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MEASUREMENT OF TECHNICAL EFFICIENCY IN PUBLIC EDUCATION: A STOCHASTIC FRONTIER PRODUCTION FUNCTION APPROACH

by

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This paper uses a stochastic frontier production function approach to measure technical efficiency in public education in Utah. A Cobb-Douglas production function is used to represent the underlying production technology. The empirical analysis shows substantial variation in inefficiency among school districts. These measures are insensitive to the specific distributional assumptions about the one-sided component of the error term. These results have significant policy implications for resource allocation in public schools.
MEASUREMENT OF TECHNICAL EFFICIENCY IN PUBLIC EDUCATION: A STOCHASTIC FRONTIER PRODUCTION FUNCTION APPROACH

Introduction

Efficiency in the public education system is a significant issue in the United States. Nationwide, real expenditure per student in public education increased over 8 percent per year between 1960 and 1993, but output, as generally measured by standardized test scores, has not increased and, in some cases (i.e., the verbal SAT score), has declined.¹ One explanation is that resources are not being utilized efficiently. There may be productive or technical inefficiency and/or allocative or price inefficiency (i.e., given the relative prices of inputs, the cost-minimizing input combination is not used). This paper evaluates the technical inefficiency in public schools using Utah school districts as a laboratory.

Two methods have been used to estimate efficiency: the nonparametric approach that developed out of mathematical programming and is commonly known as the data envelopment analysis (DEA), and the parametric approach that estimates technical efficiency within a stochastic production function model (i.e., the stochastic frontier method). DEA has been used in measuring efficiency in the public sector where market prices for output are not available. For example, Levin (1974), Bessent and Bessent (1980), Bessent et al. (1982), and Grosskopf and Weber (1989) used this method to estimate efficiency in public education. The stochastic frontier methodology was used by Barrow (1991) to estimate a stochastic cost frontier using data from schools in England. Also, Wyckoff and Lavinge (1991) estimated technical inefficiency

for elementary schools in New York, and Grosskopf et al. (1991) used the parametric approach to estimate allocative and technical efficiency in Texas school districts. The recent literature has seen a convergence of the two approaches, and their complementarity is being recognized.²

In this paper we use the stochastic frontier method to measure technical efficiency in individual school districts within the framework of an educational production function. The model is estimated using data from the 40 school districts in Utah for the academic year 1992-93. In the empirical analysis, output is measured by a standardized test score administered in the 11th grade in all districts. Two classes of inputs are included. The first, considered to be subject to control by school administrators, includes the student-teacher ratio, percentage of professional staff having an advanced degree, and expenditure per student. The second class includes such noncontrollable factors as the education level of the local population and percentage of students from low-income families. The objective of the study is to measure individual technical inefficiency at the individual school district level after accounting for the level of productivity accounted for explanatory variables.

This paper is organized as follows. First, the relevant literature is reviewed. Next, the specification of technical inefficiency is discussed within the stochastic production function framework. This is followed by a review of the estimation methods outlined by Jondrow et al. (1982). Then the data set is discussed and the empirical results presented.

Background

For a given technology, the production function defines the maximum amount of output

²The Journal of Econometrics (Lewin and Lovell 1990) devoted an entire supplemental issue to parametric and nonparametric approaches to frontier analysis.
forthcoming from a given combination of inputs. Inefficiency is measured by the difference between actual rate of output given a set of inputs and the production frontier for these input rates. Koopmans (1951) defines a technically efficient producer as one who cannot increase the production of any one output rate without decreasing another or without increasing some input. Debreu (1951) and Farrell (1957) offer a measure of technical efficiency as one minus the maximum equiproportionate reduction in all inputs that still allows continuous production of a given output rate (Lovell 1993).

