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# Probing the Mechanics of the Environmental Kuznets Curve Theory

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PROBING THE MECHANICS OF ENVIRONMENTAL KUZNETS CURVE THEORY

by

Jeremy Lynn Kidd

A dissertation submitted in partial fulfillment  
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Economics

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2009

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## ABSTRACT

Probing the Mechanics of Environmental Kuznets Curve Theory

by

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Utah State University, 2009

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The theory of the Environmental Kuznets Curve (EKC) proposes to answer important questions regarding the connections between economic growth (development) and the environment. The theory postulates the environment need not always suffer as the economy develops, and it has generated strong support and opposition. Rather than attempting to defend or debunk EKC theory, this research challenges a practice engaged in by proponents and opponents alike. Simplifying assumptions are a necessary part of economic analysis, but this research shows that any assumptions may not be universally applicable. Utilizing, in turn, a simple one-good model and then a more complicated two-good model, it is discovered that the competing assumptions utilized by proponents and opponents of the EKC theory may both be valid, depending upon the conditions present in the system being analyzed.

(120 pages)

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Jeremy Lynn Kidd

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## CHAPTER 1

### INTRODUCTION

Is there a destructive link between economic growth and the health of the environment? Has history determined the one will always be at the expense of the other? If so, what does it mean for economic and environmental policies? Can they be created to support growth without compromising the environment? The theory of the Environmental Kuznets Curve (EKC)<sup>1</sup> has been proposed as a partial answer to these questions. EKC theory proposes that degradation of the environment need not always be the result of economic growth. Instead, this theory postulates that once a society has obtained some level of national income, the environment should begin to improve as incomes continue to rise, through changes in consumption, changes in production, or both.<sup>2</sup> If the theory is correct, then it may be possible to achieve both a high standard of living and a clean environment; if incorrect, then the global environment may further degrade, despite economic development, until the earth's capacity for sustaining life has been exhausted. This research proposes to further our understanding of the accuracy of the theory by advancing the sophistication of dynamic modeling of the theory.

A vigorous debate has arisen with regard to the validity of the EKC theory and its applicability to real-world data. Several economists have provided theoretical and empirical support for EKC theory, such as Grossman and Kreuger (1991, 1995), Lucas et al. (1992), and Selden and Song (1994), yet just as many have questioned whether or not this phenomenon exists, such as Stern et al. (1996) and Ali Khan (1997). Existing literature has examined the empirical data, offering opposing views on whether the data represents an outcome predicted by EKC theory and proposing theoretical explanations for the empirical results. The

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<sup>1</sup> So named because of similarities to the work of Simon Kuznets (1955, 1963). Kuznets posited an inverted-U relationship between rising incomes and income inequality. He concluded that countries with rising incomes would see political power and income increasingly concentrated in a small portion of society. That concentration would continue until incomes had risen to a certain level, after which a middle class would begin to develop, which would dissipate the concentration of political power and income, leading to a more equal distribution of both among the population as incomes continue to rise.

<sup>2</sup> As attitudes towards environmental quality change, individuals change their consumption, and producers begin producing environmentally friendly products. This change in attitude toward the environment is often attributed to environmental amenities being a normal good. However, in the presence of diminishing marginal utility for all goods, the same result is possible under the weaker assumption of normality, as discussed in McConnell (1997). Kelly (2003) shows that the assumption of normality is a necessary condition for flow pollutants.

ongoing debate surrounding the potential for an EKC has greatly advanced our understanding of how economic development and environmental quality might be linked, but questions remain.

For example, EKC theory<sup>3</sup> was generated in response to a long-standing debate over whether environmental problems can be solved without direct government intervention, or whether the process of economic development may be sufficient to protect the environment. However, most experts, with relatively few exceptions, agree that while increased income will result in an individual's increased demand for a clean environment, an actual reduction in environmental degradation will occur only as government reacts to the collective demand for environmental quality.<sup>4</sup> Direct government action may be required as a catalyst for an EKC, or perhaps the necessary changes can begin if the government's role is limited to providing appropriate incentives.

One possible link between individual demand and government action is an open democratic process (Torras and Boyce, 1998 and Pfaff et al., 2001). In the United States, the democratic process has resulted in several environmental policies, including the Clean Air Act, the Clean Water Act, the Acid Rain Act,<sup>5</sup> and reductions in the amount of public lands open to logging. Critics of the democratic process say it moves too slowly to effectively protect the environment; fragile ecosystems can be destroyed while democratic processes march on towards a definitive conclusion. However, others defend democratic means as a necessary trade-off between protection of the environment and individual freedoms; if the environment is to be protected, it should be because individuals in society care about the future and wish it to be so.

Data for the United States is, for the most part, mixed. Several measures show reductions in the total level of environmental degradation over the period of 1970 to 2000: emissions of most airborne pollutants declined;<sup>6</sup> water quality improved in many categories;<sup>7</sup> and total land forested in the United

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<sup>3</sup> The existence of an EKC was first proposed by Grossman and Kreuger (1991). The authors did not give the phenomenon the name "Environmental Kuznets Curve," but they were the first to formally postulate a quadratic relationship between national income and pollution.

<sup>4</sup> Two prominent exceptions are Beckerman (1992) and Panayotou (1997). Aubourg et al. (2008) provide empirical evidence that greater freedom results in a flatter EKC for carbon dioxide.

<sup>5</sup> Title VII of the Energy Security Act of 1980 [P.L. 96-294].

<sup>6</sup> USEPA, National Air Pollutant Emission Trends, 1900–1998, EPA-454/R-00-002, U.S. Environmental Protection Agency, Research Triangle Park, NC. March 2000

<sup>7</sup> As evidenced by the trend towards higher water quality standards as lower standards are met.

States increased after almost a century of decline.<sup>8</sup> These statistics, combined with the rise in U.S. incomes for the same three decades, might lead a casual observer to conclude that EKC theory is accurately describing the empirical data. However, other pollutants, primarily greenhouse gas emissions, have been increasing over the same period. These discrepancies preclude any blanket assumption regarding the empirical accuracy of EKC theory and raise certain interesting questions: Does EKC theory apply only to certain measures of degradation? Hasn't the threshold level of income been reached for greenhouse gases? Are there other factors or phenomena at work?

These questions are complicated further by other complex issues, such as trade. Wealthier countries may create "pollution havens" reducing their environmental degradation by exporting polluting ("dirty") industries to poorer countries. For example, the United States may increase imports of steel rather than produce it domestically, thus reducing domestic emissions of sulfur dioxide; the United States may similarly increase its imports of lumber, reducing domestic deforestation. These actions would induce an apparent domestic EKC, but would result in greater degradation of the global environment. Of course, global degradation is not guaranteed under these conditions, but it can certainly be compounded, especially if a developed country turns to under-developed countries for the production of "dirty" goods and ultimately consumes more dirty goods than if all production remained domestic. In that case, more total production of the "dirty" goods leads to greater degradation of the global environment.

Global degradation may be greater still if the technology in underdeveloped nations is deemed "dirtier" than similar technologies in developed countries. For example, if the steel-producing technologies of under-developed countries are less advanced than those employed by the United States, then the resulting pollution may cancel any benefits the United States may receive for exporting that industry. Conversely, if the technology is cleaner, the decrease in pollution per unit of output may lead to a lower level of total degradation.

Complex issues such as this are natural ingredients of economic analysis. EKC theory combines the very complex processes of economics, trade and the environment, and the answer to whether EKC theory is correct may be much more than a theoretical concern. What to the casual observer seems an

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<sup>8</sup> USDA, U.S. Forest Facts and Historical Trends, FS-696-M, U.S. Department of Agriculture, Forest Service, Washington, DC, June 2001. There is some concern that there may still be a net decrease in "natural forest."

esoteric concern may instead directly impact the future well-being of every human being, through both economic and environmental processes.

Within the rich debate that has arisen surrounding these important issues, certain common assumptions are regularly used. Assumptions are commonplace in economics, and for good reason--assumptions allow for modeling of complex systems in a way that allows us to understand basic relationships much better. Without assumptions, much of economics study would be impossible or at least extremely difficult. However, the information we are able to glean from our models is only as good as the accuracy of our assumptions. In the EKC literature, assumptions are made regarding how society will react to such things as changes in the fixed or variable cost of abatement methods, or changes in society's tastes and preferences for consumption versus a clean environment. Most assumptions seem perfectly logical and intuitive,<sup>9</sup> and may be successful in predicting outcomes over a significant range of parameter and variable values. However, due to the complex nature of the economic and environmental systems which need to interact to create EKC phenomena, it is possible that the assumptions may not be universal—they may not hold under certain plausible circumstances.

This research utilizes single- and dual-good models to test whether certain assumptions regarding changes in parameter values yield expected results. Chapter 2 traces the evolutionary steps of EKC theory from the early 1970s through today, discussing improvements in both the empirical and theoretical literature and providing an overview of the current literature. Chapter 3 uses both a simple continuous-time, deterministic variant of the Kelly (2003) model, as well as a discrete time, deterministic model, to test whether the intuitive assumptions utilized by Kelly and others hold up under reasonable conditions. We discover that even a simple model is sufficiently complex that predicting the results of parameter change is troublesome. Specifically, we discover that there are competing effects to many parameter changes, similar to income and price effects. We also discover that changes in certain parameters can cause initial increases in pollution, but long-term decreases in pollution, an indication that the time paths of variables need not shift uniformly up or down as parameters changes.

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<sup>9</sup> Of course, not all assumptions are created equal--Maddison (2006) criticizes the implicit assumption that emissions in one country are unaffected by events in neighboring countries.

Chapter 4 extends the model by incorporating the capacity of society to choose between “green” and “dirty” production and consumption, as an additional means by which society may choose to improve its environment. Once again, we find that intuitive and commonly used assumptions do not hold under plausible circumstances, offering an additional word of caution in relying on these assumptions. The difficulty in predicting with certainty the effect of parameter change suggests that the nature of the systemic reaction to parameter change may be highly dependent upon the characteristics of individual pollutants,<sup>10</sup> the relative values to society of consumption and environmental amenities at a given point in time, etc. This potential for variation in the systemic reaction to parameter change may help explain why the current debate has been unable to reach a consensus. Chapter 5 concludes this analysis and presents possible avenues for further research.

This dissertation is not intended to be a final answer to the EKC question but rather a word of caution to fellow researchers into the mechanics of an EKC. Moreover, it is intended as a word of caution to all who intend to effect environmental change within their countries and in the world, for this research indicates that common-sense assumptions may not be correct in all circumstances. This research indicates that environmental improvement will not be achieved in the same way in all countries, and that achieving desired environmental improvements will require policymakers and other interested parties to carefully tailor environmental policies to the particular circumstances of their nation and region.

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<sup>10</sup> The fact that different pollutants have different characteristics is not surprising, but some researchers assume that a valid point with regard to one pollutant holds universally for all pollutants. As an example, Wagner (2008) argues that econometric attempts to prove the existence of an EKC for carbon dioxide are flawed. Once those arguments are made, however, Wagner assumes that his arguments are equally valid as against the existence of an EKC for all other pollutants.

## CHAPTER 2

### LITERATURE REVIEW

EKC theory has evolved over several decades from its initial intuitive conception to the complex theoretical models of today. Like all theories, EKC theory has had a number of iterations, as individual contributions have been proposed and either accepted or rejected. These contributions began in the past century with a general concern over whether post-WWII growth patterns could continue without destroying the environment. Through successive steps of empirical and theoretical debate, a quadratic relationship between income and environmental degradation has been proposed, criticized, defended, and criticized again.

Along the way, the shortcomings of each iteration have been pointed out and elaborated upon by those not sharing the same view. This process of point-counterpoint has generally improved the quality of subsequent analyses, and brought us closer to a clearer understanding of the relationship between economic growth and environmental quality. However, in many ways the underlying questions of EKC theory remain unresolved. This chapter presents, in brief form, representative works from each iterative step, outlining their contributions to our present understanding of EKC phenomena, as well as highlighting those aspects which were thought lacking by observers, critics, and sometimes supporters.

#### **2.1. Genesis of the growth/environment debate**

An important first step in the evolution of the EKC theory was the development of a growth versus environmental quality debate. Common sense informs us that living on a planet with a finite volume introduces resource constraints, and the lengthy process by which productive resources are generated and renewed only serves to strengthen that constraint. Any society living on this planet can be expected to face some form of debate over when that constraint will become binding; in other words, it is likely that every forward-looking society on earth will engage in a growth versus environmental quality debate.

One of the first examples of such a debate in modern western civilization was Thomas Malthus (1890), who expresses concern over humanity's ability to feed itself when population grew geometrically but food production grew only arithmetically. While Malthus' predictions have not been realized, the



concerns over finite resources and potentially infinite resource demand remain valid to the extent that mankind continues to inhabit a world with finite resources.

In the centuries between Malthus and 1970, the growth versus environment debate certainly continued, and it gained strength in the wake of the unprecedented growth experienced by the United States after World War II. There began to be call for curbs to economic growth in order to protect the environment out of fear that uncontrolled economic growth would lead to exhaustion of natural resources and deterioration of air and water quality.<sup>11</sup> Ruttan (1971) responds to these calls with a reminder that technological advancement has the potential to help the environment cope with continued economic growth. If applied correctly, technical forces could allow economic growth to continue without necessarily destroying the environment.

The debate that persisted through most of the 1970s can best be exemplified by the works of Meadows et al. (1972) and Beckerman (1972). Meadows et al. (1972) predicted that the exponential rate of economic growth would lead to overpopulation and the exhaustion of environmental stocks and natural resources; this, in turn, would bring economic growth to a halt. Beckerman (1972) responded, stating that poor societies would never be able to afford abatement or clean-up costs, making economic growth a requirement for a cleaner environment: greater growth provides necessary public and private funds to achieve environmental improvements.

Many related Meadows' predictions to those of Malthus more than two centuries before, and, in the end, Meadows' predictions were not realized either.<sup>12</sup> Technological change has apparently allowed the world to avoid Malthus' and Meadows' predictions. However, as Radetzki (1992) notes, technological advances may not continue indefinitely, so we may not be able to rely on it to save us every time:

The purported environmental threat posed by economic growth cannot be waved away by pointing to the failures of earlier doomsday prophets. After all, the current world economy is far larger than 20 or

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<sup>11</sup> Ruttan (1971), p. 707.

<sup>12</sup> In *Beyond the Limits*, Meadows et al. (1992) state that they were not predicting anything, merely laying before the world choices of alternative paths that the human race could follow. They also stress that while technological progress may have extended the time during which present consumption levels are possible, humanity had already overshoot Earth's capacity to support humanity in the long run. In *Limits to Growth: The 30 Year Update* (Meadows et al., 2004), they again issue their challenge to humanity to find a way to return to a sustainable level of existence by raising consumption levels of the poorest in society while reducing the ecological footprint of mankind on the earth.

200 years ago, so the probability of hitting against one or other resource constraint could be greater than in earlier periods. (p. 121)

Still, Radetzki also notes that most of the changes to the environment over time have been beneficial to mankind with relatively few negative spillover effects. Neither Beckerman nor Meadows could claim complete victory in the debate; economic growth continued mostly unabated, yet the environmental effects have been mixed through the 1970s and 1980s. The debate, however, was far from ended.

## **2.2. First empirical studies**

As economic growth progressed through the 1980s, so did the debate over whether such growth could be maintained without permanent damage to the environment.<sup>13</sup> In an attempt to shed light on the link between economic growth and the environment, the World Bank commissioned a study in the early 1990s; the goal of the study was to determine what effect free trade had on the environment during the 1980s. The economists tasked with completing the study, Grossman and Kreuger (1991) found evidence to suggest that the economic gains from trade do not result in harm to the environment. They discover that an inverted-U relationship appears to exist between rising incomes and environmental degradation, a relationship which later came to be known as the Environmental Kuznets Curve.

EKC theory is premised on the notion that, for a time, environmental degradation increases as incomes rise; however, at a certain income level, that trend changes and degradation decreases while incomes rise. Grossman and Kreuger (1991) are followed by numerous other studies that show similar results. Birdsall and Wheeler (1992) show that countries with more open trade policies saw environmental quality increase as well as incomes; dirty industries did appear to migrate to poorer countries, but clean industries appeared to migrate at an even faster rate; and large corporations in clean industries pushed for higher environmental standards in both wealthy and poor countries. Grossman and Kreuger (1995) find inverted-U relationships for a number of measures of environmental degradation, estimating the turning point of annual per-capita income at approximately \$8,000. While they admit that it is unclear whether the estimated relationship does, in fact, exist, they believe that such empirical studies were “an important first

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<sup>13</sup> Two good sources for data regarding development are the World Development Indicators, available from the World Bank website ([www.worldbank.org](http://www.worldbank.org)) and the Summers-Heston data set, also known as the Penn World Data set ([pwt.econ.upenn.edu](http://pwt.econ.upenn.edu)).

step” (p. 360).

Subsequent studies have expressed a similar inability to draw firm conclusions regarding a causal relationship between economic growth and environmental degradation. Lucas et al. (1992) found inverted-U relationships between income and per-unit-of-output toxic releases of 320 different air, water, and ground pollutants.<sup>14</sup> They offer a caution that this does not guarantee a decline in aggregate pollution levels, but such a phenomenon would be required if an EKC were to exist. Selden and Song (1994) estimate an EKC for various air pollutants, yet issue a reminder that while the long-run promises lower pollution, the nature of an EKC promises higher short-run pollution levels, and that there is no guarantee that those higher short-run levels will not be devastating to the environment. Shafik (1994) estimates a purely negative relationship between income and some measures of environmental degradation; a purely positive relationship for other measures; and EKCs for yet others.<sup>15</sup> He states that when the societal costs of environmental degradation are internalized,<sup>16</sup> abatement occurs rapidly, but if costs are externalized then abatement may never occur.

These first empirical studies set the stage for further debate. They were, in many respects, overly simplified in their assumptions, and approached an inherently dynamic process from a purely static viewpoint. As discussed below, there were also methodological concerns with these initial attempts. In spite of their weaknesses, however, these EKC pioneers offered a sufficiently convincing argument that many economists were drawn into the debate in order to correct errors in an important area of research.

### **2.3. Rebuttals to empirical studies**

In response to the first empirical studies, many economists and ecologists disputed the possibility of a positive empirical relationship between income and environmental quality. List and Gallet (1999),

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<sup>14</sup> A decrease in pollution per unit of output is merely a short-term victory, for increases in total output may cause total pollution to rise. On the other hand, a short-term increase in total output and, therefore, pollution, may eventually yield to a trend towards increasingly lower per-unit of output pollution levels.

<sup>15</sup> Lack of safe water, lack of urban sanitation, and dissolved oxygen in rivers exhibit a purely negative relationship; municipal solid waste per capita and carbon emissions per capita exhibit a purely positive relationship; and annual deforestation, total deforestation, suspended particulate matter, ambient sulfur dioxide, and fecal coliform in rivers (the latter is a cubic, rather than quadratic, relationship) exhibit an inverted-U relationship.

<sup>16</sup> Internalization of pollution’s harmful effects is more likely when those effects are concentrated locally, as opposed to those pollutants whose effects are widespread.

Munasinghe (1999), De Bruyn et al. (1998), and Dijkgraaf and Vollebergh (2005)<sup>17</sup> all criticize EKC theorists for their implicit assumption that every country must pass through a similar development path.<sup>18</sup> Due to the diffusion of technology to lesser developed nations, there is no reason to expect one EKC to describe the path all developing nations would take, as a poor country today could learn from the mistakes of countries like the United States and Great Britain. Rothman (1998) criticizes the focus of initial empirical studies on production changes, stressing that the microeconomic behavior of individual economic actors, both productive and consumptive, would be a determining factor in the development of an EKC over time.

Others have rejected the notion that income determines pollution and have suggested alternatives. For example, Kaufmann et al. (1998) offer spatial intensity of economic activity as an alternate causal variable, proposing a relationship between the concentration of economic activity in urban areas and atmospheric levels of sulfur dioxide; Unruh and Moomaw (1998) provide some evidence that historical events, such as wars, treaties, etc., correlate closely with changes in pollution levels, and might more accurately explain decreases in pollution; Magnani (2000) stresses the need to include income disparity measures; and Suri and Chapman (1998) utilize energy consumption as an explanatory variable in EKC regression analysis. The latter study indicates that developed nations may see improvements in environmental quality as incomes rise only because they are importing pollution-intensive goods from developing nations. Their results suggest that free trade, rather than offering a solution for environmental problems, is likely to exacerbate them.

Substantial criticism of the first empirical studies, such as Grossman and Kreuger (1991, 1995), has also come from those who generally accept all the theoretical foundations of EKC theory. For example, Arrow et al. (1995) claim that the environment can only handle a certain level of pollution before its capacity to repair itself is gone. They state that while countries likely do progress through an EKC-like pattern, high levels of pollution during the transition may permanently damage the environment, indicating

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<sup>17</sup> Specifically, Dijkgraaf and Vollebergh (2005) point out that moving to panel data sets, rather than simple cross-sectional analysis, has some benefits, but argue that if the model does not allow for heterogeneity, the analysis will remain fundamentally flawed.

<sup>18</sup> In fact, some argue, as did Pearce (2005), that over-emphasis on the EKC results in infliction of environmental damage on the poor “just because this is the way rich countries developed hundreds of years ago” (p. 30).

that any future reductions in pollution may not result in improvements to environmental quality. Mazzanti et al. (2007) criticize as being of limited usefulness the use of cross-country analysis while, at the same time, providing evidence through the use of panel data that an EKC is likely for a number of pollutants. Stern (1998) criticize the early debate for its undue focus on exploring empirical regularities among a large set of variables, rather than attempting to gain insight into the underlying mechanics of an EKC theoretical foundation.

These criticisms, from friend and foe alike, caused proponents of EKC theory to rethink their approach, yielding greater attention to methodological detail, a slow shift towards theoretical research, and inclusion of a greater number of variables.

#### **2.4. Second generation empirical studies**

One response to criticisms has been the development of more complex and inclusive empirical models to test for EKC-like phenomena. Hettige et al. (2000) expand the number of independent and dependent variables tested, and find an inverted-U relationship between income and the share of industrial production in national output, as well as a negative relationship between income and both end-of-pipe pollution intensity and share of all polluting sectors in industrial output. All of this seems to indicate the possibility that rising incomes could cause a decrease in degradation, leading to the possible development of an EKC. However, it is also possible that total industrial output will increase, even as its share of total GDP shrinks. Additionally, increases in total output can outweigh reductions in per-unit pollution. As such, a “race to the bottom” scenario, where pollution continues to rise, is still possible.

Wheeler (2001) also investigates the “race to the bottom” and “pollution haven” hypotheses. With respect to the latter, poorer countries do not have high enough incomes to demand higher environmental quality, and thus attract polluting industries from more affluent countries. Wheeler shows that air quality is improving in developed *and* developing nations, and concludes that informal regulation of air quality exists even where formal regulation has not been adopted. Cole (2004) finds some evidence of pollution havens, but only for some pollutants, and the evidence appeared to be of limited significance.

Anderson (2001) provides evidence to support the criticisms of Munasinghe (1999), showing that implementation of new technologies at low per-capita income levels can allow developing nations to obtain

high per-capita income levels without high emissions during the years of transition. Chaudhuri and Pfaff (2002) address Rothman's (1998) criticisms by using microeconomic data to test for the existence of an EKC at the household level. Using household data from Pakistan, they show that individuals at higher income levels choose a cleaner immediate environment. They then utilize a voting model to show how individual preferences for cleaner personal environmental conditions might be aggregated to a national policy level, leading to greater environmental quality for a nation as a whole.

Many empirical studies have begun to focus in on particular aspects of EKC theory, no doubt in at least partial response to the criticisms of Stern (1998, 2004) and others. For example, Mazzanti and Zoboli (2007) use NAMEA<sup>19</sup> panel data to investigate whether labor productivity and environmental efficiency are related. They provide evidence that improvements in labor productivity are complementary to decreases in emissions per unit of output. Johansson and Kristrom (2007) examine Swedish sulfur emissions over the span of the twentieth century, and determine that while the data roughly approximate the inverted-U shape of an EKC, that shape can be explained by the transition through four separate environmental policy regimes within Sweden.<sup>20</sup>

Other studies have begun to investigate other variables that might play a role in the emergence of an EKC. Di Viti (2007) addresses whether the various forms of common law systems or civil law systems have any advantage in achieving improvements in environmental quality with increases in income. They find that emissions tend to be higher in common law systems, but so are foreign direct investment, gross domestic product growth, gross domestic savings, and market capitalization, and that abundance of capital explains all of these. They find that the resulting low interest rates allow for implementation of environmentally friendly devices and further environmental protection. Merlevede et al. (2006) and Cole et al. (2005) both find that countries with larger average firm size are more successful in improving the environment.<sup>21</sup>

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<sup>19</sup> Dutch National Accounting Matrix including Environmental Accounts.

<sup>20</sup> These regimes encompass the following time periods: 1900-1918; 1919-1933; 1934-1967; and 1968-2002.

<sup>21</sup> Cole et al. (2005) find that large firms produce lower emissions per employee and per unit of output. Merlevede et al. (2006) find that countries with larger firms pollute more early on, but that the presence of larger firms makes it easier for countries to pass strict environmental regulations, leading to large firm countries engaging in abatement at lower income levels.

Markandya et al. (2006) focus on different environmental policy choices in various European countries, and find that policy decisions can shift the EKC to the right or left, or lower it uniformly. Auborg et al. (2008) find that countries that restructure their debt service obligations or implement democratic reforms see a shift in the turning point for an EKC. Specifically, countries with lower foreign debt obligations and greater freedom experience a flatter EKC. Park et al. (2007) include societal cultural characteristics, such as education, risk aversion, concentration of power, and materialism. They make two findings: first, that higher emphasis on education increases environmental quality, and greater materialism and concentration of power decrease environmental quality; and second, that when these cultural variables are considered, evidence of an EKC diminishes or disappears.

Richmond and Kaufmann (2006) investigate whether an EKC might arise out of a response to energy prices rather than income. The fluctuation of relative energy prices (i.e., the price of oil as compared to the price of coal), they argue, is more influential on the level of pollutants emitted than is the level of income. They conclude that rising incomes are almost assuredly insufficient to cause a decrease in environmental degradation, and that a more realistic focus for societies wanting to improve their environment would be to focus on raising energy prices. Lantz and Feng (2006) provide evidence that gross domestic product, a common measure of income in EKC analyses, is unrelated to carbon dioxide emissions, but that either population or technological<sup>22</sup> changes are more likely culprits for explaining the emergence of an inverted-U shape in carbon dioxide emissions.

These empirical studies have been somewhat successful in responding to critiques of EKC theory, and have begun to address the mechanics by which an EKC might develop. However, these empirical efforts still appear subject to the criticisms of Stern (1998), in that they are less helpful in understanding the dynamic processes involved in the development of an EKC.

## **2.5. Static theoretical models, pro and con**

In addition to the empirical analyses mentioned above, many economists have constructed static theoretical models that address Stern's (1998) criticisms, including De Bruyn et al. (1998), Gawande et al.

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<sup>22</sup> The measure of technological change is a quadratic time index, and the authors admit that a more precise measure of technology should be possible.

(2001), Antweiler et al. (2001), and Pfaff et al. (2001). Rather than specify dependent and independent variables for a regression analysis in an ad hoc manner, these models develop a theoretical “snapshot” of a society in two different time periods, allowing the researcher to see whether increasing incomes in the second period increases or decreases pollution in the second period.

Some static models indicate that a focus on rising incomes as the cause of environmental improvements may be misplaced. For example, De Bruyn et al. (1998) utilize decomposition analysis to argue that an inverted-U relationship resulted from changes in environmental policy and international environmental agreements, not by structural changes in the economy. Gawande et al. (2001) also argue against easy answers for reductions in environmental degradation, showing that an EKC can appear over time as wealthy individuals move away from areas where pollution is higher. Such movement, if not accounted for, could lead a cross-sectional study to determine that rising incomes cause declining pollution when it may be lower pollution levels causing incomes to rise in a given area.

Other static models provide support for a link between income and a clean environment. Antweiler et al. (2001) use a supply and demand analysis to look at the effect of free trade on the environment. Dividing the effects of trade into scale, technique, and composition, they find that the increase in pollution from increased production (scale) is more than offset by the shift away from polluting industries (composition) and the improvement in technology (technique), leading to overall improvements in the environment due to free trade. Pfaff et al. (2001) also support the ability of an EKC to develop, deriving a static model where households are allowed to “buy” environmental quality through changing household production methods; under reasonable conditions, a household-level EKC develops.

These theoretical static models add important insight into what might cause an EKC to develop, especially when paired with the empirical research mentioned above. However, they are limited in their ability to describe the inherently dynamic nature of the decisions that societies make over time. Rising incomes have the potential to affect a number of potentially relevant variables, such as the rate of capital accumulation, which would tend to directly impact pollution levels, but also society’s marginal valuation of a clean environment and/or capital, and consumption of various goods, including “green goods,” which would tend to have a more indirect impact. Static models such as those presented here are, by their nature,



less capable of accounting for the temporally interconnected nature of the economic and environmental variables inherent in EKC theory. However, they still can be useful in understanding certain characteristics of the EKC. For example, Plassmann and Khanna (2006b) utilize a simple static model to investigate the impact of technology and preferences on the equilibrium income-pollution path.

## 2.6. Dynamic models

Empirical and static theoretical models will certainly continue to shed light on various aspects of EKC theory. The process by which an EKC might develop is inherently dynamic, however, which lends support to the belief that dynamic models hold the greatest potential for gaining insight into the origins of an EKC.

Selden and Song (1995) offer the first dynamic analysis of EKC theory, showing that the optimal path for society may follow an inverted-U. This path becomes more likely as technology advances and less likely as pollution overwhelms the carrying capacity of the environment, as proposed in Arrow et al. (1995). Selden and Song (1995) also issue a caution that nothing guarantees a country will take the optimal path, so focusing solely on the optimal path may tell us little about the actual experiences of individual countries as they transition to higher income levels.

Stokey (1998) develops both static and dynamic models, and indicates that an EKC is much more likely with endogenous technological change. Stokey's (1998) analysis focuses solely on production changes, excluding any discussion of individuals modifying their consumption choices. While Rothman (1998) criticized the tendency to ignore either consumption or production, rather than recognizing the importance of both effects, Stokey advances the literature by introducing the concept of endogenous technical change, something missing from most EKC analyses.<sup>23</sup> Cassou and Hamilton (2004) confirm Stokey's arguments, arguing that endogenous economic growth may help the emergence of an EKC in situations where an EKC would otherwise not arise.

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<sup>23</sup> Managi (2006) argues that, at least in empirical studies, inclusion of a technology variable is necessary. Contrast that, however, with Johansson and Kristrom (2007), who admit that they do not incorporate endogenous technical change but argue, and we agree, that models do not need to incorporate endogenous technical change in order to further the state of the literature.

Anderson and Cavendish (2001) focus on policy decision making and technical change. They stress that the traditional EKC model underestimates the ability of countries to obtain a cleaner environment as incomes rise by ignoring the roles of policy and technical progress. Kelly (2003) utilizes an optimal control framework to evaluate how social planners maximize utility that is subject to a pollution byproduct. He concludes that an EKC is more likely under two conditions: first, when pollutants assimilate slowly into nature, resulting in a faster build-up of pollution; and second, when the consumption share of utility decreases, indicating a greater role for a clean environment in determining the level of utility enjoyed by individuals and society. Both of these conditions together result in higher marginal benefits of abatement and increase the likelihood of an EKC.

