High Cognitive Test Item Development and Implementation

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HIGH COGNITIVE TEST ITEM DEVELOPMENT
AND IMPLEMENTATION

by

Ashley Salisbury

A report submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF MATHEMATICS

in

Mathematics

Approved:

Dr. Brynja Kohler  Dr. James Cangelosi
Major Professor  Committee Member

Dr. John Stevens
Committee Member

UTAH STATE UNIVERSITY
Logan, Utah
2014
ABSTRACT

High Cognitive Test Item Development and Implementation

by

Ashley Salisbury, Master of Mathematics

Utah State University, 2014

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Department: Mathematics and Statistics

In secondary math classrooms there has been a movement toward discovery and problem solving based instruction. This type of instruction requires teachers to teach on what is often considered a higher level of cognition and allows students to discover more ideas and concepts on their own as opposed to traditional lecture style. Teachers with well thought-out examples, questions, and activities provide students with tools to solve problems on their own requiring students to make mathematical discoveries and connections. These skills not only benefit students in their math class but are analytical skills students can use throughout their lives.

With this change in instruction style there is also a push to change the method of assessment used in classrooms. Tests should require more of students than simple memorization of ideas and steps, the test should provide an opportunity for students to show the discoveries and connections they have made. It is also important that assessments be effective in measuring and determining what students have learned or have not learned relevant to the teacher’s objectives and concepts that have been taught. In this study, test items will be evaluated using known methods for judging item effectiveness.
ACKNOWLEDGMENTS

There have been so many who throughout the years have provided me with the support needed in order to finish this project and my master’s degree.

I would first like to thank my husband, Joshua Salisbury. I am so grateful for his love, patience and willingness to help in any way possible throughout this entire process including his help in reading and editing this never ending paper. I would also like to thank my parents, Bryan and Barbara Munns for the encouragement they have given me since kindergarten that I would graduate from college not only with my Bachelor’s degree but also with my Master’s. I am so grateful for their gentle nudges along the way to help me achieve something I often thought was not possible.

I would also like to thank my major professor, Dr. Brynja Kohler. Her ideas and guidance as I decided what my project would be has been invaluable. I have also appreciated her help with understanding the process of item construction, administration and analysis. I am also grateful to both her and Dr. James Cangelosi for their help in editing the test items.

I want to also thank the students and teachers at Layton High School who were willing to be guinea pigs and try something new.
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CHAPTER 1

INTRODUCTION

1.1 – Overview

In secondary math classrooms there has been a movement toward discovery and problem solving based instruction. This encourages students to discover ideas and concepts more on their own than traditional lecture style often allows. With these changes in instruction style there has also been a movement for change in the cognitive demand of assessment. Tests should require more of students than simple memorization of ideas and steps, the test should provide an opportunity for students to show the discoveries and connections they have made. It is also important that these assessments be effective in measuring and determining what students have learned or have not learned relevant to the objectives and concepts that have been taught.

At Layton High School there was a need for assessment items that were written at these high levels of cognition to help prepare students for the upcoming state tests at the end of the school year. The purpose of this project then became to write test items at these higher levels of cognition that the Layton High math teachers could then use as part of their assessment and as an example of higher cognitive assessment.

Many aspects contributed to the design of test items for this project. First was deciding on a common core standard and then designing an item that fit the standard. As the items were being constructed, Bloom’s taxonomy and the Cangelosi learning levels were used to help reach these higher levels of cognition.

Each item then went through a refining process which included the help of many individuals, including feedback from committee members and think alouds. Following
this the items were administered to the students and an item analysis was conducted. Based on results from the item analysis test items were once again revised.

1.2 – Objective

To create high cognitive demand test items that would fulfill the needs of the teachers at Layton High School the following objectives were used:

1. Develop open response and multiple choice test items in line with the common core curriculum standards and mathematical practice standards.
2. Construct test items that emphasize the use of higher levels of cognitive demand as determined by Bloom’s taxonomy and Cangelosi’s learning levels.
3. Use research-based test writing methods for multiple-choice and constructed response item development.
4. Perform an item analysis to determine each item’s effectiveness in measuring students’ understanding of the content and to assist in the further development of test items.
CHAPTER 2

LITERATURE REVIEW

The construction of test items is a time consuming and difficult process. There are many aspects that contribute to an effective test item that is an accurate measure of students understanding. Additionally, there is a wide variety of published literature on what is defined as an effective test item and how to best interpret the results of the item analysis. I looked at a wide variety of sources as described in this section in order to draw conclusions for my own research.

