Development of an Energy-Based Nearfield Acoustic Holography Technique

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ABSTRACT

Acoustical-based imaging techniques have found merit in determining the behavior of vibrating structures. These techniques are commonly used in numerous applications to obtain detailed noise source information and energy distributions on source surfaces. This work focuses on the continued development of the nearfield acoustical holography (NAH) approach. Source reconstructions using NAH are reliant upon accurate measurement of the pressure field at the hologram surface. For complex acoustic fields this requires fine spatial resolution and therefore demands large microphone arrays. In this paper, a technique is developed for performing NAH using energy-based measurements. Recent advancements in the area of acoustic sensing technology have made particle velocity field information more readily available. Because energy-based measurements provide directional information, a more accurate characterization of the pressure field is obtained. An analytical comparison of conventional NAH to an energy-based implementation is presented. A vibrating plate and cylinder are considered as test cases to validate the analytical results.

I. INTRODUCTION

Nearfield acoustic holography (NAH) is a methodology that enables the reconstruction of acoustic quantities in three-dimensional space from a two-dimensional measurement of the pressure field near the surface. Williams and Maynard presented a Fourier transform-based NAH method\(^1\)\(^-\)\(^3\) for separable geometries of the wave equation that has been successfully applied to a variety of radiation problems\(^4\)\(^-\)\(^6\). Two approaches are currently available for arbitrary geometry problems. The first technique solves the Helmholtz integral equation numerically via the inverse boundary element method (IBEM)\(^7\)\(^-\)\(^8\). An alternative to IBEM is the Helmholtz equation least squares (HELS)\(^9\)\(^-\)\(^10\) method which reconstructs the acoustic field using spherical basis functions.

One common aspect of all three NAH implementations is that the accuracy of reconstruction is dependent upon adequate representation of the pressure field on the measurement surface. The Fourier transform method and IBEM rely on a spatial sampling for field characterization, which can cause mid to high frequency problems to become quite cumbersome. This is due to the fact that the microphone spacing must be less than or equal to a half wavelength of the highest frequency of interest to avoid spatial aliasing. The objective of this work is to develop an
energy-based NAH approach that relaxes this requirement, thereby reducing the required number of measurement locations. This would lead to a considerable savings in data acquisition time for scanning array systems and help to reduce the inefficiency encountered at high frequencies.

The main difficulty of energy-based sensing lies in the ability to measure the acoustic particle velocity. Presently, the primary technique for particle velocity estimation is via finite difference approximations. The accuracy of this method depends on error in the pressure difference, scattering and diffraction, and microphone phase mismatch. Recently, a new particle velocity transducer known as a Microflown\textsuperscript{11} sensor has been developed which functions similar to a hot wire anemometer. The transducer consists of two thin, parallel wires five microns apart that are heated to approximately 300°C. As air particles flow across the wires heat transfer occurs. The first wire crossed will heat the air slightly which results in the second wire not being cooled to quite the same degree. This temperature difference is then used to determine the particle velocity. Jacobsen and de Bree\textsuperscript{12} showed that results comparable to finite difference intensity approximations are possible using the Microflown to measure the particle velocity. For the work presented in this paper, the Microflown sensor is used. However, the results are applicable to any energy-based sensor that measures both pressure and particle velocity.

This paper presents an energy-based NAH method where both nearfield pressure and velocity field information are used to reconstruct the field. The proposed method does not modify the currently used NAH algorithms discussed above. It does, however, provide the user with a better characterization of the field on the measurement surface to input into the chosen algorithm. Analytical results are presented to indicate the theoretical benefits of the proposed method. Experimental results for planar and cylindrical test cases are included for model validation. Because separable geometries have been chosen, the Fourier transform-based method is implemented. However, the energy-based reconstruction method presented below could also be used for arbitrary geometry problems if the IBEM is selected.

II. ENERGY-BASED RECONSTRUCTION THEORY

Current NAH reconstruction methods are based solely on measurement of the pressure field. Since pressure is a scalar quantity, it does not provide directional information for the field. Particle velocity measurements, on the other hand, supply first derivative information for the pressure field via Euler’s equation, Eq. (1).

\[
p_0 \frac{\partial \vec{u}}{\partial t} = -\nabla p
\]  

The in-plane velocities make derivative information available that is used to interpolate between measurement locations. This effectually simulates a finer mesh of pressure measurements.

