1999

Labor-Market Fluctuations in a High-Frequency Real-Business-Cycle Model

David Aadland
Utah State University

Follow this and additional works at: https://digitalcommons.usu.edu/eri

Recommended Citation
https://digitalcommons.usu.edu/eri/169

This Article is brought to you for free and open access by the Economics and Finance at DigitalCommons@USU. It has been accepted for inclusion in Economic Research Institute Study Papers by an authorized administrator of DigitalCommons@USU. For more information, please contact rebecca.nelson@usu.edu.
Economic Research Institute Study Paper

ERI #99-19
(Revised)

LABOR-MARKET FLUCTUATIONS IN A
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODEL

by

DAVID AADLAND

Department of Economics
Utah State University
3530 Old Main Hill
Logan, UT 84322-3530

March 2000
LABOR-MARKET FLUCTUATIONS IN A
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODEL

David Aadland, Assistant Professor

Department of Economics
Utah State University
3530 Old Main Hill
Logan, UT 84322-3530

The analyses and views reported in this paper are those of the author(s). They are not necessarily endorsed by the Department of Economics or by Utah State University.

Utah State University is committed to the policy that all persons shall have equal access to its programs and employment without regard to race, color, creed, religion, national origin, sex, age, marital status, disability, public assistance status, veteran status, or sexual orientation.

Information on other titles in this series may be obtained from: Department of Economics, Utah State University, 3530 Old Main Hill, Logan, Utah 84322-3530.

Copyright © 2000 by David Aadland. All rights reserved. Readers may make verbatim copies of this document for noncommercial purposes by any means, provided that this copyright notice appears on all such copies.
LABOR-MARKET FLUCTUATIONS IN A
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODEL

David Aadland

ABSTRACT

This paper investigates how the choice of timing interval within a standard real business cycle (RBC) model can help resolve several well-known labor-market puzzles. Standard quarterly RBC models have had difficulty replicating several empirical regularities arising from the US labor market. A weekly version of the RBC model is able to partially resolve several of these puzzles. The improvement in the performance of the model is due to the interaction between theoretical modifications to the standard RBC model and careful accounting for the sampling properties of US aggregate data.

JEL codes: C43, E32

Key words: Real business cycles, temporal aggregation, systematic sampling, timing interval, and labor-market fluctuations
LABOR-MARKET FLUCTUATIONS IN A
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODEL

1. Introduction

The labor market has always played a central role in business-cycle research. From Adam Smith’s division of labor to David Ricardo’s labor theory of value to John Maynard Keynes’ theory of employment, the role of workers and employers in contributing to business-cycle fluctuations has consistently been an issue of fundamental importance. Modern macroeconomic theory is no exception. In particular, standard real-business-cycle (RBC) theory has relied on the willingness of individuals to intra- and inter-temporally substitute between leisure and labor in response to technology shocks that alter the relative return to working. Early RBC models (e.g., Kydland and Prescott (1982) and Long and Plosser (1983)) failed, however, to replicate several stylized features of aggregate US labor-market data. Three of the more notable deviations between theory and measurement include: (1) the relative volatility of hours worked to output, (2) the relative volatility of hours worked to average labor productivity, and (3) the correlation between real wages (or equivalently, average labor productivity) and hours worked. These and other labor market puzzles have motivated a large amount of research attempting to bring theory and observation closer together. This paper investigates the role of the decision or timing interval in resolving some of these labor-market puzzles.

In particular, this provides an alternative explanation for addressing the magnitude of the intertemporal substitution of labor in macro models. Micro studies on the life-cycle behavior of men in the US have consistently revealed intertemporal elasticities of labor that, when used in RBC studies, are too small to replicate the magnitude of variation observed in aggregate hours worked. Pencavel (1986), in a

---

1This paper has benefited from the comments of Kevin Huang, George W. Evans, Jo Anna Gray, Mark Thoma, the Editor, an anonymous reviewer, and the participants of the 1998 Western and Midwest Economics Association annual meetings. Internet: aadland@b202.usu.edu.

2See Kydland (1995) for a survey of such research. Hall (1999) provides a more recent survey of advances in labor-market fluctuations in frictionless RBC models and contrasts the evidence with models that incorporate unemployment frictions.
survey of such panel studies, reports that the intertemporal elasticity of labor is somewhere between zero and 0.5 percent. Even using larger intertemporal labor elasticities, such as those in Mulligan (1998) who suggests that the elasticity is instead in the 1-2 percent range, are not sufficient to replicate the observed fluctuations in US hours worked within RBC models. In this paper, I provide evidence that by simply modeling agents as making decisions at finer intervals than one quarter of a year and carefully accounting for the manner that US data are collected, standard RBC models can come closer to matching observations from the US aggregate labor market. Moreover, when coupled with other standard modifications such as government consumption, time nonseparabilities, labor indivisibilities, or household production, RBC models can produce labor fluctuations in the model that are statistically indistinguishable from the US data.

It is well known that the time-series properties of economic data are sensitive to the sampling frequency. The classic documentation is by Working (1960) who shows that the time-aggregated changes in a random walk model follow a first-order moving-average process rather than a white-noise process. The primary reason that researchers evaluate macroeconomic models at quarterly or annual time intervals rather than shorter intervals such as days, weeks or months is that institutional factors set the sampling frequency of series such as the capital stock, GDP, investment and government spending. If the time-series properties of these series were not sensitive to the sampling frequency, there would not be a serious problem associated with specifying a timing interval that matched the sampling interval but was longer than the interval at which agents actually make decisions. However, since this is clearly not the case, we should be examining these models at higher frequencies than the standard quarter of a year. In order to evaluate a model at shorter timing intervals, there are essentially two options. You could (i) temporally aggregate the artificial data from a higher frequency model up to the sampling interval or (ii) temporally disaggregate the empirical data down to the model’s timing interval. Since interpolation and distribution procedures generate additional uncertainty into the analysis, I opt for the former in this paper.
Once the decision has been made to temporally aggregate the artificial data, it is important that the methods employed are consistent with those used in creating the actual data. Take the capital stock for instance. If monthly artificial data on the stock of capital is simulated from an RBC model, the time-series properties of the quarterly capital stock will be sensitive to whether the series is sampled at a single point in time or sampled at regular intervals and then averaged over time. In addition, various transformations such as natural logarithms and first differences complicate the matter further.

In this paper, I evaluate several standard RBC models at a weekly timing interval.\textsuperscript{2} I select an RBC model because of its prominence in current macroeconomic debate and because of RBC researchers’ reliance on time-series measures that have been shown to be sensitive to the unit of time measurement. I attempt to replicate the aggregation process in the actual data as close as possible, being careful to address issues associated with data transformations. I then contrast the artificial and actual data using the second-moment criteria typically used in real-business-cycle studies.

The rest of the paper proceeds as follows. In section 2, I present an overview of the RBC models, the solution technique, the process for simulating RBC data at a shorter timing interval, and the methodology for transforming the artificial data to a lower frequency. In section 3, I contrast various statistics from the US data with those generated from the high-frequency RBC models and discuss the results. In section 4, I summarize the paper’s most important findings and mention some issues for further research.

2. The High-Frequency RBC Economy

\textsuperscript{2}A daily interval was also analyzed and produced qualitatively similar results. The weekly frequency was chosen to simplify the presentation.
Over the last decade and a half, the RBC research program has experienced both remarkable success and controversy. At one end, the neoclassical growth model with a single stochastic productivity shock has proven remarkably adept at organizing macroeconomic variables and replicating various aspects of the business cycle. The skeptics, however, note shortcomings such as the absence of a role for nominal disturbances, the necessity of implausibly large productivity shocks and the use of unorthodox calibration techniques for evaluating RBC models, to mention a few. While these issues have recently received much attention, there still remain some specific puzzles that have not been thoroughly resolved. In this paper, I focus on three empirical regularities from the US labor market that standard RBC models have had difficulty replicating – the relative volatility of hours worked to output, the relative volatility of hours worked to productivity, and the correlation between real wages and hours worked. I then ask whether progress can be made toward resolving these puzzles by a willingness to analyze the RBC model at timing intervals shorter than the standard quarterly one.

A. The Models

Consider a baseline RBC model whereby a social planner maximizes the following objective function in consumption \( (c_t) \) and hours worked \( (n_t) \):

\[
E_t \sum_{j=0}^{\infty} \beta^j \left( (1-\phi) \log(c_{t+j}) + \phi V(N-n_{t+j}) \right),
\]

(1)

where \( E_t \) is the conditional mathematical expectations operator based on all information dated time \( t \) and earlier, \( \beta \) is the subjective discount rate, \( \phi \) is the weight on leisure, and \( N \) is the time endowment. The function \( V \) is assumed to be nonnegative, continuous, and twice differentiable in

\(^3\)Two useful summaries of the current state of business-cycle research are Cooley (1995) and the Handbook of Macroeconomics (1999).
leisure with $V' \geq 0$ and $V'' \leq 0$. For the baseline model, I set $V = \log(N - n_t)$, implying an intertemporal elasticity of substitution equal to one.

The objective function is maximized by choosing $c_t$, leisure $(N - n_t)$ and capital $(k_{t+1})$ streams subject to a resource constraint

$$y_t = k_t^0 (a_n n_t)^{1-\theta} \geq c_t + k_{t+1} - (1-\delta)k_t, \quad (2)$$

the law of motion for labor-augmenting technical change $(a_t)$

$$a_t = a_{t-1} \exp(\mu + \varepsilon_{at}); \quad \varepsilon_{at} \sim \text{iidN}(0, \sigma_a^2), \quad (3)$$

and the given initial capital stock, $k_0$.

B. Modifications to the Baseline RBC Model

In this section, I consider four extensions to the baseline RBC model that have been proposed in the literature to help resolve the deviations between theory and observation within the labor market. Later, I will discuss how these modifications interact with the frequency at which decisions are made within the model in order to help resolve some of the aforementioned labor-market puzzles.

i. Government Consumption

The addition of government consumption into the baseline RBC model is based on a paper by Christiano and Eichenbaum (1992). They incorporate government consumption shocks into a standard RBC framework in order to address one of the most damaging pieces of evidence against standard RBC models, that is, the production of grossly counterfactual correlations between real wages and hours worked. Whereas standard RBC models imply strong positive correlations
between real wages and hours worked, historical data from the US economy indicates a weak correlation, often referred to as the Dunlop-Tarshis observation.⁴

