2001

Are We Having Fun Yet? How Teachers use Manipulatives to Teach Mathematics

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Recommended Citation

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. Educational Studies in Mathematics, 47(2), 175-197.

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ARE WE HAVING FUN YET? HOW TEACHERS USE MANIPULATIVES TO TEACH MATHEMATICS

ABSTRACT. Teachers often comment that using manipulatives to teach mathematics is 'fun!' Embedded in the word 'fun' are important notions about how and why teachers use manipulatives in the teaching of mathematics. Over the course of one academic year, this study examined 10 middle grades teachers' uses of manipulatives for teaching mathematics using interviews and observations to explore how and why the teachers used the manipulatives as they did. An examination of the participants' statements and behaviors indicated that using manipulatives was little more than a diversion in classrooms where teachers were not able to represent mathematics concepts themselves. The teachers communicated that the manipulatives were fun, but not necessary, for teaching and learning mathematics.

With considerable research supporting the use of manipulative materials (Parham, 1983; Raphael and Wahlstrom, 1989; Sowell, 1989; Suydam, 1985; Suydam and Higgins, 1977) and widely available teacher professional development workshops focusing on their use, mathematics manipulatives are common in K-8 classrooms. As attested to by frequent references in mathematics standards, mathematics methods textbooks, in-service course offerings, professional journals and commercial resource catalogues, the use of manipulatives is well-situated in the mainstream of mathematics instruction.

Why have manipulatives become popular?

In the past century, several factors contributed to the popularity of manipulatives for mathematics instruction. Many researchers and theoreticians challenged previously held beliefs about learning, based on their beliefs that children must understand what they are learning for it to be permanent. Zoltan Dienes's (1969) work convinced researchers that the use of various representations of a concept, or 'multiple embodiments,' were needed to support students' understanding. Piaget (1952) suggested that children do not have the mental maturity to grasp abstract mathematical concepts presented in words or symbols alone and need many experiences with concrete materials and drawings for learning to occur. Bruner (1960, 1986) concluded that children demonstrate their understandings in three stages of representation: enactive (suggesting the role of physical objects), iconic

Educational Studies in Mathematics **47:** 175–197, 2001.

and symbolic. Skemp's (1987) theories supported the belief that students' early experiences and interactions with physical objects formed the basis for later learning at the abstract level. Based on theories of cognition and the social construction of knowledge (Vygotsky, 1978), more recent research by Cobb (1995) discusses cultural tools like hundreds boards, showing the complicated relationship between manipulatives and sociocultural perspectives. Current research in mathematics education views students as active participants who construct knowledge by reorganizing their current ways of knowing and extracting coherence and meaning from their experiences (Glover, Ronning and Bruning, 1990; Resnick, 1983; Simon, 1995; von Glasersfeld, 1990, 1995). The impact of theories and research connecting students' actions on physical objects with mathematical learning has had an important influence on the emergence and use of manipulatives in K-8 classrooms.

Manipulative materials are objects designed to represent explicitly and concretely mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences. Manufacturers advertise manipulatives as materials that will make the teaching and learning of mathematics 'fun' and promote their products as catalysts for engaging students in mathematical learning. Because students' abstract thinking is closely anchored in their concrete perceptions of the world (Thompson, 1992), actively manipulating these materials allows learners to develop a repertoire of images that can be used in the mental manipulation of abstract concepts.

Manipulatives are not magic

Yet these materials are not magic (Ball, 1992). Manipulatives are not, of themselves, carriers of meaning or insight. "Although kinesthetic experience can enhance perception and thinking, understanding does not travel through the fingertips and up the arm" (Ball, 1992, p. 47). It is through their use as tools that students have the opportunity to gain insight into their experience with them. Research has shown that for children to use concrete representations effectively without increased demands on their processing capacity, they must know the materials well enough to use them automatically (Boulton-Lewis, 1998). If the user is constantly aware of the artifact then it is not a tool, for it is not serving the purpose of enabling some desired activity which moves one toward a desired goal state (Winograd and Flores, 1986). Important to consider is the significance of manipulatives as potential tools and their significance as a function of the task for which a teacher conceives them being used.

Students sometimes learn to use manipulatives in a rote manner, with little or no learning of the mathematical concepts behind the procedures (Hiebert and Wearne, 1992) and the inability to link their actions with manipulatives to abstract symbols (Thompson and Thompson, 1990). This is because the manipulative is simply the manufacturer's representation of a mathematical concept that may be used for different purposes in various contexts with varying degrees of 'transparency.' Meira (1998) defines the concept of transparency of instructional devices as "an index of access to knowledge and activities rather than as an inherent feature of objects. . . a process mediated by unfolding activities and users' participation in ongoing sociocultural practices" (p. 121). The understanding or meaning of particular manipulatives become known to users in the process of using them within shared environments. Manipulatives are not necessarily transparent. We must examine how they are used by students before we can judge whether or not transparency emerges. If transparency emerges, it does so each time a manipulative is used for mathematics instruction within the context and the purpose of each particular lesson. It is the mediation by students and teachers in shared and meaningful practices that determines the utility of the manipulatives. Therefore, the physicality of concrete manipulatives does not carry the meaning of the mathematical ideas behind them. Students must reflect on their actions with the manipulatives to build meaning.

Research on manipulatives

The findings of much research has shown that students who use manipulatives during mathematics instruction outperform students who do not (Driscoll, 1983; Greabell, 1978; Raphael and Wahlstrom, 1989; Sowell, 1989; Suydam, 1986). However, some studies have shown student achievement levels to be related to teachers' experience in using manipulatives (Sowell, 1989; Raphael and Wahlstrom, 1989). Other research has focused on the effects of manipulative use on students of different ability levels (Prigge, 1978; Threadgill-Sowder and Juilfs, 1980), on the frequency of verbal interactions (Stigler and Baranes, 1988) and on students' attitudes towards mathematics (Sowell, 1989). Moreover, several studies (e.g., Gilbert and Bush, 1988; Scott, 1983; Weiss, 1994) have focused on practicing teachers' use of manipulatives and have documented the frequency of manipulative use, as well as selected beliefs of the teachers. Although meta-analyses and research reviews on manipulative use have documented how manipulatives are used in prescribed mathematics situations (Clements, 1999; Parham, 1983; Sowell, 1989; Suydam, 1985), observations and interviews of teachers using manipulatives in typical classrooms for their own purposes are lacking in the literature.

