An Application of Discrete Optimization for Developing Economically Efficient Multiple-Use Projects

United States Department of Agriculture, Forest Service

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An Application of Discrete Optimization for Developing Economically Efficient Multiple-Use Projects

J. Greg Jones
Ervin G. Schuster

RESEARCH SUMMARY

This paper presents a model formulation useful (1) for planning multiple-use projects and (2) for identifying efficient management prescriptions and aggregate emphasis projects to build into future forest planning models. The formulation is a discrete version of the continuous joint production model in economic theory. Economic efficiency can be analyzed both in terms of type of project and scale of project.

The model can be formulated and solved graphically or as a mixed-integer programming (MIP) problem. The graphic approach rather clearly depicts the nature of economic efficiency in multiple-use production and requires little in the way of equipment. It is, however, limited to problems that can be depicted in two-dimensional space. The MIP approach has the following advantages over the graphic approach: (1) it can accommodate more than two outputs, (2) inter-temporal analysis is easier to conduct, (3) capability to conduct sensitivity analysis is enhanced, and (4) it lends itself well to automation.

The MIP formulation contains decision variables that are formulated as whole decision alternatives, which assume values of 0 (do not do project) or 1 (do project). This differs from mathematical programming formulations common in forestry (for example, FORPLAN, MUSYC, and Timber RAM) in which decision variables are formulated on a per-acre basis. The advantages of the MIP formulation are that diminishing marginal productivity can be modeled and the level of site specificity is enhanced. The main disadvantage of this MIP approach is that only a limited number of management alternatives can be handled effectively, making it best suited to problems of a relatively small geographic scope, for example, a project planning area.

The MIP formulation is easy to solve and sufficiently small to be processed on a small computer. Combined with front-end data processing software, it could be useful for conducting multiple-use efficiency analysis. The potential lies not as a substitute for current forest planning methods, but rather as a tool to aid in identifying efficient management prescriptions to place in forest planning models and as a means of analyzing projects for implementation.
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INTRODUCTION

In recent history, the focus of land management economical analysis on National Forests has been in forest planning. Large-scale planning models, such as FORPLAN (Gilbert and others 1985), are being used to conduct economic analysis of multiple-use management in this planning process. For management, however, forest planning analysis has had to be conducted at a relatively low level of resolution. As a result, there may be only spatial conditions and timing sequences for implementing the general management direction identified in forest planning. This remains a need for economic analysis in project design to aid in identifying projects that efficiently implement forest plans. Clearly, if projects are not efficiently managed, the net benefit of these projects are the means by which management is implemented on the ground. Unfortunately, the economics of project planning has largely been ignored by economists and analysts. As a result, analytical techniques or models for this purpose are lacking. This may be particularly detrimental for projects with considerable multiple-use components, where efficient designs are particularly difficult to identify.

In this paper we present a model formulation we believe may be useful in planning multiple-use projects. In addition, it could have applications in forest management prescription and/or aggregate emphasis projects to build into FORPLAN models in future forest planning efforts. First, the model is presented in graphical terms for a hypothetical but realistic projects planning situation. Next, a mixed-integer mathematical programming formulation of the model is presented and solved. Then, sensitivity analysis techniques applicable to the mixed-integer programming formulation are discussed. Finally, several topics are discussed regarding the operational feasibility of this formulation.

THE CONCEPT

Gregory (1955) presented the case that an appropriate economic formulation for multiple use is the joint production model in microeconomic theory. Joint production occurs when two or more outputs are produced simultaneously (jointly) by a single production process, process, and forms, for example. The joint production model is comprised of a "production surface," which identifies the combinations of outputs that can be produced on a tract of land for by some fixed production plant, given efficient use of variable inputs. For the two-output case, this production surface is often depicted by a series of "iso-cost" (constant cost) lines. Each corresponds to a unique level for variable cost, and identifies the combinations of outputs which can be produced with that cost. Critical, overall, the values of outputs are then introduced to figure the combination of outputs on each iso-cost curve that provides the greatest total value and (ii) which of these best points the expansion maximizes net benefit. The joint production model appears to fit multiple-use management, where the intention is to produce multiple outputs from a tract of land. The problem with applying this theoretical model is that it is not yet operationally feasible in a real-world planning situation. A major impediment is the lack of adequate continuous mathematical functions relating variable cost to the values of outputs that can be jointly produced the production surface. The formulation we present is a discrete version of Gregory's joint production model that builds on an approach suggested by Muhlenberg (1964). It is comprised of a finite number of points that approximate the continuous production surface of the theoretical model. These points are believed to be more operationally feasible to estimate than continuous mathematical production relationships. Yet, this discrete formulation provides the same type of analysis as the continuous model.

