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SNOWMELT SIMULATION

by

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College of Engineering

Logan, Utah

April 1969
Abstract

The rapid growth in recent years of a variety of demands upon available water resources has lead to an increasing interest in more fundamental approaches to the science of hydrology. Accompanying this growth has been a need for an increased understanding of the snowmelt process. A completely adequate description of the entire physical process of snowmelt under all conditions is not yet available. The complex interrelated and variable nature of the snowmelt processes that occur simultaneously complicate the problem.

A preliminary mathematical model of the snowmelt process has been developed in which processes such as pack settlement rates and energy flow in the pack by means of both conduction and liquid movement are considered. Factors such as an temperature, surface albedo, and degree and direction of slope are also included. A temperature criterion is applied to predict the form of precipitation input (snow or rain) to the model. Equations of the various processes are synthesized into a dynamic model of the total system by means of an electronic analog computer. This computer was utilized primarily
because of its ability to (1) perform repetitive operations at very high speeds and (2) solve directly the several time-dependent partial differential equations included in the model. Field data from snow laboratories operated by the Corps of Engineers and highly instrumented watersheds of the Agricultural Research Service are being used to test and verify the model. Initial results have indicated close agreement between observed and computed results. Sensitivity studies have been conducted, and work is continuing to further test and improve the model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>albedo of the top surface of a snowpack</td>
</tr>
<tr>
<td>D</td>
<td>actual depth of the snowpack taking into account settlement, evaporation, and melt losses</td>
</tr>
<tr>
<td>D&lt;sub&gt;a&lt;/sub&gt;</td>
<td>depth of the snowpack taking into account settlement but neglecting evaporation and melt losses</td>
</tr>
<tr>
<td>K&lt;sub&gt;v&lt;/sub&gt;</td>
<td>the thermal diffusivity of the snow in feet&lt;sup&gt;2&lt;/sup&gt;/time</td>
</tr>
<tr>
<td>M&lt;sub&gt;rp&lt;/sub&gt;</td>
<td>rate of snowmelt due to the specific heat of rainfall</td>
</tr>
<tr>
<td>M&lt;sub&gt;r&lt;/sub&gt;</td>
<td>total melt rate of a snowpack, including both surface and ground melting</td>
</tr>
<tr>
<td>M&lt;sub&gt;rg&lt;/sub&gt;</td>
<td>groundmelt rate beneath the pack</td>
</tr>
<tr>
<td>M&lt;sub&gt;rs&lt;/sub&gt;</td>
<td>melt rate at the surface of the pack</td>
</tr>
<tr>
<td>P&lt;sub&gt;rg&lt;/sub&gt;</td>
<td>precipitation through fall rate, including stemflow</td>
</tr>
</tbody>
</table>
### Symbol Definition

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<tbody>
<tr>
<td>$\bar{\rho}$</td>
<td>average density, or average water equivalent per unit depth of the snowpack</td>
</tr>
<tr>
<td>$\bar{\rho}_i$</td>
<td>initial density of newly fallen snow</td>
</tr>
<tr>
<td>$R_I_{h}$</td>
<td>the radiation index for a horizontal surface at the same latitude as the particular watershed under study</td>
</tr>
<tr>
<td>$R_I_{s}$</td>
<td>the radiation index on a surface possessing a known degree and aspect of slope</td>
</tr>
<tr>
<td>$T_a$</td>
<td>surface air temperature in degrees Fahrenheit</td>
</tr>
<tr>
<td>$T_s$</td>
<td>the snow temperature in degrees Fahrenheit</td>
</tr>
<tr>
<td>$W_a$</td>
<td>total accumulated water equivalent of the snowpack neglecting evaporation and melt losses</td>
</tr>
<tr>
<td>$W_a(0)$</td>
<td>initial value of the total accumulated water equivalent of the snowpack</td>
</tr>
<tr>
<td>$W_s$</td>
<td>actual water equivalent of the snowpack</td>
</tr>
</tbody>
</table>

### Introduction

The rapid growth in recent years of a variety of demands upon available water resources has led to an increasing interest in more fundamental approaches to the science of hydrology. Accompanying this growth has been the need for an increased understanding of the snowmelt process. Joint investigations by the U. S. Army Corps of Engineers and the U. S. Weather Bureau (10) have contributed significantly to a more fundamental understanding of snow hydrology for
project design and streamflow forecasting. These studies have focused attention upon many of the basic phenomena involved in the snowmelt process. However, a completely adequate description of the entire physical process of snowmelt under all conditions is not yet available.

