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Kenneth S. Lyon
Utah State University

Ming Yan
Utah State University

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COMPENSATING VARIATION CONSUMER'S SURPLUS VIA SUCCESSIVE APPROXIMATIONS

By

Kenneth S. Lyon

Ming Yan
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Many economists agree that the best measure of consumer's surplus is the compensating variation consumer's surplus (CVCS); however, because the compensated demand function is not observable, there are problems of using this measure in empirical applications. There are ways around this limitation that work well under certain circumstances; however, until now there has been no solution that always works well. We introduce a successive approximations method of calculating compensating variation consumer’s surplus using data from the ordinary demand curve. In doing so we numerically identify the compensated demand function over the price interval involved. This procedure can be implemented on any ordinary demand function that is consistent with a quasi-concave utility function, and does not require that we integrate back to the utility function. We demonstrated that the error of our approximation can be made extremely "small." We also use our method to calculate CVCS for three applications in the literature where Marshallian consumer’s surplus was reported.

Keywords: consumer’s surplus, compensated demand, successive approximations

Kenneth S. Lyon
Professor, Economics
Utah State University
Logan, Utah 84322-3530

Ming Yan
Graduate Student, Economics
Utah State University
Logan, Utah 84322-3530
COMPENSATING VARIATION CONSUMER’S SURPLUS
VIA SUCCESSIVE APPROXIMATIONS

Introduction

Many economists agree that the best measure of consumer’s surplus is the compensating variation of income, also called compensating variation consumer’s surplus (CVCS), which is the area under the compensated (Hicksian) demand curve instead of that under the ordinary (Marshallian) demand curve. But when this concept is applied to empirical studies, the problem of computation arises since the compensated demand is unobservable. Empirical studies supply us with information about ordinary demand curves. Applied economists have three alternative ways around this data problem. (1) Use Marshallian consumer’s surplus (MCS) then either ignore the difference between the concepts or refer to Willig’s theoretical study (Willig 1976) which claims that in some range of income elasticity the Marshallian consumer’s surplus is a good approximation of CVCS. (2) Calculate CVCS using the expenditure function derived from the indirect utility function obtained by solving the differential equations which Roy’s identity generates. Hausman (1981) has done this for the linear and constant elasticity demand functions and has derived the explicit formulae to calculate CVCS for these functions. These formulae, however, are of limited use since they are for only two specific forms. (3) Estimate the quadratic approximation of CVCS using the Slutsky equation.

This paper introduces a fourth alternative, and gives researchers a relatively simple and highly accurate way of calculating CVCS using the parameter estimates for the ordinary demand function. In addition, this method can be used regardless of the functional form of the ordinary demand curve as long as it is consistent with income constrained maximization of a quasi-concave utility function.

When the price elasticity of demand is not very different for the ordinary and compensated demand functions these two consumer’s surpluses likewise are not very different. There are times, however, when the two elasticities and two consumer’s surpluses do differ by a large enough amount that the error of the MCS is not negligible. Examples of this are presented below in the rent control and oil import fee applications. The price elasticity of ordinary demand for a low income consumer is about -0.5 while this elasticity for the compensated demand is about -0.25.
This last elasticity follows from the Slutsky equation in elasticity form where the income elasticity is about 1.0 and the share of housing in the low income consumer’s budget is usually at least 0.25. This example illustrates that the difference between CVCS and MCS depends not only upon the income elasticity of demand but also upon the share of the commodity in the budget.

Below we give a brief review of CVCS, and then we present our method of calculating CVCS. In these calculations we use the method of successive approximations. Along with this development we show that the error of this approximation can be made extremely small. For the comparison presented it is 9.73D-14. Following this we use this method of successive approximations to analyze three consumer’s surplus applications in the recent literature. These include a rent control example, a physician immigration example, and an oil import fee example. Then the final section is a summary.

Review of CVCS

In Value and Capital, Hicks (1946, p. 40) gave the following definition of compensating variation of income:

As we have seen, the best way of looking at consumer's surplus is to regard it as a means of expressing, in terms of money income, the gain which accrues to the consumer as a result of a fall in price. Or better, it is the compensating variation of income, whose loss would just offset the fall in price, and leave the consumer no better off than before.