This literature review includes a summary on the topics, methods and procedures used in completing this project:

- Common core state standards
- Mathematical practice standards
- Bloom’s taxonomy
- Cangelosi’s learning levels
- Multiple-choice and constructed response item development
- Response formats and stimulus response
- Think alouds
- Item analysis
- Validity and reliability.

Articles and books were found through the use of Utah State University library search engines for books and articles. Recommendations of literature were also given by committee members and other graduate students master’s projects. I attempted to find
the most current and up to date articles but this did not always turn out to be the best course of action since many of the articles and books continually referred back to a single source that was written many years ago. It also became impossible to look towards literature that had been written specifically for mathematics classrooms exclusively; because there was so little available it was necessary to look at articles written for many different disciplines.

2.1 – Common Core State Standards and Mathematical Practice Standards

With the adoption of common core state standards there have been many changes in the curriculum and expectations of students. These new standards were initiated to help students graduate from high school with the necessary knowledge and skills to succeed in college and the workforce (Common Core, 2012). Because of the changes in curriculum, a corresponding change in assessment needs to be made. Assessments should be in alignment with the higher cognitive demands placed upon students with the use of discovery based learning.

Mathematical practice standards have been given to teachers to help “describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with [mathematics] as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years” (Swanson & Parrott, 2013). These standards are (Common Core, 2012):

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The standards most commonly addressed in this project were the first four. These particular standards encourage students to “break down integral parts by looking for key words, identifying main points and analyzing possible effective strategies and procedures,” to “create valid and different proofs to see the world in multiple perspectives,” and “to not only think, tell, draw, write but also reason and share their knowledge in logical and convincing arguments” (Swanson & Parrott, 2013). Through the use of the standards students’ proficiency in mathematical reasoning and their communication of mathematics in and out of the classroom will improve (Swanson & Parrott, 2013).

2.2 – Bloom’s Taxonomy and Cangelosi’s Learning Levels

To help achieve the common core state standards and mathematical practice standards in the test items there was need for a framework that could be used to help in the development and analyzing of the assessment items at higher levels of cognition. For this project two frameworks were used, Bloom’s taxonomy and Cangelosi’s learning levels.

Bloom’s taxonomy “is a tool to design, assess and evaluate student learning…It allows the instructor to gauge the level of questions on the exam” (Lord & Baviskar, 2007). The levels of Bloom’s taxonomy are the following: knowledge, comprehension, application, analysis, synthesis and evaluation. To encourage higher levels of cognitive...
thinking it is recommended that the middle and higher levels of the taxonomy be used (Lord & Baviskar, 2007).

The two lowest levels of the taxonomy are knowledge and comprehension. These levels focus on students remembering specific facts, methods and processes. Knowledge is referred to by Bloom as the “process of remembering.” Comprehension builds upon this by having students take their knowledge and be able to communicate what they know and have learned (Bloom, 1956). Ironically, these lower levels of Bloom’s taxonomy are the foundation for the majority of student evaluation techniques (Lord & Baviskar, 2007).

The top four levels are where a higher thinking level and a greater understanding of material are achieved. Application is when students can take what they have learned and are able to apply it to different situations. Analysis has students “break down objects or ideas into simpler parts and find evidence to support generalizations” (Reference Materials, 2014). Synthesis takes the opposite approach and has students bring ideas together to illustrate a connection they have not previously made or understood. The
final level is evaluation. This is where students communicate and defend their judgments of the material and/or the methods used (Bloom, 1956; Reference Materials, 2014).

To assist in the construction of test items it was beneficial to have a list of action verbs. This provides a method for those constructing the items to word the question so it achieves the desired taxonomy level. Note that some verbs may fall into multiple categories as a lot depends on the context of the question in which they are placed. A few of the verbs can be viewed in Table 1 (Reference Materials, 2014).

