Post Stall Behavior of a Lifting Line Algorithm

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Abstract

A modified lifting line algorithm is considered as a low-cost approach for calculating lift characteristics of wings above stall. The model employs a numerical lifting-line method utilizing the 3D vortex lifting law along with known 2D airfoil data to predict the lift distribution across a wing. This method is expected to be of significant importance in the design of tailsitter vertical take-off and landing (VTOL) aircraft where the aircraft experiences stall conditions during important flight maneuvers. The algorithm is presented, and results compared with published experimental data.

Nomenclature

\begin{align*}
L & \quad \text{lift vector} \\
V_\infty & \quad \text{freestream velocity vector} \\
\Gamma & \quad \text{vortex strength} \\
\rho & \quad \text{fluid density}
\end{align*}

1 Introduction

Ludwig Prandtl’s original lifting line theory stated that the lift caused by a three dimensional wing could be modeled by placing horseshoe-shaped vortices across the wing attached along the quarter chord as shown in Fig. 1. Each of these vortices would vary in magnitude, which would be at a maximum value in the center of the wing and taper out to zero at the wing tips where lift is virtually zero.

The magnitudes of each vortex could be directly translated to lift by the following vortex rule:

\[ L = \rho V_\infty \Gamma \]  

Figure 1: Prandtl’s lifting line horseshoe-shaped vortex placement.

Lifting line theory is based on the assumption that at each spanwise section of the wing, the lift generated by the section circulation can be equated to the lift generated by a similar 2D airfoil. This works well for angles of attack below stall. However, above stall this assumption breaks down. Anderson [1] mentions that if the 2D airfoil data is known above stall, an “engineering solution” may be obtained using a lifting line algorithm. Additionally, Phillips [2] suggests that his modified numerical lifting line method can converge for a wing above stall if the system of equations is extremely underrelaxed. Others have studied the use of lifting line algorithms above stall and have made various observations. The results presented in this work support Anderson’s claim that if the 2D airfoil data is known above stall, a reasonable estimate for the lift and drag on a 3D wing can be predicted. Additionally, the results presented here validate the claims of others as will be discussed.

The work completed during the previous year in-
cluded linking this algorithm with a propeller model to predict propeller effects on finite wings. The current work implements this same algorithm and studies its behavior above stall. Therefore, the algorithm is not presented in this paper. Rather, the paper focuses mainly on the assumptions and results of studying the algorithm above stall.

2 Assumptions

2.1 Potential Flow

The original lifting line theory assumes potential flow. Such an assumption is quite valid at high Reynolds numbers and at low angles of attack. However, at angles of attack near or above stall, potential flow can no longer be assumed and corrections must be made. The approach presented here, although rooted in inviscid theory, accounts for the effects of viscosity on lift, drag, and moment via semi-empirical corrections to an otherwise potential flow solution. The viscous corrections are made by using 2D viscous data for the section lift and drag behavior. However, no attempt is made to correct the potential flow effects around the tips of wings. This means that although a wing may be separated from another lifting surface by only an extremely small distance, the lift at that wing tip could drop to zero. This is not physically true. In real life, viscous effects would prohibit the lift from dropping to zero at small gaps between wings.

2.2 2D Airfoil Characteristics At Each Spanwise Wing Section

The lifting line theory assumes that the lift generated at each spanwise location along the wing is equal to that of a 2D airfoil at the same effective angle of attack. (The effective angle of attack is the sum of the incident angle of attack to the freestream velocity and the induced angle of attack resulting from the downwash caused by the trailing vortices.) This assumption is in order for wings below stall and lends to very accurate results. It has been shown [3] that flows over wings at high angles of attack (i.e. above stall) have significant three dimensional properties. More specifically, at high angles of attack, the flow separates on the upper surface of the wing and a spanwise vortex forms along the wing. Thus, in order to accurately model the aerodynamics of a wing above stall, three dimensional effects should be taken into account. Although the lifting line theory assumes nearly two dimensional flow over each spanwise section of the wing, if post-stall data for the 2D airfoil is known, the assumption that the lift generated at each spanwise location is equal to that of a 2D airfoil at the same angle of attack should still be valid. This approach has been shown [1] to prove useful as a rough estimate to calculate wing lift above stall.

