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The Effects of Financing Unfunded Social Security with Consumption Taxation when Consumers are Shortsighted

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Abstract

Using a representative-agent life-cycle model with consumer shortsightedness, I study an unfunded social security program financed via consumption taxation. Compared to financing an unfunded program with payroll taxation, I find that there is only a slight increase in well-being across planning horizons that is generated by a program with a consumption tax.

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> We suggest that both our data and the available time-series evidence are consistent with Milton Friedman's view that people save to smooth consumption over several years but, because of liquidity constraints, caution, or shortsightedness do not seek to smooth consumption over longer horizons. . . . Indeed, Milton Friedman explicitly rejected the idea that consumers had horizons as long as a lifetime in discussing the permanent income hypothesis (Carroll and Summers 1991 pp. 307, 355).

1 Introduction

The unfunded social security program in the United States is the largest such program in the world. It has been justified by many as: insurance against disability or premature death; redistribution from the wealthy elderly to the poor elderly; as a replacement for failed annuity markets; to compensate for under saving behavior. The most common justification used is the under saving for retirement (Kotlikoff 1982; Feldstein 1985; Docquier 2002; İmrohoroğlu et al. 2003; and many others). Feldstein (1985) stated more specifically, "the principal rationale for such mandatory [social security] programs is that some individuals lack the foresight to save for their retirement years" (p.303).

It is well known that the well-being of a life-cycle permanent-income consumer is reduced by the presence of a social security program, if the program has a negative net present value. This is due to the fact that the lifetime budget constraint is decreased. The existence of myopic agents in a model can result in a social security program that has an optimal tax rate greater than zero (Docquier 2002; Caliendo and Gahramanov 2009; Findley and Caliendo 2009). The tax rate in such studies is usually set to maximize a paternalistic social welfare function following behavioral economic practices (Akerlof 2002; Kanbur et al. 2006). The first study that uses a mixed economy of life-cyclers and myopic agents to estimate an optimal social security program was Feldstein (1985). Feldstein used an agent who exponentially discounts utility and one who does not discount in order to show that there is an opportunity for a welfare improvement. İmrohoroğlu et al. (2003) model quasi-hyperbolic agents following Laibson (1998) and find that in a partial-equilibrium and general-equilibrium setting it is not welfare-improving to have a social security program. Kumru and Thanopoulos (2008) models consumers with temptation preferences and show that some of the welfare loss is mitigated by a social security program. The payroll-tax financed program can have an effect on labor supply, causing less labor to be supplied as demonstrated in OLG general equilibrium models (Auerbach and Kotlikoff 1987; and Hugget and Ventura 1999).

> As an alternative to some of our proposals for benefit reductions or revenue increases, policy makers could dedicate revenue from another specific source to Social Security (Diamond and Orszag 2005, p. 5).

As foreshadowed in the quotation above, the idea of moving towards a consumption tax for general government expenditures has received some consideration by policy makers. Major changes to the U.S. social security program have not been enacted since it is viewed as politically controversial by many. To date, the majority of the consumption-tax literature is focused along two lines: using detailed taxpayer information to estimate liabilities under current and proposed regimes; and the study of theoretical economies and the effects of policy changes inside this framework. Two studies in which a flat tax was considered as a replacement for the current U.S. tax regime were conducted by Feenberg, Mitrusi and Poterba (1997) and Gentry and Hubbard (1997). There is no publicly available data set containing all necessary information, hence both studies use a combination of different public sources. Feenberg, Mitrusi and Poterba (1997) use information on income, tax liabilities and consumption while Gentry and Hubbard (1997) us data on use data on household portfolio choice. Feenberg, Mitrusi and Poterba (1997) find those lower income households bear a disproportionate share of the tax burden in comparison to the high income households. This is in contrast to the findings of Gentry and Hubbard (1997) who find that the tax liabilities could be progressive in nature, the more affluent in the economy bear a larger percentage of the tax burden. Important assumptions underlying these computational studies are that aggregate quantities and factor pricing remains constant under both regimes being considered. The line of literature which uses calibrated general-equilibrium models to examine the differences of tax liabilities under tax regime changes also exhibit mixed results with respect to the progressivity of tax burdens. Ventrua (1999) uses an OLG model with age and labor efficiency shocks to simulate heterogeneity, and he finds that the change from the current U.S. tax regime to a flat tax results in increased concentrations of wealth. Altig et al. (2001) compute the transition dynamics in moving from the current U.S. tax code to a flat tax and finds that the poor are worse off during the transition and in the new steady state as compared to before the transition. Correia (2010) uses an infinite-horizon model with heterogeneity in initial wealth and income levels. She studies the refinancing of government expenditures from an income-tax regime similar to that in the U.S. to one of a flat consumption tax. She finds that well-being increases across all initial wealth levels. For an in depth discussion of consumption taxation and optimal taxation levels, see Coleman (2000).

The idea of financing the social security program from alternative sources is not new. Gahramanov and Tang (2013) use a general-equilibrium OLG model with endogenous labor decisions and mortality risk to investigate an optimal tax policy over capital taxes, payroll taxes, income taxes, and consumption taxes. They find that the optimal policy to maintain benefits at current levels is to eliminate the payroll tax and increase a consumption tax above baseline. This leads to welfare gains across the economy. They then investigate the welfare dynamics during the transitional period, where they find that the newly retired and nearly retired face the largest welfare cost of this restructuring. They advocate for an additional transfer payment to be made during the transition to maintain utility levels for these individuals.

Findley and Caliendo (2009) study the short-term planning model of Caliendo and Aaland (2007), supplemented with an unfunded social security program that is financed with payroll taxation. They demonstrate that the program can be welfare improving for some planning horizons in general equilibrium. An open question remains as to whether or not a payroll tax is the best instrument to finance an unfunded program.

Indeed, I revisit the ability for social security to provide adequate retirement resources in the short-horizon framework used by Findley and Caliendo (2009). My contribution is the addition of an unfunded social security program financed with taxation on consumption. I find that a payroll-tax financed program can be replaced with a consumption-tax financed program. Such a move generates welfare gains in partial equilibrium, although the welfare gains are small.

2 Model

I model a representative individual who optimizes consumption and saving behavior over a short planning horizon. Time is continuous and indexed by t. The individual enters the workforce at $t = 0$. The individual retires at $t = T$, and dies at age $t = \bar{T}$. During the working period $t \in [0, T]$ the individual receives wages at rate w, and during the retirement period $t \in \left[T, \bar{T}\right]$ the individual receives social security benefits $b = Rw\nu_w + C\nu_c / \left(\bar{T} - T\right)$. The individual supplies one unit of labor inelastically while working. $R = \dfrac{T}{\bar{T} - T}$ is the worker to retiree ratio. ν_w is the payroll tax rate. ν_c is the consumption tax rate. C is aggregate consumption in the economy. Consumption at each instant is $c(t)$ and is the control variable. Any income not consumed at each instant is placed in the individual's asset account, $k(t)$, which grows at rate r. There are no borrowing constraints placed on the individual, and $k(0) = k(\bar{T}) = 0$ is assumed. The planning horizon length x is the amount of time over which an individual optimizes. I impose the restriction $x \leq \bar{T} - T$ for ease of modeling, as is customary in this model. It allows for a simple compartmentalization of the life cycle:

- Phase 1 $[0, T - x]$
- Phase 2 $[T - x, T]$
- Phase 3 $\left[T, \bar{T} - x\right]$
- Phase 4 $\left[\bar{T} - x, \bar{T}\right]$

Phase 1 is the period of the life cycle during which the individual is in the workforce and does not foresee retirement. Phase 2 is that period of the life cycle when the individual is still in the workforce, but can see the future date of retirement. Phase 3 is after the individual is retired from the workforce, but does not foresee the date of death. Phase 4 is that part of retirement when the individual can see the date of death.

Inside the short planning horizon model, the individual's behavior is time-inconsistent in Phases 1-3. This is due to the sliding planning window which moves through time with the individual. I model a naive individual, meaning that the individual does not anticipate his time-inconsistant behavior. Therefore, the individual's actual behavior is the envelope of the initial moments solved for during the optimization. The derivations of the following solutions can be found in Appendix A.

2.1 Phase 1

At any $t_0 \in [0, T-x]$ the individual solves

$$\max_{c(t)} : \int_{t_0}^{t_0+x} e^{-\rho(t-t_0)} \frac{c(t)^{1-\phi}}{1-\phi} dt \tag{1}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + w(1-\nu_w) - (1+\nu_c)c(t) \tag{2}$$

$$k(t_0) \;\; given \tag{3}$$

$$k(t_0+x) = 0, \tag{4}$$

where ρ is the personal discount rate and ϕ is the inverse elasticity of intertemporal substitution (IEIS). The solution to (1)-(4) is the optimal planned path from the perspective of $t_0 \in [0, T-x]$,

$$\hat{c}(t) = e^{gt} \left[\frac{k(t_0) e^{-rt_0} + \int_{t_0}^{t_0+x} w(1-\nu_w) e^{-rj} dj}{(1+\nu_c) \int_{t_0}^{t_0+x} e^{(g-r)j} dj} \right], \tag{5}$$

for $t \in [t_0, t_0+x]$ where $g = \dfrac{r-\rho}{\phi}$.

Following Caliendo and Aaland (2007), the actual consumption profile can be derived by replacing t_0 with t

$$c(t) = e^{gt} \left[\frac{k(t) e^{-rt} + \dfrac{w(1-\nu_w)}{r} \left(e^{-rt} - e^{-rt} e^{-rx} \right)}{\dfrac{1+\nu_c}{g-r} \left(e^{(g-r)(t+x)} - e^{(g-r)t} \right)} \right], \tag{6}$$

for $t \in [0, T-x]$. This can be more simply expressed as

$$c(t) = k(t) z_1 + w z_2, \tag{7}$$

where

$$z_1 \equiv \frac{(g-r)}{(1+\nu_c)\left(e^{(g-r)x}-1\right)} \tag{8}$$

$$z_2 \equiv \frac{(1-\nu_w)(g-r)(1-e^{-rx})}{(1+\nu_c)\left(e^{(g-r)x}-1\right)}, \tag{9}$$

and the asset account follows the path

$$k(t) = \left(e^{\Omega t}-1\right)\frac{w\left[(1-\nu_w)-(1+\nu_c)\,z_2\right]}{\Omega}, \tag{10}$$

where $\Omega = [r-(1+\nu_c)\,z_1]$.

