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VARIABLE TREND IN REAL EXCHANGE RATES

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Do bilateral real exchange rates contain stochastic trends? This paper concentrates on univariate time-series models and uses the Beveridge-Nelson decomposition method to provide evidence that real exchange rates for dollar-deutsche mark, dollar-yen, dollar-pound, and dollar-Swiss franc contain stochastic trends. Using quarterly data for the period 1971 I to 1993 IV, we find that real exchange rates are nonstationary stochastic process which do not revert to a deterministic path. Two implications of this empirical findings is highlighted in this study. First, what is perceived as excessive fluctuations in the real exchange rate may not actually be so since the equilibrium itself shifts over time. Second, the empirical validity of the purchasing power parity theory needs to be examined within the framework of an econometric model that treats the real exchange rate as containing stochastic trends.
I. Introduction

The behavior of exchange rates has been the subject of ongoing debate in recent times. The volatility of the nominal exchange rate has increased, for most economies, since the adoption of the floating exchange rate in the 1970s. With sticky price levels in the short run in countries with moderate inflation, real exchange rates have also been more variable. The volatility of the nominal and real values of the U.S. dollar in terms of German currency, the deutsche mark after the collapse of the Bretton Woods system in 1971 are illustrated in Figures 1 and 2, respectively. Most theories of exchange rate determination have not been able to explain empirically the fluctuations in the real and nominal exchange rates. As Dornbusch (1990) summarizes,

Most models have lost their ability to explain what has happened, when exchange rates moved a lot, as in the 1980s. The dollar movements in the 1980s are to open

![Figure 1. Changes in the nominal exchange rate.](image)
economy macroeconomics what the Great Depression has been to macroeconomics—a baffling, largely unexplained phenomenon (p. 185).

The volatility of the real exchange rate is generally interpreted as the failure of the Purchasing Power Parity (PPP) theory which states that the domestic price level \( P \) is equal to the nominal exchange rate \( E \) times the foreign price level \( P^* \), i.e., \( P = EP^* \) or \( E = P/P^* \). If PPP holds, then the real exchange rate \( e \) will be constant, where

\[
e = \frac{EP^*}{P}.
\]  

One of the reasons for the failure of the PPP in the short run is the macroeconomic factor of price stickiness, which means that prices require time to adjust. In the long run, the changes in the real exchange rate are typically attributed to productivity differences and other real factors. It is argued that these factors affect the real exchange rate slowly and, hence, are characterized as long-term trends (Caves, Frankel, and Jones 1996).
The PPP states that in the long run the real exchange rate should be constant. However, the fluctuations in the real exchange rate could be due to real, as well as nominal, factors. In order to understand the importance of specific sources, these fluctuations have to be decomposed into real (permanent) and nominal (transitory) components. If there have been structural shifts in the real exchange rate due to real factors, then the conventional empirical tests for PPP, which do not account for these shifts, could give misleading results. And, if the equilibrium has shifted over time due to real disturbances, then what is interpreted as the failure of the PPP may not actually be so. That is, the PPP may still hold within the framework of the equilibrium exchange rate.

The main objective of this paper is to use an alternative methodology that accounts for both these sources to explain the fluctuations in the real exchange rate. The two major theoretical models used in this paper to examine and explain the behavior of the exchange rate are Dornbusch's (1976) extended Mundell-Fleming model and Stockman's (1987) equilibrium approach.

The disequilibrium model of the Dornbusch (1976) approach relies on the variations of monetary factors to explain the fluctuations in both the nominal and real exchange rates. Fluctuations in real exchange rates are viewed against the background of a PPP-determined nominal exchange rate which relates long-run exchange rates to long-run price levels of the countries. PPP states that any change in the nominal exchange rate between two currencies is determined by the countries’ relative inflation rates. The implication is that if PPP holds, the real exchange rate remains constant. However, large short-run failures of purchasing power parity have been observed empirically.

The equilibrium approach, however, states that fluctuations in the real exchange rate are mostly due to variability of the real factors. Stockman (1987) states that, “Economic theory predicts
that real disturbances to supplies and demands for goods cause changes in relative prices, including the ‘real exchange rate’ ” (p. 12). The real disturbances, such as a change in productivity or the price of oil, have a permanent effect on real exchange rates. Stockman (1987) argues that “statistical evidence indicates that changes in nominal exchange rates and real exchange rates tend not to be followed quickly by other changes that either reinforce or reverse the original change” (p. 28). The evidence shows these changes to be permanent or persist over long periods of time.

