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by

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ABSTRACT

I consider the design of first best rural wage contracts for many tenants by an absentee landlord who delegates part of the contracting decision to his hired agent in each village. I analyze contracting in two scenarios. The first scenario is a two tiered hierarchy with no agent/tenant collusion and the second scenario is a three tiered hierarchy with agent/tenant collusion. I show that irrespective of whether the contracting is two or three tiered, when the productivities of tenants and the private information of agents across villages is perfectly correlated, the absentee landlord can always implement the first best wage contract in a Bayesian-Nash equilibrium.

JEL Classification: O12, O17

Keywords: Absentee landlord, contract, rural organization
1. Introduction

The past three decades has seen the emergence of a large literature that has analyzed the properties of contractual arrangements between landlords and tenants in agrarian economies. This literature has explored different aspects of rural contracts such as the existence of share tenancy [Stiglitz (1974), Bardhan (1984)], the role of limited liability [Basu (1992)], and the existence of permanent and spot laborers [Eswaran and Kotwal (1985)]. While it is clear that in most settings, landlords typically contract with many tenants whose productivities are positively correlated, the significance of relative performance evaluation in the design of rural wage contracts has been little studied in development economics. As such, the purpose of this paper is to analyze two instances in which relative performance evaluation is substantially in the interest of the landlord.

Specifically, I analyze two scenarios in which contracting takes place between crop growing tenants and an absentee landlord (AL) who owns land in two villages in a certain geographic area and who cannot be present on his land to supervise the hiring of tenants. The AL delegates part of the contracting decision to his hired agent in each of the two villages. The agent in each village communicates to the AL his observation of the realization of a random variable denoting the uncertain nature of tenant productivity. In the first scenario that I analyze, the agent in each village plays a passive role and the contracting is essentially a case of direct, two tiered interaction between the AL and the tenant. The AL is assumed to be unable to monitor the activities of either his agent or the tenant in each village; alternately, the cost of monitoring is assumed to be prohibitively high. Thus, in the second scenario that I analyze, I allow for the possibility that the agent and the tenant
in each village may collude to maximize the sum of the wages to be received from the AL. In this scenario, the contracting depends fundamentally on the activities of the hired agent. As such, the contracting is indirect and three tiered. The productivities of the tenants and the information of the agents is perfectly correlated. I show that in this setting, irrespective of whether the agent and the tenant in each village collude, i.e., irrespective of whether the contracting is two or three tiered, the AL can always implement the full information optimum (to be explained in section 2b) contract which extracts all the surplus from the agent and the tenant in each village.²

2. The Theoretical Framework

2a. Description of the Model

I extend previous research in multi-agent contract theory [see Sappington and Demski (1983), Demski and Sappington (1984)] and the economics of hierarchies [see Tirole (1986), Kofman and Lawarree (1993)] to model the three tiered interaction between an AL, his hired agent and a tenant in each of the two villages.

In what follows, I will focus on village A. The analysis is analogous for village B. Subscripts \( i = 1, 2, 3, 4 \) will always refer to the state of nature. Superscripts will refer to the village. Let the random variable \( \theta^A \) denote the uncertainty about tenant productivity. I assume that \( \theta \) has binary support \([\hat{\theta}^A, \tilde{\theta}^A]\), where \( 0 < \hat{\theta}^A < \tilde{\theta}^A \), and \( \Delta \theta = \tilde{\theta}^A - \hat{\theta}^A \). I shall refer to \( \hat{\theta}^A \) as the low productivity parameter and to \( \tilde{\theta}^A \) as the high productivity parameter.

The risk averse tenant in A grows a certain crop on the AL’s land, whose output and value in state \( i \) are denoted by \( x_i^A \in \mathbb{R} \). In state \( i \), the tenant chooses a level of labor effort \( e_i^A \in \mathbb{R} \). The

²The economic environment that I am analyzing consists of a three tiered hierarchy: I take this environment as given. As such, my objective is not to analyze whether this three tiered vertical structure is dominated by a two tiered vertical structure.
tenant's disutility of effort is given by $g(e^A_t)$, where \( g'(\cdot) > 0, \) \( g''(\cdot) > 0, \) and \( g(0) = 0 \). The tenant has a strictly concave and differentiable utility function \( U[T^A_u - g(e^A_t)] \), with \( \partial U'[\cdot] / \partial T^A_u \in (0, \infty), \forall T^A_u \).

\( T^A_u \in \mathbb{R} \) is the wage paid by the AL to the \( A \) tenant when he produces crop output \( x^A_t \) and the \( B \) tenant produces crop output \( x^B_t \). The \( A \) tenant's reservation utility is given by \( \hat{U}^A = U[\hat{T}^A] \), where \( \hat{T}^A \) is the tenant's reservation wage. \( \hat{U}^A \) and \( \hat{T}^A \) are common knowledge.