MODEL FORMULATION

We shall illustrate this discrete formulation of the theoretical joint production model by employing a simple but realistic example. The example pertains to a hypothetical 4,000-acre (1,619-ha) tract of forest land. This area is part of an important elk summer range and is currently overstocked with a homogeneous stand of low-quality but merchantable timber. The tree canopy is so dense that forest production is severely restricted and there is an excess of cover. The forest planning process has identified this area for a potential timber sale, the purpose of which is twofold: (1) to open up parts of the area to promote a better balance between cover/forage production and (2) to harvest timber to help meet the established cut goals for the forest.

The purpose of the model we present is to aid in identifying the type and scale of the timber sale project that most efficiently meets the two stated objectives. The scope of the problem is limited to project design. The forest planning horizon is 30 years—the length of time the cover/forage combination resulting from this management activity would be sustained. No additional harvests are scheduled for this area over the next 30 years. Finally, it is assumed that no other outputs from this area would be sufficiently affected as to warrant their inclusion in the model.

Before proceeding, we should make clear that the example we develop on the following pages is purely for illustrating the analytical approach. It would be inappropriate to reflect the management responses or subsequent results to other areas for several reasons. First, the results would be expected to be sensitive to existing conditions, such as the standing condition of this area, which could vary greatly. Second, appropriate output responses, costs, and unit values likely vary greatly as well.

The Alternatives

The five series of timber sale alternatives (A to E) presented in this section are represented on the production surface for this problem. Each series reflects a specific theme, differing in the amount of emphasis given to promoting effective wildlife habitat and timber. The model specifies that within a series, the alternatives employ common management practices and cutting unit sizes. Alternatives within a series differ only by the amount of harvesting that would be conducted, which is directly related to costs. The purpose of these alternatives is to indicate the best sequence of harvests, which would be conducted, is directly related to costs. The purpose of these alternatives is to indicate the best sequence of harvests, which would be conducted, is directly related to costs. Note that the first alternative in each series has a budget of $200,000 for the budget of the next year, and so on. A "no action" alternative (0) is also considered, it is the point against which output quantities and costs for the other alternatives are measured.

Series A—These alternatives are designed to harvest timber at the lowest possible cost; thereby yielding the greatest net dollar return to the Federal treasury. These alternatives have relatively large cutting units (35 to 40 acres [12 to 16 ha] located primarily on the basis of cost efficiency in logging and road building. Environmental constraints are satisfied, but no additional activities are undertaken for habitat improvement.

Series B—These alternatives are the same as series A, except that the costs will be closed to motorized uses by the public following harvest.

Series C—The cutting units in these alternatives are distributed in such a way as to provide an even distribution of size. Alternatives as in series B, the roads will be closed to public traffic. These alternatives differ mainly in that they place emphasis on the efficient harvest of forage and browse production.

Series D—These alternatives are characterized by smaller cutting units (average about 20 acres [8 ha]) with wildlife considerations being the primary basis for location. Roads will be closed to public access, and road slash will be cleaned up to eliminate its effect as a barrier to wildlife movement. Logging slash will be broadcast burned.

Series E—These alternatives are designed to maximize wildlife benefits while still preserving timber. Roads are small or shaped to provide a good "edge effect." As in series D, roads will be closed, road slash will be cleaned up, and logging slash will be broadcast burned.

Outputs

Two outputs are included in the model: timber and summer range effectiveness. Both are measured in terms of marginal change from the "no action" alternative.

The quantity of timber is simply the volume that would be harvested under the alternatives (cubic column in table 1). Volume is assumed to be 8.5 M ft³ (80,000 ft³) per acre across the 4,000-acre (1,619-ha) area. Although a constant volume per acre is not a requirement for this model, it is convenient for this example.

Summer range habitat effectiveness is measured in terms of change in the number of animals the 4,000-acre (1,619-ha) area can be expected to support annually (column in table 1). In order to maintain as much simplicity as possible, it is assumed that the area is classified in terms of its productivity as an average annual over the planning horizon. Later, we shall discuss an approach for handling changing outputs in a graphical formulation. Changing output quantities over time does not present any particular difficulty in the mixed-integer programming approach.