If the price of \( x_i \) decreases from \( p^0_i \) to \( p^1_i \) then it is straightforward to show that

\[
CVCS = e(p^0_i, p^0_j, U^0) - e(p^1_i, p^0_j, U^0)
\]

\[
= \int_{p^0_i}^{p^1_i} x^H_i(p, p^0_j, U^0) \, dp
\]

where \( e \) is the expenditure function, \( x^H_i \) is the compensated demand function, \( p_j \) is a vector of all other prices, and the super \( \theta \) indicates initial or original values.

CVCS by Method of Successive Approximations

We now describe a method of approximately, numerically deriving the compensated demand curve from the ordinary demand curve over some interval of price. The error of the approximation can be made extremely small, and the accuracy can be checked for some functions. We first break the interval of price into \( n \) subintervals and proceed through these bootstrapping
from one interval to the next. At each step we approximate a point on the compensated demand curve using data from the ordinary demand function. These points are then connected with straight lines yielding an approximation of the compensated demand curve that is used to calculate the CVCS. Starting from the initial point and working on the first subinterval we first seek to find using successive approximations the compensating income, \( F \), for this subinterval such that

\[
F^0 - F = \left[ x_i(p_i^0, p_{i1}^0, F) + x_i(p_i^0, p_{i2}^0, F) \right] (p_i^0 - p_i^1) / 2
\]  

(2)

where \( F^0 \) is the initial income and expenditure, \( x_i() \) is the ordinary demand function and \( p_i^1 = p_i^0 - (p_i^0 - p_i^1)/n \). This equation is suggested by

\[
e(p_i^0, p_{i1}^0, U^0) - e(p_i^0, p_{i2}^0, U^0) = \int_{p_{i1}^0}^{p_{i2}^0} x_i(p_i^0, p_{i2}^0, e(p_i^0, p_{i2}^0, U^0)) \, dp_i,
\]

(3)

Equation (3) follows directly from (1) because \( x_i^e(p_i^0, p_{i2}^0, U^0) = x_i(p_i^0, p_{i2}^0, e(p_i^0, p_{i2}^0, U^0)) \), and

![Figure 1. Successive approximation of CVCS.](image)
equation (2) is an approximation of equation (3) where the integral is approximated by the area of a trapezoid. Also $F$ is an approximation of $e(p^n_0, p^c_n, U^0)$. These relationships are identified in Figure 1 for $x$ a normal good, where we exaggerate the error by showing a single step ($n = 1$) over a relatively large decrease in price. The CVCS is of course the area in the first quadrant to the left of the compensated demand curve between $p^n_0$ and $p^l$, and the area of the trapezoid $p^n_0 p^l DA$ is $F - F$, the approximation. In addition, defining the error as the true CVCS minus the approximation, the error, which is negative in this example, is given by the area of triangle DCB minus the area between A and B bounded by the straight line and the compensated demand curve. This error can be made "small" by selecting a "large" $n$. The diagram also shows the quadratic approximation of CVCS as the area of the trapezoid $p^n_0 p^l EA$. It is clear from the diagram that for this demand curve our approximation even with $n = 1$ is superior to the quadratic approximation.

To show that equation (2) defines a contraction mapping, rename the $F$ on the left hand side $y$ and the $F$ on the right hand side $z$, and rewrite (2) as

$$y = F - [x_i(p^n_0, p^c_0, F) + x_i(p^n_0, p^c_0, z)] (p^l - p^n_0) / 2$$

(4)

We see that equation (4) maps the real numbers into the real numbers and that

$$dy/dz = (\partial x_i/\partial z) (p^l - p^n_0) / 2$$

Since the absolute value of $dy/dz$ can be made less than one by selecting $n$ sufficiently large, equation (4) satisfies the sufficient conditions to be a contraction mapping (Kolmogorov and Fomin, 1970, pp. 66-68). To find $F$ which is the fixed point of equation (4) we generate the sequence:

$$z^0 = F - Q$$

$$z^{i+1} = F - [x_i(p^n_0, p^c_0, F) + x_i(p^n_0, p^c_0, z)] (p^l - p^n_0) / 2$$

$i = 0, 1, ...$

with

$$Q = x_i(p^n_0, p^c_0, F) (p^l - p^n_0)$$

$$- [\partial x_i(p^n_0, p^c_0, F)/\partial p + x_i(p^n_0, p^c_0, F)] (p^l - p^n_0) / 2$$

(5)