The problem of synthesizing all of the various phenomena involved in the snowmelt process into a composite model to yield reliable estimates of specific snowmelt rates is difficult. The complex interrelation and variable nature of the many different phenomena occurring simultaneously further complicate the problem. In addition, many of the component parts of the process, such as the time variation of the thermal diffusivity and permeability coefficients within a snowpack, have not yet been adequately described by mathematical expressions. Thus, the search for a practical and dynamic model of the snowmelt process entails not only formulation of relationships for the many phenomena involved, but also testing to determine the relative effects of various basic parameters upon the melting process. It is anticipated that a fundamental and systematic approach will lead to the eventual development of a standard method for synthesizing snowmelt hydrographs on the basis of commonly available hydro-meteorological, physiographical, and geographical information.

Important considerations in the development of any model are the time and space increments adopted. The ultimate in modeling would utilize continuous time and infinitesimal volumes connected as
in the prototype. However, the practical limitations of this approach are obvious. The complexity of a model designed to represent a hydrologic system largely depends upon the magnitudes of the time and spatial increments utilized in the model. In particular, when large increments are applied, the scale magnitude is such that the effects of phenomena which change over relatively small increments of space and time are insignificant. For instance, on a monthly time increment interception rates and changing snowpack temperatures are neglected. In addition, sometimes the time increment chosen coincides with the period of cyclic changes in certain hydrologic phenomena. In this event net changes in these phenomena during the time interval are usually negligible. For example, on an annual basis storage changes within a hydrologic system are often insignificant, whereas on a monthly basis the magnitudes of these changes are frequently appreciable and need to be considered. As time and spatial increments decrease, improved definition of the hydrologic processes is required. No longer can short-term transient effects or appreciable variations in space be neglected, and the mathematical model therefore becomes increasingly more complex, with an accompanying increase in the requirements of computer capacity and capability.

The approach to modeling of the snowmelt process as outlined by this paper was first to consider a rather gross model in terms of time and space. With each succeeding model, definition was improved
by using reduced control volumes and time increments. In this way, the three models discussed herein were developed. The selection of a model for a particular study is entirely dependent upon the degree of refinement required by the problem solution.

**Model Development**

**Large Increments of Time and Space**

Both the complex nature of snowmelt and data limitations prevent a strictly analytical approach to this process. In particular, for the computation of melt on the basis of large time increments, such as a month, a rather empirical approach seemed most suitable. Accordingly, a relationship was proposed which states that the rate of melt is proportional to the available energy and the quantity of precipitation stored as snow. Expressed as a differential equation the relationship appears:

\[
\frac{d[W_s(t)]}{dt} = -k_s(T_a - 35) W_s(t) \quad \ldots \quad \ldots \quad \ldots \quad (1)
\]

in which

\[k_s\]

represents a proportionality constant

From an analysis of snow course data from various parts of Utah, the value of \(k_s\) was determined to be approximately 0.10 for mountain watersheds. For valley floor areas a somewhat higher value of \(k_s\) is applicable. The independent variables on the right side of Equation 1 can be expressed either as continuous functions of time or as step functions consisting of mean constant values applicable throughout
a particular time increment. In this model a time increment of one month is being utilized with the integration being performed in steps over each successive period. Thus, the final value of \( W(t) \) at the end of the period becomes the initial value for the integration process over the following period. A test of Equation 1 is illustrated by Figure 1 which indicates both predicted and actual rates of snowmelt for a watershed in Montana.

**Reducing the Space Increments**

In the snowmelt relationship of Equation 1 surface air temperature is applied as an index of available energy for the melting process. On a regional basis, air temperature is a reasonably good index of available energy at particular elevations. However, even if adequate data were available, air temperature does not provide a satisfactory means of comparing the energy flux among the different facets of a landscape. For a particular elevation Equation 1 would indicate no appreciable differences between melting rates on, for example, easterly and southerly slopes. When large units or areas are considered by a model, the effects of slope differences often tend to average. However, for small zones, slope effects are important and should be considered by a snowmelt relationship.

