January 1967


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Design and Calibration of Submerged Open Channel Flow Measurement Structures

Part 1

SUBMERGED FLOW

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Utah State University

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ABSTRACT

DESIGN AND CALIBRATION OF SUBMERGED OPEN CHANNEL FLOW MEASUREMENT STRUCTURES

PART 1, SUBMERGED FLOW

The parameters which describe submergence in flow measuring flumes are developed by a combination of dimensional analysis and empiricism. Further verification of the validity of the parameters is offered by the theoretical submerged flow equation developed from momentum relationships. The results have been verified for a flat-bottomed trapezoidal flume, rectangular flat-bottomed flume, and Parshall flume. The effects of certain boundary geometry conditions on both free (critical) flow and submerged (subcritical) flow have been investigated. Evaluations have been made regarding the most satisfactory locations for measuring flow depth, channel approach and exit conditions, roughness, entrance convergence, exit divergence, length, and scale effect.

ACKNOWLEDGMENTS

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Much of the fabrication of special test flumes was accomplished by Mr. Jed Merrill and Mr. Glen Longhurst of Technical Services, USU., Additional fabrication of special test facilities was under the direction of Mr. Kenneth Steele, Utah Water Research Laboratory, with Messrs. Gilbert Peterson, Mark Nilson, and Verl Bindrup assisting.

The cooperation and services of the Utah Water Research Laboratory were invaluable in the publication of this report. Sincere thanks are given Miss Donna Higgins for editing and Mrs. Joy Hill for typing this manuscript.

Gaylord V. Skogerboe
M. Leon Hyatt
Keith O. Eggleston
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## NOMENCLATURE

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<tr>
<td>A</td>
<td>cross-sectional area of flow</td>
</tr>
<tr>
<td>A₁</td>
<td>cross-sectional area at entrance of flume</td>
</tr>
<tr>
<td>A₂</td>
<td>cross-sectional area at throat of flume</td>
</tr>
<tr>
<td>Aₘ</td>
<td>minimum cross-sectional flow area in the throat</td>
</tr>
<tr>
<td>B</td>
<td>constriction ratio, ( b_2/b_1 )</td>
</tr>
<tr>
<td>b₁</td>
<td>bottom width at flume entrance</td>
</tr>
<tr>
<td>b₂</td>
<td>bottom width at flume throat</td>
</tr>
<tr>
<td>C</td>
<td>coefficient in the free flow equation</td>
</tr>
<tr>
<td>C₁</td>
<td>coefficient in the numerator of the submerged flow equation</td>
</tr>
<tr>
<td>C₂</td>
<td>coefficient in the denominator of the submerged flow equation</td>
</tr>
<tr>
<td>C₃</td>
<td>coefficient relating ( \phi_m(S) ) and ( f(S) )</td>
</tr>
<tr>
<td>C₄</td>
<td>coefficient of discharge</td>
</tr>
<tr>
<td>C₅</td>
<td>coefficient relating discharge, minimum flow depth in throat, and change in water surface elevation</td>
</tr>
<tr>
<td>F₁</td>
<td>hydrostatic force at section 1</td>
</tr>
<tr>
<td>F₂</td>
<td>hydrostatic force at section 2</td>
</tr>
<tr>
<td>Fₘ</td>
<td>maximum Froude number in the throat</td>
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<td>Fₛ</td>
<td>hydrostatic force on flume walls in entrance section</td>
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<tr>
<td>fₚ</td>
<td>function</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity</td>
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<td>Hₐ</td>
<td>flow depth in entrance section of Parshall flume</td>
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<tr>
<td>Hₙ</td>
<td>flow depth in throat section of Parshall flume</td>
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<tr>
<td>n₁</td>
<td>power of ( y_1 ) in the free flow equation</td>
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<tr>
<td>n₂</td>
<td>power of the submergence term in the denominator of the submerged flow equation</td>
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<tr>
<td>nₛₜ</td>
<td>value of ( n_s ) derived from relationship between ( \phi_m(S) ) and ( f(S) )</td>
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<tr>
<td>Q</td>
<td>flow rate or discharge</td>
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<td>Q₀</td>
<td>total discharge based on upstream depth of flow which has been increased due to submergence</td>
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<td>Qₜ</td>
<td>theoretical discharge</td>
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<tr>
<td>S</td>
<td>submergence; ratio of a downstream flow depth to an upstream flow depth, where the downstream measurement is the depth of flow above the flume floor at the point of upstream measurement</td>
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<tr>
<td>Sₛₜ</td>
<td>transition submergence</td>
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<td>V</td>
<td>average velocity</td>
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<tr>
<td>V₁</td>
<td>average velocity at section 1</td>
</tr>
<tr>
<td>V₂</td>
<td>average velocity at section 2</td>
</tr>
<tr>
<td>W</td>
<td>throat width of flume</td>
</tr>
<tr>
<td>y</td>
<td>flow depth</td>
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$y_m$ minimum depth of flow in flume throat, which varies in location longitudinally

\[ \Delta y \] change in water surface elevation between a section upstream and one downstream from section $m$

\[ \pi_1 \quad F_m \]

\[ \pi_2 \quad S \]

\[ \pi_3 \quad \Delta y / y_m \]

$\beta$ momentum correction coefficient

$\gamma$ specific weight of fluid

$\rho$ density of fluid
SUBMERGED FLOW INVESTIGATION

Flow Measuring Flumes

Considerable effort has been expended in developing flow measurement devices for open channels. The device most often used in water conveyance channels is a flume having a constriction, called the throat, wherein critical depth occurs. The primary advantage of a critical-depth flume is that the depth of flow upstream from the flume is a measure of the flow rate. The most widely used critical-depth flume is the Parshall flume developed by Ralph Parshall at Colorado State University. Other familiar flumes are of rectangular and trapezoidal shapes.

Most of the earlier investigations regarding measuring flumes have emphasized the development of free flow calibrations or ratings for various flume geometrics. Notable free flow investigations have been made by Cone (1917), Parshall (1926), Engel (1934), Khafagi (1942), Robinson and Chamberlain (1960), and Ackers and Harrison (1963), to mention a few. Various methods of analyzing submerged flow have been presented by Parshall (1950), Khafagi (1942), Villemonte and Gunaji (1953), Robinson and Chamberlain (1960), and Robinson (1965).

