Design of Tunable Edge Coupled Microstrip Bandpass Filters

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DESIGN OF TUNABLE EDGE-COUPLED MICROSTRIP BANDPASS FILTERS

by

Srinidhi V. Kaveri

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Electrical Engineering

Approved:

Dr. Edmund Spencer  Dr. Randy J. Jost
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UTAH STATE UNIVERSITY
Logan, Utah

2008
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Abstract

Design of Tunable Edge-Coupled Microstrip Bandpass Filters

by

Srinidhi V. Kaveri, Master of Science
Utah State University, 2008

Major Professor: Dr. Edmund Spencer
Department: Electrical and Computer Engineering

This thesis is a study of tunability of edge-coupled filters. Microstrip edge-coupled bandpass filters are planar structures and have advantages such as easy design procedures and simple integration into circuits.

Three tuning techniques were implemented. The first technique involved the loading of one open end of each coupled into tunable capacitors. The second technique used a tunable resonator in series with the edge-coupled blocks. The final design made use of tunable feedback sections.

A detailed mathematical analysis of each design was performed. MATLAB code based on the analyses was written. The MATLAB simulations were compared to Agilent Advanced Design System (ADS) simulations in order to find the minimum design parameters required to arrive at an approximate solution. ADS simulations were used to accurately determine the final design.

The tunable filters with a series capacitor and feedback were fabricated on RO4003C boards from Roger’s Corporation, having a dielectric constant of 3.55. The built boards were then tested with the HP 8510c network analyzer. The measured results were compared to the ADS simulations.

The filter with a tuning capacitor in series with the coupled sections had high insertion
loss of -20 dB and tuning range in terms of KHz. The design involving feedback had
advantages over the previous design since the insertion loss was better than -14 dB and it
had a tuning range of 91 MHz. It was observed from simulations that the design had an
adjustable tunability range and bandwidth as the width was varied.
To my parents....
Acknowledgments

Foremost, I would like to thank Dr. Edmund Spencer for his support and guidance. It was due to his encouragement that I became interested in working in the field of microwave engineering. He invested a lot of time and energy in guiding me throughout my program here at Utah State University. Working on this project required strong fundamentals in RF engineering. He had the patience to let me learn at my own pace and take one step at a time.

I would also like to thank my committee members, Dr. Randy Jost and Dr. Wei Ren, for taking the time off their busy schedules and reading my thesis. I am grateful to Dr. Jost and Dr. Bedri Cetiner for teaching courses in microwave engineering. These courses introduced me to many new things in the field of RF and microwaves. I also acknowledge Heidi Harper for helping me find components for my designs.

I thank all my friends here at Utah State University, who never made me feel away from home. I have had a blast here and will take great memories along with me.

I have had a great time working in my office, thanks to all my lab-mates, for the last two years. It is only because of them that I became enthusiastic in work.

Finally, I would like to thank my parents, my sisters Pradnya and Shilpa, my sweet niece Shreya, and all my friends back home in India. Without their love and support, I would never have been able to come to the United States for higher education.

Srinidhi V. Kaveri
Contents

Abstract .................................................................................................................. iii
Acknowledgments .................................................................................................... vi
List of Figures ....................................................................................................... ix

1 Introduction ......................................................................................................... 1
  1.1 The Microstrip Transmission Line ................................................................. 1
  1.2 Quasi-TEM Propagation ............................................................................... 1
  1.3 Z- and Y-parameters ..................................................................................... 3
  1.4 S-parameters ................................................................................................ 4
  1.5 ABCD or the Transmission Parameters ......................................................... 5
  1.6 Analysis of Periodic Structures .................................................................... 7
  1.7 Design of Third Order Chebyshev Bandpass Edge-Coupled Filter .............. 9
      1.7.1 Filter Design Using Coupled Sections .................................................... 9
      1.7.2 The Quarter-wave Transformer ............................................................. 13
      1.7.3 Simulation Results .............................................................................. 14

2 Even and Odd Modes in a Coupled Transmission Line .................................. 15
  2.1 Introduction .................................................................................................... 15
  2.2 The Capacitance Model ................................................................................ 16
  2.3 Telegrapher Equations Describing a Microwave Transmission Line ........... 17
  2.4 The Case of Coupled Lines .......................................................................... 19

3 $N^{th}$ Order Edge-Coupled Filter Analysis .................................................... 24
  3.1 Introduction .................................................................................................... 24
  3.2 Experimental Design in ADS ....................................................................... 24
  3.3 Analysis of a Single Coupled Section ............................................................ 25
  3.4 Calculation of Even and Odd Mode Characteristic Impedances ................. 31
  3.5 Calculation of the Even and Odd Mode Effective Dielectric Constant ....... 32
  3.6 ABCD Matrix of Matching Sections ............................................................ 32
  3.7 The Coupled Line Model for Equal Width Strips ......................................... 33
  3.8 Calculation of Effective Dielectric Constant ............................................... 35
  3.9 Calculation of Characteristic Impedance of the Strip .................................. 36
  3.10 Determination of $S$-parameters ................................................................. 37
  3.11 Validation of MATLAB Code .................................................................... 37
  3.12 Application of the MATLAB Code .............................................................. 38
# 4 Initial Investigation of Filter Tuning Methods

4.1 Introduction ........................................ 39
4.2 Tunable Filter Using Capacitively Loaded Coupled Sections ............... 39
4.3 Tunability with a Tuning Capacitor in Series .......................... 40

# 5 Filter Tuning Using Feedback Loop

5.1 Motivation for Feedback Loop Design ................................ 48
5.2 Design Procedure ........................................... 48
5.3 Analysis of the Tunable Filter ................................... 49
5.4 Effect of Feedback Section Width ................................... 57
5.5 Group Delay .............................................. 58

# 6 Conclusions and Future Work

6.1 Conclusions .............................................. 64
6.2 Future Work ................................................ 64

# References

................................................................. 66

# Appendices

Appendix A MATLAB Code for Feedback-Based Tunable Filter .......... 69
A.1 MATLAB Code for Analysis of Feedback Section ..................... 69
A.2 Zmatrix.m .................................................. 73
A.3 ZAsolved.m ............................................... 78
A.4 ZB solved.m ................................................. 79
Appendix B MATLAB Codes for Edge-Coupled Filter Analysis .......... 80
B.1 MLIN.m ..................................................... 80
B.2 ZtoABCD.m .................................................. 81
B.3 ABCDtoS.m .................................................. 82
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 A microstrip transmission line.</td>
<td>2</td>
</tr>
<tr>
<td>1.2 An example of TEM propagation.</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Cross-section of the microstrip showing electric field around it.</td>
<td>3</td>
</tr>
<tr>
<td>1.4 A two-port network.</td>
<td>3</td>
</tr>
<tr>
<td>1.5 Two-port network characterized by $S$-parameters.</td>
<td>5</td>
</tr>
<tr>
<td>1.6 Two-port network to be represented by $ABCD$ parameters.</td>
<td>6</td>
</tr>
<tr>
<td>1.7 Two cascaded two-port networks.</td>
<td>6</td>
</tr>
<tr>
<td>1.8 $N+1$ periodic structures cascaded in a network.</td>
<td>8</td>
</tr>
<tr>
<td>1.9 A ladder network for a third order lowpass Chebyshev filter prototype beginning with a shunt element.</td>
<td>11</td>
</tr>
<tr>
<td>1.10 MATLAB response for a third order Chebyshev microstrip edge-coupled filter.</td>
<td>14</td>
</tr>
<tr>
<td>2.1 Electric and magnetic fields of a coupled microstip line operating in even mode.</td>
<td>16</td>
</tr>
<tr>
<td>2.2 Electric and magnetic fields of a coupled microstip line operating in odd mode.</td>
<td>16</td>
</tr>
<tr>
<td>2.3 Microstrip coupled line and its equivalent capacitor model.</td>
<td>17</td>
</tr>
<tr>
<td>2.4 Even and odd modes of the coupled microstip line with magnetic and electric walls forming about the plane of symmetry.</td>
<td>17</td>
</tr>
<tr>
<td>2.5 A transmission line model for unit length.</td>
<td>18</td>
</tr>
<tr>
<td>2.6 Model of two transmission lines of unit length coupled to each other.</td>
<td>19</td>
</tr>
<tr>
<td>3.1 ADS schematic of the experimental design used for comparison.</td>
<td>25</td>
</tr>
<tr>
<td>3.2 A coupled line consisting of strips $a$ and $b$ with ports 1 and 3 as the input and output, respectively. Strip $a$ has ports 1 and 2, while strip $b$ has ports 3 and 4. Ports 2 and 4 are grounded through $Z_L$.</td>
<td>26</td>
</tr>
</tbody>
</table>
3.3 Model of a coupled line operating in even mode. .................................. 33
3.4 Model of a coupled line operating in odd mode. ................................. 34
3.5 ADS plot of $S_{11}$ and $S_{21}$ vs frequency giving the center frequency as 1.481 GHz. .......................................................... 38
3.6 MATLAB plot of $S_{11}$ and $S_{21}$ vs frequency with the code giving the center frequency as 1.515 GHz. ................................. 38
4.1 ADS schematic of the tunable filter. .......................................................... 40
4.2 MATLAB plot of $S_{11}$ vs frequency for tuning capacitance values of 1 pF and 1.5 pF. ................................................................. 41
4.3 MATLAB plot of $S_{21}$ vs frequency for tuning capacitance values of 1 pF and 1.5 pF. ................................................................. 41
4.4 ADS plot of $S_{11}$ vs frequency for tuning capacitance values of 1 pF and 1.5 pF. 42
4.5 ADS plot of $S_{21}$ vs frequency for tuning capacitance values of 1 pF and 1.5 pF. 42
4.6 $S_{11}$ tuning as the capacitor value is changed from 0.5 pF to 1.2 pF. .... 43
4.7 $S_{21}$ tuning as the capacitor value is changed from 0.5 pF to 1.2 pF. .... 43
4.8 ADS plot of variation of bandwidth with tuning capacitor value. .............. 44
4.9 ADS schematic of the tunable filter. .......................................................... 45
4.10 MATLAB plots of insertion loss and return loss for tuning capacitance values of 10 pF and 22 pF. ......................................................... 45
4.11 Layout of the filter. ........................................................................... 45
4.12 Photograph of the tunable filter. ........................................................... 46
4.13 Comparison of simulated and measured responses of $S_{11}$ vs frequency for tuning capacitance values of 10 pF and 22 pF. ......................... 46
4.14 Comparison of simulated and measured responses of $S_{21}$ vs frequency for tuning capacitance values of 10 pF and 22 pF. ......................... 46
4.15 Combined simulated and measured frequency responses for tuning capacitance values of 10 pF and 22 pF. ................................................ 47
5.1 Simulated response of the edge-coupled filter. ....................................... 49
5.2 Experimental filter network with ideal capacitors connected.  
5.3 ADS schematic of an edge-coupled filter with feedback network.  
5.4 ADS plot of variation of bandwidth as the feedback section width is increased.  
5.5 Two consecutive coupled sections with feedback network.  
5.6 Combined ADS and MATLAB simulation frequency responses for feedback tuning capacitance values of 3 pF.  
5.7 Combined ADS and MATLAB simulation frequency responses for feedback tuning capacitance values of 5 pF.  
5.8 Photograph of the fabricated board.  
5.9 Layout of the filter on ADS Momentum.  
5.10 Simulated and measured $S_{11}$ for tuning capacitance values of 3 pF and 5 pF.  
5.11 Simulated and measured $S_{21}$ for tuning capacitance values of 3 pF and 5 pF.  
5.12 Combined simulated and measured losses for tuning capacitance values of 3 pF and 5 pF.  
5.13 ADS simulation of filter tuning as the feedback capacitor value is varied from 2.7 pF to 5.2 pF in steps of 0.5 pF.  
5.14 Comparison of measured and ADS simulation for bandwidth and center frequency as the feedback capacitor value is increased.  
5.15 ADS simulation plot of variation of the value of minimum $S_{11}$ as feedback section width is varied.  
5.16 ADS simulation plot showing frequency shift as the feedback capacitor is changed from 3 pF to 5 pF as the feedback section width is varied.  
5.17 ADS simulation response of the filter with feedback section width of 0.1 mm.  
5.18 ADS simulation response of variation of center frequency as the feedback section width is increased.  
5.19 ADS simulation plot of group delay versus the frequency in GHz as the capacitance is varied from 2.7 to 5.2 pF in steps of 0.5 pF.
Chapter 1
Introduction

1.1 The Microstrip Transmission Line

A microstrip transmission line consists of a flat strip conductor suspended above a ground plane by a low-loss dielectric material. Figure 1.1 shows a microstrip of width $W$ and a substrate of dielectric constant of $\epsilon_r$ having a height $d$.

1.2 Quasi-TEM Propagation

As shown in fig. 1.2, a TEM wave is a wave which does not have field components in the direction of propagation. Hence, the longitudinal components of electric and magnetic fields, $E_z$ and $H_z$, respectively, are zero if $z$ is the direction of propagation. If we take snapshots of a TEM wave in the $z$ direction, it would appear as if a plane wave is propagating in the forward direction. These TEM waves exist only if we have two conductors since it requires equal amounts of positive and negative charge per unit length on conductors [1].