In order to incorporate government consumption, total consumption is now comprised of both public and private spending in the aggregate resource constraint (3) and enters the instantaneous utility function in the form \( c_t = c^P_t + \alpha g_t \), where \( c^P_t \) is private consumption, \( g_t \) is government consumption, and \( 0 \leq \alpha \leq 1 \). If \( \alpha = 1 \), then government and private consumption enter all equations as a sum and thus shocks to government consumption cause completely offsetting reductions in private consumption, such that government consumption shocks do not impact any real variables. On the other hand, if \( \alpha < 1 \), then government consumption reduces the marginal utility of private consumption, but not enough to offset the original government consumption shock. In this case, the presence of government consumption will impact the decision rules for real variables such as output, hours worked, etc. If \( \alpha = 0 \), government consumption is a pure resource drain and has no impact on the marginal utility of private consumption, but has its strongest effect on other real variables in the model. Following Christiano and Eichenbaum, the ratio of government consumption to technical change (\( \tilde{g}_t \)) is assumed to be governed by the following law of motion:

\[
\tilde{g}_t = \tilde{g}_{t-1} \exp((1 - \rho_g) \bar{g} + \varepsilon_{gt}); \quad \varepsilon_{gt} \sim \text{iidN} (0, \sigma^2_g).
\]

ii. Time Nonseparabilities

⁴Actually, the arguments of John Dunlop (1938) and Lorie Tarshis (1939) primarily center around the correlation between changes in real and money wage rates rather than real wages and employment. In particular, they are at odds with John Maynard Keynes' assertion in the General Theory of Employment, Interest and Money that "when money wages are rising ... real wages are falling; and when money wages are falling real wages are rising." I can find only a single paragraph in both Dunlop and Tarshis' papers where there is mention of the empirical relationship between employment and real wages. Dunlop says "there seems to be no simple relation—and especially of a causal nature—adequate to summarise the two movements without very wide margin of errors." While Tarshis states "it is surprising that there is a less close association between changes in ... real wages and man-hours. In this case, the coefficient stood at only -0.48."
The second modification to the baseline RBC model is to include a distributed lag of leisure in the utility function. Kydland and Prescott (1982) first incorporated this feature into the RBC model to capture the notion of fatigue, that is, a high level of recent work effort will tend to increase the current marginal utility of leisure and induce greater intertemporal substitution of labor.\(^5\) As in Kydland and Prescott, the instantaneous utility function is modified by letting

\[
V = \log(\Pi(L)(N - n_t)), \quad \text{where } \Pi(L) = \sum_{i=0}^{\infty} \pi_i L^i, \quad L \text{ is the lag operator, } \pi_0 = 0.5, \quad \sum_{i=0}^{\infty} \pi_i = 1, \quad \text{and the remaining coefficients decline geometrically according to } \pi_{i+1} = (1 - \eta) \pi_i, \quad i \geq 1 \text{ and } 0 < \eta < 1.
\]

iii. Labor Indivisibilities

The third modification is to consider labor indivisibilities along the lines suggested in Hansen (1985). By restricting all variation in labor hours to occur on the extensive rather than intensive margin, Hansen was able to increase the equilibrium volatility of hours worked over the business cycle while maintaining a low elasticity of substitution for labor at the individual level, as evidenced from microeconomic studies (Pencavel, 1986). The resulting aggregate instantaneous utility function, when lotteries are introduced to determine who works and who does not, is linear in leisure. This implies that \(V = N - n_t\) in (1).

iv. Home Production

The fourth and final modification is to incorporate home production into the baseline RBC model. Household production is an important feature of aggregate production in the US and, when included in a standard RBC model, can help resolve several deviations between theory and observation within the US labor market (Greenwood, et al., 1995).

\(^5\)It seems reasonable to think that the notion of fatigue captured by the distributed lag of leisure in utility may operate more acutely at higher frequencies, such as days or weeks. This provides an additional motivation for considering...
I consider a simple version of household production whereby individual utility is derived from leisure, as well as, both market consumption \((c_{m,t})\) and household consumption \((c_{h,t})\). The instantaneous utility function is

\[
(1 - \phi) \log((ac_{m,t}^e + (1 - a)c_{h,t}^e)^{1/e}) + \phi \log(N - n_{m,t} - n_{h,t}),
\]

where \(n_{m,t}\) and \(n_{h,t}\) are market and household hours worked respectively. Output is produced in both the market and household sectors using sector-specific capital and labor. The production function in each sector is Cobb-Douglas \((\theta\) is capital’s share of income in the market sector and \(\gamma\) is capital’s share of income in the household sector) with labor-augmenting technological progress. The labor-augmenting technological progress for the market and household sectors takes the form

\[
A_{j,t} = \lambda^t \bar{A}_{j,t}; \quad \bar{A}_{j,t} = \bar{A}_{j,t-1} \exp(e_{j,t}),
\]

where \(0 < \rho_j < 1\) and \(e_{j,t} \sim \text{iid}(0, \sigma_j^2)\) for \(j = h, m\). Thus, technological progress grows stochastically around an exponential trend with growth rate \(\log(\lambda)\). Finally, there is a resource constraint similar to (2) for each sector, with a common rate of depreciation \((\delta)\) in the two sectors.

C. Equilibrium Solution

I choose the procedure developed by Blanchard and Kahn (1980) for solving these rational expectations models. The method begins by transforming all nonstationary variables so they converge to a nonstochastic steady state. In our case, this can be accomplished by dividing all nonstationary variables by the technological growth variable \(a_t\) (or \(\lambda^t\) in the case of household production) and redefining the transformed variables. Next, the first-order conditions that result from maximizing (1) subject to the relevant constraints are linearized by taking a first-order Taylor-series approximation around the steady-state values of the transformed variables. Lastly, we iterate timing intervals shorter than one quarter of a year. I thank an anonymous reviewer for pointing this out.
into the future the linearized equation corresponding to the stable root of the first-order conditions. A similar procedure was outlined in King, et al. (1988). Skipping past a great deal of algebra, the decision rules for each of these models take the following form:

\[
    s_t = A_s s_{t-1} + \varepsilon_t
\]

\[
    f_t = \Pi_s s_t
\]

where \( A_s \) is the matrix of first-order autoregressive coefficients and \( \Pi_s \) is the matrix of impact multipliers. For the baseline case, the state variable is \( s_t = \hat{k}_t \), the vector of flow variables is \( f_t = (\hat{y}_t, \hat{c}_t, \hat{n}_t, \hat{i}_t)' \), \( \varepsilon_t = -\varepsilon_{st} \), \( i_t \) is gross investment, and the hat (^) over a variable refers to percentage deviations from its steady-state value.

D. Simulating Artificial Weekly Data

To evaluate the performance of the RBC models above, I generate 500 artificial data sets from the decision rules (7) and (8). The ensemble averages of various statistics from the model are then contrasted with their corresponding statistics from the US economy. In order to complete this task, the model first needs to be calibrated. Where appropriate, the quarterly generalized method of moments estimates (using establishment data) of Christiano and Eichenbaum (1992) are chosen. In the nonseparabilities model, I use the distributed lag of leisure parameters given in Prescott (1986). In the home production model, I follow Greenwood, et al. (1995) in setting \( e = 0.4 \) and making the sector specific technology shocks independent of one another. In all models, the standard deviations of the relevant shocks are chosen so that the standard deviation of per-capita output growth from the RBC model equals per-capita US real GDP growth over the period 1965:1 through 1995:4.
Given the choices for the parameters in the quarterly model, I then transform the quarterly parameter values to be consistent with a weekly version of the model. The weekly parameters associated with time (e.g., \( \rho_s, \beta \) and \((1-\delta)\)) are calculated by raising the quarterly values to the one-thirteenth power. Weekly values of other parameters are either divided by thirteen (e.g., \( \mu \) and \( N \)) or left unchanged (e.g., \( \alpha, \pi_0, \) and \( \theta \)). Lastly, the weekly values for the standard deviations of the disturbances are adjusted by dividing the quarterly value of \( \sigma_a \) or \( \sigma_m \) by \( \sqrt{13} \) and then maintaining the relative magnitudes of the variances used in the quarterly models. The entire set of parameter values for the quarterly and weekly versions of the model are presented in Table 1.

[Insert Table 1]

E. Transforming the Artificial Data

Before contrasting the weekly RBC and US data, I need to make the two comparable by (i) transforming the former out of percentage deviations from the steady state and (ii) changing the frequency of the artificial data to be consistent with the frequency of the US data. For the first task, I transform the nonstationary series into growth rates using the following approximation for all but the household-production model:

\[
\Delta \log(x_t) \equiv \Delta \hat{x}_t + \mu + \epsilon_{st},
\]

where \( \Delta = 1 - L. \)

The nonstationary variables in the household production model are also detrended using growth rates but instead the approximation is

---

6As a technical matter, it is not entirely clear how to adjust the autoregressive parameter (i.e., the \( \rho \)'s). For instance, if government spending is assumed to follow a first-order autoregressive process at a weekly frequency (as in the quarterly version of the Christiano and Eichenbaum's RBC model), then quarterly government spending will have the autoregressive parameter \( \rho \) to the thirteenth power, but will have a moving average component as well. However, in some sensitivity analysis, it was found that moderate variations in the \( \rho \)'s and the disturbance variance have little effect on the main results of this paper.

7The standard method of inducing stationarity in RBC studies is through the Hodrick-Prescott (HP) filter. Cogley and Nason (1995b) provide evidence that the HP filter can spuriously generate business cycle fluctuations in difference-
\[ \Delta \log(x_t) = \Delta \log(\hat{x}_t) - \log(\lambda). \]  

(10)

Hours worked in all versions of the model is stationary in levels and therefore is transformed

according to

\[ n_t = \bar{n} (\hat{n}_t + 1), \]  

(11)

where \( \bar{n} \) indicates the steady-state level of hours worked.

After the series have been transformed out of percentage deviations from their steady states, they need to be transformed to the lower frequencies at which the actual data are measured. It is critical that the aggregation or sampling procedure closely mimic the manner in which the actual data are created because sampling and aggregation may influence the time-series properties of the data. There are essentially two different way to reduce the frequency of a time series: temporal aggregation or systematic sampling. Temporal aggregation involves summing a flow variable (or averaging in the case of a stock) over the shorter intervals. The temporal aggregation operator is given by

\[ B_{TA} = 1 + L + L^2 + \cdots + L^{n-1} \]  

(12)

where \( n \) is the number of observations at the shorter time intervals within one time aggregate.

For our purposes, \( n \) will equal thirteen, as there are approximately thirteen weeks in a quarter (ignoring leap year). Systematic sampling involves selecting values of a stock variable at regularly spaced intervals. The systematic sampling operator is given by

\[ B_{SS} = L^j \]  

(13)

where \( j \) equals a single element out of the set \( \{0, 1, \ldots, n-1\} \). If the variable has not been logged, differenced or subject to some other type of transformation, then lowering the frequency of the series is only a matter of determining whether it is a stock or a flow and then applying the

stationary data. I detrend the actual data using both the first-difference operator and the HP filter and find that the
appropriate operator above. However, it is clear from (9) and (10) that the nonstationary variables in the models will be subject to both logarithmic and first-difference transformations.

