NEXT GENERATION PLASMA FREQUENCY PROBE INSTRUMENTATION TECHNIQUE

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Abstract—The fundamental parameter for the Earth's ionosphere, a space plasma, is its density. This density can be determined in-situ from its resonant frequency properties, which can be stimulated by an antenna operating at RF frequencies immersed in the plasma. The resonant conditions are observed through the antenna's impedance characteristics. Innovations in the Utah State University plasma impedance probe, an instrument used for making these measurements are discussed. An improved control theory model of the instrument is derived and analyzed for a variety of ionospheric conditions. Calibration measurements are compared with theoretical results.

I. INTRODUCTION

Electron density is the primary parameter of the ionosphere. Many instruments have been developed to make in-situ measurements of electron density. Potential or direct current probes measure the space environment via current collected on a surface that has been charged to a specified potential relative to the space environment. Langmuir probes, in use on spacecraft since the 1960s, are one such approach [1]. Additionally, a plasma can be made to oscillate at specific radio frequencies. The plasma parameters may then be calculated from these resonant frequencies. These resonances are determined by measuring the impedance of the plasma medium over a wide band of frequencies. Instruments that utilize the susceptibility of the plasma to radio frequency excitation are known as impedance or resonance probes [2]-[3]. A major advantage of impedance probe measurements is immunity to surface contamination, a problem that can induce significant error in potential probe measurements.

For nearly four decades, Utah State University (USU) has built an instrument known as the plasma frequency probe (PPP) for measuring electron density. [4] The instrument has undergone a series of improvements as technology has matured. The last update of the instrument was studied by Swenson [5] and Jensen [6]. The instrument has been an integral part of many sounding rocket payloads. Recent programs that have flown plasma frequency probes built by USU include CODA I (21.121), CODA II (21.128), SAL (21.117), Thunderstorm II (38.007), Thunderstorm III (36.111) and Auroral Turbulence (40.005) [7].

The latest iteration of the USU instrument, the plasma impedance probe (PIP), is actually a suite of instruments consisting of a plasma frequency probe, plasma sweep probe (PSP), and a Q probe. The instrument is scheduled to be flown on two sounding rocket programs, E-Winds (41.036-41.038, 27.144) and Peru-Hysell, on a total of six rockets. Additionally, the PIP is part of the Floating Potential Measurement Unit (FPMU) that will be flown on the International Space Station.

This monograph details the recent improvements of the PIP. Antenna impedance theory will be briefly introduced. The development of the control theory model of the PIP, including controller design, stability analysis, and simulation will be covered. A brief discussion of the digital hardware implementation follows and the paper concludes with a discussion of the calibration of the instrument.

II. ANTENNA IMPEDANCE THEORY

The upper hybrid plasma frequency \( f_p \), which is related to the plasma frequency \( f_N \) and the electron gyro frequency \( f_B \) by the following relation:

\[
\begin{align*}
\frac{1}{f_p^2} &= \frac{1}{f_N^2} + \frac{1}{f_B^2} \\
f_N &= \left( \frac{Ne^2}{4\pi^2\varepsilon_0m} \right)^{1/2} \\
f_B &= eB/2\pi m
\end{align*}
\]

where \( f_N \) is the plasma frequency, \( f_B \) and \( f_B = eB/2\pi m \). Using (1), the electron density may be computed by:

\[
N = \frac{4\pi^2(m\varepsilon_0)(f_p^2 - f_B^2)}{e^2}
\]

The plasma impedance probe utilizes a control loop to lock onto and track the upper hybrid frequency which may be used to determine electron density as seen in (2). The probe accomplishes this by measuring the impedance of a dipole antenna operating in the plasma.

Balmain derived the equations for the admittance of a short dipole in a magnetoplasma using quasi-static electromagnetic theory[8]. Balmain proposed that the observed peaks and nulls in the admittance magnitude correspond to the resonant frequencies of the plasma. The admittance magnitude at the plasma and upper hybrid frequencies are minima and in general, are easier to measure because of saturation issues. From Balmain's work, a short dipole in a plasma may be modeled as a series resonance followed by a parallel resonance. The
series resonance corresponds to the electron gyro frequency \( f_B \), while the parallel resonance corresponds to the upper hybrid frequency \( f_p \).

