}} e^{-\frac{(x - ut)^2}{4Dt}}
\]  

The first method mentioned to determine \( D \) was the same as explained above by plotting \( \log ct^{1/2} \) versus \( (x - ut)^2/t \). Another method can be shown by transforming the solution into the form for standard normal curve by the substitution \( \sigma_x = \sqrt{2Dt} \). Then

\[
c = \frac{W_0}{\sqrt{2\pi} \sigma_x} e^{-\frac{1}{2} \left( \frac{x - \bar{u}t}{\sigma_x} \right)^2}
\]  

where
Note that the exponent around $x = 0$ is $-\frac{u^2 t}{4D}$. Then the $\log_e$ of $ct^{1/2}$ was plotted versus the time elapsed since the dye was released. The slope of the plot is inversely proportional to $D$ and to the square of the mean velocity.

The work discussed above illustrates the basic solution to one-dimensional diffusion problems and shows how the diffusion coefficient can be experimentally determined. In stratified fluids the approach is somewhat similar but different in that fluids do not mix thoroughly in a direction normal to the flow and one gets an approximation to the diffusion coefficient. The process is labeled "apparent" diffusion. It will be discussed later.

The first work cited for mixing in stratified fluids is that of Keulegan (1949) who investigated the flow of a light fluid above a denser fluid in a laboratory channel. Three rectangular flumes were utilized to determine scaling effects. The sizes varied from 2 cm wide by 4 cm high, to 11.3 cm wide by 28.5 high. The length of all were about 25 times the total depth. Critical velocities were defined as that mean velocity in the upper layer which caused mixing of the lower layer with the upper layer by turbulence and wave action at the interface between the layers. He demonstrated that the form of the mixing parameter $0$
to be of the form \( \theta = \theta \left( \frac{u R_h}{\nu} \right) \) where

\[
\theta = \frac{(v g/\Delta \rho/\rho)^{1/3}}{u} \quad \ldots \ldots \ldots \ldots \quad (26)
\]

From the results of the tests,

\[
\begin{align*}
\theta &= 0.127, \quad R_c < 450 \\
\theta &= 0.178, \quad R_c > 450
\end{align*}
\]

There were other factors affecting the mixing parameter when the Reynolds number was in the turbulent range. Note that this work did not involve computation of a diffusion coefficient.

Harleman, Jordan and Lin (1959) evaluated the diffusion process in stratified fluids. The diffusion tests were performed by dividing the tank at the middle with a barrier. A salt solution was placed on one side of the barrier and fresh water was placed on the other side. The barrier was suddenly withdrawn and the diffusion of salt water into the fresh water was studied in a turbulent field. A set of screens along the bottom of the flume was driven to generate homogeneous turbulence. The type of diffusion observed in the test is similar to the lock flow problem with a barrier between fluids of different densities. Diffusion coefficients were presented.

The research of Keulegan and Brame (1960) investigated the problem of diffusion in a stratified flume driven by wind waves. This research has been summarized in the previous section of the literature review.
Masch's (1963) work was discussed in the previous section of the literature review.

Harleman (1966) formulated the basic relationships for turbulent diffusion and has shown various solutions subject to different boundary conditions. Most of these have been discussed or referenced previously. He discussed methods to study stratified flow. The normal procedure that has been used is to apply the one-dimensional diffusion equation to the stratified flow and determine an "apparent" diffusion coefficient. This coefficient is the combined effect of fluid exchange due to gravitational instability and mass transfer by turbulence. This technique was used in this work and the underlying assumptions are important to keep in mind.
CHAPTER 2
THEORETICAL DEVELOPMENT

This chapter describes the basic approach used to organize the research, and to collect and analyze the data. The general relationships for surface shear stress for both stratified and unstratified flow, the evaluation of diffusion and dispersion, and method of predicting the internal mixing in stratified flow caused by breakdown of stratification will be presented in that order.

Shear Stress Relationships

It is necessary to evaluate the shear stress on the surface in order to tie the driving force of the wind to the resulting mixing phenomena within the pond. Several different methods for measuring shear stress have been discussed previously and it was concluded that measuring the set-up (tilt) of the water surface would be the most direct and accurate method to use for both the stratified and unstratified experiments.

Unstratified flow

A basic equation for the shear stress on a section of fluid of length \( dx \) in an unstratified flow will be derived. The equation is developed for a two-dimensional flow. Applying the momentum equation to the element in Figure 2 results in the following equation
Figure 2. Fluid element for constant-density flow.

\[ u_a = \text{average velocity of air flow} \]
\[
\frac{\gamma H^2}{2} - \frac{\gamma}{2} (H + dH)^2 + \tau_s \, dx + \tau_o \, dx + pH - (p + dp)H + (p + \frac{dp}{2}) \, dH
\]

\[
= \rho \int_A (u + du)^2 \, dA - \rho \int_A u^2 \, dA \quad \ldots \quad \ldots \quad \ldots \quad (27)
\]

in which
\[
\begin{align*}
\tau_s &= \text{the surface shear stress} \\
\tau_o &= \text{the bottom shear stress} \\
H &= \text{the pond depth} \\
p &= \text{the pressure above the water surface} \\
u &= \text{the average wind velocity}
\end{align*}
\]

Expanding Equation 27 and neglecting second order terms yield

\[
- \gamma H \frac{dH}{dx} + \tau_s + \tau_o - H \frac{dp}{dx} = 2 \rho \int_A (u \frac{du}{dx}) \, dA \quad \ldots \quad (28)
\]

Now separating all the terms except shear stresses on the right hand side results in

\[
\tau_o + \tau_s = \gamma H \frac{dH}{dx} + H \frac{dp}{dx} + 2 \rho \int_A (u \frac{du}{dx}) \, dA \quad \ldots \quad (29)
\]

Neglecting the acceleration term \( u \frac{du}{dx} \), Equation 29 becomes

\[
\tau_s + \tau_o = \gamma H \frac{dH}{dx} + H \frac{dp}{dx} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)
\]

which is the same equation used by Plate and Goodwin (1965), Keulegan (1951), and others. Defining \( H' = H + H_a \) and substituting into Equation 30
Assumptions used in the deviation are hydrostatic pressure distribution and steady flow. Keulegan (1951) has shown that when acceleration terms are neglected there is at most a 5 percent error in the shear stress measurements so the approximation leading to Equation 30 is justified. The other assumptions are valid unless the measurements of set-up are made at the ends of the channel where flow is strongly curved. Flow in the ends of the channel has considerable turbulence and there is a significant momentum transfer in the vertical direction.