Note that $Q$ is the quadratic approximation of CVCS for this subinterval. This writing of $Q$ uses $x_i(p^n_0, p^c_0, U^0) = x_i(p^n_0, p^c_0, F)$ and the Slutsky equation evaluated at this same point. We progress along the sequence of $z$'s till $z^{i+1} - z^i$ is "small." Then we set $F = z^{i+1}$. The next subinterval of $p_i$ is analyzed in the same way with $F$ becoming $F$, $p^c_i$ becoming $p^c_0$, and the new $p^n_0$ calculated using
The accuracy of this approximation depends upon several items, including the curvature of the compensated demand curve. The numerical integration literature contains examples of functions where the trapezoid rule, of which this is a variation, produces inferior results to alternative numerical integration rules such as Simpson's rule (Cheney and Kincaid, 1985, pp. 150-85). We feel that these accuracy problems are far overshadowed by the confidence problems associated with the estimation of the ordinary demand functions; however, we also present an alternative approximation using Simpson's rule. It is anticipated that the Simpson's approximation will provide the more accurate results for a given \( n \) when the compensated demand curve is a curve rather than a straight line.

In the Simpson's approximation as in the trapezoid approximation, we break the interval of price change into \( n \) subintervals and proceed through these bootstrapping from one to the next, approximating at each step a point on the compensated demand curve. The major modification stems from Simpson's rule using three points on the curve being integrated, instead of just two as in the case of the trapezoid method. To start the process we use the original point as the first one, and generate the second point using the trapezoid method presented above. The third point is generated using successive approximations. After this initialization, we bootstrap along using the last two points generated and generate a new point using successive approximations. The replacement for equation (2) is

\[
F^0 - F = \left[ x_i(p_i^0, p_{i2}, F) + 4 x_i(p_i^0, p_{i2}, F^0) + x_i(p_i^0, p_{i2}, F^0) \right] (p_i^0 - p_i^0) / 3
\]

where \( F^0 \) is the initial income and expenditure, \( p_i^0 = p_i^0 - (p_i^0 - p_i^0)/n \), and \( p_i^0 = p_i^0 - (p_i^0 - p_i^0)/n \). The right hand side of equation (6) is the Simpson's numerical approximation of the integral of the compensated demand curve from \( p_i^0 \) to \( p_i^0 \) (see Cheney and Kincaid, 1985, p. 183). In the initial iteration \( F^a \) is generated using the trapezoid method and \( F \) is generated by the following successive approximations.

\[
z^0 = F^a - Q
\]

\[
z^{i+1} = F^i - \left[ x_i(p_i^0, p_{i2}, F^i) + 4 x_i(p_i^0, p_{i2}, F^a) + x_i(p_i^0, p_{i2}, z') \right] (p_i^0 - p_i^0) / 3
\]  

\( i = 0, 1, \ldots \)

with \( Q \) the quadratic approximation (equation (5)) over the interval \( p_i^0 \) to \( p_i^0 \). As before we progress along the sequence of \( z \)'s till \( z^{i+1} \) is "small," and then set \( F = z^{i+1} \). To analyze the
next subinterval of $p_i$ we move $F^a$ to $F^a$, $F$ to $F^a$, $p_i^a$ to $p_i^a$, $p_i^e$ to $p_i^e$ and calculate the new $p_i^e$ as above. The above successive approximations is then iterated to find the next $F$.

To demonstrate that both methods can be made extremely accurate we use the constant elasticity demand function and compare our approximations to the exact measure given by Hausman’s equation (23) (Hausman, 1981, p. 669). Writing the ordinary demand function as

$$x_i = \alpha p_i^e f$$

we approximate CVCS for $\alpha = .02$, $\beta = -1$, $\gamma = 2$, $p_i^0 = 2$, $p_i^1 = 1$, and $f^0 = 10$. Using Hausman’s equation (23) (actually we used the equivalent of (23) because that equation holds of $\beta \neq -1$) and double precision arithmetic on an IBM PC compatible, the exact CVCS is $1.217511413230354$. The error for the Simpson’s approximation for $n = 100$ is $3.25D-09$ and for the trapezoid approximation for $n = 1000$ is $5.29D-08$; thus, it takes more iterations using the trapezoid method to generate the same accuracy as can be generated with the Simpson’s method for this demand curve. The error for other values of $n$ are reported in Table 1.

