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Influence of Some Geometrical Parameters on Piano Key Weir Discharge Efficiency

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ABSTRACT

The Piano Key Weir (PKW) is a type of labyrinth weir with efficient hydraulic performance, especially at low heads. Since the primary parameters were extensively studied, Electricité De France (EDF) launched a new experimental program to improve the knowledge of some secondary parameters such as the overhangs, the crest shape, and the dam height. This program included the testing of a trapezoidal shape and of PKWs under submerged conditions. The data measured under free-flow conditions were also used to validate empirical correlations and FLOW-3D numerical models. This paper gives an overview of the program and highlights the main experimental and numerical results.

Keywords: Hydraulic, Piano-key-weir, Discharge, free-flow, submerged flow.

1. INTRODUCTION

The PKW (Figure 1, left) is a type of labyrinth weir (Tullis et al., 2007) with efficient hydraulic performance, especially at low heads. The reduced footprint due to the overhangs allows installing PKWs on the top of the crest of existing gravity dams, and since the first prototype of Goulours (2006), many of them (St Marc, Etroit, Malarce, etc.) were installed by EDF to upgrade the spillway capacity of these dams (Vermeulen et al., 2011).

For about ten years, many experimental and numerical studies have contributed to identify the primary parameters influencing hydraulic performance, such as the width ratio of the inlet and outlet keys (Wᵢ/Wₒ), the developed length to width ratio (L/W), and the weir height (P). Figure 1 (left) shows the main geometric parameters of a classical rectangular PKW with the standard notation defined by Pralong et al. (2011).

In 2011, Electricité De France (EDF) started a new experimental program at the National and Environmental Hydraulic Laboratory (LNHE) in Chatou to expand the knowledge on some secondary parameters effects such as the overhangs, the crest shape, and the dam height (Pᵢ). This program included the testing of a trapezoidal shape with a side-wall angle of the lateral walls to compare the hydraulic performance to the classical rectangular shape. Furthermore, since most of the experiments were carried out under free-flow conditions, the rectangular PKWs were tested under submerged conditions (Belaabed et al., 2011), (Dabling and Tullis, 2012).

The experimental data under free-flow conditions were used to validate empirical correlations and FLOW-3D numerical models.

This paper gives an overview of the experimental program and a synthesis of the results already presented (Cicero et al., 2012, 2013a, 2013b, 2013c) and of complementary tests at various dam heights.
2. EXPERIMENTAL SET-UP

2.1. Testing Apparatus

The PKW testing was conducted in a 2-m wide, 1-m deep, and 25-m long rectangular channel with discharge capacity measurements up to 500 L/s. The weir discharge \( (Q) \) was measured by an electromagnetic flow-meter of 1\% accuracy and the piezometric heads were measured at ~ 6 m upstream \( (h_u) \) and ~ 3 m downstream \( (h_d) \) of the PKW with an accuracy of +/-0.18 mm. The total upstream \( (H_u) \) and downstream \( (H_d) \) heads were calculated by adding the velocity head corresponding to the average cross-sectional velocity at the level measurement locations of \( h_u \) and \( h_d \). The discharge and the water levels were recorded for a 3 min period at a 0.5 Hz frequency. As a result, the measurements used for analysis were the average and standard deviation of 90 temporal values at steady state conditions.

2.2. PKW Design

The PKWs were fabricated of PVC, following common design principles (Cicero and Delisle, 2013a). The width \( (W) \) was the total channel width to avoid side effects and maximize the number of PKW units, which was \( N_u = 6.5 \), with half an inlet key and half an outlet key on each end.

The side wall thickness \( (T_s = 2 \) cm) was chosen to be wide enough to enable the testing of various crest shapes on the type A PKW. According to the recommended value (~35 cm) for prototypes (Vermeulen et al. 2011), the geometric scale of the models was around 1/20. The PKWs were installed on top of a platform in concrete without noses underneath the upstream overhangs. The PKWs were tested at three dam heights: \( P_d = 0.6P, 1.5P, 2P \).

