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by

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Amitrajeet A. Batabyal

ABSTRACT

In the 1992 Rio Earth Summit, developing countries (DCs) were adamant that in order to protect the environment for the future, new institutions were needed which would channel resources from the wealthy developed countries to the poor DCs. With this backdrop, I analyze the problem faced by an imperfectly informed supranational governmental authority (SNGA) who wishes to design an International Environmental Agreement (IEA). The SNGA cannot contract directly with polluting firms in the various DCs, and he must deal with such firms through their governments. Further, the SNGA is constrained by limited financial resources available for environmental protection. I study this tripartite hierarchical interaction, first for the case in which the relevant DCs are identical; I then analyze the case of heterogeneous DCs. I find that the monetary transfers necessary to induce optimal behavior by governments and firms are quite sensitive to both the timing of the underlying game and to the existence of collusion. Inter alia, my analysis suggests that IEAs are not inherently doomed due to a basic monitoring and enforcement problem arising from national sovereignty. However, the success of such IEAs is contingent on the funds available for global environmental protection.

JEL Classification: D62, D82, Q25

Key words: international environmental agreement, design, developing country
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1. Introduction

With the passage of time, it has increasingly been recognized that environmental protection is a global issue. As noted by Bernauer (1995, p. 354), the scope and significance of this issue have been amply demonstrated by the events of the 1992 Earth Summit in Rio. At this summit, it became clear that if the developed countries of the world wanted “... the environment to be secured for future generations, [then they would] have to radically assist the South in choosing a different road to development than the one they [had] currently [been] travelling on” (Rogers 1993, p. 27). Indeed, to combat the twin evils of poverty and environmental degradation, developing countries (DCs) have demanded the transfer of resources and technology from developed countries. In such a contentious setting, the success or failure to protect the environment will depend crucially on the ability of international institutions to craft effective international environmental agreements (IEAs). Given this, a key question becomes “How can international institutions, which necessarily respect the principle of state sovereignty, contribute to the solution of difficult global problems?” (Keohane, Haas, and Levy 1993, p. 6). This is the main question that I propose to analyze in this paper.

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2In this paper I shall use the terms IEA and contract interchangeably.
On the academic front, researchers have begun to study issues relating to global environmental protection in a systematic manner only very recently. As a result, many specific questions remain unanswered. What kinds of pollution abatement patterns can one expect to observe in situations in which an imperfectly informed supra-national governmental authority (SNGA) contracts with governments and polluting firms in individual DCs? What kinds of monetary transfers will be necessary to get sovereign nations to voluntarily participate in IEAs? How does the contract, which treats DCs as identical, differ from one which acknowledges the heterogeneity of DCs? How does the SNGAs inability to monitor pollution abatement in the individual countries affect the contract design question? Finally, how does the limited availability of funds affect the SNGAs contract design question? These are some of the specific questions that I shall address in this paper.

I shall build on the economics of hierarchies to study the global pollution control question as a problem in mechanism design. This perspective not only highlights the effect of key informational asymmetries on the design of contracts, but it also provides interesting insights into the kinds of pollution control arrangements one might expect to observe in an inherently hierarchical and noncooperative international environment. Although my analysis is in principle applicable to any country, the hierarchical interaction that I shall analyze is particularly relevant to DCs; as such, the reader should note that it is these countries that I have in mind in all of the subsequent analyses.

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3 See Bernauer (1995) and Keohane, Haas, and Levy (1993) for a more detailed corroboration of this claim.

4 The countries I have in mind are those which would be eligible to receive monetary transfers under the Global Environmental Facility’s (GEF) standard of per capita income of $4,000 or less. For more details see Rogers (1993, p. 155).
I now discuss the nascent literature on IEAs and then move on to provide a detailed discussion of my model.

2. International Environmental Agreements: A Brief Synopsis

Barrett (1992, 1994) has modeled IEAs as games between countries. While Barrett’s analyses are not in the design framework, he makes the important point that for IEAs to work at all, they must be self enforcing. Hoel (1992) argues against the institution of uniform emissions reduction policies in international agreements, showing that other policies yield higher levels of global welfare. Petrakis and Xepapadeas (1996) show that a large enough group of environmentally conscious countries can make self-financing side payments to a group of less environmentally conscious countries so as to produce a stable coalition which leads to lower overall pollution emissions. While these papers have certainly advanced our understanding of some aspects of "...the multi-faceted design... problem," (Black, Levi, and de Meza 1993, p. 281), many other important questions—which I discussed in section 1—remain unanswered. As such, I now discuss my modeling approach to the IEA design question.

I shall model the international environment as a multiforked, three-tiered hierarchy. Occupying the topmost tier of the hierarchy is the relevant international institution, which I shall call a SNGA. This SNGA could be an organization such as the World Bank, or the Commission on Sustainable Development (CSD) created in Agenda 21 at the Rio Earth Summit. The second and

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5Specifically in its role as an administrator of the Global Environmental Facility (GEF).
third tiers of the hierarchy consist of the government and a representative polluting firm in each DC. Each fork of the hierarchy corresponds to a single DC, and there are $N$ such forks/countries.$^6$

Three-tiered hierarchies have been studied by Tirole (1986), Kofman and Lawarree (1993), and by Batabyal (1996a, 1996b). Batabyal (1996b) compares two-tiered hierarchies with three-tiered hierarchies and shows that the timing of the game played by the relevant players does not have a significant bearing on the nature of the IEA that can be designed by a SNGA. However, to the best of my knowledge, the problem of designing IEAs for identical and heterogeneous DCs, when there are budget balance constraints and when governments and firms within a country may collude, has not been studied to date.

As such, I shall apply the theory of hierarchies to study the design of IEAs, first for identical DCs, and then for heterogeneous DCs. The reader should think of the identical DC's case as one in which the SNGA seeks to avoid the transaction costs associated with the design of country-specific IEAs. As a result, the SNGA holds all DCs, which fall within a particular criterion, to identical contractual requirements. As indicated in footnote 4, one such criterion might be the GEFs standard of per capita income of $4,000 or less. From the perspective of the SNGA, this case of identical DCs involves *ex ante* contracting.$^7$ In particular, the SNGA is constrained by an *ex ante* budget constraint. This kind of budget constraint makes sense only when all the relevant countries

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$^6$ The reader will note that in this modeling scheme, I have conferred on the SNGA, the role of principal. As such, there is a distinct asymmetry in the assumed power of the SNGA as opposed to that of governments and firms. However, given that I am interested in DCs which typically have limited bargaining power in their dealings with international organizations owing to the fact that their monetary contributions to the budgets of such organizations are minimal, this hierarchical modeling scheme appears to be appropriate. For more on the power of SNGAs over DCs, see Mosley, Harrigan, and Toye (1991).

$^7$ By *ex ante* I mean contracting which takes place with all parties holding symmetric but imperfect information about the pollution abatement technology of firms. By *ex post* I mean contracting which takes place with the players holding asymmetric information about the pollution abatement technology of the same firms.
are identical and when there is no aggregate uncertainty. As contrasted to this case, the case in which DCs are heterogeneous involves \textit{ex post} contracting for the SNGA. Here, the SNGAs’ actions are constrained by \textit{ex post} budget constraints, and the designed IEAs are country specific.

The rationale for \textit{ex ante} and \textit{ex post} contracting stems from issues including, but not limited to, the harmful atmospheric effects of sulphur and/or nitrogen emissions. The actual incidence of pollution may be domestic or transboundary.\footnote{See Crane (1993) and Paarlberg (1993) for a discussion of the relevance of international institutions when the incidence of an environmental externality is domestic.} The uncertainty in this paper arises from the SNGAs’ lack of knowledge about the quality of the pollution abatement technology available in each DC. This lack of knowledge about abatement technology quality is the source of imperfect and asymmetric information. Whereas the firm in the DC always knows the quality of its technology and the government does too in some states of nature, the SNGA is never privy to this information. The random variable denoting the private information about pollution abatement technology quality is uncorrelated across countries. In the \textit{ex ante} case, this no-correlation assumption does not have any impact because all countries are identical, and, hence, my analysis involves the study of the SNGA/government/firm interaction in a single DC. However, in the \textit{ex post} case, the no-correlation assumption means that my analysis of the three-tiered interaction between the SNGA, the government, and the polluting firm in one country is independent of the SNGAs’ dealings with some other country. Hence, without any loss of generality, I shall focus on an arbitrary country, say country $j$, in the finite set of countries. The SNGA’s task is to design incentive compatible and collusion-proof \textit{ex ante} and \textit{ex post} IEAs which can be implemented in a Bayes-Nash equilibrium.

The rest of this paper is organized as follows. In section 3, I describe the model in detail and I study the properties of the first best optimum. In section 4, I study \textit{ex ante} and \textit{ex post} contracting,
with no collusion by the firm and the government. In section 5, I study *ex ante* and *ex post* contracting with possible collusion by the government and the polluting firm. In addition to the identical DC versus heterogeneous DC interpretation of *ex ante* and *ex post* contracting that has already been provided, the reader should also think of this distinction as one involving liability. This issue concerns the potential need for limiting the *ex post* liability of the players in the various nations, in order to get them to voluntarily participate in the contracting process. In other words, as contrasted to an *ex ante* contract, an *ex post* contract is like a limited liability contract. That is, the SNGA limits the maximum loss of the relevant players in the event of an adverse state of nature.

