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SUFFER FROM DYNAMIC INCONSISTENCY?

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In this paper we have examined optimal tariffs for non-renewable natural resources in the setting of imperfect competition. We do this because Larry Karp (1984, p. 74) states that, “If the buyer attempts to exert market power, he is constrained by the dynamic optimization behavior of the seller and does not face a standard control problem.” We show that when extraction costs are a function of the remaining stock of the resource, the costate variable can be separated into a scarcity effect and a cost effect. Karp concludes that the cost effect must be left with the producer; thereby, restricting the actions of the buyer. We, however, prove that it is not necessary to pay this cost effect to the producer; hence, we conclude that the monopsonist can extract all of the rent from the seller. The optimal tariff is neither dynamically time inconsistent, nor is it “Karp’s consistent tariff.”

JEL Classification: H21, L20, Q30

Key words: Optimal control problem, optimal non-renewable resource tariff, dynamic inconsistency, imperfect competition, scarcity and cost effect
In the October 1984 issue of this Journal Larry Karp examined imperfect competition between buyers and sellers of a non-renewable resource, and reached several conclusions that are in error. He begins by modeling the producers as price takers and the buyer as a monopsony. Among his conclusions are the following: (1) "If the buyer attempts to extract market power, he is constrained by the dynamic optimization behavior of the seller and he does not face a standard control problem." (p. 74). (2) This results in the solution path being dynamically inconsistent.

(3) "Dasgupta and Heal (1979, pp. 335-336) state that a monopsonistic market for a non-renewable resource is equivalent to a monopoly, since a single buyer can offer a price arbitrarily close to extraction costs, and essentially expropriate the resource. This is true if the buyer is in a position to purchase the stock of the resource; he can then make the seller a 'take it or leave it' proposition. If, however, the buyer is able only to purchase a flow, the dynamic optimizing behavior of the seller constrains his power. In that case, the buyer cannot, in general expropriate the resource. (The exception occurs when extraction costs are constant.) Once the value of a stock is given, it makes no difference to the seller whether he sells a stock or a flow. However, the value to the seller of the stock depends on the rules of the game; one of the important rules, for the case of non-constant extraction costs, is whether the stock, or only a flow is sold." (pp. 74, 75)

We conclude that the control problem that maximizes the buyer’s objective function is a standard control problem; hence, its solution path does not suffer from dynamic inconsistency. In addition, the buyer can expropriate the resource rent even when extraction costs are not constant and the purchase is of the flow; thus, we reach the same conclusion as Dasgupta and Heal. The source of the error is a failure to sufficiently examine the seller’s optimization
problem. A portion of the problem rests in a failure to understand the characteristics of the solution path of the costate variable. When extraction costs are not constant, i.e., are a function of the remaining stock of the resource, the shadow value, costate variable, of the resource stock can be divided into a scarcity effect and a cost effect (Lyon, 1999). Both of these can be extracted by the monopsonist, resulting in a complete expropriation of the resource stock. However, if the cost effect is left with the producers as Karp does, the buyer does not face a standard control problem and the resulting solution path is dynamically inconsistent.

In examining the case of bilateral monopoly Karp portrays the buyer as weak opponent for the seller. This conclusion is generated from Table 1, where the buyer’s and seller’s payoffs are presented. The small size of the buyer’s payoff results from the buyer paying the seller the cost effect in the costate variable. The buyer does not have to leave the cost effect with the seller; hence, his bargaining position is much improved relative to that suggested by Karp.

We first state Karp’s model using his symbols, and in many places his equations. We do this to ease the comparison with the original paper. Then we analyze the efficient solution, which is the standard non-renewable resource problem. We use this as our reference model, and generate our major conclusions from it. We then show that the monopsonist’s optimal tariff will expropriate all of the rent, both the scarcity and cost effects, and we show that the efficient path and the monopsonist’s optimal path are identical. The cost function that Karp uses has marginal and average cost equal; hence there is no producers’ surplus for the buyer to extract. With the monopsonist’s optimal tariff the producers receive no rent, and we show that the necessary conditions for them to produce the quantity demanded are satisfied.

MONOPSONISTIC BUYER
In this section we state the model as presented by Karp. “The buyer’s utility of consuming at rate \( x \) is given by a concave function \( u(x) \) with continuous fourth derivatives. The domestic market is competitive, so \( u'(x) = P(x) \), where \( P \) is the price that consumers pay. Thus, \( u(x) \) is defined by \( u(x) = \int_0^x P(X) dX \). \( P(x) \) is assumed to be integrable. The government of the importing nation (or the cartel) may impose a unit tariff, given by \( q(t) \), in which case the price received by the exporters is \( P(x) - q(t) \); or they may impose an \textit{ad valorem} tariff, \( w(t) \), in which case the exporter’s price is \( s(t)P(x) \), where \( s(t) = 1/(1 + w(t)) \).” (p.76) In Karp’s analysis of bilateral monopoly the two tariffs give different results. Because we show that the buyer can extract all of the rent, there is no need to carry the \textit{ad valorem} tariff.