Before Equation 31 can be solved the unique relationship between bottom shear and the shear at the free surface must be known. Several values of the relationship have been proposed varying from $\tau_o = 0.5 \tau_s$ to $\tau_o = 0$. Recent work of Baines and Knapp (1963) and Bye (1967) have shown that bottom shear is less than one-tenth of the surface shear. Consequently, the usual convention is to assume $\tau_o = 0$ and this assumption was used during the current tests.

Equation 31 contains two unknowns: first, the gradient of the air pressure above the water surface; and second, the gradient of the free surface. The following development will demonstrate how both of these

\[ \tau_s + \tau_o = \gamma H \left( \frac{dH}{dx} + \frac{dH}{dx} \right) = \gamma H \frac{dH}{dx} \quad \ldots \ldots \quad (31) \]

in which $p = \gamma H_a$. 

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Equation 31 contains two unknowns: first, the gradient of the air pressure above the water surface; and second, the gradient of the free surface. The following development will demonstrate how both of these
unknowns can be determined by measuring the pressure at the bottom of the channel at two points separated a distance $\Delta x$ along the channel. The absolute pressure at the bottom of the channel is the sum of the hydrostatic pressure and the ambient pressure above the free surface.

From Figure 2 at point 1 the pressure at the bottom is $p_1 = \gamma_1 H_1 + p_{a_1}$ or $p_1 = \gamma H_1 + \gamma H_{a_1}$, in which $p_a = \gamma H_a$. Also at point 2 $p_2 = \gamma_2 H_2 + \gamma H_{a_2}$. Subtracting the expression for $p_1$ from the expression for $p_2$ yields $\Delta p = \gamma [\Delta H + \Delta H_a]$; now dividing by $\gamma$

$$\frac{\Delta p}{\gamma} = \Delta H + \Delta H_a = \Delta H' \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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Stratified flow

An outline of the basic development for stratified flow is presented in this section. Details of the derivation are omitted as they follow closely the development for unstratified flow. Figure 3 is the definition sketch for the variables used in this development. The basic equations of motion are applied to the upper and lower layers separately. The equation for the upper layer related the slope of the free surface $\frac{dH}{dx}$ to surface
Figure 3. Fluid element for stratified flow.

\[ \text{Density } = \rho_1 \]

\[ \text{Density } = \rho_2 \]

\[ u_a = \text{average velocity of airflow} \]

\[ \tau_s \]

\[ \tau_1 \]

\[ \tau_2 \]
shear stress \( \tau_s \), interfacial stress \( \tau_i \), density of the fluid \( \rho_1 \), height of the interface \( H_i \), and total depth of water \( H \).

A similar equation relates the previously mentioned variables to \( \tau_i \) and \( \tau_o \), the bottom shear stress. The interfacial stress \( \tau_i \) is eliminated by combining the two equations after which some simplifying assumptions are made. The assumptions are: 1) density gradients along the direction of flow are negligible, 2) the contribution of bottom shear is negligible, and 3) \( 1 + \frac{\Delta \rho}{\rho} \approx 1 \). The resulting equation is

\[
\frac{\tau_s}{\rho_1 g H} = \frac{dH}{dx} + \frac{dH}{dx} + \frac{\Delta \rho}{\rho_1} \frac{H_i}{H} \frac{dH_i}{dx} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (33)
\]

A similar relationship was developed by Keulegan and Brame (1960) for a density-stratified shear flow. All terms of this equation can be evaluated experimentally except the term involving \( \frac{dH_i}{dx} \). The interface cannot be identified accurately enough to measure directly. An indirect approach to obtain this term will be developed.

Consider a pressure tap at point 1 at the bottom of the channel.

The pressure at 1 is given by

\[
p_1 = g \rho_1 (H - H_i) + g \rho_2 (H_i) + g \rho_1 H_a \ldots \ldots \ldots \ldots \ldots \ldots (34)
\]

where the ambient air pressure is given by \( p_a = \gamma_1 H_a \). Assume \( \rho_1 \) and \( \rho_2 \) are not functions of \( x \) and that the flow is a steady, uniform flow such that a hydrostatic pressure distribution exists. Taking the derivative of \( p \) with respect to \( x \) results in
\[
\frac{dp}{dx} = g \rho_1 \frac{dH}{dx} + \frac{dH_i}{dx} (g \rho_2 - g \rho_1) + g \rho_1 \frac{dH'}{dx} \quad \ldots \ldots \quad (35)
\]

which when divided by \( g \rho_1 \) results in the following relationship,

\[
\frac{\Delta \rho}{\rho_1} \frac{dH_i}{dx} = \frac{1}{g \rho_1} \frac{dp}{dx} - \frac{dH'}{dx} \quad \ldots \ldots \quad (36)
\]

in which \( H' = H + H_a \). Both \( \frac{dp}{dx} \) and \( \frac{dH'}{dx} \) can be measured in the same manner that has been described, and the term \( \frac{\Delta \rho}{\rho_1} \frac{dH_i}{dx} \) can be calculated. When Equation 36 is substituted into Equation 33 the result is given by

\[
\frac{\tau_s}{\rho_1 g H} = \frac{dH_i'}{dx} \left( 1 + \frac{H_i}{H} \right) + \frac{H_i}{\rho_1 g H} \frac{dp}{dx} \quad \ldots \ldots \quad (37)
\]

Equation 37 was used to calculate the surface shear stress for the density-stratified flows. This equation reduces to the same equation used for homogeneous flows as \( H_i \to 0 \). The first term \( \frac{dH_i'}{dx} \) is determined from pressure taps located just below the surface waves' zone of influence in the channel. The second term \( \frac{dp}{dx} \) was measured from pressure taps located along the bottom of the channel. The final equation for stratified-flow surface shear stress developed in this section was similar to Keulegan's results except for the term including the gradient of the interface term \( \frac{dH_i}{dx} \). Keulegan's development was based on a hypothetical velocity profile and was not obtained directly from pressure measurements. The velocity at the interface was also required for his expression. The shear stress in this work was determined from pressure and set-up measurements by using Equation 37.
**Diffusion and Dispersion**

Numerous researchers have developed equations of diffusion and dispersion for pipes, open channels and open areas in the oceans. These equations and their solutions can be found in some of the research cited in the literature review. The one-dimensional equation for homogeneous diffusion was presented in the literature review with a brief discussion of how it could be applied to stratified flow.