The reasons for wanting to study collusion between the polluting firm and the LDC government are threefold. First, while the DC government participates in the IEA because it recognizes the value of such international participation, this government also acts as the polluting firm’s advocate. This aspect of the problem will give rise to scenarios in which government/firm collusion becomes a desirable option. Second, the government and the firm receive monetary transfers from the SNGA for their roles in abating pollution. Further, both these players know that the SNGA cannot monitor their activities owing to sovereignty, or for that matter, enforce the terms of the IEA in the event of a contractual breach. As such, there will be circumstances in which there are incentives for the government and the firm in each country to collude to maximize the transfers received from the SNGA. Third, as Mookherjee and Png (1995) have noted, corruption is an endemic part of public life in many DCs. This suggests a need for explicitly modeling the activities of potentially corruptible players. Due to these three reasons, an important part of this paper will consist of analyzing collusion-proof contracts.

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9 See Peterson (1993) for a discussion of some practical instances of possible government/firm collusion in an international setting.
3. The Theoretical Framework

3a. Description of the Model

Subscript \( i = 1, 2, 3, 4 \) will refer to the state of nature, and superscript \( j = 1, \ldots, N \) will refer to the country. \( \theta \) denotes the uncertainty about the quality of the pollution abatement technology that is currently available; \( \theta \) has binary support \([\tilde{\theta}, \hat{\theta}]\), where \( 0 < \tilde{\theta} < \hat{\theta} \), and \( \Delta \theta = \hat{\theta} - \tilde{\theta} \). I shall refer to \( \tilde{\theta} \) as the low abatement quality parameter and to \( \hat{\theta} \) as the high abatement quality parameter.

The risk-averse firm produces clean air, whose output and value are denoted by \( x \in \mathbb{R}_+ \). The firm chooses a level of pollution abatement \( a \in \mathbb{R}_+ \). The firm’s cost of abatement is \( g(a) \), where \( g' > 0 \), and \( g'' > 0 \), and \( g(0) = 0 \). The firm has a differentiable net payoff from pollution abatement function \( B[T_i - g(a)] \) with \( \partial B[\bullet]/\partial T_i \in (0, \infty) \), \( \forall T_i \). \( T_i \in \mathbb{R}_+ \) is the monetary transfer made by the SNGA to the firm for abating pollution in state \( i \). The firm’s reservation payoff is \( B_r = B[T_r] \), and \( T_r \) is the reservation transfer. \( B_r \) and \( T_r \) are common knowledge.

The DC government is risk averse. It has a strictly concave and differentiable utility function \( V(G) \), with \( V'(\bullet) \in (0, \infty) \), \( \forall G \). \( G \) is the monetary transfer made by the SNGA to the government for its role in participating in the IEA in state \( i \). The government’s reservation utility is \( V_r = V(G_r) \), where \( G_r \in \mathbb{R}_+ \) is the reservation transfer, and \( V_r \) and \( G_r \) are common knowledge. By employing a monitoring device, the government receives a signal, \( s \), from the firm regarding its private information and then it sends a report, \( r \), to the SNGA indicating what it observed about the firm’s pollution abatement technology quality parameter.\(^\text{10}\) In some states of nature, this monitoring

\(^{10}\)Since the main objective of this paper is not to study domestic monitoring, I shall assume that the use of this monitoring device is costless.
device malfunctions and hence in these states, the government will be unable to provide the SNGA with a useful report. Upon receiving $r$, the SNGA offers the government a transfer. Making the government's central task one of reporting is consistent with the government/SNGA interaction proposed for one specific SNGA, the Commission on Sustainable Development. As noted by Rogers (1993, p. 310), a key aspect of this interaction involves the "... Commission's... considering information provided by governments..."

The SNGA is risk neutral and he has a welfare function $U(\cdot)$, which takes the form

$$U = \sum_j (a^j + \theta^j - G^j - T^j), \quad j = 1, \ldots, N,$$

where $j$ runs over the total number of countries. Clean air produced by the firm in country $j$ is $x^j = a^j + \theta^j$. As stated, the SNGA's welfare is the difference between total clean air and the sum of government and firm transfers. In the rest of this paper, when there is no possibility of confusion, I shall suppress the country superscript. It should be understood that the focus is on country $j$. The SNGA's contract can only be conditioned on what the SNGA observes, i.e., the government's report, $r$, and the firm's production of clean air, $x$.

There are four states of nature, each occurring with probability $p_i > 0$, where $\sum_i p_i = 1$. In the ex ante case, the SNGA, the government, and the firm sign the contract holding symmetric but imperfect information about $\theta$. In the ex post case, the contract is signed after the resolution of the uncertainty about $\theta$. The firm always observes $\theta$ before choosing its abatement level. The government, on the other hand, may or may not observe the firm's private information. This depends on whether the government's monitoring device functions or malfunctions. As a result, the government's signal, $s$, may or may not be informative. I can now characterize the four states:

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The reader should note that although the government's utility function is defined only over transfers, this government does care about the firm in its country endogenously. This is because the government's transfer depends on the firm's actions when bribes are allowed. See section 5 for more details.
*State 1: The firm and the government both observe $\hat{\theta}$.

*State 2: The firm observes $\hat{\theta}$ and the government observes nothing.

*State 3: The firm observes $\hat{\theta}$ and the government observes nothing.

*State 4: The firm and the government both observe $\hat{\theta}$.

In state 1, the firm and the government observe the low abatement quality parameter. The government's monitoring device works and hence yields useful information. In state 2, the firm observes the low abatement quality parameter but the government observes nothing. In this state, the government's monitoring device malfunctions. In state 3, the firm observes the high abatement quality parameter and the government observes nothing. Once again, the government's monitoring device fails. Finally, in state 4, the firm and the government observe the high abatement quality parameter.

The timing of the game between the SNGA, the government and the firm in the *ex ante* case is as follows. First, the SNGA offers a contract to the government and the firm. Second, the firm observes $\theta$ and the government receives its signal $s$. Third, the firm chooses abatement $a$. Fourth, clean air $x$ is produced by the firm and the government sends its report $r$ to the SNGA indicating what it observed. Fifth, the SNGA compensates the government and the firm with transfers $G(x,r)$ and $I(x,r)$. When contracting is *ex post*, the uncertainty is resolved first and then the contract is signed by the players.

I shall assume that the SNGA can verify the veracity of the government report $r$. In other words, if the government's signal $s$ is noninformative, then the corresponding report $r$ reflects that fact, and the SNGA can verify that the true facts are indeed as they have been reported. In symbols, $s = 0 \Rightarrow r = 0$. On the other hand, to keep the SNGA's design problem interesting and to allow for
the possibility of government/firm collusion, I shall permit the government to lie and report that its
signal is noninformative when such is not the case.\footnote{In this scenario, lying by the government is restricted to states 1 and 4. Alternately put, reporting the wrong state is equivalent to obtaining a noninformative signal.} That is, \( s = \Theta \Rightarrow r \in \{ \Theta, 0 \} \). This completes the description of my model. I now consider the benchmark case in which perfect information is acquired by the SNGA.

3b. The First Best Optimum

In this case, the SNGA observes \( \Theta \) and the firm’s abatement choice. When this happens, the SNGA bypasses the government and contracts with the firm directly. The government receives its reservation transfer \( G_r \) and hence its reservation utility \( V_r \), in all states. The SNGA solves

\[
\begin{align*}
\max_{a^*} & \sum_{i} \{ a_i^* + \theta_i^j - g^j(a_i^j) - T_i^j - G_i \}, \\
\text{subject to} & (1a) \quad B_i^j - g(a_i^j) \geq B_i^j, \quad \forall i, \quad (1b) \quad \hat{M} \geq \sum_{i} \{ T_i^j + g^j(a_i^j) + G_i \}. 
\end{align*}
\]

Using the fact that the firm participation constraints in (1a) hold with equality, the first order necessary conditions are

\[
\frac{dg_j(a_*)}{da_j} = \frac{1}{1 + \gamma}, \quad \forall i, j,
\]

where \( \gamma \) is the multiplier on the budget constraint (1b) and \( a_* \) is the first best level of pollution abatement. We see that in the first best optimum, the marginal cost of pollution abatement for country \( j \) is set equal to the reciprocal of one plus the marginal welfare of the SNGA’s funds. The optimal level of abatement \( a_* \) is state independent; as such, the firm receives a transfer for abating pollution which is also state independent. This transfer is \( T_r + g_* \), where \( g_* = g(a_*) \).