Figure 1. Geometric parameters of a classical rectangular PKW (left) Half-element unit of a trapezoidal PKW in plan view (right).

2.2.1. Rectangular PKWs

The geometry of the three PKWs types A, B and C was the design recommended by Lemperiere et al. (2011) for the type B, i.e. with the same inlet and outlet key widths \( (W_i = W_0) \). As a result, the type C design (without upstream overhang) was the same as type B (without downstream overhang), but with a reversed placement in the channel. The main geometric parameters (Cicero et al., 2013a) are strictly the same except for the lengths of the upstream and downstream overhangs \( (B_o, B_i) \) and, consequently, for the bottom slopes of the outlet and the inlet keys \( (S_o, S_i) \).
2.2.2. Crest Shapes Tested on the PKW Type A

Four geometries (Cicero et al., 2013b) were tested on the lateral walls: a flat-top, a half-round, and a quarter-round crest, with the rounded face in both the inlet and the outlet keys. Four geometries were also tested on the upstream and the downstream crests: a flat-top, a half-round, and both rounded shapes in horizontal and also vertical.

2.2.3. Trapezoidal PKWs

Two trapezoidal PKWs (Figure 1, right) with a sidewall angle ($\alpha$) were designed, keeping the main geometric parameters of the rectangular type A with the flat-top crest. The common parameters maintained were $P$, $B_o$, $T_i$, $T_o$ and the widths of the inlet ($W_{i,d}$) and outlet ($W_{o,d}$) keys at the downstream edge, (i.e. $W_{i,d} = W_i$ and $W_{o,d} = W_o$). The resulting geometric parameters were calculated (Cicero et al., 2013c) according to the design constraints. The first design constraint (trapezoidal 1) was to maximize the sidewall angle keeping constant $L_u/W_u$, the main relevant parameter on PKW hydraulic performance. The maximum sidewall angle was found to be $\alpha = 5^\circ$.

The second design constraint (trapezoidal 2) was to maintain the upstream-downstream length ($B$), which is a parameter that influences the building cost of PKWs. So, the inlet ($B_o$) and outlet ($B_i$) overhang lengths were the same as the rectangular type A. Although the maximum sidewall angle was found to be $\alpha = 6^\circ$, we kept $\alpha = 5^\circ$ to allow comparisons with the trapezoidal 1.

The trapezoidal 1 (non-symmetric in planned view) was experimentally tested at LNHE, in the “design position” when the inlets are larger than the outlets at the upstream edge ($W_{i,u} > W_{o,u}$), and in the “reverse position” ($W_{i,u} < W_{o,u}$). The tests performed on the type A and the trapezoidal 1 (in both positions) were used to validate FLOW-3D models. Then, the hydraulic performance of the trapezoidal 2 was predicted with the validated numerical model.

3. HYDRAULIC EFFICIENCY UNDER FREE FLOW CONDITIONS

3.1. Testing Procedure and Measurement Analysis

For tests under free flow conditions, the discharge ($Q$) was increased step by step to measure the piezometric upstream level ($h_u$) when the flow conditions were stabilized. The discharge efficiency of the various geometries (PKW, dam height and crest shape) was characterized by the non-dimensional rating curves $C_w(H_u/P)$ where

$$C_w = \frac{Q}{W \sqrt{2gH_u}^{1.5}}$$  \hspace{1cm} (1)

The experimental data $C_w(H_u/P)$ were correlated by polynomial interpolations, which allowed a comparison between the discharge of the various geometries at the same upstream head. As recommended by Leite Ribeiro et al. (2012), these equations were used for $H_u > 3$cm, since we observed a wide range of measurement deviations due to viscosity and surface tension scale effects. This comparison also took into account the measurements error on $C_w$ (Cicero et al., 2013b), which were usually greater than 2%. Consequently, the discharge differences lower than 4% are considered negligible.
3.2. Overhang Effects

3.2.1. Experimental Results

Figure 2. Dam height effect on the discharges of the type B (left) and the type C (right), compared to the type A.