2.2 Phase 2

At any $t_0 \in [T-x, T]$ the individual solves

$$\max_{c(t)} : \int_{t_0}^{t_0+x} e^{-\rho(t-t_0)}\frac{c(t)^{1-\phi}}{1-\phi}dt \tag{11}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + w(1-\nu_w) - (1+\nu_c)\,c(t) \tag{12}$$

$$\frac{dk(t)}{dt} = rk(t) + b - (1+\nu_c)\,c(t) \tag{13}$$

$$k(t_0) \;\; given \tag{14}$$

$$k(t_0+x) = 0. \tag{15}$$

The planned consumption path is the solution to equations (11)-(15),

$$\hat{c}(t) = \frac{k(t_0)\,e^{-rt_0} + \int_{t_0}^{T} w(1-\nu_w)\,e^{-rj}dj + \int_{T}^{t_0+x} be^{-rj}dj}{(1+\nu_c)\int_{t_0}^{t_0+x} e^{gt}e^{-rj}dj}e^{gt}, \tag{16}$$

for $t \in [t_0, t_0+x]$. The actual path is

$$c(t) = \frac{k(t)\,e^{-rt} + \int_{t}^{T} w(1-\nu_w)\,e^{-rj}dj + \int_{T}^{t+x} be^{-rj}dj}{(1+\nu_c)\int_{t}^{t+x} e^{(g-r)j}dj}e^{gt} \tag{17}$$

for $t \in [T-x, T]$. Using z_1 from above, it can be rewritten as

$$c(t) = k(t)z_1 + \frac{w(1-\nu_w)}{r} z_1 e^{rt}\left(e^{-rt} - e^{-rT}\right) + \frac{b}{r} z_1 e^{rt}\left(e^{-rT} - e^{-r(t+x)}\right) \tag{18}$$

where

$$\begin{aligned} k(t) \quad = \quad & e^{\Omega t}\Bigg(e^{-\Omega(T-x)}k(T-x) + \frac{w(1-\nu_w)}{\Omega}\left(e^{-\Omega(T-x)} - e^{-\Omega t}\right) \\ & + \frac{w(1-\nu_w)(1+\nu_c) z_1}{r}\left[\frac{e^{-rT}}{r-\Omega}\left(e^{(r-\Omega)t} - e^{(r-\Omega)(T-x)} - \frac{1}{\Omega}\left[e^{-\Omega(T-x)} - e^{-\Omega t}\right]\right)\right] \\ & + \frac{b(1+\nu_c) z_1}{r}\left[\frac{e^{-rT}}{r-\Omega}\left(e^{(r-\Omega)(T-x)} - e^{(r-\Omega)t}\right) + \frac{e^{-rx}}{\Omega}\left(e^{-\Omega(T-x)} - e^{-\Omega t}\right)\right]\Bigg). \end{aligned} \tag{19}$$

2.3 Phase 3

At any $t_0 \in \left[T, \bar{T} - x\right]$ the individual solves

$$\max_{c(t)} : \int_{t_0}^{t_0+x} e^{-\rho(t-t_0)} \frac{c(t)^{1-\phi}}{1-\phi} dt \tag{20}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + b - (1+\nu_c)c(t) \tag{21}$$

$$k(t_0) \;\; given \tag{22}$$

$$k(t_0 + x) = 0. \tag{23}$$

The solution is the planned consumption path for $t \in [t_0, t_0 + x]$,

$$\hat{c}(t) = e^{gt}\left[\frac{k(t_0)e^{-rt_0} + \int_{t_0}^{t_0+x} be^{-rj}dj}{(1+\nu_c)\int_{t_0}^{t_0+x} e^{(g-r)j}dj}\right]. \tag{24}$$

The actual path for $t \in \left[T, \bar{T} - x\right]$ is

$$c(t) = e^{gt}\left[\frac{k(t)e^{-rt} + \frac{b}{r}\left(e^{-rt} - e^{-r(t+x)}\right)}{\frac{1+\nu_c}{g-r}\left(e^{(g-r)t} - e^{(g-r)(t+x)}\right)}\right], \tag{25}$$

which is the envelope of initial planned consumption allocations given

$$k(t) = k(T)e^{-\Omega(T-t)} + \frac{b\nu_c z_3}{\Omega}\left(1 - e^{-\Omega(T-t)}\right). \tag{26}$$

2.4 Phase 4

Since the individual can see the date of death in this phase, behavior is time-consistent. The planned consumption path from the perspective of $t_0 = \bar{T} - x$ will be the actual consumption path,

$$c(t) = e^{gt} z_4, \tag{27}$$

where

$$z_4 = \frac{g-r}{1+\nu_c}\left[\frac{k\left(\overline{T}-x\right)e^{rx} - \dfrac{b\left(1-e^{rx}\right)}{r}}{e^{g\overline{T}}\left(1-e^{-x(g-r)}\right)}\right]. \tag{28}$$

This characterizes the asset path during Phase 4 with

$$\frac{dk\left(t\right)}{dt} = rk\left(t\right) + b - \left(1+\nu_c\right)c\left(t\right) \tag{29}$$

and $k\left(\overline{T}-x\right)$ known.

2.5 Social security in the model

I will examine two options for social security financing in this model. A tax on consumption, ν_c, will be levied against all consumption in the model. I will also examine a payroll tax, ν_w, as done in Findley and Caliendo (2009). I will compare the two alternate tax regimes. The unfunded program has a balanced budget and the individual does not take into account the effects that his consumption level has on the level of benefits, such that

$$b = \frac{\nu_c\left[\int_0^{T-x} c(t)dt + \int_{T-x}^{T} c(t)dt + \int_{T}^{\bar{T}-x} c(t)dt + \int_{\bar{T}-x}^{\bar{T}} c(t)dt\right]}{\bar{T}-T} + Rw\nu_w \tag{30}$$

for $t \in [T, \bar{T}]$.

The use of a consumption tax to finance benefits creates an implicit function, since $c(t)$

is a function of b while b is a function of $c(t)$. Yet, it is possible to numerically approximate the level of benefits. Due to the inelasticity of labor supply in this model the payroll-tax portion of benefits is easily demonstrated analytically.

3 Simulation and numerical exercises

3.1 Baseline model parameters

The baseline parameters are summarized in Table(1). I set $T = 40$ and $\bar{T} = 55$ which represents an individual who enters the work force at age 25, retires at age 65, and dies at age 80. I set the real rate of return, r, to 0.035. The worker to retiree ratio is approximately 2.667. I set $r = \rho$ following convention. I set $\phi = 1$, making utility logarithmic.

3.2 Optimal tax rates

I will allow the model to determine the optimal payroll tax rate, ν_w^*, and the optimal consumption tax rate, ν_c^*, for each planning horizon length, x. The optimal rate for both programs is the rate that paternalistically maximizes lifetime utility for the individual,

$$\nu_c^* \equiv \underset{\nu_c \in [0,1]; \nu_w = 0}{\arg\max} \quad \begin{aligned} &\left\{ \int_0^{T-x} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt + \int_{T-x}^{T} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt \right. \\ &\left. + \int_T^{\bar{T}-x} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt \right\} \end{aligned} \tag{31}$$

$$\nu_w^* \equiv \underset{\nu_w \in [0,1]; \nu_c = 0}{\arg\max} \quad \begin{aligned} &\left\{ \int_0^{T-x} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt + \int_{T-x}^{T} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt \right. \\ &\left. + \int_T^{\bar{T}-x} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\mu t} \frac{c(t)^{1-\phi}}{1-\phi} dt \right\}, \end{aligned} \tag{32}$$

where μ is the social discount rate.

3.3 Individual life-cycle consumption profiles

Simulated consumption profiles using the baseline parameters in Table(1) can be seen in Figure(1) for the case of no transfer program $\nu_c = 0$ and for the case of a program with $\nu_c =$ 0.10 . The individual consumes less during Phase 1 and part of Phase 2, but has increased consumption during part of Phase 2 and all of Phases 3 and 4. The consumption tax does not distort the asset account during Phase 1 as seen in Figure(2). The tax proportionally decreases consumption during Phase 1. This non-distortion of the asset account holds for a wide array of parameters as shown in Figures(2,4,6). During Phase 2-4 the tax rate does change saving rates and consumption levels. It is important to note that the asset account with a program in place is always less than or equal to the asset account when a program is not present. This is similar to the result for the LCPI consumer in which the presence of an unfunded program causes the individual to save less for retirement.

3.4 Welfare analysis: social security vs. no program

Here, I study an unfunded program financed by a consumption tax compared to the counterfactual of no program at all. In doing this, I define a compensating variation (CV) as the percentage increase in period consumption that is needed to equalize lifetime utility without a program to the lifetime utility with an optimally parameterized social security program. In Table (2) I display the compensating variation. For all planning horizons which have a non-zero optimal tax, an unfunded program raises well-being. I also report in Table (3) that an optimally parametrized payroll-tax financed program is welfare improving, compared to no program at all.

3.5 Welfare analysis: consumption-tax financing vs. payroll-tax financing

The welfare metric that I use is that of a paternalistic social planner, where the social discount rate of μ evaluates utils over the entire life span, even though the individual is optimizing over a short-horizon. This is consistent with the majority of the behavioral economics literature.

I now define a uniquely different compensating variation, Δ, to measure the percentage increase in $c(t)$ under a particular tax regime in order to approximate the value of participating in an optimally parameterized social security program. With $\phi = 1$ the utility function becomes logarithmic, and Δ solves the following equation,

$$\begin{aligned}
& \Big\{\textstyle\int_0^{T-x} e^{-\mu t} \ln\left[(1+\Delta)\, c_{\nu_w}(t)\right] dt + \int_{T-x}^{T} e^{-\mu t} \ln\left[(1+\Delta)\, c_{\nu_w}(t)\right] dt \\
& \quad + \textstyle\int_T^{\bar{T}-x} e^{-\mu t} \ln\left[(1+\Delta)\, c_{\nu_w}(t)\right] dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\mu t} \ln\left[(1+\Delta)\, c_{\nu_w}(t)\right] dt\Big\} \\
= \quad & \Big\{\textstyle\int_0^{T-x} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt + \int_{T-x}^{T} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt \\
& \quad + \textstyle\int_T^{\bar{T}-x} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt\Big\}.
\end{aligned} \tag{33}$$

Solving for Δ gives

$$\Delta = \exp\left[\frac{U_c - U_w}{\int_0^{\bar{T}} e^{-\mu t} dt}\right] - 1 \tag{34}$$

where

$$\begin{aligned}
U_w \quad = \quad & \left\{\int_0^{T-x} e^{-\mu t} \ln\left[c_{\nu_w}(t)\right] dt + \int_{T-x}^{T} e^{-\mu t} \ln\left[c_{\nu_w}(t)\right] dt \right. \\
& \left. + \int_T^{\bar{T}-x} e^{-\mu t} \ln\left[c_{\nu_w}(t)\right] dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\mu t} \ln\left[c_{\nu_w}(t)\right] dt \right\}
\end{aligned} \tag{35}$$

$$\begin{aligned}
U_c \quad = \quad & \left\{\int_0^{T-x} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt + \int_{T-x}^{T} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt \right. \\
& \left. + \int_T^{\bar{T}-x} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt + \int_{\bar{T}-x}^{\bar{T}} e^{-\mu t} \ln\left[c_{\nu_c}(t)\right] dt \right\}.
\end{aligned} \tag{36}$$

The optimal tax rates are reported in Tables(4-9) for a range of parameter values.