The upper hybrid frequency region is the region of interest for the new PIP, so the series resonance will be ignored in this analysis. From basic circuit theory, it is well known that the impedance of a parallel RLC circuit may be described in transfer function notation as

\[
T(s) = \frac{\frac{1}{LC} s}{s^2 + \frac{1}{RC} s + \frac{1}{LC}}
\]

where \( R \), \( L \), and \( C \) are the circuit’s resistance, inductance, and capacitance respectively. Equation 3 can be rewritten in terms of the resonant frequency, \( \omega_0 \), a quality factor, \( Q \), and the resistance \( R \).

\[
T(s) = \frac{\frac{B_{00}}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}
\]

\[Q\] may be defined as

\[
Q = \frac{B W}{f_0}
\]

where \( BW \) is the 3 dB bandwidth and \( f_0 \) is the resonant frequency [9]. \( Q \) is a measure of the selectivity of the system. A system with a high \( Q \) amplifies a narrow band of frequencies. Previous estimates on \( R \) and \( Q \) for the E-region at night place \( R \) between 1 k\( \Omega \) and 120 k\( \Omega \), and \( Q \) can vary from 0.1 to 20.

III. PLASMA FREQUENCY PROBE CONTROL THEORY

A. PFP Linear System Model

The desired response of the current PFP instrument is to track changes in the upper hybrid plasma frequency with a steady state error of zero at a rate of approximately 3.6 kHz. The tracking of the upper hybrid frequency is equivalent to tracking the zero phase crossing of the phase response of Fig. 1.

In 1989, Swenson proposed a linear model of the PFP for control system analysis [5]. The model described the small signal characteristics of the PFP when the system was locked onto the parallel resonance. Swenson suggested that the system could be divided into four parts: the antenna, the phase detector, the voltage-controlled oscillator, and the loop filter. The new PFP has similar components with some subtle differences that arise from the change from analog to digital circuitry. An accurate model of the dynamics of each portion of the instrument is key to developing a suitable control methodology. The portions of the new instrument to be modeled are the antenna, the RF head, the phase detector, the analog to digital converter, and the direct digital synthesizer.

The dynamics of the antenna in plasma have been briefly mentioned in connection with Balmain’s theory. Revisiting Fig. 1, shows that the phase response of a second-order parallel resonance is nonlinear and not suitable for a linear analysis. The phase function for a second-order pole in terms of \( R \) and \( Q \) can be expressed as

\[
\angle (T(j\omega)) = - \arctan \left( \frac{\omega - \omega_0}{1 - (\frac{\omega}{\omega_0})^2} \right)
\]

Although the arctan function is nonlinear, it is readily linearized about the zero phase crossing. The inverse tangent may be approximated over a small interval by its argument. Thus, the slope of the phase curve near the resonant frequency may be approximated by

\[
m = \frac{-180^\circ Q}{f_0}
\]

The slope, \( m \), will always be negative for a second order parallel resonance as seen in Fig. 1. The antenna gain denoted, \( K_a \), in units of degrees per MHz is simply the slope of the phase curve at resonance given in (7). Estimates of \( K_a \) range from 10 deg/MHz to 3600 deg/MHz.

The RF head consists of several cascaded op-amp stages followed by a comparator. A sinusoid from a direct digital synthesizer is sent through an antenna and a reference channel simultaneously. For the antenna channel, a wide band transimpedance amplifier senses the current on the antenna and converts it to a voltage, effectively giving the impedance. The reference channel divides the input by a factor \( a \) so that a differencing operation between the antenna and reference channels
at the resonant frequency produces an impedance measurement above the noise floor of the system. The differencing operation is performed by a wide band difference amplifier and the resulting signal is sent to a logarithmic amplifier for an magnitude measurement. Both the measured antenna signal and the reference signal are sent to the comparator to be converted from sinusoids to square waves for processing in the phase detector. The RF head operates at a MHz rate, while the loop is sampled for telemetry in the low kHz rate. Due to high-speed of the RF head relative to the overall loop, no significant dynamics are assumed to be introduced into the system. Together, the antenna and RF head act as a frequency to phase transducer.

