Comparison of Control Moment Gyros and Reaction Wheels for Small Earth-Observing Satellites

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ABSTRACT
Multi-ton Earth-observing spacecraft have traditionally used control moment gyros (CMGs) to store momentum and to generate the large torques required for fast slew maneuvers. Small 3-axis controlled satellites, by contrast, will typically use cheaper and simpler reaction wheels to perform the same functions. The question then arises: which actuator is better suited for an Earth-observing mission? This paper compares the performance of each actuator subject to identical agile pointing requirements, and identifies the operating conditions in which one actuator outperforms the other.

INTRODUCTION
On almost every fine-pointing Earth-observing spacecraft, you will find either reaction wheels or CMGs. The criteria for selecting one actuator over the other may seem alien to even the majority of spacecraft engineers, mostly due to the limited CMGs on the market. The main purpose of this paper is to articulate the tradeoffs between reaction wheels and CMGs, and offer a simple decision tool that aids in identifying the correct actuator for a particular mission.

This paper begins with a mathematical description of reaction wheels and CMGs, and the mechanisms by which they produce torque. The next section addresses the existing reaction wheels and CMGs on the market and how the specifications compare for each actuator type. A case study of an agile spacecraft with stringent pointing requirements is then presented. Using simple power models and dimensional analysis, the feasibility of each actuator at different operating conditions (i.e. spacecraft size and slew angles) is shown. The results from the analysis allows mission designers to intelligently select an actuator that is better suited to their particular mission. The paper concludes with an outlook on future agile spacecraft missions, and addresses the need for actuator manufacturers to build hardware that will support the needs of Earth-observing missions over the next decade.

DYNAMIC MODELS
Although both reaction wheels and CMGs are actuators that produce torque and exchange momentum, the means by which they do so is different. The following subsections briefly describe the physical principles involved with each actuator.

Reaction Wheel Torque Equation
A reaction wheel is a brushless motor attached to a high-inertia flywheel which is free to spin along a fixed spacecraft axis (see Figure 1). It operates by producing a torque $T$ on the flywheel, causing its an-
gular momentum to increase. An equal and opposite torque, $T_{sc}$, is applied to the spacecraft:

$$T = \dot{h} = -T_{sc},$$

(1)

where $h$ is the angular momentum of the flywheel. The main specifications for a reaction wheel are its maximum torque and momentum capacity.

**CMG Torque Equation**

Like a reaction wheel, a CMG has a spinning flywheel controlled by a brushless motor. Unlike a reaction wheel, the spin axis of a CMG can rotate with the help of a second motor placed on a gimbal axis. Although different types of CMGs exist, this paper will focus on *single gimbal control moment gyro* (SGCMG), where the angular momentum can only rotate in a fixed plane (see Figure 2). As the gimbal rotates with angular velocity $\dot{\delta}$, so does the angular momentum vector. This change in angular momentum gives rise to a torque:

$$T = \dot{\delta} \times h = -T_{sc}. $$

(2)

Note that the direction of torque is always perpendicular to both the gimbal axis and the angular momentum axis, rather than along a fixed axis like that of a reaction wheel. Therefore, before commanding gimbal angular rates to achieve a desired torque, the gimbal angles of each CMG must be known.\(^7\)

With a cluster of three CMGs, there are particular sets of gimbal angles such that a pair of CMGs instantaneously share the same torque axis. In this degenerate case, referred to as a *singularity*, the cluster of actuators can only produce a net torque in a plane, rather than an arbitrary three axis torque.\(^1\) For this reason, controlling the attitude of a spacecraft with CMGs is algorithmically and computationally more complex, often making them less attractive on simple spacecraft missions.

The main specifications for a CMG are its maximum torque, momentum capacity, maximum gimbal rate and gimbal acceleration.

**DIMENSIONAL ANALYSIS**

Before considering any specific numbers we will examine the basic physics behind scaling a satellite. If all components of a multi-ton satellite are scaled down by several orders of magnitude, will agility performance worsen or improve? Likewise, how is agility affected by scaling up a Cubesat design?