I compare the utility of two identical individuals under the different tax regimes using the Δ metric. As reported in Tables(4-9), I find that the consumption-tax financed program has a higher total welfare, but only marginally. When using the Ramsey criteria for measuring welfare (such that the social planner does not discount utility, $\mu = 0$), I find that there are large gains in well-being from a consumption-tax financed program compared to a payroll-tax financed program. In this partial-equilibrium model the differences between the consumption and saving profiles are relatively small, with the paths almost laying on top of each other. But the cumulative utility gains from the consumption-tax financed program are sizable.

3.6 Robustness check of computational code

To check for potential computational errors in the simulation environment, I calculate the present value of taxes collected over a given planning horizon. If the present value of taxes is equal across tax regimes, then the behavior should be the same regardless of which tax regime is in place. I analytically solve for when each of the two regimes have the same present value of tax revenues for a given planning horizon.

The present value of taxes in a payroll-tax financed regime is

$$I_w = \int_{t_0}^{t_0+x} e^{-rt} w \nu_w dt, \tag{37}$$

and the present value of taxes in a consumption-tax financed program is

$$I_c = \int_{t_0}^{t_0+x} e^{-rt} \nu_c \hat{c}(t) dt, \tag{38}$$

where $\hat{c}(t)$ is the planned consumption path from the perspective of the planning instant t_0. Setting (37) equal to (38) yields

$$\nu_w = \frac{\nu_c}{1+\nu_c} \tag{39}$$

where the derivation is found in Appendix B. I use this equation to estimate the difference in the present values of the tax regimes within the simulations environment. I found the two calculations to be almost identical.

4 Summary and possible extensions for future work

The presence of an unfunded security program can improve well-being. A consumption-tax financed program leads to slightly higher levels of well-being as compared to a program using payroll taxation. Due to the smoothing of consumption over the life-cycle, there is an increase in lifetime utility. There is an opportunity to extend this research by allowing factor prices to adjust given behavior in the model. A general-equilibrium setting would likely lean to different quantitative results. This merits further investigation. Another interesting extension could be heterogeneity in the length of planning horizons across different individuals in the model population.

Appendix A: derivations of consumption and savings profiles

Phase 1 $[0, T-x]$

The individual solves

$$\max_{c(t)} : \int_{t_0}^{t_0+x} e^{-\rho(t-t_0)} \frac{c(t)^{1-\phi}}{1-\phi} dt \tag{40}$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + w(1-\nu_w) - (1+\nu_c)c(t) \tag{41}$$

$$k(t_0) \ \ given \tag{42}$$

$$k(t_0 + x) = 0. \tag{43}$$

Using the Maximum Principle for a one-stage problem results in the following Hamiltonian equation and optimality conditions,

$$H = e^{-\rho(t-t_0)} \frac{c(t)^{1-\phi}}{1-\phi} + \lambda(t)(rk(t) + w(1-\nu_w) - (1+\nu_c)c(t)) \tag{44}$$

$$\frac{\partial H}{\partial c} = e^{-\rho(t-t_0)} c(t)^{-\phi} - \lambda(t)(1+\nu_c) = 0 \tag{45}$$

$$\frac{\partial H}{\partial k} = r\lambda(t) = -\frac{d\lambda}{dt} \tag{46}$$

$$\frac{\partial H}{\partial \lambda} = rk(t) + w(1-\nu_w) - (1+\nu_c)c(t) = \frac{dk(t)}{dt}. \tag{47}$$

Solving the maximum condition for $c(t)$

$$c(t) = \left(e^{-\rho(t-t_0)} \frac{1}{\lambda(t)(1+\nu_c)}\right)^{\frac{1}{\phi}}. \tag{48}$$

Solving the costate equation

$$\frac{d\lambda}{dt} = -r\lambda(t) \rightarrow \lambda(t) = ae^{-rt}. \tag{49}$$

The constant of integration can be definitized such that

$$\lambda\left(t_0\right)=ae^{-rt_0} \tag{50}$$

$$a=\lambda\left(t_0\right)e^{rt_0} \tag{51}$$

$$\lambda\left(t\right)=\lambda\left(t_0\right)e^{r(t_0-t)}. \tag{52}$$

Substituting equation (52) into (48) gives

$$c(t)=\left(e^{-\rho(t-t_0)}\frac{1}{\lambda\left(t_0\right)e^{r(t_0-t)}\left(1+\nu_c\right)}\right)^{\frac{1}{\phi}} \tag{53}$$

$$=e^{gt}\left(\frac{1}{\lambda\left(t_0\right)\left(1+\nu_c\right)}e^{(r-\rho)t_0}\right)^{\frac{1}{\phi}} \tag{54}$$

where $g=\dfrac{r-\rho}{\phi}$. This can be simplified as

$$c(t)=e^{gt}A \tag{55}$$

where $A=\left(\dfrac{1}{\lambda\left(t_0\right)\left(1+\nu_c\right)}e^{(r-\rho)t_0}\right)^{\frac{1}{\phi}}$ is a transformation of the unknown constant. Solving the state equation yields

$$k\left(t\right)=e^{rt}\left[q+\int^{t}\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)c\left(j\right)\right)e^{-rj}dj\right]. \tag{56}$$

Using the boundary condition, $k\left(t_0\right)$ *given*, pins down the constant of integration

$$k\left(t_0\right)=e^{rt_0}\left[q+\int^{t_0}\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)c\left(j\right)\right)e^{-rj}dj\right] \tag{57}$$

$$k\left(t_0\right)e^{-rt_0}=q+\int^{t_0}\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)c\left(j\right)\right)e^{-rj}dj \tag{58}$$

$$q=k\left(t_0\right)e^{-rt_0}-\int^{t_0}\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)c\left(j\right)\right)e^{-rj}dj. \tag{59}$$

The particular solution is

$$k(t) = e^{rt}\left[k(t_0)\,e^{-rt_0} + \int_{t_0}^{t} \left(w\,(1-\nu_w) - (1+\nu_c)\,c(j)\right) e^{-rj}dj\right]. \tag{60}$$

Using the other boundary condition, $k(t_0 + x) = 0$,

$$e^{rt_0+x}\left[k(t_0)\,e^{-rt_0} + \int_{t_0}^{t_0+x} \left(w\,(1-\nu_w) - (1+\nu_c)\,c(j)\right) e^{-rj}dj\right] = 0 \tag{61}$$

$$k(t_0)\,e^{-rt_0} + \int_{t_0}^{t_0+x} \left(w\,(1-\nu_w) - (1+\nu_c)\,c(j)\right) e^{-rj}dj = 0 \tag{62}$$

$$k(t_0)\,e^{-rt_0} + \int_{t_0}^{t_0+x} w\,(1-\nu_w)\,e^{-rj}dj = \int_{t_0}^{t_0+x} (1+\nu_c)\,c(j)\,e^{-rj}dj. \tag{63}$$

Substituting equation (55) into (63) gives

$$k(t_0)\,e^{-rt_0} + \int_{t_0}^{t_0+x} w\,(1-\nu_w)\,e^{-rj}dj = \int_{t_0}^{t_0+x} (1+\nu_c)\,e^{gj}Ae^{-rj}dj \tag{64}$$

$$(1+\nu_c)\,A\int_{t_0}^{t_0+x} e^{(g-r)j}dj = k(t_0)\,e^{-rt_0} + \int_{t_0}^{t_0+x} w\,(1-\nu_w)\,e^{-rj}dj, \tag{65}$$

which allows us to solve for the transformation of the unknown constant

$$A = \frac{k(t_0)\,e^{-rt_0} + \int_{t_0}^{t_0+x} w\,(1-\nu_w)\,e^{-rj}dj}{(1+\nu_c)\int_{t_0}^{t_0+x} e^{(g-r)j}dj}. \tag{66}$$

Therefore, planned consumption is

$$\hat{c}(t) = e^{gt}\left[\frac{k(t_0)\,e^{-rt_0} + \int_{t_0}^{t_0+x} w(j)\,(1-\nu_w)\,e^{-rj}dj}{(1+\nu_c)\int_{t_0}^{t_0+x} e^{(g-r)j}dj}\right] \tag{67}$$

in closed-form. Since actual behavior will be decided from reoptimization at every instant, the actual paths can be mapped by replacing t_0 with t in (67). This gives the actual consumption path

$$c(t) = e^{gt}\left[\frac{k(t)\,e^{-rt} + \int_{t}^{t+x} w\,(1-\nu_w)\,e^{-rj}dj}{(1+\nu_c)\int_{t}^{t+x} e^{(g-r)j}dj}\right] \tag{68}$$

$$= e^{gt}\left[\frac{k\left(t\right)e^{-rt}+\frac{w\left(1-\nu_w\right)}{r}\left(e^{-rt}-e^{-rt}e^{-rx}\right)}{\frac{\left(1+\nu_c\right)}{\left(g-r\right)}\left(e^{(g-r)t}e^{(g-r)x}-e^{(g-r)t}\right)}\right] \tag{69}$$

$$= k(t)z_1 + wz_2 \tag{70}$$

with algebraic simplification, where

$$z_1 = \frac{\left(g-r\right)}{\left(1+\nu_c\right)\left(e^{(g-r)x}-1\right)} \tag{71}$$

$$z_2 = \frac{\left(1-\nu_w\right)\left(g-r\right)\left(1-e^{-rx}\right)}{\left(1+\nu_c\right)\left(e^{(g-r)x}-1\right)}. \tag{72}$$

Substituting equation (70) into the law of motion that governs the actual evolution of the asset account,

$$\frac{dk\left(t\right)}{dt} = rk\left(t\right)+w\left(1-\nu_w\right)-\left(1+\nu_c\right)\left(k(t)z_1+wz_2\right) \tag{73}$$

$$= k\left(t\right)\left(r-z_1\left(1+\nu_c\right)\right)+w\left(1-\nu_w\right)-\left(1+\nu_c\right)wz_2. \tag{74}$$

Solving this differential equation gives

$$k(t) = e^{(r-z_1(1+\nu_c))t}\left[q+\int^t\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)wz_2\right)e^{-(r-z_1(1+\nu_c))j}dj\right]. \tag{75}$$

