1-1-2011

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Recommended Citation

Moyer-Packenham, P. S., & Suh, J. M. (in press). Learning mathematics with technology: The influence of virtual manipulatives on different achievement groups. Journal of Computers in Mathematics and Science Teaching.

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ABSTRACT

This study examined the influence of virtual manipulatives on different achievement groups during a teaching experiment in four fifth-grade classrooms. During a two-week unit focusing on two rational number concepts (fraction equivalence and fraction addition with unlike denominators) one low achieving, two average achieving, and one high achieving group participated in two instructional treatments (three groups used virtual manipulatives and one group used physical manipulatives). Data sources included pre- and post-tests of students' mathematical content knowledge and videotapes of classroom sessions.

Results of paired samples *t*-tests examining the three groups using virtual manipulatives indicated a statistically significant overall gain following the treatment. Follow-up paired samples individual *t*-tests on the low, average, and high achieving groups indicated a statistically significant gain for students in the low achieving group, but only numerical gains for students in the average and high achieving groups. There were no significant differences between the average achieving student groups in the virtual manipulatives and physical manipulatives treatments. Qualitative data gathered during the study indicated that the different achievement groups experienced the virtual manipulatives in different ways, with the high achieving group recognizing patterns quickly and transitioning to the use of symbols, while the average and low achieving groups relied heavily on pictorial representations as they methodically worked step-by-step through processes and procedures with mathematical symbols.

Learning Mathematics with Technology:

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Mathematical learning is a complex process based on the influence and interaction of students' innate competencies, physical and sociocultural experiences, and metacognitive processes (Clements & Sarama, 2007a). Mathematical content is learned in developmental progressions characterized by mathematical concepts and processes within a larger conceptual field of mathematical knowledge (Clements, Wilson & Sarama, 2004; Steffe & Cobb, 1988; Vergnaud, 1996). Students use both internal and external representations to understand the world around them (Goldin, 2003). Research supports the use of physical (Sowell, 1989; Suydam, 1985, 1986) and virtual manipulatives for learning these concepts and processes, and some computer manipulatives (e.g., *Building Blocks*) have been shown to significantly increase students' mathematical knowledge (Clements & Sarama, 2007b).

This paper describes a teaching experiment in which different achievement groups used physical and virtual manipulatives to learn rational number concepts in four fifth-grade classrooms. The literature on the interactions among learning mathematics, virtual manipulatives, and students of different achievement groups is quite limited; therefore, this experiment sought to understand how students in different achievement groups experience mathematical learning when using virtual manipulatives.

Literature Review

Virtual manipulatives, defined by Moyer, Bolyard, and Spikell (2002), are "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Many of the virtual manipulatives currently available today were designed based on physical manipulatives that are commercially available for mathematics instruction, such as pattern blocks, tangrams, or geometric shapes and solids. Other virtual manipulatives were developed in the electronic environment with no physical counterparts. The virtual manipulatives, that are designed based on physical manipulatives, have their own unique qualities. For example, a virtual geoboard models the physical geoboard because users can place the bands on the pegs of the board to create geometric shapes. However, a unique capability in the virtual environment is that the bands placed on the pegs can be stretched and shaped beyond what is capable in the physical environment; and the areas created by the bands can be colored using a paint palette to highlight portions of the geoboard, portions of the shapes created by the bands, and overlapping portions of the shapes. Over the past 10 years publications have shown teachers using virtual manipulatives for mathematics instruction in a variety of ways (Beck & Huse, 2007; Bolyard & Moyer, 2003; Clements & Sarama, 2002; Highfield & Mulligan, 2007; Moyer & Bolyard, 2002; Moyer, Niezgoda, & Stanley, 2005; Moyer-Packenham, 2005; Reimer & Moyer, 2005; Suh & Moyer, 2007; Suh, Moyer, & Heo, 2005).