The earliest study that measured technical inefficiency in education production is Levin (1974, 1976). He used the Aigner and Chu (1968) parametric nonstochastic linear programming model to estimate the coefficients of the production frontier. He found that parameter estimation by ordinary least squares (OLS) does not provide correct estimates of the relationship between inputs and output for technically efficient schools; it only determines an average relationship. Klitgaard and Hall (1975) used OLS techniques to conclude that schools with smaller classes and better paid and more experienced teachers produce higher achievement scores. His study also estimates an average relationship rather than an individual school-specific relationship between inputs and output or a production frontier.

Among the studies on technical efficiency in public schools using the DEA method, the earliest was done by Charnes, Cooper, and Rhodes (1978) who evaluated the efficiency of individual schools relative to a production frontier. Bessent and Bessent (1980) and Bessent et al. (1982) made further refinements by incorporating a nonparametric form of the production function, introducing multiple outputs, and identifying sources of inefficiency for an individual school. Further extensions were made by Ray (1991) and McCarty and Yaiswarng (1993), who
considered controllable inputs in the first stage of the DEA model. Then the environmental (i.e., noncontrollable) inputs were regressed as independent variables in the second stage to determine the DEA efficiency score.

In these studies, the production frontier is deterministic in that all firms share a common production frontier and any deviation of a firm from that frontier is attributable to differences in efficiency. The concept of a deterministic frontier ignores the possibility that a firm’s performance may be affected by factors both within and outside its control. In response to this, the concept of a stochastic production frontier was developed and extended by Aigner, Lovell, and Schmidt (1977), Meeusen and van den Broeck (1977), Battese and Corra (1977), Battese and Coelli (1988), Lee and Tyler (1978), Pitt and Lee (1981), Jondrow et al. (1982), Kalirajan and Flinn (1983), Bagi and Huang (1983), Schmidt and Sickles (1984), and Waldman (1984).

Frontier production models have been analyzed either in the framework of the production function or in a cost-minimizing framework. Using data from school districts, Barrow’s study of schools in England (1991) tested various forms of the cost frontier and found the level of efficiency to be sensitive to the method of estimation. In their study of technical inefficiency in elementary schools in New York, Wyckoff and Lavinge (1991) estimated the production function directly and found that the index of technical inefficiency depends on the definition of educational output. For example, if output is measured by the level of cognitive skill of students rather than their college entrance test score (i.e., the ACT or SAT or any other type of composite test score consisting of reading, writing, and mathematics skills), the index of technical inefficiency based on each output measure will be different. Grosskopf et al. (1991) used a stochastic frontier and distance function to measure technical and allocative efficiency in Texas
school districts and concluded that they were technically efficient but allocatively inefficient.

**Specification of Technical Inefficiency**

In the stochastic frontier model, a nonnegative error term representing technical inefficiency is added to the classical linear model. The general formulation of the model is:

\[ y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \ldots + \beta_k x_{ik} + \varepsilon_i \]  

(1)

where \( y_i \) is output and the \( x \)'s are inputs. It is postulated that \( \varepsilon_i = v_i - u_i \), where \( v_i \sim N(0, \sigma_v^2) \) and \( u_i \sim |N(0, \sigma_u^2)| \), i.e., \( u_i \geq 0 \), and the \( u_i \) and \( v_i \) are assumed to be independent. The error term \( \varepsilon_i \) is the difference between the standard white noise disturbance \( v_i \), and the one-sided component \( u_i \). The term \( v_i \) allows for randomness across the firm and captures the effect of measurement error, other statistical noise, and random shocks outside the firm's control. The one-sided component \( u_i \) captures the effect of inefficiency (Forsund, Lovell, and Schmidt 1980). Most of the earlier stochastic production frontier studies could only calculate mean technical inefficiency of firms in the industry because they could not decompose the residual for individual observations into the two components. Jondrow et al. (1982) solved the problem by defining the functional form of the distribution of the one-sided inefficiency component and derived the conditional distribution of \( [u_i | v_i-u_i] \) for two popular distribution cases (i.e., the half normal and exponential) to estimate firm-specific technical inefficiency.