The potential insolation parameter has been proposed as a means of comparing the energy flux among the different facets of a landscape (2, 5, 8, 9). In the concept of potential insolation the earth's atmosphere is ignored. Thus, irradiation of a surface by
Figure 1. Measured and computed snowmelt rate curves for the Middle Fork Flathead River, Montana, 1947.
direct sunshine is considered to be only a function of the angle between the surface and the sun's rays. This angle, in turn, is a function only of the geometric relationships between the surface and the sun as expressed by latitude, degree of slope and aspect of the surface, and the declination and hour angle of the sun. For a given site the only variation in instantaneous potential insolation will be perfectly cyclical with time, depending upon the changes in hour angle and declination. Thus, the use of potential insolation as a parameter of a surface is sufficiently simple to make feasible its wide application.

Frequently, potential insolation for a particular surface is expressed as a percentage of the maximum possible radiation rate at the outer limit of the earth's atmosphere. The resulting dimensionless quantity is termed radiation index. Figure 2 illustrates digital computer plots of the radiation index calculated for a particular aspect and expressed as a function of slope inclination and solar declination. The latitude is 40 N. Because direct radiation is equal upon facets that show symmetry with respect to a north-south axis, two aspects are represented by this figure.

The application of the potential radiation parameter to watershed studies requires that for each zone or area under consideration the orientation and slope of an effective plane surface be defined such that this surface receives as nearly as possible the same potential insolation as is received by the particular zone.
Figure 2. Radiation index values as a function of slope inclination and time of year.
For the second model a term for radiation index is added to Equation 1 and the result appears as follows:

\[
\frac{d}{dt} \left[ W_s(t) \right] = -k_s (T_a - 35) \frac{RI_s}{RI_h} W_s(t) \quad . \quad . \quad . \quad . \quad . \quad (2)
\]

A computer plot showing snowmelt rate as computed by Equation 2 for the Circle Valley watershed (1962) is shown by Figure 3. The Circle Valley is a subbasin of the Sevier River drainage in Central Utah. Note that the plot indicates snow accumulation during the months of January, February, March, and December of that year. The points of discontinuity in the snowmelt portion of the plot result from the input to the program of mean monthly (rather than continuous) temperature values.

Small Increments of Time and Space

For this model an attempt has been made to represent the various segments of the snow ablation process in a somewhat rational manner. A thoroughly rational approach to the problem of evaluating snowmelt involves a consideration of all three processes of heat transfer, namely radiation, convection, and conduction. The relative importance of each of these processes is highly variable, depending upon conditions of weather and local environment. For example, in late spring, given a clear day and fairly open terrain, radiation is the prime factor in the melting of snow. However, under conditions of heavy cloud cover or
Figure 3. Computed accumulated snow storage equivalent on the watershed area of Circle Valley during 1962.
heavily forested terrain, radiation becomes a minor element. In the exposed areas wind is an important element in the convection process, while in heavily forested areas wind becomes a minor factor.

The sources of heat involved in the melting of snow are as follows (6, 10):

1. adsorbed solar radiation
2. net long wave (terrestrial) radiation
3. convection heat transfer from the air
4. latent heat of water vaporization by condensation from the air
5. conduction of heat from the ground
6. heat content of rain water

Rational formulas based upon the various factors listed in the preceding paragraph have been developed for snowmelt at a point. However, data required for the solution of these relationships are frequently lacking. For this reason and because of the complex nature of the process, in this model much reliance is still placed upon the empirical approach in the development of relations for predicting snowmelt. Many of the equations used in the model were developed from published charts and other available sources of information, and heavy reliance was placed upon published work in this area. While these equations have not as yet been extensively tested, it is considered that they are sufficiently general in nature to permit their broad geographic application with perhaps appropriate adjustments in certain of the constants.
The two primary causes of snowpack ablation are net evaporation from the top surface of the pack and melting of the snow. Evaporation losses from snow surfaces are estimated in the model by an empirical equation (7). Values of the independent variables in this equation are, of course, those which apply to the particular time increment being utilized in the model. For short-term melt estimates, such as on a daily basis, it is necessary to take into account the temperature of the snowpack. Significant melt will not appear at the bottom of the pack until it has reached an isothermal temperature of 32 F and its free water holding capacity has been satisfied (10). Thus, it is necessary to predict at any time, \( t \), both the temperature profile within the pack and its average free water holding capacity.