Techniques to determine diffusion and dispersion in density-stratified flow are extensions of the methods used for homogeneous fluids. The usual assumption is that the one-dimensional diffusion equation is valid for the stratified flow. The diffusion coefficient $D$ is replaced by an "apparent" diffusion coefficient $D'$. $D'$ is usually larger than $D$ for similar conditions without stratification, although there are instances when diffusion is reduced by the presence of a stable layered system. The combined effects of mass transfer by turbulent fluctuations and gravitational instability are lumped into $D'$. The methods presented to determine $D'$ for constant-density flows are applicable to stratified flows for various flow conditions. This method of analysis is not valid if the diffusion coefficient changes with time. This causes the basic convective diffusion equation to be nonlinear and the solutions given are incorrect when $D'$ is time varying.
**Critical Mixing**

There are basically two phenomena that cause mixing between the layers of a stratified flow. The first kind of diffusion occurs when an exchange of fluid between the layers causes a gradual smoothing of the density gradient at the interface. This mixing is generally caused by low levels of turbulence in the upper layer which causes small waves to travel along the interface between the layers. Eddies are formed at the interface and the two layers are mixed near the interface. As the level of turbulence increases, the wave activity at the interface approaches that of breaking waves. Eddies are shed more often and the waves grow larger. A point is reached where any increase in the turbulence and velocity of the upper layer causes the phenomena to change from a turbulent diffusion type exchange into the second mode of mixing.

This second mode of mixing is characterized by the transition of fluid from the denser, lower layer to the upper layer. This transfer is a one-way process which causes complete mixing to begin at the upwind end of the channel. The mixed front then advances downwind almost as a single progressive wave. Initial stages of wind build up are characterized by the diffusion-type mixing between the layers. At higher velocities, the mixing tends to be characterized by the complete mixing of the layers. The distinction between the two modes lies in the stability of the interface. If the interface has laminar type flow diffusion
will be the source of mixing. The exchange type mixing is obtained when a rough turbulent flow exists at the interface.

In the literature review it was shown that very little work had been done in this area. Keulegan's (1949) work was performed to study this specific problem. He demonstrated that a dimensionless relationship for stability was

\[
\theta = \left( \frac{\nu^2 g \Delta \rho}{\rho} \right)^{1/3} \frac{u}{v} \]

(38)

Data from his experiments showed that when the critical Reynolds number \( \Re_c = \frac{u R h}{\nu} \) was below 450, \( \theta \) equaled 0.127. When \( \Re_c > 450 \) \( \theta \) was equal to 0.178. The study also concludes that the dependence of \( \theta \) on \( \frac{\Delta \rho}{\rho} \) seemed more critical than on the kinematic viscosity. Keulegan's research had large differences in kinematic viscosity and one fluid flowed over the other denser fluid. The flow was induced by a hydraulic gradient in the upper layers.

The pertinent variables for a wind-generated shear flow are the densities of the layers, the total depth, the depth of the lower layer, the width of the channel, the shear velocity at the free surface, and the kinematic viscosity of the layers. In functional notation

\[
\text{Mixing} = f(\Delta \rho, \rho, H, H_1, L, B, u_*, v_1, v_2)
\]

From dimensional reasoning the following \( \pi \)-terms can be developed
\[ \text{Mixing} = f \left( \frac{\Delta \rho}{\rho}, \frac{H_i}{H}, \frac{u^*_H}{v_1}, \frac{L}{H}, \frac{B}{H}, \frac{v_1}{v_2} \right) \] \quad (39)

For the present test \( \frac{L}{H}, \frac{B}{H} \) and \( \frac{v_1}{v_2} \) were kept constant. The mixing parameter used for this test is the critical shear velocity at the onset of complete mixing. A critical Reynolds number will be found and plotted versus relative depth \( \frac{H_i}{H} \) with \( \frac{\Delta \rho}{\rho} \) as a parameter.
CHAPTER 3

EXPERIMENTAL APPARATUS AND PROCEDURES

Description of Wind-water Tunnel

The wind-water tunnel at Utah State University was a duct 4 feet high by 4 feet wide and 54 feet long. The lower 2 feet were filled with water and the upper 2 feet comprised the airflow section. Figure 4 is a sketch of the wind-water tunnel. The side walls and floor were constructed from 3/4-inch plywood sheets braced every 4 feet. Every two feet along the duct the circumference was banded by 2 x 4's. The side walls and the bottom were covered with fiberglass cloth saturated with resin. The fiberglass sealed the duct from leakage and created a hydraulically smooth surface on three sides of the facility. The top of the duct was constructed by nailing 3/8-inch plyboard sheets to the underside of two 2 x 4's which ran the full length of the tunnel. The roof was held in position by 1/8 x 1/2-inch steel straps placed between the outer box frame of the facility and the 2 x 4's of the roof section. Air leaks between the side walls and the top section were minimized by 1 1/4 x 1/4-inch neoprene seals attached to the edges of the top section.