Figure 1 provides a good basis for describing the process of estimating change in carrying capacity due to harvesting activities. Under the existing carrying capacity any given ratio of the area is assumed to be in forage production, and the remaining area is classified as an "edge point" against which output quantities and costs for the other alternatives are measured.

Second, the purpose of these alternatives is to utilize the holding capacity at the lowest possible cost; thereby yielding the greatest net dollar return to the Federal treasury. These alternatives have relatively large cutting units (35 to 40 acres [12 to 16 ha] located primarily on the basis of cost efficiency in logging and road building. Environmental constraints are satisfied, but no additional activities are undertaken for habitat improvement.

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<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Discounted total cost</th>
<th>Discounted agency cost</th>
<th>Discounted Purchaser cost</th>
<th>Size of harvest</th>
<th>Timber harvest</th>
<th>Change in elk-carrying capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A1</td>
<td>500</td>
<td>100</td>
<td>150</td>
<td>250</td>
<td>300</td>
<td>50</td>
</tr>
<tr>
<td>A2</td>
<td>600</td>
<td>120</td>
<td>180</td>
<td>300</td>
<td>360</td>
<td>60</td>
</tr>
<tr>
<td>A3</td>
<td>700</td>
<td>140</td>
<td>210</td>
<td>400</td>
<td>480</td>
<td>80</td>
</tr>
<tr>
<td>A4</td>
<td>800</td>
<td>160</td>
<td>240</td>
<td>500</td>
<td>600</td>
<td>100</td>
</tr>
</tbody>
</table>

The responses in carrying capacity presented in figure 1 were based on the relationships presented in figure 2: habitat effectiveness as a function of the percent of land in forest production, figure 3: habitat effectiveness as a function of miles of road per section, and other information presented in a recent annual report on the Montana Cooperative Elk-Logging Study (Lyon and others 1982). These relationships were selected from many alternatives as evaluated in the study mentioned. A different selection of curves would produce somewhat different results.

In applying these relationships, the potential carrying capacity under ideal conditions (40 percent of area in forest production, 40 percent in cover, and no road effects) is estimated at 160 animals per year, which is fairly high but not unrealistic. The road effects shown in figure 2 were assumed to hold only when roads are left open to motorized use by the public. Roads closed to public vehicular traffic are thought to have no effect on habitat quality once harvesting activities are completed.

One final point should be made regarding the predicted output responses. The responses in carrying capacity illustrated in figure 1 exhibit decreasing marginal physical product. Along any given series of alternatives (with the exception of series A1, as the size of harvest increases, carrying capacity increases but at a decreasing rate that is, in the slope is decreasing as scale of harvest gets larger). Slope stays positive out to a point (the maximum carrying capacity possible within each series) after which the carrying capacity decreases as size of harvest is further increased. The presence of decreasing marginal physical product is critical, for without an optimal size of cut would not exist—more would always appear better.

Values

Timber is valued as full-delivered logs at $4.10 per M bd ft. An explanation of the rationale for this basis is opposed to valuing timber as standing trees may be useful. Land managers can then accomplish management objectives by the way roads and timber sales are...
Because the change in elk-carrying capacity was based on the value of the recreational experience of elk hunting. This implicitly assumes that the change in carrying capacity presented in table 1 (last column) correctly measures the change in the number of animals that would be carried by the area. First, the value of an elk living 1 year, V, was estimated as follows:

\[ V = \beta \cdot RVD \times [RVD \cdot E] \]

where

- \( V \) = \$741.78, the RWA willingness to pay for a recreation-visitor day (RVD) of elk hunting experienced in 1982 dollars

- RVD = the number of animal elk hunting RVD's supported by an elk each year, estimated to be seven. Given these numbers, V rounded to the nearest 10 equals $220.