A. Hermite Interpolation

The chosen interpolation method is taken from the area of geometric modeling\textsuperscript{13}. For ease of programming and computability, along with other reasons specific to geometric modeling, the preferred way to perform interpolation is with parametric equations. For example, a three-dimensional curve is defined by \( x = x(r, s) \), \( y = y(r, s) \), and \( z = z(r, s) \). It is generally convenient to normalize the domain of the parametric variables, \( r \) and \( s \), by restricting their values to the closed interval between 0 and 1, inclusive.
This restriction is expressed symbolically as 
\[ r, s \in [0, 1]. \]
This interval establishes the bounding curves and the intermediate interpolation points. These curves have a natural vector representation given by Eq. (2).

\[
f(r, s) = \begin{bmatrix} x(r, s) \\ y(r, s) \\ z(r, s) \end{bmatrix}
\]

Farin\(^{14}\) points out that a piecewise lower order polynomial interpolation approach is superior in speed and accuracy to its higher order counterparts. Therefore, bicubic polynomial interpolation is selected. Hermite surface patches are chosen for interpolation between measurement locations because they match both function values and slopes at the specified corner points.

1. Curves

Bicubic Hermite surfaces are composed of an orthogonal net of cubic Hermite curves. Therefore, a preliminary discussion of these curves is necessary to provide the foundation upon which the surface interpolation is built (for a more detailed development of Hermite interpolation see Ref. 13). The algebraic form of a parametric cubic curve is given by the polynomials in Eq. (3).

\[
x(r) = a_x r^3 + b_x r^2 + c_x r + d_x \\
y(r) = a_y r^3 + b_y r^2 + c_y r + d_y \\
z(r) = a_z r^3 + b_z r^2 + c_z r + d_z
\]

The 12 scalar coefficients, called algebraic coefficients, determine a unique curve. Using vector notation to obtain a more compact form, Eq. (3) becomes

\[
f(r) = ar^3 + br^2 + cr + d
\]

where \( f(r) \) is the position vector of any point on the curve and \( a, b, c, \) and \( d \) are the vector equivalents of the scalar algebraic coefficients. The algebraic coefficients are not the most convenient way of controlling the shape of a curve, nor do they provide an intuitive sense of the curve shape. Converting to the Hermite form allows for the definition of conditions at the curve boundaries, or endpoints. Using the endpoints \( f(0) \) and \( f(1) \), the corresponding tangent vectors \( f'(0) \) and \( f'(1) \), and Eq. (4) yields the following four equations

\[
\begin{align*}
f(0) &= d \\
f(1) &= a + b + c + d \\
f'(0) &= c \\
f'(1) &= 3a + 2b + c
\end{align*}
\]

where substituting \( r = 0 \) into Eq. (4) yields \( f(0) \), and substituting \( r = 1 \) into the equation yields \( f(1) \). Differentiating \( f(r) \) with respect to \( r \) obtains \( f''(r) = 3ar^2 + 2br + c \). Substituting \( r = 0 \) and \( r = 1 \) into this yields \( f'(0) \) and \( f'(1) \) respectively, where the superscript \( r \) indicates the derivative with respect to \( r \). Solving this set of four simultaneous vector equations in four unknown vectors yields the algebraic coefficients in terms of the boundary conditions.

\[
\begin{align*}
a &= 2f(0) - 2f(1) + f'(0) + f'(1) \\
b &= -3f(0) + 3f(1) - 2f'(0) - f'(1) \\
c &= f'(0) \\
d &= f(0)
\end{align*}
\]

Substituting these equations for the algebraic coefficient vectors into Eq. (4) and rearranging terms produces Eq. (7).
This equation is simplified by making the following substitutions.

\[
\begin{align*}
B_1(r) &= 2r^3 - 3r^2 + 1 \\
B_2(r) &= -2r^3 + 3r^2 \\
B_3(r) &= r^3 - 2r^2 + r \\
B_4(r) &= r^3 - r^2
\end{align*}
\]  

Using these simplifications and subscripts to represent the endpoint \( r \) values, Eq. (7) becomes

\[
f(r) = B_1(r)f_0 + B_2(r)f_1 + B_3(r)f'_0 + B_4(r)f'_1
\]  

Equation (9) is called the geometric form, and the vectors \( f_0, f_1, f'_0, \) and \( f'_1 \) are the geometric coefficients. The \( B_i(r) \) terms are called the Hermite basis functions. Figure 1 shows each basis function as a curve over the domain of the parameter \( r \). These basis functions have three important characteristics. First, they are universal for all cubic Hermite curves. Second, they are only dependent on the parameter, making them identical for each of the three real space coordinates. Finally, they allow the constituent boundary condition coefficients to be decoupled from each other. These functions blend the effects of the endpoints and tangent vectors to produce the intermediate point coordinate values over the parameter domain.