Ansuategi and Perrings (2000) address the interaction between nations where the environment is concerned. They show that a country will choose a higher level of emissions when it does not have to bear the burden of its pollution, due to the transboundary properties of some pollutants. The primary drawback of their analysis is that it does not account for the possibility of trade. To assume that countries can affect each other negatively without admitting the potential benefits of trade on both countries leaves the analysis incomplete and less helpful. On the other hand, Janssen and van den Bergh (2004) use optimal control to measure the effect of trade on the use of extracted resources, and they find evidence for the potential existence of an EKC. However, they utilize a single social planner for both countries, maximizing the sum of the two social utility functions. This structure appears inconsistent with the nature of nations; in reality, each nation will independently seek to maximize their own utility.

Egli and Steger (2007) develop a simple model wherein individual households make the abatement choice. They justify this assumption by pointing out that in certain industries, households bear the direct cost of abatement by having catalytic converters or other technologies that reduce tailpipe emissions. Their model considers that households account for only part of the cost of pollution, so that there will be at least a small negative externality. They then aggregate to the national level, and discuss the public policy implications, such as the optimal tax in such a situation. Similar to this research, they address the impact on turning points when society's desire for a clean environment is greater than the desire for consumption, equal to the desire for consumption, and less than the desire for consumption. Their

important contribution is similar to this research, although it is much more limited in scope, and does not address the full impact of parameter change, as we do here.

Dinda (2005) creates a simple, single-good model with endogenous growth. Within the model, an EKC emerges along the non-optimal path. It is the result of insufficient allocation of capital to abatement. At low incomes, society is unable to allocate capital to abatement. This situation is non-optimal, but it is only at higher incomes that society is willing to increase investment to a level that more closely approximates the optimal path.

Each of these dynamic models has advanced the EKC debate by providing clearer understanding of what makes an EKC possible. In order to reach a conclusion regarding the EKC phenomenon, however, there is a great deal yet to be investigated. In particular, there are many assumptions made regarding the effect of certain parameters on variables in the model, and it may be incorrect to assume that the assumptions are universally correct. Criado (2008) and Aubourg et al. (2008) provide evidence that variations between regions and countries will impact the results of an EKC analysis, which should urge greater caution in applying assumptions. Moreover, each model herein includes only a few of the many variables that are likely involved in the development of empirical results predicted by EKC theory, which indicates that future research will likely investigate further those variables that are necessary for a thorough analysis.

## **2.7. Dynamic investigation of consumption and trade**

This research represents an evolutionary step in the EKC literature by further advancing the sophistication of dynamic analysis. The work of Kelly (2003) is used as a base from which to begin a more extensive investigation into the mechanics of a society's productive and consumptive choices through time, choices which might determine whether an EKC will develop.

Kelly (2003) offers an initial answer to a lingering question in the EKC literature: why do some pollutants appear to exhibit an EKC while others do not? According to Kelly, one possible explanation is that some pollutants are quickly assimilated into nature, while others remain in the air, water, or ground in hazardous form for a greater period of time, making their effects longer-lasting. Kelly develops an optimal control model that shows the possible existence of an EKC when pollutants are not fully assimilated into

nature in the current time period. Because current pollution causes a decrease in utility in both present and future time periods, slower assimilation increases the marginal benefit of additional pollution control. Therefore, a forward-thinking society will maximize the present value of a stream of utility arising in all future time periods, giving pollution abatement today additional value, and making it more likely that a society will engage in pollution control today.

Kelly utilizes a constant-population model, indicating that income levels rise as the economy grows; with increased production, pollution rises in the absence of abatement efforts. By assuming a cost function that is convex in abatement, the marginal costs of cleaning up the pollution are increasing. This would tend to result in lower abatement, but there are other pressures that might provide enough of a counter-balance to result in increasing abatement. For example, an accumulation of pollution stocks may begin to decrease utility to the point where the marginal benefits of cleaning up pollution outweigh the marginal costs, leading society to engage in more costly abatement. At any income level, if the marginal benefits to pollution control are greater than the marginal costs, then the income-pollution relationship has a negative slope; it has a positive slope at income levels where the marginal costs of pollution control are greater than the marginal benefits. By varying different pollution parameters, Kelly shows that the only assumptions needed to generate an EKC are: (1) a cost function convex in abatement; and (2) that environmental quality is a normal good.

Kelly utilizes a discrete time model that combines the above-mentioned cost function, which is convex in abatement effort, with Cobb-Douglas utility and production functions. He assumes a stochastic process, operating under substantial uncertainty regarding the results of abatement choices; he chooses initial values for the variables; and he generates “emissions curves” for a hypothetical country, simulating the path of income and emissions levels in the country over time. The path of emissions as capital stock<sup>24</sup> rises is the primary concern of the model, and Kelly presents his results as “emissions curves” and “pollution curves.” These curves show the level of emissions and pollution at every level of capital stock, and indicate a quadratic relationship for some sets of parameter values, but also strictly decreasing and strictly increasing emissions curves for other sets of parameter values.

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<sup>24</sup> Kelly and this research use per capita capital stock as a proxy for income.

To a greater or lesser extent, much of the recent EKC literature has begun to address Stern's (1998) critique that too many unanswered questions remain about the mechanics of an EKC. Stern (2004) renewed his criticism of EKC research, in general, stating that there is only flimsy statistical support for the existence of an EKC, and that a new generation of models will likely disprove the classical EKC theory and allow for greater emphasis on finding ways to improve the environment at lower levels of income.<sup>25</sup> It is too early to tell whether the predicted demise of classical EKC theory will become a reality, and there are certain areas that need illumination before Stern's (2004) "new EKC" wish can be realized. This research delves into one of those areas, specifically whether the assumptions used by many researchers regarding the impact of parameter change are universally applicable. Criado's (2008) observation that inter-regional and international variations can alter the results of an EKC analysis can also be said for other relevant variations, such as variations between industries and between pollutants themselves. By using two models of increasing complexity, we can explore the question of just how much confidence we can and should have in the assumptions we make related to the parameters of our models.

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<sup>25</sup> Constantini and Martini (2006) create a model for what they call a "Modified Environmental Kuznets Curve." In it, they use measures of sustainable development, rather than variables representing economic growth alone, as their regressors. This is likely not the type of change Stern believes is necessary, but it does appear that researchers have taken Stern's criticisms seriously.

## CHAPTER 3

## A CONTINUOUS TIME, DISCOUNTED UTILITY MODEL

**3.1. Introduction**

The conflict between proponents and opponents of EKC theory arises in large part from a dispute over the underlying assumptions used in both theoretical and empirical research. While the wide variety of assumptions cannot be simultaneously correct, that does not mean that seemingly contradictory assumptions cannot be valid along different ranges of parameter and variable values. Assume for a moment the possibility that proponents and opponents of EKC theory are all accurate, and that seeming differences are explained by the particular starting values of certain parameters and system variables. If that were the case, then a constructive dialogue might arise to determine society's options for moving to a higher level of income while protecting the environment. This chapter uses a simple model to evaluate whether a single set of assumptions need hold across all possible values of system parameters and variables.

We utilize Kelly's (2003) single-good optimal control model as a testing ground for some common assumptions regarding parameter change. In order to test one set of assumptions, we maintain other common assumptions which are not the subject of this research. We utilize the social planner construct, in which the social planner maximizes social utility by balancing choices of consumption, investment, and pollution abatement (which improves environmental quality). Specifically, the social planner balances current consumption with future consumption while simultaneously balancing consumption with a clean environment. The social planner construct is typically used for the sake of simplicity, but it is subject to certain criticisms. First, it may not be accurately descriptive of the way choices are made in a society where individual decision making, rather than central planning, governs many societal outcomes. Second, the social planner construct does not allow for the existence of negative externalities, or it guarantees that any such externalities are internalized completely. We utilize the social planner construct, even with these potential criticisms, because we aim to investigate the universality of a narrow category of assumptions utilized in EKC models. The social planner assumption is not one of the assumptions we wish to investigate, so we maintain it to allow this research to focus on those assumptions

we wish to investigate.

It is for the same reason that we choose not to use two components of Kelly's (2003) model--the stochastic effect of pollution on the environment and the desire of society to avoid abatement in order to better understand the process by which environmental degradation occurs. Kelly and Kolstad (2003) developed these components in order to allow them to better model the uncertain effects of carbon dioxide on climate change and to represent the desire of some to postpone addressing the impact of carbon dioxide until the process is better understood. While it is possible, or even likely, that these same characteristics are shared by all environmental problems, the impact of uncertainty on the process is not the focus of this research. We wish to focus very closely on a specific question, and we wish to avoid the possibility that that question will be confused by interesting, but unhelpful for our purposes, results arising solely out of the stochastic nature of Kelly's (2003) model.

### 3.2. The model<sup>26</sup>

Our model represents a primitive society, with a very low level of capital,  $k_t$ , and a pristine environment,  $\bar{m}$ . As society grows economically, capital increases but at the expense of the environment. The use of capital to produce output also results in pollution,  $m_t$ , which detracts from environmental quality. The measure of environmental quality at any time can be ascertained by looking at the difference between  $\bar{m}$  and the level of pollution,  $(\bar{m} - m_t)$ , so that as pollution approaches  $\bar{m}$ , environmental quality approaches zero. It is also possible to see  $\bar{m}$  as the carrying capacity of the environment, as described by Arrow et al. (1995), such that if pollution exceeds  $\bar{m}$ , the environment has lost some or all of its capacity to repair the harm caused by pollution, and will be forever unable to recover to its previous state.

As an example, consider the Chesapeake Bay watershed in Maryland, Virginia, Delaware, Pennsylvania, the District of Columbia, and New York. In recent years, increasing pollution runoff from farms, municipalities, or water treatment plants have led to increasing levels of nitrogen and phosphorous, which in turn have led to a decline in native species. Invasive, non-native species that thrive in high nutrient waters have begun to replace native species, and it is thought that, if a way to reverse the trend is

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<sup>26</sup> The model presented in this section will follow, as closely as possible, the model presented in Kelly (2003). Any deviations from the model will be specifically noted.

not found quickly, the Chesapeake will be pushed beyond the capacity to maintain its native species. In other words, if pollution levels rise too high, native species will be completely replaced by invasive species, and the previous ecosystem will be gone for good. The permanent loss of native species, and especially the blue crab, which many Marylanders consider a part of the state's identity, would be a terrible blow to residents of the states in the watershed.

The potential for society to suffer because of environmental harm is also represented in the model, where society gains utility,  $U$ , from two sources: consumption,  $c_t$ , and environmental improvements,  $(-m_t)$ . Society experiences diminishing marginal utility in both consumption and environmental improvements. Note that environmental quality is represented by the negative of pollution, representing that a decrease in the level of pollution is what gives society an increase in their utility. The amount of pollution emitted per unit of output,  $\sigma$ , is fixed. We call  $\sigma$  the pollution intensity parameter, but it can also be thought of as the environmental component of production technology. A value of zero for  $\sigma$  would indicate a completely clean method of production, without environmentally harmful byproducts of any kind, and a value of one would represent technology where every unit of output that could be consumed would be accompanied by an identical amount of pollution. There is a second technology component,  $A$ , which represents the efficiency of capital. A higher value of  $A$  represents a higher level of output per unit of capital. This model is a fixed technology model, so  $A$ , like  $\sigma$ , is fixed. Mazzanti and Zoboli (2007) discuss technological innovation in a similar way, as divided between innovations in productive efficiency,  $A$ , and innovations in environmental efficiency,  $\sigma$ .

Since the social planner cannot change the environmental component of production technology, and since society values a clean environment, the social planner must find some other way to raise utility through environmental improvements. She can engage in efforts to mitigate, or abate, the harmful side effects of production. This abatement process occurs simultaneous with production, so that the social planner chooses the percentage of pollution,  $u_t$  to abate, and the total output is reduced by some amount,  $G(u)$ . A value of  $u = 0$  indicates that the social planner has chosen to engage in no abatement measures, so that production output will be maximized, but pollution will occur at whatever level is indicated by  $\sigma$ , the pollution intensity parameter. A value of  $u = 1$  indicates that the social planner has chosen to abate all



pollution arising from production--the cost will be higher, but there will be no harm caused to the environment.

Production itself occurs in a process,  $F(k)$ , where capital is the only input, and where capital experiences diminishing marginal returns. Once the cost of abatement (in terms of output foregone) has been factored in and the social planner has allocated the optimal amount of net production,  $F(k)(1-G(u))$ , to consumption, the remaining production output is converted into capital at a 1:1 ratio. That amount of capital is added to the existing capital stock, increasing the amount of capital available for production in the next time period. That increase in capital is offset, somewhat, by the fact that capital depreciates at a fixed rate,  $\delta_1$ . A depreciation rate of zero indicates that capital is permanent, and a depreciation rate of one indicates that society must replace the entire capital stock each time period. Likewise, whatever pollution survives the abatement process,  $\sigma F(k)(1-G(u))$ , will add to the pollution stock. The stock of pollution decreases at the rate at which pollution is capable of being assimilated into the environment,  $\delta_2$ . An example of pollution assimilating into the environment can be seen in the limited capacity of all plants to remove carbon dioxide from the air during their respiration process. An assimilation rate of one means that there is no carryover pollution to the next time period; in other words, a pure flow pollutant. An assimilation rate of zero means that society is forever stuck with the pollution we allow into the environment, similar to the current situation with spent nuclear rods that have a half-life of thousands of years.

So, in mathematical terms, the social planner maximizes utility,  $U_t = U_t(c_t, -m_t)$ , where utility is twice differentiable and concave in consumption and environmental improvements, and that it satisfies the

Inada conditions:  $\frac{\partial U_t}{\partial c_t} > 0$ ,  $\frac{\partial^2 U_t}{\partial c_t^2} \leq 0$ ,  $\frac{\partial U_t}{\partial (-m_t)} > 0$ ,  $\frac{\partial^2 U_t}{\partial (-m_t)^2} \leq 0$ ,  $U_c(0, -m) = \infty$ ,  $U_c(\infty, -m) = 0$ ,

$U_{-m}(c, 0) = \infty$ ,  $U_{-m}(c, \infty) = 0$ . The assumption of concavity in utility allows for constant marginal utility over some interval, but disallows increasing marginal utility of consumption. Society receiving utility from environmental improvements, subject to diminishing marginal utility, is equivalent to society experiencing

a decrease in utility from an increase in pollution, subject to ever-larger decreases in utility:<sup>27</sup>  $\frac{\partial U_t}{\partial m_t} < 0$  and

$$\frac{\partial^2 U_t}{\partial m_t^2} \geq 0.$$

Society produces output at the level,  $F(k_t)$ , where  $F(k_t)$  is continuous, twice differentiable, and strictly concave in  $k_t$ , and satisfies the Inada conditions:  $\frac{\partial F(k_t)}{\partial k_t} > 0$ ,  $\frac{\partial^2 F(k_t)}{\partial k_t^2} < 0$ ,  $F_k(0) = \infty$ ,  $F_k(\infty) = 0$ .

Pollution emissions are produced at  $\sigma F(k_t)$ , and are abated as the social planner foregoes output in return for lower emissions. The social planner chooses what fraction of emissions to eliminate,  $u_t \in [0,1]$ , and the resulting loss in output,  $G(u_t)$  is continuous, twice differentiable, and convex in  $u_t$ :  $\frac{\partial G(u_t)}{\partial u_t} > 0$ ,

$$\frac{\partial^2 G(u_t)}{\partial u_t^2} > 0. \quad G(u_t) \text{ is therefore a unit cost function, representing the portion of each unit of output that is}$$

foregone in order to achieve  $u_t$ , the desired level of pollution abatement,  $G(u_t) \in [0,1]$ .<sup>28</sup> Net emissions, then, are defined by the portion of total emissions,  $\sigma F(k_t)$ , not abated through the choice of  $u_t$ :

$$e_t = (1 - u_t)\sigma F(k_t). \quad \text{The total foregone output as a result of abatement is defined by } G(u_t)F(k_t) \in [0, F(k_t)].$$

Total output is reduced by the total cost of abatement, yielding net output,  $F(k_t)(1 - G(u_t))$ . Net output can then be consumed today or invested for future consumption. Investment is denoted by  $z_t$  in society's resource constraint,  $c_t + z_t + G(u_t)F(k_t) - F(k_t) = 0$ . Therefore, according to the resource constraint, society utilizes output in three ways: consumption ( $c_t$ ); investment ( $z_t$ ); or pollution abatement  $G(u_t)F(k_t)$ .

Investment and depreciation of capital,  $\delta_1 \in [0,1]$ , define the law of motion for capital:

$$\frac{dk}{dt} = z_t - \delta_1 k_t = F(k_t)(1 - G(u_t)) - c_t - \delta_1 k_t. \quad \text{Higher levels of capital in future time periods allow for}$$

higher total output and, therefore, higher levels of consumption.

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<sup>27</sup> Of course, those large decreases in utility from environmental degradation must be outweighed by the initial increases in consumption that can only occur from production.

<sup>28</sup> The restriction of the cost function to this range requires that cost be nonnegative, and that the process of cleaning up emissions cannot cost more than the output being produced.

As a result, it is possible for a society with rising capital stocks to enjoy higher levels of consumption even while increasing abatement and improving the environment. The stock of a clean environment is measured by reference to the amount of pollution remaining from the last time period ( $m_t$ ), plus the net emissions from the current time period production. The stock of pollution depreciates as it is assimilated into nature, which it does at the rate  $\delta_2 \in [0,1]$ . Emissions can thus be thought of as investment in the future stock of pollution, with the assimilation rate depreciating that stock, so that the pollution stock in any time period is simply the cumulative effect of emissions from all past time periods, minus the amount of accumulated pollution which the environment has been able to assimilate since the last time period. Emissions in the present time period and the assimilation rate define the law of motion for the stock of a clean environment:  $\frac{d(-m)}{dt} = -e_t + \delta_2 m_t = -(1-u_t)\sigma F(k_t) + \delta_2 m_t$ .

The social planner is considered to be forward-looking, making decisions based on an infinite time horizon. Thus, the social planner faces the following problem:

$$\begin{aligned} \text{Max } W &= \int_t e^{-\rho t} U(c_t, -m_t) dt \\ \text{s.t.} \\ c_t + z_t + G(u_t)F(k_t) - F(k_t) &= 0 \\ \frac{dk}{dt} = z_t - \delta_1 k_t &= F(k_t)(1-G(u_t)) - c_t - \delta_1 k_t \\ \frac{d(-m)}{dt} = -e_t + \delta_2 m_t &= -(1-u_t)\sigma F(k_t) + \delta_2 m_t \end{aligned}$$

In other words, the social planner maximizes utility subject to a budget constraint and the laws of motion for capital investment and pollution. This is a present-value specification, which means that future utility is discounted to the present, according to the positive rate of time preference, or discount factor,  $\rho$ . The social planner, with this specification, chooses the stream of decisions for all variables at time zero. The control variables are  $c_t$  and  $u_t$ , and the state variables are  $k_t$  and  $-m_t$ .

Missing from the problem is an express transversality condition. Some believe that, for general infinite time problems, a transversality condition is not necessary.<sup>29</sup> Even if not required, however, a

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<sup>29</sup> For a justification of this assumption, please see Ferguson and Lim (1998) and Seierstad and Sydsaeter (1987). Ferguson and Lim (pp. 207-8) state that the traditional transversality condition is not a necessary condition for

transversality condition is useful in that it assures that the system does not grow without bound. That is the conventional transversality condition, and while our system does not contain an express transversality condition, the conventional transversality condition is satisfied by the assumptions of the model.<sup>30</sup>

The Present Value Hamiltonian is:

$$H = e^{-\rho t} U(c_t, -m_t) + \psi_t [F(k_t)(1 - G(u_t)) - c_t - \delta_1 k_t] - \xi_t [(1 - u_t)\sigma F(k_t) - \delta_2 m_t],$$

where  $\psi_t$  is the present-value co-state variable for capital and  $\xi_t$  is the present-value co-state variable for a clean environment. Co-state variables represent the respective societal values of a marginal unit of the state variables. It is assumed that societies will always place a positive value on an additional unit of output and a clean environment, leading to positive values for  $\psi_t$  and  $\xi_t$ .

Deriving first-order conditions for this problem leads to optimal solutions for the control variables, along with the laws of motion for the co-state variables. We assume all decisions by the social planner are made in current time, or in other words, that the decision for each time period  $t$  is made at the beginning of that time period. We thus state the current-value first-order conditions as:<sup>31</sup>

$$\frac{\partial U}{\partial c} - \lambda = 0 \tag{3.1}$$

$$-\lambda F(k) \frac{\partial G}{\partial u} + \varphi \sigma F(k) = 0 \tag{3.2}$$

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infinite time horizon problems. For finite time horizons, either the state variable must have a fixed value at the terminal time, or if the state variable is free, the co-state variable must equal zero in the terminal time period. Seierstad and Sydsaeter (pp. 233-34) state that the “natural” transversality conditions are not necessary conditions for the general case.

<sup>30</sup> Recall that the production function is strictly concave and satisfies the Inada conditions. At some point, there is no additional societal benefit to be gained from increasing the capital stock. If production has an upper limit, so, too, does consumption of that production. Abatement is expressed as a percentage of each unit of production foregone to improve the environment, so an upper limit on production guarantees an upper limit on abatement, as well. If consumption experiences an upper limit, a utility maximizing social planner cannot allow pollution to increase without bound, for each additional unit of pollution would diminish utility. Thus, pollution must have an upper limit, as well. Finally, if capital and pollution have upper limits, then the costate variables for capital and pollution, described herein, must have limits as well.

<sup>31</sup> In order to reduce clutter, we have removed time subscripts. First-order conditions define the optimal time paths for the control, state, and co-state variables; however, as a caution, we refer the reader to the previous discussion of Selden and Song’s (1995) assertion that many societies may not be progressing along their optimal path. Therefore, the following analysis would not be representative of a country’s development in the presence of significant impediments to optimal decision making; for example, in the presence of monopolistic interest groups that control government policy through force, intimidation, or economic coercion.

$$\frac{d\lambda}{dt} = \rho\lambda - \lambda \left[ \frac{\partial F}{\partial k} (1 - G(u)) - \delta_1 \right] + \varphi \left[ (1 - u)\sigma \frac{\partial F}{\partial k} \right] \quad (3.3)$$

$$\frac{d\varphi}{dt} = \rho\varphi - \frac{\partial U}{\partial(-m)} + \varphi\delta_2 \quad (3.4)$$

$$\frac{\partial k}{\partial t} = F(k_t)(1 - G(u_t)) - c_t - \delta_1 k_t \quad (3.5)$$

$$\frac{\partial(-m)}{\partial t} = -(1 - u_t)\sigma F(k_t) + \delta_2 m_t \quad (3.6)$$

Eqs. (3.1) and (3.2) are the partial derivatives of the Hamiltonian with respect to the control variables, consumption, and abatement, respectively. Eqs. (3.5) and (3.6) are the laws of motion, previously defined, for the state variables, capital, and a clean environment. Arriving at the current-value first-order conditions requires multiplying through each present value equation by  $e^{\rho t}$ . The exceptions to that rule are Eqs. (3.3) and (3.4), the laws of motion for the costate variables. We define the current-value co-state variables as  $\lambda = e^{\rho t} \psi$  and  $\varphi = e^{\rho t} \xi$ . Taking the derivative of these definitions with respect to time, we arrive at Eqs. (3.3) and (3.4).

Eq. (3.1) indicates that the level of consumption chosen will be such that the marginal value of present consumption is equated with the marginal value of capital, which represents future consumption. Note that (3.1) implies that  $\lambda > 0$  along the optimal path. Meanwhile, Eq. (3.2) indicates the level of pollution abatement that will be chosen along the optimal time path. All of the equations represent a number of tradeoffs that must be considered in choosing the optimal level of each variable. The first part of (3.2),  $-\lambda F(k) \frac{\partial G}{\partial u}$ , is the lost value to society when an additional unit of income is spent on pollution abatement, rather than consumption or investment; the second part,  $\varphi \sigma F(k)$ , is the gain to society due to abatement. Note also that (3.2) and (3.1) imply that  $\varphi > 0$ , which assumes that society will place a positive value on a clean environment along the optimal time path.

The remaining first-order conditions are the laws of motion for the state and co-state variables. Of particular interest is Eq. (3.3), the law of motion for the co-state variable of capital, which is composed of

two elements: the first part,  $-\lambda \left[ \frac{\partial F}{\partial k} (1-G(u)) - \delta_1 \right]$ , represents the value of the net marginal product of capital, after accounting for abatement costs and replacement of depreciated capital; the second part,  $\varphi \left[ (1-u)\sigma \frac{\partial F}{\partial k} \right]$ , represents the value of the harm caused by the marginal product of capital, as it increases pollution. Investment in future capital means greater potential consumption, but also greater potential pollution, and (3.3) illustrates the cost and benefit of each additional unit of capital accumulated. As one of her many tasks, the social planner must balance the gain in potential consumption that comes from investment with the potential loss in societal well-being that would result as the additional capital is put to use in making the goods society wishes to consume.

Eq. (3.4) describes the change in the marginal value of a clean environment over time. If pollution increases, then societal utility will decrease and the marginal value of a clean environment, measured by  $\varphi$ , will rise  $\left( -\frac{\partial U}{\partial(m)} > 0 \right)$ . The assimilation rate,  $\delta_2$ , impacts the level of pollution, removing a portion of the pollution stock, and therefore also impacts the value of  $\varphi$ . As society progresses along the optimal time path, production will increase, and depending upon the level of abatement chosen, so will the level of pollution. Recall that, as stated earlier in the chapter, when  $\delta_2 = 1$ , the pollution is a pure flow pollutant, and will not accumulate any stock of the pollutant across time. That means that the level of environmental quality will be degraded only to the extent of current pollution, and without any build-up of pollution, society would not experience the type of environmental degradation that would cause a significant increase in society's valuation of the environment. In other words, *ceteris paribus*, an increase in the assimilation rate would yield a lower level of pollution in every time period and, therefore, a lower value of  $\varphi$  along the optimal time path.

Of course, because this is a system of equations that are solved simultaneously, and therefore all else is not held constant, the impact of an increase in the assimilation rate may not be that simple to predict. Eq. (3.4) seems to indicate that, as the pollution stock is assimilated into nature, the marginal value of a clean environment appears to increase as well. This result is very counter-intuitive, because it proposes that an improvement in environmental quality leads to an increase in the value of the marginal unit of a

clean environment. Kelly (2003) argues that as the assimilation rate rises, the marginal benefit of abatement falls,<sup>32</sup> which could lead to higher pollution along the optimal path. Over some range of values, that is almost assuredly correct. However, it is also possible that a decrease in abatement could yield higher pollution levels and, therefore, a higher marginal value of a clean environment. In short, it is unclear at this point what the likely result of a change in the assimilation rate will be. Thus, Kelly's (2003) assumption is inherently sound, but not universal, as will be further illustrated in more depth in subsequent parts of this research.

Eq. (3.5) describes the change in capital stock over time. The social planner knows that whatever output is left over after the levels of abatement and consumption have been chosen for the current time period will be invested in capital for future time periods, and this investment decision occurs simultaneously with the consumption and abatement decisions. Additionally, every time period sees some depreciation in the then-existing capital stock. Eq. (3.6) describes the change in the stock of a clean environment over time. Emissions in the current time period increases the stock of pollution, and assimilation of some portion of the accumulated stock of pollution, emitted in all past time periods, improves the environment.

For numerical analysis, specific functional forms for utility, production, and pollution abatement cost functions are defined. Kelly (2003) utilizes the following functional forms:

$$U(c, -m) = \frac{[c^\alpha (\bar{m} - m)^{1-\alpha}]^{1-\eta} - 1}{1-\eta} \quad (3.7)$$

$$F(k) = Ak^\gamma \quad (3.8)$$

$$G(u) = b_0 + b_1 u^{b_2} \quad (3.9)$$

Utility and production are Cobb-Douglas functions; the utility function, known as the constant relative risk-aversion utility function, preserves the assumed negative marginal utility of pollution as well as the positive marginal utility for consumption, and it is used to maintain a constant level of risk aversion among members of society across consumption levels. As income in society grows, its general preference structure remains the same, so its willingness to engage in the trade-off between present and future income,

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<sup>32</sup> Kelly (2003), pp. 1380-81.

and between consumption and a clean environment remains the same.<sup>33</sup> The parameter  $\eta$  measures both relative risk aversion and the willingness to substitute consumption across time periods. Further,  $\eta \leq 1$  assures decreasing marginal utility to both consumption and a clean environment, and  $\eta < 1$  assures that consumption and environmental quality are complementary goods. The parameter  $\alpha$  represents the return in utility to the individual from consumption, and  $\bar{m}$  is the assimilative capacity of the environment. As discussed previously, that means that if pollution ever equals  $\bar{m}$ , there will be no environmental quality left for society to enjoy, and if pollution is ever greater than  $\bar{m}$ , the environment will be permanently damaged, so that all the clean-up in the world could not return the environment to its pristine state.

The production function,  $F(k)$ , assumes diminishing returns to scale in capital (i.e.,  $\gamma \in (0,1)$ ). The parameter  $A$  represents the level of technology that concerns the efficiency of capital,<sup>34</sup> as opposed to the environmental efficiency of capital,  $\sigma$ . Meanwhile, the pollution abatement cost function utilizes the following parameters:  $b_0$ , the fixed cost of pollution abatement;  $b_1$ , the variable cost of abatement; and  $b_2$ , the measure of convexity for the abatement cost function. Fixed costs and variable costs are fairly easily understood, but the meaning of the convexity parameter bears a little more explanation. The abatement cost function is convex as long as  $b_2 > 1$ , and  $u^{b_2}$  will converge to 1 as  $u$  approaches 1. A higher value of  $b_2$  means that the abatement cost function is increasingly convex, which in turn means that abatement cost is lower for all values of  $u$  less than 1, and that for low values of  $u$  marginal cost is lower, and for higher values of  $u$  marginal cost is higher. As mentioned previously, abatement cost will always be the same when full abatement ( $u = 1$ ) is chosen.

Applying the functional forms presented above, Eqs. (3.1) through (3.6) may be rewritten as:

$$\alpha c^{\alpha(1-\eta)-1} (\bar{m} - m)^{(1-\alpha)(1-\eta)} = \lambda \quad (3.1a)$$

$$\lambda (b_1 b_2 u^{b_2-1}) = \varphi \sigma \quad (3.2a)$$

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<sup>33</sup> It is certainly not impossible for a change in preferences to occur as incomes rise, but we leave that complicating factor for future researchers.

<sup>34</sup> One admitted drawback to this specification of the production function is that it assumes a constant level of technology. Future research possibilities include modifications to allow for learning-by-doing or other forms of endogenous growth, thus allowing  $A$  to change over time.



$$\frac{d\lambda}{dt} = \lambda(\rho - \gamma Ak^{\gamma-1}(1 - b_0 - b_1 u^{b_2}) + \delta_1) + \varphi[(1 - u)\sigma \gamma Ak^{\gamma-1}] \quad (3.3a)$$

$$\frac{d\varphi}{dt} = \varphi(\rho + \delta_2) - (1 - \alpha)c^{\alpha(1-\eta)}(\bar{m} - m)^{\alpha(\eta-1)-\eta} \quad (3.4a)$$

$$\frac{dk}{dt} = Ak^{\gamma}(1 - b_0 - b_1 u^{b_2}) - c - \delta_1 k \quad (3.5a)$$

$$\frac{d(-m)}{dt} = -(1 - u)\sigma Ak^{\gamma} + \delta_2 m \quad (3.6a)$$

Note from (3.1a) that if pollution ever rises beyond the maximum amount that the environment can bear, the left-hand-side becomes negative, leading society to place a negative value on an additional unit of capital. We place no specific constraint on this possibility, however, because it could only occur in a non-optimal scenario; it would be characterized by society paying to first create and then destroy productive capital. Such a scenario is not applicable to this model, which envisions a society choosing an optimal path through time; if society has a level of capital that is detrimental to societal well being, then society has made a choice that did not maximize societal utility and is, by definition, on some non-optimal time path.