Submerged flow exists in a measuring flume when a change in flow depth downstream from the flume causes a change in flow depth upstream for any particular constant value of discharge. When a change in tailwater depth does not affect the upstream depth, free flow exists. Only a flow depth upstream from the contracted section (throat) of the flume needs be measured to evaluate the discharge under free flow conditions. Two flow depths must be measured to evaluate the discharge under submerged flow conditions. The two flow depths normally measured when submerged flow exists consist of the same upstream depth used for free flow and a depth measured in the throat, although this need not be the case as will be shown later.

Problem

Many of the natural and man-made water conveyance channels in the United States and many foreign countries have a very flat gradient as a result of the local topography or design restrictions. The use of critical-depth flumes in these channels has two distinct disadvantages: (1) the energy loss occurring through the flume will be as great as the energy loss occurring over a considerable length of channel (perhaps one to three miles) and (2) the floor of the flume must oftentimes be raised above the channel bed for critical depth to occur in the throat, thereby increasing the degree of silting and increasing seepage losses upstream from the flume, particularly for discharges less than the design discharge. The operation of flumes with subcritical (submerged) flow has the advantage of reducing the head loss when compared with critical (free) flow. Furthermore, the floor of the flume can be placed on the channel bed.

Because of the limitations of critical-depth flumes, current meter measurements are the only determination of discharge made for many flat gradient channels. The time and expense of these measurements usually discourages frequent discharge measurements.

The disadvantages of critical depth flumes led to the design of a submerged rectangular flat-bottomed flume for the Delta, Melville, Abra-
ham, and Deseret Irrigation Companies in central Utah by Skogerboe, Walker, and Robinson (1965). The development of the parameters describing submerged flow was accomplished by Hyatt (1965) while investigating the design and calibration of a trapezoidal flat-bottomed flume for the same irrigation companies. The calibrations, which were made in the laboratory by means of hydraulic model, clearly proved that the discharge passing through the flume could be evaluated with an accuracy comparable to most open channel flow measuring devices by measuring the depths of flow upstream and downstream from the flume. This degree of accuracy can be maintained for submergences wherein the downstream depth is nearly as great as the upstream depth. Thus, the energy loss through a submerged flume need only be a small percentage of the upstream depth. The energy loss through the commonly used critical-depth flumes is 20 to 40 percent of the upstream depth.

**Scope of Study**

Parameters describing submergence in flow measuring flumes will be developed from dimensional analysis. A combination of empiricism and dimensional analysis will be used to develop a submerged flow discharge equation. The resulting discharge equation will be compared with the theoretical submerged flow equation developed from momentum relationships. A rectangular flat-bottomed flow measuring flume has been used to generate data necessary for establishing the parameters describing submerged flow. The form of the discharge equation describing submerged flow in a rectangular flume has been verified for a trapezoidal flat-bottomed flume and a Parshall flume. The technique for presenting submerged flow calibration (rating) curves is compared with methods used by other investigators.

The effect of certain geometric variables on the free flow and submerged flow calibration plots will be investigated. The variables to be evaluated are: entrance length, throat length, exit length, throat width, entrance convergence, exit divergence, scale factor, channel approach and exit conditions, roughness, and effect of location of flow depth measurement.

**MOMENTUM THEORY**

A theoretical submerged flow discharge equation will be developed for the flat-bottomed rectangular flume shown in Fig. 1. The momentum equation can be written between sections 1 and 2 for the control volume in Fig. 2 to arrive at a general submerged flow equation for rectangular flumes. The momentum equation can be written in the direction of flow as

\[
F_1 - F_2 - (F_{wx}) - F_f = Q_t \rho (\beta_2 V_2 - \beta_1 V_1)
\]

where \(F_1\) and \(F_2\) are the resultant forces of the pressure distributions at the two flow cross-sections, \((F_{wx})\) is the component of force in the direction of flow acting on the control volume of fluid due to the flume walls, \(F_f\) is the friction or drag force acting on the surface of the control volume, \(Q_t\) is the theoretical discharge, \(\rho\) is the density of the fluid, \(\beta_1\) and \(\beta_2\) are momentum coefficients for the two flow sections, and \(V_1\) and \(V_2\) are the average velocity at sections 1 and 2. Assuming uniform velocity distribution and neglecting the friction force
Fig. 1. Definition sketch for flat-bottomed rectangular flumes.

Fig 2. Control volume for flat-bottomed rectangular flumes.
Assuming hydrostatic pressure distribution

\[ F_1 = \gamma b_1 y_1^{2/2} \]  
\[ F_2 = \gamma b_2 y_2^{2/2} \]

where \( \gamma \) is the specific weight of the fluid, \( b_1 \) and \( b_2 \) are the width of the flume at sections 1 and 2, and \( y_1 \) and \( y_2 \) are the depths of flow at the two sections. The force of the flume walls on the control volume acting in the direction of flow occurs in the converging inlet section. Assuming the average flow depth in the converging inlet of the flume is \( y_1 \)

\[ (F_{wx}) = \gamma (b_1 - b_2) y_1^{2/2} \]

The momentum equation in the direction of flow can now be written as

\[ \frac{\gamma b_1 y_1^2}{2} - \frac{\gamma b_2 y_2^2}{2} - \frac{\gamma y_1^2 (b_1 - b_2)}{2} = \frac{Q_t \gamma (V_2 - V_1)}{g} \]

where \( g \) is the acceleration due to gravity. Assuming steady flow, the continuity equation, \( Q = AV \), can be employed.

\[ Q = b_1 y_1 V_1 = b_2 y_2 V_2 \]

Substituting the continuity equation into Eq. 6 and solving for the discharge

\[ Q_t = \sqrt{\frac{(g/2)^{1/2} b_2 (y_1 - y_2)^{1/2}}{(1 - b_2 y_2/b_1 y_1) b_2}} \]

Let the constriction ratio, \( b_2/b_1 \), be represented by \( B \) and the submergence, \( y_2/y_1 \), by \( S \). The denominator of the discharge equation can be made dimensionless by multiplying the numerator and denominator by \( y_1 - y_2 \).

\[ Q_t = \sqrt{\frac{(g/2)^{1/2} b_2 (y_1 - y_2)^{3/2}}{(1 - BS)(y_1 - y_2)^2 y_1^2}} \]

\[ Q_t = \sqrt{\frac{(g/2)^{1/2} b_2 (y_1 - y_2)^{3/2}}{S (1 + S)}} \]

For any particular flume geometry, \( b_2 \) and \( B \) become constants and the discharge (Eq. 10) is a function of \( (y_1 - y_2)^{3/2} \) and \( S \). If the submergence is held constant, the discharge becomes a function of \( (y_1 - y_2)^{3/2} \) alone. This suggests that a logarithmic plot of \( Q \) against \( y_1 - y_2 \) would yield a family of straight lines with each line representing a constant value of submergence. The lines of constant submergence would be parallel and have a slope of 3/2.