Teachers play an important role in creating mathematics environments that provide students with representations that enhance their thinking. Yet even if teachers have learned appropriate strategies for using manipulatives, their beliefs about how students learn mathematics may influence how and why they use manipulatives as they do. Teachers must reflect on students' representations for mathematical ideas and help them develop increasingly sophisticated abstract understandings (Clements, 1999). This is a challenge in teaching mathematics because many teachers lack the mathematics competencies to transform mathematical ideas into representations (Ball, 1990; 1992; Orton, 1988). Some teachers use manipulatives in an effort to reform their teaching of mathematics without reflecting on how the use of representations may change mathematics instruction (Grant, Peterson and Shojgreen-Downer, 1996).

Although research generally supports the use of manipulatives, there is evidence (e.g., Ball, 1992; Meira, 1998) that the mere presence of manipulatives does not guarantee the acquisition of conceptual understanding (Baroody, 1989). Much of this earlier work, however, employed survey techniques and failed to include direct classroom observations. This study represents an important extension of the existing research. Namely, it employs observations, interviews and self-report data to explain how and why teachers use manipulatives as they do.

METHODS

Participants

Participants in this study were 10 of 18 middle grades mathematics teachers voluntarily enrolled in a middle grades mathematics institute where they received instruction in the use of manipulatives, measuring devices, calculators and computers for teaching mathematics. Purposive sampling (Patton, 1990) was used to select 10 teachers that represented a range in teaching experience ($M = 12.2$ years, range 4–25). The participating teachers were all female, with 3 African-Americans and 7 Euro-Americans. The teachers were all public school teachers from four school systems in the same state. One of the teachers held a Master's Degree and the remaining teachers held Bachelor's Degrees. Three of the teachers possessed a mathematics major or minor in their field of study. Six of the teachers taught sixth-grade, three taught seventh-grade and one taught seventh- and eighth-grade classes. Participants were told that they would be observed, interviewed and asked to provide self-report data on their uses of manipulative materials for mathematics instruction.

Materials

Teachers participating in the study attended a 2-week mathematics summer institute at the beginning of the project and received a Middle Grades Mathematics Kit developed in collaboration with a commercial supplier and the state's department of education. The kit included: *manipulatives* such as base-10 blocks, color tiles, snap cubes, geometric solids, geoboards, dice, pattern blocks, hundreds boards, fractions bars and tangrams and *measuring devices* such as beaker sets, rocker scales, thermometers, trundle wheels, centimeter and inch tapes, Triman protractors and Triman compasses. Additionally, all teachers had calculators and teacher-made mathematics materials in their classrooms, as well as access to computers at their schools.

Procedures

A year-long study was designed to investigate how the teachers used mathematics manipulatives under normal conditions in typical classrooms. The study began during the summer with a teacher institute where teachers participated in a variety of experiences and discussions on teaching mathematics for conceptual understanding in the middle grades. Manipulative use was introduced as one of several useful pedagogical tools. Instructors discussed purposes and strategies for using manipulatives, modeled classroom dialogue and interaction and emphasized the importance of creating a shared learning context when the manipulatives were used. Teachers participated in interactive sessions that focused on understanding, reasoning, making connections, using representations and promoting discourse. Teachers had opportunities to identify representations for many middle grades concepts and discuss the challenge of using manipulatives to demonstrate abstract concepts in mathematics.

Qualitative methods were used to gather interview, observational and self-report artifact data for a period of one year. A total of 40 classroom observations and 30 semi-structured interviews were conducted with the 10 participants during the fall, winter and spring of the academic year. During the interviews, teachers were asked a variety of open-ended questions designed to capture their points-of-view. The initial interview protocol and major observation themes were developed from a pilot project and modified on the basis of subsequent classroom observations and teacher interviews using constant comparative analysis (Strauss, 1987). During classroom observations, both fieldnotes and audio-taping provided data for analysis including detailed descriptions of the teachers' activities, behaviors, interpersonal interactions and teacher quotations.

To serve as an indication of teachers' instructional practices on an ongoing basis, teachers were asked to respond to the following monthly self-report postcard requests: "Tell me about the last mathematics lesson you taught. We realize the lesson may or may not be representative of a typical lesson in your classroom. We realize the lesson may or may not have used manipulatives. Please include any plans or worksheets you used with this lesson." During the year each teacher responded to seven postcard requests for a total of 70 requests from the participants.

Analysis

The following data were used as sources in the analyses: teachers' interviews, teachers' and students' audio-taped verbalizations during classroom observations, fieldnotes of teachers' and students verbal and nonverbal behaviors during classroom observations and the self-report postcard responses of teachers' lessons. Interviews and observations were transcribed and analyzed using analytic induction (see e.g. LeCompte, Preissle and Tesch, 1993) to scan the data for themes and relationships, develop hypotheses and then modify these hypotheses on the basis of the data. Observation audio-tapes and fieldnotes were used to provide a holistic picture of the verbal and nonverbal classroom interactions that occurred. Although the quotes within the following text are verbatim, names have been changed to ensure confidentiality. Teacher pseudonyms used during the study were: Ann, Betty, Catherine, Denise, Edith, Frances, Gena, Helen, Inez and Joan.

RESULTS

What manipulatives are being used?