Let us consider $d$ as a characteristic linear dimension of a satellite. As we make the satellite larger or smaller, $d$ will vary. The mass of the satellite will vary as $d^3$ assuming its density is constant. The moment of inertia of the satellite will vary as $d^5$ (because $I \propto md^2$). Since a spacecraft maximum slew rate scales inversely with moment of inertia and linearly with momentum, the momentum capacity of the actuators must vary as $d^5$ (i.e. $h \propto d^5$). Likewise, the torque of the actuators must vary as $d^5$ to achieve a specified acceleration.

Most satellites are powered by sunlight. As a satellite is made larger or smaller, the collection area of its solar panels will vary as $d^2$. The power consumption of a reaction wheel under torque can be approximated by the rotor's angular rate multiplied by the torque. The maximum angular rate of the flywheel will tend to be constant regardless of size and so the power consumption varies as the torque, which is $d^5$.

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Since generated power scales as $d^2$, and yet power consumption of reaction wheels grows as $d^5$, a satellite with reaction wheels cannot simply be scaled up in size.

CMG power is harder to model from first principles. From a strict physical standpoint they do little mechanical work and so they should not require much power. In practice their power consumption is dominated by implementation dependent losses. An empirical survey will show that to achieve a given slew rate and acceleration, the power required for CMGs varies as $d^2$. This is a much more suitable match to the $d^2$ power generation dependency.

All else being equal, there will be a spacecraft size at which a particular design of satellite must use CMGs since the power required for reaction wheels is infeasible. In the following section, a study will be performed to identify the spacecraft size at which this occurs.
MARKET STUDY

For small commercial satellites, many more options exist for reaction wheels than CMGs. The authors easily identified over 20 available reaction wheels for purchase, but only 6 available CMGs. With each unit identified, the following specifications were recorded: momentum capacity, maximum torque, mass, volume, and power. These data points allow for comparison between the actuator types, and identifies efficiencies and inefficiencies of each.

Since a spacecraft actuator is typically selected based on its torque output and momentum storage, it is insightful to identify how reaction wheels and CMGs compare in regard to these two metrics. Figure 3 shows a plot of torque output against stored momentum. It is immediately clear from the figure that reaction wheels and CMGs occupy different regions of the torque-momentum space. For any given momentum capacity, CMGs are capable of producing significantly more torque (often orders of magnitude more). Note from the figure that the different actuators can be fully segregated with a simple boundary.

The next criteria to investigate is power consumption, since it is not clear from Figure 3 whether CMGs achieve their larger torque from a larger power supply. Figure 4 shows the torque-to-power ratio against momentum. A larger value for the ratio indicates a higher torque efficiency. The results clearly indicate that CMGs can produce significantly more torque than reaction wheels, at only a fraction of the power. Note that the power efficiency of a reaction wheel remains effectively constant across the entire range of unit sizes, while CMGs tend to get more efficient as the units get larger. Once again, it is possible to separate the actuator types into disjoint regions of the torque efficiency-momentum space. This result lends support to the generally held notion that CMGs are a more efficient actuator than reaction wheels for agile spacecraft.\(^2,4,5\)

However, in the event that a spacecraft mission requires large-angle slews and not high acceleration re-tasking, it could then be argued that momentum capacity is more valuable than torque. Surely, CMGs would offer no improvement to reaction wheels, since both actuators have the same mechanism for storing angular momentum. In fact, the additional CMG gimbal motor and hardware would add extra mass and volume, making the CMG less mechanically efficient for the maneuvers. Figure 5 shows a plot of the angular momentum efficiency with respect to volume and mass. Not surprisingly, reaction wheels outperform CMGs in this scenario. This result may seem to indicate why CMGs are typically not found on small spacecraft, which are typically much more mass and volume constrained.