A flat-bottomed trapezoidal flow measuring flume is shown in Fig. 3. The theoretical discharge equation developed for rectangular flumes (Eq.
10) approximates the trapezoidal shape when the flume bottom is very wide or the flow depths are shallow. For these limitations, the functional form of the theoretical submerged flow discharge equation for a flume having a trapezoidal shape can be written as

\[ Q_t = f \left[ (y_1 - y_2)^{3/2}, b_2, B, S \right] \quad \cdots \quad (11) \]

For the special case of a flat-bottomed trapezoidal flume with a V-shaped throat section \((b_z = 0)\), the functional form of the theoretical submerged flow discharge equation becomes

\[ Q_t = f \left[ (y_1 - y_2)^{5/2}, mb_1/y_1, B, S \right] \quad \cdots \quad (12) \]

Consequently, for any particular trapezoidal flume geometry, the theoretical value of the exponent, \(n_l\), lies somewhere between 3/2 and 5/2. The actual value of the exponent, \(n_l\), will exceed the theoretical value.

**Fig 3. Flat-bottomed trapezoidal flumes.**

**EXPERIMENTAL VERIFICATION**

Tests were initially conducted in a 5 foot wide by 5 foot deep channel with data being generated for a flat-bottomed trapezoidal flume (Hyatt, 1965), flat-bottomed rectangular flume (Skogerboe, Walker, and Robinson, 1965), shown in Fig. 4, and a Parshall flume having a throat width of
Fig. 4. Rectangular flat-bottomed flume with design discharge of 500 cfs.

Fig. 5. Dimension drawing of 2-foot Parshall flume.
Fig. 6. Submerged flow calibration curves for 2-foot Parshall flume.
Fig. 7. Experimental flat-bottomed rectangular flume.

Fig. 8. Details of experimental flat-bottomed flume.
2 feet (Skogerboe, Hyatt, England, and Johnson, 1965). The data for the flumes disclose that the slope of the lines of constant submergence is different from 3/2. The slope is actually the exponent of the flow depth in the free flow equation. The general form of the free flow equation can be written as

\[ Q = C y_1^{n_1} \]  

(13)

where \( C \) and \( n_1 \) are obtained from the straight line plot of \( Q \) against \( y_1 \) on logarithmic paper. For example, the free flow equation for the 2-foot Parshall flume illustrated in Fig. 5 is

\[ Q = 8.0 H_a^{1.55} \]  

(14)

where \( H_a \) is the depth of flow in the converging inlet section measured at a point two-thirds the length of the entrance section upstream from the flume crest. The submerged flow calibration curves for the 2-foot Parshall flume are shown in Fig. 6. The lines of constant submergence were drawn to best fit the data and resulted in slopes of 1.55 which corresponds with the exponent of \( H_a \) in Eq. 14.

A more detailed study was initiated later using the flat-bottomed rectangular flume shown in Fig. 7 and detailed in Fig. 8. Although numerous flow depths were measured, the following depths have been selected for analysis: (1) a flow depth in the flume entrance section, \( y_e \); (2) the minimum depth of flow in the throat, \( y_m \), which varies in location longitudinally; and (3) a flow depth in the flume exit section, \( y_z \). The purpose in selecting \( y_e \) and \( y_z \) was for use in an actual field installation after the calibration had been made, whereas the minimum depth \( y_m \), was measured in order to evaluate the Froude number of the flow at the roving section, \( m \), which dictates whether the flow is supercritical (free flow) or subcritical (submerged flow). A flow depth was selected in the exit section of the flume rather than the throat section because the water surface profile is more nearly horizontal resulting in a more accurate flow depth measurement.

Stilling wells were used to measure \( y_e \) and \( y_z \) (Fig. 7). A point gage was used to measure \( y_m \) since section \( m \) is not stationary. The measurement of \( y_m \) is quite difficult because of the fluctuation of the water surface. The average between the fluctuations of the water surface at section \( m \) was used in arriving at the value of \( y_m \). The fluctuation was very small at low Froude numbers (high submergence), but was very pronounced as the Froude number at section \( m \), which will be designated by \( F_m \), approached one.

Dimensional analysis can be employed to develop dimensionless parameters describing submerged flow. Applying such an analysis to the particular flume geometry under study allows the omission of any geometric terms since they are constant. The variables involved can be written as follows:

\[ V = f (g, y_2, y_m, y_7) \]  

(15)

With five independent quantities and two dimensions, three \( \pi \)-terms are necessary.
One of the desired pi-terms is the Froude number, $F_m$. For a rectangular section, $F_m$ can be expressed by

$$\pi_1 = F_m = \frac{V}{(g y_m)^{1/2}} = \frac{Q}{W g^{1/2} y_m^{3/2}}$$

(16)

where $W$ is the throat width of the flume.

The second desired pi-term is the submergence, $y_7/y_2$, which will be designated by $S$.

$$\pi_2 = S = \frac{y_7}{y_2}$$

(17)

The submergence is defined as the ratio of a downstream flow depth to an upstream flow depth, where the downstream measurement is the depth of flow above the flume floor at the point of the upstream measurement. In actuality, the ratio of any flow depth measured downstream from $y_m$ to any flow depth measured upstream from $y_m$ can be used as the submergence, $S$.