Virtual manipulatives are one of many available types of cognitive technology tools. Pea (1987) describes the features of cognitive technology tools as providing a means for users to take actions on representations of mathematical objects, reacting in response to the user by providing observable evidence of the user's actions, and sharing the cognitive load with the learner. As a cognitive technology tool, virtual manipulatives provide an externalized representation of mathematical processes, reflect mathematical properties and conventions (i.e., *mathematical fidelity*, Zbiek, Heid, Blume, & Dick, 2007), and reflect the user's strategic choices while engaged in the mathematical activity (i.e., *cognitive fidelity*, Zbiek et al., 2007). Understanding how cognitive technology tools influence students of different achievement levels may provide insight for developmental progressions that can be used to enhance mathematics instruction for students of different achievement levels.

A review of the literature indicates that there are over 50 research articles in which virtual manipulatives (or virtual manipulatives combined with physical manipulatives) have been compared with other forms of instruction (see Moyer-Packenham, Westenskow, & Salkind, 2012, for a synthesis of the effects of virtual manipulatives on student achievement). However, among these articles, there are very few studies that specifically focus on how students of different achievement levels use virtual manipulatives to learn mathematics. Only eight studies, to date, have examined virtual manipulatives with students of different achievement levels. Two of these studies were conducted by Dricky (2000) and Kim (1993), and both found no significant differences among the achievement groups when studying arithmetic and geometry concepts; although Dricky reported that the virtual manipulatives positively influenced students' time-on-task.

Results from other studies involving students of different achievement levels indicate that virtual manipulatives with multiple representations can be significantly more effective for students with high spatial abilities and students with high mathematics achievement. For example, Moreno and Mayer (1999) found pre to post test gains for high and low achieving groups in two treatments – a multiple representation (containing symbolic, pictorial and verbal) group and a symbolic only group. However, the effect size comparing high ability students in the multiple representation group to the symbolic only group produced a large effect (1.11), while the comparison of the multiple representation group to the symbolic only group for low achieving students produced a negative effect (-0.47). Additionally, when students were identified in high or low spatial groups based on a spatial test, the mean gain scores of the high spatial group were over six times greater than the low spatial group scores (Mean gain 4.46, SD=3.24 and 0.67, SD=4.73 respectively). Suh, Moyer, and Heo's (2005) observations of fifth graders also noted that the high achieving students were more efficient in finding answers, used more mental processes, and were more likely to make lists of common denominators to help themselves add fractions with unlike denominators; while the low achievers were more methodical, needed to follow each step in the computer applet tutorial, and were dependent on using the virtual manipulatives to scaffold between the pictorial and symbolic representations.

The final group of studies examining the use of virtual manipulatives by students of different achievement levels reports on students with special needs at various levels, including preschool to university remedial classes. For example, Hitchcock & Noonan (2000) reported that preschool special education children using virtual manipulatives made more progress than when they used paper and pencil. Suh and Moyer-Packenham (2008) reported that fourth grade special needs students were supported by the use of the virtual manipulatives because the tools allowed students to offload findings to the computer thereby reducing their cognitive load. Two studies reported that virtual manipulatives improved test scores for ninth- through twelfth-grade learning disabled students (Guevara, 2009) and university remedial students (Demir, 2009). As this review indicates, the research examining experiences with virtual manipulatives by students of different achievement levels is limited.

Theoretical Underpinnings & Research Questions

Virtual manipulatives are designed to include multiple representations. Goldin (2003) defines *representation* as a configuration of signs, characters, icons, or objects that stand for or represent something else. Representations common in school mathematics include: Physical or Concrete representations, such as manipulatives and three-dimensional geometric models; Visual or Pictorial representations, such as pictures, drawings, or other visual images; and, Abstract or Symbolic representations, such as letters, numbers and arithmetic operation signs. The use of and ability to translate among multiple representational systems has been shown to influence students' abilities to model and understand mathematical constructs (Cifarelli, 1998; Fennell & Rowan, 2001; Goldin & Shteingold, 2001; Lamon, 2001; Perry & Atkins, 2002). Yet little is known about how students of different achievement levels interact with these representations in the virtual environment. Some have suggested that Dual Coding Theory (i.e., information for memory is processed and stored by two interconnected systems and sets of codes) and Cognitive Load (i.e., the limitations of the amount of information held in working memory) may play a role in the types of interactions that students of different achievement levels have with tools in the virtual environment (Clark & Paivio, 1991; Mayer & Anderson, 1992; Pyke, 2003; Rieber, 1994; Sweller, 2003).

