12-2012

Behavioral Life-Cycle Models Under Alternative Demographic Regimes

Timothy F. Harris
Utah State University

Behavioral Life-Cycle Models Under Alternative Demographic Regimes

Timothy F. Harris

Master of Science- Economics

Jon M. Huntsman School of Business

Utah State University

Graduation Fall 2012

Behavioral Life-Cycle Models Under Alternative Demographic Regimes

1. Introduction

Social security in the United States will encounter financial strains due to an aging population. The decreasing worker to retiree ratio will reduce retirement benefits relative to today's benefits so long as the tax rate and eligibility age are not altered. Policy makers assume that they should make policies to maintain the current benefit levels.

In contrast to this assumption, this paper does not presume that policy makers should preserve current benefits. An analysis of the required tax rate to maintain lifetime utility across the current and future demographic regimes is undertaken rather than just a calculation of the tax rate needed to maintain present benefits. A comparison of life-cycle consumption models across the present and future demographic regimes shows that the tax rate should actually decrease 2-6 percentage points in order to equate lifetime utility. This result is opposite of the prescribed tax increase to maintain retirement benefits. This finding holds with varying assumptions of intertemporal decision making including exponential discounting (as in standard life-cycle models), hyperbolic discounting (as in Laibson, 1997), and short planning horizons (as in Caliendo & Aadland, 2006).

This counter-intuitive result is driven by the inability of a social security program (with a below market internal rate of return) to improve welfare for the models mentioned above under the present demographic regime (Caliendo (2011), Findley and Caliendo

(2008))[1]. The future demographic regime only reduces the internal rate of return and thus makes the program less advantageous. Therefore, in order to maintain utility across regimes, the pension program must be reduced.

2. Social Security Structure and Justification

The United States social security program represents the largest government transfer program in the world. In 2011 social security provided benefits to 55 million people and spent $736 billion (The Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds, 2012, p. 2). What are the justifications for such an extensive retirement program? Justification of the program on grounds of redistribution of wealth, market failures/inefficiencies, and paternalism will be addressed.

2.1 Redistribution of Wealth

One justification given for social security is that the program acts as a form of redistribution. Social security can act on both an individual level, because of the progressive nature of the benefit system, and on a generational level. Whenever the social security program stores excess income over costs, an implied redistribution of wealth occurs (Diamond, 1977, p. 278-279). Additionally, whenever the government increases benefits an indirect intergenerational transfer occurs (Blinder, 1988, p. 25). This transfer can be suitable because earlier generations are poorer on average than the younger generations throughout their respective life cycles (Diamond, 1977, p. 278-279). The intergenerational difference can be seen by examining the long run rate of real wage growth. The social security program itself is not a perfect instrument to accomplish

---

[1] An exception occurs for extremely short planning horizons in the model presented in Findley and Caliendo (2008). In this case, the findings are reversed.

societal redistribution goals. It has been pointed out that a more appropriate redistribution vehicle would not give the wealthier individuals benefits and definitely would not give them a higher percentage than the poor (Diamond, 1977, p. 279). Also, the intra-generational transfers could be thwarted by the increased longevity of the wealthy (Bagchi & Findley 2012).

2.2 Market Failure/Inefficiency

Market failure is another commonly used justification for the existence of social security. Lack of safe long-run investments causes uncertainty regarding income during retirement. The cost of fully insuring against this risk increases substantially when uncertainty regarding age of retirement and death are taken into account. If an individual is forced to retire early due to health complications or even motivation deficiencies, they can very easily find themselves unprepared for the ensuing years without wage income. Even comparatively safe investments such as mutual funds are prone to large value fluctuations (Diamond, 1977, p. 280).

The alleged annuity market failure could exist because of the relatively small demand for a luxury good. Generally, only those people that expect to live long lives buy annuities. This adverse selection causes private annuity firms to offer fewer benefits than the market demands. This problem could be a driving force in the short comings of the private annuity market, and could therefore provide reasonable justification for a public annuity program (Blinder, 1988, p. 19).

Additionally, the government has a substantially greater ability to spread and mitigate risk across generations and individuals than a private annuity firm. Similar to a private insurance program, the risk of living beyond the scope of retirement savings is subsidized by relatively premature deaths of other participants in the program. The government's comparative advantage occurs because of compulsory participation, which eliminates the main difficulty faced by private firms, adverse selection (Blinder, 1988, p. 19).

Furthermore, the government has lower administrative expenses than private annuity markets, partially due to the scale of the pension program. Even though these gains could come from large economies of scale, it is more likely a result of non-existent advertising costs and commissions from the compulsory program (Blinder, 1988, p. 23).

2.3 Paternalism

"There seems to be consensus that social security is primarily justified on grounds that people do not personally save appropriate amounts because they are shortsighted and they lack the ability to fully internalize during the working years their consumption needs for retirement" (Findley & Caliendo, 2008, p. 411).

Paternalistic motivation resulting from myopic behavior is the primary justification for a public pension program. Blinder defines paternalism as "[k]nowing that some individuals are too shortsighted, too ill-informed, or simply unwilling to face reality, the wise and benevolent government makes sure that everyone reaches retirement age with at least a minimal portion of lifetime earnings left" (Blinder, 1988, p. 28). Individuals display numerous forms of irrational behavior which could lead to under saving such as short planning horizons, hyperbolic discounting, rule of thumb savings, and hand to mouth consumption. These irrationalities could be theoretically countered by the existence of a public pension program. A "sizable fraction" of the population displays these impractical behaviors (Diamond, 1977, p. 283).

This paternalistic approach is not only displayed in the benefits provided, but also in the structure of the pension program. By legislating an appropriate retirement age for benefits, the social security program has provided an implicit incentive against premature retirement and has safeguarded against the corresponding financial deficiencies (Diamond, 1977, p. 281). Annuitized benefits during the retirement period provide a supplementary safety net for those that completely deplete private savings during the retirement period through compulsive spending or risky investments (Blinder, 1988, p. 18).

Even though these paternalistic reasons for social security are valid, the existence of such a program has corresponding adverse effects. One such example is pension induced decreased private savings. A third of beneficiaries receive 90 percent of their retirement income from social security (Diamond & Orszag, 2005, p. 7). This statistic shows that many individuals think that the government already has an institution in place to fund their retirement and therefore do not participate in private saving.

Regardless of the public pension program's shortcomings, it can be assumed that at least minimal societal improvements occur as a result of the program. However, it should be noted that diverse government programs would be able to fulfill these functions in a debatably more appropriate and direct fashion than a public pension program.