These conflicting ideas have contradictory policy implications. Some theorists suggest that the factors that cause these variations in the real exchange rate should be controlled in order to maintain a steady real exchange rate. Such policies, however, may create distortions in other markets, thus shifting the problem to other areas. The equilibrium theorists, on the other hand, suggest that these fluctuations are adjustments of the real exchange rate to disturbances in the market and, therefore, there is no need for any intervention. There have been numerous empirical studies that either support or reject the implications of each of these theories.

II. THEORETICAL FRAMEWORK

In recent literature, the real exchange rate (e) is defined as the domestic relative price of tradable goods \( (P_T) \) to nontradable goods \( (P_N) \):

\[
e = \frac{P_T}{P_N}
\]

or

\[
e = E \frac{P_T^*}{P_N}
\]

where \( E \) is the nominal exchange rate and \( P_T^* \) is the price of tradables of the foreign country.
The real exchange rate is empirically measured using the relative purchasing power approach. According to this approach, real exchange rate $e_{\text{ppp}}$ is equal to the nominal exchange rate $(E)$ corrected by the price indices which are measured by using a base year. As implied in the definition of the PPP, this is done by multiplying the nominal exchange rate by the ratio of the foreign price level $(P^*)$ to the domestic price level $(P)$, i.e.,

$$e_{\text{ppp}} = \frac{E P^*}{P}. \quad (4)$$

If the relative PPP holds, the real exchange rate will remain constant. Empirically, it has been observed that the nominal exchange rate changes, but this change is not preceded by changes in the price level. This is regarded as an example of the failure of the PPP theory. Dornbusch (1976) provides an explanation for this in his “overshooting” model. The Dornbusch model, a variant of the Mundell-Fleming model, explains the behavior of the nominal and the real exchange rates in the short run. The model traces the consequences of an increase in the domestic money supply on the nominal and the real exchange rates both in the short run and the long run. In the short run, due to sticky prices, there is an immediate depreciation of the exchange rate that is greater than the equilibrium change. The extended Mundell-Fleming model with perfect capital mobility, sluggish price adjustment, and rapid asset market or interest rate adjustment (Dornbusch 1976) explains this overshooting as a “consequence of the combination of perfect foresight and instantaneous asset market adjustment.” In the long run, the goods market adjusts and prices increase and the exchange rate returns to its equilibrium value. Therefore, PPP may not hold in the short run due to instantaneous adjustment in the asset market and sluggish adjustment in the goods market. In the long run, prices are flexible and PPP should hold.
Another explanation for the short-run failure of the PPP is "speculative bubbles." The exchange rates have fluctuated even when there are no movements in the macroeconomic fundamentals. Some economists argue that the cause of the excessive variability of the exchange rates is the expectations of the speculators (Caves, Frankel, and Jones 1996). According to Caves, Frankel, and Jones, when the exchange rate is on the speculative bubble path, it wanders away from the equilibrium value dictated by macroeconomic fundamentals because of self-confirming expectations. In the long run, however, the bubble bursts and the exchange rate returns to its equilibrium value and, therefore, PPP should hold in the long run. But empirical studies have shown that PPP does not hold in the long run.

In the long run, if variations are caused due to permanent or real shocks, the real exchange rate will not be a stationary process. The equilibrium exchange rate theory states that variability of real factors, rather than the variability in monetary factors, has been a major source of fluctuations (Stockman 1987).

Edwards (1991) states that the actual real exchange rate may respond to both monetary and real variables. He defines the equilibrium real exchange rate as the relative price of tradables to nontradables that, for given values of other variables, results in internal as well as external equilibrium. According to him, real exchange rate misalignment is the sustained deviations of the actual real exchange rate from its long-run equilibrium value.

Different rates of economic growth could cause the relative price of traded goods to nontraded goods to shift (Balassa 1964; and Caves, Frankel, and Jones 1996). If a country experiences greater economic growth, the relative price of tradables falls or that of nontradables rises. This is due to productivity increases in the tradable sector caused by the economic growth.
The relative price of nontradables could also rise due to an increase in growth, if these goods are superior goods in the consumer demand functions. Krugman (1991) disagrees with this conventional income-and-price elasticity framework that suggests that differences in the elasticities could cause substantial shifts in equilibrium real exchange rate. He argues that fast growing countries seem to face a high income elasticity of demand for their exports, while having a low income elasticity of demand for imports. The converse is true for slow growing countries. The result of this difference in income elasticities is, it turns out, just about sufficient to make trend changes in real exchange rates unnecessary (p. 42).