It is not possible to definitively determine whether the SNGA’s budget constraint binds in equilibrium. To see this, note the following. The SNGA’s welfare function exhibits constant marginal welfare in the authority’s own funds. As contrasted to this, the funds spent making
transfers do not exhibit constant marginal welfare. As a result, it is possible that in equilibrium, the SNGA will disburse only a part of $\hat{M}$ because the effect of such disbursement on clean air production drops below one before the SNGA exhausts $\hat{M}$. The second case in which the SNGA exhausts $\hat{M}$ before the effect on clean air production drops below one is also possible. Which case will prevail depends on the curvatures of the $B[l]$, and particularly the $g'(\bullet)$ functions. In the rest of this paper I shall assume that the curvatures of these functions is such that the budget constraints bind in equilibrium. From a practical standpoint, this is clearly the more relevant case. I now discuss the more interesting cases in which the SNGA cannot determine the realization of $\theta$ or the actual abatement undertaken by the firm.

4. The No Government/Firm Collusion Case

4a. Ex Ante Contracting (Identical DCs)

Since the contracting is ex ante, the SNGA, the government, and the firm share symmetric but imperfect information about $\theta$. When the government is paid $G_r$, it obtains its reservation utility $V_r$, and hence it is fully insured. Further, since I am not allowing for collusion between the government and the firm and because the SNGA can verify the government’s report, by paying $G_r$, the SNGA obtains the government’s information at least cost. This means that the three-tiered hierarchy reduces to a two-tiered hierarchy in which the government plays a passive role.

In this setting, the SNGA solves

$$\max_{\{T, a\}} \sum y p_i (a_i + \theta_i - T_i)$$

subject to

1. $\sum_i p_i B[T_i - g(a_i)] \geq B_r$,  
2. $T_3 - g(a_3) \geq T_2 - g(a_2 - \Delta \theta)$,  
3. $T_2 - g(a_2) \geq T_3 - g(a_3 + \Delta \theta)$,  
4. $\sum_i p_i [\hat{M} - \sum y^j G_r^j + T_i^j] \geq 0$. Inequality (2a) is the
firm’s participation constraint. Inequalities (2b) and (2c) are the firm’s incentive compatibility constraints. These constraints arise because the SNGA has imperfect information about \( \theta \) in states 2 and 3. These are also the states in which the government’s signal \( s \) is noninformative. Constraint (2b) says that in state 3, the firm should not claim that the state is 2. Similarly, (2c) says that in state 2, the firm should not claim that the state is 3. Inequality (2d) denotes the SNGA’s budget constraint. Note that because all DCs are identical and because there is no aggregate uncertainty, it makes sense to have an *ex ante* budget constraint. If the relevant countries are not identical, (2d) will have to be replaced—as in section 4b—by an *ex post* budget constraint. I can now solve the SNGA’s problem as stated in (2)-(2d). I am led to

**Theorem 1:** The optimal IEA is one in which (i) the SNGA obtains the government’s information at least cost, (ii) the government’s reward is \( G_r \) in all states, (iii) the pollution abatement levels satisfy \( q_1 = a_3 = a_4 > a_2 \), (iv) the firm transfers satisfy \( T_3 > T_1 = T_4 > T_2 \), and (v) at the optimum, all the constraints except (2c) bind.

**Proof:** See the Appendix.

Theorem 1 describes the pattern of pollution abatement one may expect to observe in my stylized \( N \) identical DC world. Since the SNGA acquires the government’s information in states 1 and 4 and because this information is verifiable, the firm’s abatement is the same in these two states. The optimal contract then specifies \( T_1 = T_4 \). On the other hand, in state 2 or 3, the SNGA’s information is imperfect. To prevent the firm from lying about the true \( \theta \), the optimal contract now specifies \( T_3 > T_2 \). The optimality of this contract stems in part from the feature that the SNGA rewards high abatement with a high monetary transfer and “punishes” low abatement with a low transfer. The level of abatement in the low quality state 2 is lower than the level in the other states.
This makes it less desirable to abate pollution at a low level in state 3. It is not possible to directly compare the abatement levels described in Theorem 1 with the first best level of abatement \( a_* \). This is because \( a_* \) depends on the multiplier \( \gamma \), which is specific to the first best problem.

4b. Ex Post Contracting (Heterogeneous DCs)

I now consider the case in which all the DCs are heterogeneous. The SNGA is unable to contract with the government and the firm in country \( j \) until the uncertainty about \( \theta \) has been resolved. Once again, with no collusion, the government plays a passive role. It receives its reservation utility and hence it is fully insured. The SNGA solves

\[
\max_{\{a_i, T_i\}} \sum_{i} p_i (a_i + \theta_i - T_i)
\]

subject to (1a), (2b), (2c), and (3a) \( \hat{M} - \sum_{y} \{ G_r^i + T_i^y \} \geq 0, \forall i \).

From (1a) we see that as opposed to the ex ante case, in this ex post case, it must be individually rational for the firm to contract with the SNGA in every state. Put differently, in this setting, the SNGA cannot compel the firm to abate pollution if doing so would involve making a loss. Inequality (3a) denotes the ex post budget constraints. Because the contracting countries now are heterogeneous and because the contracting is ex post, a stronger notion of budget balance—as embodied in (3a)—must be used. Inequality (3a) says that when the SNGAs contracts with DCs independently, this authority’s monetary obligations cannot exceed \( \hat{M} \) regardless of the state. Note that this ex post setting is characterized by asymmetric information. The SNGA does not know the state of nature; the firm does. The timing of the underlying game now is such that the uncertainty is resolved first and then the players contract. In this setting, the optimal contract has the properties stated in
**Theorem 2:** The three-tired hierarchy reduces to a two-tiered hierarchy in which (i) the SNGA obtains the government's information at least cost, (ii) the government's transfer equals $G_r$ in all four states, (iii) for $i \neq 2$, $a_i = (g')^{-1}\{p_i/(p_i + \gamma_i)\}$, $a_2 = (g')^{-1}\{p_2/(p_2 + \gamma_2) - D\}$, and $A_i g'(a_i) = A_3 g'(a_3) > A_4 g'(a_4) > A_2 g'(a_2)$, (iv) $T_3 - g(a_3) > T_1 - g(a_1) = T_2 - g(a_2) = T_4 - g(a_4)$, and (v) at the optimum, all the constraints except (1a, $i = 3$) and (2c) bind.\(^{13}\)

**Proof:** See the Appendix.

I now comment on some aspects of the optimal *ex ante* and *ex post* contracts. Inspecting Table 1, we notice two important differences. First, in the *ex ante* case, $a_1 = a_3 = a_4 > a_2$, and in the *ex post* case, $A_1 g'(a_1) = A_3 g'(a_3) = A_4 g'(a_4) > A_2 g'(a_2)$, where the $A_i$s are weights. These weights are functions of the respective Kuhn-Tucker multipliers and the state probabilities. In particular, we see that whereas the *ex ante* contract equalizes the actual level of abatement in states 1, 3, and 4, the *ex post* contract equalizes the weighted marginal cost of abatement in these three states. In both contracts, the level of abatement is lowest in the low abatement quality state 2.

Second, in the *ex ante* case, the gross payoffs to the firm—which satisfy $T_3 > T_1 = T_4 > T_2$—can be characterized explicitly. However, in the *ex post* case, only the net payoff to the firm can be characterized explicitly. These net payoffs satisfy $T_3 - g(a_3) > T_1 - g(a_1) = T_4 - g(a_4) = T_2 - g(a_2)$.

Because the government's report is verifiable, and because the government does not collude with the firm, optimal insurance for the firm under both regimes requires that $T_1 - g(a_1) = T_4 - g(a_4)$. Further, in these no-collusion cases, incentive problems are limited to states 2 and 3. In these states, the optimal contract must reward truth telling. As such, in the *ex ante* case, we have $T_3 > T_2$, and in the *ex post* case, we have $T_3 - g(a_3) > T_2 - g(a_2)$. Finally, I note that

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\(^{13}\)For exact representations of $D$ and the $A_i$, see the proof of Theorem 2 in the Appendix.
Table 1. The No-Government/Firm Collusion Case: *Ex Ante* versus *Ex Post* Contracting

<table>
<thead>
<tr>
<th>Contracting</th>
<th><em>Ex Ante</em></th>
<th><em>Ex Post</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollution abatement level and pattern</td>
<td>$a_1 = a_3 = a_4 &gt; a_2$</td>
<td>$A_1 g'(a_1) = A_3 g'(a_4) = A_2 g'(a_2)$</td>
</tr>
<tr>
<td>Transfers to the government</td>
<td>$g_s, \forall i$</td>
<td>$g_s, \forall i$</td>
</tr>
<tr>
<td>Payoffs to the polluting firm</td>
<td>$T_3 &gt; T_1 = T_4 &gt; T_2$</td>
<td>$T_3 - g(a_3) &gt; T_1 - g(a_1) = T_4 - g(a_4) = T_2 - g(a_2)$</td>
</tr>
</tbody>
</table>

in both cases, $T_3 - g(a_3) > T_1 - g(a_1)$. This feature of the two contracts tells us that an optimal contract will reward truth telling in the high abatement quality state when the government is unable to convey an informative report to the SNGA.