The primary difficulty of working with variables that have been transformed as natural logs is that the sum of the logs is not equal to the log of the sums.\(^8\) For example, if a monthly flow series is measured in logs, simply aggregating the observations will not produce the log of the quarterly series. A proposed solution is as follows.\(^9\) Begin by defining \(\bar{X}_{nt}\) to be the log of the product of disaggregate (or basic) variables as

\[
\bar{X}_{nt} = B_{TA} \log(x_{nt}) = \log(x_{nt}) + \log(x_{nt-1}) + \ldots + \log(x_{nt-(n-1)})
\]

\(\tau = 1, \ldots, T/n,\)

with \(\tau\) indexing aggregate time and lower cases representing variables in basic time. Now take a first-order Taylor series approximation around the sample mean \((\bar{x}_{nt})\) for each of the \(T/n\) sets of \(n\) consecutive non-overlapping basic observations, which results in

\[
\bar{X}_{nt} \approx n \log(\bar{x}_{nt}) + \frac{1}{\bar{x}_{nt}}(x_{nt} - \bar{x}_{nt}) + \ldots + \frac{1}{\bar{x}_{nt}(n-1)}(x_{nt-(n-1)} - \bar{x}_{nt}).
\]

For a temporally aggregated flow series, the sample mean for each \(n\) consecutive non-overlapping basic observations is observable and given by \(X_{nt}/n\), where \(X_{nt} = x_{nt} + \ldots + x_{n_{t-(n-1)}}\). After a few substitutions and a little rearranging, the unobservable \(\bar{X}_{nt}\) and the observable \(X_{nt}\) can be related according to

\[
\bar{X}_{nt} \approx n(\log(X_{nt}) - \log(n)). \tag{14}
\]

When a series has been first differenced, two things are required to transform the series to a lower frequency. First, the undifferenced series needs to be aggregated or sampled. This is done by
applying either $B_{TA}$ or $B_{SS}$. Second, the difference operator, $\Delta = 1 - L$, needs to be transformed into aggregate time (i.e., $\Delta^n = 1 - L^n$), which is accomplished by multiplying by $B_{TA}$:

$$B_{TA}(1 - L) = (1 + L + \cdots + L^{n-1})(1 - L) = (1 - L^n).$$  \hfill (15)

In sum then, the quarterly artificial flow or temporally aggregated data (in growth rates) will be measured as

$$(1 + L + \cdots + L^{12})(1 - L) 13(\log(X_{13}) - \log(13)),$$

and quarterly artificial stock or systematically sampled data (in growth rates) will be measured as

$$(1 + L + \cdots + L^{12})(1 - L)L^j \log(x_{13}),$$

where $j$ will depend on the week in which the actual data are sampled.

3. Simulation Results

In this section, I contrast both weekly and quarterly versions of the RBC models with the US economy. Ten variations of the RBC model were calibrated and used to simulate artificial data sets: quarterly and weekly versions of (i) the baseline RBC model; (ii) the baseline RBC model with government consumption; (iii) the baseline RBC model with time nonseparabilities; (iv) the baseline RBC model with labor indivisibilities; and (v) the baseline RBC model with home production. The results from this exercise indicate that the performance of the RBC models is sensitive to the choice of timing interval. In particular, a weekly version of these models appears to help resolve several puzzles arising from the US labor market.

A. Data

The data are taken from FRED, the Federal Reserve Economic Database of the St. Louis Federal Reserve Bank. The series selected are as follows, with FRED mnemonic in parentheses:

output is measured as real gross domestic product (GDPC92); private consumption is measured as real personal consumption expenditures on nondurable goods (PCENDC92) and service goods (PCESC92); investment is measured as the sum of real fixed private investment (FPIC92), real public gross investment \((DGIC92 + NDGIC92 + SLINVC92)\), and real personal consumption expenditures on durable goods (PCEDGC92); government consumption is measured as real government consumption expenditures and gross investment (GCEC92) less public gross investment; total hours are measured as the product of average weekly hours in private nonagricultural establishments \((AWHN OG)\) and total nonfarm payroll employees \((PAYEMS)\); and lastly, average productivity is calculated by dividing output by total hours. All the variables have been seasonally adjusted at the source; output, private consumption, investment and government consumption are measured in chain-weighted 1992 dollars; and all but average productivity have been transformed into per-capita terms using the entire civilian, non-institutionalized population that is sixteen and over \((CNP16OV)\).

In addition to selecting US variables which best match the definitions provided by the model, it is equally important to match the US sampling and aggregation procedures. Series taken from the National Income and Product Accounts (NIPA) -- output, consumption, and investment -- are collected using a wide range of sources and at varying points in the quarter; see the Bureau of Economic Analysis’ website for details (http://www.bea.doc.gov/bea/mp.htm). As an approximation, the artificial data for these series were aggregated using the temporal aggregation operator, \(B_{TA}\). Series taken from the Bureau of Labor Statistics (BLS) -- average weekly hours, employment, and the population -- are collected from the household and establishment surveys. The following excerpt is taken from the BLS website (http://stats.bls.gov:80/) regarding their sampling methods:
For both surveys, the data for a given month relate to a particular week or pay period. In the household survey, the reference week is generally the calendar week that contains the 12th day of the month. In the establishment survey, the reference period is the pay period including the 12th, which may or may not correspond directly to the calendar week.

In accordance with BLS techniques, total hours worked and population data generated from the model were thus treated as being systematically sampled from the second week of the month.

B. Comparison of the Basic and Aggregate Covariances

Before focusing on the specific results, it will be useful to derive a relationship between the basic and aggregate second-moment estimators. Begin by defining the population covariance between the growth rates of two aggregate flow variables as

\[ \Gamma_{xy}(k) = \text{cov}(\Delta^n \log(X_{nt}), \Delta^n \log(Y_{n(t-k)})) . \] (16)

In order to write \( \Gamma_{xy}(k) \) in terms of the basic covariances, \( \gamma_{xy} \), use (14) to write the log of the sums as the sum of the logs:

\[ \Gamma_{xy}(k) = \text{cov}(\Delta^n \sum_{i=0}^{n-1} \log(x_{nt-i}), \Delta^n \sum_{i=0}^{n-1} \log(y_{n(t-k)-i})). \] (17)

Next, use (15) to transform the aggregate difference operator, \( \Delta^n \), to the basic difference operator, \( \Delta \):

\[ \Gamma_{xy}(k) = \text{cov}(\sum_{i=0}^{2(n-1)} \omega_i^x \Delta \log(x_{nt-i}), \sum_{i=0}^{2(n-1)} \omega_i^y \Delta \log(y_{n(t-k)-i})), \] (18)

where \( \omega_i^x \) and \( \omega_i^y \) are the weights in the sum of the basic logs. Moving to matrix notation and assuming zero means, we can rewrite (18) as

\[ \Gamma_{xy}(k) = E\left( \omega_x g_x(0) g_y(nk) \omega_y^\prime \right) = \omega_x \gamma_{xy}(nk) \omega_y^\prime , \] (19)

where \( \omega_x = (\omega_0^x \omega_1^x \ldots \omega_{2(n-1)}^x) \) and \( g_x(nk) = (\Delta \log x_{n(t-k)} \Delta \log x_{n(t-k)-1} \ldots \Delta \log x_{n(t-k)-2(n-1)})^\prime \) and similarly for \( y \). In this case,
\[
\gamma_{xy}(nk) =
\begin{bmatrix}
\gamma_{nk} & \gamma_{nk+1} & \cdots & \gamma_{nk+2(n-1)} \\
\gamma_{nk-1} & \gamma_{nk} & \cdots & \gamma_{nk+2(n-1)-1} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{nk-2(n-1)} & \gamma_{nk-2(n-1)+1} & \cdots & \gamma_{nk}
\end{bmatrix},
\]

where \( \gamma_i \) is the cross covariance between contemporaneous \( x \) and the \( i^{th} \) lag of \( y \).

To illustrate the relationship between the aggregate and basic cross covariances, consider a simple example where \( n = 3 \) (e.g., monthly to quarterly frequency). Let \( X \) be a temporally aggregated flow variable, and \( y \) be a stock variable that is systematically sampled in the final month of the quarter (i.e., \( j = 0 \) in (13)). In this case, the contemporaneous aggregate cross covariance is

\[
\Gamma_{xy}(0) = E\left( \omega_x \gamma_x(0) \gamma_y(0)^\prime \omega_y^\prime \right) = \omega_x \gamma_{xy}(0) \omega_y^\prime
\]

where \( \omega_x = \frac{1}{3} (1 \ 2 \ 3 \ 2 \ 1) \), \( \omega_y = (1 \ 1 \ 1 \ 0 \ 0) \), and

\[
\gamma_{xy}(0) =
\begin{bmatrix}
\gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\
\gamma_{-1} & \gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 \\
\gamma_{-2} & \gamma_{-1} & \gamma_0 & \gamma_1 & \gamma_2 \\
\gamma_{-3} & \gamma_{-2} & \gamma_{-1} & \gamma_0 & \gamma_1 \\
\gamma_{-4} & \gamma_{-3} & \gamma_{-2} & \gamma_{-1} & \gamma_0
\end{bmatrix}.
\]

After some simple matrix algebra, we obtain

\[
\Gamma_{xy}(0) = \frac{1}{3} (\gamma_{-4} + 3\gamma_{-3} + 6\gamma_{-2} + 7\gamma_{-1} + 6\gamma_0 + 3\gamma_1 + \gamma_2).
\]

Notice that the contemporaneous basic cross covariance, \( \gamma_0 \), is shifted off center in (20) such that it does not receive the largest weight. Notice also that the non-contemporaneous basic cross covariance terms are also important in determining the aggregate covariance between \( X \) and \( y \). The combination of these two effects, coupled with economic theory which implies certain signs for these non-contemporaneous cross covariance terms, is responsible for my earlier statement that "the
predictions of the RBC model are sensitive to the frequency at which agents are assumed to make decisions."

C. Discussion of the Results

Table 2 presents second-moment statistics for the US economy and for several different variations of the baseline model. The statistics from the models are ensemble averages across the 500 simulations. Standard deviations for the model statistics are calculated using the variation across the 500 simulations, while standard deviations for the empirical statistics are calculated using standard formula.$^{10}$ In brackets, are the generalized Wald statistics for the null hypothesis that the model and empirical moments are equal. The Wald statistics are calculated according to

$$W = (\hat{\psi} - \bar{\psi})' (\text{est. var}(\hat{\psi}) + \text{est. var}(\bar{\psi}))^{-1} (\hat{\psi} - \bar{\psi}),$$

where $\hat{\psi}$ is a $(J\times1)$ vector of statistics from the actual US data and $\bar{\psi}$ is a $(J\times1)$ vector of statistics from the model averaged across the simulations. Under the null hypothesis, $W$ has an asymptotic distribution with $J$ degrees of freedom. Since we are interested in cases where the model is able to replicate features of the US data, single (*), double (**) and triple (***) asterisks indicate failure to reject the null hypothesis of equal statistics at the 1%, 5%, and 10% significance levels respectively.

[Insert Table 2]

Begin by noticing that the growth-rate and HP-filter results for the US economy in Table 2 are similar to the results found in other RBC studies. Investment is more volatile than output, output is in turn more volatile than consumption, and hours worked vary nearly as much as output (Prescott, 1986). Also, real wages and hours worked are negatively, but weakly correlated (Hansen and Wright, 1992) and hours worked vary slightly more than average labor productivity (Christiano

$^{10}$A technical appendix, available from the author by request, provides details on how the standard deviations of the empirical statistics were calculated.
and Eichenbaum, 1992). In general, the results are similar whether the data are detrended using first differences or the HP filter.

Now focus on the results from the quarterly version of the baseline RBC model. The successes and failures of this model are well known. In particular, the model is successful in capturing the fact that consumption varies much less than output and investment varies much more than output. Also, the model does a good job of replicating the relative volatility of average labor productivity to output. On the other hand, the model predicts too little variation in hours worked relative to output, too much variation in productivity relative to hours worked, and predicts a strong positive correlation between the real wage and hours worked whereas the data suggests a weak negative correlation. Again, these are well-known results from the standard quarterly RBC model and have been widely cited (see for example Christiano and Eichenbaum (1992) and Cooley and Prescott (1995)). Below, I discuss how a weekly decision interval and several modifications to the baseline RBC model interact to improve the fit of the model with respect to several empirical regularities from the US labor market.