Teachers returned 67 of the 70 postcards requested, a return rate of 96%. Of the 67 postcards returned describing recent lessons in their classrooms, teachers reported that some type of mathematics tool was used in 53 (79%) of the mathematics lessons. The manipulatives reported most often were hundreds boards (13 lessons), color tiles (11 lessons), snap cubes (11 lessons), centimeter cubes (8 lessons) and dice (5 lessons). Other manipulatives reported to be used in 1–2 lessons were tangrams, Cuisenaire rods, two-color counters, fraction bars, geometric solids, number tiles, pattern blocks and geoboards. Teachers also reported using calculators (7 les-

TABLE I

Mathematics tools used during 40 classroom observations

Teacher/ Grade	1	2	3	4
Ann 7th	M color counters	T fractions cards	Z protractor C calculator	M geoboards
Helen 6th	M color tiles M 100s board C calculator	T decimal cards	Z protractor	NONE
Catherine 6th	C calculator	C calculator T "I have, who has" cards	M color tiles M pattern blocks	M pattern blocks
Denise 7th	M 100s board M color tiles M dice	M cm cubes	NONE	M snap cubes
Joan 6th	NONE	Z meter stick Z ruler Z tape measure	M tangrams Z ruler	M tangrams
Frances 6th	NONE	M rainbow cubes	NONE	M snap cubes
Edith 6th	NONE	M snap cubes	NONE	M dice
Gena 6th	Z tape measures	M color tiles	M geoboards	NONE
Betty 7th/8th	M snap cubes	M pattern blocks	M snap cubes M cm cubes M color tiles	M snap cubes M cm cubes
Inez 7th	C calculators	NONE	NONE	NONE

Note. For reporting purposes, the following letters indicate the author's interpretation of the mathematics tools used in each lesson: M = manipulatives; Z = measuring devices; C = calculators; T = teacher-made cards.

sons) and measuring devices such as rulers, yardsticks, tape measures, protractors, balance scales and compasses (15 lessons).

A variety of manipulatives and other mathematics tools were used during the 40 classroom observations. Because the definition of a manipulative is often open for interpretation, the above table reports all of the mathematics tools used by teachers during the observed lessons and leaves their interpretation to the reader. Table I presents the mathematics tools used by each teacher during each lesson observation.

As the Table shows, a variance in the number of times a manipulative was used during the observed lessons ranged from no use of manipulatives (Inez) to use of a manipulative in every lesson (Betty), with most teachers falling somewhere in between these two extremes. The most frequently used manipulatives were snap cubes (6 lessons) and color tiles (5), with other manipulatives ranging from one to three lessons: pattern blocks (3 lessons), centimeter cubes (3), dice (2), geoboards (2), hundreds boards (2), tangrams (2), color counters (1) and rainbow cubes (1).

How are manipulatives being used?

In the 53 postcard reported lessons where a mathematics tool was used, teachers reported using the materials for the following concepts: place value, least common multiples, prime numbers, equivalent fractions, addition and subtraction of fractions, decimals, percent, length measurement, area, perimeter, solid geometry, congruence, symmetry, circumference, angle measurement, statistics, probability, graphing, patterns, problem solving and positive and negative integers.

During classroom observations teachers' uses of the manipulatives varied in terms of both the purpose for their use and the amount of time they were used. In 20 of the 40 lesson observations, teachers used a manipulative during instruction. Table II excludes other mathematics tools, such as measuring devices and calculators and shows only the manipulatives that were used and how they were used in each of the observed lessons.

As the table indicates, the manipulatives were used for a variety of mathematics concepts and purposes with the most common being the exploration of geometry concepts in 7 of the 20 lessons (35%) and using the manipulatives to play a game in 6 of the 20 lessons (30%).

The amount of time spent using manipulatives during the observed lessons also varied. Table III indicates the amount of time in each lesson that students in the classroom interacted with the manipulatives, with class sessions ranging from 50 to 60 minutes.

Examining the amount of time students had their hands on manipulatives during the 40 individual lessons shows that use ranged from lessons where there was no use of manipulatives to one lesson where manipulatives were used for 31 minutes of a 60 minute class session (Betty, observation 3). If these lessons are an indicator of teachers' general manipulative use, the table shows that these students used manipulatives approximately 7.38 minutes for each 57.5 minutes in mathematics classes (in 40 lessons). We might also interpret these data in another way. If we consider only those lessons where manipulatives were used, then teachers who used a manipulative during a mathematics lesson used it an average of 14.75

TABLE II

How manipulatives were used during classroom observations

Teacher/ Grade	1	2	3	4
Ann 7th	M color counters H fraction tic-tac-toe game	NONE	NONE	M geoboards H showing area in square units
Helen 6th	M color tiles, 100s board H multiplication sentences game	NONE	NONE	NONE
Catherine 6th	NONE	NONE	M color tiles, pattern blocks H perimeter problems & adding fractions	M pattern blocks H multiplying fractions
Denise 7th	M 100s board, color tiles, dice H common multiples game	M cm cubes H prime & composite bingo game	NONE	M snap cubes H finding fractional parts of a figure
Joan 6th	NONE	NONE	M tangrams H perimeter and area	M tangrams H exploring geometric shapes
Frances 6th	NONE	M rainbow cubes H simultaneous equations game	NONE	M snap cubes H finding patterns
Edith 6th	NONE	M snap cubes H exploring 3-D geometry	NONE	M dice H probability game
Gena 6th	NONE	M color tiles H finding fractional parts of a figure	M geoboards H exploring polygons	NONE
Betty 7th/8th	M snap cubes H finding averages	M pattern blocks H finding congruent shapes	M snap cubes, cm cubes, color tiles H problem solving questions	M snap cubes, cm cubes H problem solving questions
Inez 7th	NONE	NONE	NONE	NONE

Note. M = manipulatives; H = how the manipulatives were used during the lesson.

TABLE III

Amount of time manipulatives were used by students during observations

Teacher/ Grade	1	2	3	4
Ann 7th	M color counters T 6 min.	NONE	NONE	M geoboards T 8 min.
Helen 6th	M color tiles, 100s board T 14 min.	NONE	NONE	NONE
Catherine 6th	NONE	NONE	M color tiles, pattern blocks T 9 min.	M pattern blocks T 8 min.
Denise 7th	M 100s board, color tiles, dice T 5 min.	M cm cubes T 18 min.	NONE	M snap cubes T 6 min.
Joan 6th	NONE	NONE	M tangrams T 13 min	M tangrams T 16 min.
Frances 6th	NONE	M rainbow cubes T 28 min.	NONE	M snap cubes T 17 min.
Edith 6th	NONE	M snap cubes T 25 min.	NONE	M dice T 7 min.
Gena 6th	NONE	M color tiles T 9 min.	M geoboards T 23 min.	NONE
Betty 7th/8th	M snap cubes T 12 min.	M pattern blocks T 24 min.	M snap cubes, cm cubes, color tiles T 31 min.	M snap cubes, cm cubes T 16 min.
Inez 7th	NONE	NONE	NONE	NONE

Note. M = manipulatives used; T = the amount of time STUDENTS used manipulatives during the lessons.

minutes during a 50–60 minute class session (out of the 20 lessons when a manipulative was used).