The third pi-term was developed by trial and error from the trapezoidal flume study (Hyatt, 1965). The dimensionless parameter finally developed was

$$\pi_3 = \Delta y/y_m = (y_2 - y_7)/y_m$$

(18)

The free flow calibration for the rectangular flume is shown in Fig. 9. The free flow discharge equation as obtained from Fig. 9 is

$$Q = 2.87 y_2^{1.525}$$

(19)

Using the submerged flow data, the logarithm of $\pi_2$ has been plotted against $\pi_3$ in Fig. 10. The curve must pass through the point 0,0 since, as the submergence approaches 100 percent ($\log S = 0$), the difference in water surface elevation, $y_2 - y_7$, will approach zero. The relationship between $\pi_2$ and $\pi_3$ dictates the distribution of the lines of constant submergence in the submerged flow calibration plot. The relationship in Fig. 10 can be approximated by a straight line over a large range of submergence with some sacrifice in accuracy of the submerged flow calibration plot. Such an approximation is of value in gaining further insight into the characteristics of submerged flow. The dashed line in Fig. 10 is described by the equation

$$\log S = -0.274 \frac{(y_2 - y_7)}{y_m} - 0.0045$$

(20)

A logarithmic plot of $\pi_2$ against $\pi_3$ will yield a straight line relationship which can be combined with Eq. 20 to yield an approximate submerged flow discharge equation. The resulting equation contains the term $(y_2 - y_7)^{3/2}$ rather than $(y_2 - y_7)^{1.525}$ because the definition of $F_m$ contains $y_m^{3/2}$. To circumvent this situation, Fig. 11 has been prepared with $y_m$ plotted against $y_2 - y_7$ on logarithmic paper with $Q$ as the third variable. An equation relating the three variables has been developed by plotting the value of $y_m$ when $y_2 - y_7$ is equal to 1 for each line of constant discharge. These intercepts, designated $C_{im}$, have been plotted against $Q$ in Fig. 12. The empirical equation resulting from Fig. 11 is

$$y_m = C_m / (y_2 - y_7)^{0.425}$$

(21)
Fig. 9. Free flow calibration for experimental flume.
Fig. 10. Relationship between $\tau_2$ and $\tau_1$. 
Fig. 11. Plot of minimum flow depth, change in water surface elevation, and discharge.
Fig. 12. Development of relationship between minimum flow depth, change in water surface elevation, and discharge.

\[ Q = 12.6 \, C_m^{1.07} \]
The empirical equation resulting from Fig. 12 is

\[ Q = 12.6 G^{1.07} \]  \hspace{1cm} (22)

Combining Eqs. 21 and 22

\[ Q = 12.6 y_m^{1.07} (y_2 - y_7)^{0.455} \]  \hspace{1cm} (23)

An approximate submerged flow discharge equation can be obtained by combining Eqs. 20 and 23.

\[ Q = \frac{3.15 (y_1 - y_3)^{1.525}}{[-(\log S + 0.0045)]^{1.07}} \]  \hspace{1cm} (24)

The dashed lines in Fig. 13 have been obtained from Eq. 24.

The actual submerged flow calibration plot (solid lines in Fig. 13) can be obtained by combining Eq. 23 with the curve (solid line) in Fig. 10. A line of constant submergence in Fig. 13 is obtained by inserting the value of \((y_2 - y_7)/y_m\) corresponding to the desired submergence into Eq. 23, arbitrarily assigning two values of \(y_2 - y_7\), and solving for the discharge. A straight line is then drawn in Fig. 13 between the two points and the value of submergence is labeled on the line. The process is repeated for each desired line of constant submergence. The approximate solution (dashed lines) could be considered satisfactory for practical use when the submergence is less than 96 percent.

The value of submergence at which the transition from free flow to submerged flow occurs can be estimated by equating the free flow equation and the approximate submerged flow equation. For the rectangular flume studied, Eqs. 19 and 24 are set equal to one another.

\[ \frac{3.15 (y_1 - y_3)^{1.525}}{[-(\log S + 0.0045)]^{1.07}} = 2.87 y_1^{1.525} \]  \hspace{1cm} (25)

\[ 1.096 (1 - S)^{1.525} = [-(\log S + 0.0045)]^{1.07} \]  \hspace{1cm} (26)

By trial and error

\[ S = 0.883 \]  \hspace{1cm} (27)

If the free flow equation (Eq. 19) is equated against the actual submerged flow calibration (Fig. 10 and Eq. 23), the transition submergence obtained by trial and error is 0.886. Hence, free flow exists when the submergence is less than 88.6 percent, and submerged flow exists for submergences greater than 88.6 percent. A slight change in the coefficients or powers of the free flow or submerged flow equation will have a marked effect on the computation of the transition submergence.

The form of the free flow and submerged flow equations allows the calibration curves for both types of flow to be placed on a single chart such as Fig. 14, which constitutes the calibration curves for the rectangular flume studied.
Fig. 13. Submerged flow calibration curves for experimental flume.
Fig. 14. Free flow and submerged flow calibration for experimental flume.
COMPARISONS WITH THEORY

The general form of the approximate submerged flow discharge equation can be obtained from Eq. 24.

\[ Q = \frac{C_1 (\Delta y)^{n_1}}{\frac{1}{[-(\log S + C_2)]]^{n_2}}} \]  

(28)

where \( \Delta y \) is the change in water surface elevation between a section upstream and one downstream from section \( m \). An appraisal of Eqs. 10 and 28 discloses some similarities. For a particular rectangular flume with specified dimensions, the constriction ratio, \( B \), and throat width, \( b_2 \), are constants. Consequently, the theoretical discharge, \( Q_t \), becomes a function of \((y_1 - y_2)^{n_1} \) and \( S \), which is quite similar to the results obtained empirically where the discharge is a function of \((y_1 - y_2)^{n_1} \) and \( S \). The value of \( n_1 \) for rectangular flumes is greater than 3/2. As an example, the \( n_1 \) values for all Parshall flumes (Parshall, 1926) are slightly in excess of 3/2, ranging in value from 1.52 to 1.60.