Most research on the use of virtual manipulatives has focused on features in the virtual environment conducive to mathematical learning or making comparisons between virtual manipulatives and other instructional treatments. This study was designed specifically to understand how users of differing achievement levels interact with the virtual manipulatives. The study was conducted in four fifth-grade classrooms and focused on fraction concepts. The following research questions guided this teaching experiment:

(1) How does the use of virtual manipulative fraction applets during a unit on fraction addition and equivalence influence students' mathematics achievement for students of different achievement levels? Our hypothesis was that the virtual manipulatives would enhance students' mathematics achievement during the fraction unit, regardless of their different achievement levels, based on the previous results by Dricky (2000) and Kim (1993).

(2) What are the effects on average achieving students during a unit on fraction addition and equivalence for students who use the virtual manipulative fraction applets and those who do not? Our hypothesis was that the students using the virtual manipulative fraction applets would have equal or greater achievement gains than those students who did not use them during the unit, based on the small averaged effect sizes reported in a meta-analysis by Moyer-Packenham, Westenskow, and Salkind (2012) when virtual manipulatives were compared with physical manipulatives.

(3) How do the virtual manipulatives influence the way that students of different achievement levels experience the learning of fraction addition and equivalence? Our hypothesis was that students of different achievement levels would have different types of learning interactions with the virtual manipulatives during the fraction unit that would influence their learning of fraction addition and equivalence, based on the previous results by Suh et al. (2005) for students of different achievement levels.

Methods

Participants

The participants in this study were 58 fifth-grade students in four classes at the same school. The school used standardized test scores at the beginning of the academic year to place the fifth-grade students into low, average, and high achieving groups for mathematics instruction. There was one low group (N=13), two average groups (N=12 and N=12), and one high group (N=21). The low group, the high group, and one randomly assigned average group (of the two) used the virtual manipulatives during the study. The second (of the two) average group served as a control and used physical manipulatives throughout the fraction unit.

Procedures

The study occurred over two-weeks in the spring of the academic year during regular school hours, with students participating during regularly scheduled mathematics classes (60 minutes per class session). Prior to the first week of the study, students completed a 16-item mathematics pretest of fraction content knowledge. Fifth-grade state standards were used as guides to develop the assessment. These standards addressed fraction equivalence and addition of fractions with unlike denominators.

Instructional settings. During the study, lessons were conducted in the fifth-grade classrooms and a computer lab. There were 25 computers in the computer lab and a teacher computer station with a display screen. Every student had their own computer and they worked independently in the computer lab. Three of the groups (one low, one average, and one high achieving) used virtual manipulative fraction applets for five days in the computer lab during the two-week study. The fourth group (average achieving) did not use the virtual manipulatives, but did use physical manipulatives during the unit. One instructor taught all four groups during the two-week unit to reduce teacher effects.

Lessons in the computer lab began with an introduction to the virtual manipulative applet; this was followed by several mathematical tasks for the students to complete independently. Each day, students received teacher-made task sheets with instructions for using the virtual manipulatives and space to record their work. The teacher modeled how to use the virtual manipulative applets before students worked independently. Lessons in the regular classroom began with an introduction to the mathematics topic for the day; this was followed by several mathematical tasks where students used physical manipulatives. Students completed worksheets and teacher-made task sheets that provided practice with the physical manipulatives. At the end of each computer lab and classroom session, the teacher used the last 10 minutes of the class to hold a discussion with the students to elicit thinking and connect ideas that students explored during the sessions. Researchers video-recorded the class sessions.

Virtual manipulatives applets. The students in the computer lab used virtual manipulatives from the National Library of Virtual Manipulatives website (www.nlvm.usu.edu) and the National Council of Teachers of Mathematics electronic resources (www.nctm.org). Most of the applets used were *concept tutorials* that provided directions in words, numeric information presenting algorithmic conventions, dynamic pictorial models linked with numeric representations, and specific guiding feedback that constrained learners' actions to make mathematical properties explicit. These applets were selected because they included features for differentiating instruction for the students of different achievement levels.