3. Future Status of the Social Security Program

The social security program will experience reduction in benefits per retiree due to changing demographics in the United States. It is projected that the government will not be able to fully cover scheduled benefits by 2037 when the Old-Age and Survivors

Insurance and Disability Insurance (OASDI) trust fund[2] is complete exhausted. This trust fund consists of the excess benefits over expenses received over the course of the program and is used to provide benefits when the pension income is insufficient for benefits. It is further projected that in 2037, the tax income will only be able to cover 76 percent of scheduled benefits. Social security, unlike other government programs, cannot borrow in order to finance its beneficiaries and therefore will become insolvent by 2037 without benefit reductions or governmental reform (Goss, 2010, p. 112-113).

Insolvency concerns are based largely on the changing demographic of the United States. As the ratio of workers to retirees decreases, the program is forced in the long run to decrease benefits without reform. The Board of Trustees estimates the various factors that are causing this change and projects the long run worker to retiree ratio.

Over the last century, the nation has experienced an increase in life expectancy. This is in part due to greater accessibility to health care, new and better distributed medications and immunizations, improved sanitation and food preparation, and other direct effects from higher GDP and standard of living. The question faced by the Trustees is to what extent these historical factors will continue to improve and to what extent will they stagnate. It can be argued that viruses and diseases in general are becoming more resistant to medications and will cause the life expectancy to decrease. Conversely, medical research and techniques are rapidly improving. Other factors that will affect the life expectancy of individuals include the degree of health education, use of detrimental substances, obesity, pollution, exercise, discovery of new diseases,

---

[2] Although this paper is principally focused on the OASI portion, historical borrowing between the funds makes assessment of the combined funds pertinent.

violence, government spending on health care, and several other factors with unknown magnitudes (Social Security Administration Office of the Chief Actuary, 2012, p. 3).

This great uncertainty about future trends of life expectancy has led demographers to project life expectancy based on the average historic growth rate. However, there is a dispute regarding how far back the data should go in order to obtain reasonable projections. In addition to this, some demographers argue that there is a "cap" on human longevity whereas others claim that the improvement is potentially limitless (Social Security Administration Office of the Chief Actuary, 2012, p. 3-4).

Forecasts show that the more fundamental and seemingly permanent decrease in birth rates is the main driving force of increased costs. Goss stated, "This increase in cost results from population aging, not because we are living longer, but because birth rates dropped from three to two children" (Goss, 2010, p. 111). Growth in social security costs between now until 2030 is almost completely caused by decreased birth rates, and that only after 2030 the increased life expectancy will have a significant impact (Goss, 2010, p. 123-124).

There are several practical reasons for this observed decrease in the total fertility rate (TFR) including changes in social norms, increased female labor force participation, birth control, postponement of marriage, and other factors. A culmination of all of these factors has led to a decrease from the current TFR of 2.9 to a long-run (75 years) rate of 2.0 (Social Security Administration Office of the Chief Actuary, 2012, p. 3).

All of these combined factors have direct or indirect effects on the worker to retiree ratio, which is a main indicator of sustainable benefits. For the OASI portion of social security, there were 3.6 workers to retiree in 2010 and the intermediate projection for 2060 is 2.4 workers to retiree (The Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds, 2012, p. 53). This ratio and the effects of different approximations will be analyzed later on.

4. Proposals for reform

This projected decline in benefits has led to many proposals for reform. The baseline estimate is that policy makers would have to permanently increase payroll tax from 12.40 percent to 15.01 percent or cut benefits by 16.2 percent in order to be solvent in the long run (The Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds, 2012, p. 61). Additionally, more specialized proposals for structural and parametric reform have been presented.

Diamond and Orszag proposed a plan that involves parametric reform similar to those presented by the Board of Trustees. They systematically dissect the causes of the social security crisis and prescribe parametric reform options for each area. They split the causes into three main categories, improvements in life expectancy, increased earnings inequality, and the burden of legacy debt (Diamond & Orszag, 2005, p. 1).

The stance is made that major structural reforms mentioned above are not needed or desired. They argue that incremental parametric reforms can accomplish desired goals without unreasonable damage to the populace’s wellbeing. They also point out that most plans for structural change, that includes the use of individual accounts, do not take into account the need to repay the existing debt, problems of early access to retirement funds, and uncertainty associated with market investments (Diamond & Orszag, 2005, p. 2-7).

Structural reform in contrast to parametric reform is based on the idea that the social security program is fundamentally flawed and that minor changes in taxes and benefits only produce short term solutions. It is believed that the estimated required tax increase is underestimated and will require a significantly higher tax for the system to become solvent. The claim is based on the theory that an increase in tax rate could actually decrease tax revenue under certain circumstances. This could occur because as the taxes rise, individuals will choose not to work as much (according to the elasticity of labor supply) or to accept payment in non taxable forms such as increased benefits. Consequently, parametric reform is less attractive because of the large requisite tax increase (Feldstein, 2005, p. 35-36).

One proposal for reform is a plan that integrates the pay-as-you-go system into a mixed investment system. This system would keep the current taxation and add government incentivized, personal investments. The government would offer matching payments for up to a given percent which could potentially vary based on income class for greater redistributive effects. Over time, the growth of benefits would decrease in order to compensate for these incentives offered for private savings. (Feldstein, 2005, p.38).

In contrast to current private investments, the plan would stipulate where individuals could invest in order to mitigate risk and determine when individuals could have access to the funds. Recommends restrictions similar 401(k) plans that include “diversified equity and bond mutual funds” would be implemented (Feldstein, 2005, p. 37-38). Another proposal gives the example of market weighted global index fund of stocks, bond and real estate in order to ensure diversity of the investments (Kotlikoff &Sachs, 1998, p. 12).

One of the major concerns with the restructuring of the public pension program is the management of existing debt. Feldstein addresses this concern and explains that the decrease in tax income would initially cause the trust fund balance to decline, but still remains positive even with scheduled benefit payouts. After the program becomes fully established, the trust fund will actually grow significantly (Feldstein, 2005, p. 40).

5. Changes in Well-Being Across Alternative Demographic Regimes

A better understanding of the difference between the demographic regimes gives policy makers the capacity to target certain aspects of the justifications mentioned above. Paternalistic policies need to take into account specific types of individuals and how they react to long-run steady state changes. The following analysis will show which type of representative individual is affected the most and consequently who might need the most paternalistic intervention. Additionally, the analysis will show what tax policies could be implemented to achieve a paternalistic goal for the different types of individuals.