The curl of the electric field intensity in the transverse direction is zero since it results in the propagation of magnetic field in the forward direction [2] as shown,

$$\nabla_t \times \vec{E} = -j\omega\mu\vec{H}_z = 0,$$  \hspace{1cm} (1.1)

where

- $\vec{E}$ is the electric field component in $x-y$ direction,
- $\vec{H}_z$ is the magnetic field component in the $z$ direction,
- $\nabla_t$ is the transverse gradient operator in $x-y$ plane,
- $\omega$ is the radian frequency.

\[1\] http://home.sandiego.edu/ekim/e194rfs01/mstrip.pdf
\[2\] http://info.ee.surrey.ac.uk/Teaching/Courses/EFT/transmission/html/TEMWave.html
μ is the permeability of the medium.

In the case of microstrip, if we remove the ground by image theory, we get two transmission lines in parallel. Due to the presence of the dielectric, some of the fields are in air and some in the dielectric as shown in fig. 1.3 [3]. There will be different phase velocities of the fields in the two media (this is the inhomogeneity property of the microstrip). This makes it impossible for pure TEM propagation to occur.

Also, due to the abrupt change of the medium when the fields are traveling from air into the dielectric, there is a bend in the electric field lines, giving rise to a transverse curl at the boundary that is non-zero. Therefore, \( \nabla \times \vec{E} \neq 0 \).

A hybrid TE-TM mode of propagation known as the quasi-TEM propagation will exist [2]. These modes can be approximated to TEM by assuming an effective dielectric constant \( \epsilon_{eff} \), which enables us to assume that the microstrip lies in a medium of an overall dielectric constant of \( \epsilon_{eff} \). The value of this \( \epsilon_{eff} \) lies between that of air and the relative dielectric

\[ $E_x$ \] \[ $H_y$ \] \[ direction of propagation \] \[ $z$ \]

Fig. 1.2: An example of TEM propagation.
constant of the substrate.

The formulas for calculating the effective dielectric constant and the characteristic impedance of the microstrip line will be shown later.

1.3 $Z$- and $Y$-parameters

For the two-port network shown in fig. 1.4, the voltages and currents are related to each other by impedances or admittances. The voltages are represented in terms of currents through $Z$-parameters as follows:

\[
V_1 = Z_{11}I_1 + Z_{12}I_2, \quad (1.2)
\]
\[
V_2 = Z_{21}I_1 + Z_{22}I_2. \quad (1.3)
\]
In the form of matrices,

\[
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.
\] (1.4)

The $Z$-matrices are most useful for characterizing systems with networks connected in series. Similarly, the currents are related to voltages through $Y$-parameters as follows,

\[
I_1 = Y_{11}V_1 + Y_{12}V_2,
\] (1.5)

\[
I_2 = Y_{21}V_1 + Y_{22}V_2.
\] (1.6)

In the form of matrices,

\[
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.
\] (1.7)

The $Y$-matrices are most useful for characterizing systems with networks connected in shunt.

1.4 $S$-parameters

Not all networks can be analyzed using $Z$- or $Y$-parameters. They are only good for low frequencies since measuring these parameters directly at high frequencies is difficult. This is mainly due to two reasons:

1) Difficulty in defining voltages and currents at high frequencies for non-TEM transmission lines;

2) Necessity to use open and short circuits in order to find $Z$- and $Y$-parameters, which at microwave frequencies may cause instability when active elements are involved.

Hence, we define parameters closely related to power, which can be easily measured at high frequencies. For a two-port network shown in fig. 1.5, we represent the $S$-parameters
Fig. 1.5: Two-port network characterized by $S$-parameters.

in terms of the forward and reverse traveling waves $a$ and $b$, respectively, with respect to a port.

\[
\begin{align*}
  b_1 &= S_{11}a_1 + S_{12}a_2 \\
  b_2 &= S_{21}a_1 + S_{22}a_2
\end{align*}
\]

(1.8) (1.9)

In the form of matrices,

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}.
\]

(1.10)

$S_{11}$ is the reflection coefficient at port 1 only if $b_2$ is zero. This requires an absence of a source at port 2 along with a perfectly matched load (to avoid reflections coming into port 2). The same applies for $S_{22}$ with $b_1$ absent.

1.5 \textit{ABCD or the Transmission Parameters}

\textit{ABCD} parameters are widely used in systems where two-port networks are connected in cascade. An \textit{ABCD} matrix for each two-port network can be calculated and then multiplied with the other matrices to obtain the overall \textit{ABCD} matrix for the system.

Analysis is done in the forward direction. It is assumed that the current at port 2 is flowing away from the network as shown in fig. 1.6. For a two-port network, the \textit{ABCD}
parameters are defined as shown,

\[ V_1 = AV_2 + BI_2, \]
\[ I_1 = CV_2 + DI_2. \]  

In the form of matrices,

\[
\begin{bmatrix}
  V_1 \\
  I_1
\end{bmatrix} = \begin{bmatrix}
  A & B \\
  C & D
\end{bmatrix} \begin{bmatrix}
  V_2 \\
  I_2
\end{bmatrix}. \tag{1.13}
\]

Consider a system of two two-port networks cascaded as shown in fig. 1.7. Network 1 is characterized by parameters \( A_1, B_1, C_1, \) and \( D_1 \) while two-port network 2 is characterized by parameters \( A_2, B_2, C_2, \) and \( D_2 \). Note that \( I_2 \) is directed away from network 1. Hence,
we can write the equations,

\[
\begin{bmatrix}
V_1 \\
I_1 \\
V_2 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1 \\
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
V_2 \\
I_2 \\
V_3 \\
I_3
\end{bmatrix},
\] (1.14)

\[
\begin{bmatrix}
V_2 \\
I_2 \\
V_3 \\
I_3
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1 \\
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
V_3 \\
I_3 \\
V_4 \\
I_4
\end{bmatrix}.
\] (1.15)

From (1.14) and (1.15), we get

\[
\begin{bmatrix}
V_1 \\
I_1 \\
V_2 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1 \\
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1 \\
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
V_3 \\
I_3 \\
V_4 \\
I_4
\end{bmatrix},
\] (1.16)

which is of the form

\[
\begin{bmatrix}
V_1 \\
I_1 \\
V_3 \\
I_3
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_3 \\
I_3
\end{bmatrix}.
\] (1.17)

Hence, the whole system collapses to a two-port network. These \(ABCD\) parameters can be converted to \(Z\)-, \(Y\)-, and \(S\)-parameters for further analyses.

1.6 Analysis of Periodic Structures

Consider a network of \(N+1\) periodic sections cascaded as shown in fig. 1.8. From (1.14), we can write the \(ABCD\) matrix for the \(N^{th}\) section as follows,

\[
\begin{bmatrix}
V_N \\
I_N
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_{N+1} \\
I_{N+1}
\end{bmatrix}.
\] (1.18)

If we assume that the wave is traveling in the forward direction, the \(N+1\) terms are related to the preceding terms by a propagation factor \(e^{-kd}\), where \(k\) is the propagation
constant and \( d \) is the length of each section. Hence,

\[
V_{N+1} = V_N e^{-kd}, \quad (1.19)
\]
\[
I_{N+1} = I_N e^{-kd}, \quad (1.20)
\]

where \( k = \alpha + j\beta \) is the complex propagation constant to be determined. \( \alpha \) is the attenuation constant and \( \beta \) is the lossless propagation constant.

Using this in (1.18), we get

\[
\begin{bmatrix}
V_N \\
I_N
\end{bmatrix}
= 
\begin{bmatrix}
V_{N+1} e^{kd} \\
I_{N+1} e^{kd}
\end{bmatrix}
= 
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_{N+1} \\
I_{N+1}
\end{bmatrix},
\quad (1.21)
\]

From (1.21), we get

\[
\begin{bmatrix}
A - e^{kd} \\
C
\end{bmatrix}
\begin{bmatrix}
B \\
D - e^{kd}
\end{bmatrix}
\begin{bmatrix}
V_{N+1} \\
I_{N+1}
\end{bmatrix}
= 0. \quad (1.22)
\]

For a non-trivial solution, determinant of the matrix should be zero. Hence,

\[
AD - Ae^{kd} - De^{kd} + e^{2kd} - BC = 0. \quad (1.23)
\]

For a reciprocal circuit, \( AD - BC = 1 \). Applying this condition, we get

\[
\frac{A + D}{2} = \frac{e^{-kd} + e^{kd}}{2}. \quad (1.24)
\]
For a symmetrical network, $A = D$. Simplifying for this case,

$$A = \frac{e^{-kd} + e^{kd}}{2}. \quad (1.25)$$

- If $|A| < 1$, the solution for $k$ is purely imaginary. From (1.25), for this case, $A = \cos(kd)$. This implies that the wave can pass through the network without any attenuation. This band of frequencies constitutes the passband of the filter.

- If $|A| > 1$, the solution for $k$ is purely real. From (1.25), for this case, $A = \cosh(kd)$. This implies that the wave undergoes attenuation since the wave is exponentially decaying. This band of frequencies corresponds to the stopband of the filter.

1.7 Design of Third Order Chebyshev Bandpass Edge-Coupled Filter

1.7.1 Filter Design Using Coupled Sections

For an $N^{th}$ order Chebyshev filter, the transfer function is given as

$$\left|H(s)\right|^2 = \frac{H_0}{1 + \epsilon^2 C_N^2(w/w_c)}, \quad (1.26)$$

where $H_0$ is the dc attenuation, $\epsilon$ is the ripple magnitude, $w_c$ is the 3-dB corner frequency, and the Chebyshev polynomial is given by

$$C_N(w) = \cos \left[ N \cos^{-1}(w) \right]. \quad (1.27)$$

The poles of the Chebyshev transfer function lie on an ellipse [4, 5] and are given as

$$s_k = w_c(\sigma_k + j w_k), \quad (1.28)$$
where $k=1,2,...,2N$ for an $N^{th}$ order filter. $\sigma_k$ and $w_k$ are given as

\begin{align*}
\sigma_k &= -\sinh(a) \sin \left[ \frac{(2k-1)\pi}{2N} \right], \\
w_k &= \cosh(a) \cos \left[ \frac{(2k-1)\pi}{2N} \right], \\
a &= \frac{1}{N} \sinh^{-1} \left( \frac{1}{\epsilon} \right).
\end{align*}

The component values for a Chebyshev lowpass prototype are determined using the following equations:

\begin{align*}
g_0 &= 1, \\
g_1 &= \frac{2a_1}{\gamma}, \\
g_k &= \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}, k = 2,3,...,N, \\
g_{N+1} &= 1, \text{ for } N \text{ odd}, \\
g_{N+1} &= \coth^2(\beta/4), \text{ for } N \text{ even},
\end{align*}

where

\begin{align*}
a_k &= \sin \left[ \frac{(2k-1)\pi}{2N} \right], k = 1,2,...,N, \\
b_k &= \gamma^2 + \sin^2 \left( \frac{k\pi}{N} \right), k = 1,2,...,N, \\
\beta &= \ln \left[ \coth \left( \frac{A}{17.372} \right) \right], \\
\gamma &= \sinh \left( \frac{\beta}{2N} \right),
\end{align*}

where $A$ is the passband ripple in decibels given by

\begin{equation}
A = 10 \log(1 + \epsilon^2).
\end{equation}

Hence, the information required while designing a filter is the order of the filter, the
ripple factor in the passband, and the center frequency. The bandwidth is required while designing bandpass filters. We refer to the filter tables given in D.M. Pozar and G. L. Matthaei [2, 6] to find the following coefficients for a third order Chebyshev filter.

\[
\begin{align*}
g_1 &= 1.5963 \\
g_2 &= 1.0967 \\
g_3 &= 1.5963 \\
g_4 &= 1.000
\end{align*}
\]

These values are for a lowpass prototype design with source and load impedances equal to unity. The coefficients are nothing but inductances and capacitances of a lowpass filter ladder network as shown in fig. 1.9.

For filter with 50Ω impedances at the source and load, the values of inductances and capacitances need to be scaled. The design is then transformed to a bandpass version [7] by converting \( s \) to

\[
s_{bp} = \frac{s^2 + w_0^2}{sBW}. \tag{1.42}
\]

\( s_{bp} \) is the converted complex angular frequency of the bandpass filter, \( BW \) is the filter bandwidth, and \( w_0 = 2\pi f_0 \), where \( f_0 \) is the center frequency. Hence, the transfer function

Fig. 1.9: A ladder network for a third order lowpass Chebyshev filter prototype beginning with a shunt element.
of this bandpass filter $H_{bp}$ is

$$H_{bp}(s_{bp}) = H \left( \frac{s^2 + w_0^2}{sBW} \right).$$

(1.43)

The fractional bandwidth of the filter given by

$$\Delta = \frac{f_2 - f_1}{f_0},$$

(1.44)

where

$f_1$ is the lower cut-off frequency,

$f_2$ is the higher cut-off frequency.

Hence, for a center frequency of 1.5 GHz and the higher and lower cut-off frequencies at 1.6 GHz and 1.4 GHz, respectively, the fractional bandwidth is 0.133 or 13%.

Using the concept of admittance inverters, for an $N^{th}$ order filter, the admittance inverter parameters given by Pozar [2] are reproduced as follows:

for $n = 1$,

$$Z_0J_1 = \sqrt{\frac{\pi\Delta}{2g_1}};$$

(1.45)

for $n = 2, 3, \ldots, N$,

$$Z_0J_n = \frac{\pi\Delta}{2\sqrt{g_{n-1}g_n}};$$

(1.46)

and for $n = N+1$,

$$Z_0J_{N+1} = \sqrt{\frac{\pi\Delta}{2g_Ng_{N+1}}},$$

(1.47)

where $J_n$ is the admittance inverter constant for the $n^{th}$ section and $Z_0$ is the characteristic impedance of the filter.