Students used the *Grades 3-5 Number and Operations Fractions–Equivalent* applet and *Fractions–Visualizing* applet on Day 1. These applets allow students to manipulate an up and down arrow key that divides regions into multiple parts. On Day 2, students used the *Fractions–Comparing* applet to make a visual model of two different fractions by finding their common denominators. On Days 3 and 4 students used the *Fractions–Adding* applet, which presents two fractions with unlike denominators. Students rename the two fractions with common denominators, using an arrow button to search for a denominator that is common to both fractions. When the denominators are correct, students add the fractions by dragging the pieces from each addend fraction into a sum region. On Day 5, students used the *Fraction Track* game on NCTM's electronic resources. The game allows students to move markers along fraction tracks from zero to one and requires the application of equivalence and addition concepts.

Data Sources and Analysis

Data sources included pre- and post-tests of students' mathematical content knowledge and videotapes of classroom sessions. To answer research questions one and two, researchers designed a 16-item fraction pre- and post-test of mathematics content. The test was based on the state's grade level objectives for learning fraction equivalence and fraction addition with unlike denominators. Students completed the tests prior to the unit and on the last day of the unit.

Researchers analyzed pre- and post-test scores for each student group using gain scores. This took into account that the student groups under examination were starting at different levels of achievement (i.e., low, average, and high achieving groups). Therefore, an analysis comparing the groups based on mean scores for the treatment as a whole was not viable because it would prevent the identification of diagnostic information about who benefited most from the specific treatment in terms of achievement levels. (ANCOVA is not appropriate for this type of analysis because assuming equal pretest "starts" for low, average and high achieving students is not realistic.) Gain scores are a more appropriate measure for providing an analysis on the practical effects of the treatment. Therefore, the analysis of the first research question used the gain scores of the three achievement groups in the virtual manipulatives treatment using paired samples *t*-tests. The second research question focused on the test results for the two average achieving groups, one group that used the virtual manipulatives during the unit and the other group that did not. We analyzed pre- and post-test scores for the two groups using the Mann-Whitney nonparametric test for the comparison of two independent groups with a relatively small sample size.

The final research question focused on how students of different achievement levels interacted with the representations in the virtual environment during the fraction unit. Data to answer this research question were obtained using the video-recordings of class sessions, including interactions with students during their independent work. All class sessions were video recorded and all individual students were recorded during their interactions with the virtual manipulatives. These video-recordings included a record of students' direct quotes. Video-recordings were analyzed using standard qualitative analysis techniques to examine patterns and characteristics of the achievement groups (Shank, 2002; Stake, 1995; Strauss & Corbin, 1998).

Results

Pre and Post Test Results for All Achievement Groups

Our first research question asked: How does the use of virtual manipulative fraction applets during a unit on fraction addition and equivalence influence students' mathematics achievement for students of different achievement levels? We used SPSS to conduct a paired samples *t*-test to identify significant gains between students' pre- and post-test scores. Our first analysis examined gains for the three student achievement groups participating in the virtual manipulatives treatment (N=46). This analysis indicated a statistically significant (pre- to post-test) gain in student performance for all students (N=46) in the virtual manipulatives treatment, $t(45) = -2.752$, $p = .008$. Pre- and post-test scores are presented in Table 1.

Table 1

Pre- and Post-Test Scores for Different Achievement Groups Using Virtual Manipulatives

Achievement Groups		Pretest	Posttest	
Overall Group (N=46)	*M*	82.05	88.11	**
	SD	(21.18)	(13.22)	
Low Group (N=13)	*M*	70.15	81.31	*

	SD	(21.44)	(12.34)
Average Group (N=12)	*M*	78.88	85.58
	SD	(26.47)	(17.28)
High Group (N=21)	*M*	91.24	93.76
	SD	(12.71)	(8.36)

Note. *****$p < .05$; ******$p < .01$

Our next analysis for research question one examined individual achievement groups for pre- to post-test gains using follow up paired samples *t*-tests for each group. The results of the paired *t*-tests for the low-, average-, and high-achievement groups are also presented in Table 1, and showed that:

(a) There was a statistically significant (Pre *M* = 70.15, *SD* = 21.44, to Post *M* = 81.31, *SD* = 12.34) gain for the low-achieving group, *t*(12) = -2.433, p = .032;

(b) There was no statistically significant (Pre *M* = 78.88, *SD* = 26.47, to Post *M* = 85.58, *SD* = 17.28) gain for the average-achieving group, *t*(11) = -1.706, p = .116; and,

(c) There was no statistically significant (Pre *M* = 91.24, *SD* = 12.71, to Post *M* = 93.76, *SD* = 8.36) gain for the high-achieving group, *t*(20) = -0.809, p = .428.