The phase detector consists of two D flip-flops cascaded with a low pass (LP) filter. Theoretically, the phase detector can measure a full 360° of phase. Accepting the square waves produced by the comparator of the RF head, the detector encodes the phase information in a 0 to 5 Volt varying duty cycle square wave that is integrated by the filter to give a DC mean value. As with the analysis of the RF head, the D flip-flop portion of the phase detector runs at a speed that greatly exceeds the loop sampling rate and thus does not introduce poles or zeros into the system dynamics.

In the previous PFP, a simple single pole RC filter was used to integrate the output of the flip-flops. The single pole filter was prone to considerable steady state ripple introducing undesirable error. After extensive analysis, a higher order (8 pole) LP filter was chosen for the current instrument to improve the cutoff attenuation and the ripple voltage at steady state. [12] Adding an 8 pole LP filter would seem to add considerable dynamics that would be difficult to control. However, the filter settles to within 1% of its final value in approximately 30 μs and the control loop will always be sampled after the filter has settled to its steady state value, so the dynamics of the filter can be ignored in the analysis. Thus the phase detector can be modeled as a linear gain, \( K_p \), such that

\[
K_p = \frac{2.5V}{180°} \approx 13 \times 10^{-3} \frac{V}{deg} \tag{8}
\]

The analog to digital converter (ADC in short) quantizes the output voltage of the phase detector’s LP filter. The ADC converts the 0 - 5 V analog voltage into a n-bit digital word. The ADC on the current PFP provides 14-bit resolution. So the conversion from volts to counts is modeled as a linear gain, \( K_{ad} \), such that

\[
K_{ad} = \frac{2^{14} \text{ Counts}}{5V} \approx 3.28 \times 10^3 \frac{\text{Counts}}{V} \tag{9}
\]

The direct digital synthesizer (DDS) has replaced the voltage-controlled oscillator (VCO) in the current instrument. Limitations of the VCO in previous PFP instruments are well documented by Swenson [5] and Jensen[6]. The VCO added a single pole at the cutoff (-3 dB) frequency[5]. The DDS response time to the 32-bit digital frequency word that programs it is approximately 8 system clock cycles or 350 nanoseconds, which is nearly a thousand times smaller than the loop sampling time. The 144 MHz cutoff frequency of the DDS is nearly a factor of 10 larger than the highest frequency of interest of 15 MHz. Because of the fast response time and wide bandwidth, it may be assumed that the DDS does not add a pole to the system dynamics. Thus, the DDS can also be modeled as a linear gain. The gain for the DDS is equal to

\[
K_d = \frac{144 MHz}{2^{32} \text{ counts}} \approx 33.5 \times 10^{-3} \frac{\text{ MHz}}{\text{ counts}} \tag{10}
\]

Thus the PIP can be modeled as a set of four cascaded linear gains, \( K_a, K_p, K_{ad}, \) and \( K_d \). Computing the product of these four gains gives the loop gain. Looking at the dimensional analysis of the loop gain gives:

\[
K_{\text{Loop}} = K_a \cdot K_p \cdot K_{ad} \cdot K_d = \frac{\text{ deg V Cnts MHz}}{\text{ MHz deg V Cnts}} \tag{11}
\]

Hence the loop gain is unitless. Since the antenna gain, \( K_a \), is the only nonconstant gain, the loop gain \( K_{\text{Loop}} \) is a function of \( K_a \) and will vary according to the plasma parameters, \( Q \) and \( f_0 \). For ease in writing, suppose \( K_i = K_{\text{Loop}} \). Since the plant is a simple linear gain, the dynamics of the system are trivial. Connecting the system in a negative feedback loop and analyzing the system, the closed-loop transfer function becomes

\[
C(s) = \frac{K_i}{K_i + 1} R(s) \tag{12}
\]

where \( R(s) \) is the input to the system. A similar computation yields the error term, such that

\[
E(s) = \frac{1}{K_i + 1} R(s) \tag{13}
\]

Analysis of the steady-state error, \( e_{ss} \), is computed by applying the final value theorem to the error term in (13). Suppose \( R(s) \) is equal to a step input, then applying the final value theorem, the steady-state error is

\[
e_{ss} = \lim_{s \to 0} s \left( \frac{1}{K_i + 1} R(s) \right) = \frac{1}{K_i + 1} \tag{14}
\]

Thus, without some sort of control, the system response cannot meet the performance constraint of zero steady state error.

