MEASURING THE NORMALIZED RADAR CROSS-SECTION OF THE EARTH USING PENCIL BEAM SCATTEROMETERS

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ABSTRACT
Space-borne scatterometry has been used by NASA for several years to estimate the normalized radar cross-section ($\sigma_0$) of the surface of the earth. The measured value of $\sigma_0$ can then be used to study surface features such as vegetation, polar ice and ocean winds. Recently, size constraints have required NASA to switch the basic design of scatterometers from long antennas, which create a fan beam, to small parabolic dishes which create a narrow pencil beam. This change in antenna design requires new $\sigma_0$ retrieval algorithms to be developed.

$\sigma_0$ is calculated by dividing the power received by the conversion factor $X$, which is a function of the spacecraft and antenna positions. Because it is computationally expensive to calculate $X$ for each data point the $X$ factor algorithms proposes the use of a pre-computed table of nominal $X$ values for various scan angles and orbit positions. Unfortunately, the table does not take into account any variations in the orbit, or perturbations to the attitude of the spacecraft.

A perturbation correction algorithm is developed which uses the shift in baseband frequency ($\Delta f$) resulting from various perturbations to correct the nominal values of $X$. Using the combination of the $X$ factor table and the $\Delta f$ correction, $\sigma_0$ can be retrieved rapidly and accurately. This algorithm will be used to calculate $\sigma_0$ for the upcoming Quikscat and SeaWinds missions.

Consequently, NASA has developed an entirely new type of space-borne scatterometer which employs parabolic dish antennas to create a pencil beam that focuses the radar onto a single elliptical spot on the ground [Spencer et al., 1997]. Quikscat and SeaWinds are two newly developed pencil beam scatterometers that are scheduled for launch in November 1998 and early in the year 2000. In preparation for the launch of these scatterometers new algorithms must be developed for processing the data. This paper will discuss the development of the $X$ factor algorithm which will be used to extract the normalized radar cross-section ($\sigma_0$) from the radar measurements.

INTRODUCTION
Historically, space-borne scatterometers would require large horn antennas to create fan beam patterns used to scan large portions of the earth at a time. These scatterometers used the doppler shift imparted by the surface of the earth to divide the single beam into smaller resolution measurements [Naderi et al., 1991]. Over the last ten years, fan beam designs have proven very effective with the successful flights of NASA’s scatterometer (NSCAT) and the European scatterometers, ERS1 and ERS2. However, for future missions size constraints have necessitated more compact scatterometer designs.

HIGH RESOLUTION MEASUREMENTS
Before discussing the $X$ factor algorithm, it is useful to give an overview of the system design and geometry of pencil beam scatterometers. Figure 1 shows the geometry of a pencil beam scatterometer.
the spacecraft flies over the earth the antenna dish rotates causing the two illuminated spots on the ground to trace out a spiral. The outer beam is vertically polarized and traces out a swath that is 1800 km wide, while the inner beam is horizontally polarized and traces out a 1400 km wide swath. As a result of the very wide swaths a pencil beam scatterometer will be able to collect data for 90% of the earth’s surface every 24 hrs [Spencer et al., 1997].

The nominal resolution of the scatterometer is equal to the size of each illuminated spot. These spots are oval in shape with a length of about 30 km and a width of 20 km and have been given the name “eggs”. While 25 km resolution compares favorably with the resolution obtained by previous scatterometers, the developers were anxious to further increase the resolution. For this reason Quikscat and SeaWinds will transmit a linear FM chirp rather than a continuous frequency pulse. The equation for the transmitted pulse is given by [Spencer, 1998]

\[ P_t(t) = p(t) \cos(2\pi f_c + f_{dc} - \frac{1}{2} \mu(\Delta t))((\Delta t)) \]  

(1)

where

\[ \Delta t = t - t_0. \]  

(2)

In this equation, \(p(t)\) is the pulse shape, \(f_c\) is the carrier frequency and \(\mu\) is the chirp rate. The doppler compensation frequency, \(f_{dc}\), is calculated onboard the spacecraft and added to the pulse so that the doppler frequency imparted by the scatterer at the center of the pattern will be negated and the return pulse will be centered on the carrier frequency. Finally, \(t_0\) is the round trip flight time to a scatterer in the very center of the egg.