In order to gauge the difference across the current and future demographic regimes on society and some of the effects of parametric reform, three commonly cited types of consumers will be studied in a life-cycle framework. A fully rational, forward-looking representative exponential discounter will act as the baseline model. Hyperbolic discounters will be used to represent a time-inconsistent irrational individual. Lastly, a representative short term planning horizon individual will be examined in order to gauge the relevance of foresight. The purpose of this analysis is not to measure or describe transition dynamics; rather it is to explain the difference between the future and current demographic regimes.

5.1 Exponential Discounting

Stability of preferences and consistency of evaluation across time is represented by exponential discount function. The representative individual is assumed to not give heed to appetite or impulsive reactions, rather the individual will stick to the same plan that was developed though a utility maximization problem presented at day one (Ainslie, 2005, p. 635-636). Additionally, the consumer has time-consistent preferences meaning that the solution to the utility maximization problem at any future point in time is equivalent to the plan made at the original planning instant.

In this exponential life-cycle model, the individual is assumed to have no misperceptions or uncertainty about the future. The representative agent perfectly knows future wages, quantity of benefits, and also retirement and death dates. The individual is also assumed to have no bequest motive and therefore will exhaust all of his resource by the time of death. Logarithmic utility will be used to represent the representative individual's preferences.

The individual is economically born or enters the work force at $t = 0$, retires at $t = T$, and dies at time $t = \bar{T}$. The individual's only source of income during the working period, $t \epsilon [0, T]$ are wages denoted $w(t)$. For simplicity, the individual is assumed to earn an economy wide wage that has a constant[3] growth rate γ. Wages at a given point in time are given by $w(t) = w(0)e^{\gamma t}$. The individual receives interest at rate r on all income that is not consumed or put into the individual's capital stock, $k(t)$. The individual pays into the pay-as-you-go social security program at a constant tax rate θ during $t \epsilon [0, T]$ and receives benefits $b(t) = R\theta w(t)$ during the retirement

[3] In reality, the life-cycle income profile is hump shaped rather than monotonically increasing.

period $t \epsilon [T, \bar{T}]$. The worker to retiree ratio, R, along with the initial wage, wage growth rate, and interest rate are exogenous. The model will utilize an adjusted worker to retiree ratio that already takes into account population growth and mortality rates across time. Therefore, no population growth will be modeled. (A richer model could be constructed that defines R as the ratio of survivor functions).

In order to gauge the magnitude of the effect of the projected demographic change on a representative exponential discounter, the *Maximum Principle* was applied to solve for the optimal consumption path, followed by the calculation of the compensating variation. The utility maximization problem is as follows:

$$\text{Max} \int_0^{\overline{\boldsymbol{T}}} e^{-\rho t} \, \text{u}[c(t)] \, dt$$

Subject to the following constraints:

$$\frac{dk}{dt} = rk(t) - c(t)$$

$$k(\bar{T}) = 0$$

This one stage maximization problem gives equivalent optimal consumption as the two stage control problem because of the perfect foresight assumptions. After applying the *Maximum Principle*, we get the following optimal paths:

$$k^*(t) = \frac{k(0)[e^{-\rho\bar{T}+rt} - e^{(r-\rho)t}]}{e^{-\rho t} - 1} \tag{1}$$

$$c^*(t) = \frac{k(0)e^{r\bar{T}+(r-\rho)t}(-\rho)}{e^{(r-\rho)\bar{T}} - e^{r\bar{T}}} \tag{2}$$

$$k(0) = \int_0^T e^{-rt} \, w(t)dt - \int_0^T e^{-rt} \, \theta \text{w}(\text{t})dt + \int_T^{\bar{T}} e^{-rt} b(t)dt \tag{3}$$

The main metric used to describe the effects of the projected demographic change is compensating variation. Compensating variation, given by Δ, is the required percentage change in consumption at every point in time that would cause lifetime utility to be equivalent across the different demographic regimes.[4] This metric is measured from the perspective of a planner that discounts utility at a constant social discount rate η. Given the present worker to retiree ratio R_p, and a future worker to retiree ratio R_f, compensating variation is given as follows:

$$\int_0^{\bar{T}} e^{-\eta t} \ln[(1+\Delta) c_p(t)] dt = \int_0^{\bar{T}} e^{-\eta t} \ln[c_f(t)] dt \tag{4}$$

Solving forΔ:

$$\Delta = exp\left[\frac{\int_0^{\bar{T}} e^{-\eta t} \ln\left[c_f(t)\right] dt - \int_0^{\bar{T}} e^{-\eta t} \ln[c_p(t)] dt]}{\int_0^{\bar{T}} e^{-\eta t} dt}\right] - 1 \tag{5}$$

Using optimal consumption, (2), from the optimization problem and simplifying we get:

$$\Delta = \frac{k_f(0)}{k_p(0)} - 1 \tag{6}$$

Or equivalently:

$$\Delta = \frac{Present\ Value\ of\ Lifetime\ Wealth\ in\ the\ Future\ Demographic\ Regime}{Present\ Value\ of\ Lifetime\ Wealth\ in\ the\ Present\ Demographic\ Regime} - 1 \tag{7}$$

This result shows that the compensating variation is exclusively a function of the ratio of lifetime wealth in the future to current lifetime wealth.

---

[4] Gahramanov and Findley (2011) study this question for hand-to-mouth consumers.