The similarity between the denominators of Eqs. 10 and 28 can be shown by letting the denominator of Eq. 10 be represented by \( \phi_m(S) \) and the denominator of Eq. 28 by \( f(S) \).

\[ \phi_m(S) = \frac{1}{\sqrt{(1 - BS) (1 - S)^2}} \]  

(29)

and

\[ f(S) = \frac{1}{-(\log S + C_2)} \]  

(30)

Eq. 30 will be shown to approximate Eq. 29 when \( C_2 = 0 \).

\[ f(S) = 0 = \frac{1}{-\log S} \]  

(31)

The relationship between \( \phi_m(S) \) and \( f(S) \) is bounded by the constriction ratios 0.0 and 1.0 as shown in Fig. 15. The slope of each line of constant constriction ratio, which is the exponent \( n_2 \), ranges in value from 1.0 to 1.5 for corresponding \( B \) values of 0.0 and 1.0, respectively. The nomenclature \( n_2 \) is used to signify the value of \( n_2 \) derived by comparison with the pressure-momentum equation (Eq. 10) where \( f(S) \) is evaluated for \( C_2 = 0 \). The curves depicted in Fig. 15 can be written in equation form as

\[ \phi_m(S) = C_3 f(S) C_2^{n_2} = 0 = \frac{C_3}{[-(\log S)]^{n_2}} \]  

(32)

The relationship between \( \phi_m(S) \) and \( f(S) \) is not quite so simple for constriction ratios other than 0.0 and 1.0 since the lines of constant constriction ratio show a slight curvature in Fig. 15.

A very common method for utilizing a theoretical discharge equation to fit actual data is by employing a coefficient of discharge, \( C_d \), which is defined as the ratio of the actual discharge to the theoretical discharge.

\[ C_d = \frac{Q}{Q_t} \]  

(33)
Fig. 15. Relationship between $\phi_m(S)$ and $f(S)_{C_2 = 0}$. 
Fig. 16. Coefficient of discharge for experimental flume.
The submerged flow calibration plot for the experimental rectangular flume (Fig. 13) has been used in conjunction with the theoretical submerged flow discharge equation (Eq. 10) to develop the coefficient of discharge relationship for the experimental flume (Fig. 16). For any flume, \( C_d \) is a function of both \( \Delta y \) and \( S \). Since finite values of both \( \Delta y \) and \( S \) result in a unique solution of both flow depths, \( C_d \) varies as the flow depths vary. The coefficient of discharge for any flume becomes a three-dimensional plot similar in form to the submerged flow calibration curves. The slope of the lines of constant submergence in the plot of \( C_d \) is \( n_1 - 3/2 \). For example, the slope of the lines in Fig. 16 is \( 0.025 \times (1.525 - 3/2) \).

**COMPARISONS WITH OTHER METHODS**

**Parshall’s Submerged Flow Analysis**

Parshall (1950) uses a three-dimensional plot to describe submerged flow in Parshall flumes. Typical discharge correction curves developed by Parshall are shown in Figs. 17 and 18. Here, on logarithmic paper, the upstream depth of flow, \( H_a \), is plotted as the ordinate; discharge correction factor as the abscissa; and submergence, \( H_b/H_a \), as the varying parameter. The correction factor is the difference between the actual discharge, \( Q \), and the free flow discharge, \( Q_p \), obtained from tables using the measured upstream depth of flow which has been increased due to submergence. The spread between the lines of constant submergence increases as the submergence ratio approaches the transition submergence.

The lines of constant submergence drawn in Fig. 17 are not straight, but are curved greatly for low submergence values until the value of \( H_a = 1.0 \) is reached, and then tend to straighten out for larger upstream depths. For high submergence values, the lines of constant submergence are almost a straight line with a slope of about 0.565 which is close to the reciprocal of the critical flow slope (1.52) for a 1-foot Parshall flume. For Parshall flumes greater than 1 foot, Fig. 17 is still used, but the discharge value is multiplied by a correction depending upon the flume size (Parshall, 1950), even though the critical flow slope for the larger Parshall flumes becomes greater, approaching 1.60. Although not shown here, the three-dimensional plots for the 3- and 6-inch Parshall flumes have lines of constant submergence which are drawn as a straight line (Parshall, 1941).

Another inaccuracy in Parshall’s plot is that the discharge correction should be zero for the transition submergence. Such a condition requires that, as the transition submergence is approached, the submergence lines in Fig. 17 must be raised without bounds.

The data collected from the experimental flume (Fig. 8) have been plotted in Fig. 19 in the same manner used by Parshall. The spacing between the lines of constant submergence increase rapidly as the transition submergence is approached. The same data have been plotted on rectangular coordinates in Fig. 20. The advantage of obtaining the discharge correction from a rectangular coordinate plot is that the ordinate corresponds to the transition submergence and the abscissa represents 100 percent submergence. The primary disadvantage in using either Figs. 19 or 20 is that the discharge is obtained indirectly, whereas sub-
merged flow calibration curves such as shown in Fig. 14 become the “rating” for the structure and give the discharge directly.

![Discharge Correction, cfs](image1)

**Fig. 17. Diagram for computing the rate of submerged flow through a 1-foot Parshall measuring flume.**

![Discharge Correction, cfs](image2)

**Fig. 18. Diagram for determining the correction in second-feet per 10 feet of crest for submerged-flow discharge in large Parshall flumes.**
Fig. 19. Parshall's three-dimensional plot applied to experimental flume.
Fig. 20. Submerged flow discharge correction for experimental flume.
Fig. 21. Two-dimensional submerged flow plot applied to experimental flume.
Two-dimensional Submerged Flow Plot

Another approach to submerged flow analysis has been used by Robinson (1964). A two-dimensional plot is composed of three flow quantities (discharge, upstream depth, and downstream depth), but combined in such a way that only two dimensionless parameters are used. A $Q/Q_o$ ratio is plotted as the ordinate and submergence, $S$ is plotted as the abscissa (Fig. 21). The actual, or true, discharge is denoted by $Q$, and $Q_o$ is the critical flow discharge based on the upstream depth of flow which has been increased due to submergence. When the dimensionless discharge ratio, $Q/Q_o$, reaches the value of 1.0, a critical flow condition is reached because the observed discharge becomes the true discharge. When the discharge ratio equals 1.0, the curve depicted in Fig. 21 intersects the submergence scale at 88.4 percent. This would indicate the submerged flow condition is not reached until the submergence ratio is 88.4 percent or greater. The degree of contraction, sidewall slope, length of throat, and possibly flume size, all affect the transition submergence (Robinson, 1964). Hence, any change in the flume geometry will change the placement of the curve on the two-dimensional plot by moving the curve up or down and/or to the right or left. The scatter which is apparent in the data plotted in Fig. 21 is typical of most two-dimensional submerged flow plots. The real problem lies in the correct placement of the curve. In Fig. 21 the correct placement of the curve was obtained by computing $Q/Q_o$ and $y_j/y_2$ ratios from the submerged flow calibration curves for the experimental flume (Fig. 13).