<table>
<thead>
<tr>
<th>Verbs</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrange</td>
<td>Knowledge</td>
</tr>
<tr>
<td>Identify</td>
<td>Comprehension</td>
</tr>
<tr>
<td>Label</td>
<td>Application</td>
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<tr>
<td>List</td>
<td>Analysis</td>
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<tr>
<td>Match</td>
<td>Evaluation</td>
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<tr>
<td>Memorize</td>
<td></td>
</tr>
<tr>
<td>Recognize</td>
<td></td>
</tr>
<tr>
<td>Recall</td>
<td></td>
</tr>
<tr>
<td>Select</td>
<td></td>
</tr>
<tr>
<td>Classify</td>
<td></td>
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<tr>
<td>Defend</td>
<td></td>
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<tr>
<td>Describe</td>
<td></td>
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<tr>
<td>Discuss</td>
<td></td>
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<tr>
<td>Extend</td>
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<tr>
<td>Indicate</td>
<td></td>
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<tr>
<td>Infer</td>
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<tr>
<td>Predict</td>
<td></td>
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<tr>
<td>Summarize</td>
<td></td>
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<tr>
<td>Apply</td>
<td></td>
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<tr>
<td>Compute</td>
<td></td>
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<tr>
<td>Discover</td>
<td></td>
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<tr>
<td>Illustrate</td>
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<td>Interpret</td>
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<td>Modify</td>
<td></td>
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<tr>
<td>Produce</td>
<td></td>
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<tr>
<td>Show</td>
<td></td>
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<tr>
<td>Solve</td>
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<td>Analyze</td>
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<tr>
<td>Compare</td>
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<tr>
<td>Criticize</td>
<td></td>
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<tr>
<td>Examine</td>
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<tr>
<td>Identify</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td></td>
</tr>
<tr>
<td>Relate</td>
<td></td>
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<tr>
<td>Select</td>
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<tr>
<td>Arrange</td>
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</tr>
<tr>
<td>Categorize</td>
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<tr>
<td>Create</td>
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<td>Design</td>
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<td>Develop</td>
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<tr>
<td>Explain</td>
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<tr>
<td>Justify</td>
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<td>Plan</td>
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<tr>
<td>Revise</td>
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<tr>
<td>Set up</td>
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<td>Appraise</td>
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<td>Argue</td>
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<td>Defend</td>
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<tr>
<td>Evaluate</td>
<td></td>
</tr>
<tr>
<td>Judge</td>
<td></td>
</tr>
<tr>
<td>Justify</td>
<td></td>
</tr>
<tr>
<td>Rate</td>
<td></td>
</tr>
<tr>
<td>Select</td>
<td></td>
</tr>
<tr>
<td>Support</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 - Bloom's Taxonomy Action Verbs

Cangelosi’s learning levels help argue that when teaching or assessing a given concept the content, should influence the method used for teaching or assessing the material (Cangelosi, 2003). “These learning levels describe the kinds of thinking typically required in learning mathematics and are ordered according to a learning progression” (Kohler & Alibegovic). See Table 2 for a list of the Cangelosi learning levels and a description (Cangelosi, 2003).
<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct a Concept</td>
<td>“Use inductive reasoning to distinguish examples of a particular concept from non examples of that concept” (Cangelosi, 2003).</td>
</tr>
<tr>
<td>Discover a Relationship</td>
<td>“Use inductive reasoning to discover that a particular relationship exists or why the relationship exists” (Cangelosi, 2003).</td>
</tr>
<tr>
<td>Simple Knowledge</td>
<td>“Remembering a specified response (but not a multiple-step process) to a specified stimulus” (Cangelosi, 2003).</td>
</tr>
<tr>
<td>Comprehension &amp; Communication</td>
<td>“(i) Extracting and interpretation meaning from expression, (ii) using the language of mathematics and (iii) communicating with and about mathematics” (Cangelosi, 2003).</td>
</tr>
<tr>
<td>Algorithmic Skill</td>
<td>“Remembering and executing a sequence of steps in a specific procedure” (Cangelosi, 2003).</td>
</tr>
<tr>
<td>Application</td>
<td>“Use deductive reasoning to decide how to utilize, if at all, a particular mathematical content to solve problems” (Cangelosi, 2003).</td>
</tr>
<tr>
<td>Creative Thinking</td>
<td>“Use divergent reasoning to view mathematical content from unusual and novel ways” (Cangelosi, 2003).</td>
</tr>
</tbody>
</table>

Table 2 - Cangelosi Learning Levels

In an article written by Kohler and Alibegovic the following is said about student learning processes that are illustrated in the Cangelosi learning levels.

“Students construct concepts and discover relationships before they are prepared to attach conventional mathematical names and procedures to new ideas and commit those ideas to memory. Throughout the learning process students must explain their mathematical understanding using more formal notation and vocabulary as they deepen their comprehension. Finally, though not necessarily only at the end of a learning unit, students are prepared for more cognitively demanding work like the deductive thinking required in applications and truly creative work with mathematics. This is the stage when students bring
mathematical understanding into their own practice of creative problem solving” (Kohler & Alibegovic).

By implementing these learning levels in the assessment, students will be able to discover and gain further knowledge while taking the test. Thus, having students take a test will not only give them an opportunity to demonstrate what they have learned but also allow them to further their intellectual growth and continue making connections they have not previously made.