With the initial condition, $k(0) = 0$, and with $\Omega = r - \left(1+\nu_c\right)z_1$, the constant can be identified

$$0 = e^{\Omega 0}\left[q+\int^0\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)wz_2\right)e^{-\Omega j}dj\right] \tag{76}$$

$$q = -\int^0\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)wz_2\right)e^{-\Omega j}dj \tag{77}$$

which provides a closed-form solution for the asset path during Phase 1

$$k(t) = e^{\Omega t}\int_0^t\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)wz_2\right)e^{-\Omega j}dj \tag{78}$$

$$= \left(e^{\Omega t}-1\right)\frac{w\left(\left(1-\nu_w\right)-\left(1+\nu_c\right)z_2\right)}{\Omega}. \tag{79}$$

Phase 2 $[T-x,T]$

The individual can see both work income and the social security benefits flow, but he is still working. The individual solves

$$\max_{c(t)}:\int_{t_0}^{t_0+x} e^{-\rho(t-t_0)}\frac{c(t)^{1-\phi}}{1-\phi}dt \tag{80}$$

subject to

$$\frac{dk(t)}{dt}=rk(t)+w(1-\nu_w)-(1+\nu_c)c(t) \tag{81}$$

for $t=[t_0,T]$ and

$$\frac{dk(t)}{dt}=rk(t)+b-(1+\nu_c)c(t) \tag{82}$$

for $t=[T,t_0+x]$, where

$$k(t_0+x)=0 \tag{83}$$

$$k(t_0)\ \ given. \tag{84}$$

Using the Maximum Principle for two-stage problems results in the following Hamiltonians and optimality conditions,

$$H_1=e^{-\rho(t-t_0)}\frac{c(t)^{1-\phi}}{1-\phi}+\lambda_1(t)\left(rk(t)+w(1-\nu_w)-(1+\nu_c)c(t)\right) \tag{85}$$

$$H_2=e^{-\rho(t-t_0)}\frac{c(t)^{1-\phi}}{1-\phi}+\lambda_2(t)\left(rk(t)+b-(1+\nu_c)c(t)\right) \tag{86}$$

$$\frac{\partial H_1}{\partial c}=e^{-\rho(t-t_0)}c(t)^{-\phi}-\lambda_1(t)(1+\nu_c)=0 \tag{87}$$

$$\frac{\partial H_1}{\partial k}=r\lambda_1(t)=-\frac{d\lambda_1}{dt} \tag{88}$$

$$\frac{\partial H_1}{\partial \lambda_1}=rk(t)+w(1-\nu_w)-(1+\nu_c)c(t)=\frac{dk(t)}{dt} \tag{89}$$

$$\frac{\partial H_2}{\partial c}=e^{-\rho(t-t_0)}c(t)^{-\phi}-\lambda_2(t)(1+\nu_c)=0 \tag{90}$$

$$\frac{\partial H_2}{\partial k}=r\lambda_2(t)=-\frac{d\lambda_2}{dt} \tag{91}$$

$$\frac{\partial H_2}{\partial \lambda_2} = rk\left(t\right) + b - \left(1+\nu_c\right)c\left(t\right) = \frac{dk\left(t\right)}{dt}. \tag{92}$$

The two multipliers are defined as λ_1 for $t = [t_0, T]$ and λ_2 for $t = [T, t_0 + x]$ and obey the costate equations, (88) and (91), rewritten as

$$\frac{d\lambda_1}{dt} = -r\lambda_1\left(t\right) \tag{93}$$

for $t = [t_0, T]$

$$\frac{d\lambda_2}{dt} = -r\lambda_2\left(t\right) \tag{94}$$

for $t = [T, t_0 + x]$. Two-stage problems require a condition,

$$\lambda_1\left(T\right) = \lambda_2\left(T\right), \tag{95}$$

which links the multipliers at the switch point. Solving equations (93) and (94) while definitizing the constants of integration yields

$$\lambda_1\left(t\right) = a_1 e^{-rt} \rightarrow a_1 = \lambda_1\left(T\right)e^{rT} \tag{96}$$

$$\lambda_2\left(t\right) = a_2 e^{-rt} \rightarrow a_2 = \lambda_2\left(T\right)e^{rT} \tag{97}$$

Invoking the matching condition gives $\lambda_1\left(T\right) = \lambda_2\left(T\right) \rightarrow a_2 = a_1$, such that continuity exists across the switchpoint such that subscripts can be dropped.

$$\lambda\left(t_0\right) = ae^{-rt_0} \tag{98}$$

$$a = \lambda\left(t_0\right)e^{rt_0} \tag{99}$$

$$\lambda\left(t\right) = \lambda\left(t_0\right)e^{r(t_0-t)} \tag{100}$$

Solving the first maximum condition for $c(t)$ gives

$$c(t) = \left(e^{-\rho(t-t_0)}\frac{1}{\lambda\left(t\right)\left(1+\nu_c\right)}\right)^{\frac{1}{\phi}}. \tag{101}$$

Substituting in equation (100) gives

$$c(t) = \left(e^{-\rho(t-t_0)} \frac{1}{\lambda\left(t_0\right) e^{r(t_0-t)} \left(1+\nu_c\right)} \right)^{\frac{1}{\phi}} \tag{102}$$

$$= e^{gt} \left(\frac{1}{\lambda\left(t_0\right) \left(1+\nu_c\right)} e^{(r-\rho)t_0} \right)^{\frac{1}{\phi}} \tag{103}$$

where $g = \dfrac{r-\rho}{\phi}$. This can be condensed

$$c(t) = e^{gt} A \tag{104}$$

where $A = \left(\dfrac{1}{\lambda\left(t_0\right) \left(1+\nu_c\right)} e^{(r-\rho)t_0} \right)^{\frac{1}{\phi}}$ is a transformation of the unknown constant. Solving the second maximum condition for $c(t)$ yields

$$c(t) = \left(e^{-\rho(t-t_0)} \frac{1}{\lambda\left(t\right) \left(1+\nu_c\right)} \right)^{\frac{1}{\phi}} \tag{105}$$

and substituting equation (100) into (105) yields

$$c(t) = \left(e^{-\rho(t-t_0)} \frac{1}{\lambda\left(t_0\right) e^{r(t_0-t)} \left(1+\nu_c\right)} \right)^{\frac{1}{\phi}} \tag{106}$$

$$= e^{gt} \left(\frac{1}{\lambda\left(t_0\right) \left(1+\nu_c\right)} e^{(r-\rho)t_0} \right)^{\frac{1}{\phi}} \tag{107}$$

where $g = \dfrac{r-\rho}{\phi}$. This can also be simplified

$$c(t) = e^{gt} A \tag{108}$$

where $A = \left(\dfrac{1}{\lambda\left(t_0\right) \left(1+\nu_c\right)} e^{(r-\rho)t_0} \right)^{\frac{1}{\phi}}$ is also a transformation of the unknown constant. Note that (108) and (104) are identical, therefore no distinction will be made after this. Solving the first state equation gives

$$k\left(t\right) = e^{rt} \left[q + \int^{t} \left(w\left(1-\nu_w\right) - \left(1+\nu_c\right) c\left(j\right) \right) e^{-rj} dj \right] \tag{109}$$

for $t \in [t_0, T]$. Using the initial condition $k(t_0)$ *given* definitizes the unknown constant,

$$k(t_0) = e^{rt_0}\left[q + \int^{t_0} (w(1-\nu_w) - (1+\nu_c)c(j))e^{-rj}dj\right] \tag{110}$$

$$k(t_0)e^{-rt_0} = q + \int^{t_0} (w(1-\nu_w) - (1+\nu_c)c(j))e^{-rj}dj \tag{111}$$

$$q = k(t_0)e^{-rt_0} - \int^{t_0} (w(1-\nu_w) - (1+\nu_c)c(j))e^{-rj}dj. \tag{112}$$

This gives the intended asset path for $t \in [t_0, T]$,

$$k(t) = e^{rt}\left[k(t_0)e^{-rt_0} + \int_{t_0}^{t} (w(1-\nu_w) - (1+\nu_c)c(j))e^{-rj}dj\right] \tag{113}$$

Evaluate (113) at $t = T$

$$k(T) = e^{rT}\left[k(t_0)e^{-rt_0} + \int_{t_0}^{T} (w(1-\nu_w) - (1+\nu_c)c(j))e^{-rj}dj\right]. \tag{114}$$

Solving the second state equation gives

$$k(t) = e^{rt}\left[q + \int^{t} (b - (1+\nu_c)c(j))e^{-rj}dj\right] \tag{115}$$

for $t \in [T, t_0 + x]$. Using $k(t_0 + x) = 0$ identifies the unknown constant,

$$k(t_0+x) = e^{rt_0+x}\left[q + \int^{t_0+x} (b - (1+\nu_c)c(j))e^{-rj}dj\right] = 0 \tag{116}$$

$$q = -\int^{t_0+x} (b - (1+\nu_c)c(j))e^{-rj}dj. \tag{117}$$

Therefore, the particular solution is

$$k(t) = e^{rt}\left[\int_{t_0+x}^{t} (b - (1+\nu_c)c(j))e^{-rj}dj\right], \tag{118}$$

which can be evaluated at $t = T$,

$$k(T) = e^{rT}\left[\int_{t_0+x}^{T} (b - (1+\nu_c)c(j))e^{-rj}dj\right]. \tag{119}$$

Set (114) equal to (119)

$$
\begin{aligned}
& e^{rT}\left[\int_{t_0+x}^{T}\left(b-\left(1+\nu_c\right)c\left(j\right)\right)e^{-rj}dj\right] \\
= \;& e^{rT}\left[k\left(t_0\right)e^{-rt_0}+\int_{t_0}^{T}\left(w\left(1-\nu_w\right)-\left(1+\nu_c\right)c\left(j\right)\right)e^{-rj}dj\right].
\end{aligned} \tag{120}
$$

This can be rearranged

$$
\begin{aligned}
& k\left(t_0\right)e^{-rt_0}+\int_{t_0}^{T}w\left(1-\nu_w\right)e^{-rj}dj-\int_{t_0+x}^{T}be^{-rj}dj \\
= \;& \int_{t_0}^{T}\left(1+\nu_c\right)c\left(j\right)e^{-rj}dj-\int_{t_0+x}^{T}\left(1+\nu_c\right)c\left(j\right)e^{-rj}dj
\end{aligned} \tag{121}
$$

and further simplified

$$
\begin{aligned}
& \left(1+\nu_c\right)\int_{t_0}^{t_0+x}c\left(j\right)e^{-rj}dj \\
= \;& k\left(t_0\right)e^{-rt_0}+\int_{t_0}^{T}w\left(1-\nu_w\right)e^{-rj}dj+\int_{T}^{t_0+x}be^{-rj}dj.
\end{aligned} \tag{122}
$$