Now, the even and odd mode impedances of the coupled line $Z_{0e}$ and $Z_{0o}$, respectively,
are computed using (1.45)-(1.47) as follows:

\[
Z_{\text{oe}} = Z_0[1 + J_n Z_0 + (J_n Z_0)^2],
\]
\[
Z_{\text{oo}} = Z_0[1 - J_n Z_0 + (J_n Z_0)^2],
\]

for \( n = 1, 2, ..., N+1 \). From these values of \( Z_{\text{oe}} \) and \( Z_{\text{oo}} \), the widths of the coupled microstrip line are calculated for each section. The lengths are calculated from the frequency and effective dielectric constant information.

### 1.7.2 The Quarter-wave Transformer

The input impedance of a filter may not be 50\( \Omega \) at the desired center frequency. If it is terminated into 50\( \Omega \) transmission lines at either ends, there would be a lot of reflection, deteriorating the receiver performance. To match a filter to 50\( \Omega \) termination impedances, a microstrip line of quarter wavelength or an electrical length of 90° is used. Consider a quarter-wave transformer having a characteristic impedance of \( Z_0 \) terminated in load impedance \( Z_L \), which is the input impedance of the filter. The input impedance of the overall network \( Z_{\text{in}} \) is then given by

\[
Z_{\text{in}} = \frac{Z_L \cos(\beta l) + j Z_0 \sin(\beta l)}{Z_0 \cos(\beta l) + j Z_L \sin(\beta l)}.
\]

Since the length of the transformer is \( \lambda/4 \), substituting \( \beta l = \pi/2 \), we get

\[
Z_{\text{in}} = Z_0 \frac{Z_0}{Z_L}.
\]

Hence,

\[
Z_0 = \sqrt{Z_{\text{in}} Z_L},
\]

which is the characteristic impedance of the matching transformer. The microstrip line of this impedance can be designed using the information about the properties of the board.
1.7.3 Simulation Results

A program was written in MATLAB for designing a third order Chebyshev microstrip edge-coupled filter with a fractional bandwidth of 1% and a center frequency of 2.5 GHz. The results are shown in fig. 1.10.

Hence, a distributed filter can be designed using multiple couplers. This approach of $J$-inverters was not adopted since it operates only in the frequencies of interest, i.e. the admittance would be the reciprocal of the load admittance only at the desired frequency.
Chapter 2

Even and Odd Modes in a Coupled Transmission Line

2.1 Introduction

Two signals traveling along different transmission lines next to each other can be related to each other by assuming that both waves have a component that is common to both and a component that is different from each other. We call the common component to both as even and the differential component as odd.

Now, consider two parallel strips, one at voltage \( V_1 \) and other at \( V_2 \). We can call the common voltage between the two \( V_e \), corresponding to the even mode. Alternately, the differential voltage is \( V_o \), corresponding to the odd mode. As discussed earlier,

\[
V_1 = V_e + V_o, \quad (2.1)
\]
\[
V_2 = V_e - V_o. \quad (2.2)
\]

Hence, the even and odd mode voltages are given by

\[
V_e = \frac{V_1 + V_2}{2}, \quad (2.3)
\]
\[
V_o = \frac{V_1 - V_2}{2}. \quad (2.4)
\]

A similar analysis can be applied to coupled microstrip transmission lines. This implies that we have even and odd mode characteristic impedances, \( Z_{oe} \) and \( Z_{oo} \), respectively. Also, due to the inhomogeneity of the microstrip, we have different corresponding velocities of propagation, \( v_{phe} \) and \( v_{pho} \).

The electric and magnetic fields around a microstrip in even and odd mode are shown in fig. 2.1 and fig. 2.2, respectively [8].
2.2 The Capacitance Model

When we apply a voltage on one of the strips of a coupled microstrip line, there will be a charge distribution on the metal. This gives rise to a parallel plate capacitance w.r.t. ground. Also, if there exists a differential voltage between the two strips, a capacitance will be formed in between the two plates, known as the mutual capacitance. Figure 2.3 shows a microstrip coupled line and its equivalent capacitor network, where $C_{11}$ and $C_{22}$ are the parallel plate capacitance w.r.t. ground while $C_{12}$ is the mutual capacitance between the two strips. For identical width strips, $C_{11} = C_{22}$.

Figure 2.4 shows the even and odd modes of the coupled microstrip line. A magnetic wall is created along the axis of symmetry in even mode of operation. Similarly, an electric wall is created along the axis of symmetry in odd mode of operation.

This model holds true only if we consider a TEM propagation. From these values of capacitances and the velocity of propagation along the line, we can obtain the parameters like the even and odd mode effective dielectric constant and characteristic impedance [2],
2.3 Telegrapher Equations Describing a Microwave Transmission Line

Any distributed transmission line like a microstrip or a stripline can be represented as a combination of an infinite number of lumped components. Considering the lossless case, i.e. neglecting the series resistance per unit length ($R$) and the shunt conductance per unit length ($G$), we represent the line with series inductances per unit length ($L$) and shunt capacitances per unit length ($C$).

As shown in the fig. 2.5, \(^1\) we can see that the transmission line is made up of inductances and capacitances per unit length. We begin with the Kirchoff’s voltage law for the given line. The series inductance of the line would be $L \Delta z$ and the shunt capacitance

\(^1\)http://cnx.org/content/m1044/latest/
would be $C \Delta z$. We write

$$V(z,t) - L(\Delta z) \frac{dI}{dt}(z,t) = V(z + \Delta z,t), \quad (2.5)$$

$$\frac{V(z + \Delta z,t) - V(z,t)}{\Delta z} = -L \frac{dI}{dt}(z,t). \quad (2.6)$$

Considering that the line is made up of distributed components, we take the limit in (2.6) as $\Delta z \to 0$. This gives us the equation for voltage as

$$\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}. \quad (2.7)$$

The negative sign here conforms to the Lenz’s law indicating that any change in current is opposed. Similarly from the Kirchoff’s current law at node 1,

$$I(z,t) - C \Delta z \frac{\partial V}{\partial t}(z + \Delta z,t) = I(z + \Delta z,t), \quad (2.8)$$

$$\frac{I(z + \Delta z,t) - I(z,t)}{\Delta z} = -C \frac{\partial V}{\partial t}(z + \Delta z,t). \quad (2.9)$$

Taking the limit in (2.9) as $\Delta z \to 0$,

$$\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t}, \quad (2.10)$$
Fig. 2.6: Model of two transmission lines of unit length coupled to each other.

which is the equation for the current.

2.4 The Case of Coupled Lines

If we have two coupled lines, as shown in fig. 2.6, there will be mutual inductances and capacitances induced. The values of these capacitances and inductances can be given by an electromagnetic analysis of the coupled line. The telegrapher equations from (2.7) and (2.10) for two lines without the coupling being taken into account are given as

\[
\frac{\partial V_1}{\partial z} = -L_1 \frac{\partial I_1}{\partial t}, \quad (2.11)
\]

\[
\frac{\partial I_1}{\partial z} = -C_1 \frac{\partial V_1}{\partial t}, \quad (2.12)
\]

\[
\frac{\partial V_2}{\partial z} = -L_2 \frac{\partial I_2}{\partial t}, \quad (2.13)
\]

\[
\frac{\partial I_2}{\partial z} = -C_2 \frac{\partial V_2}{\partial t}. \quad (2.14)
\]
The voltage across the mutual inductance in the first transmission line is $L_m \frac{dI_2}{dt}$. Hence, the voltage equation becomes

$$V_1(z, t) - L_1(\Delta z) \frac{dI_1}{dt}(z, t) - L_m(\Delta z) \frac{dI_2}{dt}(z, t) = V_1(z + \Delta z, t),$$

(2.15)

$$\frac{V(z + \Delta z, t) - V(z, t)}{\Delta z} = -L_1 \frac{dI_1}{dt}(z, t) - L_m \frac{dI_2}{dt}(z, t).$$

(2.16)

Considering that the line is made up of distributed components, we take the limit in (2.16) as $\Delta z \to 0$. Thus,

$$\frac{\partial V_1}{\partial z} = -L_1 \frac{\partial I_1}{\partial t} - L_m \frac{\partial I_2}{\partial t}. \quad (2.17)$$

Similarly, for the second transmission line,

$$\frac{\partial V_2}{\partial z} = -L_2 \frac{\partial I_2}{\partial t} - L_m \frac{\partial I_1}{\partial t}. \quad (2.18)$$

In matrix form,

$$\begin{bmatrix} \frac{\partial V_1}{\partial z} \\ \frac{\partial V_2}{\partial z} \end{bmatrix} = - \begin{bmatrix} L_1 & L_m \\ L_m & L_2 \end{bmatrix} \begin{bmatrix} \frac{\partial I_1}{\partial t} \\ \frac{\partial I_2}{\partial t} \end{bmatrix}. \quad (2.19)$$

The voltage across the mutual capacitance $C_m$ is $V_1(z + \Delta z, t) - V_2(z + \Delta z, t)$. From the Kirchoff's current law at port 1, we get

$$I_1(z, t) - C_1 \Delta z \frac{\partial V_1}{\partial t}(z + \Delta z, t) - C_m \Delta z(\frac{\partial V_1}{\partial t}(z + \Delta z, t) - \frac{\partial V_2}{\partial t}(z + \Delta z, t)) = I_1(z + \Delta z, t).$$

(2.20)

Taking the limits in (2.20) as $\Delta z \to 0$, we get

$$\frac{\partial I_1}{\partial z} = -(C_1 + C_m) \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_2}{\partial t}. \quad (2.21)$$
Similarly for the second transmission line,

\[
\frac{\partial I_2}{\partial z} = -(C_2 + C_m) \frac{\partial V_2}{\partial t} + C_m \frac{\partial V_1}{\partial t}.
\] (2.22)

In matrix form,

\[
\begin{bmatrix}
\frac{\partial I_1}{\partial z} \\
\frac{\partial I_2}{\partial z}
\end{bmatrix} = - \begin{bmatrix}
C_1 + C_m & -C_m \\
-C_m & C_2 + C_m
\end{bmatrix} \begin{bmatrix}
\frac{\partial V_1}{\partial t} \\
\frac{\partial V_2}{\partial t}
\end{bmatrix}.
\] (2.23)

Writing the equations in the phasor form, we make the equations for voltages and currents time invariant. Hence, (2.17), (2.18), (2.21), (2.22) become of the form,

\[
d \frac{V_1}{dz} = -j \omega L_1 I_1 - j \omega L_m I_2,
\] (2.24)

\[
d \frac{V_2}{dz} = -j \omega L_m I_1 - j \omega L_2 I_2,
\] (2.25)

\[
d \frac{I_1}{dz} = -j \omega (C_1 + C_m) V_1 + j \omega C_m V_2,
\] (2.26)

\[
d \frac{I_2}{dz} = j \omega C_m V_1 - j \omega (C_2 + C_m) V_2.
\] (2.27)

Differentiating (2.24) and (2.25) w.r.t. \(z\)

\[
\frac{d^2 V_1}{dz^2} = -j \omega L_1 \frac{dI_1}{dz} - j \omega L_m \frac{dI_2}{dz},
\] (2.28)

\[
\frac{d^2 V_2}{dz^2} = -j \omega L_m \frac{dI_1}{dz} - j \omega L_2 \frac{dI_2}{dz}.
\] (2.29)

Substituting (2.26), (2.27) in (2.28), we get an equation for \(V_1\) of the form

\[
\frac{d^2 V_1}{dz^2} - a_1 V_1 - b_1 V_2 = 0.
\] (2.30)

Substituting (2.26), (2.27) in (2.29), we get an equation for \(V_2\) of the form

\[
\frac{d^2 V_2}{dz^2} - a_2 V_1 - b_2 V_2 = 0.
\] (2.31)
From (2.30), substituting for \( V_2 \) in (2.31), we get a fourth order differential equation for \( V_1 \) of the form

\[
\frac{d^4 V_1}{dz^4} - X \frac{d^2 V_1}{dz^2} - Y V_1 = 0. \tag{2.32}
\]

Similarly, from (2.31), substituting for \( V_1 \) in (2.30), we get a fourth order differential equation for \( V_2 \) of the form

\[
\frac{d^4 V_2}{dz^4} - P \frac{d^2 V_2}{dz^2} - Q V_2 = 0. \tag{2.33}
\]

If we assume, \( V_1 = V_0 e^{jkz} \) or \( V_2 = V_0 e^{jkz} \) where \( k \) is the propagation constant through the medium, substituting, we get four modes as \( k \) can take four values [8, 9, 10]. Two solutions are given for a mode called the c-mode and the two other solutions are given for a mode called the \( \pi \)-mode. The propagation constant \( k_c \) for the c-mode is given by

\[
k_c = \pm \sqrt{\frac{a_1 + a_2}{2} + \frac{1}{2}[(a_1 - a_2)^2 + 4b_1b_2]} \tag{2.34}
\]

where

\[
a_1 = (j\omega C_1)(j\omega L_1) + (-j\omega C_m)(j\omega L_m), \\
a_2 = (j\omega C_2)(j\omega L_2) + (-j\omega C_m)(j\omega L_m), \\
b_1 = (j\omega L_1)(-j\omega C_m) + (j\omega C_2)(j\omega L_m), \\
b_2 = (j\omega L_2)(-j\omega C_m) + (j\omega C_1)(j\omega L_m).
\]

Similarly, the other two solutions are given for the \( \pi \)-mode and the propagation constant is \( k_\pi \).

\[
k_\pi = \pm \sqrt{\frac{a_1 + a_2}{2} - \frac{1}{2}[(a_1 - a_2)^2 + 4b_1b_2]} \tag{2.35}
\]

If the strips are of equal widths, we have symmetric coupling. Hence, \( L_1 = L_2 \) and \( C_1 = C_2 \).
This makes the $c$ and $\pi$ modes reduce to even and odd modes, respectively [8]. Hence, the odd and even mode propagation constants are given as

\[
\begin{align*}
  k_e &= j\omega \sqrt{(L_1 + L_m)(C_1 - C_m)}, \\
  k_o &= j\omega \sqrt{(L_1 - L_m)(C_1 + C_m)},
\end{align*}
\]

where $k_e$ and $k_o$ are the odd and even mode propagation constants. The sign of the propagation constant indicates the direction of wave travel, i.e. if the value is negative, the convention is to consider the forward direction of propagation and vice versa.
Chapter 3

$N^{th}$ Order Edge-Coupled Filter Analysis

3.1 Introduction

The aim of this part of the study was to analyze the edge-coupled filter and obtain an equivalent analytical solution using the transmission line theory approach. The matrix manipulations were done and the results were plotted with the help of MATLAB. The results were then compared with Agilent ADS simulations for the same structure. A single microstrip was analyzed and then used for coupled line sections by making some approximations. A $Z$-matrix was then obtained for each coupled section and converted to the $ABCD$ equivalents. These $ABCD$ matrices were multiplied to obtain the overall transmission matrix.