The risk averse agent in \( A \) has a strictly concave and differentiable utility function \( V(G^A_u) \), where \( G^A_u \) is the wage paid to the \( A \) agent for participating in the contract. The agent's reservation utility is \( \hat{V}^A - V(\hat{G}^A) \), where \( \hat{G}^A \) is the agent's reservation wage. \( \hat{V}^A \) and \( \hat{G}^A \) are common knowledge. I assume that \( V(G^A_u) \in (0, \infty), \forall G^A_u \). By employing a monitoring device, the agent in \( A \) receives a signal \( s^A \) from the tenant regarding his productivity and then he (the agent) provides a report \( r^A \) to the AL indicating what he observes about the tenant's productivity parameter\(^3\). In some states of nature, this monitoring device malfunctions. As a result, in such states, the agent is unable to provide useful information to the AL. The AL offers the \( A \) agent a wage \( G^A_u \in \mathbb{R} \), when he reports \( r^A \), and the \( B \) agent reports \( r^B \).

The AL is risk neutral and he has a profit function defined over the output of crops in the two villages. The profit function takes the form \( \Sigma_{il} (e^l_1 \cdot \theta^l - G^l - T^l), l = A, B. \) Note that the crop output and value produced by each tenant is \( x^l - e^l_1 \cdot \theta^l, l = A, B. \) The AL's profit is a function of the total production of crops less the sum of agent and tenant wages. The AL designs the contract which he offers to the respective agent and tenant in \( A. \) The contract can only be conditioned on what the AL actually observes, i.e., the \( A \) agent's report \( r^A \), the \( B \) agent's report \( r^B \), the \( A \) tenant's crop

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\(^3\)Since the main objective of this paper is not to study the effects of intra-village monitoring, I shall assume that the use of this monitoring device is costless.
output $x^A$, and the $B$ tenant's crop output $x^B$.

There are four states of nature, each state occurring with probability $p_i > 0$, where $\sum_i p_i = 1$. The random variables $\theta^A$ and $\theta^B$ - denoting tenant productivity in each village - are perfectly correlated. The AL, the agent and the tenant sign the contract at the beginning of the growing season. That is, the players hold symmetric but imperfect information regarding $\theta^A$. The tenant always observes $\theta^A$ before choosing his effort level. Depending on whether the agent's monitoring device functions or malfunctions, the agent may or may not observe the tenant's private information. In other words, the agent's signal $s^A$ may or may not be informative. For every realization of $\theta^A$, the agent's signal $s^A \in \{0^A, 1^A\}$, where $0^A$ represents the noninformative nature of the agent's signal. The signals $s^A$ and $s^B$ are perfectly correlated. The tenant always knows the state of nature. Neither the AL nor the agent ever know the effort undertaken by the tenant. The four states are:

- **State 1:** $\theta^A_1 = \hat{\theta}^A_1, \theta^B_1 = \hat{\theta}^B_1, s^A = \hat{\theta}^A_1, s^B = \hat{\theta}^B_1$.
- **State 2:** $\theta^A_2 = \hat{\theta}^A_2, \theta^B_2 = \hat{\theta}^B_2, s^A = \hat{\theta}^A_2, s^B = \hat{\theta}^B_2$.
- **State 3:** $\theta^A_3 = \hat{\theta}^A_3, \theta^B_3 = \hat{\theta}^B_3, s^A = \hat{\theta}^A_3, s^B = \hat{\theta}^B_3$.
- **State 4:** $\theta^A_4 = \hat{\theta}^A_4, \theta^B_4 = \hat{\theta}^B_4, s^A = \hat{\theta}^A_4, s^B = \hat{\theta}^B_4$.

In state 1, tenants and agents in both villages observe the low productivity parameter. That is, the agent monitoring devices in the two villages function and hence yield useful information. In state 2, both tenants observe the low productivity parameter but the two agents observe nothing. In other words, in this state, the two agent monitoring devices malfunction. In state 3, the two tenants observe the high productivity parameter and the two agents observe nothing. Once again, the two

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4In other words, the contracting analyzed in this paper is ex ante.
agent monitoring devices malfunction. Finally, in state 4 tenants and agents in both villages observe the high productivity parameter. In other words, the two agent monitoring devices function effectively in this state. I shall assume that $p_1 > p_2$, and that $p_4 > p_3$. That is, the two monitoring devices are reliable in the sense that they are more likely to function than to malfunction.