There is agreement amongst theorists that exchange rates have fluctuated excessively in the floating exchange rate era. However, there is disagreement as to whether the sources of these fluctuations are nominal factors or real factors or both. In the face of such conflicting theoretical arguments regarding the relative importance of the sources of fluctuations, many empirical studies have been conducted. These studies have modeled the real exchange rate to be a stationary process and are based on testing the null hypothesis that the real exchange rate is a random walk. If the null hypothesis cannot be rejected, then the real exchange rate is nonstationary with the implication that the PPP does not hold and all shocks to the real exchange rate are permanent.

Empirical studies by Roll (1979), Adler and Lehman (1983), Ballie and Selover (1987), Corbae and Ouliaris (1988), Enders (1988), Layton and Stark (1990), and Mark (1990) have found the existence of unit roots in the real exchange rates or noncointegration between the nominal exchange rate and the price ratio. Although Frenkel (1978, 1981), McNown and Wallace (1989), Taylor and McMohan (1988), and Kim (1990) have found evidence supporting the PPP, most
empirical tests of PPP have been unable to reject the hypothesis that the real exchange rate follows a random walk.

Abuaf and Jorion (1990) argue that “... these results reflect the poor power of the tests employed rather than evidence against PPP” (p. 158). Monte Carlo simulations have shown that the power of various unit root tests such as Dickey-Fuller and Philips-Perron tests is very low (Enders 1995). These tests do not have the power to distinguish between a unit root process and a near unit root process. Thus, these tests will too often indicate the presence of a unit root. Further, these studies do not take into account that the trend in the real exchange rate could be stochastic rather than deterministic. The empirical studies have checked for unit roots in the real exchange rate data. The presence of unit root is taken as evidence against the PPP theory. This methodology assumes a deterministic trend. Perron (1989) found that the unit root behavior may be mimicked by a series that contains structural changes. If the data has a stochastic trend, the regression results could be misleading (Stock and Watson 1988; Nelson and Kang 1981; and Nelson and Plosser 1982).

Harvey (1989) defines trend as “... that part of the series which when extrapolated gives the clearest indication of the future long-term movements of the series” (p. 284). The trend should thus be modeled so as to best capture the long-term movements of the series. Therefore, it needs to be formulated in a way that it is flexible enough to respond to general changes in the series. If the real exchange rate has a variable trend, then it will consist of two parts: a stochastic trend and a cyclical part that is stationary. The stochastic trend will be a random walk with drift. In order to isolate the stochastic trend from the original series, a structural time series model has to be defined.

A structural time-series model is one that is set up in terms of components that have a direct interpretation (Harvey 1989). A structural model needs to be set up in such a way that each of its
components are stochastic. These structural models have a corresponding reduced form the autoregressive integrated moving average (ARIMA) representation that give identical forecasts. The Beveridge and Nelson (1981) decomposition method uses the reduced form ARIMA representation to isolate the trend and the cyclical components involving the following steps. The structural model contains a moving average term of infinite order and thus can be expressed as an ARMA process. The reduced form ARIMA is first identified and estimated. The structural model is then derived using the decomposition.

III. METHODOLOGY

There is evidence from current research on business cycles that a common stochastic trend, the cumulative effect of permanent shocks to productivity, underlies the bulk of economic fluctuations (King, Plosser, Stock, and Watson 1991). If the real exchange rate has a common variable trend, then the conclusion is that there has been structural or permanent shifts in the real exchange rate. This conclusion does not, however, necessarily imply that PPP does not hold. That is, all fluctuations of the real exchange rate do not necessarily imply a disequilibrium situation.

The next step is to find out to what extent the observed movements of the real exchange rate are due to real factors and, hence, are an equilibrium phenomenon. The movements in the real exchange rate attributed to real factors are, technically speaking, due to the innovations in the trend. The fluctuations in the real exchange, which are a disequilibrium phenomenon, are temporary in nature and are attributed to the innovations in the stationary component. The main objective of this paper is to determine empirically how much of the observed changes in the real exchange rate during a given period in a particular country is due to permanent (real) factors and how much of it is due
to temporary (monetary) factors. The policy implication is that the part of the fluctuations that is an equilibrium phenomenon is optimal and, hence, government intervention is not necessary. The part of the fluctuations which is a disequilibrium phenomenon needs to be corrected by policy actions. Before any policies are suggested it is important to first decompose the observed changes into permanent and transitory components.