Plastic windows were placed in one side of the facility to allow observation in the tunnel. These windows allowed observation of the flow over the full depth of the channel at three points along the wind-water tunnel. Two of the windows, each 2 feet 6 inches high by 3 feet long,
Figure 4. Schematic sketch of the wind-water tunnel.
were placed at either end of the flume flush with the end sections. One larger window 3 feet high by 5 feet long was inserted near the middle of the channel. The upstream end of the middle window is located 24 feet downstream from the entrance.

Transition sections are attached to each end of the main 54-foot channel to provide a smooth airflow between these sections and the main channel. A 4-foot inlet section with rounded corners channeled the air into the main section. Immediately downstream from the curved section four screens were inserted to create more uniform flow at the entrance. This increases the intensity of turbulence but causes the large eddies to be dampened out (see Fitzgerald (1963)). The inlet was 4 feet wide by 22 inches high; the roof of the inlet was flush with the roof of the main section. The bottom of the inlet continued level for 3 feet then slanted downward to form a smooth transition from the inlet to the water surface. The slope of the inlet was a 1-inch drop in 12 inches in the direction of the airflow. The slope of the outlet had a 3-inch rise in the first 12 inches of the section. At the junction between the water section and the outlet section a wave absorber was built to minimize the reflection of waves from the bulkhead. The wave absorber was constructed from a packed bed of aluminum turnings placed on the sloping section. As waves impinged on the turnings most of their energy was dissipated in turbulent flow. Little or no wave reflection was observed in the channel.
The outlet was a rectangular duct 4 feet wide and 21 inches high. It is attached to the main channel section on one end and on the other end to the suction side of the blower. Downwind from the main section a set of vanes was built to permit control of the airflow rate in the wind-water tunnel. A bleed hole 16 inches in diameter was cut into the roof of the outlet for additional flow control. When the hole was opened air was drawn through it as well as through the wind-water tunnel allowing lower airflow rates. The outlet of the blower was exhausted into the atmosphere. The blower was an FC-27 squirrel-cage blower manufactured by the Trane Company. The maximum flowrate was about 15,000 cfm.

Auxiliary equipment associated with the wind-water tunnel was:

a) piping and the diffuser tube, b) the velocity-measuring system, c) the water-surface measuring systems, and d) the sample withdrawal tubes. Each of these components are described in the following paragraphs.

Water was obtained either from the salt-mixing tanks or from the Logan City water system. During constant-density tests the water was obtained from the city lines and for stratified tests salt water was obtained from the salt-mixing tanks. Two salt-mixing tanks were used— one tank 8 feet square and 4 feet deep, the other 5 feet square and 5 feet deep. Solutions of specific densities were mixed in the tanks and placed in the main section by a bottom diffuser pipe. The diffuser pipe was a 2-inch PVC plastic pipe laid along the centerline of the main section. Two 1/8-inch diameter holes were drilled at one-foot intervals
along the pipe. These holes were drilled at 90 degrees from each other. The pipe was placed so the jets of water from the holes were at a 45-degree angle to the floor of the tunnel. A clearly defined stratification could be developed when the flowrate was kept low enough by means of the use of the diffuser pipe.

Air velocities in the tunnel were measured with a standard pitot tube and either a sensitive pressure transducer or a manometer-cathetometer combination. The transducer, built at the Utah Water Research Laboratory, had the core of a linear differential voltage transformer (LDVT) connected to a thin diaphragm which deflected when a pressure was applied to one side of the diaphragm. The output from the LDVT was amplified by a PAR H-8 phase sensitive amplifier. The calibration curve of the transducer is shown in Figure 5. A fifth degree polynomial was used to fit the data points. Maximum deviation from the curve is 0.0789 psf. This dynamic pressure corresponds to a velocity error of approximately 1.7 fps at 20 fps.

The manometer was two 2-inch diameter plastic tubes connected at one end to each other by flexible tubing. The opposite ends of the tubes were connected to the pitot tube. The Wild cathetometer could read the heights of the liquid level in these manometer tubes to an accuracy of ± 0.01 mm. Velocity profiles were taken across the entrance of the wind tunnel and mean velocities were determined for the main section. A centerline velocity 10 inches above the floor of the inlet was selected as the reference velocity. The mean velocity is
Figure 5. Calibration curve of the pressure transducer.

\[ p = -0.00974 + 0.1448E + 0.0602E^2 \\
   - 0.0131E^3 + 0.00122E^4 - 0.0000416E^5 \]
plotted against the reference velocity at the inlet section in Figure 6. The equation of the line is\[ u_m = 0.39 + 0.769 \ u_R \] in which \( u_m \) is the mean velocity and \( u_R \) is the reference velocity. Both the transducer and the cathetometer were used to obtain the mean velocities from the reference pressure of the pitot tube.

A profile of the water surface can be determined from pressure taps inserted beneath the water surface along the channel which are connected to open tubes. The profile can be determined very accurately from the height of the liquid in the tubes. The heights of the liquid in the tubes were determined with the cathetometer. Pressure taps were placed every 10 feet along the channel, however, liquid levels were usually determined at the upstream end, midpoint and downstream end of the flume. The pressure taps were connected to open tubes. The change in elevation of the water surface determined from open tubes was the slope of the water surface corrected for the pressure drop along the channel. Francis (1951) used this same method for his study of wind set-up on the water surface. Each of the open tubes used was made from 2-inch I.D. acrylic tubing with the bottom end closed except for a 1/4-inch pressure connection.

Concentrations of the dye used during the dispersion tests were determined from samples taken from the whole cross-section of the flow. An array of 1/8-inch diameter stainless steel tubes was constructed as shown in Figure 7 and was used to obtain a representative concentration at a section. A sample withdrawal array was placed every 5 feet along the
Figure 6. Mean wind velocity vs. reference velocity.
Figure 7. Schematic sketch of the dye sample collection apparatus.
channel. Holes were cut into the cross arm tubes every 3 inches across the channel. Five cross tubes were placed at 2, 7, 12, 17, and 23 inches below the water surface. When water was collected from the tubes tests showed nearly equal flow was obtained from all points of the cross-section. Flow in the tube was controlled by Tygon tubing connected to the end of the collector tube inserted through the side wall of the tunnel. When a sample was taken the Tygon tubing was lowered and the test tube was filled by gravity flow. After the sample was taken the free end of the tubing would be raised above the water surface of the wind-water tunnel. The sample withdrawal tubes were used only for the dispersion test.