Next consider the weekly version of the baseline RBC model. Notice first that the relative volatilities of private consumption and investment are unaffected by specifying a weekly decision interval. This is true across all variations on the baseline RBC model and indicates that a quarterly decision interval does not generally distort the variation in the artificial flow variables that are subject to temporal aggregation. This result occurs largely because the weekly model produces very

---

11Another possibility is to examine monthly US data since labor-market data is recorded at a monthly frequency. Since GDP is only available at quarterly intervals I measure average labor productivity by using the industrial production index as a proxy for GDP. For monthly US data, the standard deviations in the growth rates of $n$ and $y/n$ are 0.51% and 0.71%, while the contemporaneous correlation between "real wages" and $n$ is $-0.15$. A monthly version of the baseline RBC produces average standard deviations of 0.31% and 0.65% for $n$ and $y/n$, and an average correlation between real wages and $n$ equal to 0.99. However, a weekly version of the RBC model that is aggregated to the monthly frequency in a manner consistent with actual US data-collection methods produces average standard deviations for $n$ and $y/n$ equal to 0.40% and 0.70%, and an average correlation between real wages and $n$ equal to 0.44. The fact that an aggregated version of the weekly model improves the performance in these dimensions, but a monthly version of the model does not, highlights the fact that it is the careful consideration of the sampling properties of US labor-market data that primarily contribute to the results of the paper, not simply changing the frequency of decisions.
weak autocorrelation in the growth rates of the relevant variables. As a result, the quarterly variance is simply proportional to the weekly variance, such that the variance from the quarterly baseline model equals the variance from the aggregated weekly baseline model.

Second, notice the increase in the volatility of hours worked, and to a lesser degree, average productivity from the quarterly to the weekly baseline models. The increase in the volatility of these two variables is due to the fact that artificial hours worked are systematically sampled rather than temporally aggregated, as in the actual US data. To understand why this increases the volatility, consider the following heuristic example. Let \( \{x_1, x_2, \ldots, x_T\} \) be a sequence of identically and independently distributed random flow variables with a zero mean and variance \( \sigma_x^2 \). Now consider two estimates of the \( n \)-period variance of \( x \): a temporal aggregated estimate, \( B_{TA}x \), and a systematically sampled estimate, \( nB_{SS}x \). The variance of the first is \( n\sigma_x^2 \) and the variance of the second is \( n^2\sigma_x^2 \). Thus, the fact that hours worked in the US are systematically sampled from periodic weekly surveys implies that quarterly RBC models, everything else equal, will tend to understate the volatility of hours worked because they do not account for this type of sampling variation. The effect is tempered for average labor productivity because it is the ratio of a temporally aggregated variable to a systematically sampled variable.

Third, consider the contemporaneous cross correlation between hours worked and real wages. Standard RBC models with a single productivity shock produce strong positive correlations between the real wage and hours worked, 0.98 in the quarterly baseline RBC model, while the weekly baseline model reduces this correlation to 0.57. This improvement is, again, due to the nature in which hours worked are sampled. Using BLS sampling methods, equation (19), and

---

12 When measured in growth rates, the variables from the weekly model are nearly white noise. The largest average autocorrelation coefficient (of order one or greater) across the simulations for any of the variables (i.e., \( y, c^p, n, i \) or \( y/n \)) is 0.015. This is closely related to Cogley and Nason's (1995a) result that standard RBC models have weak propagation methods and thus generate very little persistence in output growth.
treated average labor productivity as a temporally aggregated variable, generates a quarterly contemporaneous covariance between real wages and hours worked equal to

\[
\Gamma_{n,y/n}(0) = \frac{1}{13} (127\gamma_{-4} + 126\gamma_{-4+1} + 123\gamma_{-4+2} + 118\gamma_{-4+3} + 111\gamma_{-4+4} + 102\gamma_{-4+5} \\
+ 91\gamma_{-4+6} + 78\gamma_{-4+7} + 66\gamma_{-4+8} + 55\gamma_{-4+9} + 45\gamma_{-4+10} + 36\gamma_{-4+11} + 28\gamma_{-4+12} \\
+ 21\gamma_{-4+13} + 15\gamma_{-4+14} + 10\gamma_{-4+15} + 6\gamma_{-4+16} + 3\gamma_{-4+17} + \gamma_{-4+18}).
\]  

As illustrated in (21), the effect of looking at the contemporaneous covariance between a temporally aggregated flow variable and a systematically sampled variable is to shift the weekly contemporaneous covariance between real wages and hours worked “off center” such that it receives a relatively small weight, placing downward pressure on the quarterly correlation between real wages and hours worked generated from the weekly model.

Now consider the modifications to the baseline RBC model and how they interact with the length of the decision interval. All four modifications tend to increase the volatility in hours worked. First, the addition of government spending introduces shifts in the labor supply, which when coupled with variation in the labor demand curve due to technology shocks, tend to increase the variation in equilibrium hours worked. Second, the introduction of a distributed lag of leisure increases the volatility of hours within the model by explicitly linking future utility of leisure to present work effort. Third, by restricting all variation in hours worked to occur on the extensive rather than intensive margin, implies a utility function for the representative agent that is linear in leisure. The large intertemporal elasticity of substitution thus generates larger variations in hours worked for a given change in technology. Fourth, and finally, the introduction of stochastic household production generates an intersectoral flow of labor as households choose to either supply labor toward household or market production, increasing the volatility of market hours worked. As evident in Table 2, all four of these modifications tend to increase the volatility of hours worked, although in the quarterly models we still reject the null of equal empirical and theoretical volatility.
in hours worked. However, when these modifications are coupled with a weekly decision interval, the volatility of hours worked are increased even further and, for the household production model, cannot statistically be distinguished from the empirical volatility for hours worked. Furthermore, the combination of a weekly decision interval and either labor indivisibilities or household production improves our ability to match the empirical observation that productivity is slightly less volatile than hours worked. In sum, the combination of theory and shorter decision intervals can help resolve two prominent puzzles arising from the labor market: (i) the relative volatility of hours worked to output and (ii) the relative volatility of average labor productivity to hours worked.

Finally, consider again the correlation between real wages and hours worked. Two of the four proposed modifications are effective in reducing this correlation. First, the introduction of government spending generates stochastic shifts in labor supply, which along with technology-driven shifts in labor demand, can reduce the correlation between equilibrium real wages and hours worked. Also, the introduction of household production shifts the supply for market labor as (positive) home technology shocks increase the rewards and flow of labor toward household production. These two modifications reduce the correlation between real wages and hours worked in the quarterly baseline model from 0.98 to 0.67 and 0.38 respectively. When these modifications are coupled with a shorter decision interval, the correlations are reduced to 0.33 and -0.05 respectively. In fact, for the weekly home production model, we no longer reject the hypothesis of equal correlations between the model and the US economy at the 10% level.

There is also a more subtle interaction between RBC theory and the shorter timing interval that help to reduce the real wage – hours worked correlation. In response to a permanent technology shock (or a persistent shock in the case of the household production model), both the real wage and hours worked increase in the period of the shock. This produces the large positive correlations in the quarterly RBC models. However, in the transition to the new steady-state,
average labor productivity will be rising while hours worked will be falling, creating a negative correlation between lagged real wages and contemporaneous hours worked (this can be seen in King and Rebelo, 1999). As can be seen in (21), these negative weekly cross covariances (i.e., the $\gamma_i$'s) will tend to reduce the quarterly contemporaneous correlation between the real wage and hours worked and bring it more in line with the US data.

The effect described above is only magnified in the time-nonseparable model. John Heaton (1993, 1995) has extensively examined the interaction between time-nonseparable preferences and time aggregation in the context of consumption expenditures and asset pricing and found them to be useful in explaining various features of time-aggregated US data. The interaction between the two is also important here, but for a slightly different reason. In this context, the presence of a distributed lag for leisure implies that high real wages in past periods will induce individuals to work more hours in those periods, which in turn will increase the marginal utility of leisure and the amount of leisure taken in the present period. This increased substitution over time will tend to produce weekly correlations between lagged values of the real wage and current hours worked that are more negative than in a standard RBC model. As a result, the quarterly artificial correlation between real wages and hours worked in (21) will be reduced even further, as evidenced by the comparison between the time-separable and time-nonseparable baseline models.

4. Conclusion

The question to ask at this point is, "What have we learned from specifying a timing interval shorter than the standard one quarter of a year?" Although the weekly RBC models considered here are certainly not rich enough to match the US data in all dimensions, one thing we have learned is that, in several dimensions, a weekly version of a standard RBC model appears to fit the US aggregate data over the last three decades better than the typically specified quarterly model. In
particular, the weekly baseline RBC model is able to partially resolve three well-known puzzles arising from the US labor market. By specifying a weekly decision interval and carefully accounting for the manner in which the US data are aggregated over time, we can improve the model’s predictions with respect to the relative volatility of hours worked to output, the relative volatility of average labor productivity to hours worked, and the correlation between the real wage and hours worked. These improvements are a result of the interaction between theoretical modifications to the baseline model that alter the nature of the labor-market fluctuations and the sampling properties of US hours worked.

This sensitivity to the timing interval also raises a number of other questions. In particular, if the choice of timing interval matters, then which interval best represents agents’ decision-making process? Is it a monthly interval? Weekly interval? Or is it an even shorter interval, such as a daily interval? And does this sensitivity appear in other macroeconomic models? If so, can it help resolve other macroeconomic puzzles? These questions, while not addressed in this paper, seem to be important areas for future research.

13 The only paper I am aware of that attempts to estimate the timing interval is Christiano (1985). In general, his technique has not been adopted in macroeconomics, presumably due to problems associated with the well-known aliasing identification problem.
Bibliography


Table 1. Parameter Values Used in RBC Simulations

<table>
<thead>
<tr>
<th>Model</th>
<th>Timing Interval</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>δ</td>
<td>β</td>
</tr>
<tr>
<td>Baseline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.021</td>
<td>0.9926</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.0016</td>
<td>0.9994</td>
</tr>
<tr>
<td>Government Spending</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.021</td>
<td>0.9926</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.0016</td>
<td>0.9994</td>
</tr>
<tr>
<td>Time Non-Separabilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.021</td>
<td>0.9926</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.0016</td>
<td>0.9994</td>
</tr>
<tr>
<td>Labor Indivisibilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.021</td>
<td>0.9926</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.0016</td>
<td>0.9994</td>
</tr>
<tr>
<td>Home Production</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.021</td>
<td>0.9926</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.0016</td>
<td>0.9994</td>
</tr>
</tbody>
</table>