A flow chart of the snow accumulation and ablation processes is shown by Figure 4. A very brief discussion of each segment of this process is presented herein.

**Free water holding capacity.** The free water holding capacity of snow is the amount of water held within the pack at the time when snow surface melt first appears at the bottom of the pack. This capacity is a function of pack density and several other factors. Since these other factors are very difficult to evaluate, it is common to use a relationship which is a function of average snow pack density alone. Figure 5(a) indicates an approximation to a relationship developed by the U. S. Corps of Engineers (10).
Figure 4. A flow chart of the snow accumulation and ablation process.
Figure 5(a). Liquid water-holding capacity of snow as a function of snow density.

Figure 5(b). Density of new snow as a function of surface air temperature.
Snowpack density. The density of a snowpack is influenced in the main by two factors, namely, the density of new snow and the compaction and settling of existing snow. Studies at the Central Sierra Snow Laboratory (10) have shown that the density of new snow varies approximately with the surface air temperature (see Figure 5(b)).

The average density of a pack may, of course, be obtained at any time, \( t \), by taking the ratio of the water equivalent to the pack depth at time, \( t \). Changes in snowpack depth are caused by melt, evaporation, snowfall, and settlement. However, it can be assumed with some degree of approximation that evaporation and melt do not cause changes in the average density of the pack. This parameter is then computed by the equation:

\[
\rho(t) = \frac{W_a(t)}{D_a(t)}
\]

in which the value of \( W_a \) at any time, \( t \), is given by summing an initial value of \( W_a \) (expressed as \( W_a(0) \)) and the accumulated precipitation over the time interval, \( t \). Thus:

\[
W_a(t) = W_a(0) + \int P_{rg} \, dt
\]

For a given initial snow depth, the value of \( D_a(t) \) is increased by subsequent snowfalls and decreased by settlement of the pack. Observations (10) indicate that the settlement rate of a snowpack can be expressed as a function of the difference between the average pack density at any time and the maximum pack density. An average maximum density for snow in a well-settled pack is approximately 60 percent (10).
On the basis of this assumption and by applying a settlement time constant, \( k_{sc} \), the following equation was developed for computing the value of \( D_a \) at any time, \( t \).

\[
\frac{d}{dt} \left[ D_a(t) \right] + k_{sc} D_a(t) = \frac{P_{rg}}{\rho_i \cdot 0.06} \left[ D_a(0) + \int_0^t P_{rg} \, dt \right]. \tag{5}
\]

The solution of Equation 5 on the analog computer yields the value of \( D_a(t) \) required in Equation 3 for estimating average density of the snowpack. It is noted that if a study is begun before the accumulation of any snow, \( D_a(0) \) is equal to zero.

A test of Equation 5 is illustrated by Figure 6, which indicates both predicted and actual depths of a snowpack. The equation was applied on the basis of a daily time increment and \( k_{sc} \) was taken as being equal to 0.10. Evaporation and melt losses were assumed to be negligible during this test period.

**Snowpack temperatures.** A procedure for predicting the temperature profile within a snowpack needs to take into account not only the conduction of heat through the snow crystals, but also heat transferred into the pack from both rain and surface melt. Consider first the development of an expression to describe temperature conditions within the snowpack for given boundary values as a function of both depth and time. If it can be assumed that lateral or horizontal heat transfer within the pack is negligible, then for a column of snow of finite diameter the conditions illustrated by Figure 7(a) apply. With respect to boundary conditions, the top surface of the
Note: Observations made at the Willamette Basin Snow Laboratory (reference 49)

Accumulated Daily Snowfall

Observed Snow Depth on Ground

Calculated Snow Depth on Ground, \( D_a \) (equation 5.21)