Table 1. Errors for the Approximations

<table>
<thead>
<tr>
<th>$n$</th>
<th>Simpson’s Approximation</th>
<th>Trapezoid Approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.27D-02</td>
<td>-1.27D-02</td>
</tr>
<tr>
<td>2</td>
<td>-4.06D-04</td>
<td>-4.06D-04</td>
</tr>
<tr>
<td>10</td>
<td>1.44D-06</td>
<td>-5.28D-04</td>
</tr>
<tr>
<td>100</td>
<td>3.25D-09</td>
<td>-5.29D-06</td>
</tr>
<tr>
<td>1,000</td>
<td>3.50D-12</td>
<td>-5.29D-08</td>
</tr>
<tr>
<td>10,000</td>
<td>9.73D-14</td>
<td>-5.29D-10</td>
</tr>
</tbody>
</table>

Three Applications of Successive Approximations of CVCS

1. Rent Control

In this section we apply our successive approximations method to a rent control example. Willis and Nicholson (1991) studied rent control in the United Kingdom and calculated the benefits from these controls using Marshallian consumer’s surplus. We have used their problem
and data sources to calculate CVCS. We use Rosenthal (1991) for the elasticity data source. The two measures are quite different because the price elasticity of demand on the compensated and ordinary demand functions are quite different.

Rosenthal's results indicate that the price elasticity and income elasticity of ordinary demand for housing in the United Kingdom are approximately -0.5 and 0.7, respectively. From Table 4 of Willis and Nicholson, we know that the share of rent in income varies from 12 to 29 percent. Using the elasticity form of the Slutsky equation and these data we calculated the price elasticity of compensated demand for housing in the United Kingdom to be in the range -0.29 to -0.42; thus, we anticipate that CVCS is quite different from MCS for a given rent control.

The rent control model used in Willis and Nicholson's study was adapted from Olsen (1972). The benefits obtained by the consumers under the rent control are shown in Figure 2.

![Figure 2. Demand and Supply of Housing.](image-url)
where the ordinary demand for housing, the compensated demand for housing, and the supply curve of housing are shown. $P_c$ is the price of housing under the rent control, and $P_m$ is the price of housing without rent control, which is the nonintervention equilibrium price. $P_d$ is the price of housing such that $Q_c$ is the quantity of housing demanded on the compensated demand curve. The MCS (Willis and Nicholson call this $\Delta B$) for the combination of rent control and rationing is

$$\Delta B = \text{Area } P_mP_cAB - \text{Area } BCD$$

and the CVCS is

$$CVCS = \text{Area } P_mP_cAB - \text{Area } BCE$$

The logic behind $\Delta B$ is given in Olsen (1972), and to explain the logic behind this measure of

Figure 3. Indifference Curve Diagram for Rent Control.
CVCS we use Figure 3. This is an indifference curve-budget line diagram for an individual where \( Y \) is all other goods and is the numeraire commodity (its price is one). Initially the individual faces the price of housing of \( P_m \) and is at the optimum point \( a \). The individual then faces rationing and moves to the point \( b \) and consumes housing \( Q_c \). If the individual were compensated by the amount \( Y_e - Y_b \) he could move to the original level of utility at the point \( e \). This can be shown to be equal to the Area \( BCE \) in Figure 2, which is the consumer's surplus portion of the welfare loss of the rationing. If the individual is at the point \( b \) and a rent control is introduced that lowers the price to \( P_c \) the individual will move to point \( c \), and is made better off by an amount equal to \( Y_c - Y_b = (P_m - P_c)Q_c = \text{Area } P_mP_cAB \). Thus the compensating variation of income for the combination of rationing and rent control that would make the individual just as well off as originally is given by equation (7). Note that if the ration quantity, \( Q_c \), were made smaller in Figure 3, the point \( c \) would move northwest along the outer budget line and the individual may end up worse off than originally with this combination of rationing and price control.

The demand function for housing used by Willis and Nicholson is in the constant elasticity form, and can be written

\[
Q = AP^b
\]

where \( Q \) is the quantity of housing demanded and \( P \) is the price of housing. \( A \) is the constant term, and \( b \) the price elasticity of demand for housing. If we assume the income elasticity is also constant, we can write the demand function as

\[
Q = BP^bY^c
\]

where \( Y \) is the money income and \( c \) is the income elasticity; thus, the correspondence between (8) and (9) is

\[
A = BY^c
\]

Using our successive approximations method and data derived from Table 4 of Willis and Nicholson we calculated the benefits under the rent control. Because \( P_mQ_m \) was reported and not just \( P_m \) and \( Q_m \) we arbitrarily set \( Q_m \) equal to 1 and \( P_m \) equal to the reported product \( P_mQ_m \). This has no effect on our results, since any other combination of the two yielding the reported product would have given the same results. The results are given in the Table 2 and Table 3. Table 2
shows the data used in our calculations and the amounts used to calculate CVCS. These are reported for three housing sectors, local authority (LA), controlled private renting (PR), and housing association (HA) properties.