Most of the times, an engineer ends up designing filters on a trial and error basis by altering parameters in simulation software like ADS. This MATLAB code allows us to understand the working of our edge-coupled filter better. We can work with the code with different types of loading and coupling combinations. Also, we can further modify the code to analyze non-trivial circuits by doing some additional matrix manipulations.

The MATLAB code was used to check designs and obtain approximate solutions. The designs were then optimized using more accurate simulation software like ADS.

3.2 Experimental Design in ADS

A third order filter was used to compare the code results with ADS results. Four identical coupled sections having an equal Q were put together as shown in fig. 3.1 and simulated. At around 1.5 GHz, the skin depth will be lower than 1 µm [3]. Hence, the conductor losses are ignored while comparing to the MATLAB code.

The dielectric attenuation coefficient $\alpha_d$ in dB/microstrip wavelength is given by T. C.
Edwards, Gupta et al., and Hammerstad et al., [3, 11, 12] as

$$\alpha_d = 27.3 \frac{\epsilon_r (\epsilon_{eff} - 1) \tan \delta}{\epsilon_{eff} (\epsilon_r - 1)},$$

(3.1)

where

- $\epsilon_r$ is the dielectric constant of the substrate,
- $\epsilon_{eff}$ is the effective dielectric constant,
- $\tan \delta$ is the loss tangent.

For RO4003C substrate from Roger’s Corporation, the loss tangent is 0.002 while the $\epsilon_r$ is taken to be 3.38. For this case, $\alpha_d$ turns out to be only 0.042 dB/microstrip wavelength.

Hence, the effect of the dielectric loss is neglected while comparing to the MATLAB code.

### 3.3 Analysis of a Single Coupled Section

For a coupled line of length $l$, we analyze each strip separately by considering the even and odd modes of propagation. We consider a single coupled section for our analysis. Port 1 and port 3 are the input and output, respectively, while ports 2 and 4 are loaded into impedances $Z_L$ as shown in fig. 3.2.
Fig. 3.2: A coupled line consisting of strips \(a\) and \(b\) with ports 1 and 3 as the input and output, respectively. Strip \(a\) has ports 1 and 2, while strip \(b\) has ports 3 and 4. Ports 2 and 4 are grounded through \(Z_L\).

Ports 1 and 2 lie on the first strip, and ports 3 and 4 lie on the other. For the first strip \(a\), we have the voltage and current equations along the \(z\)-direction as

\[
V_a(z) = V_{ae}(z) + V_{ao}(z), \\ I_a(z) = I_{ae}(z) + I_{ao}(z),
\]

where

\[
V_{ae}(z) = V_{ae}^+ e^{-jkz} + V_{ae}^- e^{jkz}, \\ V_{ao}(z) = V_{ao}^+ e^{-jkz} + V_{ao}^- e^{jkz}, \\ I_{ae}(z) = \frac{V_{ae}^+}{Z_{oe}} e^{-jkz} - \frac{V_{ae}^-}{Z_{oe}} e^{jkz}, \\ I_{ao}(z) = \frac{V_{ao}^+}{Z_{oo}} e^{-jkz} - \frac{V_{ao}^-}{Z_{oo}} e^{jkz}.
\]
Similarly for the other strip $b$

\begin{align*}
V_b(z) &= V_{be} + V_{bo}, \\
I_b(z) &= I_{be} - I_{bo},
\end{align*}

where

\begin{align*}
V_{be} &= V_{be}^+ e^{-jkz} + V_{be}^- e^{jkz}, \\
V_{bo} &= V_{bo}^+ e^{-jkz} + V_{bo}^- e^{jkz}, \\
I_{be} &= \frac{V_{be}^+}{Z_{oe}} e^{-jkz} - \frac{V_{be}^-}{Z_{oe}} e^{jkz}, \\
I_{bo} &= \frac{V_{bo}^+}{Z_{oo}} e^{-jkz} - \frac{V_{bo}^-}{Z_{oo}} e^{jkz}.
\end{align*}

We introduce the notation

\begin{align*}
\alpha &= e^{-jkl}, \\
\beta &= e^{jkl}.
\end{align*}
At port 1 and 4, \( z = 0 \). While at port 2 and port 3, \( z = l \). Using (3.14) and (3.15) in (3.4)-(3.7) and (3.10)-(3.13), we obtain

\[
\begin{align*}
V_1 &= V_e^+ + V_e^- - V_o^+ - V_o^-, \\
I_1 &= \frac{V_e^+}{Z_{oe}} - \frac{V_e^-}{Z_{oe}} - \frac{V_o^+}{Z_{oo}} + \frac{V_o^-}{Z_{oo}}, \\
V_2 &= V_e^+ \alpha + V_e^- \beta - V_o^+ \alpha - V_o^- \beta, \\
I_2 &= \frac{V_e^+}{Z_{oe}} \alpha - \frac{V_e^-}{Z_{oe}} \beta - \frac{V_o^+}{Z_{oo}} \alpha + \frac{V_o^-}{Z_{oo}} \beta, \\
V_3 &= V_e^+ \alpha + V_e^- \beta + V_o^+ \alpha + V_o^- \beta, \\
I_3 &= \frac{V_e^+}{Z_{oe}} \alpha - \frac{V_e^-}{Z_{oe}} \beta + \frac{V_o^+}{Z_{oo}} \alpha - \frac{V_o^-}{Z_{oo}} \beta, \\
V_4 &= V_e^+ + V_e^- + V_o^+ + V_o^-,
\end{align*}
\]

\[
\begin{align*}
I_4 &= \frac{V_e^+}{Z_{oe}} - \frac{V_e^-}{Z_{oe}} + \frac{V_o^+}{Z_{oo}} - \frac{V_o^-}{Z_{oo}}.
\end{align*}
\]

Rewriting this in matrix form, we get

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & -1 & -1 \\
\alpha & \beta & -\alpha & -\beta \\
\alpha & \beta & \alpha & \beta \\
1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
V_e^+ \\
V_e^- \\
V_o^+ \\
V_o^-
\end{bmatrix},
\]

(3.24)

and

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4
\end{bmatrix} =
\begin{bmatrix}
1/Z_{oe} & -1/Z_{oe} & -1/Z_{oo} & 1/Z_{oo} \\
\alpha/Z_{oe} & -\beta/Z_{oe} & -\alpha/Z_{oo} & \beta/Z_{oo} \\
\alpha/Z_{oe} & -\beta/Z_{oe} & \alpha/Z_{oo} & -\beta/Z_{oo} \\
1/Z_{oe} & -1/Z_{oe} & 1/Z_{oo} & -1/Z_{oo}
\end{bmatrix}
\begin{bmatrix}
V_e^+ \\
V_e^- \\
V_o^+ \\
V_o^-
\end{bmatrix}.
\]

(3.25)
Assuming that

\[
A = \begin{bmatrix}
1 & 1 & -1 & -1 \\
\alpha & \beta & -\alpha & -\beta \\
\alpha & \beta & \alpha & \beta \\
1 & 1 & 1 & 1
\end{bmatrix},
\]

(3.26)

and

\[
B = \begin{bmatrix}
1/Z_{oe} & -1/Z_{oe} & -1/Z_{oo} & 1/Z_{oo} \\
\alpha/Z_{oe} & -\beta/Z_{oe} & -\alpha/Z_{oo} & \beta/Z_{oo} \\
\alpha/Z_{oe} & -\beta/Z_{oe} & \alpha/Z_{oo} & -\beta/Z_{oo} \\
1/Z_{oe} & -1/Z_{oe} & 1/Z_{oo} & -1/Z_{oo}
\end{bmatrix}.
\]

(3.27)

Substituting (3.26), (3.27) into (3.24), (3.25), we obtain in matrix form

\[
\begin{bmatrix}
V_e^+ \\
V_e^- \\
V_o^+ \\
V_o^-
\end{bmatrix}
= A^{-1}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}.
\]

(3.28)

Substituting (3.28) in (3.25), we get

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4
\end{bmatrix}
= BA^{-1}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}.
\]

(3.29)
Using (3.29) we obtain a relationship between voltage and current as

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} & Z_{14} \\
Z_{21} & Z_{22} & Z_{23} & Z_{24} \\
Z_{31} & Z_{32} & Z_{33} & Z_{34} \\
Z_{41} & Z_{42} & Z_{43} & Z_{44}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4
\end{bmatrix}.
\]  (3.30)

Let us assume that the coupler section is loaded at port 2 and port 4 into $Z_L$. To be more specific, if the coupler section is loaded at port 2 and port 4 into load impedances $Z_{L_1}$ and $Z_{L_2}$, $I_2 = Y_{L_1}V_2$ and $I_4 = Y_{L_2}V_4$.

\[
\begin{align*}
V_2 &= Z_{L_1}I_2 \quad (3.31) \\
V_4 &= Z_{L_2}I_4 \quad (3.32)
\end{align*}
\]

Also, the admittance can be given as

\[
Y_{L_1} = \frac{1}{Z_{L_1}}, \quad (3.33)
\]

\[
Y_{L_2} = \frac{1}{Z_{L_2}}. \quad (3.34)
\]

Using (3.33) and (3.34) in (3.30), we can express $V_2$ and $V_4$ in terms of $I_1$ and $I_3$ as follows:

\[
\begin{bmatrix}
V_2 \\
V_4
\end{bmatrix} =
\begin{bmatrix}
(1 - Z_{22}Y_{L_1}) & -Z_{22}Y_{L_2} \\
-Z_{42}Y_{L_1} & (1 - Z_{44}Y_{L_2})
\end{bmatrix}
\begin{bmatrix}
Z_{21} & Z_{23} \\
Z_{41} & Z_{43}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_3
\end{bmatrix}.
\]  (3.35)

We also have $V_1$ and $V_3$ as

\[
\begin{bmatrix}
V_1 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{13} \\
Z_{31} & Z_{33}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_3
\end{bmatrix} +
\begin{bmatrix}
Z_{12}Y_{L_1} & Z_{14}Y_{L_2} \\
Z_{32}Y_{L_1} & Z_{34}Y_{L_2}
\end{bmatrix}
\begin{bmatrix}
V_2 \\
V_4
\end{bmatrix}.
\]  (3.36)
Substituting (3.35) in (3.36), we obtain the $V_1$ and $V_3$ in terms of $I_1$ and $I_3$, giving us the transmission matrix for the single coupled section. This is of the form

$$
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= 
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_3 \\
I_3
\end{bmatrix}.
$$

(3.37)

This $ABCD$ matrix gives the transmission parameters for the input and output of a single coupled section. Similar analysis is made for the other cascaded coupled sections. This analysis can be extended to a filter having $N$ coupled sections as

$$
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= 
\begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix}
\begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
......
\begin{bmatrix}
A_N & B_N \\
C_N & D_N
\end{bmatrix}
\begin{bmatrix}
V_{N+1} \\
I_{N+1}
\end{bmatrix},
$$

(3.38)

where $ABCD_1$, $ABCD_2$, ..., $ABCD_N$ are the transmission matrices of the $N$ coupled sections.