2.3 – Test Item Construction

When constructing test items it must be decided if the question would be most effective as multiple-choice or constructed response. To make this decision the pro and cons must be weighed and a conclusion drawn on what stimulus you want to provide the students. In an article by Haladyna, Downing and Rodriguez they state that:

“teachers have a difficult time in assessing complex abilities, such as writing or mathematical problem solving. Although we emphasize developing these complex abilities in students, as a general rule we lack the technology to write test items to assess these complex abilities. Therefore, we need better item formats and clear guidelines to help us write test items to better assess complex student learning” (Haladyna, Downing, & Rodriguez, 2002).

To help address this problem, Haladyna, Downing and Rodriguez have provided a summary of guidelines to help educators write test items, more specifically multiple-choice test items. These guidelines can be viewed in the appendix. Of those guidelines there were a few that stood out to me in my construction of the test items.
When writing test items the reading and vocabulary need to be on a level students are comfortable with. Simplifying the language to fit the level of the students will help reduce the effects of reading bias (Haladyna, Downing, & Rodriguez, 2002). In constructing the test items for this project there was a need to simplify some of the wording as well as change a few words that may have been unfamiliar to many of the students.

In the study conducted by Haladyna, Downing and Rodriguez’s on the construction of test items, there was some controversy on the use of negative stems. The use of negative stems is discouraged and when used should be written with caution. The negative term should be placed in boldfaced and capitalized typeface so that it stands out to the students (Haladyna, Downing, & Rodriguez, 2002). In the context of this project a negative stem did become necessary for one of the test items. The suggestions discussed here were utilized in order to prevent misunderstanding.

Another common dispute in literature on the construction of multiple-choice items concerns the number of distracters that should be used. Common opinions range from three to five distracters. Rodriguez found that using two distracters maximizes the reliability of a test. This allows additional items to be placed on the test since fewer distracters mean the students are able to work through the questions more quickly (Rodriguez, 2005). A lower number of distracters also provide an additional time benefit to the teacher because of the extra effort that would be required to develop more plausible distracters. Research has found that for tests with many multiple-choice questions two distracters is the most efficient in terms of time and final reliability (Rodriguez, 2005; Haladyna, Downing, & Rodriguez, 2002).
Many educators feel that multiple choice items can only be used to test lower levels of thinking. This can be true because the very construction of multiple-choice items leads to convergent thinking since test takers are required to select one of the provided answers and it can be difficult to write multiple-choice items that measure complex performance and divergent thinking (Martinez, 1999). But with practice and dedication multiple choice items “can be written to elicit complex cognitions, including understanding, prediction, evaluation and problem solving” (Martinez, 1999). This ideal is what was strived for in this project.

Constructed response items are usually associated with higher levels of cognitive demand than multiple-choice items. They often seek to have students “compose a response that has qualities of novelty and complexity” (Martinez, 1999). But even constructed response items can be written in a way that focuses on the low levels of cognitive demand (Martinez, 1999).

Unfortunately, there were fewer resources and guidelines for constructed response items compared to multiple-choice. Thus, many of the multiple-choice guidelines were used when writing the constructed response items.

When deciding if the test item being written is going to be multiple choice or constructed response it is important to consider the strengths and weaknesses of each. To help weigh the strengths and weakness, Michael Martinez has summarized their characteristics in Table 3 which I have expanded upon (Martinez, 1999). The attributes marked with a (+) are considered elements desirable in test construction and implementation.
## A Comparison of Multiple-Choice and Constructed-Response (CR) Item Formats

Modified from (Martinez, 1999)