Letting

\[
R = \begin{bmatrix}
   r^3 & r^2 & r & 1 \\
   2 & -2 & 1 & 1 \\
   -3 & 3 & -2 & -1 \\
   0 & 0 & 1 & 0 \\
   1 & 0 & 0 & 0 
\end{bmatrix}
\]

\[
\begin{align*}
M_H &= \begin{bmatrix}
   2 & -2 & 1 & 1 \\
   -3 & 3 & -2 & -1 \\
   0 & 0 & 1 & 0 \\
   1 & 0 & 0 & 0 
\end{bmatrix} \\
G_H &= \begin{bmatrix}
   f_0 & f_1 & f'_0 & f'_1 \end{bmatrix}^T,
\end{align*}
\]

the geometric form given in Eq. (9) can be transformed into the more computationally efficient matrix form, where \( M_H \) is the Hermite basis transformation matrix and \( G_H \) is the geometry matrix containing the curve boundary conditions.

\[
f(r) = RM_HG_H
\]  

The geometry matrix in Eq. (11) is altered for each segment to obtain a series of cubic Hermite curves which are combined to form a composite curve with slope continuity at the endpoints.

2. Surfaces

A large complex surface can be defined by a composite collection of simpler patches. The algebraic form of a bicubic Hermite patch
is given by the tensor product shown in Eq. (12).

\[ f(r, s) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} r^i s^j \]  

The \( a_{ij} \) are the three component algebraic coefficient vectors of the patch, where each component represents one of the three dimensions in real space. The subscripting corresponds to the order of the parameter variables that the coefficient is attributed to. Expanding Eq. (12) and arranging the \( a_{ij} \) terms in descending order produces Eq. (13), a 16 term polynomial in \( r \) and \( s \).

\[ f(r, s) = a_{33} r^3 s^3 + a_{32} r^3 s^2 + a_{31} r^3 s + a_{30} r^3 + a_{23} r^2 s^3 + a_{22} r^2 s^2 + a_{21} r^2 s + a_{20} r^2 + a_{13} r s^3 + a_{12} r s^2 + a_{11} r s + a_{10} r + a_{03} s^3 + a_{02} s^2 + a_{01} s + a_{00} \]  

Because each of the 16 vector coefficients \( a_{ij} \) has three independent components, there are a total of 48 algebraic coefficients, or 48 degrees of freedom. In matrix notation, the algebraic form is

\[ f(r, s) = RAS^T \]  

where

\[ R = \begin{bmatrix} r^3 & r^2 & r & 1 \\ s^3 & s^2 & s & 1 \end{bmatrix} \]

\[ S = \begin{bmatrix} a_{33} & a_{32} & a_{31} & a_{30} \\ a_{23} & a_{22} & a_{21} & a_{20} \\ a_{13} & a_{12} & a_{11} & a_{10} \\ a_{03} & a_{02} & a_{01} & a_{00} \end{bmatrix} \]

Since the \( a \) elements are three-component vectors, the matrix is actually a 4 x 4 x 3 array. As was found with Hermite curves, the algebraic coefficients of a Hermite patch determine its shape and position in space. Although the \( r, s \) parameter domain values are restricted between 0 and 1, the range of the variables in \( x, y, \) and \( z \) is not restricted, because the range of the algebraic coefficients is not limited. A unique point on the surface patch is generated each time a specific pair of \( r, s \) values are input into Eq. (14). These pairs of \( r, s \) values are then mapped back into real space.