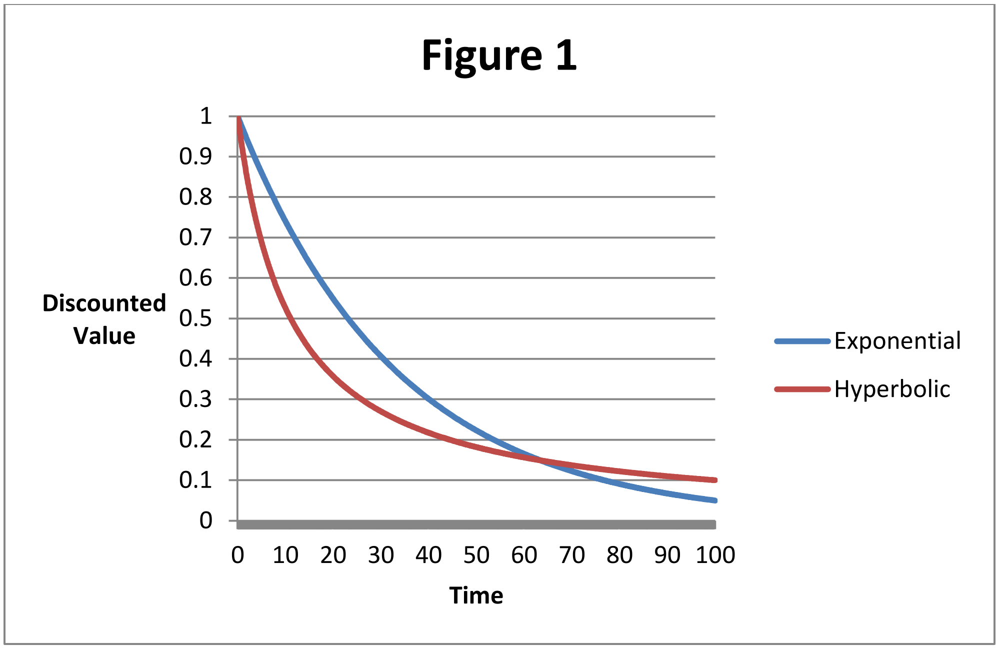
5.2 Hyperbolic Discounting Description

Hyperbolic discounters are commonly used as an example of irrationality. This time-inconsistent discounting is relevant in the study of public pensions because it characterizes the observed human behavior of impatience in short-term decisions and patience in long-term decisions which has been used to justify paternalistic intervention.

Unlike the rational exponential discounter, the hyperbolic discounter is susceptible to realistic appetites, impulses and inconsistent discounting of future events. The value of a good that is hyperbolically discounted takes on the following form:

$$Value = \frac{Value\ at\ no\ delay}{Constant + [(Impatience\ factor * Delay)]}$$

The constant is assumed to be one for this paper, and is present to prevent the discounted value from approaching infinity as the delay approaches zero. As the delay increases, the discounted value of the good also decreases but at a decreasing rate as seen in Figure 1 (Ainslie, 2005, p. 636). This form of inconsistent discounting causes *dynamic inconsistency*, meaning that the individual may reverse previous planned decisions (Prelec, 2004, p. 512).



Hyperbolic discounting implies the "tendency to prefer smaller rewards over larger ones temporarily, when the smaller reward is imminently available" (Ainslie, 2005, p. 636).

"In contrast to exponential curves, hyperbolic discount curves depict a strong by temporary tendency to prefer smaller and sooner (SS) rewards to larger and later (LL) ones, in the period just before an SS reward is due" (Ainslie, 2005, p. 637). Also, hyperbolic discount implies that small delays in the present decrease the value of good proportionately more than small delays in the future (Angeletos, Laibson, Repetto, Tobacman, & Weinberg, 2001, p. 50).

These realistic characteristics and others have caused the hyperbolic discount function to better fit choices relative to the exponential discount function in a series of animal and human experiments. The prevalence of hyperbolic discounting in animal experiments demonstrates that these tendencies are not a result of societal expectations (Ainslie, 2005, p. 637).

A single individual is actually comprised of conflicting and many times intransitive preferences. "[A]s time elapses these [perceived outcomes of preferences] shift their relationship with one another from cooperation on a common goal to competition for mutually exclusive goals" (Ainslie, 2005, p. 637).

The outcome of such competition is a function of the rationalization that occurs, and the perception of a one-time exception rather than a complete denial of rational planning. A comparison is drawn between this inner battle/rationalization and a repeated prisoner's dilemma bargaining game. In a prisoner's dilemma, cooperation is contingent on the belief that a defective act will have negative consequences in future games. Rationalization allows the individual to believe that a one time "defect" will not have any effect on the future cooperation of the rational self. However, prevalence of this rationalization or intertemporal bargaining gives rise to time preference (Ainslie, 2005, p. 642). If people perceive current decisions as indicators of future decisions or if they evaluate and internalize past moments of indiscretion, these time preference effects would decrease (Ainslie, 2005, p. 650).

This intertemporal bargaining can be the result of naïve and also rationalized procrastination. Naïve procrastination is the belief that you will start saving, dieting, exercising, etc. tomorrow. In reality, the individual will continue to push these activities to a future date unless some deadline, restriction, or change in environment causes a change in behavior. Planned or Rationalized procrastination involves an individual that is fully aware of their future selves and environment. For example, an individual could put off a task, naively believing that they will do it the next day until they are forced to do it the last moment before a deadline. A different rational individual, knowing that the task will by unpleasant, will plan to do it all on the last day rather than spreading the dissatisfaction across the entire time frame (Akerlof, 2002, p. 423-424).

Whether it is naïve procrastination, intertemporal bargaining, or rational procrastination, a hyperbolic discount function is better suited than the time consistent exponential discount function.

A further shortcoming of exponential discounting is its incapability of reasonably describing impatience. In order for short-term impatience to be included, the exponential discounter must discount the future at an extremely unrealistic and implausible rate. “Hence, exponential discounting is a theory of virtually 100% short term patience.” Hyperbolic discounting, on the other hand accommodates impatience without unreasonable long-term impatience (Rabin, 2002, p. 671).

Further evidence in support of hyperbolic discounting is given by the observed decline in retirement consumption relative to consumption during the working life. This behavior is hard to explain using an LCPI model, whereas the decline in consumption is generally observed using a hyperbolic life-cycle model (Akerlof, 2002, p. 424).

A further criticism of the standard models is that economists in general overestimate the intellectual capacity of the populace. People generally do not solve utility maximization problems in order to determine consumption let alone directly link intentions and actions. Also, people seem to “err on the side of instant gratification” rather than abide to strict plans (Laibson, Repetto, & Tobacman, 1998, p. 92-93). (Even though, the hyperbolic discounter model does involve utility maximization, the time-inconsistency aspect allows for divergence from the original planned consumption path).

Survey information and societal organizations further establish the existence of time-inconsistency. There are several surveys where individuals describe themselves to be intrinsically impulsive rather than forward looking rational planners. Additionally, the mere existence of organizations that discourage impulsive behaviors such as financial planning institutions, and Christmas clubs are evidence of impulsive or time-inconsistent behavior (Angeletos, Laibson, Repetto, Tobacman, & Weinberg, 2001, p. 93-94).