The types A, B, and C were tested at 3 dam heights, and the first results observed at low dam height (Cicero et al., 2013a) were confirmed at higher dam heights. The type B was 5% to 15% more efficient than the type A, according to the upstream head and the dam height (Figure 2). The type A was up to 15% more efficient than the type C. These results are consistent with the first PKW experiments of Ouamane and Lemperiere (2006), who observed that the type B was 9% to 12% more efficient than the type A. An explanation can be given thanks to Machiels (2012) experiments on a large scale model of type A. The latter observed a control section in the inlet key, which reduces the effective developed length and moves downstream for increasing heads. Thus, for the same upstream head, the effective length will be greater for type B than for type A and type C.

The effects of the dam height were found to be the same for all PKW types. The major effect on $C_w$ was observed at low heads ($H_u < 3$ cm) when scale effects are not negligible. For $H_u > 3$ cm, we measured the same curves $C_w(H_u/P)$ at low and intermediate dam heights and a decrease of $C_w$ at the highest dam height. This latter result was rather unexpected since Leite Ribeiro et al. (2011) and Machiels (2012) observed an increase of the discharge efficiency with the dam height. That could be due to a higher units number ($N_u = 6.5$ instead of 1.5 to 3) of the PKWs tested at LNHE.

3.3. Crest Shape Effects

The first tests performed at low (10 tests) and intermediate dam heights (4 tests) were presented in Chatou (Cicero et al., 2013b). (The various geometries characterized by the dam height and the shape of the lateral, upstream and downstream crests were detailed in a table). This program was completed by testing the 4 “uniform” crest configurations at the highest dam height. The whole tests analysis highlights the following results:

- The influence of the crest shape is mainly due to the lateral crest and strongly decreases with increasing upstream head when the lateral jets coming from the inlet keys begin to cross into the outlet keys.
- The half-round and the quarter-round shapes have better hydraulic performance than the flat-top shape, with a gain in discharge decreasing from 20 to 25% for $H_u = 3$ cm to 5% for $H_u/P > 0.4$. Anderson (2012), who also compared the half round and the flat-top shapes on a type A, found similar results: the gain in discharge decreased from 22 to 4% for $0.1 < H_u/P < 0.6$.
- The quarter-round on the lateral crest provides the same discharge efficiency whatever the position of the rounded face (in the inlet or the outlet).
The dam height has negligible effect when scale effects became negligible ($H_u > 3$ cm). For each “uniform” crest (Figure 3), the differences on $C_w$ measured at the 3 dam heights were lower than the measurement uncertainties.

![Figure 3. Effect of the dam height on the hydraulic efficiency of the type A with the flat-top crest (left) and the half-round crest (right).](image)

As a result, it is recommended to design PKWs with a half-round or a quarter-round shape on the lateral crest and a flat–top shape on the upstream and the downstream crest.

### 3.4. Validation of Empirical Correlations

The LNHE experiments were used (Cicero et al., 2013b) to validate both empirical correlations published by Leite-Ribeiro et al. (2012) for type A and by Machiels (2012) for general PKW geometries.

The “type A correlation” is based on a geometrical parametric study of PKWs tested with a half-round crest and with noses under the upstream overhangs. The predictions were compared to the measurements of the type A with the half-round crest (regardless of dam height since it has no effects). Within its limitations ($H > 0.05$ m), this correlation overestimated the discharge by 7% to 18%. Note that Anderson (2011) observed a beneficial effects (~3%) of the noses, which could reduce this relative error.

The “Machiels correlation” is more based on a physical analysis of the various geometrical effect, including the overhang and the dam height. The PKWs were tested with a flat-top crest and without noses. For the types A and C, this correlation underestimated the discharge with a maximum error of 10%. For the type B, the discharge was predicted with an error of +/- 15%, depending on the upstream head and the dam height.