**Actuator Power Models**

Before presenting the case study, it is useful to establish approximate power models for both reaction wheels and CMGs. These power models will be used to compare the efficiencies of reaction wheel-based spacecraft versus CMG-based spacecraft.

The following reaction wheel power model was determined from fitting the existing data from over 20 wheels:

\[
P_{rw} = 1000T + 4.51h^{0.47},
\]
where $T$ is the output torque in Nm, $h$ is the instantaneous angular momentum in Nm-sec, and $P_{RW}$ is the power consumption in Watts. Although the equation is oversimplified, it is a representative model for reaction wheel power over the entire range of wheels studied. Only the absolute value of torque is considered here, and regeneration is discounted.

Assume the satellite can be approximated by a uniform density cube with dimension $d$ (see Figure 7). The moment of inertia of the satellite along any principal axis is given by

$$I = \frac{1}{6} \rho d^5,$$

where $I$ is the moment of inertia, and $\rho$ is the mass density (approximated as 350 kg/m$^3$). Thus, given a satellite with moment of inertia matrix

$$I_{sc} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

it is straightforward to solve for the critical dimension of the satellite:

$$d = \left(\frac{6I}{\rho}\right)^{1/5}.$$ 

Once we have the dimensions of the spacecraft, the power available to the spacecraft in sunlight is approximated as

$$P_s = \eta P_0 d^2,$$  

**Solar Power Generation Model**

In the previous section, we modeled the power consumed by each actuator type. In this section, we will derive a simple model for solar power generation, parameterized by the satellite moment of inertia.

**Figure 5. Commercial Reaction Wheel and CMG Momentum/Volume Efficiency (Top) and Momentum/Mass Efficiency (Bottom)**

**Figure 6. Power Contour Plot for Reaction Wheels (top) and CMGs (bottom). All Contours in Watts.**
where $\eta$ is the efficiency of the solar arrays, $P_0$ is the energy density of sunlight in a low-Earth orbit, and $d^2$ is the visible cross-sectional area of the solar arrays. The values used for $\eta$ and $P_0$ are given in Table 1.

Table 1. Solar Power Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar Array Efficiency</td>
<td>$\eta$</td>
<td>0.25</td>
</tr>
<tr>
<td>Solar Energy Density</td>
<td>$P_0$</td>
<td>1360 W/m$^2$</td>
</tr>
</tbody>
</table>

In the case study presented in the next section, it is assumed that 25% of the generated power from solar arrays is allocated to the actuators. The remainder of the power is reserved for the payload, radios, attitude control sensors, and avionics hardware.

**CASE STUDY**

To understand the tradeoffs between reaction wheels and CMGs, a case study on efficiency is presented. The study centers around an Earth-observation mission with stringent tasking requirements. Rather than analyze a single baseline spacecraft with a particular desired slew profile, a wide range of spacecraft sizes and slew angles is evaluated. The goal is to find conditions for which a particular actuator is better suited for the mission than its counterpart.

**Agility Requirements**

We begin the case study by stating the satellite agility requirements. Table 2 lists the slew rate and angular acceleration requirements for the mission. By meeting these stringent requirements, a LEO spacecraft can image several targets per minute on the surface of the Earth, making it competitive with the multi-ton imaging satellites.

Table 2. Satellite Agility Requirements

<table>
<thead>
<tr>
<th>Specification</th>
<th>Requirement</th>
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<tbody>
<tr>
<td>Max Slew Rate</td>
<td>3 deg/sec</td>
</tr>
<tr>
<td>Max Acceleration</td>
<td>1.5 deg/sec$^2$</td>
</tr>
</tbody>
</table>

Naturally, the slew rate requirement can be met by selecting an actuator with sufficient momentum capacity, while the acceleration requirement is met by selecting an actuator with ample torque output.

1For simplicity, deployed solar arrays are not considered. These increase the power collected by a satellite, but also increase its moment of inertia.