“The buyer’s payoff is the discounted stream of the difference between the utility of consuming at rate \( x(t) \) and the external payment, either \( (P(x) - q)x \) or \( sP(x)x \). If \( T \) is the date at which consumption terminates and \( r \) is the discount rate, the buyer’s payoff may be written

\[
J_B = \int_0^T e^{-rt}[u(x) - (P(x) - q)x] \, dt \tag{1a}
\]

“The seller seeks to maximize profits. Let \( c(z) \) be the average cost of extracting a unit of the resource given that stock size is \( z \); then \( c(z)x \) is the instantaneous cost of extracting at the rate \( x \). Assume \( c'(z) \leq 0 \), and \( c''(z) \geq 0 \). The seller’s payoff is given by

\[
J_S = \int_0^T e^{-rt}[P(x) - q - c(z)] \, x \, dt \tag{2a}
\]

“If the seller is a monopolist, he treats \( P \) as a function of \( x \); if he behaves competitively, he takes \( P \) as given.” (p.76)

“The buyer controls \( q(t) \) or \( s(t) \), and the seller controls \( x(t) \). The buyer announces the tariff path; the seller takes this as given, and chooses \( x(t) \) to maximize (2). Both players are constrained by the non-negativity, of the stock, \( z(t) \). This constraint is given by
\[ z'(t) = -x(t), \quad z(0) = \tilde{z} \text{ given}, \quad z(t) \geq 0 \quad \text{for all } t. \]  

\[ (3) \]

In determining what tariff path to choose the buyer takes into consideration the reaction of the seller; he leads in a Stackelberg game." (p.77) Note that for the most part this is a direct quote including the equation numbers.

If the seller takes \( q(t) \) as given, then the seller’s maximization of (2a) subject to (3) include the necessary conditions

\[ e^{-rt}[P(x) - q - c(z)] - \dot{\lambda} = 0 \]  

\[ (4a) \]

and

\[ \frac{d\lambda}{dt} = e^{-rt}c'(z)x. \]  

\[ (4b) \]

where \( \dot{\lambda} \) is the costate variable. The buyer then takes this costate variable, Eq. (4b), as a constraint on his maximization problem. Eq. (4a) is then solved for \( q \) and this result is substituted into Eq. (1a) to yield

\[ J_B = \int_0^T (e^{-rt}[u(x) - c(z)x] - \lambda x) \, dt \]  

\[ (5a) \]

Karp\(^1\) uses this as the buyer’s constrained objective functional, and gives two equivalent ways of solving it. The solution to this problem is dynamically inconsistent in the sense that, just as soon as \( z \) changes the buyer desires to revise his open loop tariff. The buyer at \( t = 0 \) maximizes (5a) subject to (3) and (4b), and announces his tariff. Let \( z^*(t) \) and \( x^*(t) \) be the optimal time paths of the resource stock and extractions, respectively, and let \( T^* \) be the optimal terminal time. Then at \( t = \epsilon > 0 \) with \( z^*(\epsilon) < z^* = z^*(0) \) these *ed quantities are no longer

\(^1\)This equation corresponds to Karp’s Eq. (7a).
optimal; thus, there is a path starting at $t = \epsilon$ that is superior to continuing to follow $z^*(t)$ and $x^*(t)$ to $T^*$. The buyer then desires to revise his tariff at $t = \epsilon$. If, however, he is not allowed to revise the tariff, he is stuck with a tariff that is no longer optimal. The act of forcing the cost effect into the buyer’s maximization problem is the source of the dynamic inconsistency.

We now analyze the efficient solution, and identify the tax or tariff on extractions that would expropriate the rent from the producers. All of the rent can be expropriated including $A^*(t)$; hence, there is no reason other than altruism for leaving $A^*(t)$ with the producers.

EFFICIENT SOLUTION

The scenario can be modeled as maximizing:

$$J = \int_0^T e^{-rt}[u(x) - c(z)x] dt$$

subject to Eq. (3), and the non-negativity constraint on $x(t)$, which we assume is satisfied. We use the optimality theorem for the Hestenes Bolza problem as stated in Long and Vousden (1977) as Theorem 1, and state all of the necessary conditions for this problem. In the terminology of this theorem we have two control parameters, $T$, the optimal stopping time, and $z(T)$, the optimal resource stock at that time. In addition, we have a control variable, $x(t)$.

The necessary conditions in present value form are:

$$e^{-rt}[u'(x^*(t)) - c(z^*(t))] - \psi^*(t) = 0$$

$$\frac{d\psi^*}{dt} = e^{-rt}c'(z^*(t))x^*(t)$$

$$\frac{dz^*}{dt} = -x^*(t) \quad z^*(0) = \bar{z}$$
Transversality conditions:

\[ e^{-rT^*}[u(x^*(T^*)) - c(z^*(T^*))x^*(T^*)] - \psi^*(T^*)x^*(T^*) = 0 \]  \hspace{1cm} (7d)

\[ \psi^*(T^*) \geq 0 \hspace{1cm} \psi^*(T^*) z^*(T^*) = 0 \]  \hspace{1cm} (7e)

where the * indicates optimal quantities. The transversality condition (7d) holds for \( T^* > 0 \), and (7e) is for \( z^*(T^*) \geq 0 \). Before pressing on we note some implications of these necessary conditions. With \( T^* > 0 \) and \( c'(z) \equiv 0 \), we have \( x^*(t) > 0 \) for \( 0 \leq t < T^* \), \( x^*(T^*) = 0 \), and \( z^*(T^*) = 0 \). This follows from the fact that marginal extraction cost, \( c(z) \), never changes.