### 3.5. Effect of a Trapezoidal PKW

#### 3.5.1. Experimental Results

These tests were performed on the trapezoidal 1 only, with intermediate dam height tests presented in Chatou (Cicero et al., 2013c). Since then, the trapezoidal 1 was tested in both positions at high dam height and in the design position at low dam height only.

In the design position (Figure 4 left), the trapezoidal 1 was more efficient than the rectangular type A regardless of dam height. The gain in discharge decreased with the head and the dam height. The maximal gain was respectively 30% (low), 20% (intermediate), and 10% (high dam height). As for the rectangular PKWs (see 3.2.1): for $H_u > 3$ cm, the curves $C_w(H_u/P)$ were the same at low and intermediate dam heights (Figure 5 left) and decreased at the highest dam height.
In the reverse position (Figure 4 right), the trapezoidal 1 was less efficient than the rectangular, and the loss of discharge increased with the head and the dam height. The discharge efficiency $C_w(H_u/P)$ decreased from the intermediate to the high dam height (Figure 5 right) and significantly for $H_u/P < 0.4$.

![Figure 4](image1.png)

**Figure 4.** Dam height effect on the discharges of the trapezoidal 1 in the design (left) and reverse (right) positions, compared to the type A.

![Figure 5](image2.png)

**Figure 5.** Effect of the dam height on the hydraulic efficiency of the trapezoidal 1 in the design (left) and in the reverse position (right).

### 3.5.2. FLOW-3D Numerical Results

The numerical simulations of the rectangular type A and of both trapezoidal 1 and 2 were presented in Chatou (Cicero et al., 2013c). For the rectangular and the trapezoidal 1, the numerical results of the FLOW-3D® simulations were in rather good agreement with the physical model outcomes: the mean deviation between both models was about 8% for the rectangular PKW, 2% for the trapezoidal 1 in the design position, and 8% in the reverse position. However, the gain in discharge could be underestimated by more than 5% due to the maximal computations errors (~5% for the rectangular and 15% for the trapezoidal 1).

The FLOW-3D® model so validated allowed predicting the discharge capacities of the trapezoidal 2. Globally, this configuration was around 2% less efficient than the trapezoidal 1 due to its shorter developed crest length.  

### 4. HYDRAULIC EFFICIENCY UNDER SUBMERGED CONDITIONS

These tests were performed on the 3 PKW types at the lowest dam height ($P_d = 0.6P$). This chapter summarises the previous results (Cicero et al., 2013a) and presents a new application in § 4.3.2. (See notations in Figure 6).
4.1. Testing Procedure and Measurement Analysis

For submerged testing, the tailwater level was increased step by step while maintaining a constant flow rate to measure the discharge \( Q_s \) and both the upstream \( h_u \) and the downstream \( h_d \) piezometric levels at stabilized flow conditions. For each constant discharge, we started from free-flow conditions with a tailwater level below the crest elevation. First, the downstream level was raised by increments of 2 cm to isolate the modular submergence limit, when the free-flow conditions still applies (i.e., \( H_u = H_0 \)) whereas the downstream level exceeds the crest elevation \( (H_d > 0) \). Above this modular limit, the downstream level was raised by steps of 5 cm.

The measurements were analysed by the method of Tullis et al. (2007) previously used by Belaabed and Ouamane (2011) and Dabling and Tullis (2012). This method is based on the use of the total upstream \( (H_u) \) and downstream \( (H_d) \) heads normalized by \( (H_0) \) the total upstream head for the same discharge under free-flow conditions.

\[
\frac{H_0}{H_u} = 1 - 0.01e^{-\frac{S}{S_m}} \quad (2)
\]

\( S = \frac{H_d}{H_u} \) is the submergence factor, and \( S_m \) the modular submergence limit which can be defined, according to this equation, by \( H_0 < 0.99H_u \) for \( S > S_m \).