### 3.3. Solving the problem

Of course, the first-order equations, by themselves, tell us very little about the process they describe; we learn a good deal more from the process once it has been solved for the stationary state. The process of solving a dynamic problem, such as the one presented here, can take a number of paths, most of which require the reduction of the first-order equations to a system of differential equations composed exclusively of parameters and those variables for which there exist differential equations. That system of equations need then be solved for the steady or stationary state; in the present case the next step is to utilize that stationary state and reasonable initial values to derive the optimal path of the system to its steady (or stationary) state. This section describes the process utilized to solve the particular dynamic problem presented in this chapter.

#### 3.3.1. The system of differential equations

We begin by reducing the first-order conditions to a system of differential equations comprised solely of parameters and our state and co-state variables. Eqs. (3.1a) and (3.2a) can be solved for the two

control variables, yielding

$$c = \left[ \frac{\lambda}{\alpha} \right]^{\frac{1}{\alpha(1-\eta)-1}} (\bar{m} - m)^{\frac{-(1-\alpha)(1-\eta)}{\alpha(1-\eta)-1}} \quad (3.1c)$$

$$u = \left[ \frac{\varphi}{\lambda} \right]^{\frac{1}{b_2-1}} \left[ \frac{\sigma}{b_1 b_2} \right]^{\frac{1}{b_2-1}}. \quad (3.2c)$$

These optimal choices can then be substituted into the remaining differential equations. We lose nothing by doing so, because (3.1c) and (3.2c) define the optimal choice for consumption and abatement at any point in time, and that condition is maintained within the other equations.

Once the solutions for consumption and abatement are substituted into the remaining equations, the following system of differential equations results:

$$\frac{d\lambda}{dt} = \lambda \left( \rho + \delta_1 - \gamma A k^{\gamma-1} \left( 1 - b_0 - b_1 \left[ \frac{\varphi \sigma}{\lambda b_1 b_2} \right]^{\frac{b_2}{b_2-1}} \right) \right) + \varphi \left[ \left( 1 - \left[ \frac{\varphi \sigma}{\lambda b_1 b_2} \right]^{\frac{1}{b_2-1}} \right) \sigma \gamma A k^{\gamma-1} \right] \quad (3.3c)$$

$$\frac{d\varphi}{dt} = \varphi (\rho + \delta_2) - (1-\alpha) \left( \frac{\lambda}{\alpha} \right)^{\frac{\alpha(1-\eta)}{\alpha(1-\eta)-1}} (\bar{m} - m)^{\frac{\eta}{\alpha(1-\eta)-1}} \quad (3.4c)$$

$$\frac{dk}{dt} = A k^{\gamma} \left( 1 - b_0 - b_1 \left[ \frac{\varphi \sigma}{b_1 b_2 \lambda} \right]^{\frac{b_2}{b_2-1}} \right) - \left[ \frac{\lambda}{\alpha} \right]^{\frac{1}{\alpha(1-\eta)-1}} (\bar{m} - m)^{\frac{-(1-\alpha)(1-\eta)}{\alpha(1-\eta)-1}} - \delta_1 k \quad (3.5c)$$

$$\frac{d(-m)}{dt} = - \left( 1 - \left[ \frac{\varphi \sigma}{\lambda b_1 b_2} \right]^{\frac{1}{b_2-1}} \right) \sigma A k^{\gamma} + \delta_2 m \quad (3.6c)$$

### 3.3.2. An emissions curve

The process of finding the stationary state in a simple model can be handled with relative ease; the process involves setting each equation equal to zero, to represent the lack of movement at the stationary state, and then solving for the value of the variables in that state. In most cases, a phase diagram is then used to characterize the optimal path and the comparative statics of the system. The equations presented above, however, defy such an easy solution, and require a more complex approach.

Simulations are often used to solve complex problems; Kelly (2003) utilizes simulations, as do Farzin (1996), Howarth (2000), and Huhtala (1997). We, too, will utilize a simulation to derive the optimal path; in doing so, we utilize the algorithms provided by the mathematical program Matlab. By solving for the stationary state, we can then derive the optimal path for all variables, and check the effect of various parameter changes on the emissions path. EKC theory is often expressed in terms of the relationship between incomes and the environment. We will use capital as the measure for income, paralleling Kelly (2003). Capital is the means by which production of goods takes place, and a standard macroeconomic identity exists between real income and the total amount of goods consumed and saved. Meanwhile, emissions are defined in the model as follows:

$$e = (1 - u_t) \sigma F(k_t) = \left( 1 - \left[ \frac{\varphi \sigma}{\lambda b_1 b_2} \right]^{\frac{1}{b_2 - 1}} \right) \sigma A k^\gamma \quad (3.10)$$

As the variables progress through time along their optimal time paths, equation (3.10) allows us to track the optimal level of emissions, as well, and by plotting levels of capital against emissions, we generate an emissions curve. The time paths also allow us to generate pollution curves by plotting capital stock against pollution levels. We do this because a singular focus on emissions may ignore the long-term effects of pollution accumulation. Arrow et al. (1995) explicitly criticized such a focus on emissions, stating that zero emissions mean less if the environment has been pushed beyond its ability to recover by high pollution levels at intermediate income levels. In other words, and as discussed previously, if pollution has surpassed  $\bar{m}$ , the fact that society has reduced current emissions cannot compensate for the permanent damage done to the environment. In such a case, the environment would have been harmed beyond its ability to be repaired, so even eliminating pollution entirely could not return society to its original level of utility with respect to the environment. Once optimal emissions and pollution curves have been derived, modification of various parameters will allow us to see their impact on the optimal emissions and pollution curves, and comparison of emissions curves will then allow us to test the validity of assumptions made regarding EKCs.

### 3.3.3. The stationary state

Eqs. (3.3c) to (3.6c) cannot be easily solved without the use of an algorithm that can take small steps until arriving at the stationary state. By utilizing the program Matlab we derive the stationary state.<sup>35</sup> This first step, solving for the stationary state, is feasible using the continuous equations already derived. The next step, deriving the optimal path for the system, requires that we discretize the continuous equations. We do so by utilizing a process described by Lyon (2006), which begins with formulating our problem as a discrete rather than continuous problem. We restate the Present Value Hamiltonian as a discrete-time Bellman equation, wherein the social planner maximizes Utility subject to the laws of motion for capital and environmental quality:<sup>36</sup>

$$V(k_t, -m_t) = U(c_t, -m_t) + \beta V_{t+1}(k_t, -m_t)$$

$$V(k_t, -m_t) = U(c_t, -m_t) + \beta V_{t+1}(F(k_t)(1 - G(u_t)) - c_t + (1 - \delta_1)k_t, -(1 - u_t)\sigma F(k_t) - (1 - \delta_2)m_t) \quad (3.11)$$

The first-order equations are derived in similar fashion to the continuous equations derived above. By taking the partial derivatives of (3.11) with respect to  $c$  and  $u$ , yields (3.14) and (3.15), respectively. By the Envelope theorem:

$$\frac{\partial V_t}{\partial k_t} = \beta \frac{\partial V_{t+1}}{\partial k_{t+1}} ((1 - \delta_1) + F'(k_t)(1 - G(u_t))) + \beta \frac{\partial V_{t+1}}{\partial (-m)_{t+1}} (1 - u_t)\sigma F'(k_t) \quad (3.12)$$

$$\frac{\partial V_t}{\partial (-m)_t} = U_{-m_t}(c_t, -m_t) + \beta \frac{\partial V_{t+1}}{\partial (-m)_{t+1}} (1 - \delta_2) \quad (3.13)$$

We define  $\lambda_t = \frac{\partial V_t}{\partial k_t}$  and  $\phi_t = \frac{\partial V_t}{\partial m_t}$ , which when substituted into (3.12) and (3.13) yields (3.17) and (3.18).

Our final first-order conditions are the laws of motion for capital (3.18) and for a clean environment (3.19).

$$\alpha c_t^{\alpha(1-\eta)-1} (\bar{m} - m_t)^{(1-\alpha)(1-\eta)} - \beta \lambda_{t+1} = 0 \quad (3.14)$$

$$-\beta \lambda_{t+1} (b_1 b_2 u_t^{b_2-1}) - \beta \phi_{t+1} \sigma = 0 \quad (3.15)$$

$$\lambda_t - \beta \lambda_{t+1} ((1 - \delta_1) + \gamma A k_t^{\gamma-1} (1 - b_0 - b_1 u_t^{b_2})) - \beta \phi_{t+1} (1 - u_t) \sigma \gamma A k_t^{\gamma-1} = 0 \quad (3.16)$$

$$\phi_t - \beta \phi_{t+1} (1 - \delta_2) + (1 - \alpha) c_t^{\alpha(1-\eta)} (\bar{m} - m_t)^{\alpha(1-\eta)-\eta} = 0 \quad (3.17)$$

<sup>35</sup> Code available from the author upon request.

<sup>36</sup> Note that the budget constraint is contained within the law of motion for capital.

$$k_{t+1} - Ak_t^\gamma (1 - b_0 - b_1 u_t^{b_2}) + c_t - (1 - \delta_1)k_t = 0 \quad (3.18)$$

$$m_{t+1} - (1 - u_t)\alpha Ak_t^\gamma - (1 - \delta_2)m_t = 0 \quad (3.19)$$

Matlab is able to derive the stationary state for the system yet not for all ranges of parameter values. We choose the parameter values listed in Table 1, and the resulting stationary state, as the starting point for our analysis. The initial parameter values need to satisfy two important criteria, one functional and one theoretical. The functional criteria is that algorithmic programs like Matlab are limited in their capacity to solve complex systems, so the parameter values must be such that Matlab is capable of solving the system of equations for the stationary state. The set of parameter values that will allow for a stationary state solution is not unlimited, but neither is that set extremely small. However, meeting the functional criteria is a necessary but not sufficient condition; the theoretical criteria is needed to assure that the results are meaningful.<sup>37</sup>

The theoretical criteria is that each parameter must be within a range that allows for a meaningful, if abstract, comparison to real life. For example,  $\gamma$  needs to be somewhere between 0 and 1 because, as the parameter that reflects returns to capital, a value between 0 and 1 reflects diminishing returns to capital. The fixed and variable costs of abatement,  $b_0$  and  $b_1$ , respectively, need to be positive and, in the case of  $b_1$ , less than 1, in order to allow for the possibility that society is capable of abating 100 percent of emissions in any time period, should it choose to do so. The only constraint on  $b_2$  is that it be greater than 1, in order to assure that the cost function is convex. We require that  $0 < \eta < 1$  in order to guarantee that there be complementarity between consumption and environmental quality, and that  $0 < \alpha < 1$  in order to guarantee that society receives positive utility from consumption, subject to diminishing marginal utility. We also require that  $0 < \delta_1 < 1$  so that capital depreciates over time, but cannot be greater than 1 or else future capital would disappear before it came into existence. Likewise, we require that  $0 < \delta_2 < 1$  so that pollution assimilates at some positive rate, but so that pollution is neither a pure stock nor a pure flow pollutant. We require that  $0 < A < 1$ , to represent that capital is productive but not perfectly efficient, and that  $0 < \sigma < 1$ , to represent that production results in pollution, but that the amount of pollution

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<sup>37</sup> A number of sets of parameter variables which fulfilled the functional criteria were discarded because they failed to fulfill the theoretical criteria. For example, a number of parameter sets were unusable because they would have required that society receive increasing marginal utility, or that society experience increasing returns to capital.

Table 1

Parameter and variable values	
Parameter	Value
$\gamma$	0.45
$b_0$	0.01
$b_1$	0.08
$b_2$	3.5
$\eta$	0.9
$\alpha$	0.5
$\delta_1$	0.1
$\delta_2$	0.1
A	0.6
$\sigma$	0.45
$\rho$	0.05
$\bar{m}$	10
Stationary state	
c	0.602
u	0.8565
$\lambda$	0.9523
$\varphi$	0.4023
k	2.4455
m	0.5794

is less than the amount of output. The parameter  $\rho$  is the discount rate, so we require that  $0 < \rho < 0.10$  in order to reflect what we believe is a reasonable discount rate. Finally, we require only that  $\bar{m} > 0$ , so that the environment have some positive maximum carrying capacity. A higher value of  $\bar{m}$  means that each unit of pollution has a relatively lower negative impact on the environment as a whole.

These restrictions assure that the model will allow for a meaningful comparison to reality. However, this comparison will be only an abstract comparison, for the values are not chosen to correspond to any specific real-world circumstances. The purpose of this research is to test whether, under reasonable conditions, the expected systemic reactions to parameter change will always hold. We do not attempt to show whether those expected reactions hold when applied to any specific pollutant in any specific society. Thus, it is not of great concern that the returns to capital utilized here may not match with the average returns to capital achieved in, say, the U.S. steel industry or the Brazilian power industry. Similarly, our simulations span 100 time periods, which cannot be easily transformed into a number of years. In reality,

the path to the stationary state is likely different for each society, including the number of years that it takes to arrive at the stationary state. In other terms, the parameter and variable values, as well as the interval over which the simulation occurs, should be considered in relative rather than absolute terms.

#### 3.3.4. *The optimal time paths*

With stationary state values in hand, we can derive the optimal time path for the system. By utilizing Matlab algorithms, once again we will input initial values for capital and pollution stocks, stationary state values for our costate variables,  $\lambda$  and  $\varphi$ , and allow the values of the control variables to be determined by the system. Our choice of iterative methods for finding the best fit path from the initial time period to the stationary state is what Press et al. (1992) refer to as a “Relaxation Method.” In brief, we use Matlab to construct an estimated time path, using initial state values and stationary state costate values, and then we use the relaxation method to find the best fit path. The relaxation method constructs a matrix where the (i,j) element is the derivative of equation  $i$  with respect to variable  $j$ ; Eq.  $i$  cycles through Eqs. (3.14) to (3.19), for every time period.<sup>38</sup> The result is a matrix in block form, around the primary diagonal, which can be solved using Gaussian elimination to derive the incremental changes in variable values. The process undergoes multiple iterations until the incremental changes are sufficiently small and the estimated time path closely approximates the true time path.

We choose to simulate the optimal time path for a country from the beginning of development through the stationary state, so we choose an initial capital level of  $k_0 = 0.05$ . Before development begins, a country can be thought of as having no pollution, so we choose an initial pollution level of  $m_0 = 0$ . Combined with the terminal values of  $\lambda_{ss} = 0.9523$  and  $\varphi_{ss} = 0.4023$ , we utilize the relaxation method to derive the optimal emissions and pollution curves illustrated in Figs. 1a and 1b, respectively. The inverted-U shape exhibited in Figs. 1a and 1b are the traditional shape of the EKC. Thus, for the reasonable values chosen, an EKC is exhibited, though it is one without full abatement, so that society chooses a level of pollution less than the peak but greater than 0.

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<sup>38</sup> Thus, Eq. 15 would be (3.16) at  $t = 3$  and Eq. 61 would be (3.14) at  $t = 11$ .

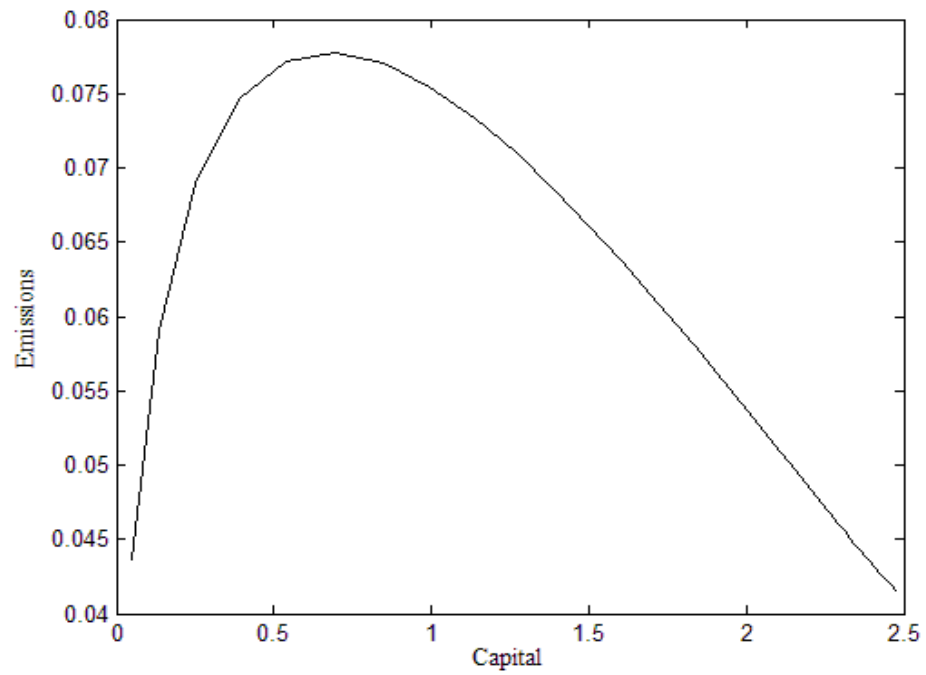


Fig. 1a. Emissions path (flow).

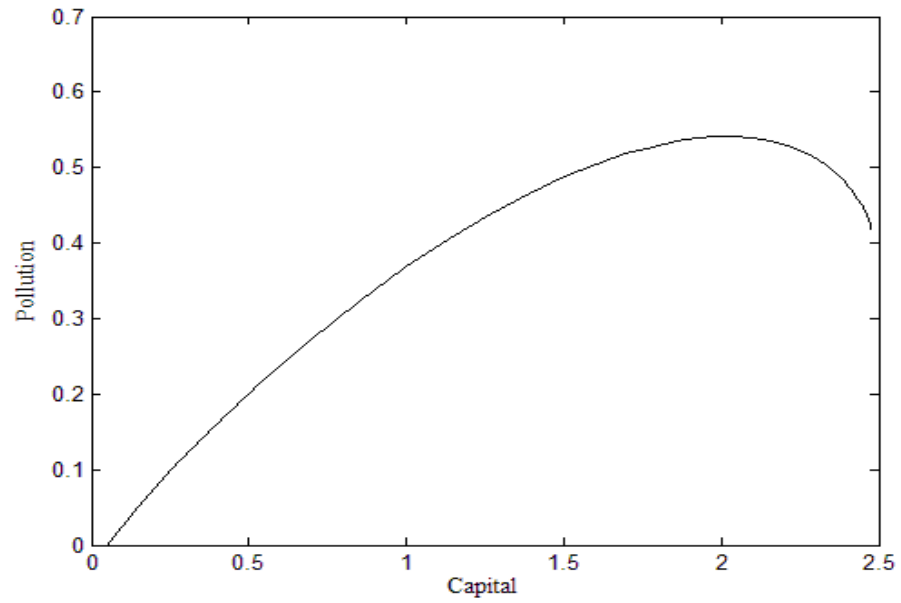


Fig. 1b. Pollution path (stock).



The time paths for the remaining variables are illustrated in Figs. 2a-2c. Fig. 2a shows that consumption rises in initial time periods, reaching a value near the stationary state value of 0.602 within the first third of the time allotted for the simulation. Fig. 2a also shows that abatement rises rapidly within the first third of the allotted time to a value near the stationary state of 0.8565. The rapid rise in consumption is fueled by the rapid rise of capital, exhibited in Fig. 2b. At  $t = 30$ , capital has reached a level above 2, where it remains throughout the rest of the simulation. Higher levels of capital allow for higher levels of consumption, but it also explains the rapid rise in pollution, seen in Fig. 2c. Additionally, as capital rises, the value of  $\lambda$  falls, and as pollution rises, and therefore environmental quality falls, the value of  $\varphi$  rises. So as society begins to progress from its initial state, it begins to accumulate capital and consume the output. As pollution rises, however, society also chooses to forego more and more potential output in order to combat the increasing pollution problem. Emissions levels peak very early in the development path, before abatement efforts can catch up to the increases in pollution. Pollution also peaks early in the society's development path, although later than emissions, due to the fact that in order to eliminate pollution, society must combine abatement efforts with the passage of time, over which pollution assimilates.

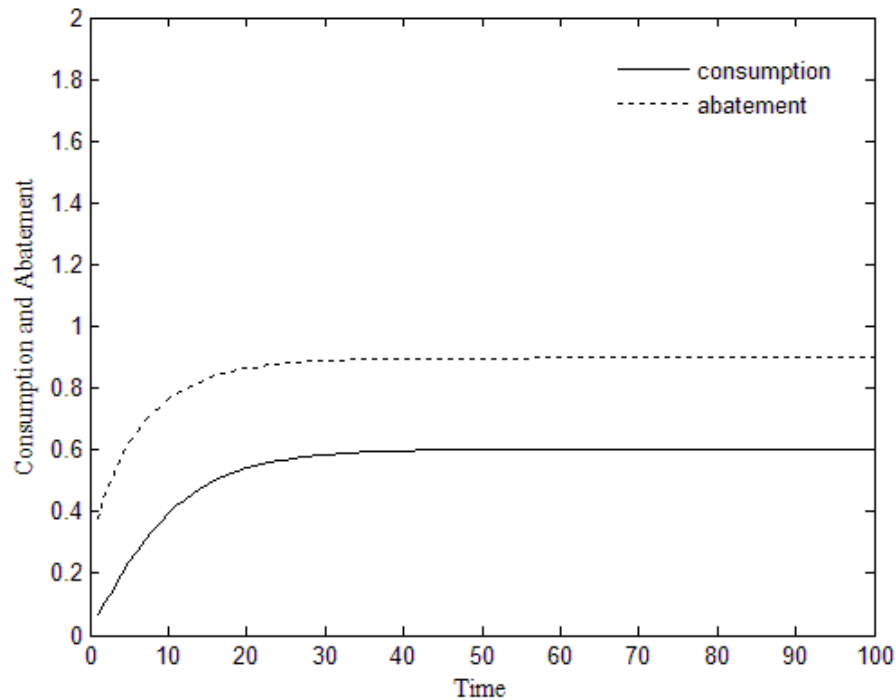


Fig. 2a. Time path of optimal consumption and abatement.

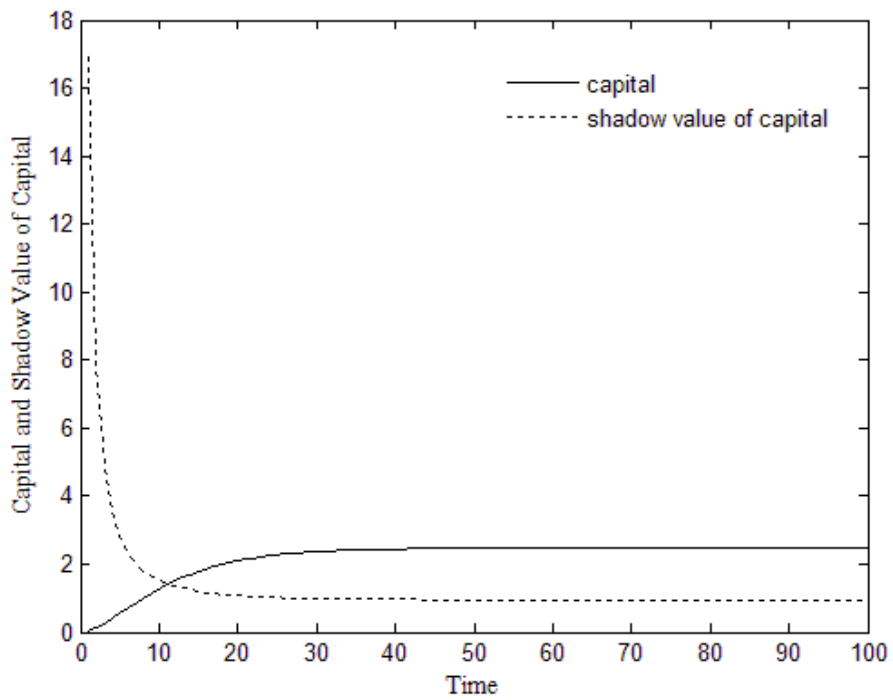


Fig. 2b. Time path of optimal capital and shadow value of capital.

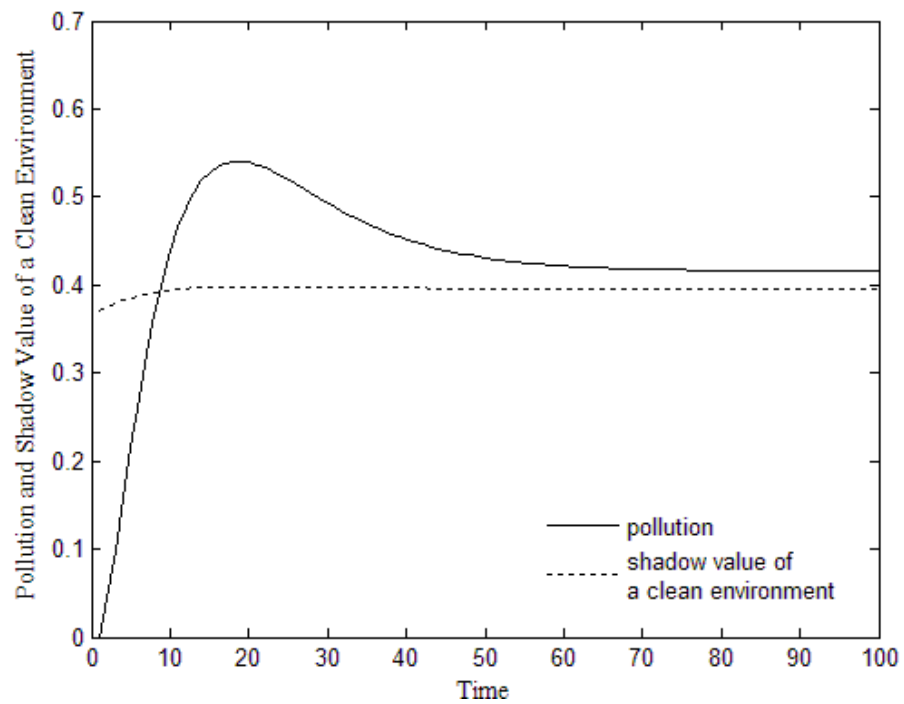


Fig. 2c. Time path of optimal pollution and shadow value of a clean environment.

### 3.3.5. *The effect of parameter change*

We pose the following question: How will the optimal emissions and pollution paths change if the parameter values are allowed to change? In our model, we assume only one type of pollution, only one technology for abating the flow of that pollution, only one technology for the production which yields that pollution, and so on. However, as pointed out by Criado (2008), different countries and/or regions will experience different production and environmental technologies, and the intuitive assumptions that accurately describe circumstances in one country or region may not be descriptive of other countries or regions.<sup>39</sup> Various pollutants may also experience different fixed and variable costs, leading to different emissions curves and pollution curves for different pollutants and countries.<sup>40</sup> This may all seem like common sense, and certainly has been discussed in the literature, but the intuitive assumptions regarding which direction the emissions curves shift with a parameter change may not be universally correct. Starting from the emissions and pollution curves derived and shown in Figs. 1a and 1b, we adjust certain parameter values,<sup>41</sup> and compare the changes in the emissions and pollution curves to the results that would be predicted under some of the more commonly used assumptions.

We begin with the variable cost of abatement,  $b_1$ . It is not difficult to imagine the variable cost of abatement differing across pollutants and across countries. There is no single abatement technology, and even within an industry, multiple types of abatement may be utilized. Take coal-fired electricity, for example, where scrubbers are used as an end-of-pipe method of removing sulfur dioxide and other particulate pollution before exhaust is released into the air. That method of abatement is used simultaneously with technology that allows for more efficient use of the energy within coal, so that there is less polluting byproduct, as a whole. The two technologies have the same purpose, but they abate air pollution in different ways, and with different costs per ton of pollutant removed from the air.

We would anticipate that when the variable cost of abatement increases, the amount of abatement

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<sup>39</sup> Plassmann and Khanna (2006b) show that while one technology cannot lead to an EKC for all pollutants, an EKC should be possible for every pollutant under the right set of preferences and technology.

<sup>40</sup> The way society views a particular pollutant may also be important. Plassmann and Khanna (2006a) argue that “consumers are unlikely to react to changes in pollution that they consider harmless” (p. 25).

<sup>41</sup> There are obviously more parameters than we test here. Kelly (2003) tests the effect of certain parameters on the emissions and pollution curves, and for the sake of comparison to an accepted model, we modify those parameters which Kelly modifies.

would decrease. As abatement decreases, the amount of pollution would likely increase in two ways. First, there should be an increase in pollution as the same amount of production resulted in a higher amount of emissions. Second, less abatement would mean higher total output, which could be used for consumption or investment in future capital. If invested in future capital, there would be even greater production in future time periods, and therefore greater pollution. As shown in Fig. 3a, an increase in  $b_1$  yields the expected result, a shift upward in both emissions and pollution corresponding to every level of capital. Fig. 3a also shows that the stationary state level of capital is slightly less after  $b_1$  increased, indicating that, at least under the conditions of this simulation, society did not use additional available output as a result of lower abatement as a means of accumulating more capital in the long run. One possible explanation is that society preferred to consume the additional output that was not used up in abatement.

Next we turn to  $b_2$ , which we call the convexity cost parameter of abatement. Of course, as mentioned previously, the abatement cost function is only convex in  $u$  if  $b_2$  is greater than 1. As  $b_2$  rises, the cost curve becomes more convex; but since  $u$  is constrained to be between 0 and 1, greater convexity means that the cost remains lower initially, but then must rise faster as  $u$  approaches 1, or as the social

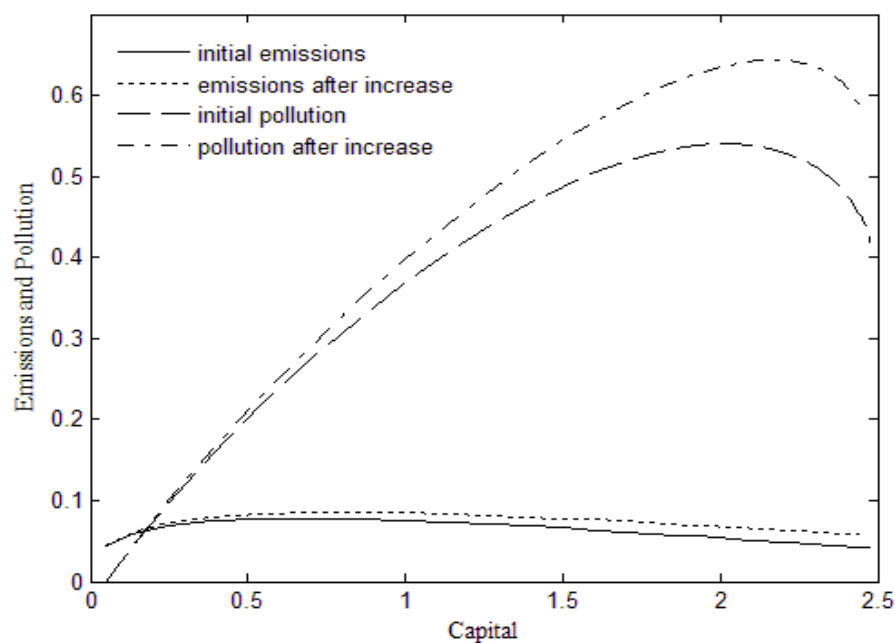


Fig. 3a. Increase in variable cost of abatement ( $b_1$ ) from 0.07 to 0.08.

planner chooses some level of abatement near full abatement. It may be helpful to think of a higher value of  $b_2$  corresponding to something closer to constant returns to scale in  $u$ .