Table 2. Calculation of CVCS under rent control

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>B*</th>
<th>Pm</th>
<th>Pm</th>
<th>Qm</th>
<th>Qm</th>
<th>PmQm</th>
<th>AreabCE</th>
<th>(Pm-P)Qm</th>
</tr>
</thead>
<tbody>
<tr>
<td>LA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td>6340</td>
<td>0.09145</td>
<td>1759</td>
<td>4443</td>
<td>1.00</td>
<td>0.7828</td>
<td>550</td>
<td>219</td>
<td>827</td>
</tr>
<tr>
<td>mean</td>
<td>6840</td>
<td>0.08912</td>
<td>1858</td>
<td>5285</td>
<td>1.00</td>
<td>0.7594</td>
<td>751</td>
<td>299</td>
<td>660</td>
</tr>
<tr>
<td>HA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td>5930</td>
<td>0.09512</td>
<td>1733</td>
<td>9839</td>
<td>1.00</td>
<td>0.6965</td>
<td>1040</td>
<td>651</td>
<td>167</td>
</tr>
<tr>
<td>mean</td>
<td>6240</td>
<td>0.09494</td>
<td>1854</td>
<td>989</td>
<td>1.00</td>
<td>0.6559</td>
<td>989</td>
<td>227</td>
<td></td>
</tr>
<tr>
<td>PR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td>7000</td>
<td>0.08388</td>
<td>1700</td>
<td>6353</td>
<td>1.00</td>
<td>0.6965</td>
<td>988</td>
<td>466</td>
<td>196</td>
</tr>
<tr>
<td>mean</td>
<td>9531</td>
<td>0.07044</td>
<td>1847</td>
<td>4222</td>
<td>1.00</td>
<td>0.7569</td>
<td>1065</td>
<td>228</td>
<td>333</td>
</tr>
</tbody>
</table>

* Price elasticity b = -0.5 and income elasticity c = 0.7. The prices are in 1988 pounds.
Source: Table 4 and Table 9, Willis and Nicholson, (1991).

Table 3 shows that the benefits of the rent control were overestimated by Willis and Nicholson because the loss of consumer’s surplus caused by the rationing was under-estimated. Note also that the absolute value of the percent error of using MCS ranges from 43 to 89 percent. In addition, it was impossible to calculate the CVCS for the housing association (HA) properties at the mean level of income because the ordinary demand curve over the range of prices involved implied a compensated demand curve that became positively sloping at high prices. The compensated demand function derived from Hausman’s (1981) expenditure function (Equation (22) of Hausman, 1981) for the constant elasticity demand function yields the same conclusion. This implies that over this range of prices the constant elasticity demand function is not consistent with income constrained utility maximization of a quasi-concave utility function. Some of the problems associated with the constant elasticity demand function are identified in Deaton and
Table 3. Comparison of benefits under the rent control

<table>
<thead>
<tr>
<th></th>
<th>CVCS</th>
<th>ΔB</th>
<th>Differences (CVCS-ΔB)/CVCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td>608</td>
<td>899</td>
<td>-48%</td>
</tr>
<tr>
<td>mean</td>
<td>361</td>
<td>518</td>
<td>-43%</td>
</tr>
<tr>
<td>HA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td>-484</td>
<td>-62</td>
<td>+87%</td>
</tr>
<tr>
<td>mean</td>
<td>***</td>
<td>-108</td>
<td>***</td>
</tr>
<tr>
<td>PR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td>-270</td>
<td>-29</td>
<td>+89%</td>
</tr>
<tr>
<td>mean</td>
<td>105</td>
<td>189</td>
<td>-80%</td>
</tr>
</tbody>
</table>