The echo return from the surface is the sum of the reflections from each individual scatterer in the illuminated area. This received echo is first mixed down to baseband using I/Q demodulation and then dechirped by mixing it with the conjugate of the transmitted chirp

\[ M(t) = e^{-j2\pi [\frac{1}{2} \mu(t-t_0) + (t-t_0)].} \]  

(3)

The resulting waveform for each scatterer is

\[ P_i(t) = P_{r,i}(t)M(t) \]  

(4)

\[ = C_i e^{j2\pi (f_{dc} - f_{bi} + \frac{1}{2} \mu(t_0 - t_i))((t - t_i)} \]  

\[ \cdot e^{-j2\pi [\frac{1}{2} \mu(t-t_0) + (t-t_0)]} \]  

(5)

\[ = e^{j2\pi f_{bi} t} \]  

(6)

where \(C_i\) is the return power and \(f_{bi}\) is the baseband frequency for an individual scatterer,

\[ f_{bi} = (f_{dc} - f_{bi} + \mu(t_0 - t_i)) t. \]  

(7)

Each scatterer on the ground generates a unique baseband frequency determined by the difference between the round trip flight time to it and the round trip flight time to a scatter in the center of the egg \((t_0 - t_i)\) and the difference between the doppler compensation frequency and the actually frequency shift imparted by the scatterer \((f_{dc} - f_{bi})\).

The eggs are “sliced” into finer resolution elements by transforming the echo into the frequency domain by the use of an FFT. The spectrum of the echo can then be broken into frequency bands that correspond to range/doppler slices on the surface of the ground. By summing the power in each frequency band the received power can then be measured for each slice. SeaWinds and Quikscat are designed to generate 12 slices. The middle 10 slices are 6 to 8 km wide and 15 to 25 km long and can be used to calculate \(\sigma^0\). The two outermost slices are much larger and will not be used for high resolution \(\sigma^0\) retrieval, but are there to ensure that the entire echo is captured.

\[ \text{CALCULATING } \sigma^0 \]

The purpose of scatterometry is to gather accurate measurements of \(\sigma^0\) for the surface of the earth. These measurements can then be used to study surface features, such as ocean winds, vegetation and polar ice. \(\sigma^0\) is related to the power received \((P_r)\) by

\[ \sigma^0 = \frac{P_r}{X} \]  

(8)

where

\[ X = \frac{P_i \lambda^2}{(4\pi)^3} \int_A \frac{G_r G_t}{R^4} dA, \]  

(9)

[Ulaby et al., 1981]. Since the power transmitted, \(P_i\), and the wavelength of the radar, \(\lambda\), are fixed over the illuminated area it is only necessary to integrate the transmit and receive gain, \(G_r\) and \(G_t\), and the range, \(R\), to each incremental area.

As the scatterometer collects data the power received must be measured and telemetered down to earth where it will be converted to \(\sigma^0\). The problem with computing \(X\) arises from the fact that data is collected at a high rate with over 190,000,000 measurements per day. With this many measurements is would be virtually impossible to integrate around each data point and calculate the \(X\) required to generate an accurate value of \(\sigma^0\). For this reason, it was decided to generate a pre-computed table of \(X\) values that can be used for the various orbit and scan positions. This look-up method of \(\sigma^0\) retrieval works very accurately as long as all the assumptions made.
Figure 2: As the orbit or attitude of the spacecraft changes the gain pattern shifts with respect to the slice lines.

when the table was calculated remain unchanged. When there are variations in the orbit or the orientation of the scatterometer, errors are introduced into the calculation of $\sigma^o$. One of the mission requirements for both SeaWinds and Quikscat is that $\sigma^o$ be measured with 0.05 dB of accuracy. In order to obtain this level of accuracy a correction must be made to the nominal table to account for scatterometer perturbations. The remainder of this paper will discuss methods for correcting $X$ without changing the nominal $X$ table.