Postponement or delay is an example of the infeasibility of exponential discounting. If an exponential discounter decides to postpone an activity, then it must be optimal to postpone that activity at a future point in time as well. This implies that any activity that is postponed will never be done regardless of time elapse, which is not realistic. Hyperbolic discounting allows for this possibility because of time effects (Prelec, 2004, p. 513).

To illustrate the applicability of hyperbolic discounting to the analysis of public pensions, Caliendo (2011) gives the following example:

> A naïve hyperbolic consumer standing at date 0 may intend to save part of his paycheck at date 1 for retirement, but when date 1 arrives he may not follow through with this plan. Unlike the impatient exponential consumer whose poverty in old age is all part of a grand plan first concocted when young, the hyperbolic consumer may unwittingly accumulate a suboptimal nest egg for retirement (p.668).

The above cited evidence for hyperbolic discounting gives strong intuitive support at first glance for a public pension program to help with the adverse effects from impulsive behavior and inadequate savings.

5.2.1 The Hyperbolic Life-Cycle Model

The hyperbolic discount function is given by $F(t - t_0) = [1 + \beta(t - t_0)]^{-1}$ where β is the discount parameter and t_0 is the planning instant. This discount function is steeper in the short term and flatter in the long term relative to the rational discount function. This present bias causes time inconsistency and preference reversal over the life-cycle (Findley & Caliendo, 2008, p. 420).

For any discount function, the growth rate of consumption is independent of social security. The only effect of social security occurs at time $t = 0$ for a hyperbolic discounter. The nature of the optimal consumption and capital paths for a hyperbolic discounter discourage an analytical solution. Therefore, an alternative, equivalent calculation of compensating variation is defined as follows because of the independence of the growth rate of consumption and social security (Caliendo, 2011).

$$(1 + \Delta)c_p(0) = c_f(0) \tag{8}$$

The optimal consumption path for $t \in [0, T]$, derived from the envelope of the planned consumption at every point in with wage growth evaluated at $t = 0$ is given by:

$$c(0) = \frac{k(0) + \frac{(1-\theta)w(0)}{(\gamma - r)}\left(e^{(\gamma - r)T} - 1\right) + \frac{R_p \theta w(0)}{(\gamma - r)}\left(e^{(\gamma - r)\bar{T}} - e^{(\gamma - r)T}\right)}{\frac{1}{\beta}\ln[1 + \beta\bar{T}]} \tag{9}$$

Substituting (9) into (8) and simplifying:

$$(1+\Delta)\left[k(0)+\frac{(1-\theta)w(0)}{(\gamma-r)}\left(e^{(\gamma-r)T}-1\right)+\frac{R_p\theta w(0)}{(\gamma-r)}\left(e^{(\gamma-r)\bar{T}}-e^{(\gamma-r)T}\right)\right]$$

$$=k(0)+\frac{(1-\theta)w(0)}{(\gamma-r)}\left(e^{(\gamma-r)T}-1\right) \tag{10}$$

$$+\frac{R_f\theta w(0)}{(\gamma-r)}\left(e^{(\gamma-r)\bar{T}}-e^{(\gamma-r)T}\right)$$

Rearranging, solving for Δ and substituting in the initial condition $k(0)=0$:

$$\Delta=\frac{\frac{(1-\theta)w(0)}{(\gamma-r)}\left(e^{(\gamma-r)T}-1\right)+\frac{R_f\theta w(0)}{(\gamma-r)}\left(e^{(\gamma-r)\bar{T}}-e^{(\gamma-r)T}\right)}{\frac{(1-\theta)w(0)}{(\gamma-r)}\left(e^{(\gamma-r)T}-1\right)+\frac{R_p\theta w(0)}{(\gamma-r)}\left(e^{(\gamma-r)\bar{T}}-e^{(\gamma-r)T}\right)}-1 \tag{11}$$

Or equivalently:

$$\Delta=\frac{Present\ Value\ of\ Lifetime\ Wealth\ in\ the\ Future\ Demographic\ Regime}{Present\ Value\ of\ Lifetime\ Wealth\ in\ the\ Present\ Demographic\ Regime}-1 \tag{12}$$

Hyperbolic discounters have the same solution to the compensating variation as exponential discounters. The exponential compensating variation is independent of ρ, and the hyperbolic compensating variation is independent of β. This is not necessarily intuitive because one could suppose that the time inconsistent hyperbolic individual would be made worse off because of the myopic behavior or present-bias. However, both individuals are affected by the same proportional amount because the worker to retiree ratio only appears in the lifetime endowment of wealth for both the hyperbolic and exponential discounters after all discounting effects have canceled out.

Given the baseline parameters given in table 1, both hyperbolic and exponential discounters will experience a 1.80 percent decline in consumption at every point in time under the future demographic regime. If the social planner does not discount future consumption, the compensating variation remains unchanged.

Table 1
Baseline calibration

Parameter name	Symbol	Value
Date of retirement	T	40
Date of death	$\bar{T}$	55
Real rate of return	r	0.035
Private discount rate	ρ	0.035
Length of planning horizon	x	3
Curvature parameter	φ	1
Initial capital stock	$k(0)$	0
Initial wage	$w(0)$	40,000
Income tax	θ	0.106
Present worker to retiree ratio	R_p	3.6
Future worker to retiree ratio	R_f	2.4
Wage growth	γ	0
Social Discount rate	η	0.035

In determining the baseline parameters, the board of trustee's worker to retiree intermediate projection of 2.4 for the year 2060 was used just taking into account Old Age and Survivors Insurance (OASI). Also, following the example of other literature, (Findley & Caliendo, 2009, p. 498) it has been assumed that the individual bears the full burden of the OASI social security tax.

5.3 Hyperbolic/Exponential Parameter Effects on Compensating Variation

The simple functional form of the compensating variation for exponential and hyperbolic individuals implies that the compensating variation is invariant to both the initial wealth as well as the discount rate. The difference between wage growth and the interest rate does affect the compensating variation. As the absolute value of the difference between the wage rate and interest rate increases, the compensating variation decreases. This occurs because the interest rate represents the market rate of return and the wage rate is directly correlated to the internal rate of return (IRR) of social security

(Feldstein, 1985, Findley and Caliendo, 2008, p. 416, Goss, 2010, p. 122). Therefore, as the difference between the interest and wage growth rate increases, the social security program gets relatively less important to the individual. This results in the need for a relatively smaller compensation in order to maintain current lifetime consumption levels in the future demographic regime.