EFFECTS OF BOUNDARY GEOMETRY

Flow Depth Location

For determining the most satisfactory location for obtaining accurate ratings under both free and submerged flow conditions, six taps were placed along the wall of the experimental flume (hereinafter referred to as FR-1 flume) as shown in Fig. 8. In addition, the flow depths $y_u$ and $y_d$ were measured upstream and downstream from the flume, respectively.

The free flow calibrations using $y_u$, $y_2$, $y_3$, and $y_4$ are shown in Fig. 22 along with the resulting free flow equations. The slope of the free flow calibrations, which is the exponent $n_1$ in Eq. 13, is the same for each of the four upstream depths. The free flow equations using $y_2$ or $y_3$ are nearly the same, the difference being less than one percent. This indicates that the same free flow rating can be used if the depth is measured somewhere between $y_2$ and $y_3$.

In evaluating the most accurate location for measuring the upstream depth, consistency plots were prepared whereby one upstream flow depth was plotted against another. Consistency plots for the upstream flow depths using subcritical flow data are shown in Figs. 23 and 24. From the consistency plots, it can be seen that $y_2$ and $y_3$ result in the most accurate straight line relationship. Although the relationship between $y_u$ and $y_2$, or $y_3$, is good (Fig. 23), upstream channel changes in a field installation could make use of $y_u$ undesirable. In Fig. 24, the relationship between
Fig. 22. Free flow calibrations for FR-1 flume.

Discharge, cfs.

$y_u$, $y_2$ and $y_4$

- $Q = 2.75 y_u^{1.525}$ cfs.
- $Q = 2.91 y_2^{1.525}$ cfs.
- $Q = 2.93 y_3^{1.525}$ cfs.
- $Q = 3.20 y_4^{1.525}$ cfs.
Fig. 23. Consistency plot of $y_n$ against $y_2$ and $y_3$ for FR-1 flume.
Fig. 24. Consistency plot of \( y_2 \) against \( y_3 \) and \( y_4 \) for FR-1 flume.
$y_2$ and $y_i$ changes with the flow rate. The effect of discharge can be attributed to the rapid change in water surface profile in the vicinity of $y_i$. Thus, a small error in locating the $y_i$ tap would result in a significant error in the flume rating.

The downstream measuring points are of importance only when the flume is operated under subcritical flow conditions. To illustrate the effect of measuring flow depths at $y_o$, $y_6$, $y_7$, and $y_o$, selected water surface profiles obtained from the data generated in the FR-1 flume are shown in Fig. 25. A rapid increase in flow depth occurs in the throat section, its location being dependent upon the flume geometry. For the FR-1 flume, the rapid increase occurs at the end of the throat section for submergences near the transition submergence. For flumes with short throats, the rapid depth increase will occur at the beginning of the exit section. As the submergence increases, the rapid depth increase moves upstream towards the front of the throat section, and the degree of depth change decreases until a relatively smooth water surface profile is obtained for the high submergence ratio as shown in Fig. 25. Accurate ratings of flat-bottomed flumes under submerged flow conditions cannot be obtained by measuring the downstream depth in the throat section since the location of both the minimum flow depth, $y_m$, and the rapid increase in depth varies with the submergence.

Data collected from the experimental (FR-1) flume indicates that the water surface profiles are relatively smooth in the vicinity of $y_i$. Submerged flow ratings for the FR-1 flume using $y_i$ as the downstream flow depth have proven satisfactory. Tests conducted with other flumes wherein the downstream flow depth was measured near the end of the flume also proved satisfactory. Consequently, measuring the downstream depth 1 or 2 feet from the flume exit is recommended.

**Channel Approach and Exit**

In the study of channel effects, the rectangular flat-bottomed FR-1 flume was again used. The bottom of this flume was placed 1 foot above the floor of the 6-by-8 foot channel located in the Utah Water Research Laboratory. The special test section shown in Fig. 26 was placed first at the flume entrance and then the flume exit. The test section had a removable bottom allowing the floor to correspond with the level of the flume bottom or channel floor, whichever was desired.

Tests conducted with the special test section placed upstream from the FR-1 flume showed no effect on the exponent, $n_1$, which remained constant at 1.525, under free flow conditions. The free flow coefficient, $C$, varied from 2.85 to 2.91 in the relationship between $Q$ and $y_2$. The error would be approximately one percent by using a free flow coefficient of 2.87 with $y_2$ in the critical flow discharge equation. This would be significant under certain circumstances, but satisfactory for most field measurements.

Since subcritical flow is dependent upon both upstream and downstream flow depths, the channel downstream from the FR-1 flume was also modified using the special test section shown in Fig. 26. The subcritical flow data generated with the test section placed both upstream and downstream
Fig. 25. Selected water surface profiles in the FR-1 flume.
Fig. 26. Test section used for evaluating channel effects on FR-1 flume rating.
Fig. 27. FR-1 flume with gravel roughness placed along sides and floor.
from the flume was evaluated by plotting $\pi_2$ against $\pi_3$ since this relationship determines the distribution of the constant submergence lines in the submerged flow calibration. The $\pi$-terms were computed using the flow depths $y_2$, $y_{int}$, and $y_7$. The channel variations did not significantly change the relationship between $\pi_2$ and $\pi_3$. Consequently, the channel variations would not be expected to significantly change the submerged flow rating for the FR-1 flume (Fig. 14).