### 3.4 Calculation of Even and Odd Mode Characteristic Impedances

For a TEM transmission line, the characteristic impedance of a lossless transmission line can be written as

$$
Z_0 = \sqrt{\frac{L}{C}},
$$

(3.39)

where $L$ and $C$ are the inductance per unit length and capacitance per unit length. Now, with air as the dielectric, the inductance remains unaffected, while the capacitance changes to $C_{air}$. The phase velocity is to the velocity of the wave in free space which is $c$. Hence,

$$
Z_{0air} = cL,
$$

(3.40)

$$
Z_{0air} = \frac{1}{cC_{air}}.
$$

(3.41)
From equations (3.39), (3.40), and (3.41) we get

\[ Z_0 = \frac{1}{c \sqrt{CC_{\text{air}}}}. \]  

(3.42)

Therefore, the even and odd mode characteristic impedances are related to the even and odd mode capacitances \( C_e \) and \( C_o \) as

\[ Z_{0e} = \frac{1}{c \sqrt{C_e C_{\text{eair}}}}, \]  

(3.43)

\[ Z_{0o} = \frac{1}{c \sqrt{C_o C_{\text{oair}}}}. \]  

(3.44)

3.5 Calculation of the Even and Odd Mode Effective Dielectric Constant

The effective dielectric constants for even and odd mode can be found out by taking the ratios of capacitances with respect to the capacitances per unit length in air.

\[ \epsilon_{\text{eff}} = \frac{C_e}{C_{\text{eair}}} \]  

(3.45)

\[ \epsilon_{\text{eff}} = \frac{C_o}{C_{\text{oair}}} \]  

(3.46)

3.6 \textit{ABCD} Matrix of Matching Sections

The phase velocity is given by

\[ v_{\text{ph}} = \frac{c}{\sqrt{\epsilon_{\text{eff}}}}, \]  

(3.47)

where \( \epsilon_{\text{eff}} \) is the effective dielectric constant. The propagation constant \( k_0 \) can be calculated as

\[ k_0 = \frac{2\pi f}{v_{\text{ph}}}, \]  

(3.48)
where $f$ is the frequency of operation. The $ABCD$ matrix $T$ is obtained for the line length $L$ given by Pozar [2],

$$
T = \begin{bmatrix}
\cos(\sqrt{\epsilon_{eff}}k_0 L) & jZ_0 \sin(\sqrt{\epsilon_{eff}}k_0 L) \\
-\sin(\sqrt{\epsilon_{eff}}k_0 L)/Z_0 & \cos(\sqrt{\epsilon_{eff}}k_0 L)
\end{bmatrix}.
$$

(3.49)

### 3.7 The Coupled Line Model for Equal Width Strips

As discussed earlier, the coupled line per unit length can be modeled by a system of three capacitors for each microstrip line. The analysis can be done separately for even and odd modes by taking into account the effect of the electric and magnetic walls. The even and odd mode models are shown in fig. 3.3 and fig. 3.4, respectively [3].

The expressions for the values of these capacitances are given by T.C. Edwards [3] after taking the fringing fields into account. The parallel plate capacitance $C_p$ and the fringing capacitance $C_f$ expressions are reproduced here as follows:

$$
C_p = \epsilon_r \epsilon_0 \frac{W}{d},
$$

(3.50)

$$
C_f = \sqrt{\epsilon_{eff}} \frac{2}{2(cZ_0)} - \frac{C_p}{2},
$$

(3.51)

where $c$ is the speed of light through vacuum and $Z_0$ is the characteristic impedance of the microstrip line.

A magnetic wall gets created at the line of symmetry (the coupled lines are of equal

![Fig. 3.3: Model of a coupled line operating in even mode.](image)
widths) when even mode fields couple across. The fringing capacitance $C'_f$ with respect to the magnetic wall is given by T.C. Edwards [3] as

$$C'_f = \frac{C_f}{1 + A(d/s) \tanh(8s/d)} \sqrt{\frac{\varepsilon_r}{\varepsilon_{eff}}},$$  \hspace{1cm} (3.52)$$

where

$$A = \exp\{-0.1 \exp(2.33 - 2.53(W/d))\}.$$ 

Now, we calculate the capacitances for the odd mode analysis. Let

$$k = \frac{s/d}{(s/d + 2(W/d))}.$$ 

If $0 \leq k^2 \leq 0.5$, we define a term $K$ such that

$$K = \frac{1}{\pi} \ln \left( \frac{2^{1 + (1 - k^2)^{1/4}}}{1 - (1 - k^2)^{1/4}} \right),$$  \hspace{1cm} (3.53)$$

else if $0.5 \leq k^2 \leq 1$,

$$K = \frac{\pi}{\ln\{2(1 + \sqrt{k})/(1 - \sqrt{k})\}}.$$  \hspace{1cm} (3.54)$$

Fig. 3.4: Model of a coupled line operating in odd mode.
The capacitance w.r.t. the electric wall in air in terms of $K$ is given as

$$C_{ga} = \epsilon_0 K. \quad (3.55)$$

The capacitance w.r.t. electric wall in the dielectric is given by

$$C_{gd} = \frac{\epsilon_0 \epsilon_r}{\pi} \ln \coth \left( \frac{\pi s}{4h} \right) + 0.65 C_f \left( \frac{0.02}{s/d} \sqrt{\epsilon_r} + 1 - \epsilon_r^{-2} \right). \quad (3.56)$$

Hence, the even and odd mode capacitances are given as

$$C_e = C_p + C_f + C'_f, \quad (3.57)$$
$$C_o = C_p + C_f + C_{ga} + C_{gd}, \quad (3.58)$$

where $C_p$ is the parallel plate capacitance with respect to the ground plane, $C'_f$ the fringing field capacitance with respect to the magnetic wall (for even mode), $C_{ga}$ and $C_{gd}$ being the capacitances with respect to electric wall (for odd mode) in air and dielectric, respectively. $C_f$ is the fringing capacitance.

The same capacitances are calculated for a strip suspended in air, i.e. for air as the substrate between the microstrip and ground. The effective dielectric constant $\epsilon_{eff}$ for this case is 1. The even and odd mode capacitances obtained with this $\epsilon_{eff}$ are $C_{eair}$ and $C_{oair}$, respectively.

### 3.8 Calculation of Effective Dielectric Constant

As mentioned in Chapter 1, we represent the microstrip transmission line by an overall dielectric constant in order to assume TEM propagation. There are a number of formulas listed for the calculation of $\epsilon_{eff}$. The most basic formula is given by Pozar [2] as follows:

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}. \quad (3.59)$$

Other formulas were given by T. C. Edwards and E. Hammerstad et al. [3, 13]. All
the formulas were coded in MATLAB and compared to the ADS Linecalc calculations. The
formula by Hammerstad and Jenson, derived assuming a static model, has a stated accuracy
of 0.2% for $\epsilon_r \leq 128$ and $0.01 \leq W/d \leq 100$ [13]. This formula was used in the code since it
turned out to be the closest to the ADS results. It is reproduced here as follows:

$$
\epsilon_{eff} = \frac{(\epsilon_r + 1)}{2} + \frac{(\epsilon_r - 1)}{2}(1 + 10/v)^{-ab},
$$

(3.60)

where $v = W/d$ and

$$
a = 1 + \frac{1}{49}\ln\left(\frac{v^4 + (v/52)^2}{v^4 + 0.432}\right) + \frac{1}{18.7}\ln\left(1 + \left(\frac{v}{18.1}\right)^3\right),
$$

$$
b = 0.564\left(\frac{\epsilon_r - 0.9}{\epsilon_r + 3}\right)^{0.053}.
$$

This formula does not take into account the dispersion [8]. The effects of dispersion were
studied and analyzed from the dispersion curves by W. J. Getsinger [14]. Dispersion affects
the results when the frequency sweeps are large. The effective dielectric constant tends to
the dielectric constant of air as the frequency increases.

3.9 Calculation of Characteristic Impedance of the Strip

The calculation of the characteristic impedance obtained by single-strip static-TEM
methods is given by Pozar [2] for two different cases as follows:

if $W/d \leq 1$,

$$
Z_0 = \frac{60}{\sqrt{\epsilon_{eff}}} \ln\left(\frac{8d}{W + 4d}\right); \quad (3.61)
$$

if $W/d \geq 1$,

$$
Z_0 = \frac{120\pi}{\sqrt{\epsilon_{eff}}[W/d + 1.393 + 0.667\ln(W/d + 1.444)]}. \quad (3.62)
$$
Again, the effect of the thickness of the conductor is not considered. The thickness leads to a small change in the calculation of $Z_0$.

### 3.10 Determination of $S$-parameters

Two $T$ matrices derived in section 3.6 are multiplied to the $ABCD$ matrices of the coupled sections obtained through our previous analysis. Here, the sequence of multiplication is to be maintained according to the attached sections. i.e. for an $N^{th}$ order edge-coupled filter

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \left[ T \right] \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix} \begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix} \ldots \ldots \begin{bmatrix}
A_{N+1} & B_{N+1} \\
C_{N+1} & D_{N+1}
\end{bmatrix} \left[ T \right],
$$

where $ABCD$ is the overall transmission matrix of the filter. We can then convert this matrix to an $S$-parameter matrix by assuming that the filter is terminated in $50\,\Omega$ impedances at both ends.

Hence, we can obtain the $S$-parameters for the filter. We can do this analysis for a filter of any order. For more complicated structures like the structures involving feedback, we can simply write the equations down and solve them simultaneously through matrix manipulations to obtain the $ABCD$ matrix.

### 3.11 Validation of MATLAB Code

The results obtained without taking the conductor and dielectric losses into account were very close to the results displayed in Agilent ADS. Spurious modes which do appear due to inhomogeneities of the microstrip [15, 16] are not shown here. ADS response shows a steeper roll-off to the left and right of the center frequency.

As shown in fig. 3.5, ADS gave a center frequency of 1.481 GHz. For the same structure, the MATLAB code gave a center frequency of 1.515 GHz. The MATLAB response is shown in fig. 3.6. A difference of 34 MHz was observed between the calculated center frequencies which was due to the more accurate formulation in ADS by considering the dispersion at
microwave frequencies.

3.12 Application of the MATLAB Code

The MATLAB code was a good way of understanding the intricacies of the microstrip. Moreover, the process of operating on individual blocks within the filter gets simplified. This is useful in analyzing complex designs which we will consider in the next two chapters.

Fig. 3.5: ADS plot of $S_{11}$ and $S_{21}$ vs frequency giving the center frequency as 1.481 GHz.

Fig. 3.6: MATLAB plot of $S_{11}$ and $S_{21}$ vs frequency with the code giving the center frequency as 1.515 GHz.
Chapter 4

Initial Investigation of Filter Tuning Methods

4.1 Introduction

Several coplanar waveguide (CPW) filters using a microstrip have been proposed in the past [17, 18, 19]. The implementation of tunable filters is important in modern-day radio receivers. Designs for tuning filters consisting of distributed coupled lines have been discussed by Saulich, Tsai et al., and Carey-Smith et al. [20, 21, 22]. In this chapter, we investigate two tuning methods for an edge-coupled filter based on microstrip.

Microstrip edge-coupled bandpass filters are planar structures and have advantages such as easy design procedures and simple integration into circuits.

4.2 Tunable Filter Using Capacitively Loaded Coupled Sections

In an edge-coupled filter, each coupled section has two open ends. Tunability was observed by loading one open end of each coupled section into a varying capacitor as shown in fig. 4.1. The other end of the capacitor was connected to ground.

From our previous analysis, we can write a program for the cascaded coupled sections by inserting $Z_{L1}$ in (3.31) as $-j/2\pi fC$, where $f$ is the operating frequency and $C$ is the tuning capacitor value. For the values of tuning capacitor as 1 pF and 1.5 pF, the MATLAB plots obtained through code for the return loss and insertion loss are shown in fig. 4.2 and fig. 4.3, respectively.

The tuning was investigated in ADS as the tuning capacitor value was changed from 1 pF to 1.5 pF. The insertion loss and return loss observed are shown in fig. 4.4 and fig. 4.5, respectively. The center frequency gets shifted from 1.224 GHz at 1.5 pF to 1.305 GHz at 1 pF.

As the tuning capacitance value was varied from 0.5 pF to 1.2 pF, the center frequency
Fig. 4.1: ADS schematic of the tunable filter.

shifted from 1.42 GHz to 1.269 GHz. The ADS simulation plots for $S_{11}$ and $S_{21}$ are shown in fig. 4.6 and fig. 4.7, respectively.

From the plot for $S_{11}$, we note that the reflection increases as we increase the tuning capacitance. This is because the input impedance of the network changes, and hence the filter is no longer matched to 50Ω. This would require a tunable matching circuit, increasing the complexity of the design.

Figure 4.8 shows ADS simulation results for filter bandwidth over the tuning range. It is observed that the bandwidth varies linearly with the tuning capacitance.

4.3 Tunability with a Tuning Capacitor in Series

The implementation and study of tunable resonators in stepped impedance resonators has been made by B. Kapilevich et al. [23]. A similar approach has been applied to edge-coupled filters. A capacitor has been inserted in series with the coupled sections. The symmetry of the filter has not been affected. No bandwidth compensation scheme has been adopted which would require tunable input and output matching sections [23].

The tuning action can be observed theoretically since the $ABCD$ matrix of the tunable resonator gets multiplied to the coupler $ABCD$ matrices. Hence, the overall $ABCD$ matrix
Fig. 4.2: MATLAB plot of $S_{11}$ vs frequency for tuning capacitance values of 1 pF and 1.5pF.