The present value of the change in elk-carrying capacity over the next 30 years for the \( j \)th alternative, \( V^{(j)} \), can be expressed in general terms as:

\[ V^{(j)} = \sum_{i=1}^{30} \frac{Q_i}{(1+r)^i} \]

where

- \( V \) = the value of an elk in year 1, expressed in constant dollars
- \( Q \) = the change in carrying capacity in year 1 for the \( j \)th alternative (last column in table 1)
- \( r \) = the discount rate in real dollars

This generalized form can be handled in the mathematical programming formulation, but must be simplified for the more restrictive graphic formulation. Let us assume no real price increase for V. Since \( Q \), is constant over time in table 1 (change in carrying capacity is constant over 30 years within each alternative), \( V^{(j)} \) can be written as:

\[ V^{(j)} = \sum_{i=1}^{30} \frac{Q_i}{(1+r)^i} \]

or

\[ V^{(j)} = \sum_{i=1}^{30} \frac{Q_i}{(1+r)^i} \]

Because V is constant across the j alternatives, it is convenient for the graphic formulation to set:

\[ V = \sum_{i=1}^{30} \frac{Q_i}{(1+r)^i} \]

Using a discount rate of 4 percent (in real dollar terms) and the previously calculated value of $220 for V, \( V^{(j)} \) equals $3,800 when rounded to the nearest hundred dollars. The present value of the change in carrying capacity, \( V^{(j)} \), can then be expressed in the familiar terms of price times quantity:

\[ V^{(j)} = 3,800 \times Q \]

Costs

Total cost for the alternatives in the second column of table 1 is in terms of change relative to no action. It has two major components. The first, Forest Service cost (third column), includes the sale-related costs that are paid with appropriated funds: sale preparation, sale administration, agency overhead, and road closure costs. The second cost component, purchaser-related costs (fourth column), include stump-to-truck, haul, broadcost burning, and road construction and reconstruction. They represent the costs that must be covered by the value of the timber when valued as delivered logs for the sale to be financially viable. Given the objective of increased forage production for improved elk habitat, activities for regenerating the timber will not be undertaken. Thus regeneration costs were not included.

GRAPHIC APPROACH

The graphic formulation presented in figure 4 follows the logic of the continuous theoretical model. The first step in developing this formulation is to construct the iso-cost curves, which identify combinations of outputs that can be produced for given levels of cost. This is simply a matter of plotting the combinations of outputs predicted for each alternative presented in table 1. The iso-cost curve labeled 200 includes the alternatives with a total cost of $200,000, the curve labeled 500, the $400,000 alternatives, and so on. The order of the series (A-E) is illustrated on the curve labeled 600, and is the same on each iso-cost line. Technically, each iso-cost curve consists only of the points representing the alternatives, because linear combinations of projects has no logical interpretation. The points are connected here merely for convenience in identifying alternatives with common cost.

Next, benefits are entered in the form of iso-benefit lines, which arise from the simple price times quantity relationship. An iso-benefit line identifies combinations of outputs that have common total present value of benefits. To illustrate, an increase in carrying capacity of 35 animals (point W) would have a present value benefit of $133,000 (35 times the $3,800 discounted unit price identified earlier. Given the price of $160 per M bd ft, the same amount of benefit would be created by harvesting 950 M bd ft of timber (point T). Each combination of outputs lying on the curve connecting points W and T has a total present value benefit of $133,000. An infinite number of iso-benefit curves could be drawn, each corresponding to a different level of total benefit. Nevertheless, location of one iso-benefit line establishes the entire family, because each has the same slope (slope equals the negative ratio of the output prices, with the price of the output on the ordinate as the denominator).

The next step is to identify which of the points along the expansion path maximizes present net value (PVN). This is most easily done by calculating PVN for each alternative on the expansion path, as illustrated in table 2. Alternative B12 is indicated as the best of the alternatives, having a PVN of $55,400. It would harvest about a thousand acres of timberland by means of 30 to 40-acre 112- to 16-acre cutting units. About 8.5 million board feet of timber would be harvested, and habitat carrying capacity would be increased by an average annual amount of 17.6 elk over the 30 years following harvest.
Intertemporal Analysis

The timber sale example contained only one intertemporal output—the carrying capacity. It was handled by assuming output quantity is constant over time, and by expressing unit value as the present value of the constant annual quantity over 30 years. In reality, multiple-use projects can be comprised of many intertemporal costs and outputs, all of which could vary in magnitude over time. Expressing output as an annual average tax in the timber sale example may not always be acceptable. Here we discuss several approaches for handling such intertemporal problems graphically. It is suggested that readers who lack a specific interest in techniques for integrating intertemporal analysis into the graphic approach skip directly to the next subsection. Discussion of Graphic Approach

Formulating a graphic model in intertemporal terms requires expressing iso-cost and iso-benefit relationships so that the benefits and costs of the alternatives are compared at a common point in time. Following custom, we express these relationships in present-value terms.