Dye for the dispersion tests was introduced as a uniform impulse at the middle of the channel. A system of three reservoirs connected to three diffuser tubes provided a uniform vertical sheet of dye across the cross-section of the channel at its midpoint. All of the diffuser tubes were 1/8-inch diameter stainless steel tubing with holes drilled at 2-inch spaces along the tube. The middle tube had holes directing jets both upward and downward. The upper tube directed a line of jets downward against the jets of the middle section. Likewise a lower diffuser tube had jets directed upward against the downward facing jets of the middle tube. This arrangement of the jets caused the dye to be spread in an even sheet from top to bottom of the channel. Movement of the dye from the middle of the channel depends upon the wind velocity, depth of stratification, and the density difference between the layers.
Experimental Procedures

Basically three different series of tests were performed in the wind-water tunnel. The first series was to determine the relationships between water surface shear stress and the mean velocity of airflow for both constant density and stratified-flow conditions. The second series was to study the dispersion and diffusion which can be attributed to wind-generated currents caused by density stratification. The third series of tests was to determine the critical mixing parameters in a stratified shear flow caused by wind shear at the free surface. However, before these three series of tests are described in detail, techniques and measurements common to all of the tests will be described.

Prior to any testing the water section of the wind-water tunnel was filled with either fresh water or with a layer of denser salt water below a layer of fresh water. Of course, in the simplest case the channel was filled to a depth of 2 feet with fresh water. Filling for these tests consisted of turning on a valve, and gradually filling the channel until the water was 2 feet in depth. The water then remained undisturbed in the channel for several hours after filling to allow any residual currents to be dissipated by internal friction.

At the beginning of a test the blower would be started with the vanes in a fully closed position. Gradually, the airflow would be increased by opening the vanes until the desired flow was obtained. If the velocity
was increased rapidly an oscillation of the water surface would be created and cause errors in water surface measurements.

The instant that air began to move across the surface water began to move in the direction of the airflow. As the water moved downwind the surface of the water began tilting so that the elevation of the downstream end of the water channel was higher than the upstream end. This increased depth in the direction of airflow caused an unbalanced hydrostatic pressure in the water. Consequently, water began to flow toward the upstream end of the channel along the bottom. Various researchers, Keulegan (1951), Francis (1951), Plate and Goodwin (1965), and O'Brien (1971), have shown that the surface water velocity is approximately

\[ 0.03 \frac{u_m}{u} \]

in which \( u_m \) is the mean velocity of the airflow.

Measurement of the elevation changes of the water surface has been done with inclined manometers (Keulegan (1951)), cathetometer sightings on large diameter, 2-inch glass or plastic tubes (Francis (1951)) or electrical point gages. The combination of a cathetometer and open plastic tubes was used for this experiment. After the air velocity had remained at a constant value for 5 to 10 minutes an equilibrium between the surface slope and the return currents was established. Then the difference in elevation of the meniscus of the tubes was determined with the cathetometer and recorded on the data sheets.

The water surface elevation could be determined at every 10-foot interval along the channel. However, during most of the test the elevations
were determined at a point 7 feet from the upstream end and at 20-foot intervals down the channel. The water surface was found to be close to a straight line (see Keulegan (1951)). Additional data were examined during the initial runs which verified that water surface was approximately linear in the middle section of the channel. Slight nonlinear variations were observed near the ends of the channel.

Filling the channel for a density-stratified run begins in the same manner as described for the constant-density case. The similarity ends when the fresh water layer is 6, 12, or 18 inches in depth. The fresh water layer was left undisturbed for several hours before the filling process was continued with salt water. During this interval salt was mixed in the salt-mixing tanks. A specific amount of salt was added to a tank in order to obtain a given density difference. Figure 8 indicates the amount of salt added by weight to obtain the density difference desired. As an example, 150 lbs of salt added to 15,000 lbs of water results in a density of 1.94377 slug/ft$^3$ for the solution. After the salt solution was prepared it was pumped to the constant head tank and allowed to flow by gravity through the diffuser pipe into the channel. It was distributed along the bottom of the channel below the fresh water and the fresh water was raised with very little mixing when the velocity in the diffuser pipe was low. Flowrate during the first one-inch depth of the salt layer must be very low to prevent mixing. Later the flowrate was increased to .10 to .20 gpm depending on the density difference ($\Delta\rho$) of the layers.
Figure 8. Amount of salt necessary to obtain desired density difference.
The filling process continued until the total depth was 2 feet. Then the stratified system was allowed to remain undisturbed for a period of time before airflow was initiated. Both procedures for start up and measurement of the water surface were the same for a density-stratified flow as for constant-density runs. However, the response of the layered system was considerably different from that of the constant-density case. Flow in the upper layer is very similar to that described for the constant-density case. Significant differences between the constant-density case and the stratified case are: a) set-up (tilt) of the interface between the two layers is in the opposite direction to the tilt of the free surface; b) flow along the interface was in a direction opposite of the airflow; and c) flow along the bottom of the channel was in the direction of the wind because of the slope of the interface and the pressure gradient. A negligible amount of mixing occurs between the layers until the air velocity becomes large enough to cause complete vertical mixing.

**Air velocity**

Mean air velocity in the channel was determined by measuring the reference velocity in the entrance by means of large diameter manometer tubes connected to the pitot tube. Elevation of the water level in the manometer tubes was read with the Wild cathetometer. The position of the reference point is 2 feet from the inlet at the centerline of the tunnel and 10 inches above the water surface. The reference velocity was related to the mean velocity of the airflow by the linear relationship shown
in Figure 6. All velocities for the various tests were determined in the manner described above. Errors associated with reading the water surfaces in the tubes caused negligible velocity errors.