Figure 6. Comparison between observed and calculated settled snow depths.
\begin{align*}
T &= T_a, \quad T_a < 32 \, \text{F} \\
T &= 32 \, \text{F}, \quad T_a \geq 32 \, \text{F}
\end{align*}

Top Surface of Snowpack

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7a.png}
\caption{An assumed snow cylinder showing boundary temperature values.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7b.png}
\caption{Thermal diffusivity of snow as a function of density.}
\end{figure}

\[ k_v(t) = 0.061 \rho(t) \]
snow is assumed equal to the surface air temperature, $T_a$, for $T_a < 32$ F.
For $T_a \geq 32$ F, the snow surface temperature is 32 F. Several studies have indicated that the temperature at the bottom of the pack is usually maintained at approximately 32 F by heat flow from the ground (3, 10).
In its simplest form the problem then is characterized by one-dimensional heat flow in an assumed homogeneous column of snow of depth $D(t)$ with insulated sides. The value of $D(t)$ is given by

$$D(t) = \frac{W_s(t)}{\rho(t)}$$

(6)

in which $\rho(t)$ is given by Equation 3, and the value of $W_s(t)$ is given by subtracting evaporation and melt losses from the precipitation reaching the ground as snow.

The heat flow equation applicable to this problem is expressed in partial derivative form as follows (4):

$$\frac{\partial^2 T_s}{\partial Z^2} = \frac{1}{K_v} \frac{\partial T_s}{\partial t}$$

(7)

in which

$Z$ = snow depth in feet

$T_s$ = the snow temperature in degrees Fahrenheit

$K_v$ = the thermal diffusivity of the snow in feet$^2$/time

The ever-changing thermal and physical properties of the snow-pack make the theory of heat flow in snow much more complicated than is the case for a homogeneous solid. Thus, the factors which affect the thermal...
diffusivity of snow are its structural and crystalline character, the degree of compaction, the extent of ice planes, the degree of wetness, and the temperature of the snow (10). However, experimental work has shown that density is a generally satisfactory index of the thermal properties of the snowpack. Figure 7(b) illustrates the following empirical relationship between density and thermal diffusivity of snow (10).

\[ K_v(t) = 0.061 \rho(t), \quad (0 < \rho \leq 0.6) \quad \ldots \ldots (8) \]

It is recognized that there is an appreciable variation in snow density with depth, and that a more accurate approach to this problem would be to divide the snowpack into finite depth increments and to consider the entire melt process in each zone. This procedure would, however, considerably complicate the model, and it is anticipated that the results of the more approximate method will be sufficiently precise to permit an evaluation of the overall approach adopted.

The solution of Equation 7 will predict both transient and steady-state temperatures with independent variables depth and time for given values of \( K_v \), the thermal diffusivity of the material. However, the analog computer has only one dependent variable, voltage, and one independent variable, time. Therefore, the standard procedure for solving a partial differential equation such as Equation 7 is to fix one of its independent variables, say distance, and to then solve the resulting ordinary differential equation. The point at which the depth \( Z \) is fixed is called a "node." By taking a sufficient number of nodes of the variable in the interval of interest,
a set of curves is obtained which represent the solution of the partial
differential equation. Thus, in this case, the partial derivatives with
respect to $Z$ are approximated by finite depth differences.

An example of finite depth increments within the snowpack is shown
by Figure 8. In this case, the snowpack has been divided into three equal
depth zones. Note that $T_1$ and $T_4$ each represent one-sixth of the total
pack depth, while $T_2$ and $T_3$ each represent temperatures in one-third
of the pack. The model then computes on a continuous basis the changing
values of $T_2$ and $T_3$ as they are influenced by the boundary values established
by $T_1$ and $T_4$. A sample temperature profile within the snowpack for a
particular time is shown by Figure 8.

As previously indicated, temperature changes within a snowpack
result not only from conduction as expressed by Equation 7, but also from
the freezing within the pack of both rainwater and melt at the snow surface.
Thus, the temperature increase from this effect within the $j$-th depth zone
of the pack is given by

$$\frac{dT_{sj}}{dt} = \frac{144}{W_s(t)/m} \left[ M_{rs}(t) + P_{rg}(t) \right]$$

in which

$m$ represents the number of depth zones into which the pack is divided.