A. Variable Trend in the Real Exchange Rate

In the 1970s, the most popular method for determining cyclical fluctuations in output was to model a time series as having a trend as a deterministic function of time. The variables are decomposed into a secular or growth component and a cyclical component. In modelling the real exchange rate the simple model containing a linear time trend is given as follows:

$$e_t = \alpha + \beta t + \epsilon_t,$$

where $e_t$ is the real exchange rate, $t$ stands for time trend, $\epsilon_t$ has mean zero, and variance $\sigma_{\epsilon t}$ and $\epsilon_t$ is serially uncorrelated. The idea behind this specification is that the potential output is measured along the trend line and the residuals measure cyclical fluctuations around the trend output. The main drawback of this model is that the trend is assumed to be a deterministic function of time. But the trend itself may vary over time.

When the time series has a variable or stochastic trend, the conventional regression analysis containing a linear time trend in the model could give misleading results (Nelson and Plosser 1982; Stock and Watson 1988). Nelson and Kang (1981) have shown that to impose a deterministic trend when one is not present may distort the apparent statistical properties of the resulting cycle. The secular movement need not be modeled by a deterministic trend. If the trend is of a stochastic nature
rather than deterministic nature, then models based on time trend residuals will be misspecified (Nelson and Plosser 1982).

The deviations from the PPP equilibrium value may be due to permanent and transitory disturbances. There may have been structural (permanent) changes in the nominal exchange rate thus causing structural (permanent) changes in the real exchange rate. This possibility will be studied using the recent advances in time-series analysis. The hypothesis that will be tested in this study is whether the trend in the real exchange rate is a random walk process. This will be done by showing that the time series of the real exchange rate belongs to the class of homogenous nonstationary ARIMA process. Then, following Beveridge and Nelson (1981), the time series will be decomposed into two components—the permanent and the stationary.

Generally, the nominal and the real exchange rates are modeled as a random walk with drift. The drift is a deterministic trend. The real exchange rate data in this study is modeled as having a variable trend. As stated earlier, this approach provides a good approximation to the long-run behavior of the real exchange rate. Stock and Watson (1988) define the "variable trend" as trend increasing in each quarter by some fixed amount on average; however, in any given quarter, the trend may deviate from its average by some unforecastable random component. This formulates the trend itself as a random walk model with drift. Suppose the variable real exchange rate $e_t$ is integrated of order one. If $e_t$ contains a stochastic trend, it can be written as,

$$e_t = \mu_t + \epsilon_t.$$  

(6)

The stochastic trend component $\mu_t$ is a random walk with a drift $\beta$ and is written as

$$\mu_t = \mu_{t-1} + \beta + u_t,$$  

(7)

where $\epsilon_t$ is stationary and is the transitory part. Both $\epsilon_t$ in equation (6) and $u_t$ in equation (7) are
white noise. Beveridge and Nelson (1981) have shown that every ARIMA representation contains a random walk stochastic trend and suggest that this might be applicable to most U.S. data. The Beveridge and Nelson (1981) representation for a general ARIMA(p,1,q) process showing the mathematical link between an ARIMA model and the stochastic trend is shown below (Stock and Watson 1988). \( y_t \) is a stationary stochastic process. Based on the Wold decomposition theorem, \( \Delta y_t \) can be represented as the sum of two mutually uncorrelated processes of which one is linearly deterministic and the other is a moving average process of infinite order and is purely indeterministic. Thus, we can write

\[
\Delta y_t = g + c_0 e_t + c_1 e_{t-1} + c_2 e_{t-2} + \ldots \\
= g + c_0 e_t + c_1 L e_t + c_2 L^2 e_t + \ldots \\
= g + \left( \sum_{j=0}^{\infty} c_j \right) e_t \\
= g + c(L) e_t.
\]