Notes: The remaining parameters are set equal to the following values for both the weekly and quarterly models: (i) Government Spending: \( \alpha = 0 \); (ii) Time Nonseparabilities: \( \pi_0 = 0.5 \) and \( \eta = 0.1 \); and (iii) Home Production: \( \gamma = 0.3245 \), \( a = 0.6 \) and \( e = 0.4 \).
Table 2. Select Second-Moment Statistics for the US and RBC Economies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Growth Rates</td>
<td>HP Filter</td>
<td>Quarterly</td>
<td>Weekly</td>
<td>Quarterly</td>
<td>Weekly</td>
</tr>
<tr>
<td>std(y)</td>
<td></td>
<td></td>
<td>0.0095</td>
<td>0.0174</td>
<td>0.0095</td>
<td>0.095</td>
</tr>
<tr>
<td>std(c_p)</td>
<td></td>
<td></td>
<td>0.0049</td>
<td>0.0089</td>
<td>0.0049</td>
<td>0.0048</td>
</tr>
<tr>
<td>std(i)</td>
<td></td>
<td></td>
<td>0.0265</td>
<td>0.0464</td>
<td>0.0190</td>
<td>0.0191</td>
</tr>
<tr>
<td>std(n)</td>
<td></td>
<td></td>
<td>0.0079</td>
<td>0.0167</td>
<td>0.0039</td>
<td>0.0041</td>
</tr>
<tr>
<td>std(y/n)</td>
<td></td>
<td></td>
<td>0.0070</td>
<td>0.0076</td>
<td>0.0066</td>
<td>0.0063</td>
</tr>
<tr>
<td>corr(y/n,n)</td>
<td></td>
<td></td>
<td>-0.1960</td>
<td>-0.1357</td>
<td>0.0744</td>
<td>0.6025</td>
</tr>
</tbody>
</table>

Notes: The variables y, c_p, i, n, and y/n refer to output, private consumption, investment, labor hours and average labor productivity respectively. Std(x) refers to the standard deviation of detrended x. Corr(x,z) refers to the cross correlation between detrended x and detrended z. Generalized Wald test are in brackets. The 1%, 5% and 10% critical values from a chi-square distribution with one degrees of freedom are 6.63, 3.84 and 2.71 respectively. A single asterisk (*), double asterisk (**), and a triple asterisk (***) refer to failure to reject the null at the 1%, 5% and 10% significance levels respectively.
Labor-Market Fluctuations in a High-Frequency Real-Business-Cycle Model

David Aadland*
Department of Economics
Utah State University

March 28, 2000

Abstract. This paper investigates how the choice of timing interval within a standard real business cycle (RBC) model can help resolve several well-known labor-market puzzles. Standard quarterly RBC models have had difficulty replicating several empirical regularities arising from the US labor market. A weekly version of the RBC model is able to partially resolve several of these puzzles. The improvement in the performance of the model is due to the interaction between theoretical modifications to the standard RBC model and careful accounting for the sampling properties of US aggregate data.

JEL Codes: C43 and E32

Keywords: Real Business Cycles, Temporal Aggregation, Systematic Sampling, Timing Interval, and Labor-Market Fluctuations

*The paper has benefited from the comments of Kevin Huang, George W. Evans, Jo Anna Gray, Mark Thoma, the Editor, an anonymous reviewer, and the participants of the 1998 Western and Midwest Economic Association annual meetings. Author’s address: Department of Economics, Utah State University, Old Main Hill, Logan UT 84322-3530. Internet: aadland@b202.usu.edu
1. Introduction

The labor market has always played a central role in business-cycle research. From Adam Smith’s division of labor to David Ricardo’s labor theory of value to John Maynard Keynes’ theory of employment, the role of workers and employers in contributing to business-cycle fluctuations has consistently been an issue of fundamental importance. Modern macroeconomic theory is no exception. In particular, standard real-business-cycle (RBC) theory has relied on the willingness of individuals to intra- and inter-temporally substitute between leisure and labor in response to technology shocks that alter the relative return to working. Early RBC models (e.g., Kydland and Prescott (1982) and Long and Plosser (1983)) failed, however, to replicate several stylized features of aggregate US labor-market data. Three of the more notable deviations between theory and measurement include: (1) the relative volatility of hours worked to output, (2) the relative volatility of hours worked to average labor productivity, and (3) the correlation between real wages (or equivalently, average labor productivity) and hours worked. These and other labor market puzzles have motivated a large amount of research attempting to bring theory and observation closer together.¹

This paper investigates the role of the decision or timing interval in resolving some of these labor-market puzzles.

In particular, this paper provides an alternative explanation for addressing the magnitude of the intertemporal substitution of labor in macro models. Micro studies on the life-cycle behavior of men in the US have consistently revealed intertemporal elasticities of labor that, when used in RBC studies, are too small to replicate the magnitude of variation observed in aggregate hours worked. Pencavel (1986), in a survey of such panel studies, reports that the intertemporal elasticity of labor is somewhere between zero and 0.5 percent. Even using larger intertemporal labor elasticities, such

¹See Kydland (1995) for a survey of such research. Hall (1999) provides a more recent survey of advances in labor-market fluctuations in frictionless RBC models and contrasts the evidence with models that incorporate unemployment frictions.
as those in Mulligan (1998) who suggests that the elasticity is instead in the 1-2 percent range, are not sufficient to replicate the observed fluctuations in US hours worked within RBC models. In this paper, I provide evidence that by simply modeling agents as making decisions at finer intervals than one quarter of a year and carefully accounting for the manner that US data are collected, standard RBC models can come closer to matching observations from the US aggregate labor market. Moreover, when coupled with other standard modifications such as government consumption, time nonseparabilities, labor indivisibilities, or household production, RBC models can produce labor fluctuations in the model that are statistically indistinguishable from the US data.

It is well known that the time-series properties of economic data are sensitive to the sampling frequency. The classic documentation is by Working (1960) who shows that the time-aggregated changes in a random walk model follow a first-order moving-average process rather than a white-noise process. The primary reason that researchers evaluate macroeconomic models at quarterly or annual time intervals rather than shorter intervals such as days, weeks or months is that institutional factors set the sampling frequency of series such as the capital stock, GDP, investment and government spending. If the time-series properties of these series were not sensitive to the sampling frequency, there would not be a serious problem associated with specifying a timing interval that matched the sampling interval but was longer than the interval at which agents actually make decisions. However, since this is clearly not the case, we should be examining these models at higher frequencies than the standard quarter of a year. In order to evaluate a model at shorter timing intervals, there are essentially two options. You can (i) temporally aggregate the artificial data from a higher frequency model up to the sampling interval or (ii) temporally disaggregate the empirical data down to the model’s timing interval. Since interpolation and distribution procedures generate additional uncertainty into the analysis, I opt for the former in this paper.
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODELS

by

DAVID AADLAND

Department of Economics
Utah State University
3530 Old Main Hill
Logan, UT 84322-3530

May 1999
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODELS

David Aadland, Assistant Professor

Department of Economics
Utah State University
3530 Old Main Hill
Logan, UT 84322-3530

The analyses and views reported in this paper are those of the author(s). They are not necessarily endorsed by the Department of Economics or by Utah State University.

Utah State University is committed to the policy that all persons shall have equal access to its programs and employment without regard to race, color, creed, religion, national origin, sex, age, marital status, disability, public assistance status, veteran status, or sexual orientation.

Information on other titles in this series may be obtained from: Department of Economics, 3530 University Boulevard, Utah State University, Logan, Utah 84322-3530.

Copyright © 1999 by David Aadland. All rights reserved. Readers may make verbatim copies of this document for noncommercial purposes by any means, provided that this copyright notice appears on all such copies.
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODELS

David Aadland

ABSTRACT

Numerous studies document that the time-series properties of economic data are sensitive to the sampling frequency. Yet, it is an almost universally accepted practice to ignore these effects in applied research. I address this issue by outlining a procedure for evaluating the performance of a real-business-cycle (RBC) model when the timing interval is chosen to be shorter than the data-sampling interval. The results indicate that a weekly version of a standard RBC model is better able to replicate certain time-series properties of the U.S. experience over the last three decades.

JEL codes: C43, E32
HIGH-FREQUENCY REAL-BUSINESS-CYCLE MODELS

1. Introduction

It is well known that the time-series properties of economic data are sensitive to the sampling frequency. The classic documentation is by Working (1960) who shows that the time-aggregated changes in a random walk model follow a first-order moving-average process rather than a white-noise process. In light of this sensitivity, it is surprising there is not more careful treatment of the timing interval in studies whose results hinge on the time-series properties of the data. To motivate more along these lines, I evaluate a standard real-business-cycle (RBC) model at a weekly timing interval. I select an RBC model because of its prominence in current macroeconomic debate and because of RBC researchers’ reliance on time-series measures that have been shown to be sensitive to the unit of time measurement.

The primary reason that researchers evaluate macroeconomic models at quarterly or annual time intervals is due to institutional factors, which determine the sampling frequency of series such as the stock of capital, GDP, investment and government spending. If the time-series properties of these series were not sensitive to the sampling frequency, there would not be a problem associated with specifying a timing interval which matched the sampling interval but was longer than the decision-making interval. However, since this is not the case, we should be

---

1 This paper has benefited from the comments of Kevin Huang, George W. Evans, Jo Anna Gray, Mark Thoma, and the participants of the 1998 Western Economics Association.

2 Daily and monthly intervals were also analyzed and produced similar results. The weekly frequency was chosen to simplify the presentation.

3 See, for example, Tiao (1972), Engle and Liu (1972), Christiano, Eichenbaum, and Marshall (1992), and Rossana and Seater (1992).
examining the properties of these models at frequencies other than the standard quarter of a year.

In order to evaluate a model at shorter timing intervals, there are essentially two options. You could (i) temporally aggregate the model or artificial data from a higher frequency model up to the sampling interval or (ii) temporally disaggregate the empirical data down to the model’s timing interval. Since interpolation and distribution procedures generate additional uncertainty into the analysis, I opt for the former in this paper.

Once the decision has been made to temporally aggregate the artificial data, it is important that the methods employed are consistent with the techniques used in creating the actual data. Take the capital stock for instance. If monthly artificial data on the stock of capital is simulated from an RBC model, the time-series properties of the quarterly capital stock will be sensitive to whether the series is sampled at a single point in time or sampled at regular intervals and then averaged over time. In addition, various transformations such as natural logarithms and first differences complicate the matter further. In this paper, I attempt to replicate the aggregation process in the actual data as close as possible, being careful to address issues associated with data transformations. I will then contrast the artificial and actual data using standard second-moment criteria, as in the seminal works of Kydland and Prescott (1982) and Long and Plosser (1983).

The rest of the paper proceeds as follows. In section 2, I present an overview of the RBC model, the solution technique, the process for simulating RBC data at a shorter timing interval, and the methodology for transforming the artificial data to a lower frequency. In section 3, I contrast various statistics from the U.S. data with those generated from the high-frequency RBC model and discuss the results. In section 4, I summarize the paper’s most important findings and mention some issues for further research.
2. The High-Frequency RBC Economy

Over the last decade and a half, the RBC research program has experienced both remarkable success and controversy. At one end, the neoclassical growth model with a single stochastic productivity shock has proven remarkably adept at organizing macroeconomic variables and replicating various aspects of the business cycle. The attraction to the RBC research program is easy to understand. It explains many features of business cycle fluctuations while adhering to the microeconomic principles, which have become fundamental to economists understanding of economic behavior. Moreover, RBC theory uses the same neoclassical growth model which has dominated the discussion of economic growth for the past few decades. At the other end, however, there are also many skeptics of the RBC philosophy. The skeptics note shortcomings such as the absence of a role for nominal disturbances, the necessity of implausibly large productivity shocks and the use of unorthodox calibration techniques for evaluating RBC models, to mention a few. While these issues have recently received much attention, there still remains some specific puzzles which have not been thoroughly resolved. In this paper, I assert that at least three of these puzzles—the correlation between real wages (or equivalently average labor productivity) and hours worked, the relative volatility of hours worked to output, and the persistence of output growth—can be partially resolved by a willingness to analyze the RBC model at timing intervals shorter than the standard quarterly one.