B. PFP Controller Design

The control system described by the transfer function in (12) is a Type 0 system with no free integrators. To force the steady state error to go to zero, an integral-type control is required. The integral controller has the form \( T(s) = \frac{K_i}{s} \) where \( K_i \) is the control gain. By adding the controller to the feedback loop, the closed loop transfer function of the system becomes

\[
C(s) = \frac{K_i K_i}{s + K_i K_i} R(s) \tag{15}
\]
and the error term becomes

$$E(s) = \frac{s}{s + K_iK_t} R(s)$$  \hspace{1cm} (16)

Suppose that $R(s)$ is a step input. Applying the final value theorem

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} (s) \left( \frac{1}{s} \left( \frac{s}{s + K_iK_t} \right) \right) = 0$$  \hspace{1cm} (17)

Hence the requirement for zero steady state error is accomplished using integral control.

The second desired performance constraint for the PFP system is tracking step changes in the ionosphere at approximately 3.6 kHz. Tracking changes at the specified rate requires the transient response of the system to settle to within 1% of its final value with a settling time $T_s < 0.278$ milliseconds. Applying a step input to (15) and taking the inverse Laplace transform of the result gives,

$$c(t) = 1 - e^{-K_iK_t t}$$  \hspace{1cm} (18)

c(t) is the expression for the transient response of the system given a step input. Solving for $K_i$, the selectable control gain requires solving the following inequality derived from the closed-loop transfer function

$$e^{-K_iK_t T_s} < 0.01$$  \hspace{1cm} (19)

Solving for $K_i$

$$K_i = \ln 0.01 \frac{-1}{K_t T_s}$$  \hspace{1cm} (20)

![Fig. 2. Step response of PFP control loop for varying integral control gains with $Q = 10$](image)

Using a nominal value of $Q = 10$, Fig. 2 demonstrates the transient response of the system described by (15) to control gains of $2^7$, $2^6$, $2^5$, and $2^4$. For $K_i = 2^6$ or $2^5$, the system meets the required settling time requirement of 0.278 seconds. As the Q of the ionosphere increases, the required control gain $K_i$ decreases due to the proportional gain increase in $K_t$ caused by Q. Similarly, for a small Q, the control gain must be increased to meet the settling time requirement. The choice of the control gain, $K_i$, is highly dependent on the Q of the ionosphere. The desired control system should converge for $0.1 \leq Q \leq 20$. So the question becomes whether the system is stable for a set control gain as Q varies.

C. PFP Stability Analysis

The integral controller selected for analysis of the PFP was assumed to be ideal. Ideal integrators are often approximated with analog circuitry, but the current PFP implements the control loop digitally. For the PFP control model, an discrete-time integrator based on the forward rectangular rule, or Euler's rule, will be used. The mathematics concerning the forward rectangular rule in digital control is well documented in [11]. Given the PFP system transfer function in (15), the discrete-time equivalent can be computed using the forward rectangular rule. The closed loop discrete transfer function in this case is

$$T(z) = \frac{K_iK_t}{z^2 + K_iK_t}$$  \hspace{1cm} (21)

where $z = e^{sT}$ and $T$ is the sampling period. Stability analysis in the z-plane requires that the poles of the discrete system lie within the unit circle. The characteristic equation of (21) is

$$\Lambda(z) = z + TK_iK_t - 1 = 0.$$  \hspace{1cm} (22)

which gives the necessary condition

$$0 < TK_iK_t < 2$$  \hspace{1cm} (23)

in order for the system to be stable. Dividing through by $K_t$, the parameter determined by the ionosphere, gives

$$0 < TK_i < \frac{2}{K_t}$$  \hspace{1cm} (24)

As seen in 24, the stability inequality, the sampling time $T$, is a critical system parameter. The PFP lock measurement has a telemetry sampling rate of 3.58 kHz, equivalent to a sampling time of approximately 0.28 ms. This sampling rate is fixed by the telemetry matrix. By using the telemetry sampling time of the loop as the parameter $T$, the only controllable parameter becomes $K_t$ and the inequality in (24) becomes

$$0 < K_t \left( \frac{7160}{K_i} \right)$$  \hspace{1cm} (25)