When \( c'(z) < 0 \), the possibility exists that \( c(z) \) will rise to the point where the extractions are more costly than they are worth, \( u'(0) = c(z^0) \) with \( 0 < z^0 < \bar{z} \). This implies \( z^*(T^*) = z^0 \), and \( \psi^*(T^*) = 0 \). This follows from (7a) evaluated at \( T^* \), (7d), and (7e). With equations for \( u \) and \( c \), parameters that satisfy the above restrictions, and values for \( \bar{z} \) and \( r \), it is possible to solve for the endogenous variables in this model. This may be a numerical solution rather than a closed form solution. Also note that Eq. (7a) and (7d) yield \( x^*(T^*) = 0 \), whether the resource is exhausted or not; however, if the cost function included a fixed cost, this conclusion would change. In the terminal time period consumption is completely choked off. When \( c'(z) = 0 \) this is by the rise of \( \psi^*(t) \), and when \( c'(z) < 0 \) it is by the rise of \( c(z) \) and/or \( \psi^*(t) \).

In the discussion above, \( \lambda^*(t) \) was the cost effect in the costate variable; thus, an examination of the cost and scarcity effects in the costate variable is useful. Define \( \zeta^*(t) \) to be the current value solution costate variable, \( \zeta^*(t) = e^{rt} \psi^*(t) \), so

\[ \frac{d\zeta^*}{dt} = r\zeta^*(t) + c'(z^*(t))x^*(t) \]  \hspace{1cm} (8)

This differential equation with terminal value of \( \zeta^*(T^*) \), has the solution:
\[ \zeta^*(t) = e^{-r(T^*-t)} \zeta^*(T^*) - \int_t^{T^*} e^{-r(s-t)} c'(z^*(s))x^*(s) \, ds. \]  

(8a)

This is generated by treating \( \frac{d\zeta^*}{dt} \) as a linear first order differential equation with a constant coefficient and variable term. See Appendix A for this derivation. This is the shadow value of the resource stock at time \( t \), and has the role in Equation (7a) of rationing the resource stock over time. It is also the rent per unit of consumption; however, as we show below it does not matter whether it accrues to the producers, or society as a whole, as long as it rations the consumption through Eq. (7a). It is the current value rate of change in the solution value of Eq. (6a) per unit change in the resource stock at time \( t \). For time zero this can be stated as \( \frac{\partial \zeta^*}{\partial z} = \zeta^*(0) \). In Equation (8a)

\[ e^{-r(T^*-t)} \zeta^*(T^*) \]  

is the Scarcity Effect, and

\[ -\int_t^{T^*} e^{-r(s-t)} c'(z^*(s))x^*(s) \, ds \]  

is the Cost Effect.

(8b) (8c)

We have the following three observations: (1) As already discussed, if the resource stock is not exhausted there will be no scarcity effect \( (\zeta^*(T^*) = 0) \), or a scarcity effect can occur only if the resource stock is exhausted. (2) The cost effect approaches zero as \( t \) approaches \( T^* \). (3) The cost effect is zero if \( c'(z) = 0 \) for all \( z \), \( c'(z) = 0 \). The scarcity effect at time \( t \) is simply the terminal scarcity value discounted to the current time \( t \), and the cost effect is the present value of the cost saving associated with the marginal unit of the resource stock. Suppose we were to inject an epsilon unit of resource into the resource stock at time \( t \). This will affect the marginal unit all along the optimal path, starting at time \( t \). In doing so it affects the extraction costs all along the path from time \( t \) on. The cost effect is the present value of these cost savings, as of time \( t \). For the case where the resource stock is exhausted and \( c'(z) < 0 \), the shadow value will contain the present value of the cost savings associated with the marginal unit and the present value of the scarcity effect of that unit. If the resource stock is not exhausted but optimal extractions take place over a positive time period, then the shadow value is due solely to the cost savings. On the other extreme, if \( c'(z) = 0 \), then the shadow value is
due strictly to scarcity. For the case where \( c'(z) < 0 \) and the extractions are halted before the resource stock is exhausted, because the costs become too high to warrant further extractions, the scarcity effect has no role in rationing the resource, but the cost effect does.

If the producers and the consumers in this model are price takers then the rent will belong to the producers. The rent in current value is \( \zeta^* (t)x^*(t) = e^{rt} \psi^*(t)x^*(t) \). If the consumers and producers are in the same country, the government can acquire this rent for society by placing a tax of \( e^{rt} \psi^*(t) \) per unit of \( x \) sold, and if we have a monopsonistic buyer a tariff can accomplish the same thing. This leaves the producers with zero rent. They will cover their extraction costs and no more. The rent will be completely expropriated. This conclusion is consistent with the above quote that referenced Dasgupta and Heal.