I now proceed to consider the effects of government/firm collusion on the optimal contract designed by the SNGA.

5. **The Government/Firm Collusion Case**

5a. *Ex Ante Contracting (Identical DCs)*

Recall that because countries are sovereign, the SNGA is unable to either monitor the actions of the government and the firm or enforce the terms of the agreement in the event of a contractual breach. Since the SNGA can never acquire the firm’s private information and must rely on the government’s report to design the optimal contract, an efficient contract must not only be individually rational and incentive compatible, but it must also be collusion-proof.\(^{14}\)

\(^{14}\)See footnote 9 as well.
I shall model collusion between the government and the firm as follows. In the *ex ante* case, before the resolution of the uncertainty regarding abatement technology quality and at the time of signing the main contract, i.e., the contract between the SNGA, the government, and the firm, the firm and the government sign a secondary contract which entails the offer and acceptance of a bribe \( b(\cdot, \cdot) \) from the firm to its government. This secondary contract is *unobservable* by the SNGA. The bribe is a function of the government’s report, \( r \), and the firm’s production of clean air, \( x \). With the offer and acceptance of this bribe, the firm’s total transfer becomes \( \{ T(\cdot) - b(r, x) \} \) and the government’s total transfer is \( \{ G(\cdot) + b(r, x) \} \).

Collusion by the firm and the government alters the incentives of the various parties and—as we shall see—the nature of the optimal contract offered by the SNGA. To see why the firm might want to bribe its government in our four-state world, consider state 4. In this state, the government is indifferent between reporting that it has observed \( \hat{\theta} \) and reporting that it has observed 0. However, the firm would prefer that the government report 0. This is one instance in which a clear rationale exists for the firm to bribe its government.

In order to formulate and solve the SNGA’s problem when there is collusion, I shall appeal to the “equivalence principle” (Tirole 1986, p. 195) and restrict myself to collusion-proof contracts. Tirole’s method involves imposing constraints in addition to the usual participation and incentive compatibility constraints. These additional constraints are designed to preclude government/firm collusion and hence make the main contract collusion-proof. Denoting the collusion-proof transfers to the government and the firm by \( \bar{G} \) and \( \bar{T} \), the SNGA solves

\[
\max_{\{\bar{G}, \bar{T}, a_i\}} \sum_{\nu \lambda} P_i (a_i + \theta_i - \bar{G}_i - \bar{T}_i)
\]  

(4)
subject to (2a)-(2d), (4a) \( \sum_i p_i V(G_i) \geq V_r \), (4b) \( \bar{G}_1 + \bar{T}_1 - g(a_1) \geq \bar{G}_2 + \bar{T}_2 - g(a_2) \), (4c) \( \bar{G}_4 + \bar{T}_4 - g(a_4) \geq \bar{G}_5 + \bar{T}_5 - g(a_5) \), (4d) \( \bar{G}_3 + \bar{T}_3 - g(a_3) \geq \bar{G}_2 + \bar{T}_2 - g(a_2 - \Delta \theta) \), and (4e) \( \bar{G}_2 + \bar{T}_2 - g(a_2) \geq \bar{G}_3 + \bar{T}_3 - g(a_3 + \Delta \theta) \).

Inequality (4a) is the government's participation constraint. Inequalities (4b) and (4c) are the core collusion constraints. Recall that in states 1 and 4 the government's signal \( s \) is informative. In these two states, the government can hide this fact. Given this, constraints (4b) and (4c) tell us that should the firm bribe its government, then the total sum of the transfers less the cost of pollution abatement in states 1 and 4 cannot be less than the corresponding totals in states 2 and 3, respectively. Finally, (4d) and (4e) tell us that it must not be possible for the government to bribe the firm. More specifically, (4d) tells us that in state 3, the government should not be able to bribe the firm to abate at the level that is optimal for state 2. Similarly, (4e) tells us that the government should not be able to bribe the firm to claim that the state is 3 when it is 2. Solving the SNGA's problem (4) subject to (2a)-(2d), and (4a)-(4e), I can state

**Theorem 3:** The optimal contract with government/firm collusion is one in which (i) \( a_1 = a_3 = a_4 > a_2 \), (ii) \( \bar{G}_4 > \bar{G}_1 > \bar{G}_2 = \bar{G}_3 \), (iii) \( \bar{T}_3 > \bar{T}_4 > \bar{T}_1 > \bar{T}_2 \), (iv) \( \bar{G}_4 + \bar{T}_4 = \bar{G}_3 + \bar{T}_3 \), and (v) all the constraints except (2c), (4b), and (4e) bind at the optimum.

**Proof:** See the Appendix.

To intuitively verify that the contract described in Theorem 3 is indeed collusion-proof, I have to show that constraints (2a)-(2d) and (4a)-(4e) are satisfied. By part (v) of Theorem 3, constraints (2a), (2b), (2d), (4a), (4c), and (4d) are satisfied. The proof of Theorem 3 tells us that constraints (2c), (4b), and (4e) hold as strict inequalities. Thus the equilibrium contract is collusion-proof.
Note that the SNGA is worse off when the government and the firm collude. This is because in the collusion case, the number of binding constraints exceeds the number of binding constraints in the no-collusion case. However, if the SNGA does offer the contract with the features described in Theorem 3, then his total monetary transfers cannot be altered by changing the government’s report or the firm’s abatement level. As such, the SNGA can be sure that his monetary obligations will be those described in Theorem 3. This is so because the equilibrium contract is collusion-proof.

I now comment on some of the noteworthy features of the contract described in Theorem 3. From Theorem 3(i) and Table 2 we see that collusion per se has no qualitative effect on the pattern of pollution abatement. Note also that it is not possible to be explicit about the deviation in the abatement levels specified in Theorem 3 from \( a^* \), because this first best level of abatement depends on a multiplier that is specific to the first best problem.

Part (ii) and Table 2 tell us that in the collusion case, the government is rewarded for the usefulness of its report. In states 2 and 3, the government reports truthfully. Thus, \( G_2 = G_3 \). On the other hand, in order to encourage the government to tell the truth about what it has observed in state 4, the government’s reward is high; by a similar line of reasoning, the government’s reward

<table>
<thead>
<tr>
<th>Table 2. Ex Ante Contracting Without and With Collusion</th>
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<tbody>
<tr>
<td>Contracting</td>
</tr>
<tr>
<td>--------------------------------------------------------</td>
</tr>
<tr>
<td>Pollution abatement level and pattern</td>
</tr>
<tr>
<td>Transfers to the government</td>
</tr>
<tr>
<td>Transfers to the polluting firm</td>
</tr>
</tbody>
</table>
in state 1 is low. The transfers in states 4 and 1 exceed those in states 2 and 3 because in states 4 and 1 the government may lie and hence the SNGA has to establish the right incentives. This is in contrast to the situation in states 2 and 3 where there is no possibility of lying. This active governmental role in the collusion case contrasts with the no-collusion case in which the government plays a passive role and receives its reservation transfer in every state.

Part (iv) of the theorem says that the total payments from the SNGA to the government and the firm in states 3 and 4 are equal. However, by part (iii) the transfers to the firm between these two states vary. Why is this so? In state 3, the firm can lie about the abatement quality parameter that it has observed and the government will not be able to tell the difference between truth telling and lying because its signal is noninformative. In order to prevent the firm from lying, the firm’s reward in state 3 is higher. On the other hand in state 4, the government’s signal is informative. Now the government has to be induced to report truthfully with a higher transfer, and the firm’s reward is correspondingly lower. From Table 2, we see that in the no-collusion case, $T_4 = T_1$. This is because the SNGA acquires the government's verifiable information at least cost and because the government reports truthfully. On the other hand in the collusion case, $\bar{T}_4 > T_1$ holds. This is because in the collusion case, the SNGA must create incentives so that the dual objectives of preventing collusion and encouraging the firm to act truthfully in the high abatement quality state are achieved.