This equation is not valid when the submergence factor \( S \) tends to 1, since the PKW no more acts as a control structure and the discharge becomes undefined. For each PKW type, the parameters \( (\alpha, S_m) \) and the limitations were given in (Cicero et al., 2013a), as well as the correlation coefficient \( R^2 \).

Within the limitations of \( S \), these equations could predict the measured ratio \( H_0/H_u \) with an average error of 2% for type A and of 3% for types B and C. Then, the discharge \( Q_s(H_0) \) were predicted by the free-flow equations, for the measured values of \( H_u \) and \( H_d \), with an average error of 3% for types A and B and of 5% for type C.

Eq. (2) allowed for comparisons between the “sensitivity to submergence” of the 3 PKWs (Figure 7 left). At constant \( S \), the ratio \( H_0/H_u \) decreased from type C to type A, meaning that the type C was less sensitive than the type A, which was less sensitive than the type B. The PKWs were also compared to linear weirs of same width \( (W = 2 \text{ m}) \) and same crest elevation \( (P + P_d = 0.355 \text{ m}) \) for identical submerged conditions \( (H_u, H_d) \).
Tullis (2011) measured the curves \( Q/Q_f (S) \) of a submerged Ogee crest, at constant discharge \( (Q_s) \), for different values of \( P_{ui} \) and \( P_{di} \), the vertical distances from the upstream and downstream aprons to the weir crest. We used (Figure 7 right) the Tullis measurements at \( Q_s = Q_{design} \) for \( H_{OD}/P_{ui} = H_{OD}/P_{di} = 0.46 \). With \( P_{ui} = P_{di} = 0.355 \) m; these data can be used for an Ogee crest profiled at a design head \( H_{OD} = 0.163 \) m. The free-flow discharge \( Q_f \) was computed by Eq. (1) with the classical empirical correlation \( C_W = 0.495(H_s/H_{OD})^{0.12} \).

For sharp and broad crested weirs, \( Q/Q_f (S) \) were computed by the Hager (1986) equations detailed in Cicero et al. (2013a), and \( Q_f \) was computed by Eq. (1) with \( C_W = 0.42 \) (sharp crest) and \( C_W = 0.327 \) (broad crest).

As a result (Figure 7 left), the sharp crested weir is the most sensitive, and the broad crested weir is the least sensitive. The Ogee crest weir and the PKW of type C have the same sensitivity to submergence.

![Figure 7. Sensitivity to submergence of PK and linear weirs (left) for identical upstream \( (H_u) \) and downstream \( (H_d) \) heads given by Ogee crest Tullis measurements (right).](image)

**4.3. Hydraulic Efficiency**

The submerged hydraulic efficiency depends on both the sensitivity to submergence and the free-flow hydraulic efficiency. Both applications are shown in § 4.3.1 and 4.3.2, respectively, to compare the 3 PKW types to linear weirs of same width and same crest elevation.

**4.3.1. Discharge Efficiency at Identical Submerged Conditions**

In this first application, we compare the discharges of PK and linear weirs for identical submerged conditions \( (H_u \) and \( H_d \) given in Figure 7, right. For PKWs, first we calculate \( H_0 \) by the sensitivity equations \( H_0/H_u(S) \), then \( Q_s (H_u) \) by the free-flow equations. For linear weirs, \( Q_s (H_u) \) is directly computed by the sensitivity equations \( Q_s/Q_f(S) \) and the free-flow equations \( Q_f (H_u) \).