As  $b_2$  increases, then an increase in abatement costs relatively less, so we should expect to see an increase in abatement. That increase in abatement should result in a lower level of emissions and pollution through time. As we look at Fig. 3b, we see that our expectations are met only at low levels of capital. Unexpectedly, the emissions curve does not simply shift upwards or down, but rather changes shape entirely. Lower initial levels of emissions and pollution are followed by higher stationary state levels of both, so that the curves cross as society approaches the stationary state. This type of reaction is not predicted by any of the traditional assumptions that are made regarding abatement costs, but that is not to say that there is not an intuitive explanation. For example, it is possible that, with an increase in the convexity of the cost function, society progresses towards the stationary state, enjoying the cleaner environment that comes from increased abatement. However, because emissions and pollution have not risen as quickly, the value of a clean environment has also not risen. As a result, society reaches its emissions turning point much later in the game, and settles into a higher level of emissions in the stationary state.

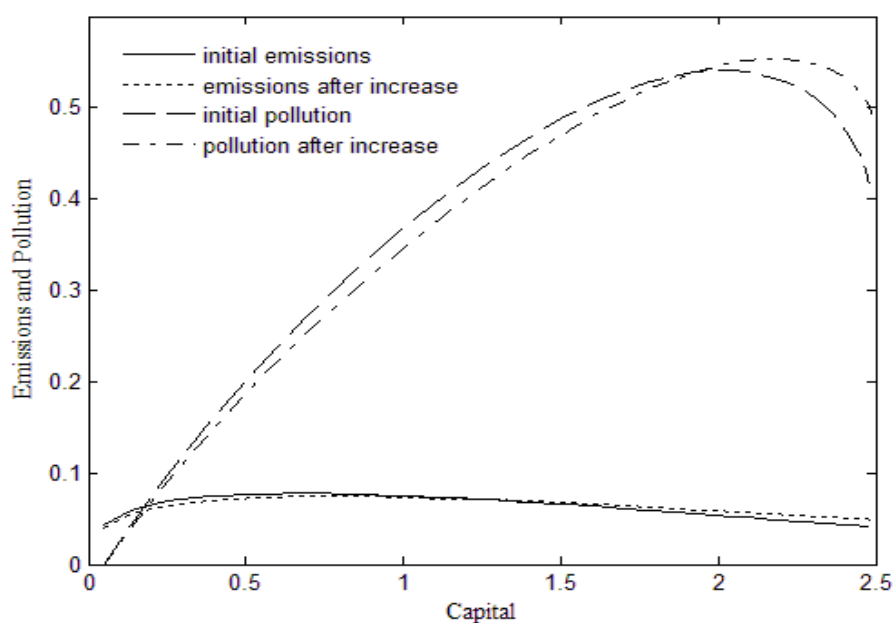


Fig. 3b. Increase in the convexity parameter of abatement ( $b_2$ ) from 3.5 to 3.51.

Next, we turn to  $\alpha$ , society's return to utility from consumption. As  $\alpha$  increases, society receives increased utility from every unit of consumption. Recall also that  $\alpha$  is the *share* of utility received from consumption or, in other words, it is the *relative* utility received from consumption, as compared to the utility received from environmental quality. Thus, as  $\alpha$  increases, the share of utility received from consumption increases and the share of utility received from environmental quality decreases. This means that the shape of the societal indifference curves is changing, and if the shape is changing, the slope of the curve or the marginal rate of substitution (MRS) is changing as well. If the budget constraint were constant, the reaction to a change in  $\alpha$  would be much easier to predict, because the optimal choice of consumption will be where the MRS is equal to the marginal rate of technical substitution (MRTS), which is the slope of the production constraint. A quick review of Eqs. (3.14) to (3.19) makes clear that a change in  $\alpha$  will impact all the equations and likely result in changes to the production constraint and therefore the MRTS. If both the MRS and the MRTS are changing, we will likely experience income and substitution effects, making prediction much more difficult.

The simplest assumption regarding the impact of an increase in the relative utility received from consumption is that consumption would increase, which would require either a decrease in abatement efforts, so that total net output rises, or if abatement efforts remain constant, it would require a smaller investment in future capital, since society invests whatever output is not consumed. Fig. 3c indicates that the social planner likely made the former choice, since the stationary state level of capital appears to be identical, but both emissions and pollution have increased through all time periods. Of course, the fact that the common, intuitive assumption proved to be accurate in this situation does not mean that its accuracy is universal; as this chapter shows, it should only be with great care that we rely heavily on the universal application of even the most intuitive assumption.

Next, we turn to the impact of a shift in the assimilation rate,  $\delta_2$ , on the emission and pollution curves. As discussed previously, the assimilation rate is the rate at which pollution will become assimilated into nature and, therefore, not a harm to the environment if no other action is taken. As an example, carbon dioxide is considered by many to be a greenhouse gas, leading to a trend of increasing warming that could have dramatic impacts on the environment. Even if no human actions are taken to reduce carbon dioxide, it

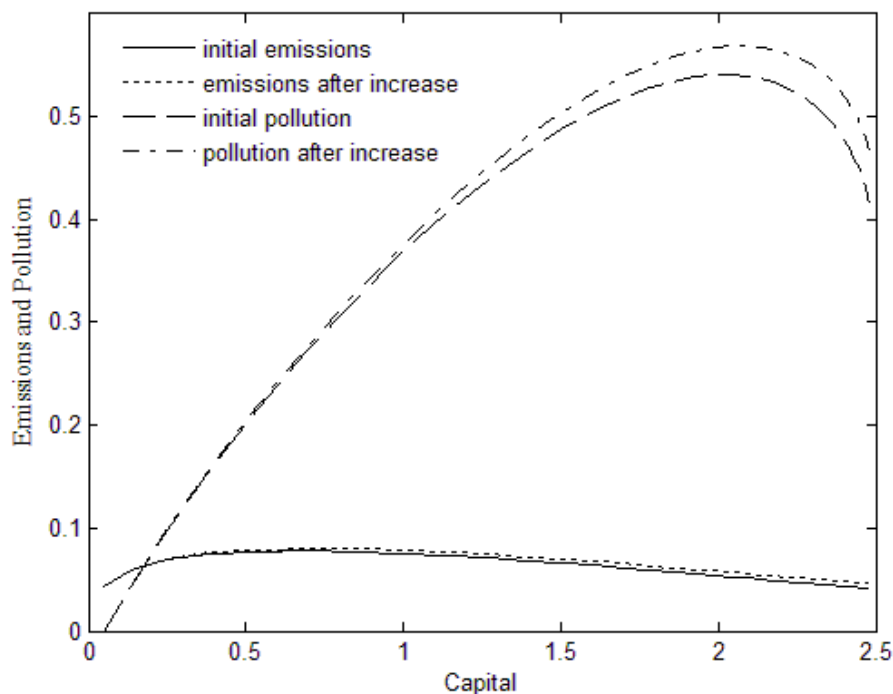


Fig. 3c. Increase in utility from consumption of  $x$  ( $\alpha$ ) from 0.5 to 0.51.

will slowly be assimilated into nature predominantly via the respiratory process of plants. As plants utilize the carbon dioxide, it is removed from the air and, in terms relevant to this research, assimilates it. The assimilation rate will likely vary across both countries and pollutants. It will vary across countries for those pollutants whose effects are more localized. For example, some countries are highly urbanized, and that urbanization will tend to reduce the total vegetation available to assimilate certain pollutants. Other countries, even industrialized countries, have maintained a significant amount of “green space,” which would tend to increase the assimilation rate for some pollutants. Of course, there are some pollutants that are not easily assimilated into the environment, and for many of these the rate will be constant across countries. An example would be the radioactive elements left over after the process of generating nuclear power; the half-life of many of those radioactive elements runs into the tens of thousands of years, and it likely matters very little where it is stored, so that the assimilation rate is likely to be very low regardless of what country or region we may be discussing.

A higher assimilation rate could discourage abatement efforts, since emissions will more rapidly dissipate into the environment; if nature is going to take care of emissions for you, why bother giving up output to take care of it? Alternatively, however, a higher assimilation rate could result in lower emissions if the cost of achieving a clean environment is lower due to the help that nature offers. Fig. 3d indicates that, for our model, the former has occurred, as emissions and pollution are both higher for every level of capital. The former may be the more likely outcome for any given set of parameter values, but the fact that the opposite outcome also has a very intuitive explanation should give us pause as to whether the assumption is accurate across the entire range of legitimate parameter values.

What the outcome is in any given circumstance depends heavily upon the opportunity costs of abatement. Along the optimal path, if the marginal improvement in the environment from one more unit of abatement is more valuable than the corresponding value of consuming the output that would be surrendered to the abatement process, then abatement will rise. A change in the assimilation rate changes the level of pollution that would otherwise exist in every time period, *ceteris paribus*. That changes the marginal benefit of abatement, but it also indirectly impacts other parts of the system and can also alter the level of consumption in every time period and therefore the marginal benefit of additional consumption.

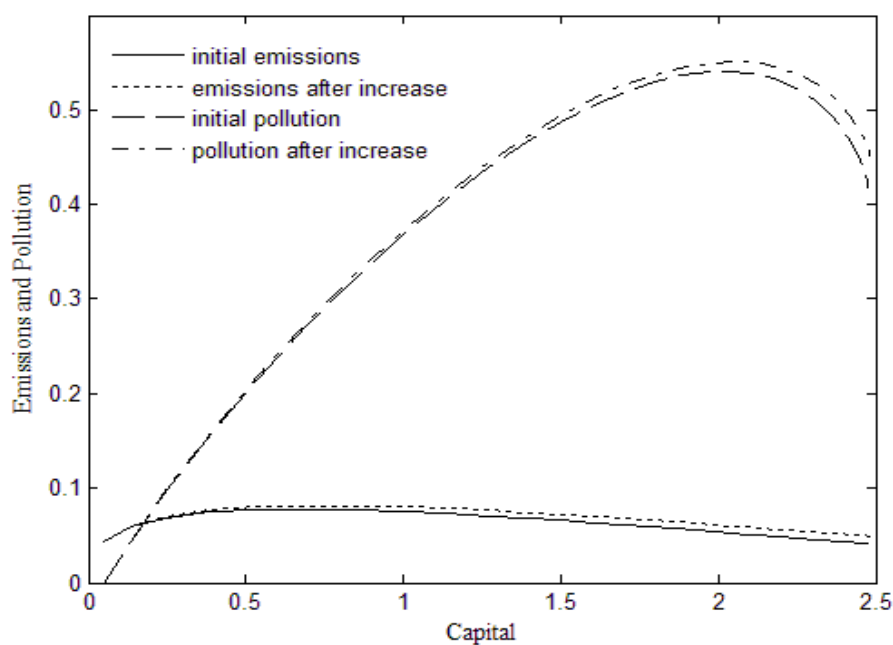


Fig. 3d. Increase in assimilation rate ( $\delta_2$ ) from 0.1 to 0.11.



Next in line is  $\gamma$ , the return to capital. While this parameter is not impacted by the type of pollutant, it will certainly vary across countries. The level of technology enjoyed by a country will certainly impact the productivity of capital, as will the relative size of the labor force.

A higher return to capital increases the benefits of production, which could then lead to an increase in total production and, if abatement remains constant, emissions. However, as noted above, a shift in the production constraint is only half of the question. Optimality requires that  $MRTS = MRS$ , and a shift in the production constraint can result in either an increase or a decrease in the optimal level of production, consumption, and abatement. It is possible that society will choose the same level of output and consumption, and spend the increased “income” on increased abatement, if the improvement in the environment will yield a higher level of utility. Fig. 3e exhibits two characteristics that indicate that the result is somewhere in between the two possible outcomes discussed: (1) emissions and pollution remain essentially the same; and (2) the stationary state level of capital has increased. The fact that emissions and pollution have increased only slightly indicates that the social planner has chosen to increase abatement, so that the environment is not degraded because of the shift. However, the social planner has not increased

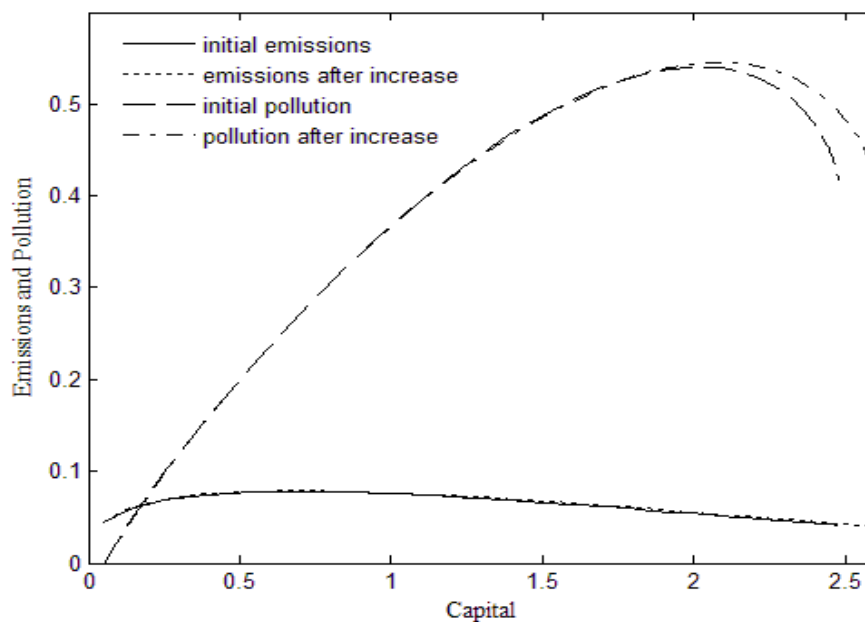


Fig. 3e. Increase in productivity of capital ( $\gamma$ ) from 0.45 to 0.46.

abatement to the point where environmental quality is higher than it was before. In other words, the social planner appears to have chosen to keep environmental quality constant. The fact that the stationary state level of capital has increased indicates that the increase in total output is sufficient to maintain environmental quality and spend more on either consumption and investment. The stationary state level of emissions is the same, but the stationary state level of pollution appears to be slightly higher. Additionally, it is clear that the turning point of pollution occurs at both a higher level of capital and pollution. It appears that greater productivity leads to a higher total burden on the environment, although for much of the time path, the social planner maintains a constant environmental quality.

Finally, we turn to the pollution intensity of production, represented by the parameter  $\sigma$ . The value of  $\sigma$  will vary according to the technology that a society enjoys, and it will vary across industries. It represents the ratio of pollution to total productive output, and different production processes will yield different levels of pollution per unit of output. Additionally, a more developed society may have productive processes that better utilize inputs so as to minimize wasteful and, often polluting, byproducts.

An increase in the pollution intensity raises the total emissions when production remains constant, yet it also increases the cost of production, which could reduce production and potentially reduce total emissions. We would anticipate that, as  $\sigma$  increases, and therefore pollution per unit of output increases, that the social planner will realize that increased pollution will cause a decrease in utility, and that the decrease in utility can be countered by giving up some output (and therefore consumption) in order to engage in greater levels of abatement. The social planner will begin to increase abatement, and therefore reduce output that can be utilized for consumption, so that the marginal utilities of consumption and environmental quality are equal, as required by the first-order conditions. Fig. 3f shows the emissions and pollution paths prior to and after the increase in  $\sigma$ . The stationary state level of emissions and pollution are lower after the increase in  $\sigma$ , but that does not tell the entire story. Fig. 3g shows a magnified section of the emissions curve, and the emissions paths cross shortly after the turning point.

As mentioned previously, the literature never considers the possibility that the emissions curves and pollution curves may cross after a parameter shift. For the purposes of most analyses, simple assumptions regarding the impact of parameter values suffice, and the assumptions which are used seem

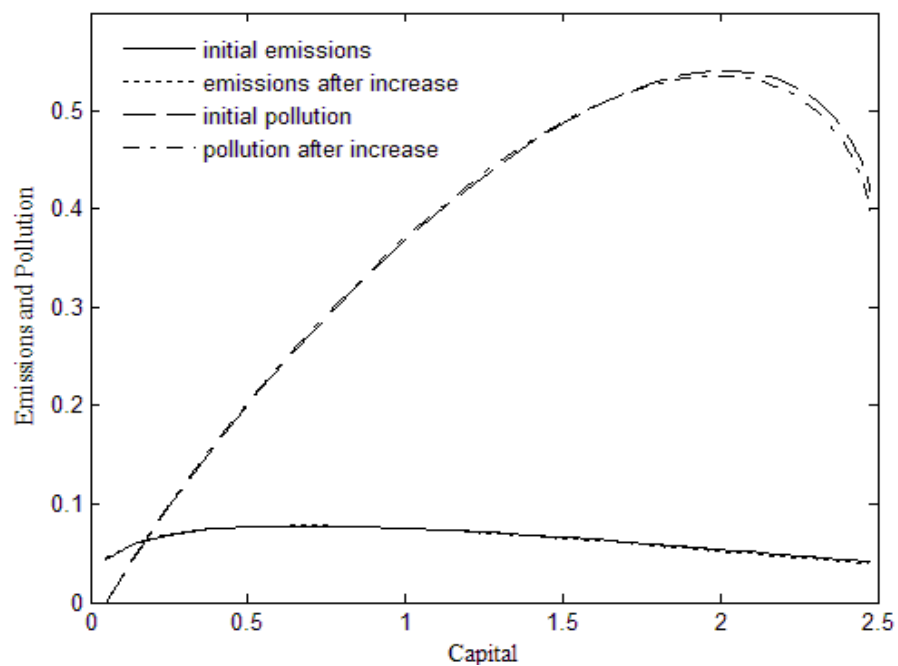


Fig. 3f. Increase in pollution intensity of production ( $\sigma$ ) from 0.45 to 0.46.

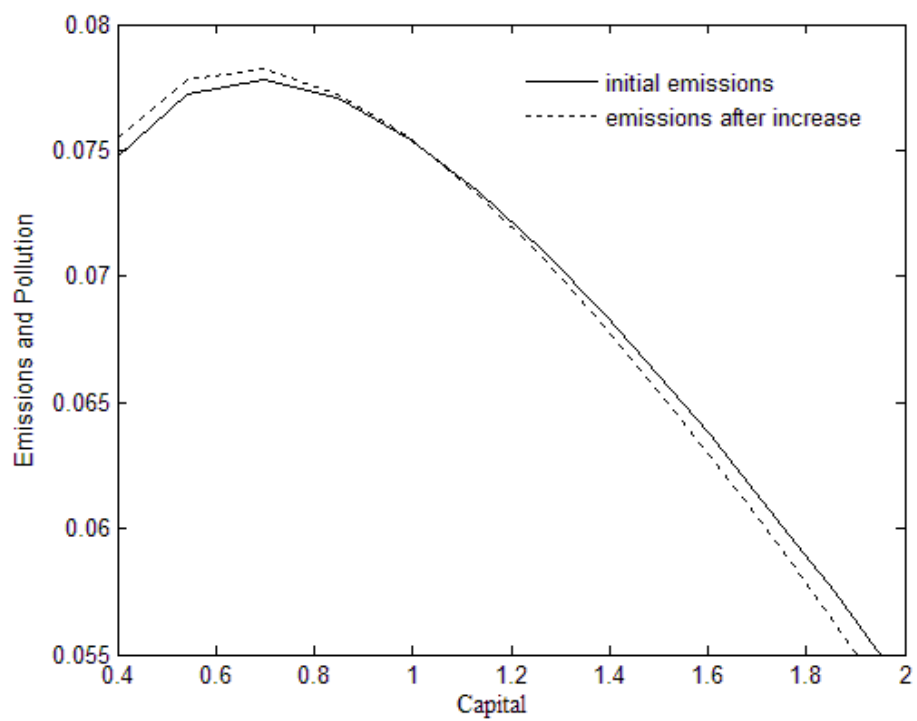


Fig. 3g. Increase in pollution intensity of production ( $\sigma$ ) from 0.45 to 0.46.

intuitive. In fact, most of them are likely accurate over a wide range of parameter values, but Fig. 3g illustrates that not only may our simple intuitive assumptions regarding a shift up or down be incorrect for any given parameter value, but it may be that the dynamic nature of the environment and economy may lead to completely unexpected results, such as the emissions curves crossing. The explanation for the emissions curves crossing can be thought of as the inverse of the explanation for the time paths crossing as the convexity parameter,  $b_2$ , increases. Recall that, when the cost function became more convex, abatement increased, and society avoided higher levels of emissions and pollution until much later in the development process, so that society settled into a higher stationary state level of emissions and pollution. The converse occurs when the pollution intensity parameter,  $\sigma$ , increases, as emissions rise faster and society reaches more quickly the maximum level of emissions it is willing to accept. Thus, society reaches its turning point earlier in the development process, and both begins the process of improving the environment earlier. And because the emissions level at the turning point is higher, society chooses a more rapid process of improving the environment, so that the slope of the emissions curve in Fig. 3g is much steeper and the emissions curves cross.

Figs. 3a through 3g illustrate that while the simple, intuitive assumptions that are regularly made regarding the impact of parameters values are often true, they are likely not universally true. Table 2 contains a summary of the predictions utilized by Kelly (2003) and others and contrasts them to the results realized in our model.

Table 2  
Simulation results

Parameter changed	$\Delta$ in emissions path		$\Delta$ in pollution path	
	Predicted	Actual	Predicted	Actual
$b_2$	(-)	(-) then (+)	(-)	(-) then (+)
$\alpha$	(+)	(+)	(+)	(+)
$\delta_2$	(+)	(+)	(+)	(+)
$\gamma$	(+)	(+)	(+)	(+)
$\sigma$	(+)	(+) then (-)	(+)	(+) then (-)
$b_1$	(+)	(+)	(+)	(+)

This is not to argue that we should not make the kind of assumptions that make economic analysis possible. Rather, we must simply be more careful in assuming that a single assumption will always be applicable, especially since we have the capability to check the validity of the assumptions. As noted by Johansson and Kristrom (2007), there is every reason to suspect that we will have to account for income and substitution effects any time environmental quality and consumption are complements.

### **3.4. Conclusion**

For too long, proponents and opponents of EKC theory have fought over the empirical results achieved on both sides. Researchers have utilized a wide variety of assumptions to arrive at their results, and those results have been just as varied as the underlying assumptions. Moreover, competing assumptions have all been based on reasonable probabilities. Even theoretical models, such as Kelly (2003), make assumptions that might not hold up under fairly standard conditions. One simple truth that underlies the EKC debate is that these contrary assumptions cannot be simultaneously correct but that does not mean that both sides cannot be correct, if the assumptions are being made along different ranges of parameter and variable values. EKC literature has, to a large extent, ignored the possibility that even the most intuitive assumption may be incorrect along some plausible range of parameter and variable values, yet understanding which assumptions are correct may vary depending on the circumstances of a given country or the characteristics of a given pollutant.

The simple model employed by this research reveals results that run contrary to some basic assumptions, although plausible explanations are offered for each seemingly counter-intuitive result. These results do not, in any way, disprove any previous research, but rather offer a reason to believe that contradictory studies need not be mutually exclusive. It is hoped that by eliminating the need to demand that any researcher has conducted a flawed study simply because the results do not match previous studies, EKC literature may progress instead to understanding why the results vary. Specifically, by taking a careful look at the conditions of the competing studies, we can gain greater insight into when we might expect each result to hold and, therefore, understand better which policies are needed at any given time in order to improve environmental quality without dramatically curtailing society's desires for consumption.

## CHAPTER 4

## DIRTY VERSUS CLEAN CONSUMPTION

**4.1. Introduction**

EKC theory poses interesting questions regarding the interaction of the economy and the environment--two inherently complex systems. Of course, greater complexity can be added as the effects of other factors, such as international trade relations, are considered. Each EKC model deals with the complexity in its own way. Beyond simple models, such as the one presented in the previous chapter, EKC literature begins to add levels of complexity that allow for the exploration of various possibilities as economics and environment collide. With increasing levels of complexity, it can be even more tempting to focus on the intriguing conclusions that can be reached, but we believe that it is just as important, or perhaps more so, to focus on the assumptions that are used when constructing additional layers of complexity. One possible layer of complexity that can be added is the opportunity for societies to produce and consume higher cost, lower polluting goods. If society, given the choice between two goods, is allowed to choose an alternative consumption good, one which results in lower levels of emissions, will such choices affect the possibility of empirical results consistent with an EKC? Copeland and Taylor (2004) point out the usefulness of a two-good model, where the goods differ in their pollution intensity, in investigating patterns of trade, and the "pollution haven hypothesis," and we believe that it is just as helpful in a single-country model.

Changes in private consumption and production choices towards "green" consumption and production are quite common in environmental economics but have not been widely used in EKC literature. One story that can be told regarding the emergence of an EKC is that, as society distances itself from subsistence level, the willingness to sacrifice consumption for additional environmental quality increases. In our previous model, that change was achievable only by an increase in abatement efforts. This chapter allows society an additional alternative, to produce and consume a different "green" type of good, one that costs more to produce<sup>42</sup> but which provides society with both consumption benefits and environmental

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<sup>42</sup> Examples of such would be wind and solar energy, which are more expensive than coal-fired electricity, and recycled paper, which tends to be more expensive than virgin paper.

benefits. Society will choose the green good to the extent that the environmental benefits of the green good outweigh the additional costs of production. We also address the primary question of this research, whether the common assumptions of EKC literature are universal across all reasonable parameter values.

Once again, we establish a continuous time, deterministic model, tracking a social planner's decisions through time. The social planner chooses production and consumption of two goods, one which contributes relatively more to environmental degradation than the other. By choosing consumption levels below production levels, the social planner can increase the future capital stock, allowing for greater future consumption. The social planner also chooses the level of abatement in each industry, by which processes total emissions are decreased. In the previous model, the social planner could improve the environment by increasing abatement efforts. Now, the social planner can shift production to the green good, which will reduce total emissions, *ceteris paribus*; the social planner can now also modify abatement levels in both industries, controlling total emissions through additional means.

#### 4.2. The model

Once again, we return to a primitive society, wherein the social planner makes multiple decisions with the ultimate goal of maximizing societal utility,  $U(c_{xt}, c_{yt} - m_t)$ . Society gets utility from consumption and environmental quality. Society gains just consumption utility from consuming the dirty good  $x$ ,  $c_{xt}$ , just environmental utility from environmental quality,  $(-m_t)$ , and both from consumption of the green good  $y$ ,  $c_{yt}$ . As a real-world example, think of the question asked by clerks at grocery stores: "paper or plastic?" Some utility is gained from use of a plastic bag to carry groceries home. Many people, however, experience additional utility from using the paper bag, not because paper is an inherently superior material for carrying most groceries, but rather because the use of the paper bag represents to the consumer an improvement in the environment as fewer non-biodegradable products reach local landfills.

Of course, most grocery stores do not charge extra if you select the paper bag, but many real-world green goods are more expensive than the corresponding dirty substitute. This fact can be explained by two factors: first, because there is some additional amount of environmental utility to be gained from consuming the green good, consumers' willingness to pay will be higher; and second, because green

production is often more expensive than dirty production, and therefore requires a higher sale price in order to cover the cost of production. This latter assumption is one of the fundamental explanations for the emergence of an EKC, that society may not be capable of cleaning up the environment until incomes rise enough to allow society to afford cleaner production processes. Moreover, the continued consumption of dirty goods even when higher utility green goods are available is evidence that there must be some additional cost associated with the green good. We assume the societal utility function to be twice differentiable and quasi-concave in consumption of both goods, and in a clean environment:  $\frac{\partial U_t}{\partial c_{xt}} > 0$ ,

$$\frac{\partial^2 U_t}{\partial c_{xt}^2} \leq 0, \frac{\partial U_t}{\partial c_{yt}} > 0, \frac{\partial^2 U_t}{\partial c_{yt}^2} \leq 0, \frac{\partial U_t}{\partial (-m_t)} \leq 0, \frac{\partial^2 U_t}{\partial (-m_t)^2} \leq 0.$$

Production,  $F(k_t)$ , is achieved as the social planner allocates capital,  $k_t$ ,<sup>43</sup> between the clean and dirty industries,  $F_t(k_t) = F_{xt}(\Delta_t k_t) + F_{yt}((1-\Delta_t)k_t)$ . The variable  $\Delta_t$ ,  $\Delta_t \in [0,1]$  represents the percentage of capital that is allocated to industry  $x$ . We assume a continuous, twice differentiable production function, strictly concave in  $k_t$ :  $\frac{\partial F_{xt}(k_t)}{\partial k_t} > 0$ ,  $\frac{\partial^2 F_{xt}(k_t)}{\partial k_t^2} < 0$ ,  $\frac{\partial F_{yt}(k_t)}{\partial k_t} > 0$ ,  $\frac{\partial^2 F_{yt}(k_t)}{\partial k_t^2} < 0$ . Implicit in this assumption is that the production process experiences constant-returns-to-scale in all inputs. Because labor is held constant, this society experiences diminishing-returns-to-scale in capital.

The production process produces a pollution byproduct,  $m_t$ , which degrades the environment and thereby reduces societal utility. The pollution intensity parameter,  $\sigma_i$ ,  $i = x, y$ ,  $\sigma_i \in [0,1]$ , represents the corresponding level of pollution emitted per unit of output. As mentioned, however, production of the clean good results in lower levels of pollution per unit of output,  $\sigma_x > \sigma_y$ . Thus, the total level of emissions for a given industry, assuming full capital allocation to that industry, would be  $\sigma_i F_i(k_t)$ ; and the total level of emissions for society, absent abatement efforts, would be  $\sigma_x F_{xt}(\Delta k_t) + \sigma_y F_{yt}((1-\Delta)k_t)$ . We assume constant values for  $\sigma_i$ , but we allow for the possibility of a cleaner environment through natural assimilation and abatement efforts.

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<sup>43</sup> As in Chapter 3, above,  $k_t$  represents the capital stock, the per-capita capital stock, and the capital-to-labor ratio.



Our social planner lacks the capacity to remove pollution from the environment once emitted. The environment, however, has the capacity to assimilate a certain percentage of pollution, which we represent by the assimilation rate,  $\delta_2 \in [0,1]$ . We assume only one pollutant, common to both production processes; therefore, natural assimilation occurs at the same rate, regardless of the source industry. The social planner is not helpless to aid the environment, however. As noted, she can shift production to the clean industry, which yields a lower level of emissions per unit of output. She can also engage in abatement efforts in each industry, eliminating some percentage of the emissions that would otherwise be emitted during the production process,  $u_{it} \in [0,1]$ ,  $i = x,y$ ,  $u_i \in [0,1]$ . While the social planner may choose different levels of abatement for each industry, she has at her disposal only one form of abatement technology, so one abatement cost function,  $G = G(u_t)$ , is applied to total production from each industry. We assume a continuous, twice differentiable cost function, strictly convex in  $u_{it}$ :  $\frac{\partial G(u_{it})}{\partial u_{it}} > 0$ ,  $\frac{\partial^2 G(u_{it})}{\partial u_{it}^2} > 0$ , where  $G(u_{it})$  is a unit cost function representing the portion of each unit of output that is relinquished in order to achieve  $u_{it}$ , the desired level of pollution abatement,  $G(u_{it}) \in [0,1]$ .