In most cases the lessons were teacher-directed: students either used the manipulatives following the teachers' directives or the manipulatives were an enrichment activity or game assigned when there was time at the end of the class period. In two lessons, teachers used a manipulative for demonstration only and these manipulatives do not appear in the table (Edith, observation 3; Inez, observation 2). For example, Edith used a circle and some one-inch square color tiles to pose a question to students about using the tiles to measure a circle's area, but the students did not use the manipulatives themselves during the lesson (Edith, observation 3).

Teachers voiced different beliefs about the purposes for using manipulatives. Edith, Inez and Denise said they used manipulatives for problem solving and enrichment. For example, Denise shared in an interview that she used manipulatives "just to have a change of pace in math. . . instead of lecturing every day, it gives some kind of break in the routine" (interview 1). Ann described the manipulatives as a reward and a privilege. Frances, Catherine and Helen, talked about a variety of purposes for manipulatives in their interviews including: to provide a visual model when introducing concepts; for student use in solving problems; and to reinforce and provide enrichment for concepts. Gena saw manipulative use as "another tool to help students solve problems, it's just like another strategy" (interview 1), while Betty saw it as "a concrete way of showing [the students] and then they can move on to not needing the manipulatives to see the pattern or the rule" (interview 1). Joan's purpose for using manipulatives in mathematics was "to make it more fun" (interview 1).

Unexamined notions about manipulative use

During the first reading the researcher reviewed 100% of the transcribed data to identify and code major themes within the transcriptions. The researcher identified two major categories for the analysis: 'fun math' and 'real math.' Through their comments, teachers defined 'fun math' as 'games,' 'enrichment,' 'an extra-time activity,' and 'a reward for behavior.' When they commented on 'procedures and algorithms,' 'basic facts,' 'preparing for tests,' 'textbooks,' and 'paper-and-pencil work,' they described this as 'real math.' The data were coded for themes and relationships related to these references.

In many instances teachers indicated that the use of manipulatives was 'fun.' Initially the term 'fun' seemed to indicate that teachers and students found enjoyment in using the manipulatives during mathematics teaching and learning. Further analysis of the data suggested that embedded in teachers' use of the word 'fun' were some unexamined notions that inhibit the use of manipulatives in mathematics instruction. Teachers made subtle distinctions between 'real math' and 'fun math,' using the term 'real math' to refer to lesson segments where they taught rules, procedures and algorithms using textbooks, notebooks, worksheets, and paper-and-pencil tasks. The term 'fun math' was used when teachers described parts of the lesson where students were having fun with the manipulatives. The analysis uncovers several of these notions about manipulative use and how these distinctions may be reflected in classroom practices.

Enjoyment in learning. Many of the teachers explained that teaching mathematics with the manipulatives made learning more fun in the sense that there was intrinsic enjoyment for students. As Catherine and Joan reported, “My personal goals for teaching mathematics are to make math fun” (Catherine, interview 1) and “It’s more fun to do fun activities... I’ve always loved to plan fun activities” (Joan, interview 2). They described students’ responses when manipulatives were used – “The students love it. They think it’s fun stuff” (Inez, interview 1) – and explained that the fun generated by using the manipulatives was promoting learning. “I think they’re really learning by having fun” (Helen, interview 2).

During lessons where manipulatives were used, students appeared to be interested, active and involved. One of the teachers noted, “Anytime we do group work or anything having to do with manipulatives, they think it’s fun time” (Catherine, interview 1). As students entered classrooms, they occasionally asked their teachers if they would be using the math textbook that day; during one observation, the teacher replied, “No, we’re going to have fun today” (Joan, observation 2).

Students also made comments during mathematics classes about particular lessons being ‘fun’ or like ‘playing a game.’ One teacher mentioned that her students “ask me about every day ‘Are we going to play a game today? Are we going to play with the blocks?’ ” (Denise, interview 2). Teachers often described games that reinforced the mathematics concepts they were teaching (“We played a game using 3 tiles and getting 15 markers.... It was to review their multiplication and division facts and equations” [Helen, interview 2]) and students saw mathematics as fun in an activity-based sense.

Rewarding experiences. Because the use of the manipulatives was so often considered enjoyable by teachers and students, many of the teachers used the manipulatives as a reward for appropriate student behaviors. Ann reported that manipulatives were a privilege and a reward for appropriate behavior. “If anything, it’s a reward for them because there have been times when I have taken them away” (Ann, interview 1). She explained that behavior determined whether or not students would use the manipulatives or the textbook. “If they’re not working or they do not follow my instructions... I would have to let them work on something else with the same concept but not being able to use the manipulatives. They may do book work” (Ann, interview 1). Inez shared this belief: “The first time a child misuses or abuses, it’s taken from them and they are separated from the class. They are not allowed to continue the activity using the manipulative for that day” (Inez, interview 1). While using the pattern blocks, Catherine communicated the message that using manipulatives was a privilege remarking to students, “I’m waiting for quiet to happen so we don’t have to go back to our textbooks” (observation 3).

For a number of the teachers, decisions to use manipulatives with certain groups of students were based not on the appropriateness of a representation for a particular mathematics concept but on whether or not students had behaved appropriately during previous lessons where manipulatives were used. If appropriate behavior had been displayed, then students were permitted to have ‘fun’ with the manipulatives. Catherine indicated that behavior played a crucial role in her decisions about using the manipulatives: “Last year I didn’t do much at all [with manipulatives] because of the kids I had. I tried a lot the first nine weeks and they just couldn’t work together, so we cut out some of it” (interview 1). Teachers’ comments indicated concern about maintaining control while the manipulatives were being used.