Each patch is bounded by four curves, and each boundary curve is a cubic Hermite curve. Applying the same subscripting notation as implemented in Eq. (9), these curves are denoted as: \( f_{00}, f_{01}, f_{10}, \) and \( f_{11}. \) As was seen for curves, the geometric form is a more convenient and intuitive way to define a patch. The geometric form is derived in the same way as for curves. The boundary conditions of the patch are used to solve for the algebraic coefficients. These conditions include the four patch corner points \( f_{00}, f_{01}, f_{10}, \) and \( f_{11}. \) and the eight tangent vectors \( f_{00}, f_{01}, f_{10}, f_{11}, f_{00}, f_{01}, f_{10}, f_{11}, f_{00}, f_{01}, f_{10}, f_{11} \) which define the boundary curves. \( B \) once again represents the Hermite basis functions, as in Eq. (9).

\[ f(r, 0) = B(r) \begin{bmatrix} f_{00} & f_{10} & f_{00}^r & f_{10}^r \end{bmatrix}^T \]

\[ f(r, 1) = B(r) \begin{bmatrix} f_{01} & f_{11} & f_{01}^r & f_{11}^r \end{bmatrix}^T \]

\[ f(0, s) = B(s) \begin{bmatrix} f_{00} & f_{01} & f_{00}^s & f_{01}^s \end{bmatrix}^T \]

\[ f(1, s) = B(s) \begin{bmatrix} f_{10} & f_{11} & f_{10}^s & f_{11}^s \end{bmatrix}^T \]  

These four curves provide 12 of the 16 vectors needed to specify the 48 degrees of freedom. Four additional vectors at the corner points, called twist vectors, are used to fully specify the patch. Mathematically, these vectors are defined as follows:
Calculating the mixed partial derivative of Eq. (13) yields

\[ \frac{\partial^2 f(r,s)}{\partial r \partial s} = 9a_{s3}r^2s^2 + 6a_{s2}r^2s + 3a_{s1}r^2 + 6a_{s2}rs^2 + 4a_{s2}rs + 2a_{s2}r + 3a_{s1}r^2 + 2a_{s2} + a_{s1} \]  

Evaluating Eq. (18) at the corner points obtains

\[
\begin{align*}
\mathbf{f}_{00} & = a_{11} \\
\mathbf{f}_{10} & = 3a_{31} + 2a_{21} + a_{11} \\
\mathbf{f}_{01} & = 3a_{13} + 2a_{12} + a_{11} \\
\mathbf{f}_{11} & = 9a_{31} + 6a_{32} + 3a_{31} + 6a_{23} + 4a_{22} + 2a_{21} + 3a_{13} + 2a_{12} + a_{11}
\end{align*}
\]

(19)

Doing the same for the remaining 12 vectors provides the remaining 12 equations required to solve for the algebraic coefficients.

Solving this set of 16 simultaneous equations from Eqs. (19) and (20) for the algebraic coefficients in terms of the geometric inputs and rearranging terms yields

\[
\begin{align*}
\mathbf{f}(r,s) &= \left[ B_1(r), B_2(r), B_3(r), B_4(r) \right] \times G_H \left[ B_1(s), B_2(s), B_3(s), B_4(s) \right]^T
\end{align*}
\]

(21)

where \( G_H \) is the Hermite geometry matrix shown in Eq. (22).

\[
G_H = \begin{bmatrix}
\mathbf{f}_{00} & \mathbf{f}_{01} & \mathbf{f}'_{00} & \mathbf{f}'_{01} \\
\mathbf{f}_{10} & \mathbf{f}_{11} & \mathbf{f}'_{10} & \mathbf{f}'_{11} \\
\mathbf{f}_{0'} & \mathbf{f}_{01'} & \mathbf{f}'_{0'} & \mathbf{f}'_{01'} \\
\mathbf{f}_{1'} & \mathbf{f}_{11'} & \mathbf{f}'_{1'} & \mathbf{f}'_{11'}
\end{bmatrix}
\]

(22)

Recalling from Eq. (11) that \( \mathbf{B}(r) \) may be expressed as \( \mathbf{R} \mathbf{M}_H \), Eq. (21) may be further simplified to obtain the conventional geometric form given by
\[ f(r, s) = RM_HG_HM_HS^T \] (23)

The remaining intricacy of the interpolation relates to converting between real and parameter space. A simple method for mapping between the two domains is presented below. Figure 2 provides an example of a set of four corner points in \( x \) and \( y \) that could be used to define a patch. In this case, the \( r \) parameter corresponds to the \( x \) direction and the \( s \) parameter to the \( y \) direction.