**PERTURBATIONAL CORRECTIONS FOR X**

The three main sources of error in $X$ are orbit perturbations, variations in the roll, pitch and yaw (attitude) of the spacecraft and quantization errors in the transmitted doppler compensation frequency. These three types of perturbations can be broken to two catagories based on the fundamental source of error in $X$. The orbit and attitude perturbations generate errors in $X$ because the spacecraft orientation with respect to the ground has changed. These will be referred to as geometric perturbations. Any variation in the transmitted frequency causes the boundaries of the slices to move with respect to the antenna pattern and will be referred to as frequency perturbations.

Correcting for Geometric Perturbations

The effect of geometric perturbations is to shift the gain pattern on the ground while the slice lines remain fixed relative to the surface. The slices do not move because the boundaries are determined by the doppler compensation frequency calculated on the spacecraft which corresponds to points of specific doppler frequencies on the ground. Figure 2 depicts the shifting of the gain pattern across the slice lines. With the gain pattern shifted, the returning baseband frequency ($f_{b,i}$) for each scatterer is going to be different than for the nominal $X$ table and must be corrected to maintain the desired accuracy of $\sigma^o$.

One other consequence of geometric perturbations is that the returning baseband frequency ($f_{b,i}$) for each scatterer is shifted as a result of the range to each scatterer changing. From Equation 7 it was shown that $f_{b,i}$ was function of $\Delta t_i$ where $\Delta t_i$ is the difference in the round trip flight time between the nominal center scatterer ($t_0$) and each scatterer ($t_i$). When the range changes $t_i$ changes resulting in a $\Delta f$ shift in $f_{b,i}$. This result is convenient because now the error in $X$ can be compared against the resulting $\Delta f$, rather than comparing it against orbit, roll, pitch, and yaw. Figure 3 shows the results of a simulation in which error was calculated for several different spacecraft attitudes and plotted versus the resulting $\Delta f$. This plot demonstrates that there is a smooth one-to-one relationship between error in $X$ for the different slices and $\Delta f$.

Figure 4: Errors in the doppler compensation frequency causes the slice lines on the surface of the earth to move.
By parameterizing $X$ in terms of $\Delta f$ a simple correction algorithm can be used that takes the nominal values of $X$ and adds a correction based on $\Delta f$. It was found that a third order polynomial best fit the error curves yielding a correction of the form

$$X_{\text{true}} = X_{\text{nom}} + A + B\Delta f + C\Delta f^2 + D\Delta f^3$$  \hspace{1cm} (10)

with $A, B, C$ and $D$ being functions of orbit latitude and scan angle, like the $X$ table itself. This method has been found to work with great accuracy and yields maximum residual correction errors of less than 0.05 dB for the inner 10 slices.

**Correcting for Frequency Perturbations**

As mentioned previously, frequency perturbations are a result of transmitting a doppler compensation frequency ($f_{dc}$) that does not exactly correspond to the doppler contributed by the surface. This is a concern because the doppler tracking algorithm is tabularized and consequently will have quantization errors as it generates the doppler compensation frequencies. The resulting frequency errors are expected to be in the range of ±3 kHz.

Frequency perturbations cause the slice boundaries shift on the surface while the gain pattern remains fixed. Figure 4 shows the shifting of the slices with respect to the illuminated egg. Like geometric perturbations, frequency perturbations result in a shift in the baseband frequency for any individual scatter. From Equation 7 it is readily apparent that the baseband frequency shift is equal to the error in the command doppler transmitted. Figure 5 shows a plot of the error resulting from command doppler errors. Again, there is a smooth one-to-one relationship between error in $X$ and $\Delta f$.