For this study, let the production function for the \( i \)th school district be represented by:

\[ y_i = A \prod_{j=1}^{k} x_{ij}^{\theta_j} e^v \]  

(2)

where \( y \) is output, \( x_j \) are exogenous inputs, \( A \) is the efficiency parameter, and \( v \) is the stochastic disturbance term. The production function in (2) is related to the stochastic frontier model by Aigner, Lovell and Schmidt (1977), who specify \( A \) as:
\[ A = a_o e^{-u} \quad u \geq 0 \]

where \( a_o \) is a parameter common to all districts and \( u \) is the degree of technical inefficiency that varies across school districts. Units for which \( u = 0 \) are most efficient. A district is said to be technically inefficient if output is less than the maximum possible rate defined by the frontier.

The term \( v \) is the usual two-sided error term that represents shifts in the frontier due to favorable and unfavorable external factors.

After including the component of inefficiency (i.e., \( e^u \)), the actual production function is written as:

\[ y_i = a_o \prod_{j=1}^{k} x_j a_j e^{-u} \quad (3) \]

If there is no inefficiency and the potential output is denoted by \( Y \), then the production function is written as:

\[ Y_i = a_o \prod_{j=1}^{k} x_j a_j e^v \quad (4) \]

Hence, the appropriate measure of technical efficiency is:

\[ \frac{\text{actual output}}{\text{potential output}} = \frac{y_i}{Y_i} = \frac{a_o \prod_{j=1}^{k} x_j a_j e^v}{a_o \prod_{j=1}^{k} x_j a_j e^v} = e^{-u} \quad (5) \]

Potential output is the maximum possible when \( u = 0 \) in equation (3). A technically efficient school district produces output (i.e., standardized test scores) that are on the stochastic production frontier that is subject to random fluctuations captured by \( v \). However, because of differences in managerial efficiency, actual performance deviates from the frontier.

Since, \( u \geq 0 \), \( 0 \leq e^{-u} \leq 1 \), and \( e^{-u} \) is a measure of technical efficiency. Thus, technical inefficiency is measured by \( 1 - e^{-u} \), where \( e^{-u} \) is technical efficiency bounded by 0 and 1.
In other words, technical efficiency lies between 1 and 0, technical inefficiency is bounded between 0 and 1.

**Method of Estimation**

This study uses the method of estimation suggested by Jondrow et al. (1982) to estimate technical inefficiency in each school district. Here, the technical inefficiency error term \( u \) is assumed to be of the half-normal type. However, other empirical results assume that \( u_i \) follows a one-parameter exponential distribution with density

\[
f(u) = e^{\frac{-u}{\alpha_u}} / \alpha_u.
\]

Results based on these both distributional assumptions about \( u \) are reported.

To estimate equation (3) by the maximum-likelihood method we need to know the probability density function \((pdf)\) of \( \epsilon_i \), which is composed of \( u_i \) and \( v_i \). The \( pdf \), mean, and variance of \( u_i \) and \( v_i \) are written as:

\[
\text{pdf of } v_i = f(v_i) = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{1}{2}\left(\frac{v_i}{\sigma_v}\right)^2}
\]

\( E(v_i) = 0 \) and \( \text{Var}(v) = \sigma_v^2 \)

\[
\text{pdf of } u_i = f(u_i) = \frac{2}{\sqrt{2\pi\sigma_u}} e^{-\frac{u_i^2}{2\sigma_u^2}}, \quad u_i \geq 0
\]

