Predictive Formula for Electron Range over a Large Span of Energies

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Predictive Formula For Electron Range over a Large Span of Energies

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I. Theory
The range, commonly known as the penetration depth, describes the maximum distance electrons can travel through a material, given an initial incident energy, before they lose all of their kinetic energy and come to a rest. The primary energy loss mechanism which causes the electrons to lose their kinetic energy is due to inelastic collisions with material. Due to the probabilistic nature of this mechanism, the Continuous Slow Drift Approximation (CSDA) is often employed to simplify the problem where the stopping power is taken as a constant.

This idea is illustrated by a Lichtenecher discharged tree pictured in Fig. 1. The tree is an example of a situation where an accelerated high voltage comes to rest and deposits energy at a given range in an insulating material. The side view of the Lichtenecher tree displays the melted plastic caused by the energy of the deposited incident electrons at a uniform penetration depth. Here the stored charge is dissipated through a discharge.

II. Original Model
The model previously developed by the Material Physics Group predicts the energy-dependent range, R(E), as a function of incident electron energy for materials found in the NIST ESTAR database. In a continuous composite analytic approximation to the range with a simple fitting parameters, the effective parameter values and bandgap's effects were quantified. A series of linear equations which were put into a matrix form giving a standard equation problem. The fitting parameters were then calculated to estimate an effective value of range for a low level model. Plotting this estimate of Nₑ versus the true value of Nₑ allows us to quantify the fit of the model as shown in Figure 6. 

Fits to these initial equations and optimum values of Nₑ were found using a data for about 20 well-known elements and compounds. Figure 5 shows several approximate fits to the range data from the ESTAR database.

Fig. 1. Range for low energy, medium energy, and high energy.

Fig. 2. Fitting parameters for the graphs in Fig. 8 as well as the linear fit statistics.

Table 1. Fitting Parameters for the graphs in Fig. 8 as well as the linear fit statistics.

III. Expanding Database
To further validate the range formulations and to lead to possible new discoveries in range penetration, the Material Physics Group’s material database needed to be expanded. A spreadsheet had previously been compiled with information on a number of materials. The spreadsheet's minor errors were corrected and both the total number of materials and database parameters for each material were extended. These considerations such as phase, color, and conductivity were added. The greater number of materials allowed for a more exact fit to be determined with the CSDA. Adding more parameters offered the opportunity of exploring possible trends that might enable a discovery of an even more exact function to describe the range.

IV. Formulation for Analytic Solution
A predictive formula such as the one seen in (7) was used to look at our single parameter Nₑ as a function of various factors. The information gathered in the analysis was fit to Nₑ in hopes of finding strong trends between variables like density, effective atomic weight, mean atomic number, plasmon energy or bandgap, conductivity, phase, and more. Continual modification to the range model using our theoretical equation (8) could lead us to universal values for A, B, C, and D. Equations were further subcategorized into grouping such as insulators, conductors, and semiconductors and liquids, solids, and gasses with the hope that different trends with different parameters would be discovered. Perhaps equations for conductors versus insulators and semiconductors would have somewhat different values for A, B, C, and D.

An analytical solution was created to greatly increase the ease with which the fitting parameters in the theoretical formula could be found. First a power law regression for Nₑ was modeled using the method of Best Estimates. To minimize the partial derivative with respect to each fitting parameter was calculated and set to zero. This gave a series of linear equations which were put into a matrix form giving a standard equation problem. The fitting parameters were then calculated to estimate an effective value of Nₑ using the power law model. Plotting this estimate of Nₑ versus the true value of Nₑ allows us to quantify the quality of the fit as shown in Figure 6.

V. Affordable Margin of Error
In order to perform range calculations, a value for the electron bandgap was needed for each material. While some material bandgaps were easier to find than others, it was necessary for a comparison to be made to see how much the fitting factor would change with a varying bandgap value. These calculations gave desirable results, showing that the fitting factor varied minimally with changing bandgaps. For an example, see Figure 9 which uses alumina (Al₂O₃) as our chosen material. It was found that the error between the values in our calculations and the values in the NIST database increased as the bandgap’s value increased from the true value.

VI. Applications
The range model developed predicts the penetration depth for various materials for different incident electrons. Its effects extend to spacecraft charging where the range is used to predict the range of incident electrons from the space environment and materials as well as the energy deposited by the electrons as they penetrate through materials. The range is also used in Electron Beam Therapy (Figure 10), the most common form of medical radiotherapy. Obtaining accurate, reliable, and efficient information on the range of electron penetration is, therefore, extremely important to the medical community.

VII. References

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