Substituting in for $c\left(t\right)$,

$$
\begin{aligned}
& \left(1+\nu_c\right)A\int_{t_0}^{t_0+x}e^{gj}e^{-rj}dj \\
= \;& k\left(t_0\right)e^{-rt_0}+\int_{t_0}^{T}w\left(1-\nu_w\right)e^{-rj}dj+\int_{T}^{t_0+x}be^{-rj}dj
\end{aligned} \tag{123}
$$

the transformation of the unknown constant is identified,

$$
A=\frac{k\left(t_0\right)e^{-rt_0}+\int_{t_0}^{T}w\left(1-\nu_w\right)e^{-rj}dj+\int_{T}^{t_0+x}be^{-rj}dj}{\left(1+\nu_c\right)\int_{t_0}^{t_0+x}e^{gj}e^{-rj}dj}. \tag{124}
$$

Inserting (124) into (108) yields planned consumption in closed-form,

$$
\hat{c}\left(t\right)=\frac{k\left(t_0\right)e^{-rt_0}+\int_{t_0}^{T}w\left(z\right)\left(1-\nu_w\right)e^{-rz}dz+\int_{T}^{t_0+x}b\left(t\right)e^{-rz}dz}{\left(1+\nu_c\right)\int_{t_0}^{t_0+x}e^{gt}e^{-rz}dz}e^{gt}. \tag{125}
$$

Replacing t_0 with t gives the actual consumption path

$$c(t) = \frac{k(t)\,e^{-rt} + \int_t^T w(1-\nu_w)\,e^{-rj}dj + \int_T^{t+x} be^{-rj}dj}{(1+\nu_c)\int_t^{t+x} e^{(g-r)j}dj} e^{gt} \tag{126}$$

$$= \frac{k(t)\,e^{-rt} + \frac{w(1-\nu_w)}{r}\left(e^{-rt} - e^{-rT}\right) + \frac{b}{r}\left(e^{-rT} - e^{-r(t+x)}\right)}{\frac{(1+\nu_c)}{(g-r)}\left(e^{(t+x)(g-r)} - e^{(g-r)t}\right)}. \tag{127}$$

Using z_1 from above, this can be simplified

$$c(t) = k(t)z_1 + \frac{w(1-\nu_w)}{r} z_1 e^{rt}\left(e^{-rt} - e^{-rT}\right) + \frac{b}{r} z_1 e^{rt}\left(e^{-rT} - e^{-r(t+x)}\right). \tag{128}$$

Inserting (128) into $\frac{dk(t)}{dt} = rk(t) + w(1-\nu_w) - (1+\nu_c)\,c(t)$ gives

$$\begin{aligned}\frac{dk(t)}{dt} &= rk(t) + w(1-\nu_w) - (1+\nu_c)\,k(t)z_1 + \frac{(1+\nu_c)\,w(1-\nu_w)}{r} z_1 e^{rt}\left(e^{-rT} - e^{-rt}\right) \\ &\quad + (1+\nu_c)\,\frac{b}{r} z_1 e^{rt}\left(e^{-r(t+x)} - e^{-rT}\right)\end{aligned} \tag{129}$$

$$\begin{aligned}&= k(t)\,\Omega + w(1-\nu_w) + \frac{(1+\nu_c)\,w(1-\nu_w)}{r} z_1 e^{rt}\left(e^{-rT} - e^{-rt}\right) \\ &\quad + (1+\nu_c)\,\frac{b}{r} z_1 e^{rt}\left(e^{-r(t+x)} - e^{-rT}\right)\end{aligned} \tag{130}$$

rewritten with $\Omega = (r - (1+\nu_c)\,z_1)$. Solving this differential equation yields a general solution,

$$\begin{aligned}k(t) &= e^{\Omega t}\left(q + \int^t \left[w(1-\nu_w) - (1+\nu_c)\left(\frac{w(1-\nu_w)}{r} z_1 e^{rj}\left(e^{-rj} - e^{-rT}\right)\right.\right.\right. \\ &\quad \left.\left.\left. + \frac{b}{r} z_1 e^{rj}\left[e^{-rT} - e^{-r(j+x)}\right]\right)\right] e^{-\Omega j}dj\right).\end{aligned} \tag{131}$$

Using the initial condition for Phase 2 definitizes the unknown constant, q, such that

$$\begin{aligned}k(T-x) &= e^{\Omega(T-x)}\left(q + \int^{T-x} \left[w(1-\nu_w) - (1+\nu_c)\left(\frac{w(1-\nu_w)}{r} z_1 e^{rj}\left(e^{-rj} - e^{-rT}\right)\right.\right.\right. \\ &\quad \left.\left.\left. + \frac{b}{r} z_1 e^{rj}\left[e^{-rT} - e^{-r(j+x)}\right]\right)\right] e^{-\Omega j}dj\right)\end{aligned} \tag{132}$$

$$
\begin{aligned}
q &= k(T-x)e^{-\Omega(T-x)} - \int^{T-x}\left[w(1-\nu_w) - (1+\nu_c)\left(\frac{w(1-\nu_w)}{r}z_1 e^{rj}\left(e^{-rf} - e^{-rT}\right)\right.\right. \\
&\quad \left.\left. + \frac{b}{r}z_1 e^{rj}\left[e^{-rT} - e^{-r(j+x)}\right]\right)\right] e^{-\Omega j} dj. \qquad (133)
\end{aligned}
$$

This yields the actual solution for the asset path

$$
\begin{aligned}
k(t) &= k(T-x)e^{-\Omega(T-x-t)} + e^{\Omega t}\int_{T-x}^{t}\left[(1-\nu_w)w - (1+\nu_c)\left(\frac{w(1-\nu_w)}{r}z_1 e^{rj}\left(e^{-rj} - e^{-rT}\right)\right.\right. \\
&\quad \left.\left. + \frac{b}{r}z_1 e^{rj}\left[e^{-rT} - e^{-r(j+x)}\right]\right)\right] e^{-\Omega j} dj \qquad (134)
\end{aligned}
$$

$$
\begin{aligned}
&= e^{\Omega t}k(T-x)e^{-\Omega(T-x)} + e^{\Omega t}\int_{T-x}^{t} w(1-\nu_w)e^{-\Omega j} dj \\
&\quad - e^{\Omega t}\int_{T-x}^{t}(1+\nu_c)\frac{w(1-\nu_w)}{r}z_1 e^{rj}\left(e^{-rj} - e^{-rT}\right)e^{-\Omega j} dj \\
&\quad - e^{\Omega t}\int_{T-x}^{t}(1+\nu_c)\frac{b}{r}z_1 e^{rj}\left(e^{-rT} - e^{-r(j+x)}\right)e^{-\Omega j} dj \qquad (135)
\end{aligned}
$$

$$
\begin{aligned}
&= e^{\Omega t}\left[e^{-\Omega(T-x)}k(T-x) + \frac{w(1-\nu_w)}{\Omega}\left(e^{-\Omega(T-x)} - e^{-\Omega t}\right)\right. \\
&\quad - \frac{w(1-\nu_w)(1+\nu_c)z_1}{r}\int_{T-x}^{t}\left[1 - e^{-r(T-j)}\right]e^{-\Omega j} dj \\
&\quad \left. - \frac{b(1+\nu_c)z_1}{r}\int_{T-x}^{t} e^{rj}\left(e^{-rT} - e^{-r(j+x)}\right)e^{-\Omega j} dj\right] \qquad (136)
\end{aligned}
$$

$$
\begin{aligned}
&= e^{\Omega t}\left[e^{-\Omega(T-x)}k(T-x) + \frac{w(1-\nu_w)}{\Omega}\left(e^{-\Omega(T-x)} - e^{-\Omega t}\right)\right. \\
&\quad - \frac{w(1-\nu_w)(1+\nu_c)z_1}{r}\left[\int_{T-x}^{t} e^{-\Omega j} dj - \int_{T-x}^{t} e^{-r(T-j)}e^{-\Omega j} dj\right] \\
&\quad \left. - \frac{b(1+\nu_c)z_1}{r}\left(\int_{T-x}^{t} e^{-rT}e^{(r-\Omega)j} dj - \int_{T-x}^{t} e^{-rx}e^{-\Omega j} dj\right)\right] \qquad (137)
\end{aligned}
$$

$$
\begin{aligned}
= \;& e^{\Omega t}\left(e^{-\Omega(T-x)}k\left(T-x\right)+\frac{w\left(1-\nu_w\right)}{\Omega}\left(e^{-\Omega(T-x)}-e^{-\Omega t}\right)\right.\\
& +\frac{w\left(1-\nu_w\right)\left(1+\nu_c\right)z_1}{r}\left[\frac{e^{-rT}}{r-\Omega}\left(e^{(r-\Omega)t}-e^{(r-\Omega)(T-x)}-\frac{1}{\Omega}\left[e^{-\Omega(T-x)}-e^{-\Omega t}\right]\right)\right]\\
& \left.+\frac{b\left(1+\nu_c\right)z_1}{r}\left[\frac{e^{-rT}}{r-\Omega}\left(e^{(r-\Omega)(T-x)}-e^{(r-\Omega)t}\right)+\frac{e^{-rx}}{\Omega}\left(e^{-\Omega(T-x)}-e^{-\Omega t}\right)\right]\right). \qquad (138)
\end{aligned}
$$

Phase 3 $\left[T,\overline{T}-x\right]$

The individual solves

$$\max_{c(t)} : \int_{t_0}^{t_0+x} e^{-\rho(t-t_0)}\frac{c(t)^{1-\phi}}{1-\phi}dt \qquad (139)$$

subject to

$$\frac{dk\left(t\right)}{dt}=rk\left(t\right)+b-\left(1+\nu_c\right)c\left(t\right) \qquad (140)$$

$$k\left(t_0\right)\;\; given \qquad (141)$$

$$k\left(t_0+x\right)=0. \qquad (142)$$

Using the Maximum Principle, the Hamiltonian and optimality conditions are

$$H=e^{-\rho(t-t_0)}\frac{c(t)^{1-\phi}}{1-\phi}+\lambda\left(t\right)\left(rk\left(t\right)+b-\left(1+\nu_c\right)c\left(t\right)\right) \qquad (143)$$

$$\frac{\partial H}{\partial c}=e^{-\rho(t-t_0)}c(t)^{-\phi}-\lambda\left(t\right)\left(1+\nu_c\right)=0 \qquad (144)$$

$$\frac{\partial H}{\partial k}=r\lambda\left(t\right)=-\frac{d\lambda}{dt} \qquad (145)$$

$$\frac{\partial H}{\partial \lambda}=rk\left(t\right)+b-\left(1+\nu_c\right)c\left(t\right)=\frac{dk\left(t\right)}{dt}. \qquad (146)$$