The half normal distribution (3) has the following mean and variance (Maddala 1977; Aigner, Lovell, and Schmidt 1977):

\[
E(u_i) = \frac{\sqrt{2}}{\sqrt{\pi}} \sigma_u \quad \text{and} \quad \text{Var}(u) = \frac{\pi - 2}{\pi} \sigma_u^2
\]
To formulate the log-likelihood function, the density function of the composite residual 
\( (v - u) \) (i.e., the joint density function of \( f(v, u) \)) is formulated and then transformed into a joint
density of \( \epsilon \) and \( u \) by integrating \( u \) from 0 to \( \infty \). Following Maddala (1977), the pdf of \( \epsilon \), which
is a composite of \( v \) and \( u \), is written as:

\[
f(\epsilon) = \frac{\sqrt{\frac{\epsilon}{\pi}}}{\sigma} \left[ 1 - F\left( \frac{\epsilon \lambda}{\sigma} \right) \right] e^{-\frac{\epsilon^2}{2\sigma^2}}
\]  

(8)

where \( \sigma^2 = \sigma_u^2 + \sigma_v^2 \), \( \lambda = \sigma_u / \sigma_v \), and \( F\left( \frac{\epsilon \lambda}{\sigma} \right) \) is the cumulative density function (cdf)
of the normal variable evaluated at \( \frac{\epsilon \lambda}{\sigma} \). The mean of this density function is called the mean technical
inefficiency and is written as:

\[
E(\epsilon) = E(u) = -\frac{\sqrt{\frac{\epsilon}{\pi}}}{\sqrt{\pi}} \sigma_u
\]  

(9)

If we have a sample of \( n \) observations, we can form the relevant log-likelihood function
as:

\[
\text{ln}L(y|\beta_1, \ldots, \beta_5, \lambda, \sigma^2) = n \text{ ln} \frac{\sqrt{\frac{\epsilon}{\pi}}}{\sqrt{\pi}} + n \text{ ln} \frac{1}{\sigma} + \sum_{i=1}^{n} \text{ ln} \left( 1 - F\left( \frac{\epsilon \lambda}{\sigma} \right) \right) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \epsilon_i^2
\]  

(10)

It should be noted that the meaning of \( 1 - F(.) \), where \( F \) is evaluated at \( \epsilon \lambda / \sigma = -\mu / \sigma \), is the
probability that a \( N(\mu, \sigma^2) \) variable is positive. Differentiating equation (10) with respect to the
unknown parameters and setting the partial derivatives equal to zero, we solve for an estimate of
\( \beta_1, \ldots, \beta_5, \lambda, \) and \( \sigma^2 \).

Individual-specific estimates of inefficiency measured by Jondrow et al. (1982), using the
conditional mean of \( u \) given \( \epsilon \), involve the following steps:

First find the joint density of \( u \) and \( v \), which may be written as:
\( f(u, v) = f(u) f(v) = \frac{1}{\pi \sigma_u \sigma_v} \exp \left[ -\frac{1}{2} \frac{u^2}{\sigma_u^2} - \frac{1}{2} \frac{v^2}{\sigma_v^2} \right] \), \( u \geq 0 \). (11)

Then, using the relationship of \( \varepsilon = v - u \), the joint density of \( u \) and \( \varepsilon \) is

\[
\mathcal{f}(u, \varepsilon) = \frac{1}{\pi \sigma_u \sigma_v} \exp \left[ -\frac{1}{2} \frac{u^2}{\sigma_u^2} - \frac{1}{2} \frac{(u^2 + \varepsilon^2 + 2u\varepsilon)}{\sigma_v^2} \right].
\] (12)

The density of \( \varepsilon \) as given by Maddala (1977) in equation (6) above and in Aigner, Lovell, and Schmidt (1977) as used by Jondrow et al. (1982) may be rewritten as:

\[
f(\varepsilon) = \frac{\sqrt{2}}{\sigma \sqrt{\pi}} \left[ 1 - F\left( \frac{\varepsilon \lambda}{\sigma} \right) \exp \left( -\frac{\varepsilon^2}{2\sigma^2} \right) \right] \text{ (Maddala)} \]

\[
f(\varepsilon) = \frac{2}{\sqrt{2\pi}\sigma} (1 - F) \exp \left[ -\frac{1}{2\sigma^2} \varepsilon^2 \right] \text{ (Aigner et al.)}
\]