5.4 Policy Implications for Exponential and Hyperbolic Discounters

Policy makers can paternalistically maintain the current level of lifetime utility for the hyperbolic and exponential consumers through a change in the tax rate. Mathematically, the policy maker would have to choose a θ_f that minimizes the compensating variation. For the exponential and hyperbolic discounters, the policy maker is able to overcome all differences in lifetime utility through a tax change.

Substituting (3) into (7) and setting $\Delta = 0$ gives the analytical problem for the exponential and hyperbolic discounters:

$$0 = \frac{\int_0^T e^{-rt}\ w(t)dt - \int_0^T e^{-rt}\,\theta_{\mathrm{f}}\mathrm{w(t)}dt + \int_T^{\bar{T}} e^{-rt} b_f(t)dt}{\int_0^T e^{-rt}\ w(t)dt - \int_0^T e^{-rt}\,\theta_{\mathrm{p}}\mathrm{w(t)}dt + \int_T^{\bar{T}} e^{-rt} b_p(t)dt} - 1 \qquad (13)$$

Integrating and solving for θ_f:

$$\theta_f = \theta_p \frac{\left[1 - e^{(\gamma-r)T}\right] + R_p\left[e^{(\gamma-r)\bar{T}} - e^{(\gamma-r)T}\right]}{\left[1 - e^{(\gamma-r)T}\right] + R_f\left[e^{(\gamma-r)\bar{T}} - e^{(\gamma-r)T}\right]} \qquad (14)$$

Taking the partial derivative with respect to R_f:

$$\frac{\partial\theta_f}{\partial R_f} = -\theta_p \left\{\frac{\left[1 - e^{(\gamma-r)T}\right]\left[e^{(\gamma-r)\bar{T}} - e^{(\gamma-r)T}\right] + R_p\left[e^{(\gamma-r)\bar{T}} - e^{(\gamma-r)T}\right]^2}{\left[1 - e^{(\gamma-r)T} + R_f\left(e^{(\gamma-r)\bar{T}} - e^{(\gamma-r)T}\right)\right]^2}\right\} \qquad (15)$$

Given the baseline parameters and $\gamma < .025$ the partial derivative will be positive. This implies that the larger the difference between the current and future worker to retiree ratios, the larger the requisite tax decrease will have to be to make the compensating variation equal zero. This occurs because as the worker to retiree ratio in the future regime decreases, the representative individual is made worse off. Any reduction in the tax rate increases welfare for a hyperbolic or exponential discounter in a pension program with a negative net present value (Caliendo, 2011, p. 671-673). Therefore a tax reduction is needed at an increasing amount when the worker to retiree ratio drops.

Using the baseline parameters, the policy maker would have to decrease taxes for exponential and hyperbolic discounters from 10.6 percent to 8.10 percent to equate equality across the different demographic regimes. Like the compensating variation, the required tax change is independent of the policy maker's discount rate or the personal discount rate.

5.5 Short Planning Horizon

Individuals have been observed to have planning horizons that are significantly less than an exponential discounter with perfect foresight. Also, individuals do exhibit savings and therefore deviate away from the hand-to-mouth consumer model. Short planning horizons models act as a "compromise" between the two extremes. Analytically, exponential and hand-to-mouth consumer models are merely limiting cases of the short planning horizon model (Caliendo & Aadland, 2006, p. 1393). There is also significant survey data that demonstrates that a large portion of the population displays planning horizons that are below 10 years (Caliendo & Aadland, 2006, p. 1394-1398).

In the short planning horizon, an individual only plans for a given time period and completely discounts any event outside of the planning horizon. This assumption is advantageous because it implies the commonly observed hump shaped consumption profile without calibrating adjustments (Caliendo & Aadland, 2006, p. 1393).

For the analysis, this paper will follow the model set up and solutions of Findley & Caliendo (2009). The model is split up into 4 different phases depending on the events that are included in the representative individual's planning horizon.

- Phase 1 is $[0,\ T-x]$
- Phase 2 is $[T-x,\ T]$
- Phase 3 is $[\ T, \bar{T}-x]$
- Phase 4 is $[\ \bar{T}-x, \bar{T}]$

Phase 1 includes the working period before the individual plans for the retirement period; Phase 2 consists of the rest of the working period where the retirement period is part of the individual's planning horizon; Phase 3 includes the retirement period where the individual cannot see until the death date; and, Phase 4 is the remaining portion of the retirement period that corresponds to the retirement period where the death date is part of the planning horizon.

Applying the *Maximum Principle* and solving for the envelope of the consumption and capital paths for the four phases, we get the following equations which further allow for the calculation of compensating variation:

5.5.1 Phase 1

$$c^1(t) = k^1(t)z_1 - e^{\gamma t}z_2\ for\ t\epsilon[0, T-x] \tag{16}$$

where

$$z_1 \equiv \frac{g-r}{e^{(g-r)x}-1} \tag{17}$$

$$z_2 \equiv \frac{(1-\theta)w(0)[e^{rx}-e^{\gamma x}](g-r)}{(\gamma-r)[e^{gx}-e^{rx}]} \tag{18}$$

$$g \equiv \frac{(r-\rho)}{\varphi} \tag{19}$$

and

$$k^1(t) = \frac{[(1-\theta)w(0)+z_2]}{(\gamma+z_1-r)}\left[e^{\gamma t}-e^{(r-z_1)t}\right] for\ t\epsilon[0,T-x] \tag{20}$$

5.5.2 Phase 2

$$\begin{aligned} c^2(t) = k^2(t)z_1 + z_1\frac{(1-\theta)w(0)e^{rt}}{(\gamma-r)}\left[e^{(\gamma-r)T}-e^{(\gamma-r)t}\right] \\ + z_1\frac{R\theta w(0)e^{rt}}{(\gamma-r)}\left[e^{(\gamma-r)(t+x)}-e^{(\gamma-r)T}\right] for\ for\ t\epsilon[T-x,T] \end{aligned} \tag{21}$$

where

$$\begin{aligned} k^2(t) = k(T-x)e^{(r-z_1)(t-T+x)} \\ + \frac{w(0)(1-\theta)e^{(r-z_1)t}}{(\gamma+z_1-r)}\left[e^{(\gamma+z_1-r)t}-e^{(\gamma+z_1-r)(T-x)}\right] \\ + [R\theta+\theta-1]\frac{w(0)e^{(\gamma-r)T+(r-z_1)t}}{(\gamma-r)}\left[e^{z_1 t}-e^{z_1(T-x)}\right] \\ + \left[1-\theta-R\theta e^{(\gamma-r)x}\right]\frac{z_1 w(0)e^{\gamma t}}{(\gamma-r)(\gamma+z_1-r)} \\ - \left[1-\theta-R\theta e^{(\gamma-r)x}\right]\frac{z_1 w(0)e^{(r-z_1)t+(\gamma+z_1-r)(T-x)}}{(\gamma-r)(\gamma+z_1-r)}\ for\ for\ t\epsilon[T \\ - x,T] \end{aligned} \tag{22}$$