Emerging distinctions

‘Fun math’ versus ‘real math’. As classroom observations and teacher interviews continued, it seemed that teachers described their use of manipulatives as fun and distinct from their regular teaching of mathematics. Although these distinctions emerged subtly, a very clear indication of this occurred halfway through the year. Describing a lesson with manipulatives, Joan said, “Sometimes I think that they are just having fun, but I don’t mind because eventually we’ll get to the *real math* part [italics added]” (interview 2). Later in the same interview Joan stated, “When we’re doing hands-on stuff they’re having more fun, so they really don’t think about it as being math” (interview 2). These comments were striking to the observer at the time and seemed to capture the emerging phenomenon of teachers using the term ‘fun’ whenever they described lessons using the manipulatives and distinguishing these from their ‘real math’ instruction. Not only did teachers appear to distinguish between ‘fun math’ lessons where manipulatives were used and ‘real math’ lessons where traditional paper-and-pencil methods were used, but they also made distinctions between parts of individual lessons. For example, the manipulatives may be used for exploration at the beginning or ‘fun math’ part of a lesson, or they may be used in an activity or a game after the mathematics content was taught; but during the teaching of specific skills or content, paper-and-pencil methods were used to teach and practice the ‘real math.’

Other teachers communicated these artificial distinctions. Denise reported that she was able to teach ‘real math’ every day, but that she could not teach ‘fun math’ every day: “I can’t do manipulatives every day…

The kids sometimes need that kind of structure where I'm in the front of the class and where they're sitting there working" (interview 2). Denise seems to be making several powerful statements about the manipulatives: they are not essential to real math instruction; they mean playing and not working; and they do not lend themselves to being a part of the structure of a mathematics class. Inez made the distinction by using activities with manipulatives to give the students and herself a break. "I look for enrichment activities and manipulative type activities that can go well with the concepts.... It's a nice break for the children and it's a nice break for me. Yesterday we used the hundreds boards.... That was a nice break from the daily things we do in the classroom" (Inez, interview 2).

Are manipulatives really important? These teachers viewed their use of manipulatives for mathematics instruction as playing, exploring, or a change of pace. With this view, manipulatives may function as part of teachers' rewards system for classroom management. By aligning manipulatives with 'fun math' teachers were, in essence, distinguishing manipulative use from their regular mathematics teaching. Manipulative use was often relegated to the end of the class period, the end of the week on Fridays, or the end of the school year when district objectives were completed. Denise taught the 'real math' first during her lessons and then made time for using manipulatives later. "Most of the time I'll teach the concept and then we'll play a game or do an activity.... When they finish their activity, if one group is ahead of the other groups, I let them sit there and play and build towers or whatever" (Denise, interview 2). Her description shows the distinction she makes during class sessions where 'teaching the concept' is the real math part and 'playing and building towers' is the fun math part.

Other teachers taught the 'real math' during the first part of the week and then allowed time for the 'fun math' with manipulatives on Fridays. "We try to always use our manipulatives on Fridays because we take our math tests on Thursdays" (Frances, interview 2). Joan explained that "Friday is free time... that's the time they can just explore whatever they can do with the blocks. I make it available for them to use... when we have free time on Fridays, or the last 15 minutes of class, if they've kept up with their work. It's their incentive to work" (interview 1). One teacher was concerned about the coverage of content students would be tested on during their criterion tests. "If I didn't have the pressure of the criterion tests, I could truly enjoy the use of the manipulatives more.... Hopefully the second half of the year things will slow down and we can do more fun things" (Edith, interview 2). The manipulatives seemed to be used only if there was extra time.

By allotting specific days and times to use manipulatives, teachers sent a clear message to their students about the importance of manipulatives in mathematics instruction. During observations, several comments indicated teachers' dissatisfaction with using the manipulatives. During one observation where color tiles were being used, Denise commented, "You can tell by looking at the blocks, but you need to get a common denominator" (observation 1). In another classroom, a student asked if she could use the fraction bars to help her complete a task and Inez responded, "If you think it would help, go ahead. It's quicker doing it mathematically. Solve it first without them" (observation 4). To another student the teacher said, "If your parent has to measure a board, they don't carry these things [fraction bars] around. Knowing to add the tops [of fractions] is much easier" (Inez, observation 4). During an interview Ann explained that even though her students "would rather do the hands-on stuff, instead of having to do the paper-and-pencil," she must use pencil-and-paper methods because "I feel they have to get the concept" (Ann, interview 1). Observations in Inez's classroom and her comments clearly reflect her beliefs: "My goal for teaching math is to have each and every child that I work with master the basic facts.... One of the skills that they need first is rote memorization. There are just some things in math that you can't teach any other way" (Inez, interview 1). Inez went on to explain, "I really don't like using manipulatives that much, but I know I need to" (interview 1). Such statements imply that in order to understand mathematics concepts, students must use textbook algorithms, thereby ignoring the possibility that through meaningful experiences with representations, such as manipulatives, students may invent their own algorithms. Even though the summer institute provided many experiences in using representations for mathematical concept construction, many teachers did not internalize this idea.

DISCUSSION

Following the script

Mathematics instruction in these classrooms followed the typical United States script or lesson 'pattern': (1) review previous material, (2) demonstrate how to solve problems for the day, (3) practice problems, and (4) correct seatwork and assign homework (Stigler and Hiebert, 1999, p. 81). Homework was collected or reviewed in the first 5–30 minutes of the class period; sometimes students wrote these problems and solutions on the board. Teachers told students to turn to a particular page in the textbook or distributed a worksheet and the lesson was introduced. Teachers demonstrated procedures for solving problems with little or no opportunity for student discussion. Problems that are part of the introduction adhere to the same general pattern. Students have class time for practice solving selected problems from textbooks and worksheets and are then given an assignment in which similar problems are completed independently. These scripts reflect the cultural activity of American mathematics teaching, where students spend most of their time acquiring isolated skills through repeated practice (Stigler and Hiebert, 1999). These cultural scripts are learned implicitly, through observation and participation.

Adherence to the coverage of state curriculum objectives was an important goal for the teachers in this study and they did not clearly see how manipulatives could be used to teach these objectives as efficiently as they had taught the objectives using paper-and-pencil approaches. Many teachers in the US are influenced by the pressures of testing to prepare procedurally competent students to be successful on largely procedural standardized tests. There is not much room for manipulatives in these procedure-driven scripts; therefore, teachers fit the use of manipulatives into existing routines for mathematics instruction. The pressure teachers feel to cover material during mathematics instruction may encourage manipulative use for 'fun math' activities such as games and enrichment, rather than supporting classroom contexts in which students make meaning from their experiences with these materials.