<table>
<thead>
<tr>
<th>Comparative Dimension</th>
<th>Multiple Choice</th>
<th>Discrete CR</th>
<th>Extended-Performance CR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cognitive Features</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cognitive Range</td>
<td>Smaller</td>
<td>Larger</td>
<td>Possibly very Large (+)</td>
</tr>
<tr>
<td>Structural Fidelity</td>
<td>Low</td>
<td>Possibly Higher</td>
<td>Possibly very High (+)</td>
</tr>
<tr>
<td>Utility for Diagnosis</td>
<td>Possibly Lower</td>
<td>Possibly Higher (+)</td>
<td>Possibly Higher (+)</td>
</tr>
<tr>
<td><strong>Item and Test Characteristics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Test-Wiseness]</td>
<td>[High]</td>
<td>[Possibly Lower]</td>
<td>[Possibly very Low (+)]</td>
</tr>
<tr>
<td>[Cueing Effect]</td>
<td>[High]</td>
<td>[Possibly Lower]</td>
<td>[Possibly very Low (+)]</td>
</tr>
<tr>
<td>[Test Anxiety]</td>
<td>[Low (+)]</td>
<td>[Possibly Higher]</td>
<td>[Possibly very High]</td>
</tr>
<tr>
<td>Scoring Reliability</td>
<td>Near Perfect (+)</td>
<td>Low to High</td>
<td>Low to Moderate</td>
</tr>
<tr>
<td>Test Reliability</td>
<td>Generally High (+)</td>
<td>[Varies]</td>
<td>Generally Low</td>
</tr>
<tr>
<td>Sampling of Content/Unit Time</td>
<td>High (+)</td>
<td>High (+)</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Economy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Item Development]</td>
<td>[More Expensive]</td>
<td>[Less Expensive (+)]</td>
<td>[Less Expensive (+)]</td>
</tr>
<tr>
<td>[Rubric Construction]</td>
<td>[Less Expensive (+)]</td>
<td>[Less Expensive (+)]</td>
<td>[More Expensive]</td>
</tr>
<tr>
<td>Administration</td>
<td>Less Expensive (+)</td>
<td>Less Expensive (+)</td>
<td>More Expensive</td>
</tr>
<tr>
<td>Scoring and Reporting</td>
<td>Less Expensive (+)</td>
<td>More Expensive</td>
<td>More Expensive</td>
</tr>
</tbody>
</table>

Table 3 - A Comparison of Test Item Types

This comparison table looks at three different elements: ‘cognitive features,’ ‘item and test characteristics’ and ‘economy.’
Cognitive features can be ascertained by examining the construction format based on their cognitive range. Usually it is assumed that constructed response items elicit more complex thinking and can test a greater range of thought processes. Multiple-choice is often assumed to elicit the lower levels of cognitive processes and is somewhat limited in the cognitions that it can reach. Many find it difficult to test productive and creative thinking using multiple-choice. As stated by Guilford, “it is alarming to contemplate what an exclusive use of answer sheet [multiple-choice] tests could do to the intellectual character of a nation” (Guilford, 1967).

Structural fidelity is the “congruence between performance called upon by the test and proficient performance in the referent domain” (Martinez, 1999). As a general rule multiple-choice questions provide the teacher with less information to understand the proficiency of any given student. Extended performance constructed response items on the other hand are better capable of highlighting the strengths and weaknesses of student understanding.

The diagnostic utility is how easy it is to locate the specific difficulties students are having (Martinez, 1999). This can be seen quickly with constructed response items because mistakes are found while grading the responses. This is often more difficult with multiple-choice since it would be necessary to determine what mistakes students made to lead them to select a particular distracter in order to discover what specific concepts students are struggling with.

The second aspect compared is item and test characteristics. One concern with multiple-choice items is that many students are able to mark the correct answer even if they do not know the material simply because they are ‘test wise’ (Martinez, 1999).
When this happens it does not give accurate information on the students’ knowledge and understanding of the material and will decrease the overall reliability of the test. Another concern of multiple choice test items that is a part of ‘test wiseness’ is that of a cueing (Schuwirth & Van Der Vleuten, 2004). Cueing occurs when students are able to guess the correct answer for other questions on the test by using the choices provided for different questions. This can be reduced by following the guidelines and being very conscious in the item construction and ordering of the test items. One positive aspect of multiple-choice that is often a downfall for constructed response items is that they reduce test anxiety. Many students become very nervous during tests and are not able to demonstrate their full understanding of the material as a result. Test anxiety can often be reduced through the use of multiple-choice items (Moskal & Leydens, 2000).

An additional aspect of item and test characteristics is that of reliability. For inter-rater reliability of a test to be high the scores assigned from two different graders would be very similar. The more complex the constructed response items become, there is a greater chance of discrepancy between the two graders’ scores. Test reliability also tends to be higher for multiple choice items because there is less discrepancy between graders and generally more content can be tested, thus resulting in more questions. Where with extended performance constructed response items there has to be a lot fewer questions because of the amount of time questions require to be completed.

The last element that is examined by the table is economy, or the value of the teacher’s time. Multiple choice items typically take the longest to construct because it is necessary to come up with plausible distracters. Constructed response items are often easier to construct but it takes longer to come up with a detailed grading rubric.
(Schuwirth & Van Der Vleuten, 2004). But then scoring is a lot quicker with multiple-choice items then it is for constructed response. Thus there is a fine balance between the time in initial construction and the time spent grading.