Putting all of this together, total gross emissions are defined by:

$$\sigma_x F_{x_t}(\Delta k_t) + \sigma_y F_{y_t}((1-\Delta)k_t),$$

and total net emissions, after abatement, are defined by:

$$e_t = (1-u_{xt})\sigma_x F_{x_t}(\Delta k_t) + (1-u_{yt})\sigma_y F_{y_t}((1-\Delta)k_t).$$

Total abatement expenditures are defined by:

$$G(u_{xt})F_{x_t}(\Delta, k_t) + G(u_{yt})F_{y_t}((1-\Delta), k_t) \in [0, (F_{x_t}(\Delta, k_t) + F_{y_t}((1-\Delta), k_t))].$$

Society must balance the desire for a clean environment with the desire for current and future consumption. Thus, total potential output,  $F_{x_t}(\Delta, k_t) + F_{y_t}((1-\Delta), k_t)$ , is reduced by the total cost of abatement, yielding the post-abatement output,  $F_{x_t}(\Delta, k_t)(1-G(u_{xt})) + F_{y_t}((1-\Delta), k_t)(1-G(u_{yt}))$ . Society can choose to utilize total potential output in one of three ways: consume it today; engage in pollution abatement; or invest the output in a larger capital stock next time period. Without further elaboration, this

decision would lead to the society's budget constraint,

$$F_{x_t}(\Delta_t k_t)(1-G(u_{x_t})) + F_{y_t}((1-\Delta_t)k_t)(1-G(u_{y_t})) - c_{x_t} - c_{y_t} - z_t = 0,$$

in which the investment choice is denoted by  $z_t$ . However, the system of equations generated in this chapter is an unstable system, so the optimal path can be difficult to derive. In order to aid in the process of deriving the optimal time path of the system, certain constraints must be imposed. The constraint we choose is that only good  $y$  may be invested in future capital development, so that the social planner allocates capital to industry  $x$  only in such amounts as are necessary to yield, after abatement, the optimal level of consumption of  $x$ . In other words, we impose the consumption constraint:

$$c_{x_t} = F_{x_t}(\Delta_t k_t)(1-G(u_{x_t})), \text{ which leads to investment being defined as } z_t = F_{y_t}((1-\Delta_t)k_t)(1-G(u_{y_t})) - c_{y_t}.$$

Investment and depreciation of capital,  $\delta_1 \in [0,1]$ , define the law of motion for capital:

$$\frac{dk}{dt} = z_t - \delta_1 k_t = F_{y_t}((1-\Delta_t)k_t)(1-G(u_{y_t})) - c_{y_t} - \delta_1 k_t. \text{ Higher levels of capital in future time periods allow}$$

for higher total output and, therefore, higher levels of consumption of both goods. With rising capital stocks, it is possible for society to enjoy higher levels of consumption of both goods even while increasing abatement and improving the environment. The state of the environment is determined by the level of post abatement emissions in any time period and the amount of pollution that has been assimilated into the environment, according to the following function:

$$\frac{d(-m)}{dt} = -e_t + \delta_2 m_t = -(1-u_{y_t})\sigma_y F_y((1-\Delta_t)k_t) - (1-u_{x_t})\sigma_x F_x(\Delta_t k_t) + \delta_2 m_t.$$

After imposing the consumption constraint, so that  $c_x$  is no longer an express variable, the forward-looking social planner faces the following problem:

$$\text{Max} W = \int_0^T e^{-\rho t} U_t(F_x(\Delta_t k_t)(1-G(u_{x_t})), c_{y_t}, (-m_t)) dt$$

s.t.

$$F_{x_t}(\Delta_t k_t) + F_{y_t}((1-\Delta_t)k_t) - q_t = 0$$

$$F_{y_t}((1-\Delta_t)k_t)(1-G(u_{y_t})) - c_{y_t} - z_t = 0$$

$$\frac{dk}{dt} = z_t - \delta_1 k_t = F_{y_t}((1-\Delta_t)k_t)(1-G(u_{y_t})) - c_{y_t} - \delta_1 k_t$$

$$\frac{d(-m)}{dt} = -e_t + \delta_2 m_t = -(1-u_{y_t})\sigma_y F_y((1-\Delta_t)k_t) - (1-u_{x_t})\sigma_x F_x(\Delta_t k_t) + \delta_2 m_t$$

This is the present value specification of the problem, so all future utility is discounted to the present time.

The control variables for the model are  $c_{yt}$ ,  $u_{xt}$ ,  $u_{yt}$ , and  $\Delta_t$ ; the state variables are  $k_t$  and  $-m_t$ .

Thus, the present-value Hamiltonian is:

$$H = e^{-\rho t} U(F_x(\Delta_t, k_t)(1 - G(u_{xt})), c_{yt}, -m_t) + \psi_t [F_{yt}((1 - \Delta_t)k_t)(1 - G(u_{yt})) - c_{yt} - \delta_1 k_t] - \xi_t [(1 - u_{yt})\sigma_y F_y((1 - \Delta_t)k_t) + (1 - u_{xt})\sigma_x F_x(\Delta_t, k_t) - \delta_2 m_t]$$

where  $\psi_t$  is the present-value, co-state variable for capital, and  $\xi_t$  is the present-value, co-state variable for a clean environment. These variables represent the value of the marginal unit of the state variables to society. The parameter  $\rho$  is the discount factor, which will be positive under an assumption of a positive rate of time preference.

We assume that the marginal product of capital will always be positive, so an additional unit of capital represents some additional amount of income. As such, society should, *ceteris paribus*, always place a positive value on one additional unit of capital. In addition, society should always place a positive value on an additional unit of a clean environment, leading to positive values for  $\psi_t$  and  $\xi_t$ .

We now develop our first-order conditions, which define equilibrium conditions for all control variables, along with laws of motion for co-state variables. We establish the equilibrium conditions for the control variables by taking the partial derivatives of the Hamiltonian with respect to the individual control variables and setting them equal to zero. We next find the laws of motion for the co-state variables by taking the negative partial derivative of the Hamiltonian with respect to the state variables. The final first-order conditions are the laws of motion for the state variables, as defined previously.

The current value first-order conditions for this model are:

$$\frac{\partial U}{\partial c_y} - \lambda = 0 \quad 4.1)$$

$$-\frac{\partial U}{\partial G} \frac{\partial G}{\partial u_x} - \varphi \sigma_x F_x(\Delta k) = 0 \quad 4.2)$$

$$-\lambda F_y((1 - \Delta)k) \frac{\partial G}{\partial u_y} + \varphi \sigma_y F_y((1 - \Delta)k) = 0 \quad 4.3)$$

$$\frac{\partial U}{\partial F_x} \frac{\partial F_x}{\partial \Delta} + \lambda \frac{\partial F_y}{\partial \Delta} (1 - G(u_y)) - \varphi \left( (1 - u_x) \sigma_x \frac{\partial F_x}{\partial \Delta} + (1 - u_y) \sigma_y \frac{\partial F_y}{\partial \Delta} \right) = 0 \quad 4.4)$$

$$\begin{aligned} \frac{d\lambda}{dt} &= \rho\lambda - \frac{\partial U}{\partial F_x} \frac{\partial F_x}{\partial k} - \lambda(F'_y((1-\Delta)k)(1-G(u_y)) - \delta_1) \\ &+ \varphi((1-u_x)\sigma_x F'_x(\Delta k) + (1-u_y)\sigma_y F'_y((1-\Delta)k)) \end{aligned} \quad (4.5)$$

$$\frac{d\varphi}{dt} = \rho\varphi - \frac{\partial U}{\partial(-m)} + \varphi\delta_2 \quad (4.6)$$

$$\frac{dk}{dt} = F_y((1-\Delta)k)(1-G(u_y)) - c_y - \delta_1 k \quad (4.7)$$

$$\frac{d(-m)}{dt} = -[(1-u_x)\sigma_x F'_x(\Delta k) + (1-u_y)\sigma_y F'_y((1-\Delta)k)] + \delta_2 m \quad (4.8)$$

Note that, as with the simple model from the previous chapter, the law of motion for the co-state variable for capital continues to include the externality of pollution. This means that this model will have the same difficulty as the simple model, in that pollution will tend to feed itself, creating a feedback loop that has the potential of continually increasing pollution levels.

Eq. (4.1) is the partial derivative of the Hamiltonian with respect to consumption of  $y$ . Eqs. (4.2) and (4.3) are the partial derivatives of the Hamiltonian with respect to abatement in industries  $x$  and  $y$ , respectively. Eq. (4.4) is the partial derivative of the Hamiltonian with respect to the capital allocation variable,  $\Delta$ . The current value forms were obtained by multiplying through each present value equation by  $e^{\rho t}$ . Eqs. (4.5) and (4.6) are the laws of motion for the costate variables,  $\lambda = e^{\rho t} \psi$  and  $\varphi = e^{\rho t} \xi$ . Finally, Eqs. (4.7) and (4.8) are the laws of motion for the state variables.

Eq. (4.1) indicates that the level of consumption of the clean good chosen will be such that the marginal value from the last unit of the clean good consumed will be equal to the marginal value of capital, which represents future consumption. Note that this decision also implicates a choice by the social planner with respect to capital allocation, as consumption of the clean good is only possible to the extent that capital is allocated to production of the clean good. Investment choices are also implicated, as investment in future consumption is defined by the difference between production of the clean good and consumption of the clean good.

Eq. (4.2) indicates the level of equilibrium pollution abatement that will take place in industry  $x$  along the optimal time path. We begin by rewriting the equation as:  $-\frac{\partial U}{\partial G} \frac{\partial G}{\partial u_x} = \varphi \sigma_x F'_x(\Delta k)$ . The left-hand

side of the equation represents the loss to society as abatement reduces the level of good  $x$  that can be consumed. The right-hand side is the gain to society as the level of pollution from industry  $x$  is reduced. Eq. (4.3) indicates the level of equilibrium pollution abatement that will take place in industry  $y$  along the optimal time path, and follows a similar pattern to that seen in (4.2). We rewrite the equation as:

$$\lambda F_y((1-\Delta)k) \frac{\partial G}{\partial u_y} = \varphi \sigma_y F_y((1-\Delta)k).$$

The right-hand side mirrors Eq. (4.2), showing the gain to society as abatement reduces the level of pollution from industry  $y$ . The left-hand side of the equation is the loss to society as abatement in industry  $y$  reduces both consumption of good  $y$  and future capital; the marginal cost

of abatement in industry  $y$ ,  $\left(\frac{\partial G}{\partial u_y}\right)$ , is multiplied by the marginal production of capital,  $(F_y((1-\Delta)k))$ , and the

shadow value of capital ( $\lambda$ ).

Eq. (4.4) indicates the optimal choice of capital allocation between industries in equilibrium along the optimal path. Before explaining the equation, we restate that any change in one variable will cause the entire system of equations to change; such is the nature of a dynamic system such as this. We begin by

$$\text{rewriting (4.5) as: } \frac{\partial U}{\partial F_x} \frac{\partial F_x}{\partial \Delta} + \lambda \frac{\partial F_y}{\partial \Delta} (1 - G(u_y)) = \varphi \left( (1 - u_x) \sigma_x \frac{\partial F_x}{\partial \Delta} + (1 - u_y) \sigma_y \frac{\partial F_y}{\partial \Delta} \right).$$

As  $\Delta$  increases (decreases), output is gained (lost) in industry  $x$ , but lost (gained) in industry  $y$ . The equation shows that society will choose an equilibrium between environmental and non-environmental goods. The left-hand side represents the non-environmental goods. By assuming an increase in  $\Delta$ , the first term represents the increase in utility from increasing consumption of good  $x$ , while the second term shows the loss in future consumption as total investment declines. The right-hand side represents environmental goods. The net change in total pollution is multiplied by the value to society of one more unit of a clean environment. If total output increases due to a change in allocation of capital, society is benefited with additional income but is burdened with a less clean environment.

First-order conditions (4.5) and (4.6) are the laws of motion for the co-state variables. As capital accumulates and production rises, income rises, making society better off, but that improvement may be offset by increases in the level of pollution. Whether society is ultimately better or worse off depends upon the level of abatement chosen by society, something that is difficult, if not impossible, to predict *ex ante*.

With regard to the value of the costate variable for capital, the increase in income tends to reduce the value of the costate variable, and the increase in pollution tends to cause the value to increase. Likewise, as pollution increases, the value of a clean environment rises, for the increase in pollution has lowered the stock of environmental quality. The level of abatement chosen by society for each industry can lessen the amount of pollution and, therefore, slow the growth of the value of the costate variable for a clean environment. The fact that pollution assimilates into the environment would initially appear to also slow the growth of the value of the costate variable, but, in fact, it may not due to the complexities of the dynamic system.<sup>44</sup> First-order conditions (4.7) and (4.8) are the laws of motion for capital and a clean environment, governed jointly by the laws of the physical world in which the society lives, as well as the choices made in terms of abatement and consumption.

Now we must define functional forms for utility, cost, and production functions. First, the utility function will continue as a Cobb-Douglas utility function, although with the additional clean good added, and we continue to have exponents that sum to one. Second, since we assume a uniform set of abatement technologies, we need only one cost function and can therefore retain the cost function used in the previous model. Third, each industry employs the same production function utilized in the previous model. The societal production function is simply the aggregate production from both industries after a determination of the capital allocation between them. Thus, the returns to capital and pollution intensity parameters for both industries will have different values to represent the increased pollution cost of the “dirty” good and the increased output cost of the “green” good.

With these points in mind, we write the functional forms as follows:

$$U(F_x(\Delta k)(1-G_1(u_x)), c_y, -m) = \frac{\left[ A(\Delta k)^{\gamma_1} (1-b_0-b_1 u_x^{b_2})^{\beta_1} c_y^{\alpha_2} (\bar{m}-m)^{1-\alpha_1-\alpha_2} \right]^{-\eta} - 1}{1-\eta} \quad (4.9)$$

$$F_x(\Delta k) = A(\Delta k)^{\gamma_1} \quad (4.10)$$

$$F_y((1-\Delta)k) = A((1-\Delta)k)^{\gamma_2} \quad (4.11)$$

$$G(u_i) = b_0 + b_1 u_i^{b_2} \quad (4.12)$$

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<sup>44</sup> For a more in-depth analysis of this seeming counter-intuitive result, please see Section 3.2.

This specification for utility allows us to vary the strength of the complementary relationship between consumption goods and a clean environment by varying the level of  $\eta$ . Any value less than one results in a complementary relationship, and as the value of  $\eta$  decreases, the complementary relationship weakens.

Applying the functional forms to our current-value first-order conditions yields:

$$\alpha_2 \left( A(\Delta k)^{\gamma_1} (1 - b_0 - b_1 u_x^{b_2}) \right)^{\alpha_1(1-\eta)} c_y^{\alpha_2(1-\eta)-1} (\bar{m} - m)^{(1-\alpha_1-\alpha_2)(1-\eta)} - \lambda = 0 \quad (4.1a)$$

$$- \alpha_1 b_1 u_x^{b_2-1} \left( A(\Delta k)^{\gamma_1} (1 - b_0 - b_1 u_x^{b_2}) \right)^{\alpha_1(1-\eta)} c_y^{\alpha_2(1-\eta)-1} (\bar{m} - m)^{(1-\alpha_1-\alpha_2)(1-\eta)} - \varphi \sigma_x (\Delta k)^{\gamma_1} = 0 \quad (4.2a)$$

$$\varphi \sigma_y ((1-\Delta)k)^{\gamma_2} + \lambda ((1-\Delta)k)^{\gamma_2} (b_1 b_2 u_y^{b_2-1}) = 0 \quad (4.3a)$$

$$\frac{\alpha_1 \gamma_1}{\Delta} \left( A(\Delta k)^{\gamma_1} (1 - b_0 - b_1 u_x^{b_2}) \right)^{\alpha_1(1-\eta)} c_y^{\alpha_2(1-\eta)-1} (\bar{m} - m)^{(1-\alpha_1-\alpha_2)(1-\eta)} + \lambda \gamma_2 k ((1-\Delta)k)^{\gamma_2-1} (1 - b_0 - b_1 u_y^{b_2}) \quad (4.4a)$$

$$+ \varphi \left( (1 - u_x) \sigma_x \gamma_1 k (\Delta k)^{\gamma_1-1} - (1 - u_y) \sigma_y \gamma_2 k ((1-\Delta)k)^{\gamma_2-1} \right) = 0$$

$$\frac{d\lambda}{dt} = \lambda(\rho + \delta_1) - \frac{\alpha_1 \gamma_1}{k} \left( A(\Delta k)^{\gamma_1} (1 - b_0 - b_1 u_x^{b_2}) \right)^{\alpha_1(1-\eta)} c_y^{\alpha_2(1-\eta)-1} (\bar{m} - m)^{(1-\alpha_1-\alpha_2)(1-\eta)} \quad (4.5a)$$

$$- \lambda A \gamma_2 (1-\Delta) ((1-\Delta)k)^{\gamma_2-1} (1 - b_0 - b_1 u_y^{b_2})$$

$$+ \varphi A \left( (1 - u_x) \sigma_x \gamma_1 \Delta (\Delta k)^{\gamma_1-1} + (1 - u_y) \sigma_y \gamma_2 (1-\Delta) ((1-\Delta)k)^{\gamma_2-1} \right)$$

$$\frac{d\varphi}{dt} = \varphi(\rho + \delta_2) - (1 - \alpha_1 - \alpha_2)(1 - \eta) \left( A(\Delta k)^{\gamma_1} (1 - b_0 - b_1 u_x^{b_2}) \right)^{\alpha_1(1-\eta)} c_y^{\alpha_2(1-\eta)-1} (\bar{m} - m)^{(\alpha_1 + \alpha_2)(\eta-1) - \eta} \quad (4.6a)$$

$$\frac{dk}{dt} = A((1-\Delta)k)^{\gamma_2} (1 - b_0 - b_1 u_y^{b_2}) - c_y - \delta_1 k \quad (4.7a)$$

$$\frac{d(-m)}{dt} = -(1 - u_x) \sigma_x A(\Delta k)^{\gamma_1} - (1 - u_y) \sigma_y A((1-\Delta)k)^{\gamma_2} + \delta_2 m \quad (4.8a)$$

### 4.3. Solving the system

#### 4.3.1. The system

We again utilize Matlab to solve the system of equations for the stationary state. After finding the set of parameters that allow for a stationary state, we make marginal changes to various parameters, and check to see what impact the changes have on the system variables. We begin by transforming the system of equations into their discrete form, following the procedure utilized in Chapter 3, to obtain the following system of discrete equations:<sup>45</sup>

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<sup>45</sup>  $\beta$  is equal to the inverse of the discount rate,  $\rho$ .

$$\alpha_2 \left( A(\Delta_t k_t)^{\gamma_1} (1 - b_0 - b_1 u_{xt}^{b_2}) \right)^{\alpha_1(1-\eta)} c_{yt}^{\alpha_2(1-\eta)-1} (\bar{m} - m_t)^{(1-\alpha_1-\alpha_2)(1-\eta)} - \beta \lambda_{t+1} = 0 \quad (4.1b)$$

$$-\alpha_1 b_1 u_{xt}^{b_2-1} \left( A(\Delta_t k_t)^{\gamma_1} (1 - b_0 - b_1 u_{xt}^{b_2}) \right)^{\alpha_1(1-\eta)} c_{yt}^{\alpha_2(1-\eta)-1} (\bar{m} - m_t)^{(1-\alpha_1-\alpha_2)(1-\eta)} - \beta \varphi_{t+1} \sigma_x (\Delta_t k_t)^{\gamma_1} = 0 \quad (4.2b)$$

$$\beta \left( -\lambda_{t+1} \left( (1 - \Delta_t) k_t \right)^{\gamma_2} (b_1 b_2 u_{yt}^{b_2-1}) - \varphi_{t+1} \sigma_y \left( (1 - \Delta_t) k_t \right)^{\gamma_2} \right) = 0 \quad (4.3b)$$

$$\begin{aligned} & \frac{\alpha_1 \gamma_1}{\Delta_t} \left( A(\Delta_t k_t)^{\gamma_1} (1 - b_0 - b_1 u_{xt}^{b_2}) \right)^{\alpha_1(1-\eta)} c_{yt}^{\alpha_2(1-\eta)-1} (\bar{m} - m_t)^{(1-\alpha_1-\alpha_2)(1-\eta)} \\ & + \beta \lambda_{t+1} \gamma_2 k_t \left( (1 - \Delta_t) k_t \right)^{\gamma_2-1} (1 - b_0 - b_1 u_{yt}^{b_2}) \\ & + \beta \varphi_{t+1} \left( (1 - u_{xt}) \sigma_x \gamma_1 k_t (\Delta_t k_t)^{\gamma_1-1} - (1 - u_{yt}) \sigma_y \gamma_2 k_t \left( (1 - \Delta_t) k_t \right)^{\gamma_2-1} \right) = 0 \end{aligned} \quad (4.4b)$$

$$\begin{aligned} \lambda_t &= \frac{\alpha_1 \gamma_1}{k} \left( A(\Delta k)^{\gamma_1} (1 - b_0 - b_1 u_x^{b_2}) \right)^{\alpha_1(1-\eta)} c_y^{\alpha_2(1-\eta)-1} (\bar{m} - m)^{(1-\alpha_1-\alpha_2)(1-\eta)} \\ & + \beta \lambda_{t+1} \left( 1 - \delta_{1+} A \gamma_2 (1 - \Delta) \right) \left( (1 - \Delta) k \right)^{\gamma_2-1} (1 - b_0 - b_1 u_y^{b_2}) \\ & + \beta \varphi_{t+1} A \left( (1 - u_x) \sigma_x \gamma_1 \Delta (\Delta k)^{\gamma_1-1} + (1 - u_y) \sigma_y \gamma_2 (1 - \Delta) \left( (1 - \Delta) k \right)^{\gamma_2-1} \right) \end{aligned} \quad (4.5b)$$

$$\varphi_t = -(1 - \alpha_1 - \alpha_2)(1 - \eta) \left( A(\Delta k)^{\gamma_1} (1 - b_0 - b_1 u_x^{b_2}) \right)^{\alpha_1(1-\eta)} c_y^{\alpha_2(1-\eta)-1} (\bar{m} - m)^{(\alpha_1 + \alpha_2)(\eta-1) - \eta} + \beta \varphi_{t+1} (1 - \delta_2) \quad (4.6b)$$

$$k_{t+1} = A \left( (1 - \Delta) k \right)^{\gamma_2} (1 - b_0 - b_1 u_y^{b_2}) - c_y + (1 - \delta_1) k \quad (4.7b)$$

$$m_{t+1} = (1 - u_x) \sigma_x A(\Delta k)^{\gamma_1} + (1 - u_y) \sigma_y A \left( (1 - \Delta) k \right)^{\gamma_2} + (1 - \delta_2) m \quad (4.8c)$$

This system of equations can be solved by Matlab once parameter values have been chosen.<sup>46</sup> The increased complexity of the system, over that which existed in the previous chapter, means that greater care must be taken in choosing the parameter values, so that a stationary state may be determined. We have included in Table 3 the values chosen for the parameters, used to determine the initial stationary state.

The first thing that will likely be apparent to the reader is that the parameters in Table 3 are, in some cases, different from those listed in Table 1. As discussed previously, the system of equations is inherently unstable, indicating that derivation of the optimal path is computationally difficult. Parameter values must be chosen so as to allow for derivation of multiple optimal time path, as each parameter is modified. Moreover, the intent of this chapter is not to directly contrast the results of the previous simple model with this more complicated model. Rather, the purpose is to conduct the same type of analysis,

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<sup>46</sup> Code available from author upon request.



Table 3  
Parameter values for examples

Parameter	Value
$\gamma_1$	0.4
$\gamma_2$	0.3
A	0.6
$\sigma_x$	0.22
$\sigma_y$	0.13
$\eta$	0.9
$b_0$	0.01
$b_1$	0.01
$b_2$	2
$\alpha_1$	0.3
$\alpha_2$	0.5
$\delta_1$	0.1
$\delta_2$	0.1
$\rho$	0.05
$\bar{m}$	26

changing the parameters and identifying how the change in parameter values impacts the optimal time paths and stationary state values of the system variables. As such, we need to assure that the parameter values are within acceptable limits.

The productivity parameters,  $\gamma_1$  and  $\gamma_2$ , are both less than 1, assuring that our assumption of diminishing returns to scale in production is met. Further,  $\gamma_1 < \gamma_2$  in order to represent the higher production cost of the clean good. Technology is between 0 and 1,  $0 < A < 1$ , to represent that capital is productive but not perfectly efficient. The pollution intensity parameters,  $\sigma_x$  and  $\sigma_y$ , are between 0 and 1, as defined above, to represent that production results in pollution, but that the amount of pollution is less than the amount of output. The pollution intensity parameter for the dirty good,  $x$ , is higher than that for the clean good,  $y$ . We require that  $0 < \eta < 1$  in order to guarantee that there is complementarity between consumption and environmental quality. The fixed and variable costs of abatement,  $b_0$  and  $b_1$  respectively, are positive, and  $b_1$  is less than 1, so that society is capable of abating 100 percent of emissions. The cost

function shape parameter,  $b_2$ , is greater than 1 to assure convexity. The utility shares of good  $x$  and good  $y$ ,  $\alpha_1$  and  $\alpha_2$ , respectively, are between 0 and 1 in order to guarantee that society receives positive utility from consumption of each good subject to diminishing marginal utility. Further,  $\alpha_1 < \alpha_2$ , representing that society receives additional utility from the environmental benefits of consuming the clean good. The depreciation rate and assimilation rates are between 0 and 1, so that capital and pollution stocks depreciate over time. The parameter  $\rho$  is set at what we believe is a reasonable discount rate. Finally,  $\bar{m}$  is obviously much higher than in the previous chapter, but the only requirement for  $\bar{m}$  is that it be greater than 0, so that the environment will have some positive maximum carrying capacity. The society represented by the parameter values in Table 3, then, lives in an environment with a much higher carrying capacity.

#### 4.3.2. Stationary state

For every set of parameters, there exists a unique optimal path, and an infinite number of non-optimal paths. Choosing parameters that fairly reflect what a representative economy may look like allows us to derive optimal solutions that give insight into the impact of various parameters on the real world, parameters over which society may be capable of exerting influence. By comparing sets of parameters and their corresponding optimal solutions, we can determine what effect changing those parameters may have on the optimal time path and stationary state level of emissions and pollution.<sup>47</sup> If we also compare the impact of those parameter changes on the stationary state levels of other variables, we may gain insight into exactly how changes in parameters impact emissions and pollution. It is possible to offer predictions as to the likely outcome resulting from a change in parameter values, as will be discussed below, yet the fact that this remains a dynamic system means that changing one parameter value may alter the time path of every variable, making precise predictions extremely difficult. Better than predictions are actual results measured as parameter values are modified and the resulting stationary states are compared.

##### 4.3.2.1. Stationary state for capital

Generally speaking, an increase in productivity parameters  $\gamma_x$  and  $\gamma_y$ , the level of technology,  $A$ ,

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<sup>47</sup> The reader will note that in the following exercise certain parameters are increased and others are decreased. This is not an indication that we believe that an increase or decrease is more likely in reality. Rather, the choice is determined by which better facilitates computation of the optimal time paths.

and the parameters representing the share of utility generated by consumption,  $\alpha_1$  or  $\alpha_2$ , correspond to an increase in the marginal benefit of capital. An increase in the productivity parameters or the level of technology would mean that the same amount of capital yields a higher level of output and that each unit of capital has a higher value to society. An increase in the utility parameters would mean that the same amount of capital yields the same level of output, but that level of output is capable of yielding a higher total utility along the optimal path, also indicating that each unit of capital has a higher value to society. Of course, the fact that each unit of capital has a higher value to society is not a sufficient condition for a higher stationary state. It also means that society can reach the same level of utility from consumption with a lower level of capital and could, in such a situation, enjoy a higher level of environmental quality. Additionally, increases in the utility parameters could also create incentives to reduce the rate of capital accumulation. An increase in the utility gained from consuming each unit of good  $x$  will tend to increase production and consumption of good  $x$ , and that can lead to a reduction in production of good  $y$ , which is needed for investment in future capital. An increase in the utility gained from consuming each unit of good  $y$  will tend to increase production and consumption of good  $y$ , but, if the increase in consumption of good  $y$  outpaces the increase in production of good  $y$ , there will be a lower amount of good  $y$  available to invest in future capital.

To further complicate the matter, increasing only one production parameter or only one utility parameter will alter the MRTS or the MRS, respectively. As the production parameters are altered, the MRTS changes, and production of one of the goods will become relatively more attractive. Similarly, as the utility parameters are altered, the MRS changes, and one good will become relatively more attractive as compared to the other good, and both goods will become relatively more attractive as compared to environmental quality. The resulting shifts in the equilibrium, where  $MRS = MRTS$ , make it difficult to predict the outcome. This difficulty is inherent in any complex dynamic system, and it is why the following analysis is important. Any parameter in such a system will directly or indirectly impact every variable, and making assumptions regarding the impact of changing a single parameter is likely to underestimate, if not completely ignore, at least one of many ways in which the parameter change affects either or both the MRS and MRTS.

An increase in the depreciation of capital,  $\delta_1$ , can be thought of as a *decrease* in the value of capital. As capital depreciates faster, the future stream of income represented by each unit of capital is reduced. The fact that the stationary state level of capital decreases as  $\delta_1$  increases is perhaps not surprising, but it should not be assumed that such is the only possible result. Instead, consider that society will realize that achieving the same level of consumption is possible by increasing capital accumulation. Such a choice will have to be balanced against other options, including obtaining the same level of consumption by reducing abatement or simply accepting lower consumption. *Ex ante*, it would be impossible to have any confidence in predicting what the outcome will be. The very real possibility that traditional assumptions, often relying heavily on price effects not income effects, may be incorrect under certain circumstances should reinforce the importance of investigating the validity of the assumptions used.

Certain other parameters can be understood as indicative of the cost of capital, such as the pollution intensity parameters,  $\sigma_x$  or  $\sigma_y$ , or the various abatement cost parameters,  $b_0$ ,  $b_1$ , and  $b_2$ . As the pollution intensity parameters increase, the same level of production results in a higher level of pollution, assuming constant abatement levels. As the cost of using capital increases, we would anticipate that society would choose a lower level of capital along the optimal path. Of course, society has the power to choose a higher level of abatement in order to counter, at least somewhat, the tendency towards increased pollution. That increased abatement would not be costless and would divert output away from consumption and, in the case of industry  $y$ , away from future capital accumulation.

Changes in the abatement cost parameters impact capital accumulation in a number of ways. The first impact is similar to the impact of a change in the pollution intensity parameters. An increase in the fixed and variable costs of capital,  $b_0$  and  $b_1$ , means that maintaining the same environmental quality at the same level of capital will be more expensive. Thus, the cost of using capital, in terms of environmental quality, has increased, which would tend to decrease capital accumulation along the optimal time path. The second impact is that as the fixed and variable costs of capital increase, the level of abatement would tend to decrease. Lower levels of abatement would then tend to lead to higher levels of production, allowing greater consumption and investment along the optimal path. An increase in the convexity parameter of abatement,  $b_2$ , on the other hand, indicates that the marginal cost of abatement remains low for a much

larger range of abatement choices. This will tend to increase the level of abatement, which in turn tends to decrease capital accumulation along the optimal path.

More complicated is the effect of an increase in the depreciation rate for pollution,  $\delta_2$ . As  $\delta_2$  increases, pollution emitted during production assimilates more quickly into the environment. Pollution left alone, then, will disappear much more rapidly, and abating pollution in the current time period yields fewer benefits as a result of the natural abatement that occurs. All of this lessens the incentive to forego consumption and investment in the current time period in order to preserve a clean environment for future time periods. Thus, *ceteris paribus*, a higher assimilation rate, will tend to increase total net production, and therefore higher consumption and investment.

Table 4a indicates that most of our predicted results have been realized. Increasing the productivity of capital in either industry results in an increase in the stationary state level of capital. Thus, the increase in capital's value to society must outweigh any of the other confounding effects mentioned above. As technology increases, and the efficiency of capital increases, society also chooses a higher stationary state level of capital. Likewise, as the pollution intensity parameter in industry  $y$  is increased, the cost of using capital in production of good  $y$  increases, and society chooses a lower stationary state level of capital. However, our predictions regarding  $\sigma_x$  and its impact on the time path of capital are not borne out by the results. It is a reasonable assumption that a decrease in  $\sigma_x$  should increase capital accumulation over time, because the pollution costs of each unit of capital have decreased. Instead, a lower pollution cost of production leads to a lower level of capital in the stationary state. There are a number of possible reasons for this outcome. It is possible that, as the pollution intensity is reduced, the level of abatement also declines. If that decline is substantial enough, then less capital is needed in order to achieve the same level of consumption (i.e., the income effects would dominate). Along the same lines, a decrease in the pollution intensity of good  $x$  means that the societal cost of good  $x$  has declined, and more of that good should be produced. If that production is the result of increased investment, then capital should increase as a result. If, however, an increase in the production of good  $x$  is primarily the result of a substitution of capital from good  $y$  to good  $x$ , then the higher productivity of capital in the dirty industry may mean that less capital is needed, so the stationary state level of capital would decline.