5.5.3 Phase 3

$$c^3(t) = k^3(t)z_1 - e^{\gamma t}z_3\ for\ t\epsilon[T,\bar{T}-x] \tag{23}$$

where

$$z_3 \equiv \frac{R\theta w(0)[e^{rx} - e^{\gamma x}](g - r)}{(\gamma - r)[e^{gx} - e^{rx}]} \tag{24}$$

and

$$k^3(t) = k(T)e^{(z_1 - r)(T - t)} + \frac{[R\theta w(0) + z_3]}{(\gamma + z_1 - r)} e^{\gamma t} - \frac{[R\theta w(0) + z_3]}{(\gamma + z_1 - r)} e^{(r - z_1)t + (\gamma + z_1 - r)T} \; for \; t\epsilon[T, \bar{T} - x] \tag{25}$$

5.5.4 Phase 4

$$c^4(t) = \frac{e^{gt}\left\{k(\bar{T} - x)e^{rx} - \frac{e^{\gamma(\bar{T} - x)}R\theta w(0)[e^{rx} - e^{\gamma x}]}{\gamma - r}\right\}(g - r)}{e^{g\bar{T}} - e^{g(\bar{T} - x) + rx}} \; for \; t\epsilon[\bar{T} - x, \bar{T}] \tag{26}$$

Compensating variation for this model is defined the same as for exponential discounter:

$$\int_0^{T-x} e^{-\eta t} \ln\left[(1 + \Delta)c_p^1(t)\right] dt + \int_{T-x}^{T} e^{-\eta t} \ln\left[(1 + \Delta)c_p^2(t)\right] dt + \int_{T}^{\bar{T}-x} e^{-\eta t} \ln\left[(1 + \Delta)c_p^3(t)\right] dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\eta t} \ln\left[(1 + \Delta)c_p^4(t)\right] dt$$
$$= \int_0^{T-x} e^{-\eta t}\ln[c_f^1(t)]dt + \int_{T-x}^{T} e^{-\eta t}\ln[c_f^2(t)]dt + \int_{T}^{\bar{T}-x} e^{-\eta t}\ln[c_f^3(t)]dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\eta t}\ln[c_f^4(t)]dt \tag{27}$$

Table 2 presents the results generated in *Excel* by approximation techniques using the baseline parameters.

Table 2

x	Δ; η=0.035	Δ; η=0
1	-4.53%	-10.35%
2	-4.30%	-9.89%
3	-4.07%	-9.38%
4	-3.84%	-8.83%
5	-3.62%	-8.26%
6	-3.42%	-7.72%
7	-3.23%	-7.21%
8	-3.07%	-6.76%
9	-2.93%	-6.36%
10	-2.81%	-6.00%
11	-2.71%	-5.68%
12	-2.61%	-5.39%
13	-2.53%	-5.14%
14	-2.46%	-4.90%
15	-2.40%	-4.69%

Given the baseline parameters, it can be observed that the compensating variation decreases in magnitude as the planning horizon for the representative agent increases. This is also true for all discount rates under 7 percent. This result can be explained as being driven by the agent's capacity to plan further into the future and mitigate the loss of utility from the demographic change.

The absolute value of the compensating variation increases significantly when the social planner does not discount the future. This occurs because the change in demographic regime only affects the retirement period and the proceeding planning horizon which is relatively more discounted by a social discounter with $\eta = 0.035$ than the periods that remain unaffected.

As the planning horizon is pushed out to the entire life span of the representative individual, the short planning horizon compensating variation will approach the compensating variation for the exponential discounter. The higher compensating variation corresponding to the short planning horizon individual is a result of the concave utility function and the corresponding utility gains by smoothing consumption.

The wage growth rate/ interest rate relationship that was described in the hyperbolic/exponential section applies to the short planning horizon model as well.

5.6 Policy Implications for Short Planning Horizons

Policy makers are concerned with the welfare of individuals during the retirement period, many times at the expense of the working period. Consequently, many of the proposed reforms involve ways of keeping retirement benefits constant by increasing taxation. Individuals who follow a short planning horizons model will not initially alter their consumption profile because of the expectation of lower benefits. They therefore offer a baseline to study the effects of raising taxation without the smoothing effect displayed by hyperbolic and exponential discounters.

To equate benefits before and after the demographic change, policy makers would have to increase taxes to 15.37% [found by solving $R_p\theta_p w(t) = R_f\theta_f w(t)$]. However, in order to maintain the same life-time utility across the two demographic regimes the tax rate would have to deviate from this baseline calculation. In order to minimize the absolute value of the compensating variation the government would alter taxes as reported in table 3.

Table 3

x	θ_f; η=0.035	Δ; η=0.035	θ_f; η=0	Δ; η=0
1	11.42%	-4.50%	27.71%	-0.36%
2	10.39%	-4.30%	27.01%	-1.29%
3	9.14%	-3.99%	25.98%	-2.33%
4	7.60%	-3.50%	24.62%	-3.43%
5	5.62%	-2.77%	22.88%	-4.50%
6	3.07%	-1.70%	20.68%	-5.45%
7	0%	-0.16%	17.90%	-6.16%
8	4.24%	0%	14.39%	-6.52%
9	5.55%	0%	9.87%	-6.35%
10	6.31%	0%	3.92%	-5.46%
11	6.82%	0%	0%	-3.72%
12	7.20%	0%	0%	-2.07%
13	7.49%	0%	0%	-0.67%
14	7.71%	0%	1.13%	0%
15	7.89%	0%	2.77%	0%

In contrast to the exponential and hyperbolic models, it is impossible for the government to achieve a compensating variation equal to zero for individuals with planning horizons shorter than 8 years using the baseline parameters. This occurs because individuals with shorter planning horizons benefit from the pensions program and sufficient value cannot be added through a decrease or increase in taxes. As the planning horizon increases, the social planner would have to increase taxes to decrease welfare under the future demographic regime to have a compensating variation equal to zero.