Shear stress determinations

Initially the shear stress was determined by a series of separate runs. Later, values of shear stress were calculated from data obtained during runs for dispersion and critical mixing. Equation 31 was used to evaluate the shear stress from the corrected set-up of the water surface for the unstratified runs. Equation 37 was used to determine values of shear stress for the stratified runs. Data values necessary to evaluate shear stress $\tau_s$ are described, respectively, for unstratified flow and for stratified flow.

Unknowns of Equation 31 are $\frac{dH'}{dx}$ and water density which was assumed to be 1.94 for these tests. This value of water density is very near the values obtained from water density determinations made at the same temperature. Values of $\frac{dH'}{dx}$ were obtained by the cathetometer and recorded to the nearest 0.01 millimeter. Values of shear stress were then calculated from Equation 31 and results are shown in Appendix A. Additional values of shear stress were obtained from all constant-density tests run throughout the experiments.

Considerably more unknown variables are required to compute the values of shear stress for density-stratified runs. There are seven unknowns that must be evaluated in Equation 37. Four of these were
determined during each test run. These were $\Delta H'$, $H_1$, $H$, and $\Delta p$.

Measurement of change of water height in the open tubes determined $\Delta H'$ and $\Delta p$. Total average water depth, $H$, and average height of density layer, $H_1$, were determined from measurements outside the plexiglass windows. Three values of $H_1$ were obtained along the channel and the mean value was used for the calculations of shear stress. The value of $dx$ for the calculations was 40 feet for all calculations made. Values of density were obtained by placing a fixed volume of water in previously weighed flasks. Then the combination of water and flask was weighed and a density determination made. Consequently, with all variables on the right side measured a value of shear stress was calculated.

**Diffusion-dispersion tests**

Dispersion tests for both constant density and stratified flow were very similar, the only differences being in the method of filling the channel and placement of the tracer dye. Techniques for filling the channel were described at the beginning of this chapter and will not be elaborated upon.

In Chapter 2 it was emphasized that the dye should be suddenly injected into the channel across the cross-section. Three diffuser tubes were used for the constant-density runs and four diffuser tubes were used for the density-stratified tests.
Initial startup of the airflow has been previously described. After the airflow and resulting water currents had reached equilibrium the required data were taken to obtain the shear stress on the free surface for that run. Next the rhodamine WT dye was released into the channel and samples were withdrawn at specific time intervals after the dye had been released. Sample intervals after the dye was released varied from 2 minutes to 15 minutes. Integrated water samples were obtained from the channel by the arrays shown in Figure 7. This array obtains an average amount of dye at the cross-section of the channel from which it was taken. Each of the samples was placed in test tubes and analyzed for fluorescence on a Turner 110 fluorometer. No interference in the reading was noted because of the sodium chloride in the solution.

Data collected for these tests were reference velocity pressure, air temperature, air pressure, corrected slope of water surface, and concentration of the dye at 10-foot intervals along the channel from the point of release as a function of time. From these data, values of \( \tau_s, u_*, x^2/t \) and \( ct^{1/2} \) were calculated. Shear stress and velocity were plotted on one graph and compared with the unstratified shear stress relationship. Values of \( \log ct^{1/2} \) versus \( x^2/t \) were used to evaluate the value of the apparent diffusion coefficient.

**Critical mixing tests for stratified flow**

Prior to any mixing tests the channel was filled as has been described previously. However, the denser water in the lower section contained
enough rhodamine WT dye to color it a bright pink. Three nominal water depths of 6, 12, and 18 inches were used for the salt layer. Also three different values of density difference were used for the tests. Consequently, each test was run for a specific density difference between the layers and a specific depth of the salt layer. After the filling of the channel the water was allowed to settle to eliminate any internal eddies of motion. There was no discernible mixing of the dyed salt layer into the clear water during this period.

Procedures for start of the air motions were more critical for the tests with small density differences and the 18-inch salt water depth than for large density differences and a 6-inch salt water depth. Consequently, the flow control vanes were opened very slowly and care was taken not to prematurely mix the two layers by abrupt increases in the air velocity. The vanes were opened to obtain a minimum velocity of about 10 fps. Then a series of readings of shear stress data were made and the behavior of the interface was observed. The values of $\Delta H'$, $H_1'$, $\rho_1'$, $\rho_2'$, and $\Delta p$ were recorded for each distinct velocity step up to and including the velocity that caused the lower layer to be mixed with the upper layer. At the point of critical mixing the denser water at the upstream end of the channel would be mixed with the water of the upper layer by turbulence and mass transport caused by the breakup of the interface. The mixing of the two layers was determined by visual observation at the windows along the side of the channel. For the tests
with maximum density difference and a salt depth of 6 inches no mixing could be obtained even with the maximum air velocity.
CHAPTER 4
DATA PRESENTATION AND DISCUSSION

This chapter will present summaries of the data collected and explain the results of the tests. Where possible the results will be compared with previous research. The basic data are presented in Appendix A. Each of the following sections will discuss the data taken and the techniques used to reduce the data. The data are then presented in the form of several graphs.

Shear Stress Tests

Results of the measurements of shear stress for both the stratified and unstratified cases will be discussed separately. Data required for the unstratified shear stress tests consisted of air temperature, barometric pressure, dynamic velocity head of the airflow, and the set-up of the free surface, corrected for pressure drop along the channel. Air temperature, barometric pressure, and the velocity head were used to calculate the reference velocity of the airflow. The mean velocity of the airflow was read from Figure 6. The shear stresses for each unstratified test were calculated from Equation 31 and the results of these calculations are shown in Figure 9. The two dashed lines on Figure 9 are values of \( \tau_s = f(u_m) \) from two previous research efforts. The values of shear stress from the present tests are higher than those of the previous
Figure 9. Curve of mean velocity versus $\tau_s$. 