Our original hypothesis was that the virtual manipulatives would enhance students' mathematics achievement during the fraction unit, regardless of different achievement levels. These results indicate that there was a significant gain overall for the students participating in the virtual manipulatives treatment. However, the low achieving students benefited most from their participation in the virtual manipulatives treatment, with statistically significant gains as an individual group.

The second research question asked: (2) What are the effects on average achieving students during a unit on fraction addition and equivalence for students who use the virtual manipulative fraction applets and those who do not? Descriptive statistics from this portion of the analysis are presented in Table 2. We used the Mann Whitney nonparametric test for comparison of two independent samples because the groups were of a relatively small sample size. The Mann Whitney statistic was found to be 55.5 and greater than the critical value at the .05 level (37), thus indicating that there was no statistically significant difference between the two groups on their pre to post test gain scores. For a parametric triangulation of the findings with the nonparametric test (Mann Whitney), a *t*-test for independent samples was also run with the same data. The results were consistent with the Mann Whitney test indicating a lack of statistical significance between the two groups, $t(22) – 1.315$, $p = .202$. Our hypothesis was that the students using the virtual manipulative fraction applets would have equal or greater achievement gains than those students who did not use them during the unit. The descriptive statistics show that the only differences between the two groups were numerical.

Table 2

Pre and Post Test Scores for Average Achievement Groups in Two Treatments

Treatment Groups		Pretest	Posttest
Physical Manipulatives	*M*	87.58	89.08
Average Group (N=12)	*SD*	(9.30)	(14.45)

Virtual Manipulatives	*M*	78.88	85.58
Average Group (N=12)	*SD*	(26.47)	(17.28)

How Different Achievement Groups Experienced the Virtual Manipulatives

The third research question asked: (3) How do the virtual manipulatives influence the way that students of different achievement levels experience the learning of fraction addition and equivalence? Our hypothesis was that students of different achievement levels would have different types of learning experiences during the fraction unit that would influence their learning of fraction addition and equivalence. Because of the unique representations within each virtual manipulative applet, the following results are organized to highlight different achievement groups interacting with each applet.

Day 1: Fractions–equivalent and fractions–visualizing applets. When the *Fractions–Equivalent* and *Fractions–Visualizing* applets were used, all achievement groups contained several students who explored with the applet to determine how many pieces they could break apart each region on the applet. Once one student began this exploration, other students around the student also wanted to see how many pieces they could make with the applet. An additional observation of all achievement groups, but more common in the high achieving group, was students creating multiple visual images of the fraction representations rapidly, going beyond the applet requirement of finding only two equivalent fractions. Students made comments like, “You don’t have to worry about taking a long time to change to different fractions.” The high achieving students quickly recognized numerical relationships among the numerators and denominators of the equivalent fractions, and no longer needed to manipulate the fraction regions to find an equivalent fraction; they could create the equivalent fraction using mental math.

During the discussion at the end of the class session, the high achieving students identified 3/6 and 4/8 as equivalent amounts because both were equal to ½. They stated ideas such as, “You have more pieces, but they’re smaller because you still have the same area of space” and “Only the numerator and denominator changes, but the number does not.” The average and low achieving groups continued to use the region models as support to find equivalent fractions throughout the tasks. They relied more on the visual aspects of the applet with comments stating, “It helps you visualize because you can see and count the number of pieces.” The average and low achieving groups used counting strategies, rather than recognizing the proportional relationships like the high achieving group. Because of this, the average and low achieving groups did not identify 3/6 and 4/8 as equivalent amounts without prompting by the instructor.