Expressing iso-cost curves in present-value terms is straightforward. Simply discount the costs of all the resources used in a project to the present. Handling intertemporal output is somewhat more difficult. Both output quantities and unit values can be changing over time: including these changes in graphical analyses is difficult for two reasons. First, a graphic approach requires that each output for an alternative be expressed as a single number. This number represents one dimension on the iso-cost curves. Carrying capacity was expressed on an average annual basis. Second, unit values must be expressed such that when multiplied by the single output response number, the product is in terms of discounted dollars. There are several ways outputs and unit values can be expressed to handle this. Either output or unit value is constant over time. To explain, let us first rewrite equation 1 and introduct the present value of elk carrying capacity in intertemporal terms.

\[ t \] 

\[ \text{Present value of output quantity: } Q_t = \frac{Q \cdot (1 + r)^t}{(1 + r)^t} \] 

\[ \text{Present value of unit value: } P_t = \frac{P}{(1 + r)^t} \] 

\[ \text{Present value of output: } Q_t P_t = \frac{Q \cdot P}{(1 + r)^t} \] 

This differs from equation 1 in that \( P_t \) is allowed to vary here. Output is expressed in the iso-cost curves as a constant annual quantity over occurring year. The reader should note that none of these approaches allows both unit value and output to vary over time. In fact, it does not appear possible to allow for this occurrence using the graphical approach. The order of multiplications and summation indicated in equation 3 must be maintained in the iso-cost curves. The first approach represented by that variable was to be accomplished: variables that represent project variables indicate those projects not selected. If readers intend to take any further discussion of MIP are referred to Hillier and Liebman [1974] or Plane and McMillan [1971].

The MIP formulation proposed: 

\[ \text{Maximize: } PV = \sum_{t=1}^{T} T C_t X_t + \sum_{t=1}^{T} DP_t V_t \] 

\[ \text{Subject to: } \] 

\[ \sum_{t=1}^{T} X_t \leq 1 \] 

\[ \sum_{t=1}^{T} X_t \geq 1 \] 

\[ \sum_{t=1}^{T} X_t \leq 0 \] 

\[ \sum_{t=1}^{T} X_t = 0 \] 

\[ \sum_{t=1}^{T} X_t = 1 \] 

\[ \sum_{t=1}^{T} X_t = 1 \] 

\[ \sum_{t=1}^{T} X_t = 0 \] 

\[ \sum_{t=1}^{T} X_t = 0 \]
the project in solution (X) to the variables measuring output (V). There is one of these rows for each constraint (or time period) i.e., for each Xj. The Yij coefficients in these rows measure the positive quantity of the j-th output produced by project X, in time period t. Equation 14 represents the set of rows that “transfer” negative output quantities from the project in solution to the variables measuring negative output (W). The Yij coefficients in these rows measure the negative quantity of the j-th output produced by project X, in time period t. Therefore, such a row need be written for each Wj present, which as explained earlier should only be a few in most applications.

WHY THIS FORMULATION?

Thoughtful readers may be wondering at this point, why output variables are not simply included in the objective function coefficients for the project variables. This would allow one to express the objective function directly in terms of output variables, thus eliminating the need for equations that state constraints on output variables.

This is done for a very good reason. The reason is that output variables are not actually available in a real situation. The output variables are functions of activities, which are not actually available in the real situation. Only the activity variables (Vij and Wij) are available. So, in order to solve the problem, we must first express the output variables in terms of activity variables.

To illustrate, assume output variable A has been included in the objective function coefficient for the project variable for project A and period t. This would allow one to express the objective function directly in terms of output A. However, this is not possible, because the output A is not actually available in the real situation. The output A is a function of activity A (V1t) and activity B (V2t), which are not available in the real situation. Therefore, the output A cannot be included in the objective function.

The reason for this formulation is that it allows one to solve the problem in terms of the activity variables, which are actually available in the real situation. By expressing the output variables in terms of activity variables, we can solve the problem in terms of activity variables, and then use the solutions to the activity variables to find the solutions to the output variables.

Solving The MIP Formulation

There are several options for solving the formulation presented by equations 11-15. One option would be to use new algorithms, or solve MIP problems, such as the branch and bound technique. This technique is based on the idea that the solution to the problem can be found by dividing the search space into smaller and smaller parts, and then solving the problem in each part. This technique is very effective, but it can be very time-consuming.

Another option is to use existing algorithms, such as those contained in the LINDO software. LINDO is a very powerful and efficient software package, and it can be used to solve very large and complex MIP problems.