**Roughness**

Rocks were placed along the walls and floor of the FR-1 flume to evaluate the effect of roughness (Fig. 27). The increased roughness created significant changes in both the critical and subcritical discharge ratings. The effect of roughness on the free flow rating is illustrated in Fig. 28. The free flow equation for the rough flume is

$$Q = 2.29 y_2^{1.59}$$

whereas, the free flow equation for the smooth (FR-1) flume is

$$Q = 2.87 y_2^{1.525}$$

Because of the rock placed inside the FR-1 flume, the cross-sectional area of the throat was reduced approximately four percent. Consequently, a portion of the difference between the free flow ratings must be attributed to the reduction in flow area. Although exaggerated roughnesses were used, it is significant that a change in roughness resulted in changing both the coefficient, $C$, and exponent, $n$, in the free flow equation.

The difference between the subcritical flow ratings for the smooth and rough flumes using selected values of submergence is illustrated in Fig. 29. The slope of the constant submergence lines has the same slope as the exponent, $n_1$, in the critical flow equations for the respective flumes. The head loss for the rough flume is greater than for the smooth flume, as would be expected. This is caused by the increased frictional resistance along the boundary of the rough flume. The head loss through the rough flume is approximately 25 percent greater than for the smooth flume. The transition submergences for the two flumes are represented by the lowest constant submergence lines drawn in Fig. 29. The transition submergence for the rough flume is much lower than for the smooth flume.

The roughness used in this experiment is greatly exaggerated compared with roughnesses normally encountered in a metal, concrete, or wood flume. The purpose of this study was not to evaluate the quantitative aspects of roughness on flume ratings, but whether or not roughness did have an effect. In a study by Skogerboe, Hyatt, and Austin (1966) regarding embankment-shaped weirs, roughness was shown to significantly change both the free flow and submerged flow ratings. The results from this study indicate that if concrete spalled, or if a metal flume became encrusted, the flume rating would be altered. Additional work is required to evaluate the quantitative changes in flume ratings brought about by various degrees of relative roughness.

**Convergence and Divergence**

The convergence of the entrance section in the FR-1 flume is 3:1. In the work of Wells and Gotaas (1956), Ludwig and Ludwig (1951) and
Hyatt (1965), it was found that the 3:1 entrance convergence was satisfactory, and consequently, was used in this study.

Divergence was studied in the variable geometry flume shown in Fig. 30. The exit divergence ratios tested were 3:1, 4:1, and 6:1. Each divergence ratio was tested under both critical and subcritical flow conditions.

With the 3:1 divergence ratio, separation occurred over the full critical flow range. As the submergence was increased, the degree of separation decreased, but still persisted to some extent at submergences above 95 percent. Operating a flume with a 3:1 exit would be undesirable since a problem in bank erosion would be likely for flumes placed in unlined channels. Also, under submerged flow conditions, the accuracy of flow depth measurement in the exit section could be unsatisfactory.

The 6:1 exit was found to function much more satisfactorily than the other divergences which agrees with the findings of Hyatt (1965). Very little separation occurred when flow conditions were near the transition

![Graph](image-url)

**Fig. 28. Critical flow ratings for smooth and rough flumes.**
submergence and separation was not noticeable at the high submergences. The performance of the 6:1 divergence under critical flow conditions is shown in Fig. 31, whereas Fig. 32 represents a submergence of 95 percent. Thus, the 6:1 divergence can be expected to function properly under the full range of expected flow conditions.

![Graph showing the effect of roughness on submerged flow calibration curves.](image)

Fig. 29. Effect of roughness on submerged flow calibration curves.
Entrance and Exit Length

An entrance length of 3 feet was found sufficient to give accurate results when the depth was measured at the two-thirds point, or 2 feet upstream from the throat constriction. In the scale effect model, an entrance of 1.5 feet was used which caused a dip to form in the water surface at the two-thirds point. Consequently, an entrance of sufficient length that a nearly horizontal water surface can be established in the vicinity of the upstream tap is considered desirable. An entrance of 6 feet was also used, which proved satisfactory. Since the practical objective of this study is to provide both flow accuracy and economy of construction, the entrance length of 3 feet is recommended.

The primary purpose of the entrance and exit section is to provide a smooth transition from the channel through the control section and back
to the channel. The entrance and exit sections also provide a rigid boundary from which accurate measurements of the upstream and downstream flow depths can be obtained.

Most flumes are designed to operate under free flow as well as submerged conditions. If such is the case, provisions to prevent erosion must be made downstream from the flume. In using the flume under subcritical flow conditions, erosion is not as serious a problem and the primary importance of exit length is to provide an accurate location for measuring the downstream depth. Exit lengths of 4, 6, and 8 feet were used, all of which proved satisfactory, but the longer lengths performed more satisfactorily than the shorter lengths. An exit length of 6 feet combined with a 6:1 divergence has the advantage of providing an exit width corresponding to the entrance width. Consequently, it is recommended that an exit length of 6 feet be used. Submerged flow conditions were always fairly smooth near the flume exit when the above recommendations were employed.

**Throat Length**

From the tests using various throat lengths, it was determined that the flume with zero throat length performed at least as satisfactorily as one with a throat length, (Figs. 33 and 34). Eliminating the throat section proved to be an advantage in preventing separation. As the flow passes through the entrance section, a contracted jet is formed (as with any flume). The contracted jet tends to assist in keeping the flow centered in the exit section (Fig. 34). Because of simplified construction and shorter overall length, the flume with no throat length is comparatively economical.
Scale Effect

A prototype and two models were constructed for the scale effect tests. One model had a scale ratio of one-fourth (SR-1 flume) and the other a ratio of one-half (SR-2 flume). The prototype structure is shown in Fig. 35. Free flow and submerged flow ratings were prepared for each of the structures. The models were used to predict the flow of the prototype.
structure according to principles of dimensional analysis for a Froude model.

Free flow calibrations for the SR-4 flume based on prototype data, and the predicted calibrations resulting from the two model flumes are shown in Fig. 36. A comparison of the results using flow depths of 1.0, 2.0, and 6.0 feet is shown in Table 1.

Table 1. Comparison of actual and predicted free flow calibrations.