Figure 8 left compares the weir discharges to the Ogee crest discharge, which is constant \( (Q_{Ogee} = Q_{design}) \). The hydraulic performance of PK and linear weir under submergence depends on the downstream head. For the chosen values of \( H_u \) and \( H_{di} \):

- The type C is less efficient than the type A. The type B is more efficient than the type A for \( S > 0.6 \) and more than the type C for \( S < 0.8 \).
- The Ogee crest is more efficient than the sharp and the broad crested weirs. The PKWs are more efficient than the Ogee crest by 30% to 60% for the type A, 10 to 70% for the type B, and 5 to 45% for the type C.
4.3.2. Upstream Level Rise at Constant Discharge

The goal of this second application is to compare, with the previous hypotheses and data, the impact on the upstream level rise \((H_u - H_d)\) of submerged weirs. The flood discharge \(Q_s = 0.290\) m\(^3\)/s is equal to the design discharge of the Ogee crest computed for \(H_{D0} = 0.163\) m.

We compute \(H_u\) according to \(S\) at constant discharge \(Q\). For PKWs, we first calculate \(H_u (Q_s)\) by the free-flow equations then \(H_u\) by the sensitivity equations \(H_u / H_d (S)\). For linear weirs, we first calculate \(Q_s\) by the sensitivity equations \(Q_s / Q_d (S)\) then \(H_u (Q_s)\) by the free-flow discharge equations.

Figure 8, right compares the rise of the upstream level \((H_u - H_d)\) computed versus \(H_d = SH_u\):
- The PKWs have less impact than the Ogee crest weir, which has less impact than the sharp and the broad crest weirs.
- The impact of the PK and linear weirs depends on the downstream level associated to the discharge \(Q_s\). For this application, both types A and B are similar and have less impact than type C for \(H_d < 0.1\) m. The sharp crest weir has less impact than the broad crest weir for \(H_d < 0.15\) m.

Since the scale factor of the PKWs was around 1/20 (see § 2.2), the prototype data of this “fictional” application are \(Q_s = 517\) m\(^3\)/s, \(W = 40\) m, and \(P_{u0} = P_{d0} = 7.1\) m. If the downstream level was \(H_d = 2\) m (−0.1 m in Figure 8 right), a type A PKW should lower the upstream level by 0.9 m, 1.60 m, and 2 m compared respectively to an Ogee crest, a sharp crest, and a broad crest weir.

![Figure 8. Discharge at identical upstream and downstream heads (left). Upstream level rise at constant discharge (right).](image)

5. CONCLUSIONS

These LNHE experiments based on PKWs of the channel width (2 m) allowed us to isolate the effects of some secondary parameters on the hydraulic performance under free flow and submerged conditions.

Under free-flow conditions:
- The overhang effect was confirmed, and the type B was from 5 to 15% more efficient than the type A, which was 15% more efficient than the type C.
- Both the sidewall angle and a round shape on the lateral crest could increase the discharge efficiency at low heads by 20 to 30%. These beneficial effects decreased down to 5% with the upstream head.
- The discharge efficiency was the same at low and intermediate dam heights and decreased at the highest dam height. Although unexpected, this latter result was clearly observed for rectangular as well as trapezoidal PKWs.
- The type A correlation overestimated the discharge by 7 to 18% without taking into account the beneficial effects of the noses (~3%).
- The general Machiels correlation underestimated the discharge of the types A and C up to 10%, and it predicted the discharge of the type B with a maximal error of +/- 15%.
- The numerical simulations with FLOW-3D models allowed predictions of the hydraulic performance of the type A and the trapezoidal 1 with an error lower than 8%.

Under submerged conditions:
- The “sensitivity to submergence” of the 3 PKW types were characterized by the general Eq. (2). The type C was less sensitive than the type A, which was less sensitive than the type B.
- The hydraulic performance of PK and linear weirs were compared on both “fictional” applications:
  - At identical submerged conditions, the type C is less efficient than the type A, which is less efficient than the type B for S > 0.6. The PKWs are more efficient than the linear weirs.
  - At constant discharge, the PKWs have much less impact than the linear weirs on the upstream level rise. With a PKW type A, the latter could be lowered by 0.90 m, 1.60 m, and 2 m compared respectively to an Ogee crest, a sharp crest, and a broad crest weir.

6. REFERENCES