The timing of the game between the AL, the $A$ agent and the $A$ tenant is as follows. First, the AL offers a contract to the agent and to the tenant in $A$ at the beginning of the growing season. Second, the tenant observes the actual realization of $\theta^A$, and the agent receives his signal $s^A$. Third, the tenant chooses $e^A$. Fourth, crop output $x^A$ is produced by the tenant and the agent sends his report $r^A$ to the AL indicating what he observed. Finally, the AL compensates the agent and the tenant in $A$ by paying wages $G^A(x^A, x^B, r^A, r^B)$ and $T^A(x^A, x^B, r^A, r^B)$.

In the remainder of this paper I shall assume that the AL can verify the veracity of the agent's report $r^A$. By this I mean that if the agent's signal $s^A$ is noninformative, then the corresponding report $r^A$ reflects that fact and the AL can verify that the true facts are indeed as they have been reported. In symbols, $s^A = \theta^A \Rightarrow r^A = \theta^A$. On the other hand, I allow for the possibility that the agent will lie and report that his signal is informative when in fact such is not the case. That is, $s^A = \theta^A \Rightarrow r^A \in \{\theta^A, 0^A\}$.5

This completes the description of the model. I now consider the benchmark case in which perfect information is acquired by the AL.

2b. The Full Information Optimum

In this case, the AL observes the tenant productivity parameter denoted by $\theta^A$ and the tenant's

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5 The reader will note that I am restricting the agent's message space in certain states. Specifically, lying by the agent is effectively restricted to states 1 and 4. Alternately put, reporting the wrong state is equivalent to obtaining a noninformative signal. A more general model would permit lying in all four states.
actual effort choice. When this happens, the AL bypasses the $A$ agent and contracts with the $A$ tenant directly. Since the agent now has no role to play, he receives his reservation utility $\hat{V}^A$ in all four states. The AL now solves

$$\max_{e^A} [e^A - \theta^A - g(e^A)].$$

(1)

The first order necessary condition requires that

$$g'(e^A) = 1, \forall \theta^A.$$ 

(2)

In other words, in the full information optimum, the marginal profit from crop production is set equal to the marginal disutility of effort. The optimal level of effort $e^A$ is the same in all states. The tenant receives a wage which is independent of the state of nature. Specifically, the total wage equals

$$\{\hat{T}^A = U^{-1}(\hat{V}^A) \} \cdot g],$$

where $g = g(e^A)$ is the disutility of effort in the first best optimum. I can now define the full information/first best optimum.

**Definition:** In the full information optimum, (a) the agent and the tenant in each village are held to their reservation utilities, (b) (2) holds, and (c) the contract is Pareto efficient in every state.

I now move on to the more interesting cases in which the AL cannot determine either the realization of $\theta^A$ or the actual effort undertaken by the $A$ tenant.

3. **Direct Contracting: The no Agent-Tenant Collusion Case**

In this section I disallow the possibility of collusion between the agent and the tenant in $A$. When the $A$ agent receives his reservation utility $\hat{V}^A$, he is fully insured. Furthermore, since I am not allowing for the possibility of collusion between the agent and the tenant as yet and because the AL can verify the agent's report, by paying $\hat{G}^A = V^{-1}(\hat{V}^A)$, the AL can obtain the $A$ agent's information
at least cost. In terms of the design of the main contract, this means that the three tiered hierarchy effectively reduces to a two tiered hierarchy in which the $A$ agent plays a completely passive role.

The AL's problem now is to solve

\[
\max_{(e_i^A, r_i^A)} \sum_{u} p_i(e_i^A + \theta_i^A - T_{iu}^A) \tag{3}
\]

subject to

\[
\sum_{u} p_i U(T_{iu}^A - g(e_i^A)) = \hat{U}^A, \tag{4}
\]

\[
p_i [T_{22}^A - g(e_2^A)] \geq p_i [T_{22}^A - g(e_2^A + \Delta \theta^4)], \tag{5}
\]

and

\[
p_i [T_{33}^A - g(e_3^A)] \geq p_i [T_{33}^A - g(e_3^A + \Delta \theta^4)]. \tag{6}
\]

Constraint (4) is the tenant's individual rationality constraint. Note that since the contracting is ex ante, we have a single probabilistically weighted constraint. Constraints (5) and (6) are the tenant's incentive compatibility constraints. These constraints stem from the fact that the AL has imperfect information about $\theta^4$ in states 2 and 3. These are also the states in which the agent's signal $s^A$ is noninformative. Constraint (5) says that in state 2, if the tenant in village $B$ applies effort $e_2^B$, then the tenant in $A$ should not apply effort $e_2^A + \Delta \theta^4$ and claim that the state is 3. Constraint (6) says that in state 3, when the tenant in village $B$ applies effort $e_3^B$, the $A$ tenant should not apply effort $e_3^A - \Delta \theta^4$ and claim that the state is 2. In other words, these two constraints are the Nash incentive
compatibility constraints requiring the $A$ tenant to tell the truth, given that the $B$ tenant is telling the truth. I can now proceed to solve the AL's problem as stated in (3) - (6). I am led to