3 Results

3.1 Below Stall

As a first check on the present numerical lifting line algorithm, inviscid estimates of wing lift coefficient for a swept wing in a uniform freestream were computed. The section lift coefficient was defined as a linear function of angle of attack, and the section parasite drag was set to zero. For this case, the algorithm exactly reproduces the results of a published numerical lifting line algorithm [4] as shown in Fig. 2.

Figure 2: 3D wing $C_L$ vs. 2D section $C_l$ for a wing with sweep. Comparison to published lifting line results [4]
3.2 Above Stall

3.2.1 Oscillations

Some numerical lifting line solutions for wings near or above stall have been known to have spanwise oscillations [5, 6] which have discouraged some from trusting these results. Von Karman is said to have proven that above stall, there are an infinite number of solutions to the lifting line equation [7]. This includes symmetrical and asymmetrical solutions. Additionally, the solutions have been shown to be greatly dependent on the initial guess for the system [1]. Thus a numerical result of the lift distribution of a wing above stall should not be accepted as singularly viable.

One attempt to remedy the oscillatory problem was conducted by Mukherjee [6] who has shown that the algorithm can be “guided” to a more controlled solution with less oscillation by using a decambering approach. This author also has initial ideas on how the solution may be guided to a less oscillatory solution. However, it is beyond the scope of the current research and will not be considered here. Here, the oscillatory behavior is simply noted and quantified. Further research should be conducted to better understand this behavior and to find methods of damping the oscillation within the solution.

Figure 3 shows resulting circulation distributions for a wing with an aspect ratio of 6 at seven post-stall angles of attack. Each circulation distribution was computed using 18 wing sections across the span in a cosine distribution. The 2D airfoil $C_l$ used for the computation can be seen in Fig. 4.

Notice the oscillatory behavior of the circulation distribution above stall. Although the distributions are obviously not correct, the integrated lift across the wing matches closely to the expected total lift on the wing which can be seen in Fig. 4.

3.2.2 Spanwise Section Distribution Effects

The author has found the solution to be highly dependent on the spanwise section distribution used for the computations. It is assumed that this phenomenon has not been realized by others [1, 6] because their...
numerical models were not able to support various spanwise section distributions.

Figure 5 shows the converged circulation distributions for a wing with an aspect ratio of 6 with two different grid densities. Both of the grid densities follow a cosine distribution.

Figure 5: Comparison of the numerical circulation distributions for a wing with two different section distributions.

To understand the effect of spanwise section distributions on the solution, the lift distribution on a symmetrical wing with an aspect ratio of 6 was solved for 13 angles of attack between 0 and 90 degrees. At each angle of attack, the wing was analyzed using 46 different distributions. These section distributions varied from 10 spanwise sections to 100 spanwise sections by increments of 2. Each of the section distributions followed a cosine distribution. At angles of attack where the 2D airfoil data has a positive lift slope, the Jacobian solver was used. At all other angles of attack, the Steepest Descent solver was used. If the solver did not converge within 1000 iterations, it was halted. Only those solutions which reached a “converged” state, meaning that the residual of the governing equation was driven sufficiently near zero, were considered. Additionally, any solutions which resulted in a total lift coefficient which varied by more than 100 percent from the 2D airfoil data were not considered. Figure 6 shows which section distributions returned solutions that met these criterion.

At each angle of attack, all of the converged solutions from the varying grid densities were averaged and their variance was quantified. Figure 7 shows the resulting average 3D wing lift coefficients at each angle of attack compared to the 2D input lift coefficient data. Figure 8 shows the variance in the averaged 3D wing lift coefficients at each angle of attack.

Notice that the variance below stall is extremely small while the variance just past stall is significantly higher. This immediate post-stall region is possibly the most difficult range of angles of attack to predict because in this region the wing is only partially stalled. At higher angles of attack where the wing is fully stalled, the variance in the solutions seems to drop. However, this drop in variance must also be partially attributed to the fact that at extremely high angles of attack, only a few grid densities converged. Still, it is significant that the variance in these solutions at high angles of attack is quite small.

From the data presented, we can conclude that although the variances in the solutions above stall are...
greater than those below stall, the total lift calculated from a circulation distribution solution above stall is a practical estimate for the total lift on the wing, even if the circulation distribution contains oscillations.