With all the advantages and disadvantages of multiple-choice and constructed response it is often difficult to select which format is best. As stated by Schuwirth and Van Der Vleuten, “no single question type is intrinsically superior…we would make the case that, rather than believing in the superiority of a particular question type, it is advisable to set up thorough quality control process, including a careful review of process and item analyses” (Schuwirth & Van Der Vleuten, 2004). To make this decision, it helps to think about the response format and the stimulus format of each equation.

In the article, _Different written assessment methods: what can be said about their strength and weakness_ by Schuwirth and Van Der Vleuten, the authors mention that one thing to consider when deciding whether to use multiple-choice or constructed response is the desired response format and stimulus response for the test item. They explain, “the stimulus indicates what the question wants the candidate to answer and thus pertains to the content of the question. The response format indicates how the response of the candidate is captured” (Schuwirth & Van Der Vleuten, 2004). Once the educator has decided the desired stimulus and response format it is then easier to decide which test item format of to use. In regards to response formats, it is important to decide for each particular concept being tested if “spontaneous generation” of the answer is necessary or if selecting the answer from a list of choices is acceptable. “Constructed response questions should be solely used to test aspects that cannot be tested with multiple-choice questions. In all other cases the loss of reliability and the higher resources-intensiveness
represent a significant downside. In such cases, multiple-choice question are not less valid than open-ended questions” (Schuwirth & Van Der Vleuten, 2004). Between stimulus and response format the most important is stimulus. It should first be decided how we want the students to answer the question and then determine which response format would best capture the desired stimulus (Schuwirth & Van Der Vleuten, 2004).

Even with all the test writing guidelines provided there will always be situations that fall outside of what is recommended. It is then up to the educator to use their best judgment in how to write the test items so they best assess the learning of their students. As stated by Haladyna, Downing and Rodriguez,

“Each item as it is being written presents new problems and new opportunities. Just as there can be no set formulas for producing a good story or a good painting, so there can be no set of rules that will guarantee the production of good test items. Principles can be established and suggestions offered, but it is the item writer’s judgment in application (and occasional disregard) of these principles and suggestions that determines whether good items or mediocre ones will be produced” (Haladyna, Downing, & Rodriguez, 2002).

2.4 – Think Alouds

Once the test item has been constructed it is then recommended to test it on a small select group of individuals before it is administered to the entire sample. The think aloud process allows for information to be collected to estimate the difficulty, the amount of time it will take to complete the item, discover any errors or things that may be unclear and cause confusion. It is beneficial to have individuals discuss their thought processes out loud because “metacognition can be used to monitor solution processes and to
regulate the problem solving episodes of analyzing and exploring a task, making a solution plan, implementing the plan and verifying the answer” (Jacobse & Harskamp, 2012).

For many individuals it is difficult to discuss their ideas and thought process out loud especially when they get stuck and are not sure how to proceed. Encourage them to continue to discuss their ideas out loud without leading them to the answer. Some possible guiding questions that can be used are: ‘Can you show me?’, ‘Is there another step after that?’, ‘Could you draw me a picture?’, ‘Can you explain what you just did?’ ‘Can you think of any mistakes someone else solving the problem might make? Why?’ If they ever ask a question it is best to try to redirect using another question (Schoenfeld & Ball, 2007).

I had the opportunity to observe a think aloud being held live by, Deborah Ball, an educational researcher at the University of Michigan, during a conference I attended at UC Berkeley. At this conference I learned how beneficial it is to listen to someone verbally talk through the test items. This not only helps find mistakes but helps me better understand the thought processes my test items are causing students to exhibit (Ball, 2013).

2.5 – Item Analysis

Item analysis is a method in which the test creator can receive more formal feedback on their test questions so appropriate revisions can be made to improve the test items for future use (Rodriguez, 2005). Item analysis includes examining the item difficulty, item discrimination and the effectiveness of distracters.
As discussed by Niko and Brookhart there are six reasons to conduct an item analysis. The first is to examine if your item functions as intended. Did it assess the desired concepts? Was it the correct level of difficulty? Does it distinguish between those who know the material and those who do not? If it was a multiple-choice item, how well did the distracters function? All of these questions are heavily addressed in this project’s item analysis (see Item Development and Results).

The next three reasons apply more to a classroom teacher who is conducting the item analysis for students in their own classroom. These reasons are to provide feedback to the students and open opportunities for class discussion, to receive feedback on what concepts their students struggled with and what parts of the curriculum could be improved (Nitko & Brookhart, 2007).

Revising the assessment items is the fifth reason for conducting an item analysis. When revisions occur they strengthen the item for the next time it is used. Revising is also a less time consuming process than a complete re-write would be. Because of the item analysis it easier to determine which items need the most revisions and what needs to be done to each item so that it can be more effective. This also provides an efficient method for teachers to develop a strong test bank from which they can pull questions from in the future (Nitko & Brookhart, 2007).