2. Benefits From Physician Immigration

Our second application is an analysis of the benefits to consumers from physician immigration. In a study of physician immigration to the US from 1966 to 1971, Svorny (1991) used Marshallian consumer’s surplus as the measure of consumer gains. In this section we report our calculated CVCS and compare the results. The relevant supply and demand functions are shown in Figure 4. In this figure $D$ is the original demand for physician services in the United States, $P_0$ and $Q_0$ are average annual earnings and number of active physicians, respectively. $SRS$ and $LRS$ are the short-run and the long-run supply of physicians before the liberalization of immigration of foreign trained physicians, respectively. The resulting initial long-run equilibrium point is $Q_1, P_1$. The demand for physician services shifted upward through time to $D'$ as income increased. The liberalization of physician immigration shifted the short-run supply of physicians to $SRS'$. These shifts resulted in a new equilibrium at $Q_f, P_f$. In the absence of this liberalization
of immigration the new short-run equilibrium point would have been $Q_0$, $P_0$. (For detail, see Svorny, 1991).

Svorny calculated consumer’s surplus for the years 1967-1972 using the formula

$$CG(t) = [P_{0}(t)Q_{0}(t) - P_{1}(t)Q_{1}(t)]/(1 + a)$$

where $a$ is the price elasticity of demand for physician services in the US. This formula calculates MCS for a constant price elasticity demand function and gives Area $P_{0}P_{1}AB$ in Figure 4. For this same price decrease CVCS is given by Area $P_{0}P_{1}CB$.

Like Svorny we use a constant elasticity demand function and use Feldstein (1964) as our data source. Svorny’s demand function can be written

$$Q(t) = KP^{a}(t)$$

where $a$ is the price elasticity of demand. Our form is
\[ Q(t) = n(t) l(t) P^a(t) Y_a^b(t) \]

where \( n \) is the population, \( Y_a \) is average personal income, \( b \) is the income elasticity of demand, and \( I \) is the constant term in the individual demand function. We aggregate the individual demand functions because the elasticity data are for the these demand functions and we use the average individual to simplify the problem. We use the same price elasticity as Svorny -0.19 and an income elasticity of 0.56 (Feldstein, 1964). Our price and quantity data come from Svorny and our population and income data from the *US Statistical Abstracts*. With these data \( I(t) \) can be calculated. These data for the years 1967-1972 are reported in Table 4.

Table 4. Solutions to demand functions

<table>
<thead>
<tr>
<th>Year</th>
<th>( P_1'(t) )</th>
<th>( P_0'(t) )</th>
<th>( Y_a(t) )</th>
<th>( Q_1(t) )</th>
<th>( Q_0(t) )</th>
<th>( n(t)^2 )</th>
<th>( l(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>34730</td>
<td>35133</td>
<td>3167</td>
<td>322045</td>
<td>321340</td>
<td>199.1</td>
<td>1.291 \times 10^{-4}</td>
</tr>
<tr>
<td>1968</td>
<td>36104</td>
<td>36998</td>
<td>3295</td>
<td>330732</td>
<td>329199</td>
<td>199.3</td>
<td>1.305 \times 10^{-4}</td>
</tr>
<tr>
<td>1969</td>
<td>36931</td>
<td>38398</td>
<td>3374</td>
<td>338942</td>
<td>336443</td>
<td>201.3</td>
<td>1.313 \times 10^{-4}</td>
</tr>
<tr>
<td>1970</td>
<td>35684</td>
<td>38086</td>
<td>3392</td>
<td>348328</td>
<td>344043</td>
<td>203.2</td>
<td>1.324 \times 10^{-4}</td>
</tr>
<tr>
<td>1971</td>
<td>35202</td>
<td>39407</td>
<td>3413</td>
<td>359423</td>
<td>351799</td>
<td>206.2</td>
<td>1.338 \times 10^{-4}</td>
</tr>
<tr>
<td>1972</td>
<td>32506</td>
<td>40317</td>
<td>3611</td>
<td>371000</td>
<td>356126</td>
<td>208.2</td>
<td>1.305 \times 10^{-4}</td>
</tr>
</tbody>
</table>

1 Prices and income are in 1967 dollars.

2 Population is in million.

Table 5. CVCS vs. MCS

<table>
<thead>
<tr>
<th>Year</th>
<th>cvcs_j(t)</th>
<th>CVCS(t)=n(t)cvcs_j(t)</th>
<th>MCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>0.6507</td>
<td>129.55</td>
<td>129.65</td>
</tr>
<tr>
<td>1968</td>
<td>1.4789</td>
<td>294.75</td>
<td>295.01</td>
</tr>
<tr>
<td>1969</td>
<td>2.4601</td>
<td>495.21</td>
<td>495.40</td>
</tr>
<tr>
<td>1970</td>
<td>4.0896</td>
<td>831.02</td>
<td>831.46</td>
</tr>
<tr>
<td>1971</td>
<td>7.2433</td>
<td>1493.57</td>
<td>1494.98</td>
</tr>
<tr>
<td>1972</td>
<td>13.6045</td>
<td>2832.46</td>
<td>2837.29</td>
</tr>
</tbody>
</table>

CVCS and MCS are all in 1967 million dollars.