We now show that a per unit tax\(^2\) or tariff of \( q(t) = P(x^*(t)) - c(z^*(t)) - e^{rt}g \) will make the producers’ supply function horizontal at \( c(z^*(t)) \). In this last expression, \( g \) is a gift to the sellers per unit of \( x \) extracted. The term \( e^{rt} \) makes the gift have a constant present value, and is included to facilitate the proof. We show that the cost effect can be extracted and any portion of the scarcity effect can be left with the producers. Of course, we let \( g \) go to zero. With this constraint the sellers become passive producers just covering costs. The seller’s problem becomes maximize Eq. (2a) subject to (3) and

\[
P(x) - q - c(z) = e^{rt}g
\]

The producers are modeled as price takers; therefore, they treat \( P(x), q \) and \( g \) as exogenous. We write the Hamiltonian and Lagrangean functions as, respectively

\[
H_S = e^{-rt}[P(x) - q - c(z)]x - \lambda x, \tag{9a}
\]

\(^2\)The tax would be relevant if producers and consumers are in the same country. Also recall that \( P(x) = u'(z) \).
\[ L_S = H_S + \nu [P(x) - q - c(z) - e^{\nu t}g]. \]  

(9b)

Two of the necessary conditions are:

\[ e^{-\nu t}[P(x(t)) - q(t) - c(z^0(t))] - \lambda^0(t) = g - \lambda^0(t) = 0, \]  

(9c)

\[ \frac{d\lambda^0}{dt} = e^{\nu t}c'(z^0(t))x^0(t) + \nu^0(t)c'(z^0(t)) \]  

(9d)

where the super \(^0\) indicates the seller’s optimal path. Note that because the constraint \([P(x) - q - c(z)] = e^{\nu t}g\) is binding for every \(t\) then by (9c) \(\lambda^0(t) = g\) for all \(t\), and \(d\lambda^0/dt = 0\) for all \(t\). This implies that \(e^{\nu t}x^0(t) + \nu^0(t) = 0\). The current value multiplier, \(\partial L_S^*/\partial g = -e^{\nu t}\nu^0(t)\), is the instantaneous value of the per unit gift, \(g\), and is equal to \(x^0(t)\). At time \(t\) a unit increase in the per unit gift of one unit will increase the total gift by \(x^0(t)\).

From these necessary conditions we conclude that the cost effect can be extracted from the seller, and that we can leave any portion of the scarcity effect with the seller that we desire. Since positive economics provides no rationale for leaving any of the scarcity effect with the seller we set \(g = 0\), and this implies that \(\lambda^0(t) = 0\) for all \(t\). Rather we set \(g\) arbitrarily close to zero, since with \(g = 0\) the producers are just as well off not producing as producing. This tips the decision in favor of producing. Note that this agrees with Dasgupta and Heal’s (1979, p. 335) conclusion. “The key point to note is that there is no reason for him (the monopsonist) to offer a price for the unextracted resource which is more than ‘infinitesimal’.”

The producers receive no rent as indicated by \(\lambda^0(t) = 0\) for all \(t\). The only thing the producers do is supply the inputs to extract the resource, and they do this at cost. The form of the extraction cost function makes this easy to achieve. Note that these necessary conditions
are independent of the extraction rate. This is because marginal cost is independent of this rate. At any time \( t_0 \), the producers have a horizontal supply function at the level \( c(z(t_0)) \); thus, demand determines the quantity as is the usual case when supply is horizontal.

Karp states that (p. 74), "If the buyer attempts to exert market power, he is constrained by the dynamic optimization behavior of the seller and he does not face a standard control problem." He also states (p.75), "If, however, the buyer is able only to purchase a flow, the dynamic optimizing behavior of the seller constrains his power. In that case, the buyer cannot in general expropriate the resource. (The exception is when extraction costs are constant.)" Thus, Karp recognized that all of the rent could be extracted when \( c'(z) = 0 \), and there is only a scarcity effect in the costate variable; however, he failed to recognize that it can also be extracted when \( c'(z) < 0 \), and the cost effect is also present. This appears to be due to not considering this possibility.

This makes it clear that the cost effect in the costate variable, Eq. (8c), is not a necessary part of the return to the producers; thus, we do not add Eq. (4b) as a constraint to the buyer’s optimization problem. The tariff is, therefore, dynamically consistent. We next analyze the case of bilateral monopoly.

**BILATERAL MONOPOLY**

When the seller is monopolistic and the buyers are competitive, the seller will not only receive all of the resource rent, but will also extract some of the consumers' surplus. To illustrate the relative magnitudes we use the numerical problem introduced by Karp, where (p.90) "the cost of extracting the last unit of the resource is equal to the choke price . . . " That is, \( P(0) = c(0) \). This implies that the terminal optimal costate variable is zero, which implies that the scarcity effect is zero; thus, the costate variable has only a cost effect.