The sixth reason is that it provides feedback to the teacher on their construction of the test items. “Probably the most effective way to improve your item writing skills is to analyze the items and understand the way students respond to them and the use this information to revise items and try them again with students” (Nitko & Brookhart, 2007).
2.6 – Validity/Reliability

When compiling test items and constructing rubrics it is important to look at the overall picture and observe the validity and reliability of both the test and the rubric.

When examining validity, it is important to carefully review each test item (Schuwirth & Van Der Vleuten, 2004). To help in evaluating the validity of a test and scoring rubric Moskal and Leydens have provided some guiding questions (see Table 4).

<table>
<thead>
<tr>
<th>Question to Examine Each Type of Validity Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Content</strong></td>
</tr>
<tr>
<td>1. Do the evaluation criteria address any extraneous content?</td>
</tr>
<tr>
<td>2. Do the evaluation criteria of the scoring rubric address all aspects of the intended content?</td>
</tr>
<tr>
<td>3. Is there any content addressed in the task that should be evaluated through the rubric, but is not?</td>
</tr>
<tr>
<td><strong>Construct</strong></td>
</tr>
<tr>
<td>1. Are all of the important facets of the intended construct evaluated through the scoring criteria?</td>
</tr>
<tr>
<td>2. Is any of the evaluation criteria irrelevant to the construct of interest?</td>
</tr>
<tr>
<td><strong>Criterion</strong></td>
</tr>
<tr>
<td>1. How do the scoring criteria reflect competencies that would suggest future or related performances?</td>
</tr>
<tr>
<td>2. What are the important components of the future or related performance that may be evaluated through the use of the assessment instrument?</td>
</tr>
<tr>
<td>3. How do the scoring criteria measure the important components of the future or related performance?</td>
</tr>
<tr>
<td>4. Are there any facets of the future or related performance that are not reflected in the scoring criteria?</td>
</tr>
</tbody>
</table>

Table 4 - Questions to Examine Each Type of Validity Evidence

Without reliability it is impossible to have validity. Reliability refers to how consistent the test scores are. The construction of a good rubric can help in improving
the reliability of the test scores. Inter-rater reliability it is how close the scores would match if two different people were grading (Moskal & Leydens, 2000). The easiest way to achieve inter-grader reliability is to use multiple choice items because constructed response questions usually lower the reliability. This occurs for a couple of different reasons the first being that it takes longer to answer a constructed response question and thus there are fewer questions on the test. Ideally, the more questions on the test the higher the reliability will be. The second reason is that multiple choice questions have a high inter-rater reliability due to the inability to give partial credit on a given problem. For constructed response items there is more variability in how the points can be assigned (Schuwirth & Van Der Vleuten, 2004). The use of a good rubric can help increase the reliability when scoring constructed response items.

There are a few different tests that can be used to determine the reliability. The first is Kuder-Richardson 20. This is used when your test consist of dichotomously scored items. If the test consists of constructed response questions it is necessary to use coefficient alpha (Reynolds, Livingston, & Willson, 2009).

There are many different aspects that have to be considered when developing effective test items and ensuring that your test has good validity and reliability. All these different little pieces can often feel overwhelming. Michael Rodriquez says it best when he said, “Item-writing is an art. It requires uncommon combination of special abilities. It is mastered only through extensive and critically supervised practice. It demands, and tends to develop, high standards of quality and a sense of pride in craftsmanship” (Rodriguez, 2005).
CHAPTER 3

METHODS AND PROCEDURES

3.1 – Sample

To develop and analyze the higher cognitive demand test items there were multiple things that needed to occur. These included selecting a sample, developing the test items, constructing a grading rubric, administering the test items, determining an evaluation criterion, conducting an item analysis and making appropriate revisions to the test items.

The sample of student’s was taken from Layton High School. Three teachers of Secondary Math II volunteered to have their honor classes participate. This resulted in five classes, approximately 160 students. Permission was obtained from the administration and a letter was provided to the parents to inform them that their student would be participating in a study on the analysis of test items (see Appendix for Letter of Information). None of the parents requested their student not participate.

3.2 – Test Item Construction

For the content of the test items there were three chapters selected with the intention of approximately one problem for each section of a chapter (Integrated Math II: A Common Core Math Program, Pittsburg). From the school’s textbook, the three chapters that were selected were Chapter 8 on trigonometry which would be tested during the time frame of November 21-26, 2013, Chapters 9 and 10 were tested together and included topics such as circles, arcs and sectors of circles. These two chapters were tested during the time frame of December 12-17, 2013.
As the test items were developed, there were many details that went into the construction and review process to ensure the items were ready for administration. Once the test items were administered, the item analysis was conducted and final revisions were made. A summary of this process can be viewed in Figure 2.