Finally, part (v) tells us that (4b) does not bind at the optimum. This is because when the firm observes the low abatement quality parameter, the government’s report does not make a difference since the firm voluntarily prefers to abate pollution at the low level.
5b. Ex Post Contracting (Heterogeneous DCs)

I now study the case of heterogeneous DCs in which there is collusion between the DC government and the firm. The SNGA is unable to get the relevant players to contract until the uncertainty about \( \theta \) has been resolved. Denoting the collusion-proof transfers to the government and the firm by \( G \) and \( T \), the SNGA solves

\[
\max_{\{G, T, a_i\}} \sum_{i=1} p_i (a_i + \theta_i - G_i - T_i)
\]

subject to (1a), (2b), (2c), (3a), (4b)-(4e), and (5a) \( V(G) \geq V_r \), \( \forall i \). The optimal contract has the properties stated in

**Theorem 4:** (i) For \( i \neq 2 \), \( a_1 = (g^')^{-1} \{ p_1/(p_1 + \gamma_1) \}, \ a_2 = (g^')^{-1} \{ p_2/(p_2 + \gamma_2) - D \}, \) and \( \tilde{A}_2 g^'(a_1) = \tilde{A}_3 g^'(a_2) = \tilde{A}_4 g^'(a_3) = \tilde{A}_4 g^'(a_4) \), \( \theta_3 - g(a_3) > \theta_1 - g(a_1) = \theta_2 - g(a_2) = \theta_4 - g(a_4) \), (iii) \( \tilde{G}_3 > \tilde{G}_1 = \tilde{G}_2 = \tilde{G}_3 = G_r \), and (iv) at the optimum all the constraints except (1a, \( i = 3,4 \)), (2c), (4e), and (5a, \( i = 4 \)) bind.\(^{15}\)

**Proof:** See the Appendix.

Part (iv) of the theorem tells us that with the exception of (1a, \( i = 3,4 \)), (2c), (4e), and (5a, \( i = 4 \)), all the other constraints are satisfied. The proof of Theorem 4 tells us that constraints (1a, \( i = 3,4 \)), (2c), (4e), and (5a, \( i = 4 \)) hold as strict inequalities. Hence, the contract described in Theorem 4 is indeed collusion-proof.

A comparison of the optimal ex ante and ex post contracts when there is government/firm collusion can be made with the aid of Table 3. There are four essential differences. First, while the ex ante contract equalizes abatement levels in states 1, 3, and 4, the ex post contract equalizes the

\(^{15}\)For an exact representation of \( D \) and the \( \tilde{A}_i \), see the proof of Theorem 4 in the Appendix.
Table 3. The Government/Firm Collusion Case: Ex Ante Versus Ex Post Contracting

<table>
<thead>
<tr>
<th>Contracting</th>
<th>Ex Ante</th>
<th>Ex Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollution abatement level and pattern</td>
<td>$a_1 = a_3 = a_4 &gt; a_2$</td>
<td>$\tilde{A}_4 g'(a_4) &gt; \tilde{A}_3 g'(a_3) = \tilde{A}_2 g'(a_2)$</td>
</tr>
<tr>
<td>Transfers to the government</td>
<td>$\tilde{G}_4 &gt; \tilde{G}_1 &gt; \tilde{G}_2 = \tilde{G}_3$</td>
<td>$\tilde{G}_4 &gt; \tilde{G}_1 = \tilde{G}_2 = \tilde{G}_3$</td>
</tr>
<tr>
<td>Payoffs to the polluting firm</td>
<td>$\tilde{T}_5 &gt; \tilde{T}_4 &gt; \tilde{T}_1 &gt; \tilde{T}_2$</td>
<td>$\tilde{T}_3 - g(a_3) &gt; \tilde{T}_4 - g(a_4) &gt; \tilde{T}_1 - g(a_1) = \tilde{T}_2 - g(a_2)$</td>
</tr>
</tbody>
</table>

weighted marginal cost of abatement in these three states. The weights $\tilde{A}_i$ are functions of the state probabilities and the Kuhn-Tucker multipliers. Second, while the ex ante contract specifies $\tilde{G}_4 > \tilde{G}_1 > \tilde{G}_2 = \tilde{G}_3$, the ex post contract specifies $\tilde{G}_4 > \tilde{G}_1 = \tilde{G}_2 = \tilde{G}_3$. In both cases, the government’s signal is noninformative in states 2 and 3. As such, the optimal ex ante and ex post contracts specify $\tilde{G}_2 = \tilde{G}_3$. Further, the government can lie about its signal in states 1 and 4. In order to induce truth telling by the risk-averse government, both optimal contracts specify $\tilde{G}_4 > \tilde{G}_1$. Third, in the ex ante case, $\tilde{T}_3 > \tilde{T}_4 > \tilde{T}_1 > \tilde{T}_2$, whereas in the ex post case, $\tilde{T}_3 - g(a_3) > \tilde{T}_4 - g(a_4) = \tilde{T}_1 - g(a_1) = \tilde{T}_2 - g(a_2)$. In both cases, the highest gross and net payments, respectively, are in state 3. This encourages the firm to tell the truth about $\theta$ when the government is unable to convey an informative report to the SNGA. Finally, while in the ex ante case, 6 constraints—(2a), (2b), (2d), (4a), (4c), and (4d)—bind at the optimum, in the ex post case, 13 constraints—(1a, $i \neq 4$), (2b), (3a), (4b), (4c), (4d), and (5a, $i \neq 4$)—bind at the optimum. This means that the SNGA’s expected welfare when he designs country specific IEA’s can be no greater than when he designs a single contract for all the relevant DCs. This also tells us that when there is government/firm collusion, the SNGA will prefer to design a single contract rather than $N$ country-specific contracts.
Finally, consider the differences in the optimal *ex post* contract, without and with collusion. The essential features of these two contracts are illustrated in Table 4. From this table, we see that collusion has no qualitative impact on the pattern of equilibrium pollution abatement. However, the quantitative impact is almost certainly different because, in general, the weights $A_i$ and $\tilde{A}_i$ will be unequal. The transfers to the government are almost unchanged; the only change—$\tilde{G}_4 > \tilde{G}_1$—in the collusion case—reflects the need to establish incentives so that the government reports truthfully in the high abatement quality state $4$. The net payoff to the firm in both contracting regimes exhibits the same qualitative pattern. In the collusion case though, the transfers are designed so that the equilibrium contract is collusion-proof.

6. Conclusions

In this paper I analyzed the question of environmental protection for identical and heterogeneous DCs within the framework of the directives set forth in the various agreements reached at the 1992 Rio Earth Summit. I modeled the institutional setting for the underlying

Table 4. *Ex Post* Contracting Without and With Collusion

<table>
<thead>
<tr>
<th>Contracting</th>
<th>Without Collusion</th>
<th>With Collusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollution abatement level and pattern</td>
<td>$A_1 g'(a_1)=A_2 g'(a_1)=A_4 g'(a_4) &gt; A_2 g'(a_2)$</td>
<td>$\tilde{A}_1 g'(a_1)=\tilde{A}_3 g'(a_3)=\tilde{A}_4 g'(a_4) &gt; \tilde{A}_2 g'(a_2)$</td>
</tr>
<tr>
<td>Transfers to the government</td>
<td>$G_i \forall i$</td>
<td>$\tilde{G}_4 &gt; \tilde{G}_1 = \tilde{G}_3 = \tilde{G}_r$</td>
</tr>
<tr>
<td>Net payoffs to the polluting firm</td>
<td>$T_3 - g(a_3) &gt; T_1 - g(a_1) = T_2 - g(a_2)$</td>
<td>$\tilde{T}_3 - g(a_3) &gt; \tilde{T}_1 - g(a_1) &gt; \tilde{T}_2 - g(a_2)$</td>
</tr>
<tr>
<td></td>
<td>$= T_4 - g(a_4)$</td>
<td>$= \tilde{T}_4 - g(a_4)$</td>
</tr>
</tbody>
</table>
problem as a three-tiered hierarchy with $N$ forks, and then I studied the nature of the optimal, budget balanced *ex ante* and *ex post* contracts, without and with collusion. A number of policy conclusions emerge.

First, although it is generally more desirable to account for the heterogeneity of DCs by designing country-specific IEAs, the SNGA will prefer not to do so. Alternately put, the SNGA will prefer to treat DCs as identical and design a single contract. This is because the SNGA’s payoff, when he contracts *ex ante*, is typically higher than his payoff when he contracts with the various DCs *ex post*. However, it should be noted that in the context of DCs, unless the SNGA can limit the *ex post* liability of the players, nations may well refuse to participate in *ex ante* contracting schemes. This tells us that there is a potential conflict between the kind of IEA a SNGA is likely to want and the kind of IEA that is likely to be favored by DCs.

Second, because of the nature of the IEA design problem, and because most of the incentive constraints and all the budget constraints bind, we typically cannot expect that the SNGA’s designed contracts will elicit the first best level of abatement. However, this question cannot be definitively resolved because the contractually specified abatement levels cannot be directly compared across the five contracting scenarios.

Third, the qualitative features of the optimal IEAs depend on the timing of the underlying game between the SNGA, the government, and the firm. Whereas in the *ex ante* case, the pattern of abatement and the gross payoffs to the government and the firm can be characterized explicitly, such is not the case in the *ex post* case. Further, while the *ex ante* contract equalizes abatement levels in three of the four states, the *ex post* contract equalizes the weighted marginal cost of abatement in these three states. This means that if the SNGA cannot get the relevant parties to
contract *ex ante*, the resulting contract will be more complex. In this more complex contract, the level of pollution abatement will depend on the marginal welfare of the SNGA’s funds and on the state probabilities.

Fourth, several observers, such as Rogers (1993, p. 236), have worried that many of the Earth Summit directives “... offer a back door option by which signatories can excuse themselves at a later date if the going gets too tough.” The implementability of *ex post* contracts should diminish such concerns because an *ex post* contract can be viewed as a limited liability contract. In this sense, as compared to an *ex ante* contract, an *ex post* contract is more likely to be renegotiation proof.