The disadvantage of the continuous programming approach is that it may not yield integer solutions if additional function coefficients are added to the constraints. However, this is not a serious problem, because the solutions are approximations, and they are usually very good.

Sensitivity Analyses

Output responses, costs, and unit values included in such a model are predicted future outcomes, and thus are not known with certainty. Sensitivity analyses can aid the analyst in dealing with uncertainty. It can help determine the range of predicted outcomes over which an alternative identified as optimal remains optimal. Sensitively analyses can also be used to identify what other alternatives are preferred when predicted outcomes are outside the limits for which a given alternative is optimal.

Unfortunately, most of the postoptimization techniques used in linear programming for sensitivity analyses are not applicable to integer programming. One reason is that integer programming is a non-linear problem, and the techniques used for linear programming are not appropriate. Another reason is that integer programming is a non-convex problem, and the techniques used for linear programming are not appropriate.

In summary, sensitivity analyses can be used to determine the range of predicted outcomes over which a given alternative identified as optimal remains optimal. Sensitivity analyses can also be used to identify what other alternatives are preferred when predicted outcomes are outside the limits for which a given alternative is optimal. Sensitivity analyses can also be used to identify what other alternatives are preferred when predicted outcomes are outside the limits for which a given alternative is optimal.

Table 3—Formation of the timber sale example as an MIP problem

<table>
<thead>
<tr>
<th>Row name</th>
<th>A2</th>
<th>A4</th>
<th>E18</th>
<th>TIMB</th>
<th>WILD</th>
<th>NWILD</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVN</td>
<td>-200.0</td>
<td>-400.0</td>
<td>0.0</td>
<td>-1,800.0</td>
<td>0.140</td>
<td>3.8</td>
<td>-3.8</td>
</tr>
<tr>
<td>EON 12</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>TVOL</td>
<td>1.470</td>
<td>2,333</td>
<td>0.0</td>
<td>9,504.0</td>
<td>1.0</td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>WVOL</td>
<td>42.0</td>
<td></td>
<td></td>
<td>1.0</td>
<td></td>
<td></td>
<td>-1.0</td>
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<tr>
<td>NWIVOL</td>
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<td>17.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2.0</td>
</tr>
</tbody>
</table>

*Variables A2 through E18 are treated as 0-1 integer variables.*
Table 4. The unit values over which project B12 remains optimal

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Lower limit</th>
<th>Highest value</th>
<th>Project selected if unit value is below the identified lowest value</th>
<th>Project selected if unit value is above the identified highest value</th>
</tr>
</thead>
<tbody>
<tr>
<td>WILD</td>
<td>27,806.00</td>
<td>0.10</td>
<td>C8</td>
<td>C14</td>
</tr>
<tr>
<td>TIMB</td>
<td>139.56</td>
<td>143.05</td>
<td>B10</td>
<td>B12</td>
</tr>
</tbody>
</table>

UNIT VALUES

Several types of sensitivity analyses for unit values are potentially useful. The choice depends on the question being asked. The effect of some specific change in unit value on a previously optimal solution is best determined by making that change in the formulation and solving. This is accomplished by changing the objective function coefficient or the output variables associated with the change in unit value. This can be done easily with a text editor because only a few numbers would change. The model is then resolved using standard procedures. No knowledge of the more sophisticated postoptimization procedures is needed.

Analysts may also be interested in determining the range in unit values over which a particular solution remains optimal. This could be calculated by systematically changing unit values and resolving, but this process would likely require a large number of solutions. An easier approach would be to use a postoptimization technique available in most linear programming packages which can perform this directly. To illustrate, the EXCHANGE procedure in FMPS was used to calculate the range in unit values over which the figure 5 solution remains optimal. The results are summarized in Table 4. The lowest and highest unit values for WILD are, respectively, 82,770 and 85,220. As long as the unit value for WILD is within this range, project B12 is preferred, assuming other parameters constant.

In addition to the range in unit values, linear programming ranges can be expected to identify what project would be preferred if the unit value falls below or rises above the indicated range. (See, for example, the two columns of Table 4. For example, if the unit value for WILD were to fall below 82,770, then project B10 would be preferred. This does not imply that B10 is preferred for all unit values less than 82,770, but rather for some range, whose lower unit is unspecified and whose upper limit is 82,770.