**Theorem 1:** The AL can implement the full information optimum contract in a Bayesian-Nash equilibrium. This contract has the following features: (a) the AL obtains the agent's information at least cost, (b) the agent's wage equals $\hat{G}^A - V^i(\hat{V}^A)$ in all states of nature, (c) the effort levels satisfy $e_i^A = (g')^{-1}(l) = e_i^A$, $\forall i$, (d) the wage paid by the AL to the tenant satisfies $T_{11}^A = T_{22}^A = T_{33}^A = T_{44}^A$, and (e) the contract is Pareto efficient in every state.

**Proof:** See the Appendix.

Comparing Theorem 1 with the definition of the full information optimum provided in section 2b, it is easy to verify that the contract specified in Theorem 1 does indeed implement the first best. Further, Theorem 1 describes the pattern of effort application one may expect to observe in our stylized two village setting when the AL does not know the tenant's productivity and he must design an optimal contract which takes into account the organizational hierarchy. Since the AL acquires the agent's information in states 1 and 4 and because this information is verifiable, the tenant can be required to apply effort at the first best level. The optimal contract then specifies equal wages to the tenant in these two states.

On the other hand when the state is 2 or 3, the AL's information is imperfect. This notwithstanding, Theorem 1 tells us that because the private information of the tenants in $A$ and $B$ is perfectly correlated, the AL can exploit this fact to great advantage. Specifically, the AL can require that the first best level of effort be applied in these two states as well. As such, the wages to the tenant are the same in all four states. The two "out of equilibrium" wages satisfy

$$[T_{21}^A < T_{33}^A \cdot g(e_2^A - \Delta \theta^4) - g(e_3^A)],$$

and

$$[T_{32}^A < T_{22}^A \cdot g(e_3^A + \Delta \theta^4) - g(e_2^A)].$$

Intuitively, we can think
of the AL placing the two tenants in a Prisoner's dilemma game in states 2 and 3. In this game,
telling the truth, i.e., applying the "correct" level of effort is the unique Nash equilibrium. As such,
the existence of multiple equilibria is not an issue. Theorem 1 tells us that the first best
implementation result of Sappington and Demski (1983) extends to ex ante contractual settings as
well.

I stress that these results depend crucially on the perfect correlation of (a) the agent signals
and (b) the private information of the tenants in the two villages. The reader can verify for himself
that the full information optimum can be implemented by the AL in a dominant strategy equilibrium
as well.

4. Indirect Contracting: The Agent-Tenant Collusion Case

Recall that the AL is assumed to be unable to monitor the activities of agents and tenants in
A and B. Since the AL can never acquire the tenant's private information and must rely on his agent's
report \( r^A \) to design the optimal wage contract, it is of considerable interest to determine the nature
of the equilibrium contract that can be implemented by the AL when his agent and the tenant in
village A collude to maximize the sum of the wages to be received from the AL.

I model collusion between the agent and the tenant as follows. Before the resolution of the
uncertainty regarding the productivity parameter and at the time of signing the main contract, the
agent and the tenant in each village sign a secondary contract which entails the offer and acceptance
of a monetary bribe from the tenant to the agent. Naturally, this secondary contract is unobservable
by the AL. The bribe can only be conditioned on what the tenant and the agent observe, i.e., the bribe
is a function of the agent's report \( r^A \) and the tenant's crop output \( x^A \). With the payment and the
receipt of the bribe, the tenant's total wage becomes \( \bar{T}^A = T^A(\bullet) - b^A(x^A, r^A) \), and the agent's total
wage becomes $\widetilde{G}^A - G^A(\ast) + b^A(x^A, r^A)$. I shall not concern myself with the question of how the surplus from the bribe is divided. For my purposes it is only necessary to stipulate that this secondary contract is in fact signed by the agent and the tenant.

Collusion by the agent and the tenant alters the incentives of the various parties but not - as we shall see - the nature of the optimal contract offered by the AL. To see why the tenant in $A$ might want to bribe the village agent, consider state 4. In this state, the agent is indifferent between reporting that he has observed $\hat{e}^A$ and reporting that he has observed $\theta^A$. The tenant on the other hand would prefer that the agent report $\theta^A$. This is one instance in which a clear rationale exists for the tenant to bribe the village agent.