In this problem \( P(x) = \alpha - x, c(z) = \kappa - z, \bar{z} = 1, r = 0.1 \), with \( \alpha = \kappa \). As Karp says
"The calculations are straight forward, but very tedious." The manipulations are shown in Appendix B. In the solution to the differential equations the constant of integration for the positive root is zero and the optimal terminal time is infinite. Table 1 shows the numerical results.

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>Perfect competition</th>
<th>Monopolist (competitive buyers)</th>
<th>Monopsonist (competitive sellers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer's payoff</td>
<td>0.0569</td>
<td>0.0350</td>
<td>0.3649</td>
</tr>
<tr>
<td>Seller's payoff</td>
<td>0.3079</td>
<td>0.3209</td>
<td>0</td>
</tr>
<tr>
<td>Total payoff</td>
<td>0.3649</td>
<td>0.3559</td>
<td>0.3649</td>
</tr>
</tbody>
</table>

In the competitive scenario the seller receives 84 percent of the total payoff while the buyer received only 16 percent. In the monopoly scenario the seller receives 90 percent of the total payoff, and in the monopsony scenario the buyer receives 100 percent of the payoff. This illustrates that the seller has the advantage as compared to the buyer in the first two scenarios, but this is reversed in the monopsony scenario.

This sets the stage for the bilateral monopoly discussion. If the buyer is the leader in a Stackelberg game then we will get the monopsony result of Table 1, and if the buyer uses Karp’s consistent tariff the seller’s payoff is 0.2654 and the buyer’s is 0.0769 for a total of 0.3423. Since the seller receives more payoff under competition, Karp concludes that the seller would behave competitively, and he also concludes that, (p.91) “if the seller behaves competitively, the follower has no choice but to do likewise.” As discussed above, however, the buyer will not do likewise. Instead the buyer will set the tariff so that the price received by the seller is arbitrarily close to zero, giving the producer the incentive to produce. This will

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3These manipulations are shown in the appendix because, even though they are straight forward, I do not get the same numbers that Karp did.
expropriate the entire resource rent as indicated in the last column of Table 1. The problem, however, is that the seller is not likely to be a passive follower in this one-on-one situation, nor is it likely that buyer will be a passive follower. The relative gains and losses are too great for either one to be passive. The most likely outcome is a negotiated one, having similar characteristics to the following. The buyer as the leader sets a tariff equal to say one-half of the costate variable, $q(t) = \frac{1}{2}\zeta^*(t)$ where $\zeta^*(t)$ is the current value form of $\psi^*(t)$ of Eq. (7c). The producer using the maximization problem of Eq. (9a)-(9d) follows the lead.

**SUMMARY**

In this paper we have examined optimal tariffs for non-renewable natural resources in the setting of imperfect competition. We do this because Larry Karp (1984, p. 74) states that, “If the buyer attempts to exert market power, he is constrained by the dynamic optimization behavior of the seller and does not face a standard control problem.” We show that when extraction costs are a function of the remaining stock of the resource, the costate variable can be separated into a scarcity effect and a cost effect. Karp concludes that the cost effect must be left with the producer; thereby, restricting the actions of the buyer. We, however, prove that it is not necessary to pay this cost effect to the producer; hence, we conclude that the monopsonist can extract all of the rent from the seller. The optimal tariff is neither dynamically time inconsistent, nor is it “Karp’s consistent tariff.”

When the seller is a monopolist and the buyers are competitive the seller will receive all of the rent and some of the consumers’ surplus, and when the buyer is a monopsonist and the sellers competitive, the buyer can extract all of the rent. In the case of bilateral monopoly, if the buyer is the leader in a Stackelberg game with the seller a passive follower, the buyer will extract all of the rent. If we reverse their roles, we get the monopoly results. Neither one, however, is likely to be a passive follower in this one-on-one situation. The most likely outcome for this case is a negotiated one.
APPENDIX A

Proof of Equation (5)

We start with Equation (8)

\[
\frac{d\zeta^*}{dt} = r\zeta^*(t) + c'(z^*(t))x^*(t)
\]  \hspace{1cm} (A1)

with \(\zeta^*(T^*)\) given by the transversality condition. This can be written

\[
\frac{d\zeta^*}{dt} - r\zeta^*(t) = c'(z^*(t))x^*(t)
\]

which is a linear first order ordinary differential equation with variable term. This differential equation has the integrating factor \(e^{rt}\) and general solution

\[
\zeta^*(t) = e^{rt}[A + \int e^{-rt} c'(z^*(t))x^*(t) \, dt], \quad \text{where } A \text{ is the constant of integration.} \quad (A2)
\]

Define:

\[
F(t) := \int e^{-rt} c'(z^*(t))x^*(t) \, dt
\]

Thus, Equation (A2) can be written

\[
\zeta^*(t) = e^{rt}(A + F(t))
\]

and

\[
\zeta^*(T^*) = e^{T^*}(A + F(T^*)) \quad \text{where } \zeta^*(T^*) \text{ is determined by the transversality condition.}
\]

Solving for \(A\) yields,
Using this result we get,

\[ \zeta^*(t) = e^{-r(T^*-t)} \zeta^*(T^*) - e^{T}[F(T^*) - F(t)] , \]

and

\[ \zeta^*(t) = e^{-r(T^*-t)} \zeta^*(T^*) - e^{T} \int_{t}^{T^*} e^{-rS} c'(z^*(t)) x^*(t) \, ds \]

which is the desired result.