Table 4a  
 Change in stationary state capital stock (k) due to parameter changes

	Initial stationary state	New stationary state	Change
Increase $\gamma_x$ from 0.4 to 0.41	1.9522	1.9673	(+)
Increase $\gamma_y$ from 0.3 to 0.31	1.9522	2.0244	(+)
Increase A from 0.6 to 0.61	1.9522	1.9989	(+)
Decrease $\sigma_1$ from 0.22 to 0.21	1.9522	1.9517	(-)
Increase $\sigma_2$ from 0.13 to 0.14	1.9522	1.9515	(-)
Decrease $\eta$ from 0.9 to 0.5	1.9522	1.9522	even
Increase $b_0$ from 0.01 to 0.011	1.9522	1.9494	(-)
Increase $b_1$ from 0.01 to 0.011	1.9522	1.9511	(-)
Increase $b_2$ from 2 to 2.1	1.9522	1.9519	(-)
Increase $\alpha_1$ from 0.3 to 0.31	1.9522	1.972	(+)
Decrease $\alpha_2$ from 0.5 to 0.495	1.9522	1.9583	(+)
Increase $\delta_1$ from 0.1 to 0.11	1.9522	1.7738	(-)
Increase $\delta_2$ from 0.1 to 0.2	1.9522	1.9536	(+)

Increases in the costs of abatement lead to a decrease in the stationary state level of capital because the cost of using capital in an environmentally friendly manner has now increased. Using capital in a non-environmentally friendly way will result in lower utility as the environment is degraded. In sum, there is an increase in the cost of using capital, so society chooses a lower stationary state level of capital. Likewise, increasing the convexity of the abatement cost function means that the marginal cost of abatement has decreased and abatement is therefore less costly. An increase in abatement would lessen the level of output potentially available for investment, and therefore lessen the stationary state level of capital. Note, however, that both increases and decreases in the cost of abatement can yield the same impact on the stationary state level of capital, reinforcing the fact that we should be very careful in our assumptions regarding the impacts of parameter change.

As society receives greater utility from the production of good  $x$ , it necessarily receives less utility from a clean environment, so society is willing to choose a higher stationary state level of capital, and likely a corresponding higher level of pollution. However, a decrease in the utility society receives from good  $y$  also yields a higher level of capital in the stationary state. The decrease means that society gains greater utility from a clean environment, yet society chooses a higher level of capital, which can result in a more degraded environment. However, society is also capable of increasing abatement in both industries to

counter the impacts of a higher capital stock.

As capital depreciates at a higher rate, society may take certain steps in an attempt to counter the more rapid deterioration of the capital stock, but, at least in this case, it is not fully effective, and the stationary state level of capital decreases. Finally, as pollution assimilates at a faster rate, society reacts to the lower cost of using capital in production by choosing a higher stationary state level of capital.

#### 4.3.2.2. *Stationary state for consumption*

We turn next to the impact of parameter change on the levels of consumption for both goods. An increase in the productivity factors for each industry gives society the opportunity to consume more. *Ceteris paribus*, whichever industry experiences the increase in productivity will see an increase in output. Of course, the system is dynamic, so it is highly unlikely that everything else will remain constant. As productivity rises, society could choose a higher level of abatement, leaving consumption constant but allowing society to experience higher environmental quality. Alternatively, society may shift capital to the other industry, thus allowing for an increase in both goods. One additional possibility is that society will choose a higher level of investment, so as to allow greater consumption in future time periods. *Ex ante*, it is difficult to predict with certainty the changes society will choose to implement in reaction to productivity changes.

An increase in the technology parameter means that capital, in general, is more efficient, regardless of the industry in which it is put to use. As such, an increase in technology would allow for greater production in both industries, so we would anticipate that there would be an increase in consumption of both goods. Increases in the pollution intensity parameters means that production and, therefore, consumption of each good becomes more expensive in terms of the environmental costs of consumption. We should therefore expect a reduction in consumption of the good for which there has been an increase in pollution intensity. As society shifts capital, we would also expect an increase in consumption of the good for which there has been no increase in pollution intensity.

As the fixed and variable costs of abatement increase, we would expect consumption of both goods to decrease, as it becomes more expensive to produce and, therefore, consume in an environmentally friendly way. As the abatement cost function becomes more convex, the marginal cost of abatement would

decrease, so producing in an environmentally friendly way becomes cheaper, and we would expect greater production of both goods, which could then be consumed. An increase in the depreciation rate for capital suggests more capital would need to be saved in order to maintain consumption at the current level. Since investment is only possible to the extent that consumption is foregone, it is likely that an increase in the capital depreciation rate will cause a decrease in consumption of both goods. Finally, as the pollution assimilation rate increases, current pollution is less costly, so society could reduce abatement in both industries and achieve greater output with the same level of capital, which could then be consumed.

Comparing these reasonably standard predictions with the actual results, we note certain disparities. Rather than being troubling, however, these disparities merely confirm what was discovered in the previous chapter--standard assumptions regarding complex dynamic systems, though seemingly reasonable, should be confirmed through further investigation before being relied upon. Each parameter impacts the optimal path of multiple variables directly, and all variables indirectly; even the most logically crafted assumption can fail in the face of increasing complexity. By looking at Tables 4b and 4c, we see that an increase in  $\gamma_x$  causes an increase in consumption of  $x$  and a decrease in consumption of  $y$ , but that an increase in  $\gamma_y$  yields an increase in consumption of both goods. Thus, in the case of  $\gamma_x$ , substitution effects appear to dominate, while in the case of  $\gamma_y$ , income effects dominate. An increase in technology yields an increase in consumption of both goods, as more efficient capital allows for increased production of both goods.

Changes in the pollution intensity parameters have some unexpected results. As industry  $x$  becomes less pollution intensive, consumption of good  $x$  has become less costly to the environment, and yet society chooses a lower level of consumption of good  $x$  and a higher level of consumption of good  $y$ , which has now become relatively more environmentally costly. If we focus on consumption alone, there is little to explain why this should occur, but the dynamic nature of the system can confound otherwise intuitive predictions. Table 4d indicates that a decrease in the pollution intensity of industry  $x$  causes capital to be shifted away from industry  $x$  and into industry  $y$ , indicating that consumption of good  $x$  will decrease. When industry  $y$  becomes more pollution intensive, consumption of  $y$  decreases, which is expected, and there is no change in consumption of  $x$ .



Table 4b

Change in stationary state consumption of good x ( $c_x$ ) due to parameter change

	Initial stationary state	New stationary state	Change
Increase $\gamma_x$ from 0.4 to 0.41	0.5195	0.5222	(+)
Increase $\gamma_y$ from 0.3 to 0.31	0.5195	0.5216	(+)
Increase A from 0.6 to 0.61	0.5195	0.5333	(+)
Decrease $\sigma_1$ from 0.22 to 0.21	0.5195	0.5194	(-)
Increase $\sigma_2$ from 0.13 to 0.14	0.5195	0.5195	even
Decrease $\eta$ from 0.9 to 0.5	0.5195	0.5195	even
Increase $b_0$ from 0.01 to 0.011	0.5195	0.5192	(-)
Increase $b_1$ from 0.01 to 0.011	0.5195	0.5193	(-)
Increase $b_2$ from 2 to 2.1	0.5195	0.5194	(-)
Increase $\alpha_1$ from 0.3 to 0.31	0.5195	0.5253	(+)
Decrease $\alpha_2$ from 0.5 to 0.495	0.5195	0.5214	(+)
Increase $\delta_1$ from 0.1 to 0.11	0.5195	0.4984	(-)
Increase $\delta_2$ from 0.1 to 0.2	0.5195	0.519	(-)

Table 4c

Change in stationary state consumption of good y ( $c_y$ ) due to parameter change

	Initial stationary state	New stationary state	Change
Increase $\gamma_x$ from 0.4 to 0.41	0.44	0.4385	(-)
Increase $\gamma_y$ from 0.3 to 0.31	0.44	0.4443	(+)
Increase A from 0.6 to 0.61	0.44	0.4505	(+)
Decrease $\sigma_1$ from 0.22 to 0.21	0.44	0.4401	(+)
Increase $\sigma_2$ from 0.13 to 0.14	0.44	0.4399	(-)
Decrease $\eta$ from 0.9 to 0.5	0.44	0.44	even
Increase $b_0$ from 0.01 to 0.011	0.44	0.4394	(-)
Increase $b_1$ from 0.01 to 0.011	0.44	0.4401	(+)
Increase $b_2$ from 2 to 2.1	0.44	0.4401	(+)
Increase $\alpha_1$ from 0.3 to 0.31	0.44	0.4382	(-)
Decrease $\alpha_2$ from 0.5 to 0.495	0.44	0.4393	(-)
Increase $\delta_1$ from 0.1 to 0.11	0.44	0.423	(-)
Increase $\delta_2$ from 0.1 to 0.2	0.44	0.4407	(+)

As  $b_0$  increases, consumption of both goods declines, as was predicted, yet an increase in either  $b_1$  or  $b_2$  causes an increase in consumption of good y and a decrease in consumption of good x. This disparity arises out of the strength of secondary effects; as  $b_1$  increases, the variable costs of abatement are increasing, which creates stronger substitution effects and causes society to substitute out of the dirty good, which requires more abatement, and into the clean good. As  $b_2$  increases, the cost structure of abatement

Table 4d  
Change in stationary state allocation of capital ( $\Delta$ ) due to parameter change

	Initial stationary state	New stationary state	Change
Increase $\gamma_x$ from 0.4 to 0.41	0.3574	0.3623	(+)
Increase $\gamma_y$ from 0.3 to 0.31	0.3574	0.3481	(-)
Increase $A$ from 0.6 to 0.61	0.3574	0.3575	(+)
Decrease $\sigma_1$ from 0.22 to 0.21	0.3574	0.3573	(-)
Increase $\sigma_2$ from 0.13 to 0.14	0.3574	0.3575	(+)
Decrease $\eta$ from 0.9 to 0.5	0.3574	0.3574	even
Increase $b_0$ from 0.01 to 0.011	0.3574	0.3574	even
Increase $b_1$ from 0.01 to 0.011	0.3574	0.3571	(-)
Increase $b_2$ from 2 to 2.1	0.3574	0.3572	(-)
Increase $\alpha_1$ from 0.3 to 0.31	0.3574	0.3637	(+)
Decrease $\alpha_2$ from 0.5 to 0.495	0.3574	0.3595	(+)
Increase $\delta_1$ from 0.1 to 0.11	0.3574	0.3544	(-)
Increase $\delta_2$ from 0.1 to 0.2	0.3574	0.3562	(-)

changes, reducing the cost of abatement, and yet the result is the same as an increase in  $b_1$ , which represents an increase in the cost of abatement. An increase in the convexity of the abatement cost function will have less of an impact on the cost of abatement in industry  $x$ , due to the fact that abatement in industry  $x$  is near 1, as shown in Table 4e. In industry  $y$ , however, abatement is lower, so raising it to a higher power reduces the abatement cost more dramatically. Because abatement is now less costly in industry  $y$  than in industry  $x$ , consumption of good  $y$  is now less costly than consumption of good  $x$ , and society appears to make the choice to substitute out of  $x$  and into  $y$ .

An increase in the capital depreciation rate yields the expected result, as it becomes much harder to maintain the same level of capital. A reduction in the level of capital leads to a reduction in output and, therefore, consumption. A decrease in the pollution assimilation rate leads to a decrease in consumption of  $x$  and an increase in consumption of good  $y$ . As noted above with regard to the cost function convexity parameter, a change in the cost of abatement is not felt evenly in both industries. That disparity leads society to substitute out of consumption of  $x$  and into consumption of  $y$ . Society reacts to the increase in the depreciation rate by choosing to take advantage of the lower cost of production by saving more of it for future consumption, and it can only do so through production and investment of good  $y$ .

Table 4e  
 Change in stationary state abatement in industry  $x$  ( $u_x$ ) due to parameter change

	Initial stationary state	New stationary state	Change
Increase $\gamma_x$ from 0.4 to 0.41	0.973	0.9777	(+)
Increase $\gamma_y$ from 0.3 to 0.31	0.973	0.9769	(+)
Increase $A$ from 0.6 to 0.61	0.973	0.9974	(+)
Decrease $\sigma_1$ from 0.22 to 0.21	0.973	0.9309	(-)
Increase $\sigma_2$ from 0.13 to 0.14	0.973	0.9737	(+)
Decrease $\eta$ from 0.9 to 0.5	0.973	0.973	even
Increase $b_0$ from 0.01 to 0.011	0.973	0.9715	(-)
Increase $b_1$ from 0.01 to 0.011	0.973	0.8888	(-)
Increase $b_2$ from 2 to 2.1	0.973	0.9344	(-)
Increase $\alpha_1$ from 0.3 to 0.31	0.973	0.9086	(-)
Decrease $\alpha_2$ from 0.5 to 0.495	0.973	0.9989	(+)
Increase $\delta_1$ from 0.1 to 0.11	0.973	0.9352	(-)
Increase $\delta_2$ from 0.1 to 0.2	0.973	0.5878	(-)

#### 4.3.2.3. Stationary state for capital allocation

As parameters change, the optimal levels of consumption and production of the clean and dirty goods will also change, leading the social planner to change the allocation of capital between industries. For example, as the productivity of capital in either industry increases, the value of capital within that industry increases, and we would expect that the social planner would shift capital into that industry in order to take advantage of the increased productivity. As technology increases, and capital becomes more productive, the value of capital, generally, would increase, which could lead society to desire a greater amount of capital. Because the only way to increase the capital stock is to produce more of good  $y$ , so we would predict that the value of  $\Delta$  would decrease, since  $\Delta$  represents the portion of capital stock in industry  $x$ . We note, however, that income effects may dominate, in that the increased efficiency of capital allows for the same output at lower levels of capital, so that investment in future capital is not as essential, leading to a higher value of  $\Delta$ .

As the pollution intensity parameters in each industry increase, the cost of allocating capital to that industry increases, so as industry  $x$  yields higher levels of pollution, we would expect less capital in industry  $x$ . Conversely, as industry  $y$  yields higher levels of pollution, we would expect more capital in industry  $x$ . We would not expect great change from alterations in the abatement cost parameters, as any

such changes impact abatement in both industries in the same way. However, increasing costs of abatement mean that the cost of using capital has increased, so we would expect that there might be a slight increase in capital being allocated to industry  $x$ . As society receives greater utility from a particular good, we would predict that the social planner would shift production to that industry. An increase in the capital depreciation rate means that society is losing capital at a faster rate, so we would predict that the social planner would shift more capital away from industry  $x$ , and towards industry  $y$ , where it can be invested in future capital accumulation. Finally, an increase in the pollution assimilation rate means that the cost of using capital in production has decreased, so we would expect to see a shift of capital to industry  $y$ , where it can be invested in future capital.

By looking at Table 4d, we again see that our results are mixed. As each individual industry becomes more productive, capital shifts to that industry, as predicted. As capital becomes generally more efficient, however, society shifts more production into industry  $x$ . The value of capital has increased, but instead of shifting more capital towards industry  $y$ , where more capital can be accumulated, the social planner shifts capital to the industry which promises greater current consumption. Recall, however, that these results are for the stationary state, not the time path. The increased efficiency of capital has allowed society to accumulate more capital over time, shown in Table 4a, indicating that allocation to industry  $y$  was higher along the optimal time path, settling down to a lower stationary state level that allows for greater utility from greater consumption of  $x$ .

Decreasing the pollution intensity in industry  $x$  decreases the cost of allocating capital to industry  $x$ , but society chooses to shift capital away from industry  $x$  and to industry  $y$ . However, when industry  $y$  becomes more pollution intensive, the social planner shifts more capital towards industry  $x$ . In both circumstances, allocating capital to industry  $y$  has become relatively more costly to the environment, and yet seemingly equivalent stimuli result in different outcomes.

Increasing the fixed cost of abatement yields no change in capital allocation, but increasing the variable cost of abatement and the convexity of the cost function both result in a shift towards industry  $y$ . Increasing the variable cost affects both industries in the same way, but a change in the convexity of the abatement cost curve does not. The convexity of the abatement cost function means that the marginal cost

of abatement increases as abatement approaches 1. Increasing the convexity amplifies the effect, so that the marginal cost of abatement in industry  $x$ , which has an abatement level near 1, is much greater than the marginal cost of abatement in industry  $y$ , where abatement is relatively low. As it becomes increasingly more costly to produce good  $x$  in an environmentally friendly way, society shifts production away from industry  $x$ .

The remaining parameter changes yield the predicted results. When society receives relatively more utility from consuming  $x$ , either because of an increase in  $\alpha_1$  or a decrease in  $\alpha_2$ , the social planner shifts more capital to production of  $x$ . When capital depreciates at a faster rate, the social planner shifts capital to industry  $y$ , where it can be invested in future capital stock, and when pollution assimilates faster, society shifts capital to industry  $y$ , in order to accumulate more capital since using capital is less environmentally costly.

#### *4.3.2.4. Stationary state for abatement*

We turn next to the effect of parameter changes on the level of abatement in either industry. An increase in the productivity factor of either good should increase the amount of income that society has to spend on either consumption, investment, or abatement. That increase in income should allow the social planner to choose a higher level of abatement in the industry in which the productivity increase is realized. The same amount of capital now generates more output, and some of that potential output can be utilized, instead, in abatement, leading to the same level of output but with a lower level of emissions. Alternatively, the social planner may also reallocate capital between industries, so that society can achieve increased abatement in the industry that has not seen an increase in productivity. An increase in technology will allow for greater output in both industries, and that increase may be spent on increased abatement in both industries.

A slightly different choice faces society when the pollution intensity of each industry changes. As more pollution is produced for the same level of output, society may desire to reduce pollution, but it may do it through abatement in either industry. It may decide to engage in abatement in the industry that is now a greater source of pollution, or it may decide that the cheaper method of abating pollution is through abatement in the industry that has not seen an increase in pollution intensity, and is therefore relatively

cleaner. As the fixed and variable costs of abatement increase, we would predict that the amount of abatement in both industries would decline, and an increase in the convexity parameter would decrease the marginal cost of abatement over most of the reasonable values of  $u_x$  and  $u_y$ , so that we would expect an increase in abatement in both industries.

As society begins to receive greater utility from consumption of goods  $x$  or  $y$ , there will tend to be an increase in demand for those goods, which creates a disincentive to abate in the industry that produces the good from which society now receives a greater level of utility. There will also be a corresponding disincentive to engage in abatement efforts in the industry that produces the good for which society's preferences have remained constant. The latter disincentive arises out of the fact that society's increased demand for one good means a corresponding decrease in the utility society gains from a clean environment. As the capital depreciation rate increases, we would expect that the social planner would decrease abatement, especially in industry  $y$ , so that more output would be available to replenish the stock of capital. Finally, as the pollution assimilation rate increases, the value of engaging in abatement declines, for the environment's natural capacity to eliminate pollution on its own has increased. We would therefore predict that abatement would decline in both industries.

By looking at Tables 4e and 4f, we see that, in this case, an increase in  $\gamma_x$  results in society choosing to spend the additional resources on higher levels of abatement in industry  $x$ , achieving desired reductions in pollution in that industry while increasing consumption of  $x$ , as shown in Table 4b. The additional productivity is not used to increase abatement in industry  $y$ , but Table 4g indicates that total pollution has declined, so the reduction in abatement in industry  $y$  is more than offset by the increase in abatement in industry  $x$ . An increase in  $\gamma_y$  results in an increase in abatement in both industries, indicating that society makes the choice to expend some of its additional resources on achieving a cleaner environment in both industries. A similar result is achieved as an increase in technology results in greater income, which society chooses to expend on greater abatement in both industries.

A decrease in the pollution intensity of industry  $x$  leads to an increase in abatement in industry  $y$ , and a decrease in abatement in industry  $x$ , as industry  $y$  is now the relatively higher polluter. In other words, society chooses to address the problem directly at the source, by increasing abatement in the

Table 4f

Change in stationary state abatement in industry y ( $u_y$ ) due to parameter change

	Initial stationary state	New stationary state	Change
Increase $\gamma_x$ from 0.4 to 0.41	0.2979	0.2969	(-)
Increase $\gamma_y$ from 0.3 to 0.31	0.2979	0.3009	(+)
Increase A from 0.6 to 0.61	0.2979	0.3048	(+)
Decrease $\sigma_1$ from 0.22 to 0.21	0.2979	0.2985	(+)
Increase $\sigma_2$ from 0.13 to 0.14	0.2979	0.3211	(+)
Decrease $\eta$ from 0.9 to 0.5	0.2979	0.298	(+)
Increase $b_0$ from 0.01 to 0.011	0.2979	0.2975	(-)
Increase $b_1$ from 0.01 to 0.011	0.2979	0.2722	(-)
Increase $b_2$ from 2 to 2.1	0.2979	0.3185	(+)
Increase $\alpha_1$ from 0.3 to 0.31	0.2979	0.2828	(-)
Decrease $\alpha_2$ from 0.5 to 0.495	0.2979	0.3075	(+)
Increase $\delta_1$ from 0.1 to 0.11	0.2979	0.2868	(-)
Increase $\delta_2$ from 0.1 to 0.2	0.2979	0.1794	(-)

Table 4g

Change in stationary state pollution (m) due to parameter change

	Initial stationary state	New stationary state	Change
Increase $\gamma_x$ from 0.4 to 0.41	0.6171	0.6126	(-)
Increase $\gamma_y$ from 0.3 to 0.31	0.6171	0.6208	(+)
Increase A from 0.6 to 0.61	0.6171	0.5973	(-)
Decrease $\sigma_1$ from 0.22 to 0.21	0.6171	0.6611	(+)
Increase $\sigma_2$ from 0.13 to 0.14	0.6171	0.6403	(+)
Decrease $\eta$ from 0.9 to 0.5	0.6171	0.6171	even
Increase $b_0$ from 0.01 to 0.011	0.6171	0.6188	(+)
Increase $b_1$ from 0.01 to 0.011	0.6171	0.7346	(+)
Increase $b_2$ from 2 to 2.1	0.6171	0.6439	(+)
Increase $\alpha_1$ from 0.3 to 0.31	0.6171	0.7045	(+)
Decrease $\alpha_2$ from 0.5 to 0.495	0.6171	0.5794	(-)
Increase $\delta_1$ from 0.1 to 0.11	0.6171	0.6504	(+)
Increase $\delta_2$ from 0.1 to 0.2	0.6171	0.5782	(-)

industry that is relatively higher polluting. An increase in the pollution intensity of industry y, however, leads society to increase abatement in both industries. Of course, it is possible to think of increasing the pollution intensity of y as equivalent to decreasing the pollution intensity of x, and to expect similar results from both. The results shown in Tables 4e and 4f indicate that such expectations in a complex system may go unrealized depending upon the starting point and the magnitude of the changes.

The predicted reaction to increases in the fixed and variable costs of abatement are realized, in that increased costs of abatement result in decreased abatement. As the convexity of the cost function increases, however, abatement in industry  $x$  decreases, but abatement in industry  $y$  increases. As discussed above, an increase in the convexity of the abatement cost function means that the marginal cost of abatement is decreasing in abatement. Abatement in industry  $x$  is already almost complete, while abatement in industry  $y$  is still relatively low, resulting in greater potential for cost savings by increasing abatement in industry  $y$ . If abatement is going to be increased in industry  $y$ , society can reduce abatement in industry  $x$  by some amount and still enjoy the same level of environmental quality.

As  $\alpha_1$  increases, we see the predicted results materialize, with a decrease in the abatement level achieved in both industries, which allows for higher net production of  $x$  to be consumed. As  $\alpha_2$  decreases, society chooses a higher level of abatement in both industries. Any decrease in the consumption utility parameters results in an increase in the utility share for a clean environment. Thus, as society gains less utility from consumption of  $y$ , it increases abatement in order to gain more of what it now values at a relatively higher rate, environmental quality. Increases in the capital depreciation rate, and the pollution assimilation rate cause abatement in both industries to decline, as predicted, and in response to the higher cost and lower benefit of abatement.

#### 4.3.2.5. *Stationary state for pollution*

The impact of parameter changes on emissions will be discussed below, but it is also important to look at the impact on the stock of pollution, as well. Increases in productivity would be expected to give rise to an increase in output, with the possible increase in investment, and therefore even greater production in the future, with corresponding increases in pollution. The same result can be expected from an increase in technology. Of course society's preference for a clean environment must be taken into account, and some amount of the increased wealth will almost assuredly be spent on environmental improvements through abatement. *Ex ante*, it is impossible to know whether or not the increased abatement expenditures will be enough to offset the increase in production which we expect.

Increased pollution intensity would be expected to increase pollution as well, as the same production amount yields greater pollution. Society will be willing to expend some amount of additional



resources on abatement, but it is unlikely that society will be willing to expend sufficient resources to maintain the same level of pollution. Because consumption and a clean environment are complementary goods, we would expect some new equilibrium at some level of pollution greater than the original stationary state level. As the cost of abatement increases, whether fixed or variable costs, abatement should decrease and pollution rise, and as the abatement cost function becomes more convex, abatement should increase and pollution go down.

Increased utility from consumption should lead to lower abatement and, therefore, higher pollution, as society values a clean environment less, both in absolute and relative terms. As capital depreciates at a faster rate, society must rebalance its mix of consumption and a clean environment in order to maintain a stable stock of capital. Increased investment will likely reduce both consumption and abatement, so we would expect that pollution would increase. As pollution assimilates faster into the environment, we would expect that the level of pollution would drop. There may be additional, less direct reasons for why pollution would decline in the stationary state, but the simplest explanation is simply that more of it is disposed of by the environment in every time period, making it harder to build up pollution stock, even if society desired a higher stationary state level of pollution.

Many of the predicted results for stationary state pollution were realized but not all. Table 4g shows that an increase in the productivity of industry  $x$  and an increase in technology result in lower pollution over time, while an increase in the productivity of industry  $y$  yields an increase in pollution. This may seem odd that an increase in the productivity of the green industry yields a higher level of pollution, but an increase in the productivity of  $y$  also increases the possibilities for investment in future capital stocks. Society is willing to expend some of the additional income it receives from the productivity boost or technological advance on abatement, enough to counter the potential for increased pollution, but it is unwilling to sacrifice too much investment.

Decreased pollution intensity in industry  $x$  causes an increase in pollution, as society pulls back on abatement in the dirty good, shown in Table 4e. Increased pollution intensity in industry  $y$  increases pollution as expected. Increases in fixed and variable costs of abatement also increase pollution, as expected, although an increase in the convexity of the abatement cost function causes an increase in

pollution, as the resulting decrease in abatement in industry  $x$  outweighs the increase in abatement in industry  $y$ . Increased utility from consumption of  $x$  leads to increased pollution, as abatement decreases in response to a decline in utility from a clean environment. A decrease in utility from consumption of  $y$  means higher utility from a clean environment and corresponding lower pollution. Increased capital depreciation leads to increased pollution as society is forced to increase output in order to maintain a stable capital stock. Increased pollution assimilation leads to lower abatement, as shown in Table 4f, but the natural ability of the environment to dispose of pollution more than compensates for the decreased abatement.

Of course, one of the things which the previous chapter revealed is that a discussion of stationary state values is only half of the story, for it is possible that the time paths, pre- and post-parameter change, may cross at some point during the time period of the simulation. Therefore, we now turn to an analysis of the time paths.

#### 4.3.3. *Optimal time paths*

After solving for the stationary state, Matlab is capable of deriving an optimal path from the initial values of  $k_0 = 0.5$  and  $m_0 = 0$  to the stationary state values described in the above tables, and using the parameter values described in Table 3. Once the optimal time paths for all variables have been derived, we can determine the optimal level of emissions for every level of capital by using the equation:

$$e_t = (1 - u_{xt})\sigma_x F_{xt}(\Delta k_t) + (1 - u_{yt})\sigma_y F_{yt}((1 - \Delta)k_t) \quad (4.13)$$

Fig. 4a shows the optimal emissions path, and Fig. 4b shows the optimal pollution path. The emissions path shows the upside-down U shape characteristic of an Environmental Kuznets Curve, similar to the results of the previous chapter. Unlike the previous chapter, however, the quadratic form of the pollution curve is not as pronounced. For most of the time path, pollution appears to be monotonically increasing, and it is only as society approaches the stationary state that the pollution curve appears to be reaching its peak.<sup>48</sup> Emissions are decreasing as capital increases, yet the low assimilation rate means that pollution

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<sup>48</sup> Note that this phenomenon is not the same as that described by Lieb (2004). Lieb described stock pollutants as increasing monotonically, while flow pollutants exhibited EKC properties. While Figs. 4a and 4b are the flow and stock of pollution in the model, they are different ways of measuring a single pollutant. Lieb, on the other hand, was discussing the characteristics of different pollutants.

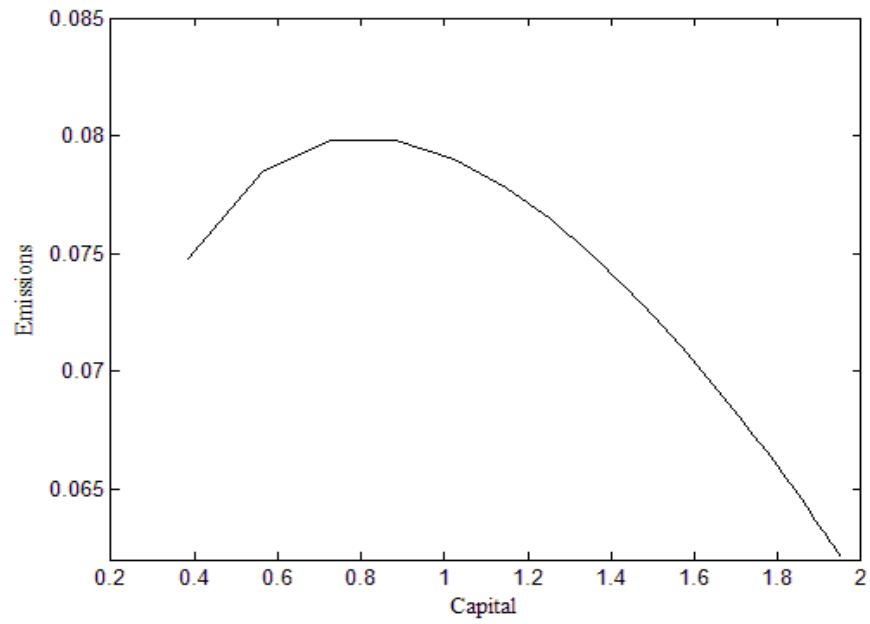


Fig. 4a. Emissions path.