<table>
<thead>
<tr>
<th>Depth (feet)</th>
<th>SR-4 (proto.) Q, cfs</th>
<th>SR-2 (1/2 model) Q, cfs</th>
<th>Error, %</th>
<th>SR-1 (1/4 model) Q, cfs</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10.8</td>
<td>11.0</td>
<td>1.9</td>
<td>11.3</td>
<td>5.6</td>
</tr>
<tr>
<td>2.0</td>
<td>31.3</td>
<td>32.3</td>
<td>2.9</td>
<td>33.7</td>
<td>7.4</td>
</tr>
<tr>
<td>6.0</td>
<td>17.0</td>
<td>178.0</td>
<td>4.6</td>
<td>191.0</td>
<td>12.3</td>
</tr>
</tbody>
</table>

The error in predicting discharge under free flow conditions increases as the depth or flow rate increases. Also, the predicted prototype rating resulting from the SR-2 model is more accurate than the predicted rating from the SR-1 flume, as would be expected.

To compare the subcritical flow ratings for the SR-4 flume along with the predicted ratings obtained from the model flumes, Fig. 37 has been prepared using selected submergences of 90, 95, and 98 percent. As with the free flow calibrations, the SR-2 flume more accurately predicts the prototype submerged flow calibration than does the SR-1 flume. Also, the prediction error increases as the submergence increases, indicated in Table 2.

Table 2. Comparison of actual and predicted submerged flow calibrations.

<table>
<thead>
<tr>
<th>S</th>
<th>SR-4 (proto.) Q, cfs</th>
<th>SR-2 (1/2 model) Q, cfs</th>
<th>Error, %</th>
<th>SR-1 (1/4 model) Q, cfs</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10.8</td>
<td>11.0</td>
<td>1.9</td>
<td>11.3</td>
<td>5.6</td>
</tr>
<tr>
<td>2.0</td>
<td>31.3</td>
<td>32.3</td>
<td>2.9</td>
<td>33.7</td>
<td>7.4</td>
</tr>
<tr>
<td>6.0</td>
<td>17.0</td>
<td>178.0</td>
<td>4.6</td>
<td>191.0</td>
<td>12.3</td>
</tr>
</tbody>
</table>

The transition submergence, $S_t$, did not change for the prototype or model flumes. The transition submergence for each of the flumes was 87 percent. This gives more evidence that $S_t$ is largely a function of the constriction ratio, since the constriction ratio is the same for the models and the prototype.

The scale effect study indicates the degree of error that might be expected when a model is used to predict the actual prototype calibration plots. The effect is more pronounced as the model scale is reduced. Relatively large models were used in this study. When possible, a direct calibration of a flume should be made in order to obtain accurate results. If a model calibration is required, then a field check of the prototype structure should also be undertaken to correct the rating.
Fig. 35. Prototype flume, SR-4, for evaluating scale effect.
Fig. 36. Comparison of actual free flow rating with model predictions for SR-4 flume.
Fig. 37. Comparison of actual submerged flow rating with model predictions for SR-4 flume.
SUMMARY

The general form of the free flow calibration equation for a flume can be written as

$$Q = C Y^{n_1}$$

The analysis of data for a rectangular flume has led to the development of an approximate submerged flow equation which can be written in general form as

$$Q = \frac{C_1 (\Delta y)^{n_1}}{- \left( \log S + C_2 \right)^{n_2}}$$

The use of Eq. 28 has been found to be valid for a trapezoidal flat-bottomed flume, a rectangular flat-bottomed flume, and a Parshall flume. Of particular significance is that the exponent, $n_1$, appears both in the free flow equation and the submerged flow equation. Because of this, the free flow and submerged flow equations developed for any particular flume geometry can be set equal to one another, and the solution will yield the value of submergence at which the transition from free flow to submerged flow occurs. Also, since $n_1$ appears in both equations, calibration curves for both types of flow can be placed on a single graph.

The theoretical submerged flow discharge equation developed from momentum relationships for a rectangular flat-bottomed flume is

$$Q_t = \frac{(g/2)^{1/2} b_2 (y_1 - y_2)^{3/2}}{\sqrt{(1 - BS) (1 - S)^2}}$$

When Eqs. 28 and 10 are compared for a rectangular flume, the exponent $n_1$ is found to exceed 3/2. The denominators of the two equations are compatible if $C_2$ equals zero. The empirical analysis does show that $C_2$ approaches zero as the submergence, $S$, approaches one. The comparison of the denominators of the two equations showed that $n_2$ varies between 1.0 and 1.5.

The flow parameters have been developed for the calibration of flumes operating under submerged flow conditions. The effects of certain boundary geometry conditions (e.g., effect of location of flow depth measurement, channel approach and exit, roughness, entrance convergence, exit divergence, entrance length, exit length, throat length, and scale effect) on the critical and subcritical discharge ratings for flow measuring flumes have been evaluated.

The location of flow depth measurement affects the accuracy of the rating curves for a flume. The depths should be measured at sections in the flume where the water surface is relatively smooth and level. The most inaccurate location for measuring the flow depth is in the throat because of abrupt changes in the water surface elevation. The depths should not be measured outside of the flume in the approach or exit channel, unless the channel is stable and will not change.

The channel changes did not affect the ratings as long as the depth measurements were made within the flume. The ratings will be affected
if the depths are measured in an unstable channel. The effects of high velocity of approach were not evaluated. (This is one area of study which should be undertaken.)

In the roughness study, it was found that an exaggerated roughness of rocks affected the flow through a measuring flume. Also, the roughness reduced the transition submergence.

The entrance and exit sections in a flow measuring flume are used to provide a smooth transition from the channel to the control section and back to the channel again. The convergence, divergence, and length are important in performing this function. An entrance section having a 3:1 convergence and a 3-foot length was effective in providing a smooth water surface profile for accurate measurement of the upstream flow depth. The exit divergence could not be as great because separation occurred. A 6:1 divergence with a 6-foot length prevented separation and provided a suitable location for measuring the downstream flow depth.

The throat is the most critical portion of the flume and must be constructed with care. The throat is the “control” which determines to a large extent the flow characteristics of the flume. Flumes having no throat length were found as satisfactory in performance as flumes having a throat length.

The scale effect studies disclosed a certain degree of inaccuracy in using models for predicting prototype ratings. From the overall results of the tests it is evident that the prototype structure should be calibrated to obtain accurate results. If model studies are conducted for a particular flume geometry, the rating for the prototype structure should be field checked.
REFERENCES