APPENDIX B

In this appendix we solve two numerical problems. We first solve them analytically, and then apply the specific linear equations, \( u'(x) = P(x) = \alpha - x \), \( c(z) = \kappa - z \), \( \bar{z} = 1 \), \( r = 0.1 \), with \( \alpha = \kappa \). To find the efficient path we solve the current value form of Eq. (7a) - (7e). As defined in connection with Eq. (8) we define \( \zeta^*(t) \) to be the current value solution costate variable, \( \zeta^*(t) = e^{T} \psi^*(t) \), so Eq. (7a) - (7e) become

\[ u'(x^*(t)) - c(z^*(t)) - \zeta^*(t) = 0 \quad (B1a) \]

\[ \frac{d\zeta^*}{dt} = r\zeta^*(t) + c'(z^*(t))x^*(t) \quad (B1b) \]

\[ \frac{dz^*}{dt} = -x^*(t) \quad z^*(0) = \bar{z} \quad (B1c) \]

Transversality conditions:
Evaluating (B1a) at $T^*$ and combining with (B1d) yields

$$u(x^*(T^*)) - c(z^*(T^*))x^*(T^*) - \zeta^*(T^*)x^*(T^*) = 0$$  \hspace{1cm} (B1d)

$$\zeta^*(T^*) \geq 0 \hspace{1cm} \zeta^*(T^*) z^*(T^*) = 0$$  \hspace{1cm} (B1e)

which implies that $x^*(T^*) = 0$. Because this problem has (p.90) “the cost of extracting the last unit of the resource is equal to the choke price…” $[P(0) = c(0)]$, (B1a) evaluated at $T^*$ and (B1e) yield $\zeta^*(T^*) = z^*(T^*) = 0$. Suppose $z^*(T^*) > 0$, then (B1a) implies $\zeta^*(T^*) > 0$, and (B1e) implies $z^*(T^*) = 0$; hence we have a contradiction. With $z^*(T^*) = 0$ (B1a) implies $\zeta^*(T^*) = 0$.

Substitute the linear equations into (B1a) and solve for $x(t) = z(t) - \zeta(t)$. Substituting this into (B1b) and (B1c) yields,

$$\frac{d\zeta^*}{dt} = (1+r)\zeta^*(t) - z^*(t)$$  \hspace{1cm} (B1b')

$$\frac{dz^*}{dt} = \zeta^*(t) - z^*(t)$$  \hspace{1cm} (B1c')

This is a pair of first order, linear differential equations with an initial condition, $z^*(0) = \tilde{z} = 1$, and two terminal conditions $z^*(T^*) = \zeta^*(T^*) = 0$ where $T^*$ is also endogenous. The eigen values, $Er$, and normalized eigen vectors, $Ev$, are
\[ Er = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 0.3702 \\ -0.2702 \end{bmatrix} \quad Ev = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \begin{bmatrix} 0.8077 & 0.5895 \\ 0.5895 & 0.8077 \end{bmatrix} \]

Note that because the two differential equations are homogeneous, the particular integral is null; hence, the general solutions are

\[
\zeta^*(t) = c_1 v_{11} e^{r_1 t} + c_2 v_{12} e^{r_2 t} \quad \text{(B2a)}
\]

\[
z^*(t) = c_1 v_{21} e^{r_1 t} + c_2 v_{22} e^{r_2 t} \quad \text{(B2b)}
\]

where \( c_1 \) and \( c_2 \) are constants of integration. The equations at the boundaries are

\[
\zeta^*(T^*) = 0 = c_1 v_{11} e^{r_1 T^*} + c_2 v_{12} e^{r_2 T^*} \quad \text{(B3a)}
\]

\[
z^*(T^*) = 0 = c_1 v_{21} e^{r_1 T^*} + c_2 v_{22} e^{r_2 T^*} \quad \text{(B3b)}
\]

\[
z^*(0) = \bar{z} = c_1 v_{21} + c_2 v_{22} \quad \text{(B3c)}
\]

By substitution we can confirm that \( c_1 = 0, T^* \) infinite, and \( c_2 = \bar{z}/v_{22} \) is a solution. We now confirm that it is the only solution. If \( c_2 = 0 \) then (B3a) and (B3b) imply \( c_1 = 0 \) but this is not consistent with (B3c); hence \( c_2 \neq 0 \). This implies by (B3a) and (B3b)

\[
\frac{c_1 v_{11}}{c_2 v_{12}} = \frac{c_1 v_{21}}{c_2 v_{22}} = -e^{(r_2 - r_1)T^*}
\]