Fig. 4.3: MATLAB plot of $S_{21}$ vs frequency for tuning capacitance values of 1 pF and 1.5pF.

of the filter shown in fig. 4.9 is

$$
\begin{bmatrix}
A' & B' \\
C' & D'
\end{bmatrix}
= \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix}
\begin{bmatrix}
A_t & B_t \\
C_t & D_t
\end{bmatrix}
\begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix},
$$

(4.1)

where $ABCD_t$ is the tuning resonator transmission matrix, $ABCD_1$ and $ABCD_2$ correspond to the transmission matrices of the first and second halves of the filter, respectively.
Fig. 4.4: ADS plot of $S_{11}$ vs frequency for tuning capacitance values of 1 pF and 1.5 pF.

In case of a capacitor $C$ as the tuning element in series

$$\begin{align*}
ABCD_t &= \begin{bmatrix} 1 & Z_t \\ 0 & 1 \end{bmatrix}, \\
Z_t &= -j/(2\pi fC).
\end{align*}$$

(4.2)

(4.3)

Fig. 4.5: ADS plot of $S_{21}$ vs frequency for tuning capacitance values of 1 pF and 1.5 pF.

For this schematic, the MATLAB results were obtained. For the values of tuning capacitor as 10 pF and 22 pF, the MATLAB plots obtained through code for the return
The tuning was investigated in ADS as the tuning capacitor value was changed from 10 pF to 22 pF.

A board was built (RO4003C substrate was used) on the milling machine by generating a layout as shown in fig. 4.11. Figure 4.12 shows a picture of the board. Two capacitors of values 10 pF and 22 pF were used and the results were observed on the network analyzer. The data was then plotted using ADS data display tool. The insertion loss and return loss observed were compared to the ADS simulation results as shown in fig. 4.13 and fig. 4.14, respectively.

Matching the capacitive element to the coupled sections is not straight-forward since
the width of the matching section required is greater than 25 mm. This would be very
difficult to implement in this design. In this process, a lot of reflection occurs and much
less energy gets transferred to the other half of the filter. This in-turn affects the tunability
of the filter.

The center frequency of the simulated response drops from 1.543 GHz to 1.540 GHz
while the board results show a shift in center frequency from 1.586 GHz to 1.580 GHz as
we change the tuning capacitance from 10 pF to 22 pF. We see a greater variation in the
center frequency compared to the simulated results.

The overall simulated and measured response of the tunable filter for the tunable
capacitor of values 10 pF and 22 pF in series are shown in fig. 4.15.
Fig. 4.9: ADS schematic of the tunable filter.

(a) MATLAB plot of $S_{11}$ vs frequency.

(b) MATLAB plot of $S_{21}$ vs frequency.

Fig. 4.10: MATLAB plots of insertion loss and return loss for tuning capacitance values of 10 pF and 22pF.

Fig. 4.11: Layout of the filter.
Fig. 4.12: Photograph of the tunable filter.

Fig. 4.13: Comparison of simulated and measured responses of $S_{11}$ vs frequency for tuning capacitance values of 10 pF and 22 pF.
(a) ADS plot of $S_{21}$ vs frequency.

(b) Measured response of $S_{21}$ vs frequency obtained from board.

Fig. 4.14: Comparison of simulated and measured responses of $S_{21}$ vs frequency for tuning capacitance values of 10 pF and 22pF.

(a) ADS simulation response as the frequency varies from 1.46 GHz to 1.6 GHz.

(b) Measured response as frequency varies from 1.3 GHz to 1.7 GHz.

Fig. 4.15: Combined simulated and measured frequency responses for tuning capacitance values of 10 pF and 22pF.
Chapter 5
Filter Tuning Using Feedback Loop

5.1 Motivation for Feedback Loop Design

In the design employing capacitively loaded open ends of coupled sections, we get good tunability. But to compensate for the bandwidth change and increased return loss, we need tunable matching sections [23, 24]. This increases the complexity of the design. Also, having four varactors can introduce more insertion loss. Moreover, the periodicity of the structure depends heavily on the accurate tuning of each varactor. This is difficult with the component parasitics at high frequencies. Hence, there was a motivation to reduce the number of varactor diodes without compromising on the tunability.

The design using a tunable element in series with the filter sections gave very low tunability. The matching section design for the capacitor was difficult. As a result, there was increased reflection, affecting the tunability. The measured results showed an insertion loss of about 20 dB. Though the design was simple, it suffered from many drawbacks.

5.2 Design Procedure

There was a possibility of electronically tuning the edge-coupled filter using a varactor diodes if they were connected in the feedback. To begin with, a third-order edge-coupled filter was designed and the center frequency measured was 1.552 GHz. The response of the filter is shown in fig. 5.1. On addition of the capacitive feedback sections from one open ended port of a coupled section to an open ended port of the previous coupled section as shown in fig. 5.2, the center frequency shifted.

We cannot use only capacitors in feedback, because of the geometry of the structure. The design required insertion of microstrip transmission lines connected from one end of the coupler to the capacitor and vice versa as shown in fig. 5.3.
A plot of filter bandwidth vs feedback section width was obtained using ADS. The width of the strips was determined for a bandwidth of 20 MHz. From fig. 5.4, we see that the bandwidth of 20 MHz with feedback capacitor value of 3 pF is 4.56 mm. Hence, the width of the microstrip lines was obtained. The characteristic impedance of the microstrip line corresponding to this width is 50Ω.

The length of the feedback sections was determined by taking the geometry of the network into account.

5.3 Analysis of the Tunable Filter

The analysis of an edge-coupled filter with a feedback network connected between consecutive cascaded coupled sections is not straightforward since the logic of multiplying $ABCD$ matrices of each coupled line as we move in the forward direction no longer holds true. This is because the input and output of each coupled line are dependant on the next
or the previous coupled section depending on the section under consideration. This leads us to the analysis of a coupled line network with the $ABCD$ matrix of the feedback section embedded into it.

We begin our analysis with only two coupled sections, A and B, connected with a feedback network as shown in fig. 5.5. The output of the first coupled section A at port 3 is connected to the input of the next coupled section B. Energy is fed back from coupled section B at port 4 to coupled section A at port 2 via a capacitive feedback loop. The impedance matrix of the coupled section A is as follows:

$$
\begin{bmatrix}
V_{1A} \\
V_{2A} \\
V_{3A} \\
V_{4A}
\end{bmatrix}
= 
\begin{bmatrix}
Z_{A11} & Z_{A12} & Z_{A13} & Z_{A14} \\
Z_{A21} & Z_{A22} & Z_{A23} & Z_{A24} \\
Z_{A31} & Z_{A32} & Z_{A33} & Z_{A34} \\
Z_{A41} & Z_{A42} & Z_{A43} & Z_{A44}
\end{bmatrix}
\begin{bmatrix}
I_{1A} \\
I_{2A} \\
I_{3A} \\
I_{4A}
\end{bmatrix}.
$$

(5.1)

This impedance matrix is a function of even and odd mode voltages, $\alpha$ and $\beta$.

But now port 2 is playing no part in our analysis because it has an open circuited end and in no way affects the other voltages and currents once the input-output relationship of
the coupled section gets set. Hence, we delete the second row throughout and the second column of the impedance matrix. This leads us to

\[
\begin{bmatrix}
V_{1A} \\
V_{3A} \\
V_{4A}
\end{bmatrix} =
\begin{bmatrix}
Z_{A11} & Z_{A13} & Z_{A14} \\
Z_{A31} & Z_{A33} & Z_{A34} \\
Z_{A41} & Z_{A43} & Z_{A44}
\end{bmatrix}
\begin{bmatrix}
I_{1A} \\
I_{3A} \\
I_{4A}
\end{bmatrix}.
\]

(5.2)

We now break the matrices into blocks and solve the equations in order to get them rearranged into a form that enables us to get \(V_{4A}\) and \(I_{4A}\) on the L.H.S. of (5.2). This makes our analysis easier in the next round since \(V_{4A}\) and \(I_{4A}\) are related to \(V_{2B}\) and \(I_{2B}\) by an
$ABCD$ matrix of the feedback element.

\[
\begin{bmatrix}
    V_1A \\
    V_3A \\
    I_3A
\end{bmatrix} = [A]_{3 \times 3} \begin{bmatrix}
    I_{1A} \\
    V_{4A} \\
    I_{4A}
\end{bmatrix} \tag{5.3}
\]

Similar analysis can be made for coupled line B. After switching columns and solving matrix blocks, we write the matrix relations in a form

\[
\begin{bmatrix}
    V_1B \\
    I_{1B} \\
    V_{3B}
\end{bmatrix} = [B]_{3 \times 3} \begin{bmatrix}
    V_{4B} \\
    I_{4B} \\
    I_{3B}
\end{bmatrix}. \tag{5.4}
\]

As mentioned earlier, we write that $V_{4A}$ and $I_{4A}$ are related to $V_{2B}$ and $I_{2B}$ by a transmission matrix of the feedback element.

\[
\begin{bmatrix}
    V_{4A} \\
    I_{4A}
\end{bmatrix} = \begin{bmatrix}
    A' & B' \\
    C' & D'
\end{bmatrix} \begin{bmatrix}
    V_{4B} \\
    I_{4B}
\end{bmatrix} \tag{5.5}
\]

Using (5.3), we express the input output relationship in terms of $V_{4B}$ and $I_{4B}$ by incorporating the $ABCD$ matrix. This leads us to

\[
\begin{bmatrix}
    V_1A \\
    V_3A \\
    I_3A
\end{bmatrix} = [A_f]_{3 \times 3} \begin{bmatrix}
    I_{1A} \\
    V_{4B} \\
    I_{4B}
\end{bmatrix}, \tag{5.6}
\]

where

\[
A_f = \begin{bmatrix}
    A_{11} & A_{12}A' + A_{13}C' & A_{12}B' + A_{13}D' \\
    A_{31} & A_{22}A' + A_{23}C' & A_{22}B' + A_{23}D' \\
    A_{41} & A_{32}A' + A_{33}C' & A_{32}B' + A_{33}D'
\end{bmatrix} \tag{5.7}
\]
The next simplification we can make is by the fact that

$$V_{3A} = V_{1B}, \quad (5.8)$$

and

$$I_{3A} = -I_{1B}. \quad (5.9)$$

Substituting these conditions in (5.6) and (5.4), we now write the equations for $V_{3A}$ and $I_{3A}$ for section A and $V_{1B}$ and $I_{1B}$ for section B.

$$\begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ -B_{21} & -B_{22} \end{bmatrix} \begin{bmatrix} V_{4B} \\ I_{4B} \end{bmatrix} + \begin{bmatrix} B_{13} \\ -B_{23} \end{bmatrix} I_{3B} \quad (5.10)$$

$$\begin{bmatrix} V_{3A} \\ I_{3A} \end{bmatrix} = \begin{bmatrix} A_{f22} & A_{f23} \\ A_{f32} & A_{f33} \end{bmatrix} \begin{bmatrix} V_{4B} \\ I_{4B} \end{bmatrix} + \begin{bmatrix} A_{f21} \\ A_{f31} \end{bmatrix} I_{1A} \quad (5.11)$$

Equating the R.H.S. of the (5.10) and (5.11), we get

$$\begin{bmatrix} V_{4B} \\ I_{4B} \end{bmatrix} = \left[ A_{f22} - B_{11} \ A_{f23} - B_{12} \\ A_{f32} + B_{21} \ A_{f33} + B_{22} \right]^{-1} \left\{ -\begin{bmatrix} A_{f21} \\ A_{f31} \end{bmatrix} I_{1A} + \begin{bmatrix} B_{13} \\ B_{23} \end{bmatrix} I_{3B} \right\}. \quad (5.12)$$

This leads us to an equation of the form

$$\begin{bmatrix} V_{4B} \\ I_{4B} \end{bmatrix} = [X]_{2 \times 1} I_{1A} + [Y]_{2 \times 1} I_{3B}. \quad (5.13)$$
Now that we can express $V_{4B}$ and $I_{4B}$ in terms of $I_{1A}$ and $I_{3B}$, we can substitute in the equations for $V_{1A}$ and $V_{3B}$ from (5.6) and (5.4), respectively. Hence,

$$V_{1A} = A_{f11}I_{1A} + \begin{bmatrix} A_{f12} & A_{f13} \end{bmatrix} \begin{bmatrix} V_{4B} \\ I_{4B} \end{bmatrix},$$

(5.14)

$$V_{3B} = \begin{bmatrix} B_{31} & B_{32} \end{bmatrix} \begin{bmatrix} V_{4B} \\ I_{4B} \end{bmatrix} + B_{33}I_{3B}.$$  

(5.15)

Substituting for $V_{4B}$ and $I_{4B}$ from (5.13) in (5.14) and (5.15), we get the impedance matrix for the two coupled sections connected in feedback.

$$\begin{bmatrix} V_{1A} \\ V_{3B} \end{bmatrix} = [Z]_{2 \times 2} \begin{bmatrix} I_{1A} \\ I_{3B} \end{bmatrix},$$

(5.16)

where

$$Z = \begin{bmatrix} (A_{f11} + A_{f12}X_{11} + A_{f13}X_{21}) & (A_{f12}Y_{11} + A_{f13}Y_{21}) \\ (B_{31}X_{11} + B_{32}X_{21}) & (B_{33} + B_{31}Y_{11} + B_{32}Y_{21}) \end{bmatrix}.$$ 

(5.17)

Hence, the analysis of the two coupled sections connected by a tunable feedback section condenses to a two port network analysis. This $Z$-matrix can be converted to an equivalent $ABCD$ matrix $ABCD_{fb}$. In our case of a third order filter with four sections, the next two coupled sections are also connected to each other using feedback. This block will have a similar $ABCD$ matrix as we look from port 2. The $ABCD$ matrix of this section $ABCD_{fb}'$ is given as

$$ABCD_{fb}' = \begin{bmatrix} A'_{fb} & -B'_{fb} \\ -C'_{fb} & D'_{fb} \end{bmatrix},$$

(5.18)
where

\[
\begin{bmatrix}
A'_{fb} & B'_{fb} \\
C'_{fb} & D'_{fb}
\end{bmatrix}
= \begin{bmatrix}
A_{fb} & B_{fb} \\
C_{fb} & D_{fb}
\end{bmatrix}^{-1}
\]  \hspace{1cm} (5.19)

This $ABCD$ matrix is multiplied to the $ABCD$ matrix of the other block and then by multiplying the $ABCD$ matrices of the matching sections, the overall $ABCD$ matrix of the tunable filter is computed. The $S$-parameters of the filter are found by converting the overall $ABCD$ to its $S$-matrix equivalent.