The decomposition is a linear one, and the second moments of the process determine the decomposition. A simple linear deterministic process is taken as a proxy for the true generating mechanism. \( L \) is the lag operator, and \( LY_t = y_{t-1} \). In general, a polynomial in the lag operator can be written as \( a(L) = 1 + a_1 L + a_2 L + \ldots \), where \( a_1, a_2, \ldots \) are constants. There may not be any obvious interpretation of \( L \) and, in this case, there is none. Here \( L \) simply conveys the information in the sequence. In the following presentation we write \( y_t \) for \( e_t \), the real exchange rate to facilitate exposition. Since \( c(L) \) is of infinite order, the MA(\( \infty \)) process can be expressed as an ARMA model of the form

\[
a(L) \Delta y_t = f + b(L)e_t,
\]

where \( f \) is a constant and \( a(L) \) and \( b(L) \) are lag polynomials of order \( p \) and \( q \), respectively. Since the actual amount of data is limited, this formulation has less number of parameters. This is what is
known as the principle of parsimony (Box and Jenkins 1976). Inverting \( a(L) \) we can write equation (8) as

\[
\Delta y_t = g + c(L) \epsilon_t,
\]

where \( g = f/\sum_{j=0}^{p} a_j \) and \( c(L) = b(L)/a(L) \). Note that the C weights, i.e., \( c_1, c_2, \ldots \) can be obtained by \( c(L) = b(L)/a(L) \). Now, recursively, substituting lagged \( \Delta y_t \) and assuming \( y_o = 0 \) and \( \epsilon_r = 0 \) for \( r \geq 0 \), the final expression is given as

\[
y_t = g t + h \sum_{r=1}^{q} c_j \epsilon_r + d(L) \epsilon_t
\]

where \( h = \sum_{j=0}^{q} c_j \Rightarrow h = \frac{\sum_{j=0}^{q} b_j}{\sum_{i=0}^{p} a_i} \) and \( d = -\sum_{i=0}^{q} c_j \). We can write equation (10) as

\[
y_t = y_t^p + y_t^s
\]

where \( y_t^p = g t + h \sum_{r=1}^{q} \epsilon_r \) and \( y_t^s = d(L) \epsilon_t \) or

\[
y_t^p = g + y_{t-1}^p + h \epsilon_t,
\]

where \( y_t^p \), the stochastic trend, is a random walk with a drift \( g \). The permanent and stationary components are both proportional to the disturbance term \( \epsilon_t \) and are, thus, perfectly correlated. An increase in the trend component will result in a decrease in the stationary component. This implies that the change in the stationary component will either augment or offset part of the permanent component, depending on the increase or decrease in the stationary component.

**B. Computation of the Variable Trend**

Beveridge and Nelson (1981) define the permanent part as that part of \( y_t \) which will stay in the future. Obviously, one has to look into the future value of \( y_t \) at some future date. To get the forecasted value, the Box-Jenkins method of identification is used. So now the next step is to
identify and estimate the appropriate ARIMA models for the data sets. Then the Beveridge and Nelson decomposition is applied to isolate the permanent and stationary components. The forecasted value is used to find the permanent part of \( y_t \). The forecasted value for \( k \) periods ahead is the conditional expectation of \( y_{t+k} \) given the information available at time \( t \).

\[
E_t y_{t+k} = \mu_0 + \mu_1 y_t + \mu_1^2 + \ldots + \mu_1^{j-1} + \mu_1^j y_t.
\]

For example, for one period ahead forecast

\[
E_t y_{t+1} = \mu_0 + \mu_1 y_t.
\]

Using this forecast function we get

\[
y_{t+k} = y_t + k \mu + \sum_{i=1}^{k} \epsilon_{t+i} + \beta_1 \sum_{i=1}^{k} \epsilon_{t+i-1} + \beta_2 \sum_{i=1}^{k} \epsilon_{t+i-2},
\]

and

\[
E_t y_{t+1} = \mu_0 + y_t + \beta_1 \epsilon_t + \beta_2 \epsilon_{t-1},
\]

\[
E_t y_{t+1} = \mu_0 + y_t + \beta_1 \epsilon_t + \beta_2 \epsilon_{t-1}.
\]

Since

\[
E_t \epsilon_{t+i} = 0 \text{ for } i > 0,
\]

all forecasts for \( K > 1 \) will be equal to

\[
E_t y_{t+k} = k \mu_0 + y_t + (\beta_1 + \beta_2) \epsilon_t + \beta_2 \epsilon_{t-1}.
\]