If the question to be asked is how does the preferred project change over a wide range in unit values, then parametric programming can be used to good advantage. Parametric programming involves reformulating the objective function from:

\[ Z = \sum c_j x_j \]

(14)

a general expression for equation 11, to:

\[ Z = \sum (c_j + \theta x_j) \]

(15)

Here, \( \theta \) represents constant changes to be applied to the objective function coefficients \( c_j \). The symbol \( \theta \) represents a scalar that, when multiplied times the \( x_j \) values, results in proportional change in the objective function coefficients. In the parametric programming procedure, \( \theta \) is incremented upward, starting at zero (where equations 16 and 17 are equivalent) to some user-specified upper limit. In this process, the values for \( \theta \), where the optimal solution changes, are identified.

To illustrate the use of parametric programming, assume we desire to investigate how the preferred alternatives change over the range of timber prices from $120 per M bd ft to $200 per M bd ft. All else remaining equal. The changes that would be made to the matrix presented in Table 3 are as follows: First, change the objective function coefficient for TIMB from 0.10 to 0.12 (8120 expressed in thousands). Next, a new corresponding to \( \theta \) in equation 17 must be added to the matrix. Because the objective function coefficient for TIMB is the only coefficient to be changed in this analysis, the only nonzero coefficient in this new \( \theta \) row would be the coefficient for TIMB. Set this coefficient equal to 0.12. The scalar \( \theta \) then measures the percentage of change in the original form from the starting price of $120 per M bd ft.

The results from this parametric programming analysis are summarized in Table 5. Project C2 is optimal over the range in timber prices from $120 to $129.33 per M bd ft. As timber price was increased from $129.33 per M bd ft, the optimal solution moves out series B of project alternatives. The selection of the scale of project within series B appears to be sensitive to timber price. However, the type of harvesting in series B is clearly preferred over the approach in the other series of alternatives over the range in timber prices.

Table 5. Preferred alternatives and the range in timber prices over which they are optimal

<table>
<thead>
<tr>
<th>Project alternative</th>
<th>Range in timber price over which project is optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project</td>
<td>Dollars per M bd ft</td>
</tr>
<tr>
<td>C2</td>
<td>120.00 - 129.33</td>
</tr>
<tr>
<td>B4</td>
<td>129.33 - 130.71</td>
</tr>
<tr>
<td>B6</td>
<td>130.71 - 134.74</td>
</tr>
<tr>
<td>B8</td>
<td>134.74 - 137.15</td>
</tr>
<tr>
<td>B10</td>
<td>137.15 - 139.56</td>
</tr>
<tr>
<td>B12</td>
<td>139.56 - 143.06</td>
</tr>
<tr>
<td>B14</td>
<td>143.06 - 145.73</td>
</tr>
<tr>
<td>B16</td>
<td>145.73 - 147.07</td>
</tr>
<tr>
<td>B18</td>
<td>147.07 - 200.00</td>
</tr>
</tbody>
</table>

All other parameters held constant at the levels in Table 1.
 Outputs

In the model formulation depicted by equations 11-15, it is typical to find at least at some level by most, if not all, projects. It would seem that the question most frequently asked regarding output would be, 'Is there a systematic underestimating or overestimating outputs across the project superstructures?' For the present, it can be assumed that, for each such a systematic change can be expressed as a percentage of change from the previously predicted outputs, investi- gating this is relatively easy. The suggested approach would be to modify the coefficient for the output variables in the output equations (13 and 14) and resolve the model. This process is best explained via an example. Assume we desire to determine if a 10 percent increase in ek- car-rying capacity over that already predicted would affect which project is chosen. This 10 percent increase would be approximately the coefficient for WILD in row WVOlv stable 3 from 1.0 to 0.9. This 10 percent increase would require a 10 per- cent larger quantity allocated to WILD to maintain the equality of row WVOlv. The model would then be re- solved to determine the effect of the change. In this instance, the 10 percent increase in ek- carrying capacity had no effect on the project chosen (B12). The only effect was the value of the objective function increased to 862,900.