$$\tau_s = 0.00484 - 0.0007365 u_m + 0.0000347 u_m^2$$
researchers when based on some velocity $u_m$. However, there are many complicating factors in the relationship between $\tau_s$ and $u_m$. The primary reason for the differences is the sizes of channels used by various researchers. The 4-foot by 4-foot channel cross-section used in the present tests is much larger than the channels used by Francis, Keulegan, and Fitzgerald. Keulegan used a channel about 6 inches by 11 inches high; Fitzgerald used a channel 6 inches wide by 6 inches high. In contrast, Hidy and Plate used a 2.5-foot by 3-foot channel and obtained higher values of shear than the others mentioned. There is a scaling factor in the relationship of shear stress to velocity. For example, Van Dorn (1953) obtained values of shear stress on an open pond very close to that of Keulegan, when referenced to air velocity measured 10 meters high. The shear stress coefficient was shown by Van Dorn (1953) to increase if the velocity was measured nearer to the water surface. Another difference between the tests was the fact that the mean velocity was measured at a height of 10 meters above the ground by Van Dorn and the mean velocities in wind-water tunnels were obtained from the flowrates and cross-sectional area in the tunnels. The flow in these wind tunnels had a core of fluid with nearly potential flow characteristics surrounded by growing boundary layers on all sides of the core. A sound basis to calculate the basic relationship between shear stress and wind velocity for this condition has not been developed. Consequently, small differences between these velocities could cause
substantial changes in the relationship between $\tau_s$ and velocity. The relationship obtained from the tests is

$$\tau_s = 4.84 \times 10^{-3} - 7.3 \times 10^{-4} \frac{u}{m} + 3.47 \times 10^{-5} \frac{u^2}{m}$$

The size variation of the channels have the same relationship as the values of shear stress for a given velocity, i.e., Keulegan's being the smallest $\tau$-value and the author's being the largest for a given velocity. Before more reliable data can be obtained more sensitive measurement techniques and equipment must be developed. Additional factors affecting this relationship are length of the channels, wave action on the water surface, and the stratification of the fluid.

In summary it is apparent that the relationship between shear stress and velocity is generally difficult to establish. However, it is not the purpose of this work to establish such a general relationship. Even though the shear stress-velocity relationship found in this work is unique to this study, the shear stress on the surface is believed to be accurate. Hence the internal circulation of the water, which is dependent on surface shear stress, should be accurately determined.

Several more data elements are required for the density-stratified shear stress tests. The additional elements required are the density of the lower layer, the density of the upper layer, and the depth of the lower layer. Values of shear stress were calculated using the data in Appendix A and Equation 37, which was developed in Chapter 2. Results
of these computations are shown in Figure 10.

Several general observations can be made about the relationships. First, the data were obtained during the tests for critical mixing and there are no data values at wind velocities beyond where the two layers became mixed. Second, for a given wind velocity, the surface shear stress increases as the density differences between the layers decreases. Third, the surface shear stress decreases as the depth of the salt layer increases.

Values of shear stress lie both above and below that obtained for the constant-density case depending upon the height of the salt layer and the density difference. The shear stress was found to increase over that of the constant-density case as the density differences approached zero. This result was caused by the instability of the interface for these conditions. For a given wind velocity the slope of the interface increases as the density difference becomes smaller. Consequently, the free surface was tilted in the opposite direction and the set-up obtained was found to be much larger than that observed for the constant-density case. This leads to a higher surface shear stress. These data were not compared with any other work, because the only overlapping areas of previous tests are in the regions of very low velocities. However, in this region the results of these tests are in good agreement with other researchers.

In summary, shear stress measurements reported here are slightly higher than those obtained by previous researchers for the
Figure 10. Curves of mean velocity vs. shear stress for selected density differences and layer thickness.
constant-density tests. The data obtained in the stratified tests indicate that the relationship between velocity and shear stress was significantly influenced by the density difference and the ratio of the depths of the density layers. Good relationships were obtained for each of these parameters. Further efforts beyond the scope of this work will be necessary to obtain a general relationship between shear stress and air velocity. A clear understanding of the momentum exchange between the wind and the water is required to develop the generalization.

**Diffusion-Dispersion**

Data for all diffusion tests on both stratified and unstratified flow were collected and analyzed in the same way. The shear stress data associated with the diffusion-dispersion tests were collected and computed as outlined in the previous section. Only the data required and the method of handling dispersion data will be outlined here.

The dye concentration $c(x,t)$ was determined for different values of the depth of the density layer, density difference, and for different mean velocities in the airflow section. For each test, apparent diffusion coefficients both upwind and downwind from the point of release were computed. This was done by determining the slope of the regression curve of $\log ct^{1/2}$ versus $x^2/t$ which is equal to $\frac{\log e}{4D}$. 
Dispersion

Twenty-two separate diffusion-dispersion tests were run for the unstratified flow. Two or more replications were run at the same reference velocity. The 22 runs were separated into seven different groups. Each run within a group had very nearly the same velocity. Mean velocities and mean diffusion coefficients were determined for each group. From the quadratic least squares fit in Figure 9, the surface shear stress was determined for each mean velocity. A shear-velocity Reynolds number is calculated and the diffusion coefficient is found in the manner outlined above. The results are plotted as a non-dimensional diffusion coefficient $D^*$ on Figure 11. The dimensionless diffusion coefficient was formed by dividing $D$ by the product of depth and shear velocity. The minus-curve, or the curve for the diffusion upwind from the point of injection, has a larger diffusion coefficient than the downwind section. This effect is caused by the shape of the water velocity profile in the channel. About two-thirds of the total depth is flowing upwind and the dye is quickly mixed throughout the whole depth. But for the downwind flow only a thin layer near the surface is moving at a high velocity. This shallow layer moves downwind with reduced lateral mixing of the dye throughout the flow. Although the diffusion coefficient of only the shallow, high velocity layer had a higher value than the upwind mean $D$-value, when averaged over the entire depth it had a lower $D$-value.
Figure 11. Curve of $D^*$ as function of $R_{u*}$ for unstratified flow.
The curves indicate a drop in \( D^* \) as the Reynolds number increases. This is because the dimensional \( D \) is approaching a constant value which is a function of the intensity of turbulence, while the wind velocity continues to increase. In a wind-driven experiment such as this, the point where the wind-driven turbulence approaches a constant value can be shown on a plot of \( D \) versus Reynolds number and is shown in Figure 12. The upwind \( D \) is approximately two times as large as the downwind \( D \). This would tend to cause an accumulation of a pollutant at the upwind end of a pond when the wind was blowing. This behavior has been observed on the sewage lagoons at Logan, Utah.