Using the elements of \( Ev \) we see that the only value of \( c_1 \) that satisfies the first equality is \( c_1 = 0 \), and this implies \( T^* \) is infinite.
The total payoff is (we suppress the asterisk and $t$ for brevity)\(^4\)

\[
J_T = \int_0^\infty e^{-\eta} [u(x) - c(z)x] \, dt
\]

\[
= \int_0^\infty e^{-\eta} [\alpha x - \frac{1}{2} x^2 - (\kappa - z)x] \, dt
\]

\[
= \int_0^\infty e^{-\eta} [\alpha x - \frac{1}{2} x^2] \, dt
\]

\[
= \int_0^\infty e^{-\eta} [-r_2 (c_2 v_{22} e^{r_2 t})^2 - \frac{1}{2}(r_2 c_2 v_{22} e^{r_2 t})^2] \, dt
\]

\[
= \int_0^\infty \exp((2r_2 - r) t) (c_2 v_{22})^2 (-r_2 - \frac{1}{2} r_2^2) \, dt
\]

\[
= - (c_2 v_{22})^2 (-r_2 - \frac{1}{2} r_2^2) / (2r_2 - r)
\]

(B4a)

The resource rent which in this problem accrues to the sellers is

\[
RR = \int_0^\infty e^{-\eta} \zeta x \, dt
\]

\[
= \int_0^\infty e^{-\eta} (-r_2) c_2 v_{12} v_{22} e^{2r_2 t} \, dt
\]

\[
= r_2 c_2^2 v_{12} v_{22} / (2r_2 - r)
\]

(B4b)

The return to the buyer is

\[
J_B = J_T - RR.
\]

(B4c)

The manipulations for the monopoly model when he is faced by price-taker buyers are

\(^4\)A Matlab m-file to calculate all of these values is stored at [www.usu.edu/mathecon/consistency/payoff.m]
very similar to those just performed. The objective functional is,

$$ J_M = \int_0^T e^{-rT} [P(x) - c(z)] x \, dt $$

and the necessary conditions in current value form are

$$ P' (x^{\oplus}(t)) x^{\oplus}(t) + P(x^{\oplus}(t)) - c(z^{\oplus}(t)) - \Lambda^{\oplus}(t) = 0 $$

(B6a)

$$ \frac{d\Lambda^{\oplus}}{dt} = r\Lambda^{\oplus}(t) + c'(z^{\oplus}(t))x^{\oplus}(t) $$

(B6b)

$$ \frac{dz^{\oplus}}{dt} = -x^{\oplus}(t) \quad z^{\oplus}(0) = \bar{z} $$

(B6c)

Transversality conditions:

$$ [P(x^{\oplus}(T^{\oplus})) - c(z^{\oplus}(T^{\oplus}))] x^{\oplus}(T^{\oplus}) - \Lambda^{\oplus}(T^{\oplus}) x^{\oplus}(T^{\oplus}) = 0 $$

(B6d)

$$ \Lambda^{\oplus}(T^{\oplus}) \geq 0 \quad \Lambda^{\oplus}(T^{\oplus}) z^{\oplus}(T^{\oplus}) = 0 $$

(B6e)

where $\Lambda$ is the current value costate variable, and the super $\oplus$ indicates the optimal path.

Manipulations of these yield $\Lambda^{\oplus}(T^{\oplus}) = z^{\oplus}(T^{\oplus}) = 0$.

Substituting the linear equations into (B6a) and solving yields $x(t) = z(t)/2 - \Lambda(t)/2$.

Substituting this result into (B6b) and (B6c) gives

$$ \frac{d\Lambda^{\oplus}}{dt} = (\frac{1}{2} + r)\Lambda^{\oplus}(t) - \frac{1}{2}z^{\oplus} $$
The Eigen values and Eigen vectors associated with this pair of homogeneous, linear, first order differential equations are

\[
E r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 0.2791 \\ -0.1791 \end{bmatrix} \quad E v = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \begin{bmatrix} 0.8416 & 0.5401 \\ 0.5401 & 0.8416 \end{bmatrix}
\]

We have the same boundary conditions as in the competitive model, and they yield, as before, \( c_1 = 0 \) and \( c_2 = z/v_{22} \) with \( T^\oplus \) infinite. These yield

\[
A^\oplus(t) = c_2 v_{12} e^{r_2 t}
\]

\[
z^\oplus(t) = c_2 v_{22} e^{r_2 t}
\]

\[
x^\oplus(t) = -r_2 c_2 v_{22} e^{r_2 t}
\]

The total payoff is given by Eq. (B4a) and (B4b), with the new parameter values. The monopolist’s payoff is given by (B5) and manipulates into

\[
J_M = \int_0^\infty e^{-\eta} \left[ (\alpha - x)x - (\kappa - z)x \right] \, dt
\]

\[
= \int_0^\infty e^{-\eta} \left[ zx - x^2 \right] \, dt
\]

\[
= \int_0^\infty e^{-\eta} \left[ -r_2 \left( c_2 v_{22} e^{r_2 t} \right)^2 - (r_2 c_2 v_{22} e^{r_2 t})^2 \right] \, dt
\]
The return to the buyer is

\[ J_B = J_T - J_M. \]