In order to formulate and solve the AL's problem when there is collusion, I shall use a method due to Tirole (1986, pp. 192-197; 1988, pp. 461-462). Specifically, I shall appeal to the "equivalence principle" and restrict myself to contracts that are collusion-proof. The method essentially involves setting up constraints in addition to the usual individual rationality and incentive compatibility constraints for the agent and the tenant. I stress that in this section, I am considering simultaneous collusion in both villages. The equilibrium contract designed by the AL for $A$ is collusion-proof on the assumption that if the resulting contract were not constrained to be collusion-proof, agent-tenant coalitions would form in both villages. The reader will note that this assumption of "simultaneous collusion" is weaker than the assumption which requires the wage contract for $A$ to be collusion-proof whether or not there is collusion in $B$. I can now formulate the AL's problem. The AL solves

$$\max_{(e^A, \theta^A, \bar{T}^A)} \sum_{i} p_i (e^A_i - \theta^A_i - \bar{T}^A_i)$$  \hspace{1cm} (7)
subject to (4), (5), (6) with $T^A_u$ replaced with $\tilde{T}^A_u$,

$$\sum_{\omega} p(\tilde{G}^A_u) \geq \tilde{v}^A,$$

(8)

$$p_2[\tilde{G}^A_{22} + \tilde{T}^A_{22} - g(e^A_2)] \geq p_2[\tilde{G}^A_{23} + \tilde{T}^A_{23} - g(e^A_1 + \Delta \theta^A)],$$

(9)

$$p_3[\tilde{G}^A_{33} + \tilde{T}^A_{33} - g(e^A_3)] \geq p_3[\tilde{G}^A_{23} + \tilde{T}^A_{23} - g(e^A_2 - \Delta \theta^A)],$$

(10)

$$p_1[\tilde{G}^A_{11} + \tilde{T}^A_{11} - g(e^A_1)] \geq p_2[\tilde{G}^A_{22} + \tilde{T}^A_{22} - g(e^A_2)],$$

(11)

and

$$p_4[\tilde{G}^A_{44} + \tilde{T}^A_{44} - g(e^A_4)] \geq p_3[\tilde{G}^A_{33} + \tilde{T}^A_{33} - g(e^A_3)].$$

(12)

Constraint (8) is the agent's individual rationality constraint. Constraint (9) tells us that the agent should not be able to bribe the tenant to lie in state 2 and apply effort at the level appropriate for state 3. Similarly, (10) tells us that the agent should not be able to bribe his tenant to apply effort in state 3 at the level appropriate for state 2. Constraints (11) and (12) are the core collusion constraints. The purpose of these two constraints is to make the solution to the AL's problem collusion-proof. Recall that in states 1 and 4 the agent's monitoring device functions and as such his signal $s^A$ is informative. Thus in these two states, the agent can hide this fact. Given this, constraints (11) and (12) are telling us that should an optimal secondary contract between the agent and the tenant arise, then the total wage bill less the disutility of effort in states 1 and 4 cannot be less than the corresponding totals in
states 2 and 3 respectively. Solving the AL’s problem (7) subject to (4), (5), (6), and (8) - (12), I can state

Theorem 2: In the three tier hierarchy with agent-tenant collusion, the AL can implement the full information optimum wage contract in a Bayesian-Nash equilibrium. This contract has the following features: (a) $e_i^A = (g')^{-1}(1) = e_i$, $\forall i$, (b) $\bar{G}_{11}^A = \bar{G}_{22}^A = \bar{G}_{33}^A = \bar{G}_{44}^A$, (c) $\bar{T}_{11}^A = \bar{T}_{22}^A = \bar{T}_{33}^A = \bar{T}_{44}^A$, (d) only the agent and the tenant individual rationality constraints bind, and (e) the contract is Pareto efficient in all four states.

Proof: See the Appendix.

To verify that the contract specified in Theorem 2 is indeed collusion-proof, I have to show that constraints (4) - (6) and (8) - (12) are satisfied. By part (d) of the Theorem, constraints (4) and (8) are satisfied. Because $\bar{T}_{23}^A$, $\bar{T}_{32}^A$, $\bar{G}_{23}^A$, and $\bar{G}_{32}^A$ do not enter the AL’s profit function or the agent and tenant utility functions, they can be chosen by the AL so as to ensure strict inequality in (5), (6), (9), and (10). Thus these four constraints are satisfied. Finally, by parts (a), (b), and (c) of the Theorem and the reliability assumption, i.e., $p_1 > p_2$ and $p_4 > p_3$, it follows that constraints (11) and (12) are also satisfied. Thus the contract specified in Theorem 2 is collusion-proof. By comparing Theorem 2 with the definition of the first best optimum provided in section 2b, it is easy to check that the contract specified in Theorem 2 does indeed implement the first best.