We calculated CVCS using these data and our successive approximations method. The resulting CVCS and Svorý’s MCS are reported in Table 5. As can be seen, there is little difference between the CVCS and the MCS in this example. This small difference is the result of the price elasticity of ordinary demand ($a = -0.19$) and of compensated demand ($a^{H} = -0.18$) being almost the same. In 1967, the total expenditure on the physician services was 56.70 dollars per person while the income per person was 3167 dollars; thus, the elasticity of the compensated demand is

$$a^{H} = a + \delta b = -0.19 + (56.70/3167) \cdot 0.56 = -0.18$$

This example illustrates that there are cases where CVCS and MCS are not very different. CVCS, however, is still the measure that is of interest, and since CVCS can always rather easily be calculated, we suggest that it should be.

3. The Deadweight Losses from a U.S. Oil Import Fee

Our final application is to deadweight losses of an oil import fee. Walls (1991) studied these losses for a hypothetical $5 per barrel fee on crude oil imported into the United States using Marshallian consumer’s surplus. We use her data and calculate CVCS with our successive approximation method. We find large differences between the two measures.
The U.S. is modeled as a price-taker for imported oil, with the $5 fee increasing the domestic price from $14.27 to $19.27. The domestic supply and demand are both modeled so to take into account the difference between long-run and short-run curves, with both becoming more elastic as the length of time increases. This dynamic element give rise to deadweight loss changing through time as the imposition of the import fee becomes more distant in the past.

The fee gives rise to a loss of consumer’s surplus, an increase in producer’s surplus, and an increase in tariff revenues. The deadweight loss (DWL) is the sum of these three. The producer’s surplus and tariff revenue of the $5 fee are, respectively,

\[ \text{PS}_t = 5 \left[ \frac{1}{2} (Q_{US,1} - Q'_{US}) \right] \]
\[ \text{TR}_t = 5 (Q' - Q'_{US}) \]

where \( Q_{US,1} \) is the domestic output without the tariff, \( Q'_{US} \) is domestic output with the tariff, and \( Q' \) is total consumption from domestic production and imports with the tariff all at time \( t \).

The Marshallian consumer’s surplus of the fee is

\[ \text{MCS}_t = \int_{P^N}^{P^T} f(P) dP \]

where \( P \) is a vector of current and past oil prices affecting consumption at time \( t \), \( P^N \) is the vector of prices with the tariff, \( P^T \) is the vector of prices with the tariff, and \( f(P) \) is the demand function for oil in the U.S. Walls’ demand function is a lagged double logarithm form

\[ \ln Q_t = \ln \beta_0 + \beta_1 \ln Y_t + \beta_2 \ln P_t + \beta_3 \ln P_{t-1} + \ldots + \beta_{10} \ln P_{t-10} + e_t \]

where \( Y_t \) is U.S. real GNP at time \( t \), \( P_t \) is the average inflation-adjusted wellhead price of oil, and \( e_t \) is a second-order auto-correlated error term with a zero mean and constant variance. The results of the estimation of the coefficients are summarized as follows:

\[ \beta_0 = 2.1733 \quad \beta_1 = 1.0906 \quad \beta_2 = -0.0300 \quad \beta_3 = -0.0329 \quad \beta_4 = -0.0358 \]
\[ \beta_5 = -0.0388 \quad \beta_6 = -0.0417 \quad \beta_7 = -0.0447 \quad \beta_8 = -0.0476 \quad \beta_9 = -0.0505 \]
\[ \beta_{10} = -0.0535 \quad \beta_{11} = -0.0564 \quad \beta_{12} = -0.0594 \]

Walls forecasts supply and demand over the period 1988-1998 using a 3 percent annual increase in GNP and constant oil prices of $14.27 without an import fee and $19.27 with the fee. Using the information given, we were able to calculate CVCS\(_t\) for these years. The results of these calculations are shown in Table 6.