The preference for following the typical mathematics teaching script in the United States may also be an illustration of Elbow's (1986) observation that "much teaching behavior really stems from an unwarranted fear of things falling apart" (pp. 71–72). Using manipulatives deviates from the US teaching script, perhaps so much so that it incites student discussion and interaction with peers and materials, causing a disruptive environment as opposed to one in which students are working paper-and-pencil problems in an orderly routine. Although teachers gave verbal assent to the notion that manipulatives could be used to teach mathematics concepts, their actual lessons reflected traditional teaching routines with manipulatives used primarily to supplement the lesson script.

The summer institute was useful in the sense that the teachers made more use of the manipulatives than teachers in general might have done. However, the inservice failed to go beyond increasing use of the materials. Teachers were unsuccessful in using the manipulatives to engage students in making sense of mathematics; rather the use of the manipulatives focused more on reinforcement of previously learned content or activities that were 'fun.' These data give concrete meaning to Smith's (1996) ideas that teachers do not embrace a more constructivist approach because they might sense a loss of efficacy in doing so. Perhaps the teachers resist a constructivist epistemology and misinterpret the role of manipulatives in mathematics, or perhaps they do not see the potential of manipulatives for creating experiences in which students construct their own mathematical representations when used appropriately.

Teachers conceptions of mathematics and manipulatives

A central construct of this study is the relationship between teachers' conceptions of mathematics, teachers' conceptions of manipulatives and of purposes for using the manipulatives (Khoury and Zazkis, 1994; Thompson, 1985; Tirosh, 2000). A teacher's beliefs are intertwined with her/his actions. There is not simply some correlation; beliefs orient action and actions condition beliefs on an on-going basis. To alter the lesson script requires reflection on one's own teaching of mathematics as a cultural practice (Stigler and Hiebert, 1999) and the willingness to adopt practices that encourage conceptual, as opposed to merely mechanical understandings of mathematics. Yet this is a challenging goal because, as Stigler and Hiebert (1999) note, "Compared with other countries, the United States clearly lacks a system for developing professional knowledge and for giving teachers the opportunity to learn about teaching" (p. 12–13).

The teachers in this study communicate many underlying assumptions about how and why they use manipulatives in their classrooms. As this study shows, we must acknowledge these assumptions and examine the how and why of manipulative use that may inhibit teachers from using these concrete representations effectively. Purchase of these materials by teachers and their school systems and professional development workshops on their use are of little value if we do not take into account teachers' beliefs about manipulative use and its effects on their use in classroom situations.

Some teachers buy in to the notion that the main purpose for using manipulatives is 'fun math.' By using manipulatives for 'fun math,' teachers artificially set up a classroom situation in which materials may not be used effectively. If "teachers act as if student interest will be generated only by diversions outside of mathematics" (Stigler and Hiebert, 1999, p. 89), they send a message to students that explorations and conjectures with representations are not connected to real mathematical learning. Comments to students that trivialize manipulative use in both subtle and overt ways devalue the potential of these materials as representations for learning mathematics concepts. Focusing their use on fun limits the possibilities for students' explorations of meaningful mathematical content in engaging and interesting ways.

Teachers' conceptions of mathematics are grounded in their understanding of the content of mathematics. Research emphasizes the important and significant impact of teachers' understandings of mathematics on the mathematical thinking that takes place in the classroom (Ma, 1999). Ma's (1999) international study demonstrated that although American elementary school teachers showed correct procedural knowledge of problems in whole-number subtraction and multiplication, they were less skilled than the Chinese teachers in more advanced topics. For example, only one of the 23 American teachers provided a conceptually correct but pedagogically problematic representation for a problem involving division by fractions $(1{}^{3}/_{4})\ {}^{1}/_{2} = 3{}^{1}/_{2}$). This research highlights the importance of teachers having depth, breadth and thoroughness in their understanding of mathematics. "Teachers with this deep, vast and thorough understanding do not invent connections between and among mathematical ideas, but reveal and represent them in terms of mathematics teaching and learning" (Ma, 1999, p. 122). It is not clear that the teachers in this study were able to make connections or represent mathematics ideas in meaningful ways while using the manipulatives.

Using representations effectively

The effective use of manipulatives for mathematics instruction is more complicated than it might appear. Mathematical relationships must be imposed on the materials. The student's own internal representation of ideas must somehow connect with the external representation or manipulative. However, manipulatives are externally generated as manufacturers' representations of mathematical ideas; therefore, meaning attached to the manipulatives by manufacturers is not necessarily transparent to teachers and students (Meira, 1998). For example, fraction tiles are the designer's attempt to represent her/his idea of fractions by using concrete objects. It is a false assumption to believe that mathematical relationships are abstracted from empirical objects.

Goldin and Shteingold (2001) define a representation as a "sign or a configuration of signs, characters, or objects" that can "stand for (symbolize, depict, encode, or represent) something other than itself" (p. 3). Although individuals may share the tools used for representation – *symbols* (such as pictures or tally marks) and *signs* (such as spoken words or written numerals) (Kamii, Kirkland and Lewis, 2001; Piaget, 1951), to Piaget (1951), representation is what each individual does. In other words, an individual's knowledge is based on the making of relationships and comes from the learner's own mental actions. Mathematical reasoning is fostered when teachers focus on children's constructive abstraction (children's thinking) rather than on representation (Kamii, Kirkland and Lewis, 2001). For example, to a young child the numeral '8' may represent nothing and to an adult it represents a number of things. Therefore, individuals may represent the same or different ideas about the numeral 8. This process is not empirically observable: it comes from the individual's own thinking and understanding of connections formed through mental actions (Kamii and Warrington, 1999; Piaget, Inhelder and Szeminska, 1948/1960). Although using representations may support the knowledge construction process, much of the representation must come from the student.

Studies investigating teachers' knowledge of representations have demonstrated that many teachers lack the knowledge of how mathematical ideas are transformed into representations (Ball, 1990; Orton, 1988). To further complicate the issue of using manipulatives to represent abstract relationships, students' interpretations of a representation may also be different from the one introduced by the teacher (Ball, 1992). In one report (Baroody, 1989) a teacher was using Cuisenaire rods to represent one concept and the students constructed a model for the colored rods that made sense to them but was different from the concept the teacher intended them to learn. Students had constructed their own mathematical meaning from the activity but had not learned the target concept of the lesson. In another study, when students were given free access to manipulatives, they spontaneously and selectively used these materials to mediate their own learning (Moyer, 1998). However, some of the teachers in the study had difficulty following the students' thinking and their uses of the representations. The use of manipulatives for games on Fridays does not provide mathematical experiences that support students' constructive abstraction.