Day 2: Fractions–comparing applet. When using the *Fractions–Comparing* applet, students in the high achieving group used their knowledge of multiples and numerical relationships to determine common denominators to compare the two given fractions on the applet. Comments during this session included, “When I find this denominator I need to know common multiples.” The average and low achieving students were much slower and more methodical as they clicked through the possible choices of multiples on the applet to find a common denominator. Some of the average students knew the multiples, but they used the applet to confirm their thinking. They appeared less confident of their knowledge of the multiples. The applet directions ask students to “find different names” for the two fractions that are given in order to compare the two fractions. In the low achieving group, several students were observed finding equivalent fractions to the given fractions, but not common denominators. For example, when the two fractions were 5/8 and 1/3, instead of finding 24 as the common denominator of both fractions, several low achieving students entered into the applet 5/8=10/16 and 1/3=2/6 (e.g., 24 would be a common denominator for 5/8 and 1/3).

Days 3 and 4: Fractions–adding applet. When students used the *Fractions–Adding* applet, all groups were influenced by the built in constraints in the applet that did not allow students to add the two fractions together until they renamed each fraction using a common denominator. The applet quickly taught the high achieving students the addition procedure and provided guidance and immediate feedback that confirmed that students were following the procedures on the applet. The high achieving students did not need the visual models to find an equivalent fraction, so they simply entered in the numbers on the applet. They also did not need to move the fraction pieces on the applet to the sum circle or square because they quickly observed the sum of the two fractions without employing this step. The average achieving students also developed some efficiency strategies by Days 3 and 4 in the computer lab. For example, they were observed writing multiples of given denominators on their task sheets or typing in the numbers for common denominators on the applet first and then using the applet models to check their thinking; the low achieving students did not do this at all. The low achieving students engaged in multiple trial and error interactions. They entered multiple wrong answers into the applet and through guidance and feedback provided by the applet, the low achieving students experimented until they understood the addition procedure. Initially, rather than finding the common denominator for the two fractions, some of the low achieving students found an equivalent fraction for each given fraction (as they did with the *Fractions–Comparing* applet on Day 2 in the computer lab), but this was not a common denominator. For example, for the exercise 1/2 + 1/3 = __, instead of finding 6 as the common denominator, students wrote that ½ = 2/4 and 1/3 = 2/6. However, they could not use these equivalent fractions in the applet because of the built-in constraints. This individual struggle seemed to help them learn the procedure for finding the common denominator. The low achieving students made comments about the guidance provided such as, “If you write it on paper you’ll get the problem wrong. But here (pointing to the screen) you can’t get it wrong unless you’re not careful” and “You put it right here (pointing to the screen). It won’t fit here, so you know it’s not eighths; that’s not the denominator.” This trial and error process helped the low achieving students learn the procedures at their own pace.

Day 5: Fraction track game. During the *Fraction Track* game, there were observable differences among the groups. By Day 5 in the computer lab, both the high and average achieving groups recognized the equivalent fractions and could use this knowledge to be strategic in the game. One student commented, “Whenever you hit a number that is a factor of the denominator, you can use it in its place.” Not only did they recognize the amount they had on the number line in the applet, but they also recognized the amount that remained (the residual) to get to one whole. Both groups used some mental addition and subtraction strategies. However, several students in the high achieving group were the only ones to notice that the fractions on the game board in the applet were equivalent in a vertical alignment. Because the low achieving students did not connect their work with equivalent fractions to the activities in the game, they did not realize that one-fourth could be used in place of two-eighths (and vice versa) on the game board. The low achievers often filled up each line on the board and then waited to get the exact remaining amount (rather than using an equivalent amount); they did not know what to do when they got a number that did not complete any of their remaining lines on the game board.

Limitations. One limitation of this study was that the achievement of the high achieving group was near the ceiling effect, with scores of 91% out of 100 on the pre-test. This left little room for the effects of the treatment to be observed in the high achieving group. This type of ceiling effect in a study may have allowed more room for the low achieving group to improve most between the pre- and post-tests during the study.

The study was exploratory and reports results from only four classrooms with 58 students, all receiving instruction from the same teacher. It offers no generalizations about the effects virtual manipulatives will have on other fifth-grade students in other classrooms. However, with the limited amount of studies focusing on how students of different achievement levels experience mathematics learning while using virtual manipulatives, this study is a contribution to the literature. We also acknowledge limitations in our data. The assessments used were teacher-made tests, and therefore, not standardized. Although the pre- and post-tests were similar, there may have been discrepancies in the levels of difficulty on each assessment.