Due to the uncertainty of $K_t$, ideally a larger upper bound on the inequality would ease the constraint on the control gain. Although the loop output is sampled at 3.58 kHz, internally the sampling rate could occur at a much faster rate. Suppose that instead of using the telemetry sampling time as $T$, the discrete-time integrator sampling period will be used for $T$. Since the system filter takes 30 μs to settle to a final value, a sampling
time of 40 µs will be chosen to ensure that the filtered value is stable. Increasing the inner loop sampling rate as such, changes the numerator of the upper limit of (25) to 50000, effectively giving a margin for $K_i$ that is seven times larger than if the integrator were to run at the telemetry loop sampling rate.

The worst case inequality occurs when the Q of the ionosphere is large, causing $K_t$ to increase and the right hand upper bound of (24) to decrease, thereby tightening the constraint on the product $TK_i$. Recalling the gain computed in (20), it can quickly be shown that for a Q of 20, the inequality fails and the system is unstable. The gain in (20) is optimal in the transient domain, allowing tracking of the worst case transient problem when Q = 0.1. However, as Q increases, the inherent gain of the ionosphere manifested through the antenna increases and the system becomes unstable for large control gains. Adaptive control might remedy the problem by continuously updating the control gain based on the Q value sampled from the ionosphere measurements, but cost and time limitations inhibit the development of such a controller at this time.

D. PFP Simulation

To aid in the choice of a suitable control gain and the actual digital design of the PFP, a simulation testbed shown in Fig. 3 was developed in Simulink to model the effect of variations in Q and $K_i$ on the control system, as well as address the issues due to additive noise and system response to a sinusoidal input.

The testbed in Fig. 3 shows the various gains discussed previously. The accumulator block contains a discrete-time integrator that uses the forward Euler integration method. An initializing gain is provided as input to the accumulator as an initial condition. A noise source has been added to simulate the effect of additive Gaussian white noise on the phase measurement. The sinusoidal modulation block has been added to model the loop response to changing sinusoidal frequencies. A saturation block has been placed on the output of the antenna to clamp the dynamic output range of the antenna from $-90^\circ$ to $+90^\circ$, the range of values that the actual antenna can physically return. There is also a gain stage preceding the analog to digital converter which represents an adjustable analog gain stage present in the hardware that precedes the sampling operation. The actual quantization of the ADC is performed in the A/D sampling block with a sample rate of 3.6 kHz, the rate at which the telemetry system samples the loop.

Extensive simulations were performed using various combinations of Q, R, fo, and $K_i$. A 1 MHz step change in the upper hybrid frequency was used as the system input. The noise source was set to be zero mean with unit variance. The simulations revealed several interesting results. For the smallest value of Q, a control gain greater than $2^{16}$ yielded an unstable system. For a Q of 20, a gain of $2^{14}$ manifested system instability. Using Q = 10, simulations were performed to check the step response of the system and the frequency error due to several control gains. The results are presented in Fig. 4.

The simulation clearly demonstrates that the system trades transient response for accuracy. Fig. 4 shows that as the control gain is increased, the system responds rapidly to the input as expected, but this occurs at the expense of steady state error manifested as
ever, the transient response for a control gain of 1 is the desired response of approximately a frequency error for the unity gain controller is less than Fig. 4. that could achieve the desired transient response, the measurement. Reliable, accurate data is the driving force in analog to digital conversion. The A/D converter produces a 14-bit phase word, but a 32-bit frequency word is used to update the sinusoidal output of the DDS. Including the system and control gains, the control loop is able to produce essentially a 28-bit word for the DDS. This translates to approximately 2 degrees of phase error and less than 1% frequency error without additive noise. The unity gain controller mitigates this error and provides excellent noise immunity for the PFP.

The simulation testbed aided greatly in the decision process for the control gain, but it should be noted that the unity gain controller is not optimal because it does not fulfill both performance constraints. An optimal controller for the system would continuously update the control gain, $K_i$, to respond to ionosphere dynamics. A large gain could be used for pull in purposes, slowly reducing to unity gain when the system is near lock. This configuration would provide an optimal transient and error response. For future missions, a gain scheduled controller or adaptive controller based on these principles should be investigated to allow the system to meet all of the required performance constraints.