---

A. The Model

The RBC model used in this paper is a variant of Christiano and Eichenbaum’s (1992) model, hereafter referred to as the CE RBC model. The CE RBC model incorporates government consumption shocks into a standard RBC model in order to address one of the most damaging piece of evidence against standard RBC models, that is, the production of grossly counterfactual correlations between real wages and hours worked. Whereas standard RBC models imply strong positive correlations between real wages and hours worked, historical data from the US economy indicates a weak correlation, an observation coined the Dunlop-Tarshis observation.\(^5\)

In this model, a social planner maximizes the following objective function in consumption \((c_t)\) and hours worked \((n_t)\):

\[
E_t \sum_{j=0}^{\infty} \beta^j (c_{t+j} - \phi (N - n_{t+j}))^{\sigma} / (1 - \sigma),
\]

where \(E_t\) is the mathematical expectations operator at time \(t\) based on all information dated time \(t-1\) and earlier, \(\beta\) is the subjective discount rate, \(\phi\) is the weight on distributed leisure, \(\sigma\) is the curvature parameter, and \(N\) is the time endowment. Total consumption is the sum of both public and private spending and enters the instantaneous utility function in the form \(c_t = c^p_t + \alpha g_t\),

where \(c^p_t\) is private consumption, \(g_t\) is government consumption, and \(0 \leq \alpha \leq 1\). If \(\alpha = 1\), then

\(^5\) Actually, the arguments of Dunlop (1938) and Tarshis (1939) primarily center around the correlation between changes in real and money wage rates rather than real wages and employment. In particular, they are at odds with Keynes’ assertion in the *General Theory of Employment, Interest and Money* that “when money wages are rising … real wages are falling; and when money wages are falling real wages are rising.” I can find only a single paragraph in both Dunlop and Tarshis’ papers where there is mention of the empirical relationship between employment and real wages. Dunlop says “there seems to be no simple relation—and especially of a causal nature—adequate to summarise the two movements without very wide margin of errors.” While Tarshis states “it is surprising that there is a less close association between changes in … real wages and man-hours. In this case, the coefficient stood at only -0.48.”
government and private consumption enter all equations as a sum and thus shocks to government consumption cause completely offsetting reductions in private consumption. If \( \alpha < 1 \), then government consumption reduces the marginal utility of private consumption, but not enough to offset the original government consumption shock. If \( \alpha = 0 \), government consumption is a pure resource drain and has no impact on the marginal utility of private consumption.

The objective function is maximized by choosing \( c_t^p \), leisure \((N - n_t)\) and capital \((k_{t+1})\) streams subject to a resource constraint

\[
y_t = k_t^0 (a_t n_t)^{1-\theta} + c_t^p + g_t + k_{t+1} - (1-\delta)k_t,
\]

(2)

the law of motion for labor-augmenting technical change \((a_t)\)

\[
a_t = a_{t-1} \exp(\mu + \varepsilon_{at}); \quad \varepsilon_{at} \sim \text{iid}N(0, \sigma^2_a),
\]

(3)

the law of motion for the ratio of government consumption to technical change \((\tilde{g}_t)\)

\[
\tilde{g}_t = \tilde{g}_{t-1} \exp((1-\rho)\tilde{g} + \varepsilon_{gt}); \quad \varepsilon_{gt} \sim \text{iid} \mathcal{N}(0, \sigma^2_{\tilde{g}}),
\]

(4)

and the given initial capital stock, \( k_0 \).

The primary difference between the CE RBC model and the one presented here is that this model includes a distributed lag of leisure in the utility function (see Kydland and Prescott, 1982). As a result, high work effort in the near past will increase the marginal utility of leisure today. The distributed lag is given by \( \Pi(L) = \sum_{i=0}^{\infty} \pi_i L^i \), where \( L \) is the lag operator, \( \pi_0 = 0.5 \), \( \sum_{i=0}^{\infty} \pi_i = 1 \), and the remaining coefficients decline geometrically according to \( \pi_{i+1} = (1-\eta)\pi_i \), \( i \geq 1 \) and \( 0 < \eta < 1 \). A convenient representation for this distributed lag is

\[
b_t = (1-\eta)b_{t-1} + n_{t-1}, \quad \text{where} \quad b_t = \sum_{i=1}^{\infty} (1-\eta)^{i-1}n_{t-i} \quad \text{and} \quad b_0 \quad \text{is given. This specification for leisure will be important for addressing the real wage/hours worked correlation.}
I choose the procedure outlined by Blanchard and Kahn (1980) for solving this rational expectations model. The method begins by transforming all nonstationary variables so they converge to a nonstochastic steady state. In our case, this can be accomplished by dividing all nonstationary variables (i.e., \(k_t, y_t, g_t\), and \(c_t\)) by the technical progress variable \(a_t\) and redefining the transformed variables (i.e., \(\bar{x}_t = x_t / a_t\)). Next, the first-order conditions that result from maximizing (1) subject to (2), (3) and (4) are linearized by taking a first-order Taylor-series approximation around the steady state values of the transformed variables. Lastly, we iterate into the future the linearized equation corresponding to the stable root of the first-order conditions. Skipping past a great deal of algebra, the decision rules for this problem take the following form:

\[
\begin{align*}
    s_t &= A s_{t-1} + \varepsilon_t \\
    f_t &= \Pi s_t
\end{align*}
\]

where \(s_t = (k_t \hat{g}_t \hat{b}_t)^t\), \(f_t = (\hat{y}_t \hat{c}_t \hat{n}_t \hat{i}_t)^t\), \(\varepsilon_t = (-\varepsilon_{at} 0 \varepsilon_{gt})^t\), \(A\) is a (3\(\times\)3) matrix of first-order autoregressive coefficients, \(\Pi\) is the (4\(\times\)3) matrix of impact multipliers, \(i_t\) is gross investment, and the hat (\(^\wedge\)) over a variable refers to percentage deviations from their steady-state values.

B. Simulating Artificial Weekly Data

To evaluate the performance of the RBC model above, I begin by generating 500 artificial data sets from the decision rules (5) and (6). Then the ensemble averages of various statistics from the model are contrasted with their corresponding statistics from the U.S. economy. In order to complete this task, values for the parameter vector

\[
\Gamma = \left( \beta \ \alpha \ N \ \pi_0 \ \sigma \ \rho \ \eta \ \theta \ \delta \ \phi \ \mu \ \bar{g} / \bar{c} \ \sigma_a \ \sigma_g \right)
\]
must be assigned. I choose the quarterly generalized method of moments estimates of Christiano and Eichenbaum (1992), with $\sigma_a$ and $\sigma_g$ rescaled so that the standard deviation of per-capita output growth from the RBC model equals per-capita U.S. real GDP growth over the period 1965:1 through 1995:4.

In order to simulate the artificial weekly data sets from the RBC model, I first transform the quarterly values of $\Gamma$ to be consistent with a weekly version of the model. The weekly parameters associated with time (i.e., $\rho$, $\beta$ and $(1-\delta)$) are calculated by raising the quarterly values to the one-thirteenth power. Weekly values of other parameters are either divided by thirteen (i.e., $\mu$, $\bar{g}$ and $N$) or left unchanged (i.e., $\alpha$, $\pi_0$, $\sigma$, $\eta$, $\theta$, and $\phi$). Lastly, the weekly values for the standard deviations of the disturbances (i.e., $\sigma_a$ and $\sigma_g$) are adjusted by dividing the quarterly value of $\sigma_a$ by $\sqrt{13}$ and then maintaining the relative magnitudes of the two variances estimated in Christiano and Eichenbaum. The entire set of parameter values for the quarterly and weekly versions of the model are presented in Table 1, and the resulting coefficient matrices for the RBC model are presented in Table 2. The models using these values will hereafter be referred to as the weekly and quarterly baseline models.

C. Transforming the Artificial Data

Before contrasting the weekly RBC and U.S. data, I need to make the two comparable by (i) transforming the former out of percentage deviations from the steady state and (ii) changing

\footnote{As a technical matter, it is not entirely clear how to adjust the parameter $\rho$. If government spending is assumed to follow a first-order autoregressive process at a weekly frequency (as in the quarterly version of the CE RBC model), then quarterly government spending will have the autoregressive parameter $\rho$ to the thirteenth power, but will have a moving average component as well. Looking at it from another perspective, if government spending is assumed to follow a first-order autoregressive process at the quarterly frequency, then it will not follow the same process at the weekly frequency. However, in some sensitivity analysis, it was found that moderate variations in $\rho$ and the disturbance variance have little effect on the main results of this paper so a weekly AR(1) process is maintained.}
the frequency of the artificial data to be consistent with the frequency of the U.S. data. For the first transformation, I make use of the following approximation

\[ \Delta \log(x_t) \approx \Delta \hat{x}_t + \mu + \varepsilon_{at}, \]  

(7)

where \( \Delta = 1 - L \), and transform the nonstationary series from the model (i.e., \( y_t, g_t, i_t \), and \( c_t^p \)) into growth rates.\(^7\) The stationary series from the model, \( \hat{n}_t \), is transformed according to

\[ n_t = \bar{n} (\hat{n}_t + 1). \]  

(8)

After the series have been transformed out of percentage deviations from their steady states, they need to be transformed to the lower frequencies at which the actual data are measured. It is critical that the aggregation or sampling procedure closely mimic the manner in which the actual data are created because sampling and aggregation may influence the time-series properties of the data. There are essentially two different ways to reduce the frequency of a time series: temporal aggregation or systematic sampling. Temporal aggregation involves summing a flow variable (or averaging in the case of a stock) over the shorter intervals and forming one aggregate. The temporal aggregation operator is given by

\[ B_{TA} = 1 + L + L^2 + \cdots + L^{n-1} \]  

(9)

where \( n \) is the number of observations at the shorter time intervals within one time aggregate. Temporal averaging of a stock series simply involves applying \( B_{TA} \) and dividing by \( n \). For our purposes, \( n \) will equal thirteen as there are approximately thirteen weeks in a quarter (ignoring leap year). Systematic sampling involves selecting values of a stock variable at regularly spaced intervals. The systematic sampling operator is given by

\(^7\)The standard method of inducing stationarity in RBC studies is through the Hodrick-Prescott (HP) filter. Cogley and Nason (1995b) provide evidence that the HP filter can spuriously generate business cycle fluctuations in difference stationary data. I detrend the actual data using both the first-difference operator and the HP filter and find that other than the autocorrelations for output growth, the results are similar using either method to detrend the data.
where \( j \) equals a single element out of the set \( \{0, 1, \ldots, n - 1\} \). If the variable has not been logged, differenced or subject to some other type of transformation, then lowering the frequency of the series is only a matter of determining whether it is a stock or a flow and then applying the appropriate operator above. However, it is clear from (7) that the nonstationary variables in the RBC model will also be subject to both logarithmic and first-difference transformations.