If the AL does indeed offer the contract with the characteristics described in Theorem 2, then his total wage bill cannot be altered by changing the agent’s report or the tenant’s effort level. As such, the AL can be sure that his monetary obligations will be those described in Theorem 2. This is so because the equilibrium contract is collusion-proof. Alternately put, the AL offers the best contract from the set of feasible contracts that are constrained to be collusion-proof.
Theorem 2 says that like the direct contracting (no collusion) case, there exists a first best wage contract that can be implemented by the AL in a Bayesian-Nash equilibrium. This is a strong result. This result tells us that the two state, two tier, first best implementation result of Sappington and Demski (1983) generalizes substantially. Further, the equilibrium contract is Pareto efficient in every state, the first best level of effort can be required in every state, and the wage paid to the agent and the tenant in \( A \) are equal in all states. The two "out of equilibrium" wages to the \( A \) agent satisfy 
\[
[\tilde{G}_{23}^A < \tilde{G}_{13}^A - \tilde{T}_{33}^A - \tilde{T}_{23}^A + g(e_1^A - \Delta \theta^4) - g(e_3^4)], \text{ and } [\tilde{G}_{12}^A < \tilde{G}_{22}^A - \tilde{T}_{22}^A - \tilde{T}_{12}^A + g(e_2^A - \Delta \theta^4) - g(e_2^4)].
\]
The "out of equilibrium" wages to the tenant in \( A \) satisfy the inequalities stated at the end of section 3 with \( T \) replaced by \( \tilde{T} \).

The intuition for the results of Theorem 2 lies in viewing the contract as an incentive scheme in which the AL effectively places the agents and the tenants in the two villages in Prisoner's Dilemma games. By appropriately designing the "out of equilibrium" wages, the AL is able to ensure that misrepresentation of private information does not pay. As such, "telling the truth" constitutes a unique Nash equilibrium in this game for both the agents and the tenants.

5. Conclusions

In this paper I have studied the design of first best rural wage contracts in perfectly correlated agrarian environments. I showed that when the private information of the agents and the productivities of the tenants in the two villages are perfectly correlated, the AL can use this fact to extract all the surplus from the agent and the tenant in each village.

In most rural agrarian settings, the productivities of tenants in villages that are located close to each other are likely to be strongly correlated on account of factors such as the weather and land quality. The analysis of this paper tells us that in the limiting case of perfect correlation, irrespective
of whether the contracting is direct (two tiered) or indirect (three tiered), the AL loses nothing from his inability to monitor; indeed he can always implement the full information optimum wage contract.

The line of research pursued in this paper can be extended in a number of different directions. I suggest two possible extensions. First, examining the wage contracting problem in a multi-period setting will enable one to analyze issues such as credibility and commitment. Clearly, these are important issues in long term contracting. Second, the analysis of the present paper can be extended to study hierarchical contracting with positively but imperfectly correlated private information. Examining positive but imperfect correlation in a multi-tenant hierarchical setting will enlarge the scope of the present analysis by allowing for the analysis of issues such as implementation via augmented and/or dominant strategy mechanisms. I am currently pursuing some of these issues and I hope to report my results shortly.
Appendix

This appendix contains the proofs of the two Theorems stated in the text of the paper. Both proofs involve Kuhn-Tucker analysis.

Proof of Theorem 1: Let $\alpha_i$, $\beta_i$, and $\beta_2$ denote the multipliers corresponding to (4), (5) and (6) respectively. Upon writing the Lagrangian, we see that for $i = 1, 4$, this Lagrangian depends on $e_i^4$ only through $[e_i^4 - T_i^4]$ and $[T_i^4 - g(e_i^4)]$. Thus, for $i = 1, 4$, it suffices to maximize $[e_i^4 - g(e_i^4)]$ over $e_i^4$. This yields $e_i^4 = e_i^4 - e_i^4 = (g')^{-1}(1)$. That is, the first best level of effort obtains in states 1 and 4. The remaining first order conditions are (A1) $(\alpha_iU'(\bullet) + \beta_i)g'(e_i^4) - \beta_i(p_i/p_2)g'(e_i^4 - \Delta \theta^4) = 1$, (A2) $(\alpha_iU'(\bullet) - \beta_i)g'(e_i^4) - \beta_i(p_i/p_2)g'(e_i^4 - \Delta \theta^4) = 1$, (A3) $\alpha_iU'(\bullet) = 1$, (A4) $\alpha_iU'(\bullet) - \beta_i = 1$, (A5) $\alpha_iU'(\bullet) - \beta_2 = 1$, and (A6) $\alpha_iU'(\bullet) = 1$. I now proceed by means of six steps.