In the case of rain falling on snow the temperature of the rain is
assumed to be equal to that of the surface air. For air temperatures in
excess of 32 F there might be some question that the heat given up by the
Notes: 1. Number of depth zones, $m = 3$.
2. Boundary conditions:
   \[ T_4 = T_a, \quad T_a < 32 \, \text{F} \]
   \[ T_4 = 32 \, \text{F}, \quad T_a \geq 32 \, \text{F} \]
   \[ T_1 = 32 \, \text{F} \]

Figure 8. An example of finite depth increments within the snowpack.
rain in cooling to 32 F should be included in the above equation. However, this heat contributes to surface melt, and is therefore included in the melt equation. In the event that the snowpack is not yet isothermal at 32 F, the precipitation and melt (now both at a temperature of 32 F) move downward through the pack until freezing occurs. If at any time, t, during a finite period of integration, the temperature at one level reaches 32 F, heat being given up by freezing water within the pack immediately begins to influence the temperature at the adjacent lower level. Similarly, as soon as the pack is brought to the isothermal state, any further water present in the pack does not freeze but rather enters the free water storage capacity of the snow as estimated by the relationship of Figure 5(a). When this capacity is satisfied, free water (in addition to groundmelt) appears at the bottom of the pack.

If, after a period of melting, the air temperature falls below the freezing point, heat contributions from Equation 9 cease and pack temperatures at each level are computed only by Equation 7. Under this situation the quantity of free water held by the pack is again set to zero. 

Snowpack melt. Basically the calculation of melt is based on a degree-day factor. The base temperature selected for this computation is 32 F. Thus, the rate of melt before any adjustments are made is given by (10, 11):

\[ M_{rs} = F(T_a - 32) \]

in which

\( F \) is a rate factor expressed as inches per unit of time per degree F above 32 F
As in the case of the evapotranspiration equation, the air temperature parameter is an index of the total insolation received on a regional basis, and the value of $F$ in Equation 10 will therefore vary with the degree of slope and aspect of the land surface. To provide an adjustment of this variation, the radiation index parameter was again utilized so that

$$F = k \frac{R_I}{R_{I_h}}$$

in which $k_m$ is a constant of proportionality.

As before noted, in the northern hemisphere, the ratio $R_I / R_{I_h}$ decreases on northerly slopes with decreases in solar declination, while for southerly slopes the ratio increases with decreasing declination. Thus, on northerly slopes Equation 11 will yield less melt per degree of temperature above 32 F in the winter months than in the spring months. The reverse of this situation will apply for south-facing slopes. These results are in agreement with actual observations (1, 10).

Now, by combining Equations 10 and 11, the melt equation becomes

$$M_{rs} = k m \frac{R_I}{R_{I_h}} (T_a - 32)$$

The effect of vegetative cover on snowmelt can be taken into account by the use of a solar radiation transmission coefficient for vegetation. Studies by the Corps of Engineers (7, 10, 11) indicate a relationship between the effective or weighted cover coefficient, $C_v$ and the vegetation
transmission coefficient, $K_v$, to be of the form:

$$K_v = \exp(-4C_v) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (13)$$

The melt equation is now written thus:

$$M_{rs} = k_m K_v \left( \frac{RI}{RI_h} \right) (T_a - 32) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (14)$$

The albedo, or reflectivity, of the snowpack is important in estimating the amount of solar radiation absorbed by the pack. Albedo is expressed as the ratio of reflected shortwave to incident radiation on the snow surface. Values range from about 0.80 for newly fallen snow to as little as 0.40 for melting, late-season snow (6, 10, 11).