Once we get the forecasts using the Beveridge and Nelson (1981) method, we can now decompose the series into its trend and cyclical components. The cyclical component is given by,

\[
y_t - \mu_t = -(\beta_1 + \beta_2) \epsilon_t - \beta_2 \epsilon_{t-1}.
\]

If \( \epsilon_t \) changes, then change in trend is \( 1 + \beta_1 + \beta_2 \). Then, using the betas and epsilons, we can find the cyclical and trend components. This is similar to the Stock and Watson (1988) approach. The betas correspond to the C(L) in equation (10). The Beveridge and Nelson (1981) method is
operationalized by the following:

\[ y_{t-k} = \Delta y_{t-k} + \Delta y_{t-k-1} + \Delta y_{t-k-2} + \ldots + \Delta y_t. \]

The trend is the current value of \( y_t \) plus the sum of all the forecasted changes in the series

\[ \lim_{k \to \infty} E(y_{t+k}) = \lim_{k \to \infty} E(\Delta y_{t+k} + \Delta y_{t+k-1} + \Delta y_{t+k-2} + \ldots + \Delta y_{t+1}) + y_t. \]

Therefore, the cyclical part can be calculated as

\[ y_t - \lim_{k \to \infty} (E(y_{t+k} - k \mu)) = \lim_{k \to \infty} E(\Delta y_{t+k} + \Delta y_{t+k-1} + \Delta y_{t+k-2} + \ldots + \Delta y_{t+1}) - k \mu. \]

The construction of an ARIMA(p, d, q) model of a stationary series consists of a three-step procedure. It involves: (i) model identification, (ii) model estimation, and (iii) diagnostic checks on model adequacy. The identification process involves the selection of appropriate values for p, d, and q. Based on the correlograms of the series, the orders of p and q are determined and the ARIMA model is specified. This step requires judgement rather than the use of any clear cut rules. The next step is the estimation of the model using appropriate statistical tools. The third step is diagnostic checking for the adequacy of the tentative model. There are various tests for this, however, the LM test is considered the most reliable (Maddala 1992).

**IV. MEASUREMENT OF REAL EXCHANGE RATE**

The process of construction of indexes is associated with numerous problems. The problems due to severe data constraints are compounded by the problems faced while trying to find proxies and deciding on which indexes to use. Some authors have suggested domestic consumer price index (CPI) as a proxy for nontradable prices and a foreign wholesale or producer price index as a proxy for the world price of tradables. In this paper two indexes of the bilateral real exchange rate have been constructed for domestic and foreign countries as suggested by Edwards (1991):
where WPI* is wholesale price index of the foreign country (Germany, Japan, United Kingdom, and Switzerland); CPI* is the consumer price indexes of the foreign country; and CPI is the consumer price index of the domestic country (U.S.A.). The real exchange rate indexes have been constructed using quarterly data on nominal exchange rate and price indexes of the two countries for the period 1971:1 to 1993:IV from *The Encyclopedia of World Economics.*

### V. ESTIMATION RESULTS AND ANALYSIS

The autocorrelation function and the partial autocorrelation of the first differences of the log of el and e2 for the four countries were examined. They were identified and estimated as ARIMA processes. The Beveridge and Nelson (1981) decomposition is now applied to these data sets. Using equations (12) and (13) we can compute the permanent component (h) of the log real exchange rate. The results of the estimated models for each of the four countries have been summarized below.

**Germany e1**

\[
\Delta e_t = 0.0021 + \epsilon_t - 0.143 \epsilon_{t-1} - 0.282 \epsilon_{t-2} - 0.354 \epsilon_{t-3} + 0.768 \epsilon_{t-4}, \text{ S.E } = 0.0749
\]

**Germany e2**

\[
\Delta e_t = 0.0035 + \epsilon_t - 0.219 \epsilon_{t-2} - 0.383 \epsilon_{t-3} + 0.659 \epsilon_{t-4}, \text{ S.E } = 0.0783
\]
Japan e1

$$\Delta \epsilon_t = -5.229 + 0.875 \Delta \epsilon_{t-1} + \epsilon_t - 0.407 \epsilon_{t-1}, \text{ S.E. } = 0.089$$

Japan e2

$$\Delta \epsilon_t = -5.084 - 0.951 \Delta \epsilon_{t-1} + \epsilon_t - 0.391 \epsilon_{t-1}, \text{ S.E. } = 0.105$$