An important observation is that the curves in Figure 5 are very similar to the curves shown in Figure 3. While the mechanisms for error in each case are very different, the error curves are similar because the gain pattern has shifted relative to the position of the slices and the amount of shift is similar in both cases. In fact, the purpose of plotting the error versus $\Delta f$ is that $\Delta f$ gives a quantitative measurement of the relative position of the gain pattern with respect to the slices. Small deviations of $\Delta f$ from either or both sources will cause relatively similar errors in $X$. This enables the use of the $\Delta f$ correction (Eq. 10) for both geometric and frequency perturbations with only one set of coefficients, $A, B, C$ and $D$. 

Figure 3: Errors in each of the 12 slices due to geometric perturbations plotted versus the resulting baseband frequency shift. Each scatter plot traces a smooth curve that can be parameterized as function of $\Delta f$. 

![Diagram showing errors in each of the 12 slices due to geometric perturbations plotted versus the resulting baseband frequency shift.](image-url)
Figure 5: Errors in $X$ for each of the 12 slices due to command doppler errors. The error is plotted against the resulting baseband frequency shift. Each scatter plot traces a smooth curve that can be parameterized as function of $\Delta f$.

X Correction Scheme

Using the $\Delta f$ method the nominal value of $X$ for any slice can be corrected with a knowledge of the baseband frequency shift. The frequency shift is obtained by calculating the baseband that would be received from some nominal point and the baseband that is actually received from the same point with the perturbations taken into account. Once $\Delta f$ has been calculated the nominal $X$ value can be obtained from a table and corrected using the pre-computed coefficients also obtained from the table.

While $\Delta f$ works very well for the simulated cases, there are potential problems that must be understood. The first potential problem is that the parameterized curves for the geometric and frequency perturbations are very similar around $\Delta f = 0$ but begin to diverge farther out in frequency. The measurement of $\Delta f$ only gives the shift of the slices with respect to the antenna pattern and does not relate any information about how these actually shift. In the case of the frequency perturbations, shifts cause the range to each slice to change, while for geometric perturbations the range to the slices are unaffected.

This can have second order effects on the calculation of $X$ and causes the correction plots to diverge for large frequency shifts.

Another source of error is the possibility that both geometric and frequency perturbations occur in such a way that there is no net frequency shift. For example, consider the case where an attitude error causes the antenna pattern to shift out while quantization error in the doppler tracking algorithm causes the slice boundaries to shift the same distance. Insipite of the change in the location and range of the illuminated spot on the surface, $\Delta f = 0$ because the relative position of the slices and gain pattern has not changed. This will add some error to the measurement of $\sigma^s$ because no correction is made even though the antenna is not at the nominal pointing. Fortunately, the magnitude of this error has been found to be quite small.

As a final test, a simulation was run in which several perturbations were applied to the spacecraft and the corrected values of $X$ were compared with the true values of $X$. The results of this simulation are given in Figure 6. From the results of the simulation it is evident that the $\Delta f$ correction algorithm
will keep errors in $\sigma^o$ within the 0.05 dB mission requirement.

**SUMMARY**

As a result of the extremely high data rates of future scatterometers it is computationally impossible to integrate around each data point in order to obtain an accurate estimate of $\sigma^o$. For this reason, $\sigma^o$ will be calculated using a table of pre-computed $X$ values. However, the table cannot compensate for deviations in the nominal orbit and attitude and a correction scheme must be devised to correct the nominal $X$ as the scatterometer is perturbed. The $\Delta f$ correction method uses the baseband frequency shift to quantify the perturbations and applies a third order polynomial correction based upon $\Delta f$. This combination allows $\sigma^o$ to be retrieved at high data rates and with the required accuracy.

The $X$ table and correction algorithm have been developed at BYU for use by NASA/JPL during the Quikscat mission scheduled for launch in November 1998.

**REFERENCES**