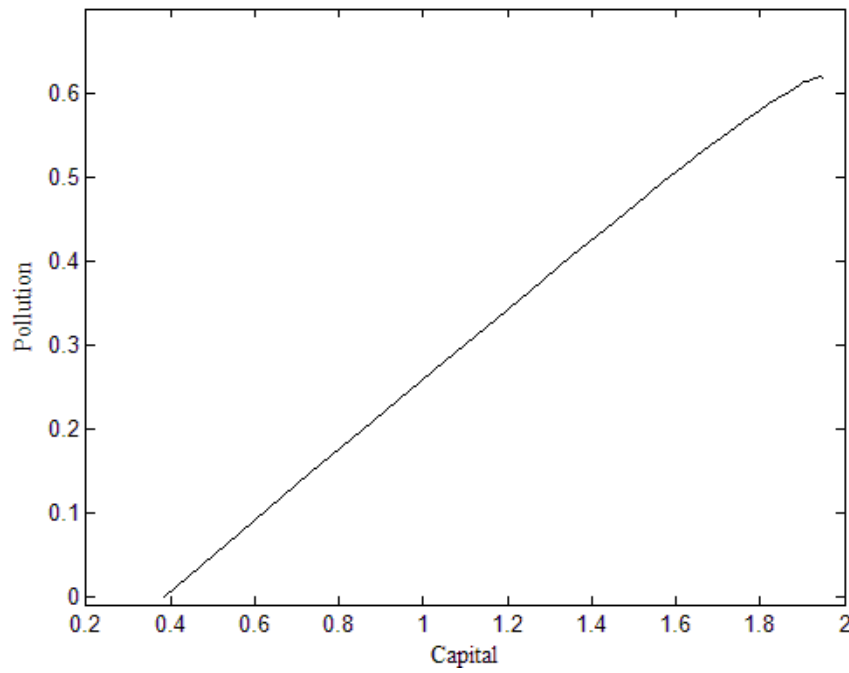


Fig. 4b. Pollution path.

remains in the environment for a relatively long time, so that pollution plateaus at the same time, and at approximately the same rate as capital, resulting in the path exhibited in Fig. 4b.

The optimal time paths of the other variables are also informative and are represented in Figs. 5a through 5e. Fig. 5a shows consumption of both goods rising rapidly to their stationary state, with consumption of  $x$  remaining at a higher level than consumption of  $y$ . Consumption of  $x$  stabilizes at 0.5195, and consumption of  $y$  stabilizes at 0.44. This higher level of consumption of  $x$  is partially explained by the higher utility gained from consumption of  $x$ , as well as the need to forego consumption of  $y$  in order to invest in future capital. Those effects are countered somewhat by the increased harm to the environment from production of  $x$ . Fig. 5b shows abatement in both industries, rising to their stationary state, with abatement in industry  $x$  settling at a much higher level, 0.973, than abatement in industry  $y$ , 0.2979. This disparity is explained by the higher pollution intensity in industry  $x$ , with society choosing to expend more to make the dirty good less environmentally destructive. Fig. 5c shows the time path of pollution and the shadow value of a clean environment. As pollution rises quickly and stabilizes at its stationary state level, the shadow value of a clean environment also rises and stabilizes. The time path for the shadow value is quite flat, even flatter than in the previous chapter. This is explained by the fact that  $\bar{m}$ , the carrying capacity of the environment, is much higher in this society than that represented in the previous chapter. Fig. 5d shows the time path of capital and the shadow value of capital. Capital reaches its stationary state level of 1.9522 at approximately half way through the total time span of the simulation. Fig. 5e shows the time path of capital allocation, with the allocation shifting quickly until approximately one-third of capital is allocated to production of  $x$ .

#### 4.3.4. *Effect of parameter change*

As the discussion above demonstrated, a change in the values of the parameters can have unexpected impacts on the variables. Figs. 6a through 6m show the effect of those parameter changes on the emissions path. Note that there may be seeming disparities in the shape of the initial emissions curve. These disparities are merely the result of changes in the range of the x-axis and y-axis, in order to adequately show the change resulting from the parameter change.

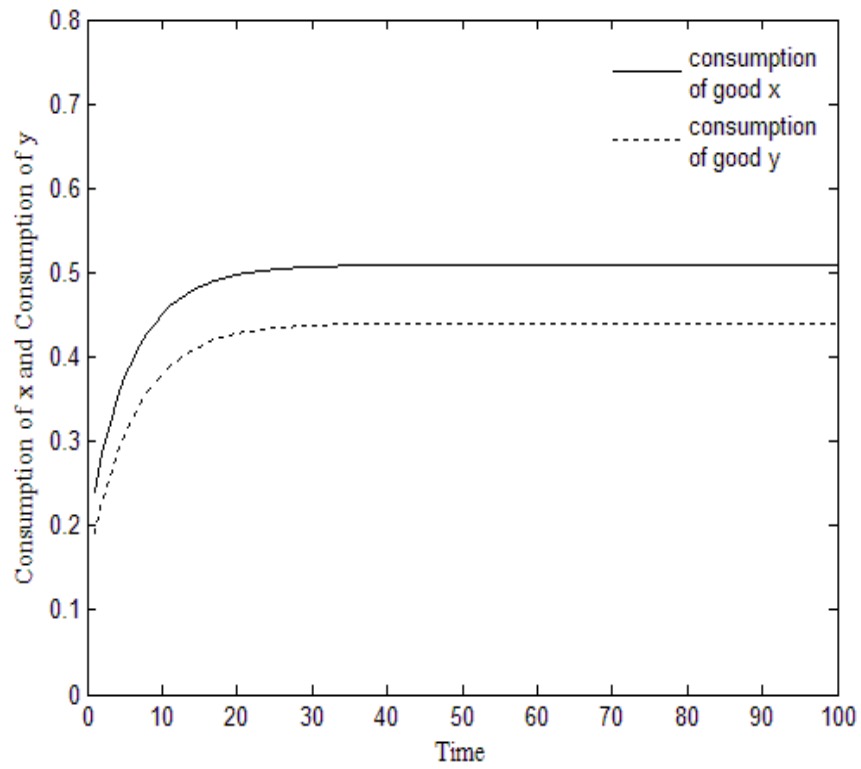


Fig. 5a. Time path of optimal consumption of x and y.

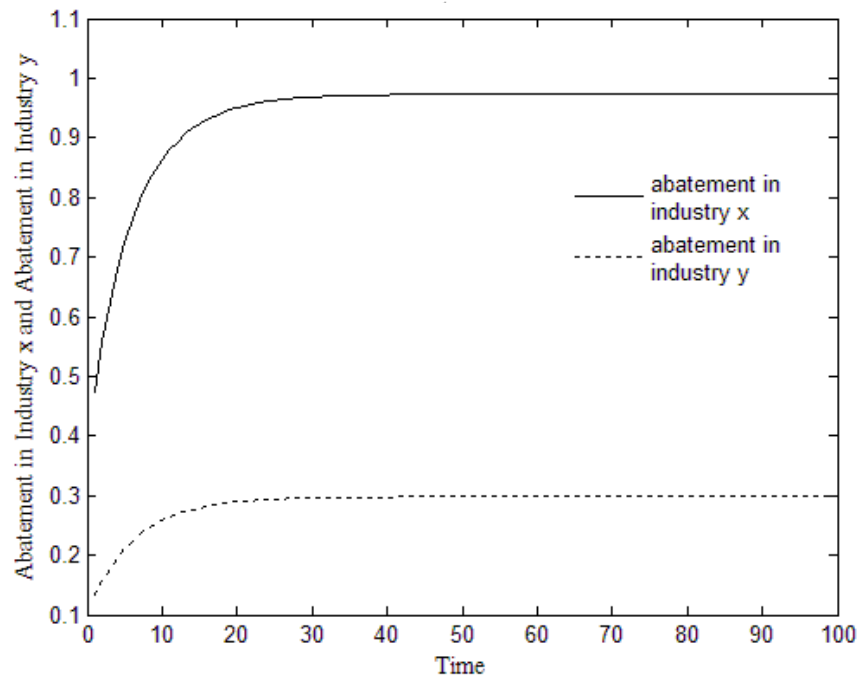


Fig. 5b. Time path of optimal abatement.

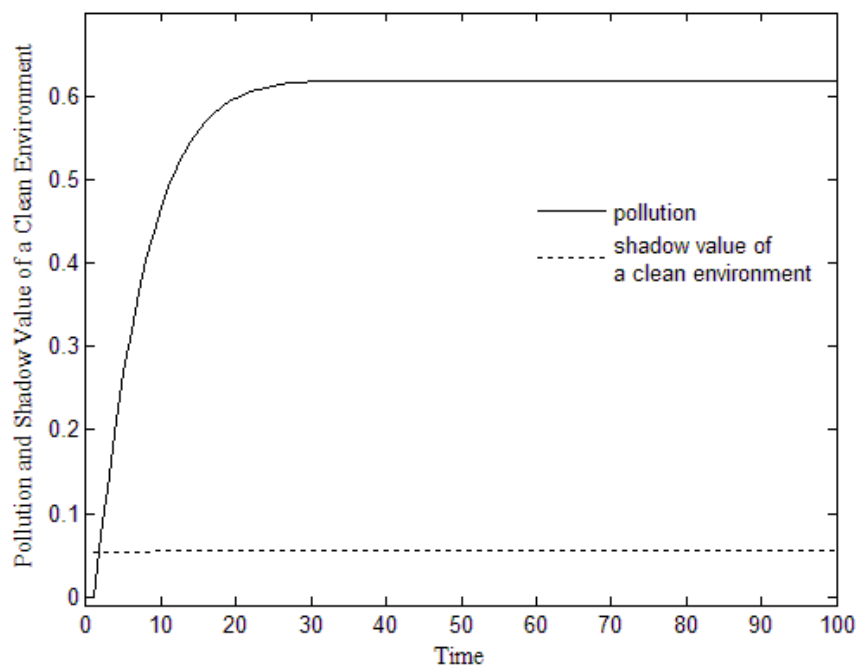


Fig. 5c. Time path of optimal pollution and shadow value of a clean environment.

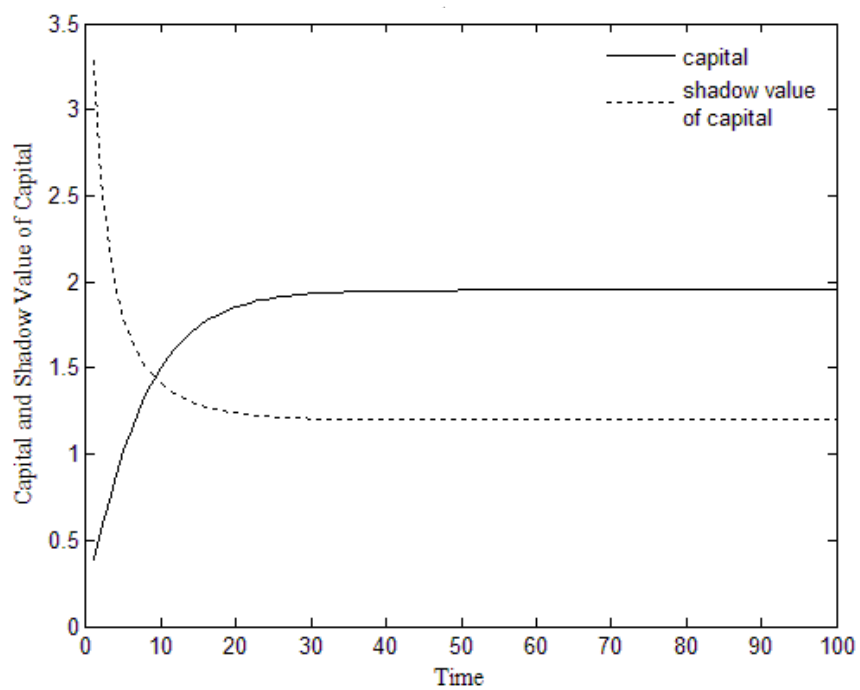


Fig. 5d. Time path of optimal capital and shadow value of capital.

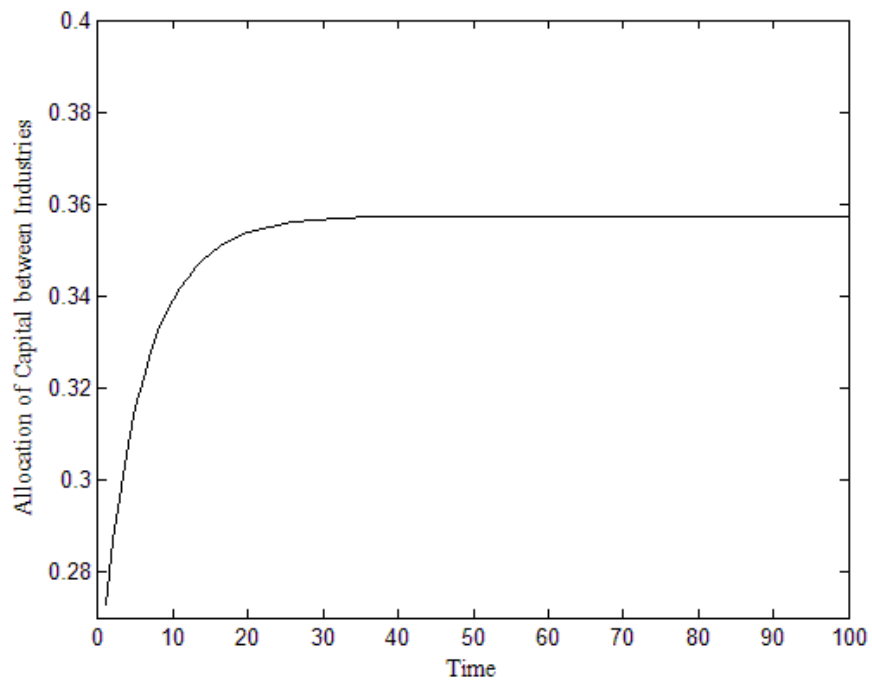


Fig. 5e. Time path of optimal allocation.

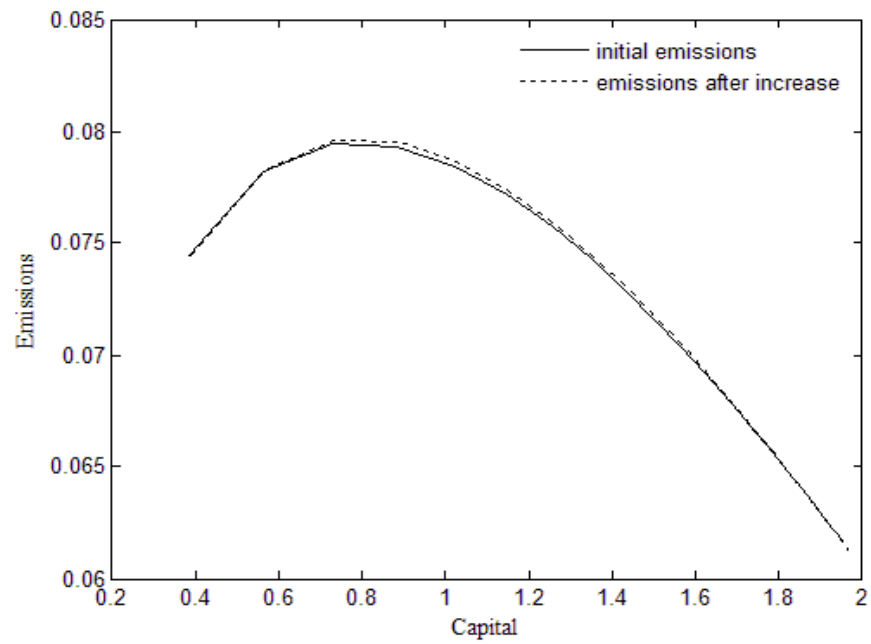


Fig. 6a. Increase in productivity in industry x ( $\gamma-1$ ) from 0.4 to 0.41.

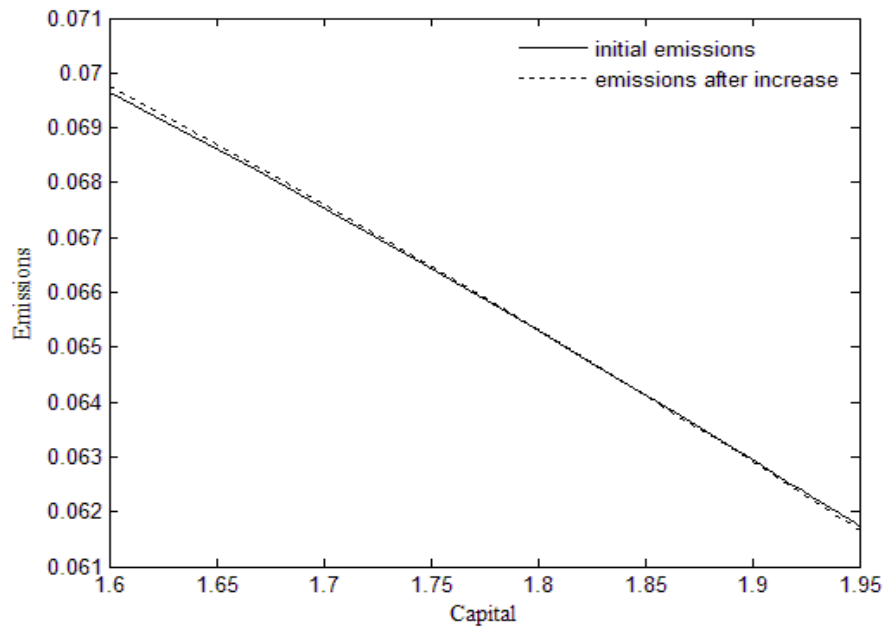


Fig. 6b. Increase in productivity in industry x ( $\gamma_1$ ) from 0.4 to 0.41.

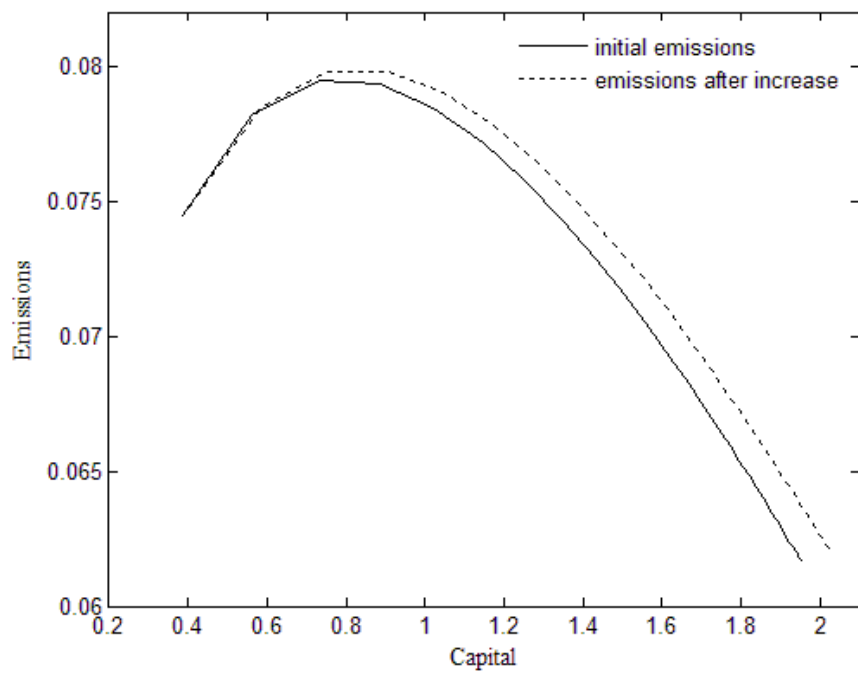


Fig. 6c. Increase in productivity in industry y ( $\gamma_2$ ) from 0.3 to 0.31.



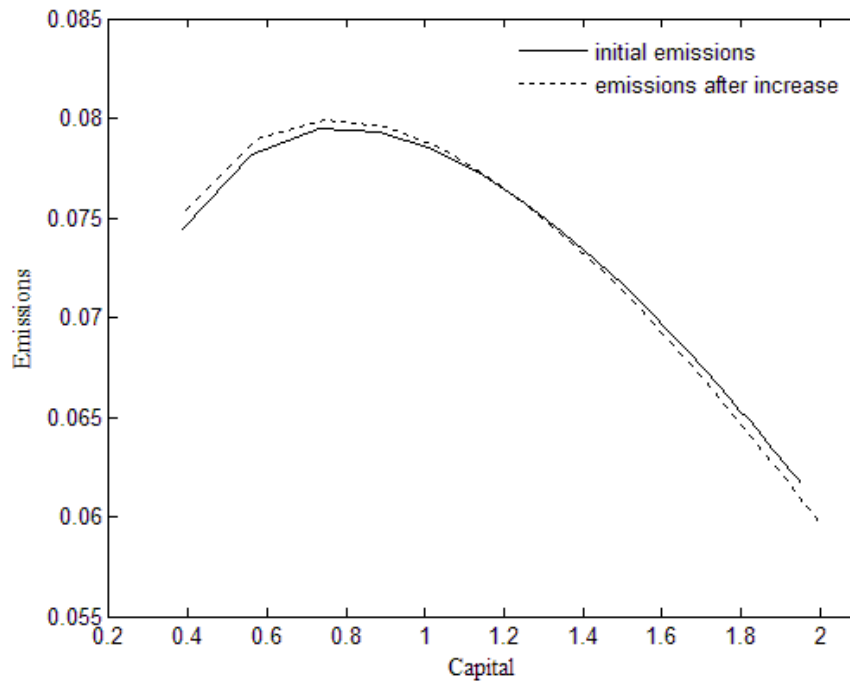


Fig. 6d. Increase in technology (A) from 0.6 to 0.61.

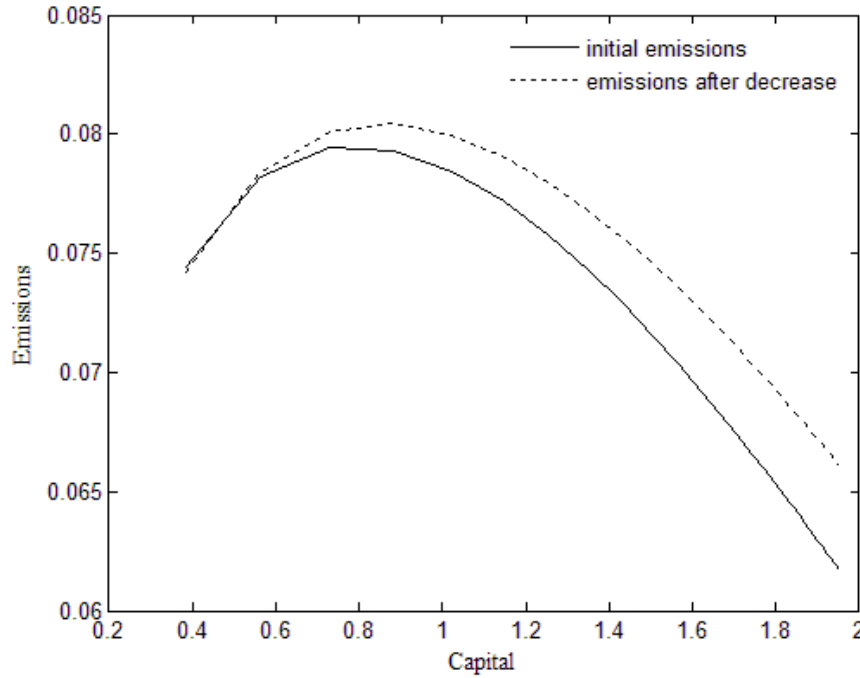


Fig. 6e. Decrease in pollution intensity in industry x ( $\sigma_x$ ) from 0.22 to 0.21.

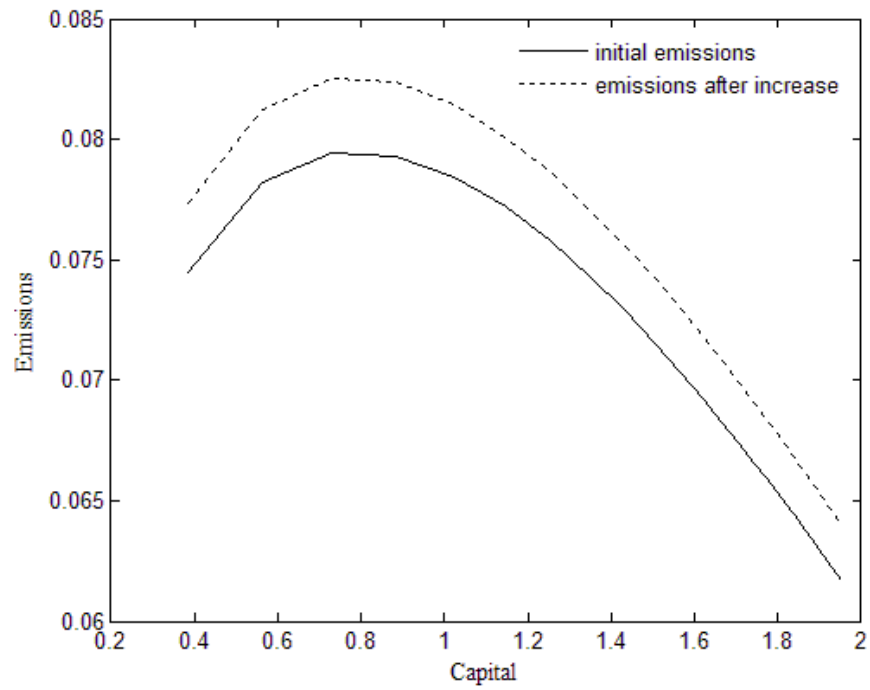


Fig. 6f. Increase in pollution intensity in industry y ( $\sigma_y$ ) from 0.13 to 0.14.

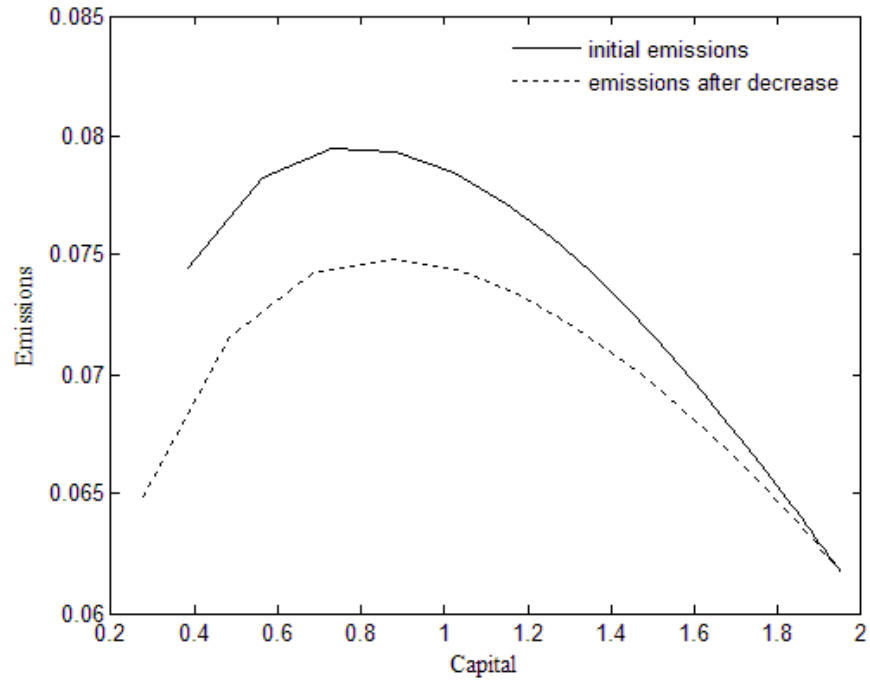


Fig. 6g. Decrease in complementarity ( $\eta$ ) from 0.9 to 0.5.

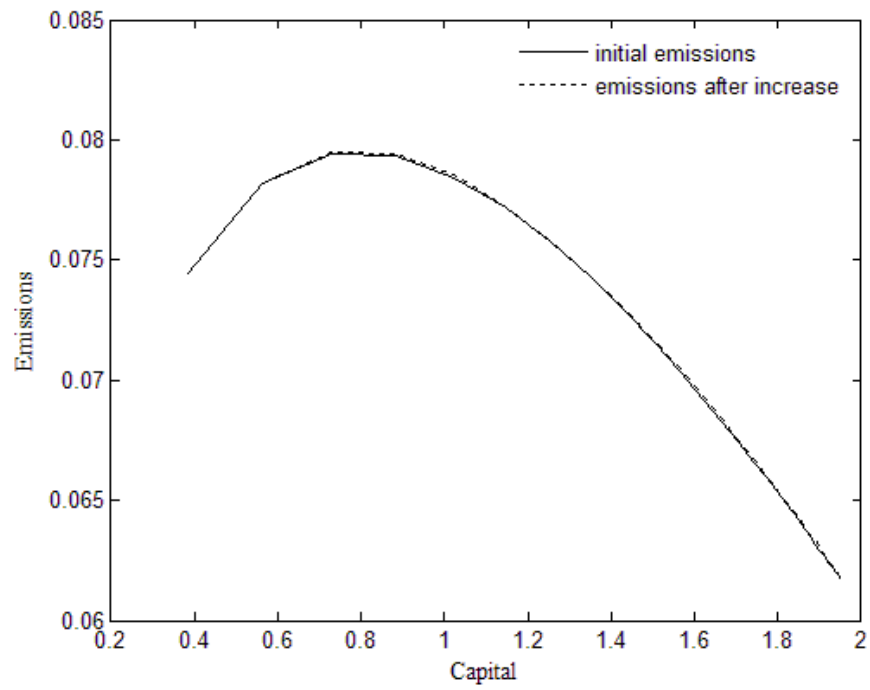


Fig. 6h. Increase in fixed cost of abatement ( $b_0$ ) from 0.01 to 0.011.

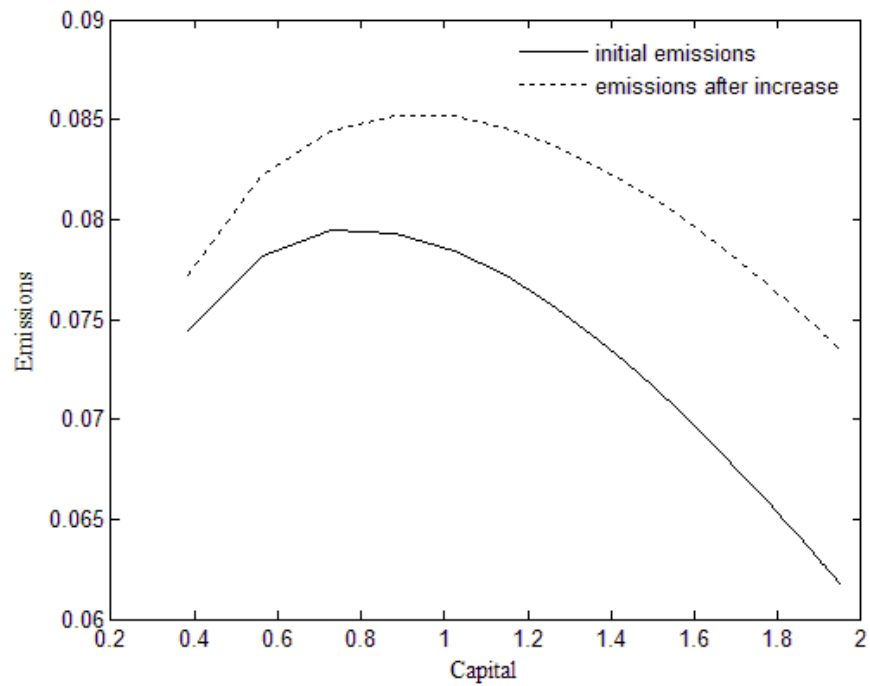


Fig. 6i. Increase in variable cost of abatement ( $b_1$ ) from 0.01 to 0.011.