Discussion

In research studies where students of all achievement levels are included in the analysis as a whole group, it is difficult to determine how students of different achievement levels are influenced by instructional treatments. This prevents researchers from obtaining diagnostic information about who benefits most from a specific treatment in terms of student achievement levels, and how different achievement levels experience the treatment.

As the results of this study reveal, overall, the three groups using virtual manipulatives demonstrated statistically significant gains between the pre- and post-tests used in the study. Further examination of the individual gains of each achievement group (low-, average-, and high-achieving) using virtual manipulatives indicated that the low-achieving group had statistically significant pre- to post-test gains, while the average- and high-achieving groups did not. Using gain scores in our study measured a very important practical effect of the treatment on student achievement in terms of different achievement levels. The two average achieving groups (one using virtual manipulatives and the other using physical manipulatives) had no statistically significant differences in their performance on the pre- and post-tests.

While the results of the pre- and post-testing for each of the four achievement groups are interesting, they occlude what we believe is the more important result of this teaching experiment. Namely, that the students in different achievement groups had different types of experiences with the virtual manipulatives. The video-recordings and conversations with students in the computer lab provide a glimpse into the different types of interactions students in different achievement groups experienced when working with the virtual manipulatives. While all of the groups engaged in explorations and some creative activity with the applets upon initial introduction, a number of their work habits and routines with the virtual manipulatives were different.

The high achieving group used more mental math strategies, identified multiples and factors, saw patterns more quickly, sometimes ignored the pictorial models in the applets, recognized equivalence and proportional relationships, applied equivalence understanding to use as a strategy in the fraction game, and focused on symbolic features in the applets to complete mathematical tasks. The average achieving group used some mental math strategies later in the fraction unit, used a step-by-step methodical process to find/check multiples and common denominators, relied on the pictorial models in the applets, used counting strategies rather than proportional relationships, recognized equivalence relationships later in the fraction unit, applied equivalence understanding to use as a strategy in the fraction game, and relied on pictorial and symbolic features in the applets to complete mathematical tasks. The low achieving group used a step-by-step methodical process to find multiples and common denominators, relied heavily on the pictorial models in the applets, used counting strategies rather than proportional relationships, did not recognized equivalence relationships, experienced confusion with common denominators, and engaged in multiple trial and error interactions with the pictorial and symbolic features in the applets to complete mathematical tasks.

Why did the Interactions of Different Achievement Groups have a Positive Influence on Their Learning?

Although each group experienced the virtual manipulatives in different ways, they all experienced learning gains during the experiment. So how did different types of learning experiences lead to learning gains for all three of the different achievement groups that used the virtual manipulatives? One answer to this question may lie in the findings from a recent meta-analysis by Moyer-Packenham, Westenskow, and Salkind (2012). This meta-analysis revealed evidence of five interrelated virtual manipulative affordances that promote mathematical learning: focused constraint, creative variation, simultaneous linking, efficient precision, and motivation. While each of these five interrelated affordances may have influenced the different achievement groups, some affordances may have had a greater impact than others for each of the high, average and low achieving groups. In other words, students at different achievement levels may be influenced by different affordances in the virtual manipulatives.

In the affordance *efficient precision*, the virtual manipulatives contain precise representations allowing accurate and efficient use. This affordance seemed to be most influential and beneficial for the high achieving students. For example, the high achieving groups were able to recognize patterns quickly and then proceeded to skip or ignore pictorial and guiding features in the applets. The applets contained efficiency features that allowed the user to quickly produce multiple examples or to skip elements within the applet (e.g., students did not need to use the pictorial elements to get the numerical elements correct). The applets allowed the high achieving students to learn the mathematical concepts and processes, see patterns and relationships, and use the virtual manipulatives with efficiency.

In contrast, the affordance *focused constraint*, in which the virtual manipulatives constrain student attention on mathematical objects and processes, seemed to be most influential and beneficial for the average and low achieving students. For example, the average and low achieving students used multiple trial-and-error attempts to determine common denominators and to find common denominators so that they could add two fractions together. The constraining, guiding, and feedback features supported the low and average achievement groups throughout their mathematical interactions. The guiding and support features were available to students as long as these support features were needed. This was especially evident for the average achieving students, who seemed to rely on the pictorial and symbolic models initially, and during later class sessions, they did not need this pictorial support at the same level as they had on Days 1 and 2.

