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(the last integral has been taken from a table\textsuperscript{11}). Therefore, $E_0$ behaves like $(\ln H)^2$ for large values of $H$. A more careful, tedious, but straightforward study of (3), with the use of majorizations and minorizations, gives the following more precise result for the asymptotic behavior of $E_0$:

$$E_0 = mc^2 + (\alpha/4\pi)mc^2\left[\ln(2\pi^2 H/mc^2) - C - \frac{3}{2} + A + \cdots\right], \quad (5)$$


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**Interpretation of a Unified Theory of Gravitation and Symmetry Breaking**

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The formalism of Moen and Moffat is interpreted as a Yang-Mills theory set in a space-time generally endowed with curvature and torsion.

In a recent paper,\textsuperscript{1} Moen and Moffat describe the possibility of a generalized definition of "parallel" transport of a vector nonet [an element of the tensor representation of the combined group of space-time and $U(3)$ transformations] resulting in (a) a connection between space-time and internal symmetries without reference to a "supergroup" and (b) unitary symmetry breaking induced by the presence of a zero-mass boson (to first approximation). We show that it is possible to interpret the formalism in this work as an extended Yang-Mills theory. From this point of view we see that a total symmetry group is already "embedded" in the theory, and that the character of the background space-time is sufficient to break the internal symmetry.

To see how it may be possible to make the aforementioned interpretation, we first review some aspects of a local gauge theory set in a curved background. At the outset there is, presumably, a matter field which displays a unitary symmetry characterized by\textsuperscript{2}

$$\psi'(x) = S^{-1}(x)\psi(x). \quad (1)$$

The entities generically designated $S$ are taken to be matrix representations of elements of a group of internal transformations, and are by assumption functions of the space-time coordinates of the event point at which the transformation is made. The internal degrees of freedom of the $\psi$ field are thus adjustable at all other points of space-time, in keeping with the requirements of a local picture of interaction. To ensure the invariance of the dynamical structure of this system, it is necessary to introduce auxiliary field operators $B_\mu$ that couple universally with the various $\psi$ components, and which transform under local internal group action as

$$B'_\mu = S^{-1}(B_\mu S - \nabla_\mu S). \quad (2)$$

Here $\nabla_\mu$ denotes the relevant space-time covariant derivative with respect to the $\mu$th coordinate.

In a sense, the $B_\mu$ fields are like components of an affine connection\textsuperscript{3}; as a consequence, we may define a totally covariant derivative operator expressed symbolically as

$$D_\mu = \nabla_\mu + B_\mu. \quad (3)$$

$D_\mu$ commutes with both space-time and internal transformations, and serves to establish a meaning for a parallel transport of fields with mixed indices. In terms of the vector nonets mentioned in I, the operation of $D_\mu$ provides, for example,

$$D_\mu A'^{\sigma} = \nabla_\mu A'^{\sigma} + B_\mu A'^{\sigma} = \partial_\mu A'^{\sigma} + \left[ \left( \begin{array}{c} \sigma \\ \mu \nu \end{array} \right) \right] A'^{\nu} + B_\mu A'^{\sigma}, \quad (4)$$

where Greek indices refer to space-time structure, Latin indices to internal.

Now, the covariant derivative defined in I is just such an operator, that is, it measures the effect of the total variation of fields. As expressed in that work, the