The deadweight losses in the Walls’ study are underestimated, because for price increases
of a normal good $|CVCS|$ is greater than $|MCS|$. This yields an underestimate of the deadweight loss. Notice that the percent error ranges from 87.2 percent to 11.1 percent during the first 6 years and from 9.6 percent to 5.5 percent during the last 5 years. The percent error becomes smaller through time because the demand function became more elastic, and the percent difference between the ordinary and compensated demand curves for a normal commodity is largest when demand is inelastic.

Table 6. Consumer Surplus, Producer Surplus, Tariff Revenue and Deadweight Loss from a $5 U.S. Oil Import Fee (calculated in 1987 billion U.S. dollars)

<table>
<thead>
<tr>
<th>Year</th>
<th>CVCS,</th>
<th>MCS,</th>
<th>PS,</th>
<th>TR,</th>
<th>DWL</th>
<th>DWL'</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>-30.51</td>
<td>-30.31</td>
<td>13.63</td>
<td>16.65</td>
<td>-.234</td>
<td>-.03</td>
<td>87.2%</td>
</tr>
<tr>
<td>1989</td>
<td>-30.72</td>
<td>-30.63</td>
<td>14.36</td>
<td>15.84</td>
<td>-.516</td>
<td>-.43</td>
<td>16.7%</td>
</tr>
<tr>
<td>1990</td>
<td>-31.43</td>
<td>-31.35</td>
<td>13.69</td>
<td>17.11</td>
<td>-.627</td>
<td>-.55</td>
<td>12.3%</td>
</tr>
<tr>
<td>1991</td>
<td>-33.03</td>
<td>-32.93</td>
<td>13.17</td>
<td>18.95</td>
<td>-.911</td>
<td>-.81</td>
<td>11.1%</td>
</tr>
<tr>
<td>1992</td>
<td>-35.44</td>
<td>-35.28</td>
<td>12.70</td>
<td>21.47</td>
<td>-1.266</td>
<td>-1.11</td>
<td>12.3%</td>
</tr>
<tr>
<td>1993</td>
<td>-37.79</td>
<td>-37.58</td>
<td>12.28</td>
<td>23.84</td>
<td>-1.672</td>
<td>-1.46</td>
<td>12.7%</td>
</tr>
<tr>
<td>1994</td>
<td>-40.05</td>
<td>-39.85</td>
<td>11.89</td>
<td>26.10</td>
<td>-2.058</td>
<td>-1.86</td>
<td>9.6%</td>
</tr>
<tr>
<td>1995</td>
<td>-42.42</td>
<td>-42.22</td>
<td>11.55</td>
<td>28.37</td>
<td>-2.503</td>
<td>-2.30</td>
<td>8.1%</td>
</tr>
<tr>
<td>1996</td>
<td>-44.82</td>
<td>-44.60</td>
<td>11.23</td>
<td>30.57</td>
<td>-3.024</td>
<td>-2.80</td>
<td>7.4%</td>
</tr>
<tr>
<td>1997</td>
<td>-45.56</td>
<td>-45.36</td>
<td>10.95</td>
<td>31.17</td>
<td>-3.443</td>
<td>-3.24</td>
<td>5.9%</td>
</tr>
<tr>
<td>1998</td>
<td>-46.85</td>
<td>-46.63</td>
<td>10.69</td>
<td>32.20</td>
<td>-3.958</td>
<td>-3.74</td>
<td>5.5%</td>
</tr>
</tbody>
</table>

1 $\Delta MSC, \Delta PS, \text{ and } TR, \text{ are from Table III of Walls (1991).}$
2 $\text{DWL} = CVCS, + PS, + TR,$
3 $\text{DWL'} = MCS, + PS, + TR,$
4 $\text{Difference} = (\text{DWL} - \text{DWL'})/\text{DWL}$

Summary

In this paper we have introduced a successive approximations method of calculating
compensating variation consumer's surplus using data from the ordinary demand curve. In doing so we numerically identify the compensated demand function over the price interval involved. This procedure can be implemented on any ordinary demand function that is consistent with a quasi-concave utility function, and does not require that we integrate back to the utility function. We demonstrated that the error of our approximation can be made extremely "small" by dividing the interval of price into a large number of subintervals. We also used our method to calculate CVCS for three applications in the literature where Marshallian consumer's surplus was reported. Two of these identified relatively large errors in using MCS; thus, a relative simple method, as this one is, of calculating CVCS has value.

References