Solving the maximum condition gives

$$c(t)=\left(e^{-\rho(t-t_0)}\frac{1}{\lambda\left(t\right)\left(1+\nu_c\right)}\right)^{\frac{1}{\phi}}, \qquad (147)$$

and solving the costate equation yields

$$\frac{d\lambda}{dt}=-r\lambda\left(t\right)\rightarrow\lambda\left(t\right)=ae^{-rt}, \qquad (148)$$

where the unknown constant can be rewritten

$$\lambda(t_0) = ae^{-rt_0} \tag{149}$$

$$a = \lambda(t_0) e^{rt_0} \tag{150}$$

such that

$$\lambda(t) = \lambda(t_0) e^{r(t_0 - t)}. \tag{151}$$

Substituting (151) into (147),

$$c(t) = \left(e^{-\rho(t-t_0)} \frac{1}{\lambda(t_0) e^{r(t_0-t)} (1+\nu_c)} \right)^{\frac{1}{\phi}} \tag{152}$$

$$= e^{gt} \left(\frac{1}{\lambda(t_0)(1+\nu_c)} e^{(r-\rho)t_0} \right)^{\frac{1}{\phi}}, \tag{153}$$

where $g = \dfrac{r-\rho}{\phi}$. The notation can be compressed for simplicity, such that

$$c(t) = e^{gt} A \tag{154}$$

where $A = \left(\dfrac{1}{\lambda(t_0)(1+\nu_c)} e^{(r-\rho)t_0} \right)^{\frac{1}{\phi}}$ is again a tranformation of the unknown constant of integration. Solving the state equation yields

$$k(t) = e^{rt} \left[q + \int^{t} (b - (1+\nu_c) c(j)) e^{-rj} dj \right] \tag{155}$$

for $t \in [t_0, t_0 + x]$. Using the initial condition, $k(t_0)$ *given*, identifies q

$$k(t_0) = e^{rt_0} \left[q + \int^{t_0} (b - (1+\nu_c) c(j)) e^{-rj} dj \right] \tag{156}$$

$$q = k(t_0) e^{-rt_0} - \int^{t_0} (b - (1+\nu_c) c(j)) e^{-rj} dj. \tag{157}$$

The intended asset path is therefore

$$k(t) = e^{rt} \left[k(t_0) e^{-rt_0} + \int_{t_0}^{t} (b - (1+\nu_c) c(j)) e^{-rj} dj \right]. \tag{158}$$

Using the boundary condition, $k(t_0 + x) = 0$,

$$e^{r(t_0+x)} \left[k(t_0) e^{-rt_0} + \int_{t_0}^{t_0+x} (b - (1+\nu_c) c(j)) e^{-rj} dj \right] = 0, \tag{159}$$

which is simplified as

$$k(t_0) e^{-rt_0} + \int_{t_0}^{t_0+x} be^{-rj} dj = \int_{t_0}^{t_0+x} (1+\nu_c) c(j) e^{-rj} dj. \tag{160}$$

Substituting (154) in for $c(t)$ gives

$$k(t_0) e^{-rt_0} + \int_{t_0}^{t_0+x} be^{-rj} dj = \int_{t_0}^{t_0+x} (1+\nu_c) e^{gj} A e^{-rj} dj \tag{161}$$

$$(1+\nu_c) A \int_{t_0}^{t_0+x} e^{(g-r)j} dj = k(t_0) e^{-rt_0} + \int_{t_0}^{t_0+x} be^{-rj} dj, \tag{162}$$

where the tranformation of the unknown constant is identified

$$A = \frac{k(t_0) e^{-rt_0} + \int_{t_0}^{t_0+x} be^{-rj} dj}{(1+\nu_c) \int_{t_0}^{t_0+x} e^{(g-r)j} dj}. \tag{163}$$

Therefore, the planned consumption path is

$$\hat{c}(t) = e^{gt} \left[\frac{k(t_0) e^{-rt_0} + \int_{t_0}^{t_0+x} be^{-rj} dj}{(1+\nu_c) \int_{t_0}^{t_0+x} e^{(g-r)j} dj} \right]. \tag{164}$$

Replacing t_0 with t yields the actual consumption path,

$$c(t) = e^{gt} \left[\frac{k(t) e^{-rt} + \int_{t}^{t+x} be^{-rj} dj}{(1+\nu_c) \int_{t}^{t+x} e^{(g-r)j} dj} \right] \tag{165}$$

$$= e^{gt} \left[\frac{k(t) e^{-rt} + \frac{b}{r} \left(e^{-rt} - e^{-r(t+x)} \right)}{\frac{(1+\nu_c)}{(g-r)} \left(e^{(g-r)t} - e^{(g-r)(t+x)} \right)} \right]. \tag{166}$$

Using z_1 from above and reducing the fraction,

$$c(t) = k(t)z_1 + \frac{\frac{b}{r}(g-r)\left(1-e^{-rx}\right)}{(1+\nu_c)\left(e^{(g-r)x}-1\right)} \tag{167}$$

$$= k(t)z_1 + bz_3$$

where

$$z_3 = \frac{(g-r)\left(1-e^{-rx}\right)}{r\,(1+\nu_c)\left(e^{(g-r)x}-1\right)}. \tag{168}$$

The actual law of motion for Phase 3 is

$$\frac{dk(t)}{dt} = rk(t) + b - (1+\nu_c)\,c(t) \tag{169}$$

$$= rk(t) + b - (1+\nu_c)\left[k(t)z_1 + bz_3\right] \tag{170}$$

$$= k(t)\left(r - (1+\nu_c)\,z_1\right) + b\left(1 - (1+\nu_c z_3)\right). \tag{171}$$

Rewriting this with $\Omega = r - (1+\nu_c)\,z_1$ and then solving gives a general solution

$$k(t) = e^{\Omega t}\left[q + \int^{t} b\left(1-(1+\nu_c z_3)\right)e^{-\Omega j}dj\right]. \tag{172}$$

Using the actual initial condition for Phase 3 identifies the unknown constant

$$k(T) = e^{\Omega T}\left[q + \int^{T} b\left(1-(1+\nu_c z_3)\right)e^{-\Omega j}dj\right] \tag{173}$$

$$q = k(T)e^{-\Omega T} - \int^{T} b\left(1-(1+\nu_c z_3)\right)e^{-\Omega j}dj. \tag{174}$$

The actual asset path is therefore

$$k(t) = e^{\Omega t}\left[k(T)e^{-\Omega T} + \int_{T}^{t} b\left(1-(1+\nu_c z_3)\right)e^{-\Omega j}dj\right] \tag{175}$$

$$= e^{\Omega t}\left[k(T)e^{-\Omega T} + \frac{b\left(1-(1+\nu_c z_3)\right)}{\Omega}\left(e^{-\Omega T}-e^{-\Omega t}\right)\right] \tag{176}$$

$$= k(T)e^{-\Omega(T-t)} + \frac{b\left(1-(1+\nu_c z_3)\right)}{\Omega}\left(e^{-\Omega(T-t)}-1\right). \tag{177}$$

Phase 4 $\left[\overline{T}-x,\overline{T}\right]$

There is no time inconsistency in this phase. The path can be easily acquired by evaluating (164) at $t_0 = \overline{T} - x$,

$$c(t) = e^{gt}\left[\frac{k\left(\overline{T}-x\right)e^{-r\left(\overline{T}-x\right)} + \int_{\overline{T}-x}^{\overline{T}} be^{-rj}dj}{(1+\nu_c)\int_{\overline{T}-x}^{\overline{T}} e^{(g-r)j}dj}\right] \tag{178}$$

$$= e^{gt}\left[\frac{k\left(\overline{T}-x\right)e^{-r\left(\overline{T}-x\right)} - \dfrac{b\left(e^{-r\overline{T}}-e^{-r\left(\overline{T}-x\right)}\right)}{r}}{(1+\nu_c)\left(e^{(g-r)\overline{T}}-e^{(g-r)\left(\overline{T}-x\right)}\right)}\right](g-r) \tag{179}$$

$$= e^{gt}(g-r)\left[\frac{k\left(\overline{T}-x\right)e^{-r\left(\overline{T}-x\right)} - \dfrac{be^{-r\overline{T}}\left(1-e^{rx}\right)}{r}}{(1+\nu_c)e^{(g-r)\overline{T}}\left(1-e^{-x(g-r)}\right)}\right] \tag{180}$$

$$= e^{gt}\frac{(g-r)}{(1+\nu_c)}\left[\frac{k\left(\overline{T}-x\right)e^{rx} - \dfrac{b\left(1-e^{rx}\right)}{r}}{e^{g\overline{T}}\left(1-e^{-x(g-r)}\right)}\right]. \tag{181}$$

Defining

$$z_4 = \frac{(g-r)}{(1+\nu_c)}\left[\frac{k\left(\overline{T}-x\right)e^{rx} - \dfrac{b\left(1-e^{rx}\right)}{r}}{e^{g\overline{T}}\left(1-e^{-x(g-r)}\right)}\right], \tag{182}$$

actual consumption can be rewritten

$$c(t) = e^{gt}z_4. \tag{183}$$

Coupled with $\dfrac{dk(t)}{dt} = rk(t) + b - (1+\nu_c)c(t)$ and $k\left(\overline{T}-x\right)$ *given*, (183) characterizes the asset path during Phase 4.

Appendix B: derivation of the present value of taxes

Assuming $t_0 = 0$ for simplicity of demonstration, which corresponds to Phase 1, the present value of taxes paid over a short horizon in the payroll-tax financed regime is

$$I_w \equiv \int_0^x e^{-rt} \nu_w w dt \tag{184}$$

$$= \frac{\nu_w w}{r}\left(1 - e^{-rx}\right). \tag{185}$$

The present value of taxes paid in the consumption-tax financed regime is

$$I_c \equiv \int_0^x e^{-rt} \nu_c \hat{c}(t) dt \tag{186}$$

$$= \frac{\nu_c \hat{c}(0)}{g-r}\left(e^{(g-r)x} - 1\right) \tag{187}$$

where

$$\hat{c}(t) = \hat{c}(0)e^{gt}, \tag{188}$$

and where

$$\hat{c}(0) = \frac{w(1-\nu_w)\int_0^x e^{-rt}dt}{(1+\nu_c)\int_0^x e^{(g-r)t}dt} \tag{189}$$

$$= \frac{\dfrac{w(1-\nu_w)}{r}\left(1 - e^{-rx}\right)}{\dfrac{(1+\nu_c)}{(g-r)}\left(e^{(g-r)x} - 1\right)}. \tag{190}$$

This can be rewritten

$$I_c = \frac{\nu_c}{g-r}\left(e^{(g-r)x} - 1\right)\frac{\dfrac{w(1-\nu_w)}{r}\left(1 - e^{-rx}\right)}{\dfrac{(1+\nu_c)}{(g-r)}\left(e^{(g-r)x} - 1\right)} \tag{191}$$

$$= \frac{\nu_c w(1-\nu_w)}{(1+\nu_c)r}\left(1 - e^{-rx}\right). \tag{192}$$

Comparing only one tax regime at a time, $\nu_w = 0$ such that

$$I_c = \frac{\nu_c w}{(1+\nu_c)r}\left(1 - e^{-rx}\right) \tag{193}$$

Setting I_c equal to I_w with a scaler inserted suggests

$$I_w \Gamma = I_c, \tag{194}$$

or rewritten with substitution

$$\frac{\nu_w w}{r}\left(1 - e^{-rx}\right)\Gamma = \frac{\nu_c w}{(1+\nu_c) r}\left(1 - e^{-rx}\right). \tag{195}$$

This can also be written as

$$\Gamma = \frac{\nu_c}{(1+\nu_c)\nu_w}, \tag{196}$$

or as

$$\nu_w = \frac{\nu_c}{(1+\nu_c)} \tag{197}$$

with $\Gamma = 1$.