**Slew Definitions**

Figure 8 shows a generic slew profile for the spacecraft. Note that the spacecraft must accelerate until the maximum slew rate is achieved, and then decelerate until momentarily coming to rest on the target. This profile is sequentially repeated, alternating between off-nadir slew angles of $\pm \theta$ degrees while the satellite is in sunlight. Based on the requirements in Table 2, for sufficiently small slew maneuvers, the satellite might not reach its maximum slew rate.

**Case Study Results**

Using the slew profile definition in Figure 8 combined with the power models in Equations 3 and 4, the average power consumed by each actuator type was calculated. A contour plot of power versus moment of inertia and slew angle is shown in Figure 9.

Figure 9. Power Consumption vs/ Slew Angles for Small Spacecraft. All Contours in Watts.

Note that for very small moments of inertia (< 0.1 kgm$^2$) the spacecraft with reaction wheels requires much less power than CMGs. This is due to the fact that only a sufficiently small torque is necessary to achieve the acceleration requirement. Conversely, for small slew angles and larger spacecraft—where
the satellite must maintain a large average torque output—CMGs quickly become more power efficient compared to reaction wheels.

Figure 10 illustrates the regimes in which one actuator outperforms the other with respect to power. It is evident that once a spacecraft exceeds a moment of inertia of roughly 1 kgm$^2$, a CMG offers considerable power savings.

![Figure 10. Actuator Efficiency Regimes](image)

In fact, for particular spacecraft sizes and maneuvers, reaction wheels become an infeasible choice of actuator. By infeasible, we mean that there is not sufficient power generation from the solar arrays to supply the wheels with the necessary power to perform the slews. Figure 11 shows the feasibility regime of reaction wheels. Note that reaction wheels are better suited for small spacecraft missions performing large angle maneuvers, rather than large spacecraft missions with quick re-tasking requirements.

![Figure 11. Reaction Wheel Feasibility on an Agile Earth-Observation Spacecraft](image)

**MARKET EVOLUTION**

This study shows that based on the existing market offerings, Earth-observing satellites exceeding a mass of 30 kg should strongly consider using CMGs, and at roughly 100 kg there is little choice but to use CMGs. What is not clear is whether this threshold is based on fundamental physics alone, or upon the implementation details of those devices that are currently sold.

It is only in the last decade that high-performance attitude control systems on microsatellites have been contemplated. Prior to this, few satellite missions were demanding actuators with high torque.

There are fundamental physical reasons why reaction wheels cannot work for very large Earth-observing satellites. However, microsatellite wheels optimized for high torque may appear to push the feasibility envelope up to slightly larger spacecraft. Four-wheel regenerative cluster architectures may also be used to mitigate the peak power consumption problem.

We should also expect to see a wider variety of small CMG offerings as demand for agile microsatellites increases. As hardware becomes cheaper and more reliable, and as CMG steering laws become more computationally tractable, a large fraction of the small satellite community will turn to CMGs as the preferred actuator on microsatellite missions.

**CONCLUSION**

In the design phase of an agile, Earth-observation mission, a satellite team is presented with the option of flying reaction wheels or CMGs. Many reaction wheels exist on the market, making them a compelling choice over CMGs. Reaction wheels are also less mechanically complex and their operation is much simpler compared to CMGs, which suffer from singularities. However, after studying the efficiencies of each actuator type, it was shown that CMGs offer considerably more torque at a fraction of the power. The torque efficiency becomes a driving factor on large spacecraft with demanding acceleration requirements.

Over the next decade, there will be more demand for small, agile spacecraft that can collect more images, and hence offer a sizable return on investment. That demand will prompt some hardware manufacturers to develop efficient reaction wheels for high-torque applications. Many hardware manufacturers will also begin development on CMGs for this new breed of agile, mid-size spacecraft. In doing so, new opportunities will materialize for small satellites that have
heretofore been reserved for their larger counterparts.

References


3 Lappas, V., A Control Moment Gyro (CMG) Based Attitude Control System (ACS) for Agile Small Satellites, University of Surrey, 2002.