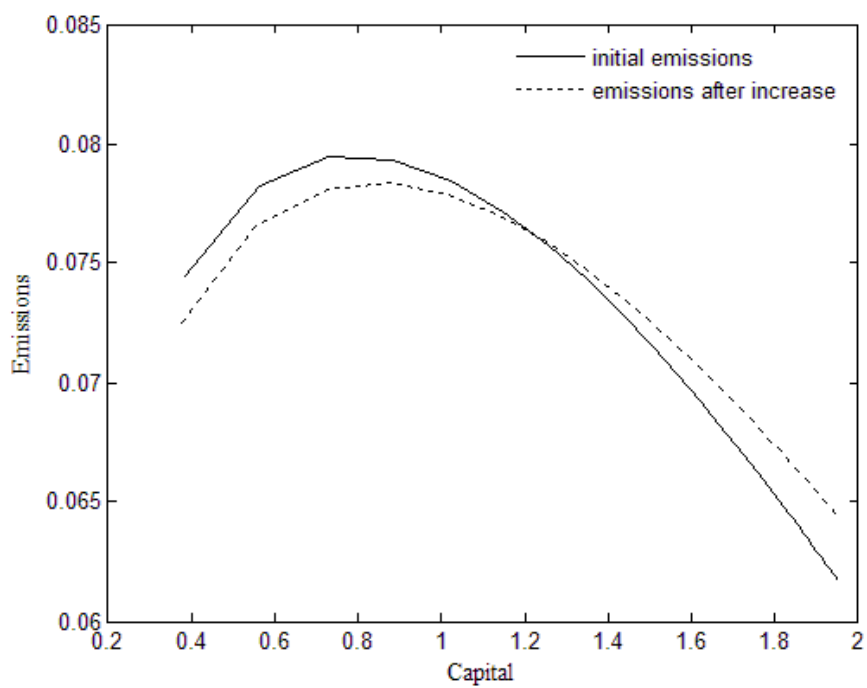


Fig. 6j. Increase in convexity of abatement cost function (b2) from 2 to 2.1.

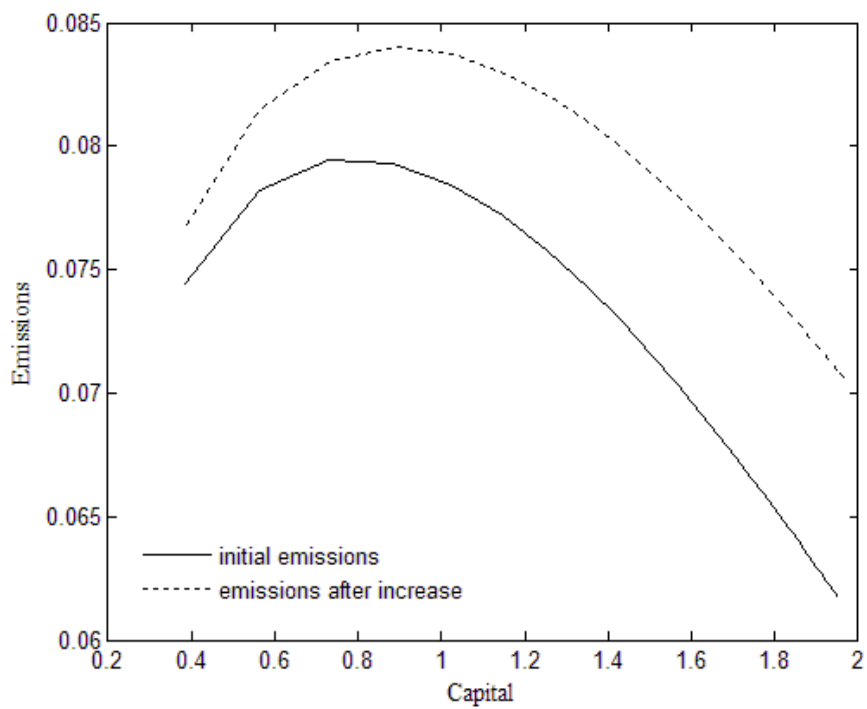


Fig. 6k. Increase in utility from consumption of  $x$  ( $\alpha-1$ ) from 0.3 to 0.31.

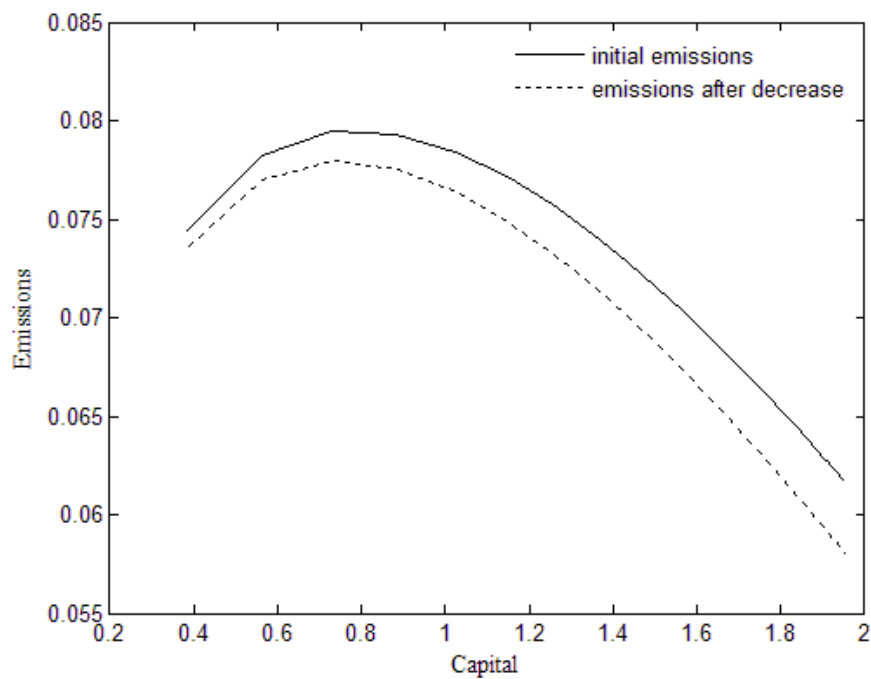


Fig. 6l. Decrease in utility from consumption of  $y$  ( $\alpha=2$ ) from 0.5 to 0.495.

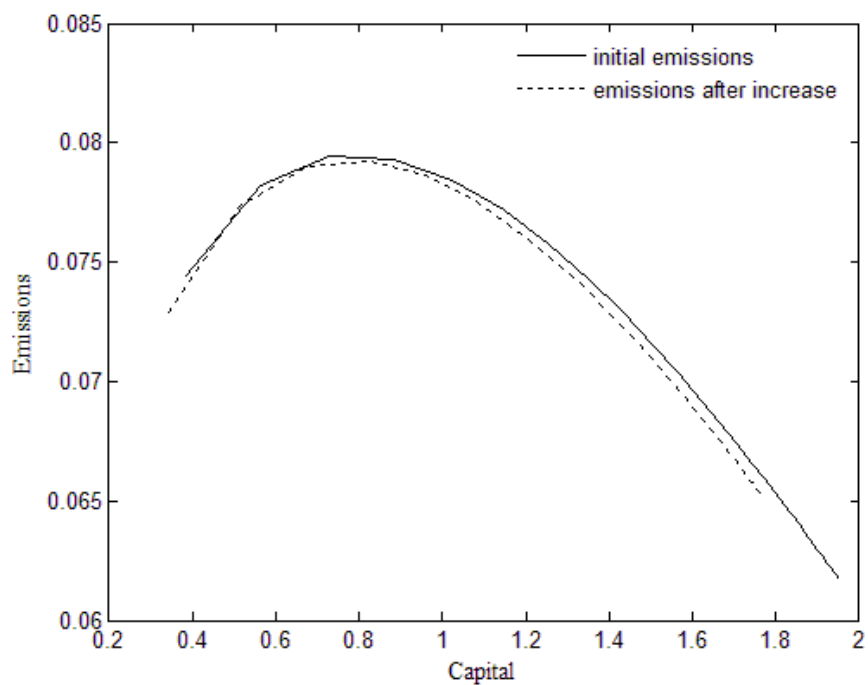


Fig. 6m. Increase in capital depreciation ( $\delta=1$ ) from 0.1 to 0.11.

We turn first to an increase in productivity in industry  $x$ . We know from the discussion above that, in the stationary state, such an increase results in higher levels of capital, a higher percentage of that capital being allocated to industry  $x$ , with a corresponding increase in consumption of  $x$ , but also an increase in abatement in industry  $x$ . We also know that consumption of  $y$  declines, likely because society invests more of good  $y$  in future capital, which is apparently then shifted to industry  $x$  to take advantage of the increased productivity. As we discovered in the previous chapter, however, simply because there is a particular change in the stationary state level does not mean that the entire time path shifts in that direction. Any or all of the variables then could have been higher during the transition to the stationary state, yet the variable could have settled down to a lower stationary state than before the change.

When it comes to emissions, we would expect that the increase in productivity in industry  $x$  would increase emissions, as more capital is shifted to industry  $x$ , which has a higher pollution intensity. Fig. 6a shows emissions curves before and after the increase. The initial reaction is a decrease in emissions, which later turns into an increase in emissions at peak levels of emissions, but then shifting again to a decrease in emissions at the far right of the emissions curve. This latter phenomenon is difficult to see from Fig. 6a, so we amplify the far right section of the emissions curve in Fig. 6b. That the emissions curves cross not once, but twice, is unexpected, yet can be explained by remembering that in order to make the best use of the increased productivity in  $x$ , the social planner will shift more production initially to  $y$ , in order to accumulate sufficient capital that more capital may be allocated to industry  $x$ . Once more capital has been accumulated, it is shifted to industry  $x$ , and pollution rises to a higher peak. That higher peak, however, triggers higher abatement in industry  $x$ , so that society is able to achieve a lower stationary state level of emissions.

We turn next to an increase in productivity in industry  $y$ . From our discussion above, we know that an increase in the productivity of capital in industry  $y$  will result in an increase in consumption of both  $x$  and  $y$  in the stationary state, as the increase in productivity allows for greater investment. That investment yields higher levels of capital, which allows for greater consumption, but also greater abatement in both industries. That would lead us to expect that emissions would decline in the stationary state.

However, we also know that the stationary state level of pollution is actually higher under a higher productivity in industry  $y$ , which means we should be cautious in making any predictions.

As the productivity of capital in industry  $y$  is increased, there is an initial decrease in emissions, but a higher level of emissions in the long run. Note, however, that the increase in emissions in the stationary state is not as great as the increase in total emissions over time, because the increase in capital stock shifts the emissions curve out. At every level of capital, emissions are higher, but emissions settle to a similar level in the stationary state. Because industry  $y$  is the only industry whose output can be used to invest in future capital, an increase in productivity may cause, over time, an increase in capital formation. An increase in productivity in industry  $y$  will cause a shift in capital to industry  $y$ , which initially will result in lower emissions, since production of good  $y$  is less pollution intensive. However, a constant increase in production leads to an increase in the total capital stock, which allows for greater production and consumption of both goods. An increase in production of good  $x$  is thus facilitated, which results in an increase in emissions along the optimal path.

By turning to an increase in technology, we know that we will see an increase in the stationary state level of capital, as well as increases in consumption of  $x$  and  $y$  and abatement in industries  $x$  and  $y$ . In essence, an increase in technology increases the efficiency of capital, essentially boosting societal income, allowing society to realize increases in consumption while also paying for increased abatement to protect the environment, which is the other source of utility for society. *Ex ante*, it is difficult to know whether the rising pollution from increased production of  $x$  and  $y$  will be effectively countered by the increase in abatement.

Fig. 6d indicates that, initially, society produces at a higher level, which yields higher levels of emissions. Society reaches a higher peak of emissions, which yields an increase in society's valuation of environmental quality, and leads to the increased abatement in both industries. Thus, while society reaches a higher peak, the stationary state level of emissions are lower and at a higher level of capital.

We turn next to a decrease in pollution intensity in industry  $x$ . As industry  $x$  becomes relatively more environmentally friendly, we would expect emissions to decrease. However, from our discussion above, we know that a decrease in pollution intensity in industry  $x$  has some unexpected results, such as

decreasing consumption of  $x$  and increasing consumption of  $y$ , even though the relative environmental quality characteristics of the two goods have been shifted in favor of  $x$ . The social planner does choose to decrease abatement in industry  $x$  and increase abatement in industry  $y$ , and the stationary state level of pollution increases.

Fig. 6e shows an increase in emissions with a decrease in  $\sigma_x$ ; the increase in emissions coincides with the increase in stationary state levels of pollution shown in Table 4g. With a decrease in pollution intensity of production of  $x$ , society chooses lower abatement, which results in higher emissions in the long run, although at low levels of capital emissions are actually higher.

Further complicating the analysis are the results from an increase in the pollution intensity of industry  $y$ . These results illustrate with greater clarity the potential perils of making intuitive assumptions without further verification. An increase in the pollution intensity of industry  $y$  and a decrease in the pollution intensity of industry  $x$  shift the relative environmental benefits of the two industries in the same direction. A comparison of Figs. 6e and 6f show that the general trend in emissions is the same from the two changes, although the results depicted in the relevant portions of Tables 4a through 4g show that society takes a different route to the same destination. The one fundamental difference in the emissions curve is that society experiences lower emissions levels at the initial portions of the emissions curve with a decrease in pollution intensity of industry  $x$ .

Fig. 6f shows an increase in emissions even as society chooses higher levels of abatement in both industries, indicating that society desires to counter the increased pollution intensity with increased abatement, but that increased abatement is not sufficient to effectively counter the increased pollution intensity. It does, however, require that society give up a portion of the stationary state capital stock, as shown in Table 4a.

By turning to a decrease in  $\eta$ , this represents a decrease in the complementarity between goods. Most importantly for the purposes of this discussion, it represents a decrease in the complementarity between consumption and a clean environment. From our discussion above, we know that the only impact a decrease in complementarity has in the stationary state is an increase in abatement in industry  $y$ . However, as we look at Fig. 6g, we notice that there is a significant shift downward in emissions. The



choice of increased abatement in industry  $y$  appears to be greater along the emissions path than it is at the stationary state, since the emissions curves before and after the decrease in complementarity converge to the original stationary state level of emissions.

We next address an increase in the fixed cost of abatement, which we know will result in a decrease in abatement in both industries, as well as decreases in consumption of  $x$  and  $y$ . That is an indication that society would like to maintain the same level of environmental quality and is willing to sacrifice some amount of consumption in an attempt to stay as close as possible to the original levels of abatement. Because this is an increase in the fixed cost of abatement, rather than variable cost, the cost does not increase with the level of abatement, so society must incur an additional cost so long as some positive level of abatement is to be chosen. Society cannot maintain the same level of environmental quality, and the stationary state level of pollution increases as a result, but it is not as dramatic an increase in pollution as we would expect with an increase in the variable cost of abatement. We would anticipate that the emissions curve would shift up, and looking at Fig. 6h, we see that is precisely what has occurred. The change is not dramatic, indicating that society may have been mostly successful in maintaining a constant level of environmental quality. The biggest change is the stationary state level of capital, as abatement cost increases, as at least a portion of the additional cost comes at the expense of investment, rather than consumption and abatement.

Increasing the variable cost of abatement has many of the same impacts as an increase in the fixed cost of abatement. The primary difference from our previous discussion of the stationary state levels is that an increase in the variable cost of abatement causes a shift in production and consumption to good  $y$ . The reason behind this result is that a shift from the dirty good to the clean good is an imperfect substitute for abatement, in that a shift of one unit of capital from industry  $x$  to industry  $y$  will yield lower gross emissions. Of course, this does not take into account the fact that abatement is much higher in industry  $x$ , but it is an attempt by society to achieve a cleaner environment by alternative means as abatement becomes more costly.

By looking at Fig. 6i, we see that emissions increase much more significantly after an increase in the variable cost than after an increase in the fixed cost of abatement. This result is not surprising, as

discussed previously, because an increase in the fixed cost just increases the choice of engaging in *any* level of abatement, whereas an increase in the variable cost of abatement increases the marginal cost of abatement at every level of abatement.

We next turn to an increase in the convexity of the abatement cost function, which, as discussed previously, indicates that the abatement cost function slopes upward at a less severe rate. This effectively reduces the marginal cost of abatement at low rates of abatement, while increasing the marginal cost of abatement as abatement nears complete abatement. We know that as the abatement cost function becomes more convex, the social planner shifts capital into industry  $y$ , allowing increased consumption of  $y$ . This shift is a result of the relative costs of abatement in the two industries; abatement in industry  $x$  is approaching full abatement, which means that the marginal cost of abatement is much higher in  $x$  than in  $y$ , where abatement is approximately 0.3, as shown in Table 4f and Fig. 5b. A reduction in the level of abatement in  $x$  and a corresponding increase in the abatement level in  $y$  (adjusted to account for the lower pollution intensity in industry  $y$ ) could maintain the same level of environmental quality. Doing so without further changes, however, would alter the relative levels of consumption and environmental quality, and virtually assure that society was not at the optimal level of utility. Because the social planner's primary goal is maximization of utility, she shifts production towards industry  $y$  at the same time as abatement in industry  $y$  is increased.

Our initial prediction regarding emissions is that an increase in the convexity of the abatement cost function, and corresponding decrease in the marginal cost of abatement, would cause a decrease in emissions. However, the reaction of various variables to the change, especially the fact that increased convexity of the abatement cost function results in an increase in the stationary state level of pollution, makes us question the correctness of our initial prediction.

Fig. 6j shows that the emissions curve exhibits a similar reaction as that demonstrated in Chapter 3, in that emissions are initially lower, as predicted, but that the stationary state level of emissions are higher, as indicated by the higher stationary state level of pollution. Society is faced with marginal abatement costs that are declining in abatement, which increases the incentives to abate. Society chooses to do precisely that, but that choice and the resulting maintenance of a clean environment for a longer period

of time leads to a delay in the rise of the costate variable for environmental quality. As a result, we see from Table 4d that more production is moved to industry  $y$  and abatement is increased in industry  $y$ , yet society lowers its abatement efforts in industry  $x$ , and as a result faces higher emissions and pollution levels in the stationary state.

Next, we turn to an increase in  $\alpha_1$ , the share of utility that arises from consumption of  $x$ . An increase in the utility parameter for good  $x$  shifts the MRS, making  $x$  more attractive relative to  $y$  and both goods more attractive relative to a clean environment. We would anticipate that such a shift in the MRS would cause an increase in production and consumption of good  $x$ , along with a reduction in abatement, both of which are consistent with the results seen above. However, we note that Tables 4a and 4d indicate that the stationary state capital stock has increased, and more capital is allocated to industry  $x$ . Because the capital stock is increased only as production from industry  $y$  is invested in the future capital stock, we know that society must have allocated a larger portion of capital to industry  $y$ , in order to achieve the higher capital stock.

Fig. 6k shows the expected increase in emissions. The shift of capital to industry  $y$ , required for the increased stationary state capital stock, does not decrease emissions even at early stages, indicating that the decrease in abatement in industry  $y$  is likely at the earliest stages of development.

We next turn to a decrease in utility from consumption of  $y$ , which alters the MRS, increasing the relative importance of a clean environment in society's utility function. This should increase abatement in both industries and reduce emissions. The change also increases the relative desirability of consumption of  $x$ , which could counter, in part, the improvements in environmental quality that we would expect from the increase in the relative importance of a clean environment. From the discussion above, we know that the stationary state level of pollution decreases after the change, and the downward pressures on emissions are likely stronger. We also know that the change results in an increase in consumption of  $x$ , a decrease in consumption of  $y$ , and increased abatement in both industries.

Fig. 6l confirms our predictions, as emissions decrease over the entire emissions curve. The increase in abatement in industry  $x$  appears sufficient to counter the potential for increased emissions as production and consumption shift to industry  $x$ .

By turning to an increase in the capital depreciation rate, we know from the discussion above that the social planner needs to engage in more investment in order to achieve the same level of production and consumption. Capital is shifted away from industry  $x$ , because investment cannot come from industry  $x$ . Consumption of  $x$  therefore declines, but so does consumption of  $y$ , as more production is needed for investment. Abatement also decreases, as another means to pay for the increased investment. All of this would indicate an increase in emissions, as would the fact that the stationary state level of pollution increases.

Fig. 6m shows a downward shift in the emissions curve, although the decrease appears to be relatively small. The stationary state capital stock is significantly lower, as it is impossible to maintain the same capital stock in the face of increased depreciation. That reduction in capital is part of the reason why emissions are lower even as abatement declines in both industries. It may initially appear contradictory that the emissions curve shifts down yet the stationary state level of pollution is higher. However, note that the terminal point on the post-change emissions curve in Fig. 6m has both a lower capital level and a higher emissions level, so that the higher stationary state level of pollution is not contradictory.

Finally, we come to the change in emissions from an increase in the depreciation rate for pollution, or the assimilation rate. As the assimilation rate increases, the benefit of abatement may have decreased, because more of what we allow into the environment this period will be taken care of by nature in every future period. Alternatively, if the assimilation rate has increased, then the cost of achieving a clean environment has now decreased, increasing society's incentive to engage in the effort of achieving that cleaner environment. From the previous discussion, we know that an increase in the assimilation rate reduces the stationary state level of abatement in both industries, lending support to the former explanation. However, the latter explanation may also have some truth, as the social planner takes other actions to achieve a clean environment, shifting capital away from the dirty industry and towards the clean industry. This leads to a decrease in consumption of  $x$  and an increase in consumption of  $y$  and an increase in investment. The reduction in abatement, however, does not result in an increase in the stationary state level of pollution, which is the result of an increasing capacity of the environment to assimilate pollution without specific abatement efforts by the social planner.

By looking at Figure 6n, we see that an increase in the assimilation rate causes an increase in emissions over time, exactly as predicted. Even though the stationary state level of pollution decreases, emissions increase, as a higher percentage of the emissions shown above are readily assimilated into the environment.

Recall that there were significant predictive difficulties in Chapter 3 with regard to the impact of parameter change. The many uncertainties regarding the reaction of a complex system to a change in a single parameter leads to the conclusion that it is only with great care that we should assume universality of even the most intuitive assumptions. This chapter increased the complexity of the system, through the inclusion of a second industry with different consumptive and pollution characteristics. Other changes were necessary, such as changes in the baseline parameter values, but one thing remained consistent from the previous chapter to this chapter, and that is the uncertainty regarding parameter values. Even the additional analysis that was conducted regarding the impact of parameter values on the stationary state levels of the variables was insufficient to allow prediction with certainty what the reaction of the system

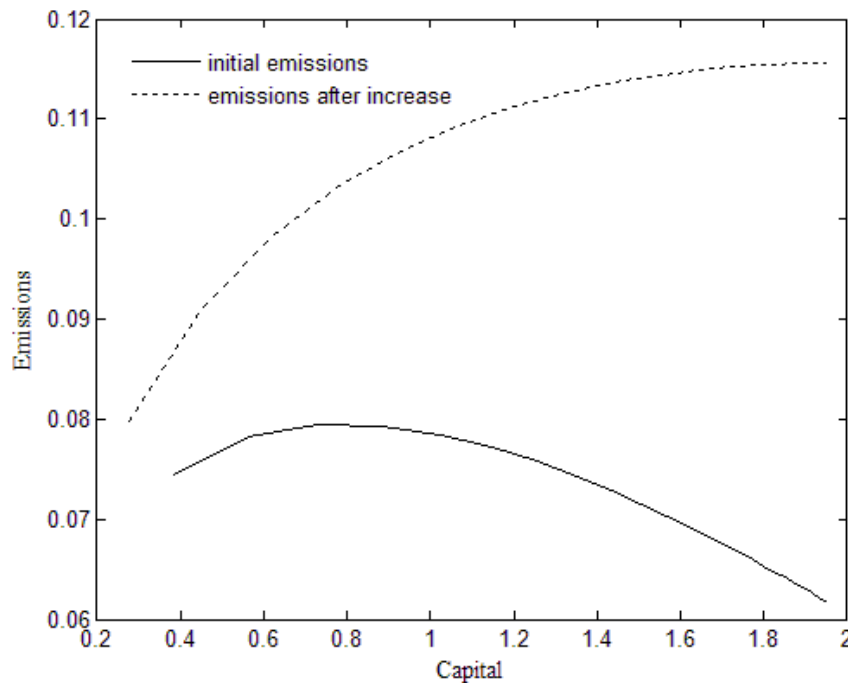


Fig. 6n. Increase in pollution assimilation ( $\delta_2$ ) from 0.1 to 0.2.

would be to changes in parameter values. Table 5 summarizes our *ex ante* predictions, as compared with the results achieved.

Table 5 illustrates that, regardless of the intuitive strength of the assumptions we use in analyzing these complex systems, there are a number of alternative conditions under which the common assumptions are incorrect. Although the variation in baseline parameters from Chapter 3 to this chapter preclude a simple side-by-side analysis, the results from this chapter's analysis reveal not only similar uncertainty, but also the intriguing result that an increase in  $\gamma_x$  causes the pre- and post-increase optimal emissions paths to cross, not once but twice. These results urge further caution, and recommend further analysis regarding the impact of parameter change.

#### 4.4. Conclusion

Certain trends in U.S. history indicate that individuals, advocates, and government entities may be beginning to take seriously the potential benefits of a shift towards green production, as described in this chapter. Curbside recycling is becoming more and more commonplace in cities around the United States, yet individual and group voluntary efforts to recycle have been around much longer. The number of products proudly proclaiming their use of recycled materials is another example, and even private corporations have begun to take steps to improve the environment, based solely on their desire to capture a

Table 5  
Simulation results

Parameter changed	Change in emissions path	
	Predicted	Actual
Increase $\gamma_x$ from 0.4 to 0.41	(+)	(-) then (+) then (-)
Increase $\gamma_y$ from 0.3 to 0.31	(+)	(-) then (+)
Increase A from 0.6 to 0.61	(+)	(+) then (-)
Decrease $\sigma_1$ from 0.22 to 0.21	(-)	(-) then (+)
Increase $\sigma_2$ from 0.13 to 0.14	(+)	(+)
Decrease $\eta$ from 0.9 to 0.5	(-)	(+)
Increase $b_0$ from 0.01 to 0.011	(+)	(+)
Increase $b_1$ from 0.01 to 0.011	(+)	(+)
Increase $b_2$ from 2 to 2.1	(-)	(-) then (+)
Increase $\alpha_1$ from 0.3 to 0.31	(+)	(-)
Decrease $\alpha_2$ from 0.5 to 0.495	(-)	(-)
Increase $\delta_1$ from 0.1 to 0.11	(+)	(+)
Increase $\delta_2$ from 0.1 to 0.2	(+)	(+)

larger share of an increasingly environmentally conscious market. Advertising campaigns by local and state governments encourage water and power conservation. Many government and non-governmental entities have attempted to educate the public as to the potentially devastating consequences of global warming that could arise from choosing certain types of consumption. These efforts can be seen as either an attempt to alter society's preferences for green goods, which in our model is represented by the parameter  $\alpha_2$ , or possibly an attempt to alter society's general preferences for a clean environment, roughly approximated by the variable  $\phi$ . While this model does not allow for external modifications, mid time stream, to the parameter or variable values, that does not mean that such modifications are not possible.

This chapter advances our understanding of the processes by which society chooses both the economic and environmental outcomes that they must live with. It does so by expanding the traditionally simple models to include a new alternative in consumption, one that is a substitute, although imperfect, for direct abatement efforts. In addition, it enlightens us as to how changes in parameter values might affect the complex economic and environmental systems that produce real-world results. Efforts by many to alter society's preferences for environmental quality have been successful in recent years, although there is reason to suspect (as described by some EKC researchers) that much of the perceived change in preferences may be merely a change in budgetary constraints, which in turn allow for greater leeway to pursue environmental preferences which may have always existed but which took the proverbial "back seat" to economic concerns.

More to the point, these efforts have costs, as well as environmental benefits, and this chapter illustrates that our intuitive assumptions regarding which parameters are important in achieving environmental improvements may be too narrow. Policy decisions regarding the environment should be made from an educated perspective, and that includes an understanding of all possible routes to obtain the desired goal. The results of this chapter show that there may be previously unemphasized routes. If these routes can achieve desired results without the high political and economic costs that can accompany current policy debates over the environment, perhaps environmental quality can be more readily and more affordably obtained.

## CHAPTER 5

### CONCLUSIONS

EKC literature is extensive and multifaceted, starting with simple econometric models that offered the first insights into the possibility of an EKC. Those initial studies have given rise to a wide range of theoretical models, static and dynamic, covering everything from household consumption/pollution choices to international trade and its impact on local and global pollution levels. The initial studies have also given rise to an even wider range of empirical models, testing the significance of not only the regressors used but also the empirical methods employed. Time series, cross-sectional studies, and panel data have all been utilized in the search for an answer to the question of whether and how economic growth might be beneficial to the environment. Sadly, in the 24 years since the initial wave of EKC papers, no consensus has been reached, and many EKC researchers appear convinced that no consensus is possible.

EKC theory holds the promise of being more than merely a theoretical puzzle suitable for debate in the halls of academia. If traditional EKC theory, the modified EKC of Stern (2004), or any number of alternate theories that have arisen from EKC theory are correct, then there is potential to achieve greater environmental quality while maintaining or improving economic conditions for the world's populations. The ongoing debate has led to a more rigorous application of econometric tools to ever-broadening data sets, and has stretched the intellect of interested economists to investigate the various circumstances which might give rise to an EKC. That debate proceeds apace, continuing to yield greater insights, and one hopes that the elusive consensus is still possible.

One thing has been largely missing from the debate, however, close scrutiny of the assumptions that have been commonly used to describe the likely reaction of economic and environmental systems to changes in various parameters. Differences exist between pollutants and between countries. It is possible that, within a complex system, the impact of parameter change will vary depending upon the starting values. Far too often that possibility is not expressly considered, yet the possibility that there may be significant differences in the parameter values may help explain why researchers arrive at different outcomes under reasonably similar circumstances. It is possible that a consensus has been elusive because EKC researchers have not been working from the same starting point.



This research has as its primary purpose to show the necessity of taking greater care in assuming the universality of common assumptions within the context of a complex system. Subordinate to that purpose, but still important, is to show that emissions can be reduced and the environment improved through changes to parameters that do not appear on the surface to be directly linked to environmental quality. Within the simple model of Chapter 3, intuition was shown to be an imperfect tool in predicting the reaction of the system to a change in such parameters as technology, abatement cost, and societal preferences. The introduction of a more complicated model in Chapter 4 illustrated that certain steps can be taken to improve our understanding of the likely reactions. An EKC was present, to a greater or lesser extent, in each simulation. More important than merely providing support for the traditional quadratic form of the EKC, however, this research shows how important it is for society to understand the conditions under which it operates. Society is much more likely to achieve environmental quality in all circumstances if it can better understand the likely reactions to changes in production or abatement technology, or in society's preferences for consumption and a clean environment.

This research is not intended as any form of final answer, but rather to focus attention on a previously unheralded area of EKC research. Also important, it offers strong advice to societies who wish to improve their environmental quality and to understand their own circumstances before assuming that what worked for other countries will necessarily work if applied universally. Technologies will differ across countries and, possibly, even intranationally across regions. So, too, will the assimilative capacities of various ecosystems, especially as they interact with different forms of pollution. These differences may be substantial obstacles in achieving the desired environmental improvements if a country simply models its programs after the successful programs of another country without conducting further analysis.

A consensus as to the validity of EKC theory may lie far into the future, but further progress towards that consensus can be facilitated as consideration is given to the principles stated in this research, that seeming opposite results may not be the result of failure on the part of one or other researcher, but rather differences in the parameter values associated with the particular pollutant, country, or region being analyzed. EKC models will, and should, become increasingly complex, further incorporating the complexities of trade, or even non-optimality. As these complexities are added, it will also further

complicate an understanding of how parameter change impacts predicted outcomes. However, if given serious consideration, it should be possible, with time, to identify ranges over which general assumptions are applicable, and more useful policy recommendations will then be possible.

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## CURRICULUM VITAE

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#### Published Papers

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### Research Support

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Kidd, Jeremy. *Too Much, Too Little, or Just Right?: Information Sharing in a Post-9/11 World*.

Kidd, Jeremy. *Tort, Contract, or Hybrid: Medical Malpractice Reform Under Corrective Justice*.

Kidd, Jeremy. *Bound by the Rules of the Game: Bridging the Gap Between Scalia and Dworkin*.

Kidd, Jeremy. *"I Do, I Do, I Do": Why the Constitution should Protect Religious Polygamists*.

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