Further, teachers should be aware that some manipulatives may be useless. Although the colorful design of these materials makes them aesthetically pleasing, some of these devices have little relationship to mathematical concepts. When using materials whose design is not based on sound mathematical ideas, it is easy to understand why teachers have difficulty connecting the manipulatives with the mathematics.

CLOSING COMMENTS

How and why manipulatives are used by teachers is a complex question. In previous studies where manipulatives were used, we viewed mere use itself as a positive sign. But simply using manipulatives is not enough if we do not consider *how* classroom teachers are using them. "If teaching is a system, then each feature, by itself, doesn't say much about the kind of teaching that is going on. What is important is how the features fit together to form a whole" (Stigler and Hiebert, 1999, p. 75). Individual features of a lesson, such as the use of manipulatives, can only be evaluated as effective in terms of how they connect and relate with other features and fit into the overall lesson. Teachers' use of manipulatives in this study is intertwined with issues of mathematical content knowledge, teachers' awareness of the conceptual structure of mathematics and the ability to teach this content to students by pulling from a wide variety of concept areas to help students understand new ideas. This includes the ability to use a variety of representations and to encourage students' uses of representations as well.

The development of the student's internal representation of ideas, tested on the external representations or manipulatives, is at the heart of what it means to learn mathematics. Connecting the learner's own mental actions with the shared tools used for representation is the challenge for mathematics teachers. Although using representation supports knowledge construction, the internal representation of ideas comes from the learner. Manipulatives may serve as tools for teachers to translate abstractions into a form that enables learners to relate new knowledge to existing knowledge. This requires teachers to guide students to translate between representations in the form of mathematical objects, actions and abstract concepts so that students can see the relationship between their knowledge and new knowledge. Although one may assume that this is an easy thing for teachers to do, in fact, it is not. It is a challenge for teachers to (1) interpret students' representations of their mathematical thinking, (2) reveal and represent connections among mathematical ideas and, (3) develop appropriate concrete contexts for learning mathematics.

This study highlights the underlying beliefs teachers utilize as filters to interpret information about the use of manipulatives and to make value judgments about the utility of these materials for teaching mathematics. For example, research clearly shows that certain kinds of material uses aid learning (or at least measured achievement), yet teachers often ignore this research. Perhaps we are not clear about the results; or perhaps we do not systematically build on them to provide ways of practice which are useful and become part of our program of teacher education. This study also provides important feedback about the effectiveness of professional development efforts in using manipulatives for teaching mathematics. For these teachers, the manipulatives served a limited, but useful, purpose. Perhaps in the design of professional development we do not take into account the contingencies under which teachers practice and children learn; or perhaps it is too great a challenge to insert manipulatives into the American 'script.' Designing professional development that provides teachers with deeper understandings of concepts such as representation and its role in mathematics teaching and learning may shape not only how the manipulatives are used but also what beliefs motivate teachers to use them.

ACKNOWLEDGEMENT

Special thanks to Madeleine Gregg, Constance Kamii and Elizabeth Milewicz for their feedback on this paper.

REFERENCES

Ball, D.L.: 1990, 'The mathematical understandings that preservice teachers bring to teacher education', *Elementary School Journal* 90, 449–466.

Ball, D.L.: 1992, 'Magical hopes: Manipulatives and the reform of math education', *American Educator* 16(2), 14–18, 46–47.

Baroody, A.J.: 1989, 'Manipulatives don't come with guarantees', *Arithmetic Teacher* 37(2), 4–5.

Boulton-Lewis: 1998, 'Children's strategy use and interpretations of mathematical representations', *Journal of Mathematical Behavior* 17(2), 219–237.

Bruner, J.S.: 1960, *The Process of Education*, Harvard University Press, Cambridge, MA.

Bruner, J.S.: 1986, *Actual Minds, Possible Worlds*, Harvard University Press, Cambridge, MA.

Clements, D.H.: 1999, ' "Concrete" manipulatives, concrete ideas', *Contemporary Issues in Early Childhood* 1(1), 45–60.

Cobb, P.: 1995, 'Cultural tools and mathematical learning: A case study', *Journal for Research in Mathematics Education* 26(4), 362–385.

Dienes, Z.P.: 1969, *Building Up Mathematics*, Hutchison Education, London.

Driscoll, M.J.: 1983, *Research Within Reach: Elementary School Mathematics and Reading*, CEMREL, Inc., St. Louis.

Elbow, P.: 1986, *Embracing Contraries: Explorations in Learning and Teaching*, Oxford University Press, New York.

Gilbert, R.K. and Bush, W.S.: 1988, 'Familiarity, availability and use of manipulative devices in mathematics at the primary level', *School Science and Mathematics* 88(6), 459–469.

Glover, J.A., Ronning, R.R. and Bruning, R.H.: 1990, *Cognitive Psychology for Teachers*, Macmillan, New York.

Goldin, G. and Shteingold, N.: 2001, 'Systems of representations and the development of mathematical concepts', in A.A. Cuoco and F.R. Curcio (eds.), *The roles of Representation in School Mathematics*, NCTM, Reston, VA, pp. 1–23.

Grant, S.G., Peterson, P.L. and Shogreen-Downer, A.: 1996, 'Learning to teach mathematics in the context of system reform', *American Educational Research Journal* 33(2), 509–541.

Greabell, L.C.: 1978, 'The effect of stimuli input on the acquisition of introductory geometry concepts by elementary school children', *School Science and Mathematics* 78, 320–326.

Hiebert, J. and Wearne, D.: 1992, 'Links between teaching and learning place value with understanding in first grade', *Journal for Research in Mathematics Education* 23, 98–122.

Kamii, C., Kirkland, L. and Lewis, B.A.: 2001, 'Representation and abstraction in young children's numerical reasoning', in A.A. Cuoco and F.R. Curcio (eds.), *The Roles of Representation in School Mathematics*, NCTM, Reston, VA, pp. 24–34.