Switzerland e1

$$\Delta \epsilon_t = 0.0025 + \epsilon_t - 0.257 \epsilon_{t-2} - 0.2996 \epsilon_{t-3} + 0.540 \epsilon_{t-4}, \text{ S.E. } = 0.0846$$

Switzerland e2

$$\Delta \epsilon_t = 0.007 + \epsilon_t - 0.277 \epsilon_{t-2} - 0.299 \epsilon_{t-3} + 0.551 \epsilon_{t-4}, \text{ S.E. } = 0.089$$

United Kingdom

$$\Delta \epsilon_t = 0.015 - 0.296 \Delta \epsilon_{t-1} + \epsilon_t - 0.263 \epsilon_{t-2} - 0.256 \epsilon_{t-3} + 0.561 \epsilon - t - 4, \text{ S.E. } = 0.091$$

Now for 100 periods ahead forecast for Germany e1 we get

$$\mathbb{E}(\Delta y_{t+100} + \Delta y_{t+99} + \Delta y_{t+98} + \ldots + \Delta y_{t+1}).$$

or $$\mathbb{E} \epsilon_{t+100} = 100(0.0021) + 0.992 \epsilon_t.$$ The permanent component, as calculated from equations (10) and (11), have been summarized in Table 1. For the observation 1971q2, the stochastic portion of the trend is $$\epsilon_{1971q2} + 0.992 \epsilon_{1971q2},$$ and the cyclical part is 0.008.

The importance of the trend component in each of these real exchange rates can be seen in Table 1. If $$\epsilon$$ has a pure stochastic trend with no stationary component, then a 1% increase in $$\epsilon$$ above its forecasted value in any given quarter will increase the long-run forecast of $$\epsilon$$ by exactly 1%. This is because any shock to this system will be purely permanent. On the other hand, if $$\epsilon$$ is a purely stationary process with a deterministic trend, then the long-run forecast will increase by 0%. All shocks will be transitory and, therefore, there will be no impact on the long-run forecast value.
Table 1. Long-Run Predicted Increase in Real Exchange Rate

<table>
<thead>
<tr>
<th>Real Exchange Rates Data</th>
<th>Univariate Statistical Model</th>
<th>Long-Run Increase in $\text{e}$ Predicted from a 1% Unforeseen Increase in $\text{e}$ in One Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany e1</td>
<td>ARIMA(0,1,4)</td>
<td>0.992</td>
</tr>
<tr>
<td>Germany e2</td>
<td>ARIMA(0,1,4)</td>
<td>1.057</td>
</tr>
<tr>
<td>Japan e1</td>
<td>ARIMA(1,1,1)</td>
<td>0.677</td>
</tr>
<tr>
<td>Japan e2</td>
<td>ARIMA(1,1,1)</td>
<td>0.640</td>
</tr>
<tr>
<td>Switzerland e1</td>
<td>ARIMA(0,1,4)</td>
<td>0.983</td>
</tr>
<tr>
<td>Switzerland e2</td>
<td>ARIMA(0,1,4)</td>
<td>0.975</td>
</tr>
<tr>
<td>United Kingdom e1</td>
<td>ARIMA(4,1,3)</td>
<td>1.480</td>
</tr>
</tbody>
</table>

The data for the four countries provide evidence of a stochastic trend rather than a deterministic one. For each of these countries the long-run forecast of $\text{e}$ changes by a positive amount much greater than 0% for a 1% unforeseen increase in $\text{e}$. This indicates that there is some permanent component in this change. For example, if the $\text{e}_1$ for United States-Germany (\$/DM) grows by an unforeseen 1%, then 0.992% of that growth is due to the innovations in the trend and 0.008% is due to the stationary innovations. As these innovations are perfectly correlated, they either augment or partially offset each other. If the trend increases by 0.992%, the cyclical component increases by 0.008% initially. It will die off gradually, leaving a net increase of 0.992% in the long-run forecasted value of $\text{e}$.

From the results we also see that in the cases of United States-Germany, United States-United Kingdom, and United States-Switzerland, the permanent components are larger than the stationary components. Although the permanent component is smaller for the United States-Japan case, the results from all four countries indicate that the equilibrium has shifted over
time due to the real shocks. The deviations away from this shifting equilibrium are due to the nominal factors. The conventional methods of testing to see if PPP holds have used a deterministic trend as the long-run equilibrium value around which the real exchange rate fluctuates. If this equilibrium is varying over time, then the magnitude of the fluctuations in the real exchange rate may have been overstated. So what is perceived as excessive fluctuations in the real exchange rate may not actually be so. And further, the results provide evidence that the PPP may hold within the framework of the equilibrium exchange rate. The long-run equilibrium real exchange rate is determined by real factors and is stochastic rather than deterministic.