### 3.2.3 Lifting Line Limitation

At this point, an insightful realization about the limitations of the lifting line theory is worthy of note. Namely, that as a finite wing approaches 90° angle of attack, the lifting line theory is less able to take 3D effects into account in the lift, drag, and moment calculations.

This realization came as a result of a study of drag coefficients above stall. A 2D airfoil usually has a drag coefficient of about 2 at 90° angle of attack. However, the drag coefficient of a finite wing at 90° is usually around 1.2. Figure 9 compares the 2D drag data published by Pope [8] and used in the current model with the finite wing results for a wing with an aspect ratio of 5.536. Notice that the finite wing drag coefficient nears the 2D airfoil drag coefficient at high angles of attack. At 90°, where the 3D drag should be significantly lower than the 2D drag, the lifting line model computes the drag to match that of the 2D airfoil.

The reason for this phenomenon can be understood by understanding the effect of circulation. If a portion of the wing is at an angle of attack near 90°, it has no circulation, and thus produces no downwash on other sections of the wing. The drag coefficient at any given section of the wing is calculated from the local angle of attack. Therefore, if there is no downwash, the local angle of attack is the same as the freestream angle of attack, and the 2D airfoil drag coefficient is taken as the section drag coefficient. This means that as the 3D wing approaches 90° angle of attack, the 3D drag coefficient should likewise approach the 2D drag coefficient data, which is the case in the numerical results presented in Fig. 9.

This phenomenon is the same for lift and moment calculations. Therefore, as a finite wing nears 90°, its lift, drag, and moment calculations using lifting line theory approach that of its 2D airfoil. This trend can be seen in Fig. 4 and Fig. 7. Notice that as the wing approaches 90° angle of attack, the 3D results increasingly match the 2D results. This behavior can also be seen in the results presented in the following section. Similar results were found for moment calculations.

It is important to realize that this characteristic of lifting line theory is a result of the lift on a wing section (which is directly proportional to the circulation of the wing section) approaching zero at 90°. Thus, if an airfoil had 2D lift characteristics that approached zero at 50° rather than at 90°, this phenomenon would occur near 50° rather than at 90°.

### 3.3 NACA 0015 Test Case

To validate the model above stall, numerical results were compared to experimental values published by Critzos [9] and Anderson [1] for a NACA 0015 airfoil.
Critzos published 2D lift, drag, and moment data taken by Pope [8] whose original publication was not readily available. However, Critzos reports that the data was taken at a Reynolds number of $1.23 \times 10^6$ and that the data was published without correction factors because the experimentalists found (through some tests and assumptions) that correction factors were not necessary. Anderson published numerical and experimental lift data for a finite wing with an aspect ratio of 5.563 from $0^\circ$ to $50^\circ$ at a Reynolds number of $2 \times 10^6$. However, he does not reveal the 2D data used for his numerical model. Thus Pope’s 2D data was used as input to the current numerical model and the results were compared to Anderson’s experimental and numerical results. Figure 10 compares the 2D data from Pope and the 3D numerical results of the current model with the 3D experimental and numerical results published by Anderson.

![Figure 10: Experimental vs. numerical results for a finite wing of aspect ratio 5.536.](image)

From the discrepancy between Anderson’s finite-wing numerical results and the numerical results of the current model, it is apparent that Anderson’s 2D airfoil lift data was slightly different than Pope’s lift data. This can partly be attributed to the difference in Reynolds numbers of the two data sets. However, below $12^\circ$, the current model matches Anderson’s experimental values better than his own finite-wing numerical model does. Notice that as Anderson’s numerical results approach $40^\circ$, his numerical model seems to diverge and over-predict lift. However, the results of the numerical model presented here show that this model is capable of predicting reasonable values for lift across the entire regime of angles of attack.

## 4 Conclusion

The lifting line algorithm has proven to be extremely fast and accurate for 3D wings below stall. This provides a very useful tool for the initial design stages of conventional aircraft. For wings above stall, the lifting line method can be used with caution. It has shown to produce reasonable values for the integrated lift on a wing. However, these solutions are often plagued with oscillations in the circulation distribution, which make the resulting lift, drag, and moment distributions difficult to believe. Further research may be able to alleviate these drawbacks to convergence above stall. Additionally, it has been found that the lifting line theory has significant limitations in predicting 3D effects on a wing when the 2D airfoil lift data approaches zero. However, the results produced at high angles of attack are within reason, and can be used with caution.

## References