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Table 1

Baseline parameters

Retirement date	T	40
Date of death	$\overline{T}$	55
planning horizon	x	10
IEIS	ϕ	1
Personal discount rate	ρ	0.035
retiree to worker ratio	R	2.667
wage	w	40000
real rate or return	r	0.035

Table 2

No program vs. consumption-tax financed program

	W_0	W^*_w	CV
1	236.3025252	252.6870847	34.70285%
2	246.2265692	252.7763546	12.64679%
3	249.5221449	252.8793832	6.29422%
4	251.1668483	252.9992367	3.38774%
5	252.152197	253.1384632	1.80939%
6	252.8081444	253.2981998	0.89499%
7	253.2759645	253.4785241	0.36897%
8	253.6262497	253.6790217	0.09600%
9	253.898201	253.899142	0.00171%
10	254.1153312	254.1153312	0
11	254.292599	254.292599	0
12	254.4399751	254.4399751	0
13	254.564361	254.564361	0
14	254.6706858	254.6706858	0
15	254.7625637	254.7625637	0

$$\rho = 0.035,\ \mu = .035,\ r = 0.035,\ \bar{T} = 55,\ T = 40$$

Table 3

No program vs. payroll-tax financed program

	W_0	W_c^*	CV
1	236.3025252	252.6862788	34.70088%
2	246.2265692	252.7732336	12.64040%
3	249.5221449	252.8726371	6.28118%
4	251.1668483	252.9881505	3.36690%
5	252.152197	253.123226	1.78118%
6	252.8081444	253.2802075	0.86199%
7	253.2759645	253.4606859	0.33642%
8	253.6262497	253.6661419	0.07256%
9	253.898201	253.898201	0
10	254.1153312	254.1153312	0
11	254.292599	254.292599	0
12	254.4399751	254.4399751	0
13	254.564361	254.564361	0
14	254.6706858	254.6706858	0
15	254.7625637	254.7625637	0

$\rho = 0.035$, $\mu = .035$, $r = 0.035$, $\bar{T} = 55$, $T = 40$

Table 4

x	ν_c^*	U_c^*	ν_w^*	U_w^*	Δ
1	.125	252.6871	.112	252.6863	0.00330%
2	.116	252.7764	.104	252.7732	0.01279%
3	.106	252.8794	.096	252.8726	0.02765%
4	.094	252.9992	.086	252.9882	0.04544%
5	.079	253.1385	.073	253.1232	0.06246%
6	.063	253.2982	.059	253.2802	0.07376%
7	.045	253.4785	.042	253.4607	0.07312%
8	.025	235.6790	.022	253.6661	0.05279%
9	.004	254.8991	0	253.8982	0.00386%
10	0	254.1153	0	254.1153	0
11	0	254.2926	0	254.2926	0
12	0	254.4400	0	254.4400	0
13	0	254.5644	0	254.5644	0
14	0	254.6707	0	254.6707	0
15	0	254.7626	0	254.7626	0

Note. $\rho = 0.035$, $\mu = .035$, $r = 0.035$, $\bar{T} = 55$, $T = 40$

Table 5

x	ν_c^*	U_c^*	ν_w^*	U_w^*	Δ
1	.174	308.4714	.149	308.4695	.00657%
2	.165	308.5540	.142	308.5463	.02570%
3	.155	308.6604	.134	308.6437	.05608%
4	.142	308.7936	.124	308.7654	.09448%
5	.126	308.9556	.112	308.9151	.13564%
6	.109	309.1477	.097	309.0962	.17233%
7	.089	309.3703	.079	309.3119	.19544%
8	.066	309.6234	.058	309.8617	.19325%
9	.042	309.9069	.033	309.8617	.15141%
10	.016	310.2202	.004	310.2047	.05193%
11	0	310.5562	0	310.5562	0
12	0	310.8565	0	310.8565	0
13	0	311.1156	0	311.1156	0
14	0	311.3416	0	311.3416	0
15	0	311.5403	0	311.5403	0

Note. $\rho = 0.025$, $\mu = 0.025$, $r = 0.035$, $\bar{T} = 55$, $T = 40$

Table 6

x	ν_c^*	U_c^*	ν_w^*	U_w^*	Δ
1	.089	211.4763	.082	211.4761	.00123%
2	.080	211.5726	.074	211.5716	.00477%
3	.070	211.6777	.066	211.6756	.01039%
4	.059	211.7964	.056	211.7913	.01668%
5	.046	211.9263	.044	211.9219	.02148%
6	.031	212.0735	.030	212.0691	.02193%
7	.015	212.2356	.013	212.2327	.01427%
8	0	212.4104	0	212.4104	0
9	0	212.5603	0	212.5603	0
10	0	212.6770	0	212.6770	0
11	0	212.7700	0	212.7700	0
12	0	212.8452	0	212.8452	0
13	0	212.9070	0	212.9070	0
14	0	212.9585	0	212.9585	0
15	0	213.0018	0	213.0018	0

Note. $\rho = 0.045$, $\mu = 0.045$, $r = 0.035$, $\bar{T} = 55$, $T = 40$

Table 7

x	ν_c^*	U_c^*	ν_w^*	U_w^*	Δ
1	.374	565.3000	.272	562.7388	4.76679%
2	.371	565.3004	.271	562.9414	4.38237%
3	.366	565.3026	.269	563.1766	3.94103%
4	.359	565.3085	.266	563.4393	3.45695%
5	.349	565.3209	.262	563.7199	2.95371%
6	.337	565.3431	.257	564.0081	2.45685%
7	.323	565.3782	.250	564.2955	1.98807%
8	.306	565.4298	.242	564.5765	1.56368%
9	.288	565.5014	.232	564.8478	1.19540%
10	.267	565.5965	.221	565.1082	0.89184%
11	.245	565.7188	.207	565.3572	0.65964%
12	.220	565.8716	.190	565.5949	0.50444%
13	.194	566.0587	.170	565.8220	0.43127%
14	.166	566.2832	.146	566.0400	0.44327%
15	.137	566.5486	.116	566.2936	0.46484%

Note. $\rho = 0.035$, $\mu = 0$, $r = 0.035$, $\bar{T} = 55$, $T = 40$

Table 8

x	ν_c^*	U_c^*	ν_w^*	U_w^*	Δ
1	.374	565.2580	.272	562.7530	4.65972%
2	.371	565.2970	.271	562.9989	4.26686%
3	.365	565.3624	.268	563.3064	3.80884%
4	.356	565.4555	.264	563.6687	3.30197%
5	.344	565.5783	.258	564.0726	2.77548%
6	.329	565.7334	.251	564.5032	2.26184%
7	.311	565.9231	.242	564.9476	1.78939%
8	.291	566.1499	.230	565.3959	1.38025%
9	.268	566.4161	.216	565.8412	1.0579%
10	.243	566.7788	.198	566.2785	0.91387%
11	.215	567.1300	.177	566.7046	0.77635%
12	.186	567.5262	.152	567.1175	0.74581%
13	.154	567.9690	.121	567.5568	0.75219%
14	.121	568.4595	.083	568.0839	0.68528%
15	.087	568.9987	.037	568.7190	0.5984%

Note. $\rho = 0.025$, $\mu = 0$, $r = 0.035$, $\bar{T} = 55$, $T = 40$

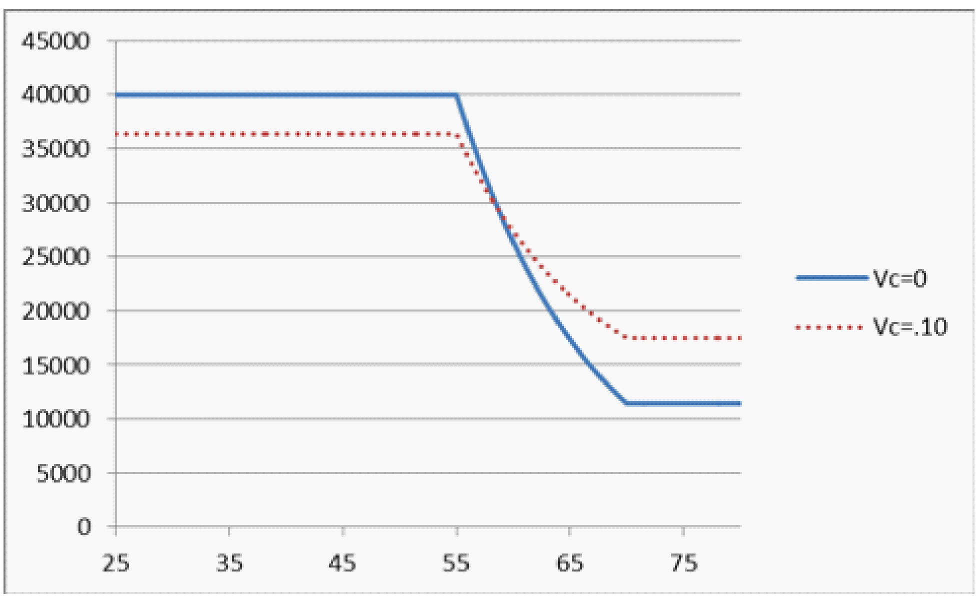
Table 9

x	ν_c^*	U_c^*	ν_w^*	U_w^*	Δ
1	.374	565.2870	.272	562.7246	4.76908%
2	.372	565.2491	.272	562.8844	4.39332%
3	.368	565.1886	.270	563.0481	3.96862%
4	.362	565.1085	.268	563.2125	3.50739%
5	.354	565.0122	.266	563.3713	3.02842%
6	.345	564.9038	.262	563.5181	2.55147%
7	.334	564.7871	.258	563.6479	2.09280%
8	.321	564.6665	.253	563.7588	1.66405%
9	.306	564.5464	.248	563.8506	1.27308%
10	.290	564.4312	.241	563.9246	0.92547%
11	.272	564.3258	.232	563.9828	0.62569%
12	.253	564.235	.222	564.0275	0.37794%
13	.232	564.1636	.211	564.0611	0.18659%
14	.209	564.1167	.196	564.0857	0.05648%
15	.184	564.0995	.179	564.1033	−0.00702%