The slopes at the endpoints must also be transformed to the parameter domain. This is accomplished by scaling the \( r \) derivatives by \( \Delta x \) and the \( s \) derivatives by \( \Delta y \) as shown in Eq. (25) for the corner point corresponding to \( r = s = 0 \):

\[ f_{00}^r = \frac{\partial f_{00}}{\partial x} \frac{\Delta x}{\Delta r} \]
\[ f_{00}^s = \frac{\partial f_{00}}{\partial y} \frac{\Delta y}{\Delta s} \]  (25)

where \( \Delta r \) and \( \Delta s \) equal one because they are restricted to vary from zero to one. The form is the same for the remaining three corner points of the patch.

Because measurements with energy-based sensors do not in general provide enough information to calculate twist vectors, they have been set to zero for this investigation. The Hermite geometry matrix from Eq. (22) then becomes

\[
G_H = \begin{bmatrix}
    f_{00} & f_{01} & f_{00}' & f_{01}' \\
    f_{10} & f_{11} & f_{10}' & f_{11}' \\
    f_{00}' & f_{01}' & 0 & 0 \\
    f_{10}' & f_{11}' & 0 & 0 \\
\end{bmatrix}
\]  (26)

This limits the patches to having only first derivative continuity at their edges. The results presented below indicate that adequate reconstructions are still obtained with this simplification. Figure 3 shows a sample bicubic Hermite patch and the required inputs at each corner point \( f_{rs} \). Each patch represents the rectangular area between four corner point locations. The above interpolation is repeated for each segment of the surface and all the patches combined.
III. ANALYTICAL IMPLEMENTATION

With the surface interpolation completed, the chosen NAH algorithm is applied. An analytical model is developed to investigate the theoretical benefits of the energy-based reconstruction method. This model requires first that a synthetic acoustic field be created from a hypothetical source. The field is then sampled and the chosen algorithm implemented. The error is then evaluated on the estimation plane by comparing the actual and reconstructed fields.

A. Synthetic Field Creation

A rectangular, simply supported plate is chosen as the hypothetical source because it has a simple closed-form radiation equation. The plate shown in Fig. 4 is driven by a harmonic point source acting normal to the plate at its center \(x_0, y_0\). The surface displacement \(w\) for the plate as a function of angular frequency \(\omega\) is given by Eq. (27)\(^3\)

\[
w(x, y, \omega) = \frac{F}{\rho h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{mn}(x_0, y_0) \Phi_{mn}(x, y) \frac{\omega^2 - \omega_m^2}{\omega^2 - \omega_n^2} (27)
\]

\[
\Phi_{mn}(x, y) = \frac{2}{\sqrt{L_x L_y}} \sin \left( \frac{m\pi x}{L_x} \right) \sin \left( \frac{n\pi y}{L_y} \right) (28)
\]

The pressure at a point in space, \(p(x, y, \omega)\), is computed by summing the contribution from each \(dx\,dy\) area element. Radiation from the plate is simulated using a discrete summation of Eq. (29) for a 32 x 32 grid of point sources on the plate. The field is then sampled at chosen measurement locations to obtain the pressure and gradient information to be used...
for interpolation. The selected NAH algorithm is then applied to reconstruct the field.

**FIG. 5.** Description of the geometric quantities used in Rayleigh's integral.

**B. Error Evaluation**

The reconstruction error is evaluated by first calculating the pressure field at the measurement and estimation planes directly using Eq. (29). The direct calculation of the pressure field at the estimation plane serves as a reference against which the NAH reconstruction is compared. The reconstruction error is quantified by differencing the NAH estimation and the direct calculation at the estimation plane. The standard deviation of these residuals is then computed and normalized by the maximum pressure field value to obtain a single value representing the whole field error. This error is then compared for the energy-based and pressure only reconstructions. The number of sensors used to populate the measurement array is varied in both dimensions in order to determine the possible reduction in sensor count using energy-based measurements.

**C. Results**

The results below correspond to the synthetic field generated by a 30.5 cm x 45.7 cm x 0.3175 cm plate vibrating in the 3, 3 mode, as shown in Fig. 4. These dimensions are chosen to match the dimensions used for the experimental validation presented in Sec. IV. The measurement plane is set to 5 cm and the estimation plane to 2 cm above the plate. Figures 6(a) and (b) show the resulting normalized whole field estimation error plots for measurement array sizes ranging from 10 x 10 to 20 x 20 for conventional and energy-based NAH reconstruction.