For the case where the social planner does not discount the future, the tax rate which minimizes the absolute value of the compensating variation actually exceeds the baseline tax rate for planning horizons smaller than 9 years. The intuition is that if the social planner does not discount, the program is more advantageous for individuals with short planning horizons. Therefore, a higher tax rate (meaning greater implementation of the program) provides a higher level of welfare as defined by the social planner. The slight increase in the tax rate as the planning horizon approaches 15 years is a punitive action that disallows the future regime from having a higher lifetime utility than the current demographic regime.

In the case of exponential and hyperbolic discounters where the program (with negative net present value) is never beneficial, it seems intuitive to let the benefits drop and not increase taxation. It has been observed that individuals who benefit from social security now will also benefit under future demographics (Findley & Caliendo, 2009, p. 507). This observation makes it seems reasonable a priori to assume that if the social security improves lifetime utility for a certain type of individual, such as short planning horizon agents, that the government should take action to maintain the benefits from the program through a tax increase.

The model projections show that the baseline compensating variation actually increases for all planning horizons if the government raises taxes to 15.3% to equate benefits across the demographic change. In addition, it can be shown that tax reductions actually decrease, improve, the compensating variation for all planning horizons except for those with a one year planning horizon assuming the baseline parameters.

Additionally, in the case of individuals with a planning horizon greater than 7 years, a complete discontinuation of the program would not only reduce the compensating variation, but would make them better off than the individuals under the current demographic regime.

As would be expected, a decrease in the worker to retiree ratio makes the program less advantageous for the short planning horizon individual. Therefore, there is evidence that participation in the program, or tax rate, should decrease and not increase to keep benefits constant for individuals who represent short planning horizons.

6. Implications

The equality of the compensating variation for the hyperbolic and exponential discounter, along with the greater compensating variation for a representative with a short planning horizon, illustrates that the effect of a change in demographic regimes depends on the agent's foresight and not bounded rationality.

7. Conclusion

This paper explores the implications of the United States social security program under two separate demographic regimes. The compensating variation diverges significantly depending on the model and the degree of foresight implied therein. Counter intuitively, it has been shown that irrationality, modeled through hyperbolic discounting, does not affect the compensating variation. In order to keep lifetime utility constant between the two demographic regimes, policy makers would have to decrease taxation for all representative individuals except for those who are currently benefited the most by the social security program. It has additionally been shown that an increase in taxes to maintain current benefits only increases the divergence in lifetime utility across the representative periods.

References

Ainslie, G. (2005). Précis of breakdown of will. *Behavioral and Brain Sciences*, *28*, 635-673. Retrieved from http://www.Pioeconomics.com

Akerlof, G. A. (2002, June). Behavioral macroeconomics and macroeconomic behavior. *The American Economic Review, 92(3)*, 411-433. Retrieved from http://links.jstor.org/sici?sici=0002-8282%28200206%2992%3A3%3C411%3ABMAMB%3E2.0.CO%3B2-N

Angeletos, G., Laibson, D., Repetto, A., Tobacman, J., & Weinberg, S. (2001, Summer). The hyperbolic consumption model: Calibration, simulation, and empirical evaluation. *Journal of Economic Perspectives*, *15(3)*, 47-68.

Bagchi, S. & Findley, T. S. (Spring 2012) Differential increases in longevity and social security reform.

Blinder, A. S. (1988). Why is the government in the pension business? In S. M. Wachter (Ed.), *Social security and private pensions: Providing for retirement in the twenty-first century* (pp. 17-43). University of Pennsylvania: Lexington Books.

Caliendo, F., & Aadland, D. (2006, July 14). Short-term planning and the life-cycle consumption puzzle. *Journal of Economic Dynamics & Control*, *31*, 1392-1415. doi:10.1016/j.jedc.2006.05

Caliendo, F. N. (2011). Time-inconsistent preferences and social security: Revisited in continuous time. *Journal of Economic Dynamics & Control*, *35*, 668-675. doi:10.1016/j.jedc.2010.12.006

Diamond, P. A. (1977, June). A framework for social security analysis. *Journal of Public Economics*, *8*, 275-298.

Diamond, P. A., & Orszag, P. R. (2005). Saving social security: The diamond-Orzag plan. *The Economists' Voice*, *2*(1), 1-8. Retrieved from http://www.bepress.com/ev/vol2/iss1/art8

Feldstein, M. (1985, May). The optimal level of social security benefits. *The Quarterly Journal of Economics*, *100*(2), 303-320. Retrieved from http://www.jstor.org/stable/1885383

Feldstein, M. (2005, Spring). Structural reform of social security. *Journal of Economic Perspectives*, *19*(2), 33-35.

Findley, T. S., & Caliendo, F. N. (2008, May 15). The behavioral justification for public pensions: a survey. *Journal of Economic Finance*, *32*, 409-425. doi:10.1007/s12197-008-9036-4

Findley, T. S., & Caliendo, F. N. (2009, May 8). Short horizons, time inconsistency, and optimal social security. *Int Tax Public Finance*, *16*, 487-513. doi:10.100797-009-9115-2

Gahramanov, E. & Findley, T. S. (2012) A welfare-theoretic analysis of a decrease in public pension benefits.

Goss, S. C. (2010). The future financial status of the social security program. *Social Security Bulletin*, *70(3*), 111-125. Retrieved from http://www.socialsecurity.gov/policy

Kotlikoff, L. J. & Sachs, J. (1998) The personal security system: a framework for reforming social security, Federal Reserve Bank of St. Louis, issue Mar, 11-13

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics*, 62:443-477

Laibson, D. I., Repetto, A., & Tobacman, J. (1998). Self-control and saving for retirement. *Brookings Papers on Economic Activity*, *1*, 91-196.

Prelec, D. (2004). Decreasing impatience: A criterion for non-stationary time preference and “hyperbolic” discounting . *Scandinavian Journal of Economics*, *106(3)*, 511-532. doi:10.1111/j.1467-9442.2004.00380.x

Rabin, M. (2002). A perspective on psychology and economics. *European Economic Review*, *46*, 657-685. Retrieved from www.elsevier.com/locate/econobase

Social Security Administration Office of the Chief Actuary. (2012,April 23). *Long-range demographic assumptions for the 2012 trustees report*. Washington, DC: U.S. Government Printing Office

The Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds. (2012). *The 2012 Annual Report of the board of trustees of the federal old-age and survivors insurance and federal disability insurance trust funds* (112th Congress, 2d Session-House Document 113-102). Washington, DC: U.S. Government Printing Office.