Fifth, the research of this paper tells us that a SNGA can indeed circumvent the monitoring and enforcement problem stemming from national sovereignty by designing collusion-proof contracts.

With talk of rising disparity between the South and the North and the increasingly acrimonious nature of international discussions regarding the use of environmental resources, the design question studied in this paper takes on a particular significance. This is in no small measure due to the fact that the implementation of such agreements will do more to engender and maintain international security than will most strategic or unilateral policy measures.
Appendix

In this appendix, I provide the proofs of the four theorems stated in the text of the paper. All the proofs involve Kuhn-Tucker analysis.

**Proof of Theorem 1:** I shall proceed by means of ten steps. The Lagrangian to (2)-(2d) is

\[
\mathcal{L} = \sum_{i} p_i (x_i - T_i) + \alpha \left[ \sum_{i} p_i \beta \left( \bullet \right) - B_r \right] + \beta \left[ T_3 - g(\bullet) - T_2 + g(\bullet) \right]
\]

\[
\gamma \left( \sum_{i} p_i \left[ \hat{M} - \sum_{i} \left( G_i^{j} + T_i^{j} \right) \right] \right),
\]

where \( \alpha, \beta, \) and \( \gamma \) are the multipliers corresponding to (2a), (2b), and (2d), respectively. The first-order necessary conditions are

(a1) \( \alpha \left( \frac{\partial B[\bullet]}{\partial T_1} \right) = 1 + \gamma, \)

(a2) \( \alpha \left( \frac{\partial B[\bullet]}{\partial T_2} \right) = 1 + (\beta/p_2) + \gamma, \)

(a3) \( \alpha \left( \frac{\partial B[\bullet]}{\partial T_3} \right) = 1 - (\beta/p_3) + \gamma, \)

(a4) \( \alpha \left( \frac{\partial B[\bullet]}{\partial T_4} \right) = 1 + \gamma, \)

(a5) \( \alpha B[\bullet] g'(a_1) = 1, \)

(a6) \( \alpha B[\bullet] g'(a_2) = 1 + (\beta/p_2) g'(a_2 - \Delta \theta), \)

(a7) \( \alpha B[\bullet] + (\beta/p_3) g'(a_3) = 1, \)

and (a8) \( \alpha B[\bullet] g'(a_4) = 1. \)

**Step 1:** (2d) binds at the optimum.

**Proof:** This follows by assumption. See the related discussion in section 3b.

**Step 2:** (2a) binds at the optimum.

**Proof:** From (a1) \( \alpha = 0 \Rightarrow \gamma = -1. \) This is impossible. Thus \( \alpha > 0. \)

**Step 3:** \( a_1 = a_4, \ T_1 = T_4. \)

**Proof:** (a1) and (a4) give \( T_1 - g(a_1) = T_4 - g(a_4). \) Using this and manipulating (a5) and (a8), I get \( a_1 = a_4. \)

Now it follows that \( T_1 = T_4. \)

**Step 4:** (2b) binds at the optimum.

---

\(^{16}\)I shall check later to see that (2c) is satisfied.
Proof: \( \beta = 0 \), (a2), and (a3) tell us that \( T_3 - g(a_3) = T_2 - g(a_2) \). Using this equality in (2b) gives 
\[ 0 > g(a_2) - g(a_2 - \Delta \theta) \, . \] This is impossible. Thus, \( \beta > 0 \). 

Step 5: \( a_4 > a_2 \).

Proof: (a4) and (a8) give \( (1 + \gamma)g'(a_4) = 1 \). (a2) and (a6) tell me that \( (1 + \gamma)g'(a_2) < 1 \). From these two expressions, I get \( g'(a_4) > g'(a_2) \Rightarrow a_4 > a_2 \). 

Step 6: \( a_3 = a_4 \).

Proof: From (a3), (a4), (a7), and (a8), I get \( (1 + \gamma)g'(a_3) = (1 + \gamma)g'(a_4) \Rightarrow a_3 = a_4 \). 

Step 7: \( T_3 > T_2 \).

Proof: This claim follows because \( \beta > 0 \), and \( a_3 > a_2 \). 

Step 8: \( T_3 > T_1 \).

Proof: (a1) and (a3) give \( T_3 - g(a_3) > T_1 - g(a_1) \). Because \( a_1 = a_3 \), I conclude that \( T_3 > T_1 \). 

Step 9: \( T_4 > T_2 \).

Proof: (a2) and (a4) give \( T_4 - g(a_4) > T_2 - g(a_2) \). Because \( a_4 > a_2 \), it follows that \( T_4 > T_2 \). 

Step 10: \( a_1 = a_3 = a_4 > a_2 \).

Proof: This follows from steps 3, 5, and 6. 

Finally, I shall check to see that (2c) is satisfied. This is equivalent to showing that 
\[ g(a_3 + \Delta \theta) - g(a_3) > g(a_2) - g(a_2 - \Delta \theta) \, . \] This inequality holds because \( a_3 > a_2 \), \( \Delta \theta > 0 \), \( g' > 0 \), and \( g'' > 0 \). This completes the proof of Theorem 1. 

Proof of Theorem 2: I shall proceed by means of seven steps. The Lagrangian is

\[ L = \sum \phi_i P_i (x_i - T_i) + \sum \phi_i a_i \{ B_i \{ \bullet \} - B_r \} + \beta \{ T_3 - g(\bullet) - T_2 + g(\bullet) \} + \sum \phi_i \gamma_i [\tilde{M} - \tilde{M}_r] \{ G_i \{ \bullet \} + T_i \} \, . \quad (b) \] 

\[ 17 I \ shall \ check \ later \ to \ see \ that \ (2c) \ is \ satisfied. \]
where $\alpha_i$, $\beta$, $\gamma_i$, $i = 1, \ldots, 4$ are the multipliers corresponding to (1a), (2b), and (3a), respectively. The first-order necessary conditions are

(b1) $\alpha_1 \{ \partial B[\cdot]/\partial T_1 \} = \gamma_1 + p_1$, 
(b2) $\alpha_2 \{ \partial B[\cdot]/\partial T_2 \} = \beta + \gamma_2 + p_2$, 
(b3) $\alpha_3 \{ \partial B[\cdot]/\partial T_3 \} = \gamma_3 - \beta + p_3$, 
(b4) $\alpha_4 \{ \partial B[\cdot]/\partial T_4 \} = \gamma_4 + p_4$, 

and

(b5) $\alpha_1 B'[\cdot]g'(a_1) = p_1$, 
(b6) $\alpha_2 B'[\cdot]g'(a_2) = \beta g'(a_2 - \Delta \theta) + p_2$, 
(b7) $\{ \alpha_3 B'[\cdot] + \beta \} g'(a_3) = p_3$, and 
(b8) $\alpha_4 B'[\cdot]g'(a_4) = p_4$.

Step 1: The budget constraints bind at the optimum.

Proof: This follows by assumption. See the related discussion in section 3b.

Step 2: For $i \neq 3$, $\alpha_i > 0$.

Proof: From (b1) $\alpha_1 = 0 \Rightarrow \gamma_1 = -p_1$, which is impossible. Thus $\alpha_1 > 0$. From (b2) $\alpha_2 = 0 \Rightarrow \gamma_2 = -(p_2 + \beta)$, which is impossible, irrespective of whether $\beta > 0$. Thus, $\alpha_2 > 0$. From (b4) $\alpha_4 = 0 \Rightarrow \gamma_4 = -p_4$, which is impossible. Thus $\alpha_4 > 0$.

Step 3: (2b) binds at the optimum.

Proof: From (b3) $\beta = 0 \Rightarrow \alpha_3 > 0$. Using this result in (2b) gives $0 > g(a_2) - g(a_2 - \Delta \theta)$. This is impossible. Thus, $\beta > 0$.

Step 4: $(1a, i = 3)$ is slack at the optimum.

Proof: $\alpha_2 > 0$, $\beta > 0$ tell us that $T_3 - g(a_3) = T_2 - g(a_2) + \{ g(a_2) - g(a_2 - \Delta \theta) \} = T_r + \{ D \}$, $D > 0$.

In turn, this tells us that $T_3 - g(a_3) > T_r \Rightarrow \alpha_3 = 0$.

Step 5: $T_3 - g(a_3) > T_1 - g(a_1) = T_2 - g(a_2) = T_4 - g(a_4)$.

Proof: This follows because $\alpha_1 > 0$, $\alpha_2 > 0$, $\alpha_3 = 0$, $\alpha_4 > 0$.

Step 6: For $i \neq 2$, $a_i = (g')^{-1} \{ p_i/(p_i + \gamma_i) \}$, $a_2 = (g')^{-1} \{ p_2/(p_2 + \gamma_2) - D \}$, 
$D = \{ p_2/(p_2 + \gamma_2) - g'(a_2) \}$. 