This procedure of obtaining the $S$-matrix was coded up in MATLAB (refer to Appendices A and B). The results were then compared with a similar ADS schematic as shown in fig. 5.6 and fig. 5.7. The bandwidth obtained in the MATLAB simulation was smaller compared to the ADS simulation results.

For a feedback capacitor of value 3 pF, the center frequency obtained through MATLAB was 1.533 GHz while the ADS results showed it to be 1.553 GHz.

For a feedback capacitance of 5 pF, the center frequency obtained through MATLAB was 1.49 GHz while the ADS results showed it to be 1.478 GHz.

Figure 5.8 shows the board built on the milling machine (RO4003C substrate was used, with $\epsilon_r = 3.55$ and $\tan\delta = 0.0027$) from the generated layout displayed in fig. 5.9 with two sets of capacitances of 3 pF and 5 pF and the results were observed on the HP 8510c network analyzer. The data was then plotted using ADS data display tool.

The results were compared to the ADS simulation results. The insertion loss and return loss observed are shown in fig. 5.10 and fig. 5.11, respectively. The center frequency of the simulated response drops from 1.553 GHz to 1.478 GHz while the board results show a shift in center frequency from 1.534 GHz to 1.448 GHz as we change the feedback capacitance from 3 pF to 5 pF.

The ADS simulated bandwidth varies from 20.65 MHz to 20.27 MHz while the experimental board bandwidth varies from 28 MHz to 32 MHz as the feedback capacitance is changed from 3 pF to 5 pF. This increase in bandwidth in board design is because of the
fringing capacitances caused by open-ended effects [3, 25, 26, 27].

The schematic shows a minimum $S_{11}$ of -7.8 dB and maximum $S_{21}$ of -5.6 dB for feedback capacitance of 5 pF, while the board results show minimum $S_{11}$ of -9.993 dB and maximum $S_{21}$ of -11.216 dB for the same value of capacitance. For a 3 pF capacitance, ADS results showed a minimum $S_{11}$ of -8.45 dB and maximum $S_{21}$ of -5.654 dB while the board results showed minimum $S_{11}$ of -9.887 dB and maximum $S_{21}$ of -13.124 dB. It is observed that the experimental board results show more insertion loss than the simulated results. The combined responses are showed in fig. 5.12.

The tuning of the filter is from 1.473 GHz to 1.564 GHz, an overall range of 91 MHz.
5.4 Effect of Feedback Section Width

This tuning range can be increased by increasing the width of the feedback microstrip lines. But in this process we would be affecting the response of the filter. It is observed that reducing the width of the feedback microstrip lines actually improves the return loss as
shown in fig. 5.15 since we would be matching it to the high input impedance as seen from
the feedback section. But it would also imply that we are bypassing the filtering action.
This affects the tuning since lower amount of energy passes through the feedback sections
as shown in fig. 5.16. The simulated response for width of the feedback microstrip sections
as 0.1 mm is shown in fig. 5.17. The minimum $S_{11}$ obtained is -15.822 dB. Figure 5.18
displays the variation of center frequency as the feedback microstrip line width is tuned.

5.5 Group Delay

Group delay is a measure of the time taken by a signal to travel from the input port to
the output port. We observe from fig. 5.19 that the group delay remains near constant as
the feedback capacitor is varied from 2.7 pF to 5.2 pF. High group delay is a characteristic
of edge-coupled filter, but the consistency is important over the tuning range.
Fig. 5.10: Simulated and measured $S_{11}$ for tuning capacitance values of 3 pF and 5 pF.

(a) ADS response displaying shift in $S_{11}$. (b) Measured response displaying shift in $S_{11}$.

Fig. 5.11: Simulated and measured $S_{21}$ for tuning capacitance values of 3 pF and 5 pF.

(a) ADS response displaying shift in $S_{21}$. (b) Measured response displaying shift in $S_{21}$. 
(a) Combined ADS results displaying shift in response.

(b) Combined measured results displaying shift in response.

Fig. 5.12: Combined simulated and measured losses for tuning capacitance values of 3 pF and 5 pF.

(a) Simulated $S_{11}$ tuning.

(b) Simulated $S_{21}$ tuning.

Fig. 5.13: ADS simulation of filter tuning as the feedback capacitor value is varied from 2.7 pF to 5.2 pF in steps of 0.5 pF.
Fig. 5.14: Comparison of measured and ADS simulation for bandwidth and center frequency as the feedback capacitor value is increased.

Fig. 5.15: ADS simulation plot of variation of the value of minimum $S_{11}$ as feedback section width is varied.
Fig. 5.16: ADS simulation plot showing frequency shift as the feedback capacitor is changed from 3 pF to 5 pF as the feedback section width is varied.

Fig. 5.17: ADS simulation response of the filter with feedback section width of 0.1 mm.

Fig. 5.18: ADS simulation response of variation of center frequency as the feedback section width is increased.
Fig. 5.19: ADS simulation plot of group delay versus the frequency in GHz as the capacitance is varied from 2.7 to 5.2 pF in steps of 0.5 pF.
Chapter 6
Conclusions and Future Work

6.1 Conclusions

A novel tuning technique was implemented through the use of feedback. This filter gave more design flexibility over the other designs which involved loading open ends into tunable elements and series tunable sections. The design was simple and easy to fabricate.

The mathematical analysis was done for each design and verified by writing a MATLAB code. The effects due to conductor and dielectric losses were ignored in the code. ADS was used to accurately arrive at the final design.

The design with tuning elements attached to an open end of each coupled section required tunable matching sections in order to compensate for the reflection across the tuning range. The design involving a tunable capacitor in series with the coupled sections had high insertion loss of about -20 dB.

Comparisons were made between the ADS simulation and the board results for the designs involving feedback and a series tunable section. The measured and simulated responses showed good agreement.

6.2 Future Work

The project based on the tunable filter using feedback can be further optimized using optimization algorithms. The Particle Swarm Optimization or Genetic Algorithms can be incorporated in the MATLAB code. The program can then be used for obtaining the desired bandwidth and center frequency.

The feedback section width corresponding to the filter bandwidth is obtained using ADS simulation plots. Derivation of a mathematical equation describing the bandwidth-feedback section width variance can be more useful and can be easily implemented in a
MATLAB code.

In the design using tunable capacitor in series, inserting matching sections between the coupled sections and the capacitor was difficult. Using lumped components for this purpose will lead to better results since the lumped components can be tuned easily.

The spurious mode formed due to the inhomogeneity of the microstrip affects the filter performance. In future, the design can be modified to diminish the effect of spurious modes.

A design using an amplifier in feedback to the filter was attempted. Mathematically, the center frequency should shift as the gain of the amplifier is varied since the poles of the transfer function change. This design can be tried to design a multi-band filter.
References


Appendices
Appendix A

MATLAB Code for Feedback-Based Tunable Filter

A.1 MATLAB Code for Analysis of Feedback Section

This code does the entire analysis of the feedback based tunable filter. The matching section transmission lines are termed as 'mlin'.

```matlab
clear;
close all;
c_light = 3e8;
epsilon_r = 3.55;
epsilon0 = 8.854e-12;
d = 0.8e-3; %Height of substrate
%
% Coupler Sections
coupler_L1 = 29.44e-3; %first coupler length
coupler_L2 = 29.32e-3; %second coupler length
s1 = 0.9084e-3; %first coupler spacing
s2 = 2.985e-3; %second coupler spacing
W1 = 1.418e-3; %first coupler width
W2 = 1.456e-3; %second coupler width
%
%ABCD of feedback network
```
Lf1 = 21.98e-3; %first strip length
Wf1 = 4.56e-3; %first strip width
Lf2 = 22.38e-3; %second strip length
Wf2 = 4.56e-3; %second strip width
Lf3 = 27.84e-3; %third strip length
Wf3 = 4.56e-3; %third strip width

% Fix
C = 3e-12; %tuning capacitance value

freq = 1.3e9:0.001e9:1.7e9;

for ii = 1:size(freq,2);

Z_Amatrix = Zmatrix(epsilon_r, coupler_L1, s1, W1, d, freq(ii));
% analysis of coupler to get the Z matrix of the 4 port
Z_Amatrixopen = [Z_Amatrix(1,1) Z_Amatrix(1,3) Z_Amatrix(1,4);...
Z_Amatrix(3,1) Z_Amatrix(3,3) Z_Amatrix(3,4);...
Z_Amatrix(4,1) Z_Amatrix(4,3) Z_Amatrix(4,4)];
Z_a = ZAsolved(Z_Amatrixopen);
% rearranging the Z matrix after ignoring the effect of the open port 2
% on the coupler

Lm = 27.84e-3; %MLIN length
Wm = 6.44e-3; %MLIN width
Mlin = MLIN(epsilon_r,Lm,Wm,d,freq(ii)); %ABCD MLIN

ABCDf1 = MLIN(epsilon_r,Lf1,Wf1,d,freq(ii));
ABCDf2 = MLIN(epsilon_r,Lf2,Wf2,d,freq(ii));
ABCDf3 = MLIN(epsilon_r,Lf3,Wf3,d,freq(ii));
ABCDc = [1 -j/(2*pi*freq(ii)*C);0 1]; %T matrix for tuning capacitor

ABCDf = ABCDf1*ABCDf2*ABCDc*ABCDf3;

Z_A = [Z_a(1,1) Z_a(1,2)*ABCDf(1,1)+Z_a(1,3)*ABCDf(2,1)...
      Z_a(1,2)*ABCDf(1,2)+Z_a(1,3)*ABCDf(2,2) ; ...
      Z_a(2,1) Z_a(2,2)*ABCDf(1,1)+Z_a(2,3)*ABCDf(2,1)...
      Z_a(2,2)*ABCDf(1,2)+Z_a(2,3)*ABCDf(2,2) ; ...
      Z_a(3,1) Z_a(3,2)*ABCDf(1,1)+Z_a(3,3)*ABCDf(2,1)...
      Z_a(3,2)*ABCDf(1,2)+Z_a(3,3)*ABCDf(2,2)];

%-----------------------------------------------------
%coupled section B

Z_Bmatrix = Zmatrix(epsilon_r,coupler_L2,s2,W2,d,freq(ii));
%analysis of coupler to get the Z matrix of the 4 port
Z_Bmatrixopen = [Z_Bmatrix(1,1) Z_Bmatrix(1,3) Z_Bmatrix(1,4);...
                 Z_Bmatrix(3,1) Z_Bmatrix(3,3) Z_Bmatrix(3,4);...
                 Z_Bmatrix(4,1) Z_Bmatrix(4,3) Z_Bmatrix(4,4)];
Z_B = ZBsolved(Z_Bmatrixopen);
%rearranging the Z matrix after ignoring the effect of the open port 2
%on the coupler
% solution for V4B and I4b in terms of I1A and I3B
Q = inv([Z_A(2,2)-Z_B(1,1) Z_A(2,3)-Z_B(1,2); Z_A(3,2)+Z_B(2,1)...
Z_A(3,3)+Z_B(2,2)]);
R = - Q*[Z_A(2,1); Z_A(3,1)];
S = Q*[Z_B(1,3); -Z_B(2,3)];

Zfinal = [(Z_A(1,1)+Z_A(1,2)*R(1,1)+Z_A(1,3)*R(2,1))...
(Z_A(1,2)*S(1,1)+Z_A(1,3)*S(2,1));...
(Z_B(3,1)*R(1,1)+Z_B(3,2)*R(2,1)) (Z_B(3,3)+Z_B(3,1)*S(1,1)+...
Z_B(3,2)*S(2,1))];
ABCD1 = ZtoABCD(Zfinal);

% ABCD of the I/O of half the coupler
ABCDfinal1=ABCD1;
ABCDfinal2=[ABCD1(1,1) -ABCD1(1,2); -ABCD1(2,1) ABCD1(2,2)];

% with matching mlins the final ABCD
ABCDIO = Mlin*ABCDfinal1*ABCDfinal2*Mlin;

% Smatrix = ABCDtoS(ABCDIO,50);
S11(ii) = Smatrix(1,1);
S12(ii) = Smatrix(1,2);
S21(ii) = Smatrix(2,1);
S22(ii) = Smatrix(2,2);
figure;
subplot(2,1,1)
plot(freq*1e-9, 20*log10(abs(S11)))
title('plot of $S_{11}(dB)$ vs frequency(GHz)');
grid on;
xlabel('Frequency(GHz)');
ylabel('S_{11} (dB)');

subplot(2,1,2)
plot(freq*1e-9, 20*log10(abs(S21)))
title('plot of $S_{21}(dB)$ vs frequency(GHz)');
grid on;
xlabel('Frequency(GHz)');
ylabel('S_{21} (dB)');

figure;
plot(freq*1e-9, 20*log10(abs(S11)),freq*1e-9, 20*log10(abs(S21)),'r')
grid on
title('Plot of Loss in dB vs frequency in GHz');
xlabel('frequency in GHz');
ylabel('Loss in dB');
xlim([1.3 1.7])