**FIG. 6.** (a) The estimation error for conventional NAH reconstructions varying the number of sensors in the x and y directions. (b) Estimation error for energy-based NAH reconstructions varying the number of sensors in the x and y directions.
These plots indicate that the inclusion of field directional information in the reconstruction significantly improves the ability to reconstruct the field accurately. In fact, a 10 x 10 array of energy-based measurements has a whole field error of 0.0326, which is slightly lower than the 0.0433 error for a 20 x 20 array of pressure measurements. These results show that the number of measurement locations can be reduced by about 75% when energy-based sensing equipment is used. This reduction seems reasonable since twice the information is being used in each direction. If a three channel probe is used to measure the field, a channel count reduction of 25% would also be realized for non-scanning systems. These results represent the theoretical optimal performance of the conventional and energy-based reconstructions because the measurements have zero positioning, amplitude, and phase error.

IV. EXPERIMENTAL VALIDATION

A. Planar Test Case

An experimental setup is designed to approximate the simply supported plate used in the analytical investigation. Figure 7(a) shows the 30.5 cm x 45.7 cm x 0.3175 cm aluminum plate. It is attached along its edges to a heavy steel frame using cone point set screws to approximate the simply supported boundary condition. A 20 mm diameter piezoelectric patch, shown in Fig. 7(b), is used to excite the plate at its center. The plate and a measurement grid are suspended in an anechoic chamber for data acquisition. A single Microflown ultimate sound probe (USP) is used to scan the field to obtain the pressure and in-plane velocities required for reconstruction. The plate is excited at 1090 Hz corresponding to the 3, 3 operating shape. The field is sampled at 2 cm and 5 cm from the plate as in the analytical case. The vertical and horizontal step distance is set to 5 cm and the plate is overscanned in both directions yielding a 50 cm x 80 cm overall measurement array size. The 2 cm measurement again serves as the reference against which the NAH reconstructions are compared.

Figure 8 shows the reference pressure as measured on the 2 cm estimation plane. An 11 x 17 array of pressure measurements at 5 cm is used to reconstruct the pressure at the estimation plane using the traditional Fourier NAH method. The pressure only reconstruction is then compared to the energy-based reconstruction using a 6 x 9 array of measurements spanning the same area. The resulting reconstructions are shown in Figs. 9(a) and (b).
Conventional NAH and the energy-based NAH are both able to accurately reconstruct the pressure field on the estimation plane. The normalized whole field estimation error for conventional NAH is 0.051, while the error for the energy-based reconstruction is 0.039. The energy-based reconstruction is slightly more accurate than the conventional reconstruction with 70% fewer measurements.

B. Cylindrical Test Case

A cylindrical ABS plastic tube is used for this test case. The tube dimensions are: 10.2 cm inner diameter, 10.8 cm outer diameter, 50.8 cm length. Simply supported boundary conditions are approximated at the tube ends using tapered conical plugs. The tube is driven at 1524 Hz with the same 20 mm piezoelectric patch used for the plate. This excitation corresponds to the 3, 3 operating shape. Scans are made at 2 cm and 4 cm radial distances from the outer surface of the tube. Figure 10 shows the reference field at 2 cm. The resulting reconstruction from an 11 x 14 array of pressure measurements at 4 cm is presented in Fig. 11(a). The vertical step distance is 10.2 cm and the incremental rotation angle is 27.7 degrees. The energy-based reconstruction shown in Fig. 11(b) is obtained using a 7 x 7 array of measurements.
V. CONCLUSIONS

Based on the analytical and experimental results presented in this work, the number of measurements required to obtain comparable reconstruction results to conventional NAH is reduced by up to 70% if both pressure and in-plane velocities are measured. For cases where sub-arrays of sensors are required to scan the field, the proposed energy-based reconstruction method reduces significantly the amount of repositioning, and therefore time, required. The sub-arrays could also be increased in size up to three and a half times if the same number of sensors is used. It should also be noted that the proposed interpolation method is applicable to other reconstruction methods, such as IBEM, that rely on a spatial sampling of the pressure field.

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