The primary difficulty of working with variables that have been transformed as natural logs is that the sum of the logs is not equal to the log of the sums. For example, if a monthly flow series is measured in logs, simply aggregating the observations will not produce the log of the quarterly series. A proposed solution is as follows. Begin by defining \( \bar{X}_{nt} \) to be the log of the product of disaggregate (or basic) variables as

\[
\bar{X}_{nt} = B_{TA} (L) \log(x_{nt}) = \log(x_{nt}) + \log(x_{nt-1}) + \ldots + \log(x_{nt-(n-1)})
\]

with \( \tau = 1, \ldots, T / n \), for each of the \( T/n \) sets of \( n \) consecutive basic observations, which results in

\[
\bar{X}_{nt} \approx n \log(\bar{x}_{nt}) + \frac{1}{\bar{x}_{nt}} (x_{nt} - \bar{x}_{nt}) + \ldots + \frac{1}{\bar{x}_{nt}} (x_{nt-(n-1)} - \bar{x}_{nt}).
\]

For a temporally aggregated flow series, the sample mean for each \( n \) consecutive non-overlapping basic observations is observable and given by \( X_{nt} / n \), where

\[
X_{nt} = x_{nt} + \ldots + x_{nt-(n-1)}.
\]

After a few substitutions and a little rearranging, \( \bar{X}_{nt} \) and \( X_{nt} \) can be

---

\(^8\) Using a state-space framework, Harvey and Pierse (1984) deal with this problem by assuming that the observable logged aggregates are normally distributed and then use the extended Kalman filter to handle the implied nonlinear observation equation. See Harvey (1989, pp.160-62) for more details on the extended Kalman filter.
related according to

\[ \tilde{X}_{n_t} \equiv n(\log(X_{n_t}) - \log(n)). \quad (11) \]

When a series has been first differenced, two things are required to transform the series to a lower frequency. First, the undifferenced series needs to be aggregated or sampled. This is done by applying either \( B_{TA} \) or \( B_{SS} \). Second, the difference operator, \( \Delta = 1 - L \), needs to be transformed into aggregate time (i.e., \( \Delta^n = 1 - L^n \)), which is accomplished by multiplying by \( B_{TA} \):

\[ B_{TA} (1-L) = (1+L+\cdots+L^{n-1})(1-L) = (1-L^n). \quad (12) \]

Therefore, quarterly artificial flow data (in growth rates) will be measured as

\[ 13(1+L+\cdots+L^{12})(1-L)(\log(X_{13t}) - \log(13)), \]

and quarterly artificial stock data (in growth rates) will be calculated as

\[ (1+L+\cdots+L^{12})(1-L)L^j \log(x_{13t}), \]

where \( j \) will depend on the week in which the actual data are sampled.

3. Simulation Results

In this section, I contrast both weekly and quarterly versions of the RBC model with the U.S. economy. Several variations of the RBC were calibrated and used to simulate artificial data sets: (i) the quarterly baseline model; (ii) the weekly baseline model; (iii) the weekly baseline model without government (\( \alpha = 1 \)); and (iv) the weekly baseline model without a distributed lag of leisure in the utility function (\( \pi_0 = 1 \)). The predominant conclusion from this exercise is that

\[ ^9 \text{Of course, for systematically sampled stock series, logarithms do not present the same difficulty.} \]
the performance of RBC models is sensitive to the choice of timing interval. In particular, a weekly version of the baseline RBC model appears to replicate the U.S. experience over the past three decades better than standard quarterly versions of the model.

A. Data

The data are taken from FRED, the Federal Reserve Economic Database of the St. Louis Federal Reserve Bank. The series selected are as follows, with FRED mnemonic in parentheses:

output is measured as real gross domestic product (GDPC92); private consumption is measured as real personal consumption expenditures on nondurable goods (PCENDC92) and service goods (PCESC92); investment is measured as the sum of real fixed private investment (FPIC92), real public gross investment (DGIC92 + NDGIC92 + SLINVC92), and real personal consumption expenditures on durable goods (PCEDGC92); government consumption is measured as real government consumption expenditures and gross investment (GCEC92) less public gross investment; total hours are measured as the product of average weekly hours in private nonagricultural establishments (AWHNOC) and total nonfarm payroll employees (PAYEMS); and lastly, average productivity is calculated by dividing output by total hours. All the variables have been seasonally adjusted at the source; output, private consumption, investment and government consumption are measured in chain-weighted 1992 dollars; and all but average productivity have been transformed into per-capita terms using the entire civilian, non-institutionalized population that is sixteen and over (CNP16OV).

In addition to selecting U.S. variables which best match the definitions provided by the model, it will be equally important to match the U.S. sampling and aggregation procedures. Series taken from the National Income and Product Accounts (NIPA) -- output, consumption, and investment -- are collected using a wide range of sources and at varying points in the quarter;
see the Bureau of Economic Analysis’ website for details (http://www.bea.doc.gov/bea/mp.htm).

As an approximation, the artificial data for these series were aggregated using the temporal aggregation operator, $B_{TA}(L)$. Series taken from the Bureau of Labor Statistics (BLS)—average weekly hours, employment, and the population—are collected from the household and establishment surveys. The following excerpt is taken from the BLS website (http://stats.bls.gov:80/) regarding their sampling methods:

For both surveys, the data for a given month relate to a particular week or pay period. In the household survey, the reference week is generally the calendar week that contains the 12th day of the month. In the establishment survey, the reference period is the pay period including the 12th, which may or may not correspond directly to the calendar week.

In accordance with BLS techniques, total hours worked and population data generated from the model were thus treated as being systematically sampled from the second week of the month.

B. Discussion of the Results

As mentioned in the introduction, a weekly version of the RBC model is better able to replicate the business cycle than a quarterly version in three dimensions: the autocorrelation of output growth; the weak (negative) correlation between hours worked and real wages; and the relative volatility of labor hours to output. As will be shown below, all three results depend on the pattern of weekly own- and cross-correlations of the various series.

Before focusing on the numbers in Table 3, it will be useful to derive a relationship between the basic and aggregate second-moment estimators. Begin by defining the population covariance between the growth rates of two aggregate flow variables as

$$
\Gamma_{xy}(k) = \text{cov}(\Delta^n \log(X_{nt}), \Delta^n \log(Y_{n(t-k)})).
$$

(13)
In order to write $\Gamma_{xy}(k)$ in terms of the basic covariances, $\gamma_{xy}$, use (11) to write the log of the sums as the sum of the logs:

$$\gamma_{xy}(k) = \text{cov}(\Delta_n^{1/2n} \sum_{i=0}^{n-1} \log(x_{n(i-1)}), \Delta_n^{1/2n} \sum_{i=0}^{n-1} \log(y_{n(i-k)-i})).$$

(14)

Next, use (12) to transform the aggregate difference operator, $\Delta_n$, to the basic difference operator, $\Delta$:

$$\gamma_{xy}(k) = \text{cov}(\sum_{i=0}^{2(n-1)} \omega_i^x \Delta \log(x_{n(i-1)}), \sum_{i=0}^{2(n-1)} \omega_i^y \Delta \log(y_{n(i-k)-i})).$$

(15)

where $\omega_i^x$ and $\omega_i^y$ are the weights in the sum of the basic logs. Moving to matrix notation, we can rewrite (15) as

$$\gamma_{xy}(k) = \mathbb{E} \left[ \mathbf{g}_x(0) \mathbf{g}_y(nk) \right] \omega_y \mathbf{K} \omega_x \gamma_{xy}(nk) \omega_y,$$

(16)

where $\omega_x = (\omega_0^x, \omega_1^x, \ldots, \omega_{2(n-1)}^x)$, $\mathbf{g}_x(ns) = (\Delta \log x_{n(i-s)} \Delta \log x_{n(i-s)-1} \ldots \Delta \log x_{n(i-s)-2(n-1)})'$ and similarly for $y$. In this case,

$$\gamma_{xy}(nk) = \begin{bmatrix}
\gamma_{nk} & \gamma_{nk+1} & \cdots & \gamma_{nk+2(n-1)} \\
\gamma_{nk-1} & \gamma_{nk} & \cdots & \gamma_{nk+2(n-1)-1} \\
\vdots & \ddots & \ddots & \vdots \\
\gamma_{nk-2(n-1)} & \gamma_{nk-2(n-1)+1} & \cdots & \gamma_{nk}
\end{bmatrix},$$

where $\gamma_i$ is the cross covariance between the contemporaneous $x$ and the $i^{th}$ lag of $y$.

Consider a simple example where $n = 3$ (e.g., monthly to quarterly frequency), $X$ is a temporally aggregated flow variable, and $y$ is a stock variable that is systematically sampled in the final month of the quarter (i.e., $j = 0$ in (10)). In this case, the first- and second-order aggregate cross covariances are
\[ \Gamma_{xy}(0) = E [g_x(0) g_y(0)'] \omega_y \gamma_{xy}(0) \omega_y' \]

\[ \Gamma_{xy}(3) = E [g_x(0) g_y(3)'] \omega_y \gamma_{xy}(3) \omega_y' \]

where \( \omega_x = \frac{1}{3} (1 \, 2 \, 3 \, 2 \, 1) \), \( \omega_y = (1 \, 1 \, 1 \, 0 \, 0) \),

\[ \gamma_{xy}(0) = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \]

and \( \gamma_{xy}(3) = \begin{bmatrix} \gamma_4 & \gamma_5 & \gamma_6 & \gamma_7 \\ \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \\ \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \\ \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \\ \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \end{bmatrix} \)

After some simple matrix algebra, we obtain

\[ \Gamma_{xy}(0) = \frac{1}{3} \gamma_{-4} + 3\gamma_{-3} + 6\gamma_{-2} + 7\gamma_{-1} + 6\gamma_0 + 3\gamma_1 + \gamma_2 g \]

\[ \Gamma_{xy}(3) = \frac{1}{3} \gamma_{-1} + 3\gamma_0 + 6\gamma_1 + 7\gamma_2 + 6\gamma_3 + 3\gamma_4 + \gamma_5 g \]

Notice that the contemporaneous basic cross covariance, \( \gamma_0 \), does not receive the largest weight in the contemporaneous aggregate cross covariance.

Table 3 presents second-moment statistics for the U.S. economy and for several different versions of the model. The statistics from the models are ensemble averages across the 500 simulations. Standard deviations for the model statistics are calculated using the variation across the 500 simulations, while standard deviations for the empirical statistics are calculated using standard formula; see the appendix for further details. In brackets, are the generalized Wald statistics for the null hypothesis that the model and empirical moments are equal. The Wald statistics are calculated according to

\[ W = (\hat{\psi} - \bar{\psi})' \text{est. var}(\hat{\psi}) + \text{est. var}(\bar{\psi}) g (\hat{\psi} - \bar{\psi}) , \]
where $\tilde{\psi}$ is a $(J \times 1)$ vector of statistics from the actual U.S. data and $\bar{\psi}$ is a $(J \times 1)$ vector of statistics from the model averaged across the simulations. Under the null hypothesis, $W$ has an asymptotic distribution with $J$ degrees of freedom. Since we are interested in cases where the model is able to replicate features of the U.S. data, single (*), double (**), and triple (***), asterisks indicate *failure to reject* the null hypothesis of equal statistics at the 1%, 5%, and 10% significance levels, respectively.

First, notice that in Table 3, the growth rates and HP filter results for the U.S. economy are similar to the results found in other papers. Investment is more volatile than output, output is in turn more volatile than consumption, and hours worked vary nearly as much as output (Prescott, 1986). Also, real wages and hours worked are negatively, but weakly correlated (Hansen and Wright, 1992). And finally, output growth displays low-order positive autocorrelation (Cogley and Nason, 1995a). Generally, the results are similar whether the data are detrended using first differences or the HP filter, except for the larger positive autocorrelation for output growth using the HP filter. The HP filter is known to generate additional persistence in a difference-stationary series (Cogley and Nason, 1995b).