Proof: From (b1) and (b5) I get \( q = (g')^{-1}\{p_1(p_1 + \gamma_1)\} \). From (b3) and (b7) I get \( a_3 = (g')^{-1}\{p_3(p_3 + \gamma_3)\} \). From (b4) and (b8), I get \( a_4 = (g')^{-1}\{p_4(p_4 + \gamma_4)\} \). Finally, from (b2) and (b6) I get \( a_5 = (g')^{-1}\{p_2(p_2 + \gamma_2 - D)\} \).

Step 7: \( A_i g'(a_i) = A_3 g'(a_3) = A_4 g'(a_4) > A_2 g'(a_2) \), where \( A_i = \{1 + (\gamma_i/p_i)\} \).

Proof: From the proof to step 6, it follows that \( \{1 + (\gamma_2/p_2)\} g'(a_2) < 1 = \{1 + (\gamma_1/p_1)\} g'(a_1) \), \( i = 1, 3, 4 \).

Finally, I shall check to see that (2c) is satisfied. This can be verified in a manner analogous to that employed in the proof of Theorem 1. This completes the proof of Theorem 2.

**Proof of Theorem 3:** I shall proceed by means of 13 steps. The Lagrangian is

\[
\mathcal{L} = \sum_{i=1}^{n} p_i (x_i - G_i - T_i) + \mu \left\{ \sum_{i=1}^{n} p_i V(\bullet) - V_r \right\} + \alpha \left\{ \sum_{i=1}^{n} p_i B(\bullet) - B_r \right\} + \beta \left\{ \tilde{T}_3 - g(\bullet) - \tilde{T}_2 + g(\bullet) \right\} + \epsilon \left\{ \tilde{G}_4 + \tilde{T}_4 - g(\bullet) - \tilde{G}_3 - \tilde{T}_3 + g(\bullet) \right\} + \kappa \left\{ \tilde{G}_3 + \tilde{T}_3 - g(\bullet) - \tilde{G}_2 - \tilde{T}_2 + g(\bullet) \right\} + \gamma \left\{ \sum_{i=1}^{n} p_i [\tilde{M} - \sum_{i=1}^{n} \{G_i + \tilde{T}_i\}] \right\},
\]

where \( \mu, \alpha, \beta, \epsilon, \kappa, \) and \( \gamma \) are the multipliers corresponding to (4a), (2a), (2b), (4c), (4d), and (2d), respectively. The first-order necessary conditions are

1. \( \mu \mathcal{L}'(G_1) = 1 + \gamma \),
2. \( \mu \mathcal{L}'(G_2) = 1 + \gamma + (\kappa/p_2) \),
3. \( \mu \mathcal{L}'(G_3) = 1 + \gamma + (\epsilon - \kappa)/p_3 \),
4. \( \mu \mathcal{L}'(G_4) = 1 + \gamma - (\epsilon/p_4) \),
5. \( \alpha \mathcal{L}'[\bullet] \partial T_1 = 1 + \gamma, \)
6. \( \alpha \mathcal{L}'[\bullet] \partial T_2 = 1 + \gamma + (\beta + \kappa)/p_2, \)
7. \( \alpha \mathcal{L}'[\bullet] \partial T_3 = 1 + \gamma + (\epsilon - \beta - \kappa)/p_3, \)
8. \( \alpha \mathcal{L}'[\bullet] \partial T_4 = 1 + \gamma - \epsilon/p_4, \)
9. \( \alpha \mathcal{L}'[\bullet] g'(a_1) = 1, \)
10. \( \alpha \mathcal{L}'[\bullet] g'(a_2) = 1 + \{(\beta + \kappa)/p_2\} g'(a_2 - \Delta \theta), \)
11. \( \alpha \mathcal{L}'[\bullet] + (\beta - \epsilon + \kappa)/p_3 \) \( g'(a_3) = 1, \)
12. \( \alpha \mathcal{L}'[\bullet] + \epsilon/p_4 \) \( g'(a_4) = 1. \)

Step 1: (2d) binds at the optimum.

Proof: This follows by assumption. See the related discussion in section 3b.
Step 2: (4a) binds at the optimum.

Proof: Substituting \( v = 0 \) in (c1) yields \( \gamma = -1 \). This is impossible. Thus, \( v > 0 \).

Step 3: (2a) binds at the optimum.

Proof: Substituting \( \alpha = 0 \) in (c5) gives \( \gamma = -1 \). This is impossible. Thus, \( \alpha > 0 \).

Step 4: (2b) binds at the optimum.

Proof: Pick \( \beta = 0 \), (c2), (c3), (c6), and (c7) give (c13) \( V'( \bar{G}_2')/V'( \bar{G}_3') = B' [ \bar{T}_2 - g(a_2)]/BB' [ \bar{T}_3 - g(a_3)] \).

From (2b) I get \( \bar{T}_3 - g(a_3) > \bar{T}_2 - g(a_2 - \Delta \theta) > \bar{T}_2 - g(a_2) \). Substituting this in (c13), I get \( \bar{G}_3 > \bar{G}_2 \).

Using \( \bar{G}_3 > \bar{G}_2 \) and \( \bar{T}_3 - g(a_3) > \bar{T}_2 - g(a_2) \) I get \( \kappa = 0 \). Now (c6) and (c7) give \( \bar{T}_2 - g(a_2) > \bar{T}_3 - g(a_3) \). This last inequality violates (2b). Thus, \( \beta > 0 \).

Step 5: (4d) binds at the optimum.

Proof: Recall that \( \beta > 0 \), (c2), and (c3) tell us that \( V'( \bar{G}_3') > V'( \bar{G}_2') \Rightarrow \bar{G}_3 > \bar{G}_2 \). This is impossible. Thus, \( \kappa > 0 \Rightarrow \bar{G}_3 = \bar{G}_2 \).

Step 6: (4c) binds at the optimum.

Proof: \( \epsilon = 0 \), (c2), (c3), and \( \bar{G}_2 = \bar{G}_3 \) tell us that \( \nu V'( \bar{G}_2') = \nu V'( \bar{G}_3') \). This is impossible. Thus, \( \epsilon > 0 \).

Step 7: \( \bar{G}_4 > \bar{G}_1 \).

Proof: \( \epsilon > 0 \), (c1), and (c4) tell us that \( V'( \bar{G}_4') < V'( \bar{G}_1') \Rightarrow \bar{G}_4 > \bar{G}_1 \).

Step 8: \( a_1 = a_3 = a_4 = (g')^{-1} \{ 1/(1 + \gamma) \} \).

Proof: From (c5) and (c9) I get \( a_1 = (g')^{-1} \{ 1/(1 + \gamma) \} \). From (c7) and (c11) I get \( a_3 = (g')^{-1} \{ 1/(1 + \gamma) \} \). From (c8) and (c12) I get \( a_4 = (g')^{-1} \{ 1/(1 + \gamma) \} \).

Step 9: \( a_2 < a_i \), \( i = 1, 3, 4 \).
Proof: From (c5) and (c9) I get \((1 + \gamma)g'(a_1) = 1\). From (c6) and (c10) I get \((1 + \gamma)g'(a_2) < 1\).

From these two expressions it follows that \(g'(a_2) < g'(a_1) \Rightarrow a_2 < a_1\). 

**Step 10:** \(\bar{G}_1 > \bar{G}_2\).

**Proof:** (c1) and (c2), give \(V'(\bar{G}_1) = (1 + \gamma)/\nu\), and \(V'(\bar{G}_2) = \{(1 + \gamma)/\nu\} + \kappa/p_2\nu\). Since \(V''(\bullet) < 0\), it follows that \(\bar{G}_1 > \bar{G}_2\). 

**Step 11:** \(\bar{G}_4 > \bar{G}_1 > \bar{G}_2 = \bar{G}_3\).

**Proof:** This follows from steps 5, 7, and 10.

**Step 12:** \(\bar{G}_4 + \bar{T}_4 = \bar{G}_3 + \bar{T}_3\).

**Proof:** This follows because \(\epsilon > 0\), and \(a_3 = a_4\).

**Step 13:** \(\bar{T}_3 > \bar{T}_4 > \bar{T}_1 > \bar{T}_2\).

**Proof:** (c5), (c6), (c8), and \(B''(\bullet) < 0\) give \(\bar{T}_4 - g(a_4) > \bar{T}_1 - g(a_1) > \bar{T}_2 - g(a_2) \Rightarrow \bar{T}_4 > \bar{T}_1 > \bar{T}_2\).

Combining this with steps 11 and 12 yields \(\bar{T}_3 > \bar{T}_4 > \bar{T}_1 > \bar{T}_2\).

Finally, I need to check that (2c), (4b), and (4e) are satisfied. The fact that (2c) holds as a strict inequality can be verified in a manner analogous to that employed in the proof of Theorem 1.

Given this, (4e) also holds as a strict inequality because \(\bar{G}_2 = \bar{G}_3\). Finally, (4b) is satisfied because \(\bar{G}_1 > \bar{G}_2\) and because \(\bar{T}_1 - g(a_1) > \bar{T}_2 - g(a_2)\). This completes the proof of Theorem 3.