Step 1: At the optimum, (4) binds.

Proof: I have to show that $\alpha_i > 0$. This follows from (A3) and (A6).

Step 2: $T_{11}^4 = T_{44}^4$.

Proof: This follows from (A3) and (A6).

Step 3: $\beta_i = \beta_2 = 0$.

Proof: Suppose not. Then either (i) $\beta_1 > 0$, $\beta_2 = 0$, (ii) $\beta_1 = 0$, $\beta_2 > 0$, or (iii) $\beta_1 > 0$, $\beta_2 > 0$.

Substituting (A4) into (A1), and (A5) into (A2), I get (A7) $g'(e_2^4) - 1/[g'(e_2^4 - \Delta \theta^4)] = \beta_2(p_2/p_3)$, and (A8) $g'(e_2^4) - 1/[g'(e_2^4 - \Delta \theta^4)] = \beta_2(p_2/p_3)$ respectively. Since $g'(e_2^4) \leq 1$, and $g'(e_2^4) \leq 1$, (A7) and (A8) can hold iff $\beta_1 = \beta_2 = 0$. This rules out cases (i), (ii), and (iii).

Remark: The intuition for the above result should be clear. Since $T_{33}^4$ and $T_{32}^4$ do not enter the profit or the utility functions, they can be chosen by the AL so as to ensure that (5) and (6) hold as strict inequalities.
Step 4: \( e_2^A - e_1^A = e_1^A = (g')^{-1}(1) \).

Proof: This follows on substituting \( \beta_1 = 0 \) in (A8) and \( \beta_2 = 0 \) in (A7).

Step 5: \( T_{11}^A - T_{22}^A - T_{31}^A - T_{44}^A \).

Proof: Substitute \( \beta_1 = \beta_2 = 0 \) and \( e_2^A - e_1^A = e_1^A \) into (A4) and (A5) and then compare (A4) and (A5) with (A3) and (A6).

Step 6: The equilibrium contract is Pareto efficient in every state.

Proof: I note that in state 3 - as in every other state - \( \{\partial U[\star]/\partial T_{33}\}/(U[\star]g'(e_1^A)) = 1 \). That is, the marginal utility from wage receipts equals the marginal disutility of effort. This completes the proof of Theorem 1.

Proof of Theorem 2: Let \( \alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \delta_1, \) and \( \delta_2 \) be the multipliers corresponding to (4), (8), (5), (6), (9), (10), (11), and (12) respectively. Writing the Lagrangian, it is straightforward to check that - as in the proof of Theorem 1 - for \( i = 1, 4, e_i^A = e_i^A = (g')^{-1}(1) \). The remaining ten first order conditions are (A9) \( \{\alpha_i U[\star] + \beta_1 + \gamma_1 + \delta_1\}g'(e_1^A) - (\beta_2 + \gamma_2)(p_1/p_2)g'(e_2^A - \Delta \theta^4) = 1 \), (A10) \( \{\alpha_i U[\star] - \beta_2 + \gamma_2 - \delta_2\}g'(e_2^A) - (\beta_1 + \gamma_1)(p_2/p_3)g'(e_1^A - \Delta \theta^4) = 1 \), (A11) \( \alpha_i U[\star] + \delta_1 = 1 \), (A12) \( \alpha_i U[\star] - \beta_1 + \gamma_1 - \delta_1 = 1 \), (A13) \( \alpha_i U[\star] + \beta_2 + \gamma_2 - \delta_2 = 1 \), (A14) \( \alpha_i U[\star] + \delta_2 = 1 \), (A15) \( \alpha_2 V(\star) + \gamma_1 - \delta_1 = 1 \), (A16) \( \alpha_2 V(\star) + \gamma_2 - \delta_2 = 1 \), and (A17) \( \alpha_2 V(\star) - \delta_2 = 1 \). I now proceed by means of eleven steps.

Step 1: \( \beta_1 = \gamma_1 = \beta_2 = \gamma_2 = 0 \).

Proof: Substituting (A12) and (A13) into (A9) and (A10) respectively, I get (A19) \( \{g'(e_2^A) - 1\}/(g'(e_2^A - \Delta \theta^4)) = (\beta_2 + \gamma_2)(p_1/p_2) \), and (A20) \( \{g'(e_1^A) - 1\}/(g'(e_1^A + \Delta \theta^4)) = (\beta_1 + \gamma_1) \) (\( p_2/p_3 \)) respectively. Now \( g'(e_2^A) < 1 \), and \( g'(e_1^A) < 1 \) together tell us that (A21) \( (\beta_2 + \gamma_2) = 0 \) and (A22) \( (\beta_1 + \gamma_1) = 0 \). Finally, (A21) and (A22) tell us that \( \beta_1 = \gamma_1 = \beta_2 = \gamma_2 = 0 \).
Remark: Constraints (5) and (6) do not bind because $\bar{T}_{23}^A$ and $\bar{T}_{32}^A$ can be chosen by the AL so as to ensure that these constraints hold as strict inequalities. Similarly, because $\bar{G}_{21}^A$ and $\bar{G}_{32}^A$ do not enter the AL’s profit function or the A agent’s utility function, they can be set so as to ensure strict inequality in (9) and (10).