covariant derivative of a contravariant vector nonet is
\[ A_i^i = \partial_\alpha A^{\alpha i} + h_{ij}^k \Gamma^{\alpha i}_{\beta j} A^\beta k, \]
which the \( h_{jk}^i \) given in terms of the conventional \( f_{jk}^i \)
and \( \delta_{jk}^i \) of \( U(3) \) symmetry\(^4\) as
\[ h_{ij}^k = (1 - \alpha) f_{jk}^i + \alpha \delta_{jk}^i. \]
The right-hand side of Eq. (5) is obviously
\[ \partial_\alpha A^{\alpha i} + h_{ij}^k \Gamma^{\alpha i}_{\beta j} A^\beta k, \]
where the sum on \( a \) is 1–8. Consideration of the transforma-
tion law
\[ \Gamma^{\nu}_\mu = \frac{\partial x^\rho}{\partial x^\mu} \frac{\partial x^\sigma}{\partial x^\nu} \]
shows that, under change of coordinates, only the unitary scalar component of \( \Gamma^{\nu}_\mu \) transforms as a connection while the remaining internal components transform as space-time tensors. Hence, it is plausible to interpret (5) as (4) by allowing the identifications
\[ A_i^i \rightarrow D_i A_i^i, \]
\[ h_{ij}^k \rightarrow \delta_{ij} B_{ij}^k, \]
and
\[ h_{ij}^k \rightarrow \delta_{ij} B_{ij}^k. \]
In fact, the second replacement is already given in I [Eq. (59)]. After interpretation, assuming as in I that internal transformations may be made path-independent, we are always able to select an internal basis such that the third term of (7) is zero.\(^5\) Consequently, the total divergence of a vector density nonet \( \Xi_{\mu i} \),
\[ D_{\nu} \Xi_{\mu i} = \partial_\nu \Xi_{\mu i} + 2 \left[ \frac{\sigma}{[\mu\nu]} \right] \Xi_{\mu i} + B_{ij}^k \Xi_{\nu j}, \]
becomes
\[ D_{\nu} \Xi_{\mu i} = \partial_\nu \Xi_{\mu i} + 2 \left[ \frac{\sigma}{[\mu\nu]} \right] \Xi_{\mu i}. \]
As a result, the conservation law
\[ D_{\nu} \Xi_{\mu i} = 0 \]
yields
\[ \tilde{F}(i) = \int \partial_\nu \Xi_{\mu i} d^3x = -2 \int \left[ \frac{\sigma}{[\mu\nu]} \right] \Xi_{\mu i} d^3x. \]
The symmetry of the \( F \)-spin operators is broken, even in the case of zero Yang-Mills fields, by the unconventional space-time structure available in our hypotheses. The right-hand side of (10) vanishes, we note, both in the event of a torsion-free space-time and when the torsion present is completely antisymmetric.

Let us examine, in the light of our interpretation, statements (a) and (b) given initially. The assumptions in I appear tacitly to include a supergroup, namely, the direct product of space-time and internal groups. One then sees a trivial combination of the two sets of symmetries, a situation manifested in the vanishing of the Yang-Mills fields. On the other hand, symmetry breaking is still possible as a result of the assumed torsion. The torsion acts as an independent field which couples to the current \( \Xi_{\mu i} \) to break the unitary symmetry, but the unambiguous identification of a particle with this field is problematic.\(^6\)

A slight modification in the unitary transformation laws given in I provides a nontrivial local gauge picture, replete with symmetry breaking even in the ordinary Minkowski background. If we let the vector nonets transform internally as
\[ \tilde{\delta} A_{\nu i} (x) = i \epsilon(x) L_{ij} A_{\nu j} (x), \]
the variation of \( A_{\nu i} \), gives a "connectionlike" law for
\[ \tilde{\delta} h_{ij}^k \Gamma_{\mu \nu} \]
\[ \tilde{\delta} (h_{ij}^k \Gamma_{\mu \nu}) = i \epsilon(x) \left( L_{ij}^m h_{mn}^k \Gamma_{\mu \nu} - L_{ij}^m h_{mn}^k \Gamma_{\mu \nu} \right) \]
\[ - i \left( \partial_\nu \epsilon(x) L_{ij}^m \delta_{\mu j} \right). \]
Since the parameters \( \epsilon(x) \) are taken as scalar-valued functions of space-time, (12) is the statement in I language of the infinitesimal version of (2). With the wider generality, (8) and (9) imply
\[ \tilde{F}(i) = -2 \int \left[ \frac{\sigma}{[\mu\nu]} \right] \Xi_{\mu i} d^3x = -2 \int B_{ij}^k \Xi_{\nu j} d^3x, \]
which indicates that the coupling of the Yang-Mills field to the current density alone is sufficient to break the symmetry. In the usual theory\(^2\) massless spin-1 bosons are associated with the \( B_{ij}^k \) fields; these can be held responsible for the breaking (13). The prototype (and as yet singular) example is, as mentioned in I, that of the electromagnetic potentials \( A_{\mu} \).