A.2 Zmatrix.m

This function does the analysis of the coupler to get a $Z$-matrix of the four-port network. This function is also used in the other filter designs.
```matlab
function Z = Zmatrix(epsilon_r,coupler_L1,s1,W1,d,freq);
c_light = 3e8;
epsilon0 = 8.854e-12;
W2 = W1;
s2 = s1;
coupler_L2 = coupler_L1;
v = W1/d;
g = s1/d;
a = 1+log((v^4+ (v/52)^2)/(v^4+0.432))/49 + log(1+(v/18.1)^3)/18.7;
b = 0.564*((epsilon_r-0.9)/(epsilon_r+3))^0.053;
epsilon_Eff1 = (epsilon_r+1)*0.5+0.5*(epsilon_r-1)*(1+10/v)^(-a*b);
epsilon_Eff2 = 1;

if W1/d<1
    Zo1=(60/sqrt(epsilon_Eff1))*log((8*d/W1)+(W1/(4*d)));
end

if W1/d>1
    Zo1=120*pi/((sqrt(epsilon_Eff1))*((W1/d)+1.393...
        +0.667*log((W1/d)+1.444)));
end

if W2/d<1
    Zo2=(60/sqrt(epsilon_Eff2))*log((8*d/W1)+(W1/(4*d)));
end

if W2/d>1
```

\[ \text{Zo2} = 120\pi / \left( \sqrt{\epsilon_{\text{Eff2}}} \right) \left( (W1/d) + 1.393 \ldots + 0.667 \log((W1/d) + 1.444) \right); \]

end

% effective permittivity of the medium in order to do a TEM analysis
Vph1 = c\_light/sqrt(\epsilon_{\text{Eff1}}); % phase velocity
Vph2 = c\_light/sqrt(\epsilon_{\text{Eff2}}); % phase velocity

%------------------------------------------------------------------------

% Edward’s formulas effective dielectric const. for even and odd modes
%
\text{Cp1} = \epsilon_r \epsilon_0 W1/d; % parallel plate capacitance
\text{Cf1} = 0.5 \left( \sqrt{\epsilon_{\text{Eff1}}} / (c\_light \times \text{Zo1}) - \text{Cp1} \right); % fringing capacitance
\text{A1} = \exp(-0.1 \exp(2.33 - 2.53 \times (W1/d)));
\text{Cfe1} = \text{Cf1} \times \sqrt{\epsilon_r / \epsilon_{\text{Eff1}}} / \left( 1 + \text{A1} \times (d/s1) \times \tanh(8\times s1/d) \right);
% fringing capacitance near magnetic wall
\text{Cp2} = \epsilon_0 W2/d; % parallel plate capacitance
\text{Cf2} = 0.5 \left( \sqrt{\epsilon_{\text{Eff2}}} / (c\_light \times \text{Zo2}) - \text{Cp2} \right); % fringing capacitance
\text{A2} = \exp(-0.1 \exp(2.33 - 2.53 \times (W2/d)));
\text{Cfe2} = \text{Cf2} \times \sqrt{1 / \epsilon_{\text{Eff2}}} / \left( 1 + \text{A2} \times (d/s2) \times \tanh(8\times s2/d) \right);
% fringing capacitance near magnetic wall

% calculations for odd mode synthesis
k1 = (s1/d)/((s1/d)+2*(W1/d));
k2 = (s2/d)/((s2/d)+2*(W2/d));
if $k_1^2 \geq 0$ && $k_1^2 \leq 0.5$,

$$K_1 = \frac{(1/\pi) \log(2*(1+(1-k_1^2)^{(1/4)})/(1-(1-k_1^2)^{(1/4)}))}{1}$$

else $k_1^2 \geq 0.5$ && $k_1^2 \leq 1$,

$$K_1 = \frac{\pi \log(2*(1+sqrt(k_1))/(1-sqrt(k_1)))}{1}$$
end

if $k_2^2 \geq 0$ && $k_2^2 \leq 0.5$,

$$K_2 = \frac{(1/\pi) \log(2*(1+(1-k_2^2)^{(1/4)})/(1-(1-k_2^2)^{(1/4)}))}{1}$$

else $k_2^2 \geq 0.5$ && $k_2^2 \leq 1$,

$$K_2 = \frac{\pi \log(2*(1+sqrt(k_2))/(1-sqrt(k_2)))}{1}$$
end

$C_{ga1} = \varepsilon_0 K_1$; % Capacitance w.r.t. electric wall in air

$C_{gd1} = (\varepsilon_0 \varepsilon_{r}/\pi) \log(\coth((\pi/4)*(s_1/d)))...
+0.65*F_1*((0.02/(s_1/d))*sqrt(\varepsilon_{r})+1-(\varepsilon_{r})^(-2))$;

$C_{e1} = C_p+C_f+C_{fe1}$;

$C_{o1} = C_p+C_f+C_{ga1}+C_{gd1}$;

$Z_{oe1} = 1/(V_{ph1}\varepsilon_{Eff1}C_{e1});$

$Z_{oo1} = 1/(V_{ph1}\varepsilon_{Eff1}C_{o1});$

$C_{ga2} = \varepsilon_0 K_2$; % Capacitance w.r.t. electric wall in air
\[ C_{gd2} = \left( \frac{\varepsilon_0}{\pi} \right) \log \left( \coth \left( \frac{\pi}{4} \frac{s_2}{d} \right) \right) + 0.65 \times C_f2 \times \left( \frac{0.02}{s_2/d} + 1 \right) \]

\[ C_{e2} = C_p2 + C_f2 + C_{fe2} \]

\[ C_{o2} = C_p2 + C_f2 + C_{ga2} + C_{gd2} \]

\%---------------------------------------------------------------

\[ Z_{oe1} = \frac{1}{c_{light} \sqrt{C_{e1} \times C_{e2}}} \]

\[ Z_{oo1} = \frac{1}{c_{light} \sqrt{C_{o1} \times C_{o2}}} \]

\%coupling in dB-----------------------------------------------

\[ \varepsilon_{Eff1e} = \frac{C_{e1}}{C_{e2}} \]

\[ \varepsilon_{Eff1o} = \frac{C_{o1}}{C_{o2}} \]

\[ c_{phase1e} = \frac{c_{light}}{\sqrt{\varepsilon_{Eff1e}}} \]

\[ c_{phase1o} = \frac{c_{light}}{\sqrt{\varepsilon_{Eff1o}}} \]

\[ Z_{L1} = 100000 \]

\[ Y_1 = \frac{1}{Z_{L1}} \]

\[ Z_{L2} = 100000 \]

\[ Y_2 = \frac{1}{Z_{L2}} \]

\%\[ Z_0 = 50 \]

\[ k_{01e} = \frac{2 \pi \times freq}{c_{phase1e}} \]

\[ k_{01o} = \frac{2 \pi \times freq}{c_{phase1o}} \]
\[
\begin{align*}
\alpha_{1e} &= \exp(-j k_{0_1e} \cdot \text{coupler}_L); \\
\beta_{1e} &= \exp(j k_{0_1e} \cdot \text{coupler}_L); \\
\alpha_{1o} &= \exp(-j k_{0_1o} \cdot \text{coupler}_L); \\
\beta_{1o} &= \exp(j k_{0_1o} \cdot \text{coupler}_L);
\end{align*}
\]

\[
A_1 = \begin{bmatrix}
1 & 1 & 1 & 1 \\
\alpha_{1e} & \beta_{1e} & \alpha_{1o} & \beta_{1o} \\
-\alpha_{1o} & -\beta_{1o} & 1 & -1 \\
1 & 1 & -1 & -1
\end{bmatrix};
\]

\[
B_1 = \begin{bmatrix}
1/Z_{oe1} & -1/Z_{oe1} & 1/Z_{oo1} & -1/Z_{oo1};
\alpha_{1e}/Z_{oe1} & -\beta_{1e}/Z_{oe1} & \alpha_{1o}/Z_{oo1} & -\beta_{1o}/Z_{oo1};
\alpha_{1e}/Z_{oe1} & -\beta_{1e}/Z_{oe1} & -\alpha_{1o}/Z_{oo1} & \beta_{1o}/Z_{oo1};
1/Z_{oe1} & -1/Z_{oe1} & -1/Z_{oo1} & 1/Z_{oo1}
\end{bmatrix};
\]

\%--------------------------------------------------------------------------

InvB1 = inv(B1);
Z = A1*InvB1;

A.3 ZAsolved.m

This function does the returns the \(Z\)-matrix of the coupled line A after rearranging the rows and columns and ignoring the effect of port 2.

function YYY = Z_A(Z1);
Z = [Z1(1:3,1) Z1(1:3,3) Z1(1:3,2)];
A = Z(1:2,1:2);
i = Z(1:2,3);
k = Z(3,1:2);
j = Z(3,3);
a = [(A-i*inv(j)*k) i*inv(j) ; -(inv(j))*k inv(j)];
YYY = [a(1:3,1) a(1:3,3) a(1:3,2)];
A.4 ZBsolved.m

This function does the returns the $Z$-matrix of the coupled line B after rearranging the rows and columns and ignoring the effect of port 2.

```matlab
function YYY = Z_B(Z1);
Z2 = [Z1(1:3,3) Z1(1:3,2) Z1(1:3,1)];
Z3 = [Z2(1:3,2) Z2(1:3,1) Z2(1:3,3)];
A = Z3(1:2,1:2);
i = Z3(1:2,3);
k = Z3(3,1:2);
j = Z3(3,3);
a = [(A-i*inv(j)*k) i*inv(j) ; -(inv(j))*k inv(j)];
YYY = [a(1,1:3); a(3,1:3); a(2,1:3)];
YYY = [YYY(1:3,3) YYY(1:3,2) YYY(1:3,1)];
```
Appendix B

MATLAB Codes for Edge-Coupled Filter Analysis

This appendix consists of the functions required for an edge-coupled filter analysis.

B.1 MLIN.m

This function returns the ABCD matrix of the microstrip transmission line for a given width and length.

function T = MLIN(epsilon_r,L,W1,d,freq)

c_light = 3e8;
epsilon0 = 8.854e-12;
v = W1/d;
a = 1+log(((v^4+(v/52)^2)/(v^4+0.432))/49 + log(1+(v/18.1)^3))/18.7;
b = 0.564*((epsilon_r-0.9)/(epsilon_r+3))^-0.053;

epsilon_Eff1 = (epsilon_r+1)*0.5+0.5*(epsilon_r-1)*(1+10/v)^(-a*b);
%epsilon_Eff1 = (epsilon_r+1)*0.5+0.5*(epsilon_r-1)*(1+10*d/W1)^(-0.555);
if W1/d<1
    Zo=(60/sqrt(epsilon_Eff1))*log((8*d/W1)+(W1/(4*d)));
end

if W1/d>1
    Zo=120*pi/((sqrt(epsilon_Eff1))*((W1/d)+1.393+0.667*log((W1/d)+1.444)));
end
\[ c_{\text{phase}} = \frac{c_{\text{light}}}{\sqrt{\epsilon_{\text{Eff1}}}}; \]

\[ k_0 = \frac{2\pi f_{\text{freq}}}{c_{\text{phase}}}; \]

\[ T = \begin{bmatrix} \cos((\sqrt{\epsilon_{\text{Eff1}}})k_0L) & jZ_0\sin((\sqrt{\epsilon_{\text{Eff1}}})k_0L); \ldots \; j\sin((\sqrt{\epsilon_{\text{Eff1}}})k_0L)/Z_0 \cos((\sqrt{\epsilon_{\text{Eff1}}})k_0L) \end{bmatrix}; \]

### B.2 ZtoABCD.m

This function converts the \( Z \) matrix into an equivalent \( ABCD \) matrix.

```matlab
function ABCD=ZtoABCD(Z)

dim=size(Z);
ABCD=zeros(dim);

if(length(dim)<3)
   N=1;
else
   N=dim(3);
end

for(n=1:N)
   z11=Z(1,1,n);
   z12=Z(1,2,n);
   z21=Z(2,1,n);
   z22=Z(2,2,n);

   a=z11;
   b=-det(Z(:,:,n));
   c=1;
```
d=-z22;

ABCD(:, :, n) = [a, b; c, d] / z21;
end

B.3 ABCDtoS.m

This function converts the $ABCD$ matrix into an equivalent $S$-matrix.

function S = ABCDtoS(abcd_param, Z0)

dim = size(abcd_param);
S = zeros(dim);

if (length(dim) < 3)
    N = 1;
else
    N = dim(3);
end

for (n = 1:N)
    a = abcd_param(1, 1, n);
    b = abcd_param(1, 2, n) / Z0;
    c = abcd_param(2, 1, n) * Z0;
    d = abcd_param(2, 2, n);

    delta = a + b + c + d;

    s11 = a + b - c - d;
    s12 = 2 * (a*d - b*c);
s21=2;
s22=-a+b-c+d;

S(:,:,n)=[s11,s12;s21,s22]/delta;
end