Kamii, C. and Warrington, M.A.: 1999, 'Teaching fractions: Fostering children's own reasoning', in L.V. Stiff and F.R. Curcio (eds.), *Developing mathematical reasoning grades K-12: 1999 Yearbook*, NCTM, Reston, VA, pp. 82–92.

Khoury, H.A. and Zazkis, R.: 1994, 'On fractions and non-standard representations: Pre-service teachers' conceptions', *Educational Studies in Mathematics* 27(2), 191–204.

LeCompte, M.D., Preissle, J. and Tesch, R.: 1993, *Ethnography and Qualitative Design in Educational Research*, 2nd ed., Academic Press, San Diego, CA.

Ma, L.: 1999, *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*, Erlbaum, Mahwah, NJ.

Meira, L.: 1998, 'Making sense of instructional devices: The emergence of transparency in mathematical activity', *Journal for Research in Mathematics Education* 29(2), 121–142.

Moyer, P.S.: 1998, 'Using mathematics manipulatives: Control- versus autonomy-oriented middle grades teachers (Doctoral dissertation, The University of North Carolina at Chapel Hill, 1998), *Dissertation Abstracts International, 59-07A*, 2406.

Orton, R.E.: 1988, *Using Representations to Conceptualize Teachers' Knowledge*, Paper presented at the Psychology of Mathematics Education-North America, DeKalb, IL.

Parham, J.L.: 1983, 'A meta-analysis of the use of manipulative materials and student achievement in elementary school mathematics', *Dissertation Abstracts International* 44A, 96.

Patton, M.Q.: 1990, *Qualitative Evaluation and Research Methods*, SAGE, Newbury Park, CA.

Piaget, J.: 1951, *Play, Dreams and Imitation in Childhood*, Reprint, W.W. Norton and Co., New York.

Piaget, J.: 1952, *The Child's Conception of Number*, Humanities Press, New York.

Piaget, J., Inhelder, B. and Szeminska, A.: 1960, *The Child's Conception of Geometry*, Routledge and Kegan Paul, London. (Original work published 1948). 1948/1960.

Prigge, G.R.: 1978, 'The differential effects of the use of manipulative aids on the learning of geometric concepts by elementary school children', *Journal for Research in Mathematics Education* 9(5), 361–367.

Raphael, D. and Wahlstrom, M.: 1989, 'The influence of instructional aids on mathematics achievement', *Journal for Research in Mathematics Education* 20(2), 173–190.

Resnick, L.B.: 1983, 'Mathematics and science learning: A new conception', *Science* 220, 477–478.

Scott, P.B.: 1983, 'A survey of perceived use of mathematics materials by elementary teachers in a large urban school district', *School Science and Mathematics* 83(1), 61–68.

Simon, M.A.: 1995, 'Reconstructing mathematics pedagogy from a constructivist perspective', *Journal for Research in Mathematics Education* 26(2), 114–145.

Skemp, R.R.: 1987, *The Psychology of Learning Mathematics*, Erlbaum, Hillsdale, NJ.

Smith, J.P.: 1996, 'Efficacy and teaching mathematics by telling: A challenge for reform', *Journal for Research in Mathematics Education* 27(4), 387–402.

Sowell, E.J.: 1989, 'Effects of manipulative materials in mathematics instruction', *Journal for Research in Mathematics Education* 20(5), 498–505.

Stigler, J.W. and Baranes, R.: 1988, 'Culture and mathematics learning', in E.Z. Rothkopf (ed.), *Review of Research in Education*, American Educational Research Association, Washington, D.C., pp. 253–306.

Stigler, J.W. and Hiebert, J.: 1999, *The Teaching Gap*, The Free Press, New York. Strauss, A.: 1987, *Qualitative Analysis for Social Scientists*, Cambridge University Press, New York.

Suydam, M.N.: 1985, *Research on Instructional Materials for Mathematics*, ERIC Clearinghouse for Science, Mathematics and Environmental Education, Columbus, OH. (ERIC Document Reproduction Service No. 276 569).

Suydam, M.N.: 1986, 'Manipulative materials and achievement', *Arithmetic Teacher* 33(6), 10, 32.

Suydam, M.N. and Higgins, J.L.: 1977, *Activity-Based Learning in Elementary School Mathematics: Recommendations from Research*, ERIC Center for Science, Mathematics and Environmental Education, College of Education, Ohio State University, Columbus, OH.

Thompson, A.G.: 1985, 'Teachers' conceptions of mathematics and the teaching of problem solving', in E.A. Silver (ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives*, Erlbaum, Hillsdale, NJ.

Thompson, P.W.: 1992, 'Notations, conventions and constraints: Contributions to effective uses of concrete materials in elementary mathematics', *Journal for Research in Mathematics Education* 23(2), 123–147.

Thompson, P.W. and Thompson, A.G.: 1990, *Salient Aspects of Experience with Concrete Manipulatives*, International Group for the Psychology of Mathematics Education, Mexico City.

Threadgill-Sowder, J.A. and Juilfs, P.A.: 1980, 'Manipulatives versus symbolic approaches to teaching logical connectives in junior high school: An aptitude x treatment interaction study', *Journal for Research in Mathematics Education* 11(5), 367–374.

Tirosh, D.: 2000, 'Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions', *Journal for Research in Mathematics Education* 31(1), 5–25.

von Glasersfeld, E.: 1990, 'An exposition of constructivism: Why some like it radical', in R.B. Davis, C.A. Maher and N. Noddings (eds.), *Constructivist Views on the Teaching and Learning of Mathematics*, National Council of Teachers of Mathematics, Reston, VA, pp. 19–29.

von Glasersfeld, E.: 1995, 'A constructivist approach to teaching', in L.P. Steffe and J. Gale (eds.), *Constructivism in Education*, Erlbaum, Hillsdale, NJ, pp. 3–15.

Vygotsky, L.S.: 1978, *Mind in Society: The Development of Higher Psychological Processes*, Harvard University Press, Cambridge, MA.

Weiss, I.R.: 1994, *A Profile of Science and Mathematics Education in the United States: 1993*, Horizon Research, Inc., Chapel Hill, NC.

Winograd, T. and Flores, F.: 1986, *Understanding Computers and Cognition: A New Foundation for Design*, Ablex, Norwood, NJ.

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