Step 2: (8) binds at the optimum.

Proof: I have to show that $\alpha_2 > 0$. Suppose not. Then from (A15) and (A16) I get $\gamma_1 = 2$, which is impossible in view of Step 1 above. I conclude that $\alpha_2 > 0$. ♦

Step 3: (4) binds at the optimum.

Proof: I have to show that $\alpha_1 > 0$. Suppose not. Then from (A11) and (A15) I get $\alpha_1 V(\cdot) = 0$, which is impossible. I conclude that $\alpha_1 > 0$. ♦

Step 4: The equilibrium contract is Pareto efficient in states 1 and 4.

Proof: I have to show that the marginal rate of substitution between the wage and effort equals unity.

By differentiating the Lagrangian w.r.t. $\bar{T}_{11}^A$ and $e_1^A$, I get $\partial U[\cdot]/\partial \bar{T}_{11}^A/\{U[\cdot]|g(e_1^A)\} = 1$. Similarly, by differentiating the Lagrangian w.r.t. $\bar{T}_{44}^A$ and $e_4^A$, I get $\partial U[\cdot]/\partial \bar{T}_{44}^A/\{U[\cdot]|g(e_4^A)\} = 1$. Hence the claim follows. ♦

Step 5: $e_2^A - e_4^A = e_3^A - e_4^A$.

Proof: This follows on substitution of $\beta_1 = \gamma_1 = \beta_2 = \gamma_2 = 0$ in (A19) and (A20). ♦

Step 6: $\delta_1 = \delta_2 = 0$.

Proof: First, suppose that $\delta_1 > 0$. Dividing (A11) by (A23) $\{\alpha_1 U[\cdot] + \delta_1\} = 1$, I get (A24)

$$\{\alpha_1 U[\cdot] + \delta_1\}/\{\alpha_1 U[\cdot]\} = \{\alpha_1 U[\cdot] + \delta_1\}/\{\alpha_1 U[\cdot] + \delta_1\}.$$  

Now, because $\partial U[\cdot]/\partial \bar{T}_{11}^A = U[\cdot]$, (A24) holds for any $\delta_1$. Then (11) holds with equality and (A15) and (A16) tell us that $\bar{G}_{11}^A > \bar{G}_{22}^A$. Similarly, (A23) and (A9) tell us that $\bar{T}_{11}^A - g(e_1^A) > \bar{T}_{22}^A - g(e_2^A)$. Substituting these
values into (11), we see that (11) cannot hold with equality. Thus I conclude that \( \delta_1 = 0 \). A similar line of reasoning using (A17), (A18), (A10), and (A25) \( \{a_iU[\bullet] - \delta_2\} = 1 \), tells us that \( \delta_2 = 0 \) as well.

*Step 7:* \( \overline{T}_{11}^A = \overline{T}_{44}^A \).

*Proof:* This follows from (A11), (A14), and the result of Step 6.

*Step 8:* \( G_{11}^A = G_{44}^A \).

*Proof:* This follows from (A15), (A18), and the result of Step 6.

*Step 9:* \( \overline{T}_{22}^A = \overline{T}_{33}^A = \overline{T}_{11}^A = \overline{T}_{44}^A \).

*Proof:* Compare (A11), (A12), (A13), and (A14), using the results of Steps 1, 5, and 6.

*Step 10:* \( \overline{G}_u^A = (\gamma)^{-1}\{1/\alpha_x\}, \forall i \).

*Proof:* The claim follows on substituting \( \gamma_1 - \gamma_2 - \delta_1 - \delta_2 = 0 \) into (A15), (A16), (A17), and (A18).

*Step 11:* The equilibrium contract is Pareto efficient in states 2 and 3.

*Proof:* Use the results of Steps 1, 5, and 6, in (A9), (A10), (A12), (A13), to note that \( \{\partial U[\bullet]/\partial T_{22}^A\}/\{U[\bullet]\} = 1 \), and that \( \{\partial U[\bullet]/\partial T_{33}^A\}/\{U[\bullet]\} = 1 \). This completes the proof of Theorem 2.
References