Cost

Change in virtually any underlying cost examples, labor costs or equipment costs would change the objective function coefficient for each project alternative. Therefore, for reasons discussed earlier, shadow prices provide little information regarding how cost changes might affect an optimal solution. The effect of potential changes in costs is best analyzed using parametric programming procedures. The general formulation for parametric programming described by equation 17 also applies here. The only difference is that here the ur row to be added to the model presented in the table was the pur- chaser costs presented in the fourth column of table 1. The signs of these parameters specifies the por- tion of the analysis dealing with cost increases and positive- for the assumption of a decrease. If such a sys- tematic change can be expressed as a percentage of change from the previously predicted outputs, investi- gating this is relatively easy. The suggested approach would be to modify the coefficient for the output variables in the output equations (13 and 14) and resolve the model. This process is best explained via an example. Assume we desire to determine if a 10 percent increase in ek- car-rying capacity over that already predicted would affect which project is chosen. This 10 percent increase would be approximately the coefficient for WILD in row WVOlv stable 3 from 1.0 to 0.9. This 10 percent increase would require a 10 per- cent larger quantity allocated to WILD to maintain the equality of row WVOlv. The model would then be re- solved to determine the effect of the change. In this instance, the 10 percent increase in ek- carrying capacity had no effect on the project chosen (B12). The only effect was the value of the objective function increased to 862,900.

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A second attractive feature of this MIP formulation is the small size and simplicity—at least when compared to other mathematical programming formulations used in forestry. It is easy to solve and sufficiently small to be processed on a small computer. Given the front-end software described above, we believe there is little question that the MIP approach for solving discrete joint production models would be operationally viable. It should be no more difficult to use than simulation programs, which are commonly used by resource managers with little or no training in operations research.

Summing It Up

As we have discussed, the discrete joint production model provides the same type of analysis as the continuous joint production model of economic theory. It provides the capability to analyze the economic efficiency of multiple-use management. Both in terms of type of project and scale of project (for example, in the timber sale example both the type of cutting alternatives and amount of harvesting were included in the analysis).

The graphic approach to solving these discrete models has the advantage of requiring little in the way of equipment—only paper, pencil, and a straightedge. Little or no start-up time is involved—no need to write computer software or to learn how to use existing software. In addition, it rather clearly depicts the nature of economic efficiency in multiple-use production. The graphic approach, however, has some real limitations enumerated earlier (limited to two outputs and difficulty in conducting intertemporal analysis). Because of these, the graphic approach will likely be limited to special applications.

The MIP approach provides some important advantages over the graphic approach. It lends itself well to automation. With appropriate software, users relatively inexperienced in computer modeling could conceivably build and solve such a model very efficiently. Next, the mathematical programming formulation provides some very useful sensitivity analysis capability. Finally, the MIP approach is not limited to two outputs and can handle intertemporal analysis more easily.

The discrete joint production model provides a somewhat different type of analysis than what resource allocation mathematical programming formulations common in forestry generally provide. In "ordinary" linear programming formulations, output is a linear function of acres treated, for each decision variable. Questions regarding scale of activities can be addressed only rather crudely by varying the level at which constraints are imposed. The discrete joint production model, on the other hand, can evaluate both output and cost relationships, making it a more effective approach by analyzing questions of scale. This can be important, particularly when wildlife and recreation outputs are among the joint products.

A second difference is that the spatial arrangement of activities can be identified more precisely in the discrete joint production model. This is advantageous when location of an activity affects cost or outputs.

Third, the discrete joint production model requires that the user consider fewer alternatives than what can be considered in "ordinary" linear programming formulations. In some respects, the model we have presented has characteristics of both simulation and optimization. Like simulation, it requires the user to formulate whole alternatives. But it does provide some of the optimization and sensitivity analysis capabilities of mathematical programming. Because of the limited number of alternatives that can be handled effectively, the joint production model is best suited to problems of a relatively small geographic scope.

In conclusion, we believe the modeling approach presented in this paper is a practical and useful tool for conducting multiple-use efficiency analysis. The potential lies not as a substitute for current forest planning methods, but rather as a tool to aid in identifying efficient management prescriptions to place in forest planning models, and as a means of analyzing projects for implementation. It would be most effective when spatial arrangement of activities is important, and when outputs or costs are nonlinear with respect to acres treated.

REFERENCES


A discrete version of the continuous joint production model in economic theory is presented for use in designing multiple-use projects and identifying efficient management prescriptions for forest planning. Data requirements are less demanding than the continuous theoretical model, yet some of the more important features are maintained. Models can be formulated graphically or as mixed-integer programming problems that are easily solved via computerized routines.

KEYWORDS: economic analysis, multiple-use, decision making, mixed-integer programming

The Intermountain Station, headquartered in Ogden, Utah, is one of eight regional experiment stations charged with providing scientific knowledge to help resource managers meet human needs and protect forest and range ecosystems.

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