Essentially, the high achieving students found equivalent fractions and learned the fraction addition procedures as a result of the *efficient precision* in the virtual manipulatives, while the average and low achieving students found equivalent fractions and learned the fraction addition procedures as a result of the *focused constraint* in the virtual manipulatives. Perhaps this effect is explained by Kaput (1992) who stated that a constraint-support structure in a virtual environment "frees the student to focus on the connections between the actions on the two systems [notation and visuals], actions which otherwise have a tendency to consume all of the student's cognitive resources even before translation can be carried out" (p.529). The high achieving students were freed to focus on the connections and relationships, which they did rapidly, while the average and low achieving students received sustained support from the constraints in the applets throughout the fraction lessons. The methodical trial and error activity of the low and average achieving students provided multiple examples that students could work through at their own pace.

In previous research comparing instructional treatments where students used physical manipulatives only, computer simulations only, and a combination of the two, researchers concluded that not all students are influenced in the same manner by different instructional treatments (Berlin & White, 1986). The specific feedback in written form on the screen may have served the function of correcting or highlighting students' errors, thereby making students more aware of their own misconceptions. The numerical and written feedback also provided a model for students on how to write fractions in numbers, words, and in pictures. This feedback was immediate and individual. By providing these models of how to write and represent fractions, the applets were essentially teaching students accurate mathematical terminology and notation. Because students worked at their own pace, they were able to complete the number of examples appropriate for them and at a speed where they could understand what they were seeing and doing. This kept the advanced students interested and engaged and allowed less able students the opportunity, through trial and error, to understand the concepts.

These results connect with other findings and highlight the importance of differentiating for different achievement levels during mathematics instruction. For example, in a study conducted by Threadgill-Sowder and Juilfs (1980), the researchers found that the lower achieving children showed more improvement in recognizing geometric patterns when using physical manipulatives. In the current study, we also found that lower achieving students were influenced by the treatment using the virtual fraction manipulatives. These similar results may indicate that lower achieving students benefit more from visual and physical models that scaffold their mathematics learning and support their conceptions of content in meaningful ways.

Conclusion

There are several aspects of this study significant to the use of virtual manipulatives for mathematics instruction with students of different achievement levels. The mathematics pre- and post-tests showed significant gains overall in this relatively small sample of students, with lower achieving students showing significant gains as an individual group following the virtual manipulatives treatment. While these testing gains are worth noting, what may be more important is the results on how different achievement groups were influenced by and benefited from interacting with the virtual manipulatives.

One aspect to consider is which applets are useful to students of different achievement levels and how interrelated affordances may influence different students during mathematics instruction. Some applets may provide higher achieving students with multiple examples so that they can quickly recognize patterns, while other applets can provide constraints and guiding feedback for lower achieving students who need more support and guidance. In addition, there are virtual manipulative applets that contain multiple affordances.

Another aspect demonstrated by this study is that there are multiple affordances within each virtual manipulative applet, and that one or more of these affordances may be more influential and beneficial for one achievement group while another affordance (within the same virtual manipulative applet) may be more influential and beneficial for another achievement group. This result may explain why students of all achievement levels using virtual manipulatives outperform students participating in comparison treatments in a number of different studies. Essentially, the multiple affordances built in to the virtual manipulatives provide “something for everyone” and a way for students at each achievement level to learn the mathematical concepts and procedures. The different impacts on students of different achievement levels may be a factor that is important for the design of mathematics instruction that uses technology. These different effects may have been caused by the visual/pictorial models that helped students to understand the concepts. Or students may have been helped by the pictorial models being linked with the mathematical symbols so that they saw two different forms of representation while students were working. The virtual manipulatives also provided opportunities to practice using a visual model that could be changed and manipulated. Students do not have this opportunity for practice with dynamic visual representations when they view pictorial images on textbook pages or worksheets. Our hope is that this study encourages teachers and researchers to examine more deeply how specific affordances produce positive effects with students of different achievement levels.

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