We next outline the manipulations for bilateral monopoly with Karp’s consistent tariff. The buyer’s objective functional (see Karp p. 89) is,

\[ J_B = \int_0^\infty e^{-\gamma t} [u(x) + P'(x)x^2 - c(z)x] \, dt \]  \hspace{1cm} \text{(B7)}

and necessary conditions in current value form

\[ u'(x^\odot(t)) + P'(x^\odot)x^\odot + 2P'(x^\odot)x^\odot - c(z^\odot(t)) - \phi^\odot(t) = 0 \]  \hspace{1cm} \text{(B8a)}

\[ \frac{d\phi^\odot}{dt} = r\phi^\odot(t) + c'(z^\odot(t))x^\odot(t) \]  \hspace{1cm} \text{(B8b)}

\[ \frac{dz^\odot}{dt} = -x^\odot(t) \hspace{1cm} z^\odot(0) = \tilde{z} \]  \hspace{1cm} \text{(B8c)}

Transversality conditions:

\[ u(x^\odot(T^\odot)) + P'(x^\odot)x^\odot - c(z^\odot(T^\odot))x^\odot(T^\odot) - \phi^\odot(T^\odot)x^\odot(T^\odot) = 0 \]  \hspace{1cm} \text{(B8d)}
This yields the same general form for the differential equations

\[
\frac{d\phi^\Theta}{dt} = (\frac{1}{2} + r) \phi^\Theta(t) - z^\Theta/3
\]

\[
\frac{dz^\Theta}{dt} = \phi^\Theta/3 - z^\Theta/3
\]

The solution to these is

\[
\phi^\Theta(t) = c_2 v_{12} e^{rt}
\]

\[
z^\Theta(t) = c_2 v_{22} e^{rt}
\]

\[
x^\Theta(t) = - r_2 c_2 v_{22} e^{rt}
\]

with \( c_2 = \bar{z}/v_{22}, \ r_2 = -0.1393, \ v_{12} = 0.5031, \) and \( v_{22} = 0.8642. \) The total payoff is given by (B4b) using the new parameter values, and the resource rent is given by (B4c) and the new parameter values. The buyer’s payoff is given by (B7) minus the resource rent,

\[
J_B = \int_0^\infty e^{-r} [u(x) + P'(x)x^2 - c(z)x] \ dt - RR,
\]

\[
= \int_0^\infty e^{-r} [\bar{\alpha}x - \frac{1}{2}x^2 - x^2 - (\kappa - z)x] \ dt - RR,
\]

\[
= \int_0^\infty e^{-r} [\bar{z}x - 1.5x^2] \ dt - RR,
\]
The seller's return is

\[ J_S = J_T - J_B. \]

REFERENCES


Do Optimal Non-Renewable Resource Tariffs Suffer from Dynamic Inconsistency?

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In this paper we have examined optimal tariffs for non-renewable natural resources in the setting of imperfect competition. We do this because Larry Karp (1984, p. 74) states that, “If the buyer attempts to exert market power, he is constrained by the dynamic optimization behavior of the seller and does not face a standard control problem.” We show that when extraction costs are a function of the remaining stock of the resource, the costate variable can be separated into a scarcity effect and a cost effect. Karp concludes that the cost effect must be left with the producer; thereby, restricting the actions of the buyer. We, however, prove that it is not necessary to pay this cost effect to the producer; hence, we conclude that the monopsonist can extract all of the rent from the seller. The optimal tariff is neither dynamically time inconsistent, nor is it “Karp’s consistent tariff.”

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Optimal control problem, Optimal non-renewable resource tariff, Dynamic inconsistency, Imperfect competition, Scarcity and cost effect
Do Optimal Non-Renewable Resource Tariffs Suffer from Dynamic Inconsistency?

In the October 1984 issue of this Journal Larry Karp examined imperfect competition between buyers and sellers of a non-renewable resource, and reached several conclusions that are in error. He begins by modeling the producers as price takers and the buyer as a monopsony. Among his conclusions are the following: (1) "If the buyer attempts to extract market power, he is constrained by the dynamic optimization behavior of the seller and he does not face a standard control problem." (p.74). (2) This results in the solution path being dynamically inconsistent.

(3) "Dasgupta and Heal (1979, pp. 335—336) state that a monopsonistic market for a non-renewable resource is equivalent to a monopoly, since a single buyer can offer a price arbitrarily close to extraction costs, and essentially expropriate the resource. This is true if the buyer is in a position to purchase the stock of the resource; he can then make the seller a 'take it or leave it' proposition. If, however, the buyer is able only to purchase a flow, the dynamic optimizing behavior of the seller constrains his power. In that case, the buyer cannot, in general expropriate the resource. (The exception occurs when extraction costs are constant.) Once the value of a stock is given, it makes no difference to the seller whether he sells a stock or a flow. However, the value to the seller of the stock depends on the rules of the game; one of the important rules, for the case of non-constant extraction costs, is whether the stock, or only a flow is sold." (pp. 74, 75)

We conclude that the control problem that maximizes the buyer's objective function is a standard control problem; hence, its solution path does not suffer from dynamic inconsistency. In addition, the buyer can expropriate the resource rent even when extraction costs are not constant and the purchase is of the flow; thus, we reach the same conclusion as Dasgupta and Heal. The source of the error is a failure to sufficiently examine the seller's optimization