IV. DIGITAL HARDWARE IMPLEMENTATION AND INTEGRATION

The new iteration of the plasma impedance probe is really an integrated suite of instruments capable of performing a frequency lock measurement via the PFP as explained earlier in this paper, as well as plasma frequency sweep and $Q$ measurements. The integration of the three instruments has been made possible by recent advances in electronic technology. Direct digital synthesis coupled with complex programmable logic devices and multi-channel analog to digital converters allow the current PFP to perform plasma frequency, sweep, and $Q$ measurements seamlessly.

The plasma sweep probe (PSP) is one of the instruments integrated into the PIP. The PSP for the EWinds mission performs a 128 point sweep from 0.1 to 15 MHz.
The control logic of the PIP is embedded in a programmable logic device manufactured by Altera. Programmable logic devices (PLD in short) are complex digital devices that can be used to implement combinatorial logic and finite state machines. The Altera ACEX family used in the new PIP has a significant gate density and is in-system reprogrammable. The high gate density allows all of the science functions to be integrated in a single chip conserving space and power. In-system programmability allows for significant design modifications after the system has been assembled, giving design flexibility in testing and calibration. Finite state machines implemented in VHDL are used for system control and command. Coupled with memory, these finite state machines give the PIP the ability to interleave the PFP, PSF, and Q measurements seamlessly.

V. CALIBRATION

The PIP instrument was calibrated by making extensive measurements of a set of well-known passive loads. Ten resistors ranging from 40 Ω to 51 kΩ, 11 capacitors ranging from 10 pF to 27 nF, and six inductors ranging from 100 µH to 2.4 mH were first measured using accurate laboratory equipment. The load values were carefully selected to test the entire dynamic range of the instrument. A brief examination of the sweep data for the passive loads showed that the instrument seemed for the most part to be working as expected. For an unknown reason, the magnitude measurement dropped at higher frequencies for the resistors. Further investigation showed that the difference amplifier on the RF head was having difficulty performing at higher frequencies. The cause of this problem is unknown at the present and will require more analysis. Also, data was collected from several different RLC loads with known resonant frequencies to check the ability of the instrument to lock onto the upper hybrid or parallel resonance. These RLC loads were built carefully to simulate the expected Q values of the ionosphere. The instrument locked onto all of the loads exhibiting very stable behavior at the lock frequency. The noise on the lock frequency was estimated to be on the order of hundreds of Hertz as predicted in the model. The transient response of the instrument was as expected. It should be noted that many measurements were also made of the system with no load attached, an open circuit configuration, in order to understand the inherent impedance of the instrument. Due to time constraints for the EWinds mission, a frequency response calibration with a dynamic calibrator was not performed. The dynamic calibrator is an electrically tunable parallel RLC circuit that uses a variactor diode, a variable inductor, or both to modulate the resonant frequency. A dynamic calibration will be performed on the two Peru-Hysell payloads in order to empirically determine the actual slew rate limit of the...
instrument.

Each load was attached to the PIP via a custom built set of test cables. The PFP, PSP, and Q probe data were saved via an ITAS telemetry decommutator to disk for data analysis on a PC using Matlab. Analysis of the calibration data is currently underway. The data collected from the calibration will be used in a nonlinear least squares curve fit to derive the instrument parameters. These parameters will be used in the data analysis of the flight data to give as accurate results as possible.

VI. CONCLUSION

A knowledge of plasma density and density disturbances are required to understand radio frequency communication with spacecraft. The plasma impedance probe, an instrument to measure plasma density, has been updated and improved by students at Utah State University. Improvements to the PIP include the development of a linear model of the instrument used for control theory analysis. This model has encouraged the development of a new digitally implemented frequency lock algorithm to accurately track the upper-hybrid frequency of a space plasma. The dynamic response, performance, and accuracy results of the updated instrument to this point have been encouraging. During testing and calibration, future improvements for the instrument such as adaptive gain control were identified, that should allow the probe to meet all of the required performance constraints. Hopefully, the measurements obtained with this new probe in the upcoming EWinds, Peru-Hysell, and FPMU missions will allow scientists and engineers to understand dynamic local phenomena of the ionosphere, such as equatorial plasma bubbles more completely.

REFERENCES