**Proof of Theorem 4:** I shall proceed by means of 13 steps. The Lagrangian is

\[
\mathcal{L} = \sum_{i,j} P \{ (x_i - \bar{G}_i - \bar{T}_i) + \sum_{a} \alpha_i \{ B[\bullet] - B_r \} + \sum_{a} \nu_i \{ V(\bullet) - V_r \} + \beta \{ \bar{T}_3 - g(\bullet) - \bar{T}_2 + g(\bullet) \} + \\
\epsilon_1 \{ \bar{G}_1 + \bar{T}_1 - g(\bullet) - \bar{G}_2 - \bar{T}_2 + g(\bullet) \} + \epsilon_2 \{ \bar{G}_4 + \bar{T}_4 - g(\bullet) - \bar{G}_3 - \bar{T}_3 + g(\bullet) \} + \\
\kappa \{ \bar{G}_3 + \bar{T}_3 - g(\bullet) - \bar{G}_2 - \bar{T}_2 + g(\bullet) \} + \sum_{a} \gamma_i \{ \bar{M} - \sum_{a} \{ \bar{G}_i^j + \bar{T}_i^j \} \} \]
\]

\((d)\)

I will check later to see that (2c) and (4e) are satisfied.
where \( \alpha_i, \nu_i, \beta, \epsilon_i, \kappa, \gamma_i, i = 1, 2, 3, 4, l = 1, 2 \) are the multipliers associated with (1a), (5a), (2b), (4b), (4c), (4d), and (3a), respectively. The first-order necessary conditions are

\[(d1) \quad \nu_1 V'(G_1) = p_1 - \epsilon_1 + \gamma_1, \quad (d2) \quad \nu_2 V'(G_2) = p_2 + \epsilon_1 + \kappa + \gamma_2, \quad (d3) \quad \nu_3 V'(G_3) = p_3 + \epsilon_2 - \kappa + \gamma_3, \]
\[(d4) \quad \nu_4 V'(G_4) = p_4 - \epsilon_2 + \gamma_4, \quad (d5) \quad \alpha_1 \{ \partial B_1 [s] / \partial \bar{T}_1 \} = p_1 - \epsilon_1 + \gamma_1, \]
\[(d6) \quad \alpha_2 \{ \partial B_2 [s] / \partial \bar{T}_2 \} = p_2 + \beta + \epsilon_1 + \kappa + \gamma_2, \quad (d7) \quad \alpha_3 \{ \partial B_3 [s] / \partial \bar{T}_3 \} = p_3 + \epsilon_2 - \beta - \kappa + \gamma_3, \]
\[(d8) \quad \alpha_4 \{ \partial B_4 [s] / \partial \bar{T}_4 \} = p_4 - \epsilon_2 + \gamma_4, \quad (d9) \quad \{ \alpha_4 B^/[s] + \epsilon_i \} g'(a_4) = p_4. \]

**Step 1:** The budget constraints bind at the optimum.

**Proof:** This follows by assumption. See the related discussion in section 3b.

**Step 2:** (5a, \( i = 2 \)) and (1a, \( i = 2 \)) bind at the optimum.

**Proof:** From (d2) \( \nu_2 = 0 \Rightarrow \gamma_2 = -(p_2 + \epsilon_1 + \kappa) \). This is impossible irrespective of whether \( \epsilon_1 \geq 0 \) and \( \kappa \geq 0 \). Thus, \( \nu_2 > 0 \). From (d6) \( \alpha_2 = 0 \Rightarrow \gamma_2 = -(p_2 + \beta + \epsilon_1 + \kappa) \). This is impossible irrespective of whether \( \beta > 0 \), \( \epsilon_1 > 0 \), and \( \kappa > 0 \). Thus, \( \alpha_2 > 0 \).

**Step 3:** (5a, \( i = 1 \)) and (1a, \( i = 1 \)) bind at the optimum.

**Proof:** (d1) tells us that \( \nu_1 = \epsilon_1 = 0 \) is impossible. (d1) and (d5) tell us that either (i) \( \alpha_1 = \nu_1 = 0 \), or (ii) \( \alpha_1 > 0 \), \( \nu_1 > 0 \). If (i) holds, then (4b) is slack and \( \epsilon_1 = 0 \). But this is impossible. Thus, \( \alpha_1 > 0 \), \( \nu_1 > 0 \).

**Step 4:** (5a, \( i = 3 \)) binds at the optimum.

**Proof:** From (d3), \( \nu_3 = 0 \Rightarrow \kappa = p_3 + \epsilon_2 + \gamma_3 \). Substituting this in (d7), I get \( \alpha_3 B^/[s] + \beta = 0 \Rightarrow \alpha_3 = \beta = 0 \). If the state 3 participation and the incentive compatibility constraints...
are slack at the optimum, then the SNGA can increase his welfare by lowering $\overline{T_3}$. But this violates $\gamma_3 > 0$. Thus, $\nu_3 > 0$. ■

**Step 5:** (2b) binds at the optimum.

**Proof:** If $\beta = 0$, then (d3) and (d7) tell us that $\alpha_3 > 0$. Using this in (2b) yields
$$0 > g(a_2) - g(a_2 - \Delta \theta).$$
This is impossible. Thus, $\beta > 0$. ■

**Step 6:** (4d) binds at the optimum.

**Proof:** $\beta > 0 \Rightarrow \overline{T_3} - g(a_3) = \overline{T_2} - g(a_2 - \Delta \theta)$. Because $\overline{G_2} = \overline{G_3}$, I conclude that $\kappa > 0$. ■

**Step 7:** (1a, $i = 3$) is slack at the optimum.

**Proof:** $\beta > 0 \Rightarrow a_2 > 0 \Rightarrow \overline{T_3} - g(a_3) = \overline{T_2} - g(a_2) + \{g(a_2) - g(a_2 - \Delta \theta)\} = T_r + \{D\}, D > 0$. In turn, this tells us that $\overline{T_3} - g(a_3) > T_r \Rightarrow \alpha_3 = 0$. ■

**Step 8:** (1a, $i = 4$) and (5a, $i = 4$) are slack at the optimum.

**Proof:** (d4) and (d8) tell us that either (i) $\alpha_4 > 0$, $\nu_4 > 0$, or (ii) $\alpha_4 = \nu_4 = 0$. If (i) holds, then (4c) is violated. Thus, $\alpha_4 = \nu_4 = 0$. ■

**Step 9:** $\overline{G_4} > \overline{G_1} = \overline{G_2} = \overline{G_3} = \overline{G_r}$.

**Proof:** This follows because $\nu_1 > 0$, $\nu_2 > 0$, $\nu_3 > 0$, $\nu_4 = 0$. ■

**Step 10:** (4b) and (4e) bind at the optimum.

**Proof:** $\alpha_1 > 0$, $\alpha_2 > 0$, $\nu_1 > 0$, $\nu_2 > 0 \Rightarrow \epsilon_1 > 0$. (d4) and $\nu_4 = 0 \Rightarrow \epsilon_2 > 0$. ■

**Step 11:** $\overline{T_3} - g(a_3) > \overline{T_4} - g(a_4) > \overline{T_1} - g(a_1) = \overline{T_2} - g(a_2)$.

**Proof:** This follows because $\alpha_4 = 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, $\epsilon_2 > 0$. ■

**Step 12:** For $i \neq 2$, $a_i = (g')^{-1}\{p_i(p_i + \gamma_i)\}$, $a_2 = (g')^{-1}\{p_2(p_2 + \gamma_2) - D\}$, $D = \{p_2(p_2 + \gamma_2) - g'(a_2)\}$. 

Proof: From (d5) and (d9), (d7) and (d11), and (d8) and (d12), I get
\[ q_i = (g')^{-1}\left\{ \frac{p_i}{p_i + \gamma} \right\}, \quad i = 1, 3, 4. \]
From (d6) and (d10), I get
\[ a_2 = (g')^{-1}\{ p_2(p_2 + \gamma) - D \}. \]

Step 13: \( \tilde{A}_1 g'(a_1) = \tilde{A}_3 g'(a_3) = \tilde{A}_4 g'(a_4) = 1 \\geq \tilde{A}_2 g'(a_2), \tilde{A}_1 = \{1 + (\gamma/p_i)\}. \)

Proof: From (d5) and (d9), (d7) and (d11), and (d8) and (d12), I get
\[ \{1 + (\gamma_1/p_1)\} g'(a_1) = \{1 + (\gamma_3/p_3)\} g'(a_3) = \{1 + (\gamma_4/p_4)\} g'(a_4) = 1. \]
From (d6) and (d10), I get
\[ \{1 + (\gamma_2/p_2)\} g'(a_2) < 1. \]
Hence, the claim follows.

I now check to see that (2c) and (4e) are satisfied. The satisfaction of (2c) can be verified as in the proof of Theorem 1. Having shown that (2c) is satisfied, to verify that (4e) is satisfied, it suffices to note that from step 9, \( \overline{G_2} = \overline{G_3} \). This completes the proof of Theorem 4.
References