Second, notice that whereas the quarterly baseline model generates very little autocorrelation in output growth, the weekly model generates noticeable first-order autocorrelation, but little second-order autocorrelation. This effect, noted in Cogley and Nason (1995b), is a reflection of a result first noted by Working (1960). Working shows that the changes in a random walk process, when subject to temporal aggregation, become an MA(1) process with limiting MA coefficient equal to 0.25. Since output growth in the weekly baseline RBC model is approximately a white noise process, temporal aggregation is then able to account for the positive first-order autocorrelation coefficient. Moreover, when considering the first two
autocorrelation coefficients jointly, the Wald test indicates the at the 5% significance level we reject the hypothesis that the quarterly model and U.S. autocorrelations are equivalent, but fail to reject the hypothesis for the weekly models.

Third, the standard deviations of the weekly RBC series generally provide a better match to the standard deviations of the US aggregates than do the quarterly RBC series. In particular, the baseline weekly model does a superior job in matching the standard deviation of real wages. Whereas we reject the hypothesis of equal volatility for the quarterly model, we fail to reject that they are equal under the weekly model. Furthermore, the result is not driven by either the presence of government nor a distributed lag for leisure as indicated by the last two columns in Table 3.

Another aspect where the weekly version of the model appears to provide an improvement is in the volatility of hours worked relative to that of output growth (although strictly speaking, we reject the null of equal standard deviations for all specifications of the model). Under growth rates, the standard deviation of hours worked in the U.S. is 0.0079, whereas the quarterly baseline model generates only 60% as much volatility in hours worked, with a standard deviation of 0.0049. This lack of volatility in hours worked has been a known failure of standard RBC models. It has been especially perplexing given the relatively small labor supply elasticities of substitution estimated in panel studies (Pencavel, 1986) and the large role intertemporal substitution of labor is known to play in generating business cycle fluctuations.\footnote{Introducing labor indivisibilities into the model, whereby all the variation in total hours worked takes place along the employment margin rather than the hours-per-worker margin (Hansen, 1985), is now the standard method for increasing the volatility of hours worked to output. However, this specification tends to generate too much volatility in hours worked.} The weekly baseline model combining both government consumption and a
distributed lag for leisure increases the standard deviation of hours worked to 0.0057 and may provide another avenue for increasing the low observed volatility in quarterly studies.

Fourth, and finally, consider the contemporaneous cross correlation between hours worked and real wages. Since standard RBC models with a single productivity shock imply the labor demand curve will shift stochastically along a fixed, upward-sloping labor supply curve, the correlations between the real wage (or equivalently average labor productivity) and hours worked will be strongly positive. The introduction of additional factors such as government consumption shocks, which will shift the labor supply curve, can help to mitigate this problem. The correct combination of productivity and government consumption shocks could then potentially better mimic the weak correlation between real wages and hours worked. Christiano and Eichenbaum (1992) had some success in this dimension. Simply choosing a weekly rather than a quarterly timing interval, provides an alternative explanation, without need to introduce new shocks into the model. Recall from above that both the BLS establishment and household surveys generate employment data by systematically sampling from data in approximately the second week of each month. Mimicking this sampling structure for the artificial data generated by the weekly baseline model and using equation (16) gives a quarterly contemporaneous coefficient equal to

$$
\Gamma_{n, y/n}(0) = \frac{1}{13} (127\gamma_{-4} + 126\gamma_{-4\pm1} + 123\gamma_{-4\pm2} + 118\gamma_{-4\pm3} + 111\gamma_{-4\pm4} + 102\gamma_{-4\pm5} \\
+ 91\gamma_{-4\pm6} + 78\gamma_{-4\pm7} + 66\gamma_{-4\pm8} + 55\gamma_{-4\pm9} + 45\gamma_{-4\pm10} + 36\gamma_{-4\pm11} + 28\gamma_{-4\pm12} \\
+ 21\gamma_{-4\pm13} + 15\gamma_{-4\pm14} + 10\gamma_{-4\pm15} + 6\gamma_{-4\pm16} + 3\gamma_{-4\pm17} + \gamma_{-4\pm18})
$$

Two effects are responsible for failing to reject equal correlations between real wages and hours worked in the weekly baseline and U.S. data. First, the presence of a distributed lag for leisure implies negative cross correlations between lagged real wages and contemporaneous hours worked, $\gamma_i$, for $i < 0$. That is, high real wages in past periods will induce individuals to
work more hours in the present period, which in turn will increase the marginal utility of leisure and the amount of leisure taken in the present period. This relationship is shown in Table 4, which shows the negative, albeit small, correlations between hours worked and lagged real wages. Second, the fact that the hours worked are systematically sampled implies that the weekly contemporaneous correlations between real wages and hours worked, $\gamma_0$, will receive a relatively small weight in (17). Notice also that in Table 4, the cross correlations are dominated by the contemporaneous coefficient, and as a consequence, reducing its relative weight in (17) will place downward pressure on the quarterly correlation between real wages and hours worked.

4. Conclusion and Future Directions

The question to ask at this point is what, if anything, have we learned from specifying a timing interval shorter than the standard one quarter of a year. One thing we have learned is that a weekly version of a standard RBC model appears to fit the U.S. aggregate data over the last three decades better than the typically specified quarterly model. In particular, the weekly baseline RBC model is able to partially resolve three empirical regularities that standard RBC models have had a difficult time replicating. First, temporal aggregation of the RBC model provides a better fit for the low-order positive autocorrelation observed in the growth of U.S. GDP per capita. Second, the Dunlop-Tarshis observation that real wages and hours worked are weakly correlated in the U.S. economy is not statistically rejected by a weekly version of the RBC model. And third, by choosing a weekly rather than quarterly timing interval, the relative volatility of hours worked to output in the RBC economy is increased.

Although these observations are interesting in their own right, I would argue that the superiority of the weekly RBC model is less important than the apparent sensitivity of RBC
models to the choice of timing interval. After all, there is no compelling reason why behavior by individuals in an economy should be better represented by quarterly decision-making intervals than monthly, weekly, daily, or even hourly intervals. Short of discovering the natural timing, it would seem that researchers should be checking the performance of RBC and other dynamic macro models at frequencies other than those at which the data are collected.

This sensitivity to the timing interval also opens up a number of questions. In particular, if the choice of timing interval matters, then which interval best represents agents' decision-making process? Is it a weekly interval? Daily interval? And does this sensitivity appear in other macroeconomic models? If so, can it help in resolving other puzzles? These questions, while not addressed in this paper, seem to be important areas for future research.
References


References


Appendix

This appendix covers two issues. One, it shows how to calculate the variances of the quarterly productivity and government consumption shocks in order to match the sample variance of the growth in real GDP per capita. And two, it shows, how the estimated variances for the various second-moment statistics were calculated.

I. Calculating Quarterly Shock Variances

As is common in RBC studies, the variances of the disturbances will be chosen such that the theoretical and empirical variances of GDP growth are equal. Using (5) - (7), we can write the variance of GDP growth as

\[
\text{var}(\Delta \log(y_t)) = \text{var}(\Delta \hat{y}_t) + \text{var}(\mu + \varepsilon_{a,t}) + 2 \text{cov}(\Delta \hat{y}_t, \mu + \varepsilon_{a,t})
\]

\[
= \text{var}(\pi_1 \Delta \hat{s}_t) + \sigma_a^2 - 2\pi_1\sigma_a^2
\]

\[
= \pi_1 \text{var}(\Delta \hat{s}_t) \pi_1' + \sigma_a^2 (1 - 2\pi_1)
\]

(A.1)

where \( \pi_j \) represents the jth row of \( \Pi \), \( \pi_{ij} \) represents the (i,j) element of \( \Pi \), and \( \Delta = 1 - L \). Since \( \Delta \hat{s}_t \) is a stationary vector autoregressive process, we can write the variance of \( \Delta \hat{s}_t \), \( \Sigma \), as

\[
\text{vec}(\Sigma) = [I - (A \otimes A)]^{-1} \{\text{vec}(Q_d) - [(A \otimes I) + (I \otimes A)]\text{vec}(Q)\},
\]

(A.2)

where \( Q_d \) is the variance of \( \Delta \varepsilon_t \), \( Q \) is the variance of \( \varepsilon_t \), \( \otimes \) is the Kronecker product, and the \( \text{vec}(x) \) operator stacks the columns of \( x \) into a single column vector; see Hamilton (1994). Using (A.1), (A.2), the parameter values in Table 1, the estimated variance of the growth in US GDP (see Table 3), and the estimated variance ratio, \( \sigma_a = 0.75 \sigma_g \), from Christiano and Eichenbaum (1992), we arrive at the quarterly estimates: \( \sigma_a = 0.0100 \) and \( \sigma_g = 0.0134 \).
II. Calculating the Variances of the Second-Moment Statistics

The estimated variances for US second-moment statistics are calculated as follows. First, the estimated variance for the standard deviation, \( s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} \), is approximated by taking a first-order Taylor series approximation of \( s \) around \( \sigma^2 \):

\[
s \approx \sigma + 0.5(\sigma^2)^{-0.5} (s^2 - \sigma^2),
\]

where \( n \) refers to the sample size and \( \sigma^2 \) is the variance of \( x \). The variance of (A.3) is:

\[
\text{var}(s) \approx \frac{0.25}{\sigma^2} \text{var}(s^2),
\]

which under the assumption of normally distributed \( x \) and substituting in \( s^2 \) for \( \sigma^2 \) gives

\[
\text{est. var}(s) \approx \frac{0.25}{s^2} \frac{2s^4}{n-1} = \frac{0.5s^2}{n-1}.
\]

Second, the estimated variance of the contemporaneous cross correlation coefficient, \( r_{xy}(0) \), is given by

\[
\text{est. var}(r_{xy}(0)) \approx \frac{1}{n} \left[ \sum_{v=-\infty}^{\infty} \mathbf{d}(v) r_{yy}(v) + r_{xy}(v)r_{xy}(-v) \right] + \frac{1}{n} \left[ \sum_{v=-\infty}^{\infty} r_{xy}^2(0) \mathbf{c}(v) + 0.5r_{xx}^2(v) + 0.5r_{yy}^2(v) \right] - 2 \frac{1}{n} \sum_{v=-\infty}^{\infty} r_{xy}(0) \mathbf{d}(v) r_{xy}(v) + r_{xy}(-v)r_{yy}(v),
\]

where \( r_{xx}(j) \) refers to the jth autocorrelation coefficient for \( x \) (Box and Jenkins, 1976, p.376-77).

Third, and finally, the estimated covariance between autocorrelation coefficients is approximated by

\[
\text{est. cov}(r(j), r(j+1)) \approx \frac{1}{n} \left[ \sum_{v=-\infty}^{\infty} \mathbf{b}(v) (r(v+j) + r(v+2j+1)) + 2r(j)r(j+1)r^2(v) \right] - \frac{2}{n} \sum_{v=-\infty}^{\infty} \mathbf{b}(v) r(v) r(v+j+1) + r(j+1)r(v)r(v+j),
\]

where the subscripts have been omitted for simplicity (Box and Jenkins, 1976, p.376-77).