Therefore, following Jondrow et al. (1982), the conditional density of \( u \) given \( \varepsilon \), for half normal distribution, is:

\[
f(u|\varepsilon) = \frac{f(u, \varepsilon)}{f(\varepsilon)} = \frac{1}{1 - F\left( \frac{\varepsilon \lambda}{\sigma} \right)} \exp \left[ -\frac{1}{2\sigma^2} \left( \frac{u + \sigma_u^2 \varepsilon}{\sigma_v^2} \right)^2 \right] \text{ } u \geq 0
\] (13)

where \( \sigma^2 = \frac{\sigma_u^2 \sigma_v^2}{\sigma^2} \) and the mean of this conditional density function is written as:

\[
E[u_i|\varepsilon_i] = \mu_u + \sigma_u \frac{f(u_u/\sigma_u)}{1 - F(\mu_u/\sigma_u)}
\] (14)

where \( u_u = -\sigma_u^2 \varepsilon / \sigma^2 \), \( \mu_u / \sigma_u = \varepsilon \lambda / \sigma \), and \( f(.) \) and \( F(.) \) represent the standard normal density and cumulative distribution functions, respectively.

**The Data Set**

Relevant data for the 40 school districts in Utah were developed from reports prepared by
the Utah State Office of Education (1992-93) and the Utah Education Association (1993).

Output (\(y_i\)) is measured by the average 11th grade standardized test score for each school district. Use of multiple outputs, such as the proportion of students passing each of the reading, writing, and mathematics skills tests, are common in DEA studies (for example, see Grosskopf and Weber 1989 and McCarty and Yaiswarng 1993), but, because such data were not available, we use a single-output measure. The inputs are the student-teacher ratio (\(x_1\)), expenditure per student other than staff salaries (\(x_2\)),\(^3\) percentage of professional staff with an advanced degree (\(x_3\)), percentage of students who qualify for Aid to Families with Dependent Children (AFDC) subsidized lunch (\(x_4\)), and percentage of district population having completed high school (\(x_5\)).

The first two variables, \(x_1\) and \(x_2\), are proxies for the level of instructional inputs, \(x_3\) measures quality, and \(x_4\) and \(x_5\) measure the socioeconomic status of the students. In the single equation model, the first three variables (\(x_1, x_2, x_3\)) are subject to control by management, while \(x_4\) and \(x_5\) measure local socioeconomic characteristics that are beyond such control.\(^4\) The summary statistics for both inputs and output are reported in Table 1.

Following Schmidt and Lovell (1979) and Battese and Coelli (1988), a Cobb-Douglas functional form of the production function is postulated. The production function in log linear form is written as:

\[
\ln y_i = \alpha_0 + \beta_1 \ln x_1 + \beta_2 \ln x_2 + \beta_3 \ln x_3 + \beta_4 \ln x_4 + \beta_5 \ln x_5 + v - u
\]  

\(^3\)This is used as a proxy for instructional input following McCarty and Yaiswarng (1993).

\(^4\)There have been a number of studies of economies of scale in school operation where school or school district size has been shown to have a significant negative effect on cost or expenditure per student. As there may be a similar scale effect here, a specification of the model was estimated that included average school size as an explanatory variable but it had very little effect on the measure of inefficiency.
Table 1. Summary Statistics of Utah School Districts