Data for the stratified runs were limited by two factors: First, because of the length of time required for one test, only one value of \( H_1 \) was used during the stratified diffusion runs. Second, valid data could only be collected up to the point where mixing occurred between the layers. As a result of these limitations only the data for density differences of \( \Delta \rho = 0.0143 \) and \( \Delta \rho = 0.0071 \) were used. Figures 13 and 14 are graphs of the dimensionless diffusion coefficient versus the shear-velocity Reynolds number. Also shown on the graphs are the curves for the constant-density case. The stratified diffusion coefficients both upwind and downwind from the point of dye release approach the constant-density case as the point of mixing is approached. The high diffusion coefficient at low velocities may have been influenced by the fact that the dye was not as evenly dispersed by wind-driven turbulence, hence
Figure 12. D as a function of shear velocity Reynolds number.
Figure 13. Downstream dimensionless diffusion coefficient as function of shear velocity Reynolds number.
Figure 14. Upstream dimensionless diffusion coefficient as function of shear velocity Reynolds number.
the sampling procedure may not have been representative. The turbulence due to the wave action did not penetrate the total depth of the flow and the dye moved in narrow bands without much mixing between these bands. This occurrence allowed concentrated areas of dye to move rapidly along the interface driven by the density currents at the interface.

From Figures 13 and 14 it can be seen that the increased diffusion due to stratification gradually decreases as mixing is approached. Consequently, the diffusion within a stratified body of water is increased over the constant-density case provided that the winds are mild. This effect decreases as higher shear stress occurs on the water surface. The explanation of this "apparent" increase in diffusion is that the flow was laminar and parallel to the channel for the lower velocities and that for the higher velocities the flow was not only parallel to the channel but had considerable vertical mixing within each layer.

**Critical Mixing**

Closely associated with the problem of longitudinal diffusion is vertical mixing between the layers. During this series of tests the stratified fluid was placed in the channel. Three different values of density were used for the tests and depths of the salt layer of 6, 12, and 18 inches were employed. Critical shear stress was defined as that shear stress which caused vertical mixing between the two layers. Only the critical shear stress, the value of density difference, and original
depth of the salt layers are used to obtain a relationship that indicates minimum values of shear required to mix the layers.

The rationale behind the method of presentation was derived from dimensional analysis arguments as shown in Chapter 2. To further clarify the concept of a critical shear velocity, consider the case when a constant value of shear stress is maintained at the surface of a stratified pond. Next, allow the density difference between the two layers to decrease until the shear stress becomes great enough to cause complete vertical mixing. This would be the critical shear stress for that $\Delta \rho / \rho$. The tests for critical shear velocity were conducted by varying the value of shear stress and maintaining a constant-density difference. Data in Figure 15 show critical shear-velocity Reynolds numbers plotted versus $\frac{H_1}{H}$ with $\Delta \rho / \rho$ as a parameter. The curves of the figure indicate that as the density difference is decreased the fluids are mixed at a lower shear-velocity Reynolds number. Also as the depth of the salt layer is decreased, mixing becomes more difficult to initiate or maintain. This is a direct result of the velocity profile in the channel. The velocity near the bottom of the channel is much smaller than it is nearer the surface. Consequently, the velocity gradient and resulting shear stress in the water are less. Also the interface is farther below that zone of influence of wind-generated waves when $H_1$ is small. All these factors promote a more stable interface. The effect of the density variation on mixing will be expanded upon.
Figure 15. Critical mixing Reynolds number vs. density difference and layer thickness.
Keulegan (1949) studied the mixing and stability at the interface of a density-stratified fluid. Results from his tests indicated the relationship given previously in Equation 38. For conditions above Reynolds number of 450 for the channel, $\theta$ was 0.178. For conditions below $\text{Re} = 450$, $\theta$ was found to be 0.127. The general conclusions resulting from Keulegan's investigation were similar to those obtained from the present study. Because of the fact that different fluids were used and because of an inability to measure water velocities, no direct comparison can be made between the data sets.
CHAPTER 5

CONCLUSIONS

The following general conclusions can be made about the results of this experimental investigation:

1. Surface shear stress is not a unique function of mean velocity. The relationship is dependent on the physical size and shape of the experimental facility. The basic relationship by $\tau_s$ and velocity can be established for a given facility. The resulting $\tau_s$ values can be used as the driving force for the internal mixing phenomenon, i.e., the relationship between $\tau_s$ and velocity need not be general so long as $\tau_s$ is correct.

2. For constant-density flows diffusion coefficients tend toward stabilization at higher values. As the turbulence of the flow becomes established, the diffusion mechanism tends toward constancy. The upwind diffusion coefficient is about twice that of the downwind diffusion coefficient.

3. For stratified flows, the dimensionless apparent diffusion coefficient $D'$ is highest at lower shear-velocity Reynolds numbers. As the velocity is increased and breakdown of stratification is approached, the diffusion coefficients approach the values for constant density. For strongly stratified flows, $D'$ can be 5 to 7 times those values for constant-density flows.

4. The shear-velocity Reynolds number at which the stratification breaks down and complete mixing occurs is well established. As logic
would dictate, the breakdown occurs sooner when the relative density difference is smaller and when the relative depth of the higher density layer is greater. Figure 15 shows these interrelationships which can be used for predictive purposes.
SELECTED REFERENCES


Table 1. Calibration data for pressure transducer.

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Table 2. Mean velocity and reference velocity in wind-water tunnel.

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Table 3. Mean velocity shear stress data for unstratified flow.

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<th>Mean Velocity (fps)</th>
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Table 4. Shear stress data for stratified flow.

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Table 5. Dye dispersion data for unstratified flow.

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D_L = Diffusion Coefficient

U_* = Shear Velocity

d = Depth of Flow
Table 6. Dye dispersion data for stratified flow.

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