The results obtained in this paper can be explained better if we look at the specific permanent and transitory shocks to the equilibrium real exchange rate. The difference in the magnitude of the transitory shocks in case of Japan may be explained by the monetary policies of Japan in the recent past.

VI. SUMMARY AND CONCLUSIONS

The sources of fluctuations in the real exchange rate has been the subject of debate amongst economists. The exchange rate theories based on the PPP have attributed these movements to monetary shocks. The equilibrium theory disagrees and states that these fluctuations are reactions of the market to real shocks. The problem arises while adopting remedial policies, if any, for the excessive fluctuations. The next step is to turn to empirical studies to see which theory is supported by the data. Unfortunately, these studies do not throw any further light on this issue. There are studies supporting each of these views.
In the existing literature, empirical studies used conventional methods, such as checking for unit roots in the series. If the null hypothesis, that there is unit root in the data, could not be rejected, then that was taken as evidence against PPP theory. These studies used a deterministic trend in their methodology and that could have given misleading results. This paper used the concept of variable trends to study the sources of fluctuations of the real exchange rate in terms of permanent and transitory components. The variable trend captures the long-run movements of the series. If there were any real shocks to the series, these would show up in the stochastic part of the trend. All nominal shocks would be reflected in the cyclical component. The series was decomposed into a variable trend (permanent component) and a cyclical part (transitory component) using the Beveridge and Nelson (1981) method.

The results show that the fluctuations in the real exchange rate are due to real as well as nominal shocks. The innovations in the permanent component are a major portion of the overall disturbances in the case of all countries except Japan. These differences can be explained by looking into the relative importance of productivity shocks and monetary shocks. The conclusion from these results is that the equilibrium value of the real exchange rate is shifting due to real shocks. The fluctuations in the real exchange rate are overstated due to incorrect modelling. The equilibrium exchange rate determined by real factors is stochastic in nature. Further, this study provides evidence for the PPP theory within the framework of a stochastic long run real exchange rate.
REFERENCES


Appendix

This appendix shows the Beveridge and Nelson (1981) decomposition of an ARIMA(p,1,q) process into its permanent and stochastic components. Using Wold's decomposition theorem, we get

\[ \Delta y_t = g + c_0 \varepsilon_t + c_1 \varepsilon_{t-1} + \ldots \]  
(A.1)

\[ y_t = g + y_{t-1} + c_0 \varepsilon_t + c_1 \varepsilon_{t-1} + \ldots \]  
(A.2)

Now, recursively substituting, we get

\[ y_{t-1} = g + y_{t-2} + c_0 \varepsilon_{t-1} + c_1 \varepsilon_{t-2} + \ldots \]  
(A.3)

\[ y_t = gt + (1 + c_1 L + c_2 L^2 + \ldots) \sum_{i=0}^t \varepsilon_t \]  
(A.4)

\[ y_t = gt + c_0 \sum_{i=0}^t \varepsilon_t + c_1 \sum_{i=0}^t \varepsilon_{t-1} + c_2 \sum_{i=0}^t \varepsilon_{t-2} + \ldots + c_1 \varepsilon_t - c_1 \varepsilon_{t-1} + c_2 \varepsilon_t - c_2 \varepsilon_{t-1} + \ldots \]  
(A.5)

\[ y_t = gt + \sum_{j=0}^\infty c_j \sum_{i=1}^t \varepsilon_t + d(L) \varepsilon_t. \]  
(A.6)

The final expression is given as

\[ y_t = gt + h \sum_{i=1}^t \varepsilon_t + d(L) \varepsilon_t, \]  
(A.8)

where \( h = \sum_{j=0}^\infty c_j \) and \( d = -\sum_{j=1}^\infty c_j \), or, we can write equation (A.8) as

\[ y_t = y_{tp} + y_{ts}, \]  
(A.9)

where \( y_{tp} = gt + h \sum_{i=1}^t \varepsilon_t \) and \( y_{ts} = d(L) \varepsilon_t \).