Note. $\rho = 0.045$, $\mu = 0$, $r = 0.035$, $\bar{T} = 55$, $T = 40$

Figure 1

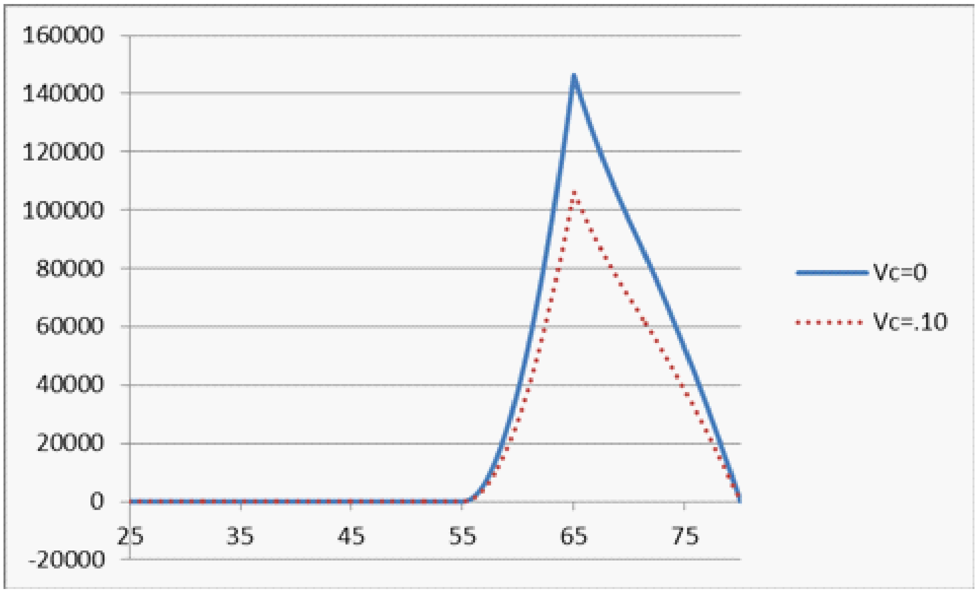
Consumption profiles with and without consumption-tax financed program



Note. $\rho = 0.035$, $r = 0.035$, $\bar{T} = 55$, $T = 40$, $x = 10$

Figure 2

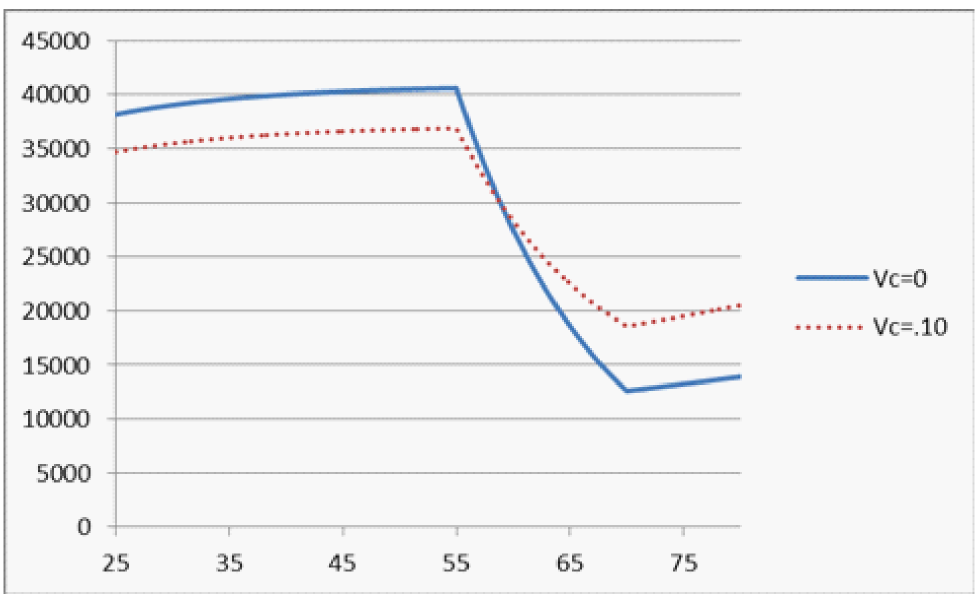
Asset accounts with and without consumption-tax financed program



Note. $\rho = 0.035$, $r = 0.035$, $\bar{T} = 55$, $T = 40$, $x = 10$

Figure 3

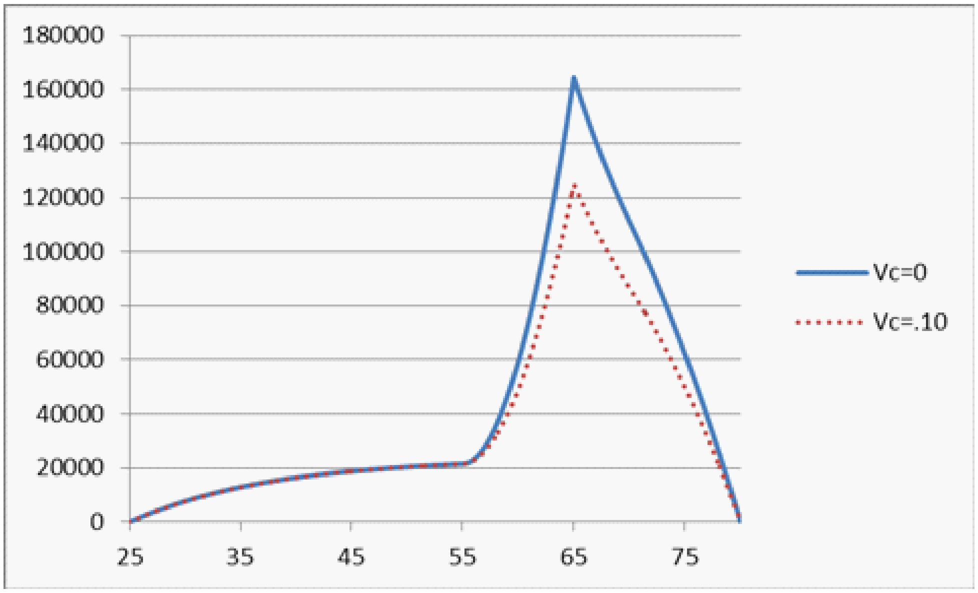
Consumption profiles with and without consumption-tax financed program



Note. $\rho = 0.025$, $r = 0.035$, $\bar{T} = 55$, $T = 40$, $x = 10$

Figure 4

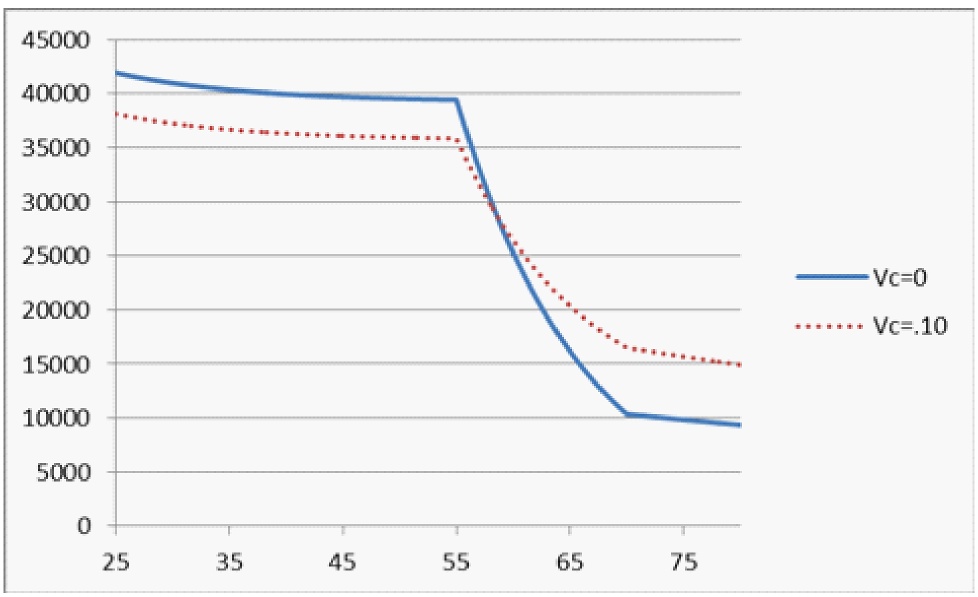
Asset accounts with and without consumption-tax financed program



Note. $\rho = 0.025$, $r = 0.035$, $\bar{T} = 55$, $T = 40$, $x = 10$

Figure 5

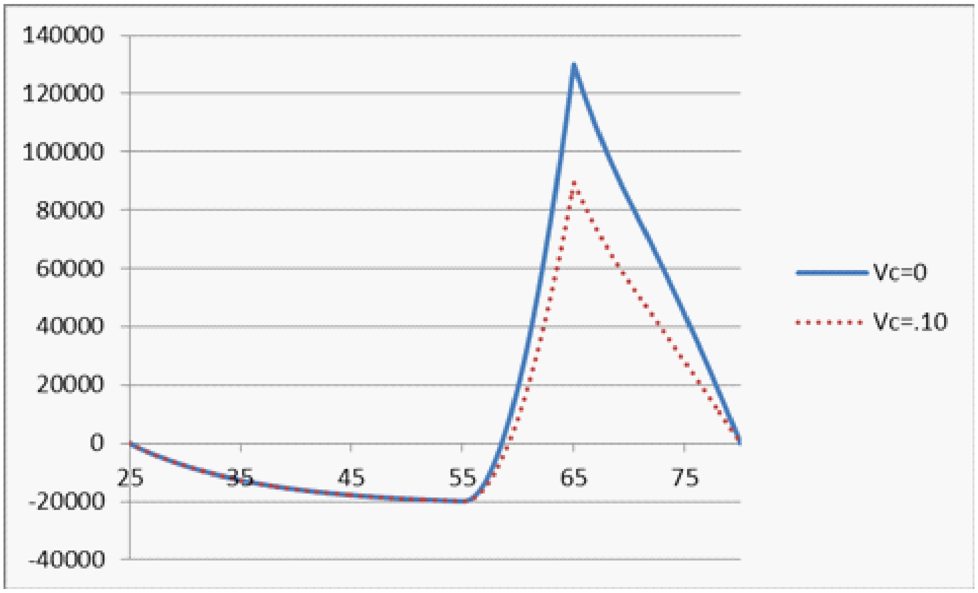
Consumption profiles with and without consumption-tax financed program



Note. $\rho = 0.045$, $r = 0.035$, $\bar{T} = 55$, $T = 40$, $x = 10$

Figure 6

Asset accounts with and without consumption-tax financed program



Note. $\rho = 0.045$, $r = 0.035$, $\bar{T} = 55$, $T = 40$, $x = 10$