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average test score</td>
<td>52.100</td>
<td>7.523</td>
<td>30.000</td>
<td>68.000</td>
</tr>
<tr>
<td>Student-teacher ratio</td>
<td>22.119</td>
<td>3.441</td>
<td>11.600</td>
<td>27.560</td>
</tr>
<tr>
<td>Percentage of professional staff with advanced degree</td>
<td>26.858</td>
<td>10.427</td>
<td>3.310</td>
<td>46.970</td>
</tr>
<tr>
<td>Expenditure per student other than staff salaries</td>
<td>2,661</td>
<td>899</td>
<td>1,798</td>
<td>6,428</td>
</tr>
<tr>
<td>Percentage of population with high school diploma</td>
<td>82.817</td>
<td>6.137</td>
<td>59.700</td>
<td>91.600</td>
</tr>
<tr>
<td>Percentage of students receiving subsidized lunch</td>
<td>25.650</td>
<td>10.618</td>
<td>5.000</td>
<td>51.000</td>
</tr>
</tbody>
</table>

where $y_i$ is the educational output (i.e., average test score), the $x_i$'s are the inputs described above, and $\nu_i \sim N(0, \sigma^2_\nu)$ and $u_i \sim N(0, \sigma^2_u)$. The condition that $u_i \geq 0$ allows production to occur below the stochastic production frontier.

The following relationships between output and each explanatory variables are hypothesized:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Hypothesized Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-teacher ratio</td>
<td>$\beta_1$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>Percentage of professional staff with advanced degree</td>
<td>$\beta_2$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>Expenditure per student other than staff salary</td>
<td>$\beta_3$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>Percentage of population with high school education</td>
<td>$\beta_4$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>Percentage of students receiving subsidized lunch</td>
<td>$\beta_5$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>
Empirical Results

Maximum-likelihood estimates of the parameters based on half normal and exponential distributions of $u$ are reported in Table 2. Except for expenditure per student, all the coefficients have the correct signs, but only the coefficients on student-teacher ratio, expenditure per student, and percentage of population with high school education are significant at the 0.05 or lower probability level in either equation. Of these, the most important is the percentage of the local population having a high school education where a 1% change is associated with a 0.98% to 1.13% change in test scores. This indicates the importance of the environment for learning provided in the home. The negative sign on the student-teacher ratio is as expected and confirms the conventional wisdom that smaller classes are more conducive to better learning.

The net effect of greater spending per student is negative, suggesting that some school districts are more efficient than others. Further, it is consistent with the criticism that simply spending more money per student does not guarantee better student performance. Finally, the welfare variable has the expected negative sign, but the coefficient is not statistically significant.

These results are consistent with those obtained by Walberg and Fowler (1987) with regard to the positive relationship between quality of instructional staff and test score. However, they found that the student-teacher ratio (i.e., the class size) is among the weakest variables influencing learning and tends to be associated with lower scores; our results indicate otherwise.

A comparison of technical inefficiency (i.e., $1-e^{-u}$) based on half normal and exponential distributions of the one-sided component of the disturbance are compared and contrasted in columns (3) and (5) of Table 3, and a frequency distribution of these measures is shown in Table 4. While there are differences in the measures of inefficiency between the half normal and
Table 2. Parameter Estimation  
Dependent Variable: Ln (Test Score)

<table>
<thead>
<tr>
<th>Variables</th>
<th>MLE (Half Normal)</th>
<th>MLE (Exponential)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.4306</td>
<td>0.8835</td>
</tr>
<tr>
<td></td>
<td>(1.183)</td>
<td>(0.643)</td>
</tr>
<tr>
<td>ln (student-teacher ratio)</td>
<td>-0.4238*</td>
<td>-0.2633</td>
</tr>
<tr>
<td></td>
<td>(-4.829)</td>
<td>(-1.779)</td>
</tr>
<tr>
<td>ln (percentage of professional staff with advanced degree)</td>
<td>0.0363</td>
<td>0.0494</td>
</tr>
<tr>
<td></td>
<td>(0.767)</td>
<td>(1.154)</td>
</tr>
<tr>
<td>ln (average expenditure per student other than staff salaries)</td>
<td>-0.143*</td>
<td>-0.0505</td>
</tr>
<tr>
<td></td>
<td>(-2.182)</td>
<td>(-0.532)</td>
</tr>
<tr>
<td>ln (percentage of population with high school diploma)</td>
<td>1.1290*</td>
<td>0.9775*</td>
</tr>
<tr>
<td></td>
<td>(5.872)</td>
<td>(3.616)</td>
</tr>
<tr>
<td>ln (percentage of students receiving subsidized lunch)</td>
<td>-0.0010</td>
<td>-0.0282</td>
</tr>
<tr>
<td></td>
<td>(-0.024)</td>
<td>(-0.563)</td>
</tr>
<tr>
<td>$\sigma = (\sigma_v^2 + \sigma_u^2)^{1/2}$</td>
<td>0.197*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.302)</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td></td>
<td>8.5474*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.355)</td>
</tr>
<tr>
<td>Log of the likelihood function</td>
<td>35.9298</td>
<td>32.2008</td>
</tr>
<tr>
<td>$\sigma^2_n$</td>
<td>0.0388</td>
<td>0.0137</td>
</tr>
<tr>
<td>$\sigma^2_v$</td>
<td>0.0000</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