This decrease in albedo may be expressed as a function of time in the differential form

$$\frac{dA}{dt} = -k_a \left[ A(t) - 0.04 \right], \quad [0.40 \leq A(t) \leq 0.80] \quad \ldots (15)$$

The integrated form of this equation is as follows:

$$A(t) = 0.40 \left( 1 + e^{-k_a t} \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (16)$$

The value of the time constant, $k_a$, is about 0.20. Both Equations 15 and 16 are subject to the two conditions that a fall of new snow returns the value of $A(t)$ to 0.80, and the occurrence of rain causes the value of $A(t)$ to equal 0.40. A comparison of Equation 16 with an experimentally determined curve (12) is shown by Figure 9.
Figure 9. A comparison between observed and computed snow surface albedo.
The effect of albedo is now included by modifying Equation 14 as follows:

\[ M_{rs} = k \frac{RI}{m} K \left( \frac{S}{RI_h} \right) (T_a - 32) (1 - A). \ldots \ldots \ldots \ldots \ldots (17) \]

Finally, as previously indicated, the adjustment of this equation to include the effect of rain falling on a snowpack is accomplished by assuming the temperature of the rain to be equal to that of the air. Thus,

\[ M_{rp} = (T_a - 32) \frac{P_{rg}}{144} \ldots \ldots \ldots \ldots \ldots \ldots (18) \]

The equation for surface melt then becomes

\[ M_{rs} = k \frac{RI}{m} K \left( \frac{S}{RI_h} \right) (T_a - 32) (1 - A) + (T_a - 32) \frac{P_{rg}}{144} \ldots (19) \]

It should be noted that for each watershed zone a uniform depth of snowpack is assumed to exist throughout the entire winter and snowmelt season. Thus, no adjustment is made for partial snow cover over a zone.

A preliminary estimate of the value of the constant \( k_m \) in the preceding equations was obtained by applying Equation 17 to the watershed of the Central Sierra Snow Laboratory (10, 11). This watershed is situated at a latitude of 39°22' north, has a mean elevation of approximately 7,500 feet, and an average aspect of about 11 degrees west of due south, and an average slope of approximately 13 percent. An average value of \( F \) (Equation 10) for melt at a point under no forest cover during the snowmelt period has been determined as being 0.106 inch per day per degree day, based on mean daily temperatures. The average declination of the sun during the
snowmelt period was 10 degrees. From this information values of $RI_s$ and $RI_h$ were established as being equal to 57 and 55 respectively. The vegetation transmission coefficient, $K_v$, applicable to the snow courses was estimated to be 0.43 and the value of the albedo during the melt period was assumed at 0.40.

Now substituting in Equation 17, \( 0.106 = k_m \left[ \frac{0.43}{0.57} \right] (1 - 0.40) \) from which $k_m = 0.40$.

The value of $k_m$ computed by the same procedure for the Willamette Basin Snow Laboratory was also approximately equal to this figure.

A uniform groundmelt rate, $M_{rg}$, of 0.02 inch per day is assumed to occur (1, 6, 10) and this quantity is added to the portion of the surface melt appearing at the bottom of the pack. Thus, total melt, $M_r$, is given by

\[ M_r = M_{rs} + M_{rg} \]  

From this point both snowmelt water and rain falling directly on bare ground enter the soil at an infiltration rate characteristic of the soil, with any surplus water forming surface runoff. The surface runoff component first must satisfy an estimated depression storage requirement for the zone. Additional surface runoff beyond this requirement is then routed off the watershed.

**Model testing.** As indicated by Figures 6 and 9, individual relationships used in this model have been tested to a limited extent against observed data. The model as a whole has not been tested. However, it
has now been programmed on the analog computer and extensive testing and verification studies using experimental field data will be undertaken in the very near future. Some preliminary output curves are shown by Figure 10.

**Summary and Conclusions**

In this paper three mathematical models of the snowmelt process have been proposed. The basis of each model is a fundamental and logical mathematical representation of the various phenomena involved in terms of the time and space increments adopted for the model. Each succeeding model contains improved relationships and definition in terms of time and space, and therefore is capable of more accurately representing the prototype system.

Modeling or simulation can provide considerable insight into the system being studied in terms not only of the various processes involved, but also of the relative importance of these processes. Thus, through this approach, it is possible to expose critical areas where data and perhaps theory are lacking, and to establish guidelines for more fruitful and meaningful research and data gathering activities in the future.

Testing and verification of any model requires reliable field data. Sound operational models of the snowmelt process coupled with adequate field data will represent a significant contribution to the proper management and use of our water resources.
Figure 10. Sample output from the snowmelt model.
Selected References