*t-statistics are in parentheses.
*—indicates coefficients are significant at 5% or lower probability.
Table 3. Measures of Technical Inefficiency of Individual School Districts Using Half Normal and Exponential Distribution of $u$

<table>
<thead>
<tr>
<th>District Code</th>
<th>District Size</th>
<th>Half Normal</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inefficiency</td>
<td>Rank</td>
</tr>
<tr>
<td>11</td>
<td>1,576</td>
<td>0.0004</td>
<td>1</td>
</tr>
<tr>
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Table 4. Distribution of School Districts by Degree of Technical Inefficiency

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<th>Technical Inefficiency Range</th>
<th>Number of Districts</th>
<th>Half Normal Distribution</th>
<th>Exponential Distribution</th>
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<td>Range of technical inefficiency index</td>
<td>0.0004 – 0.3772</td>
<td>0.0188 – 0.3300</td>
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</table>

Exponential distributions, the rankings are very similar. The exponential method estimates lower technical inefficiency in 33 of the 40 cases. The correlation coefficient for the two rankings is 0.975. The mean inefficiency is 0.1373 for the half normal estimates and 0.1059 for the exponential function. The size of the district (i.e., number of students) also is shown in Table 3. There is no obvious relationship between size and efficiency discernible from these data.

Depending on the measure used, 15 to 25 of the school districts have inefficiencies that measure 0.10 or less. This probably should be construed as being good performance given the nature of the production system and the constraints within which resource allocation decisions can be made, especially with regard to personnel, many of whom have rather strong employment security. Unfortunately, there are seven and nine districts estimated by the half normal and exponential methods, respectively, where actual output is more than 20% below potential. In the worst case, output is 33% to 38% below the potential.

\(^5\)The correlation coefficient of two rankings is 0.975.
Summary

This study has attempted to measure efficiency in each of the 40 school districts in Utah using two alternative assumptions about the distribution of the one-sided component $u$ of the disturbance term $\epsilon$. The empirical results indicate: (1) substantial variation in efficiency among districts with the technical inefficiency measure $1 - e^{-u}$ ranging from 0.0004 to 0.3772 for the half normal distribution and from 0.0188 to 0.3300 for the exponential distribution; (2) very close consistency between the rankings of technical inefficiencies based on the two distributions (e.g., the correlation coefficient is 0.975); and (3) the primary factor explaining student test-score performance in order of importance are: percentage of local population who have completed high school, the student-teacher ratio, and expenditure per student, although the latter has an unexpected negative coefficient.

Of the 40 school districts, 15 to 25 (depending on the distributional assumption) are operating with inefficiency levels less than 0.10 (i.e., actual output is no more than 90% of potential output). However, seven to nine districts are estimated to have a technical inefficiency measure greater than 0.20. These data imply substantial variation in managerial or organizational capacity among these districts and the need for some to substantially improve performance.
References