![Flow Chart of Test Item Development](image)

Figure 2 – Flow Chart of Test Item Development

As can be viewed on the top row of Figure 2 the requirements for each test item included its alignment with the content taught in the students’ textbooks and class which were in alignment with common core state standards and the mathematical practices standards. Bloom’s taxonomy and Cangelosi’s learning levels were used to help achieve the desired level of cognition.
Following an examination of the student’s textbook, items were constructed based on the provided material. Additionally, a special effort was made to ensure that the test items’ notation and vocabulary was consistent with that found in the textbook. It was then necessary to decide how the concepts of a chapter could be combined so students could discover relationships and learn while taking the test.

Each test item was constructed to fulfill one or more common core state standards. If a standard was more encompassing than the test item allowed for, the item was developed to fulfill a portion of the standard. Once the test items were constructed they were assigned mathematical practice standards.

When the initial idea for each item had been drafted, the test items were then constructed so that they reached one of Bloom’s highest four levels: application, analysis, synthesis and evaluation. It was challenging to move away from constructing test items that fell under the knowledge and comprehension levels and push toward higher levels of cognitive demand. To reach these desired levels each item went through several drafts and rewordings.

Cangelosi learning levels were also applied to each question during its development process to diversify the types of test items. Emphasis was placed upon the levels that reached the higher levels of cognitive demand.

When the test items were completed they were sent to Dr. Brynja Kohler and Dr. James Cangelosi for further review. Their feedback addressed parts of the question that were unclear, identifying items that were not in alignment with common core standard, use of incorrect notation and inaccurate interpretation of mathematical definitions.
The items were then analyzed further by conducting think alouds. It was decided that participants with some math experience would be best so that the items could be completed and information gained by the think alouds. Friends and family members that met this criterion were selected to participate in each think aloud, making it easier to work together and coordinate times. The math experience of the individuals who participated included: a recent college graduate in History, a freshman majoring in Statistics, a high school senior who is taking AP Statistics and a high school math teacher. The test items were once again revised based on feedback from the think alouds. The feedback from the think alouds resulted in correction of mistakes that had not previously been recognized and the reordering and rewording of some questions.

Upon conclusion of the think alouds a rubric was then constructed. This involved assigning points based upon the amount of work each item required and the estimated difficulty of that item. The rubric also included a breakdown of what was required to receive a given number of points for each item. Many of these breakdowns were decided based on observations made during the think aloud.

3.3 – Test Item Administration & Grading

During the test administration phase of the study teachers were sent the test items, a copy of the solutions and a grading rubric at least one week before they were administered. This provided the teachers with an opportunity to request any changes and allow them time to gather or develop test items to cover any additional material they wished to be tested. This allowed for at least part of the test to be a style the students were familiar with. In order to encourage a legitimate effort in completing the test it was decided that the study portion of the test would be included on the students’ grades.
Upon completion of the test administration the study portion of the test was returned to me for grading in order to reduce variability. Points were assigned based on the criteria from the rubric. It was also at this point students were also given an identifier based on their teacher and alphabetical order. Teachers were given their students’ results on the study portion of the test and then provided me with students’ results on their portion of the test.

3.4 – Item Analysis

Students’ overall scores were then entered into an Excel spreadsheet and sorted from highest to lowest based on their overall percentage. It was necessary to use percentages since each of the three teachers written portions of the test were worth a different amount of points. The students were sorted into the upper, middle and lower groups based on their overall percentage. The advantage of using the combined score is that it ideally would give a better representation of the students’ knowledge. A disadvantage was that the teachers’ portion of the test added a source of outside bias into the item analysis.

Based on a derivation by Kelley in 1939, the upper and lower 27% of the sample is used to compute the item analysis (Kelley, 1939). There was some slight deviation to account for convenient breaking points in both the upper and lower group that would result in the groups being the same size. For the individuals in these two groups, detailed information for each item was entered into the spreadsheet for use in the item analysis. Analysis was conducted only on the test items written for the study. The item analysis included the item difficulty and item discrimination values for each test item. For test reliability, Kuder-Richardson 20 and coefficient alpha were conducted.
To compute the item analysis for the multiple choice items each student in the upper and lower group was assigned a zero if they missed the question and a one if they answered correctly. For those who answered incorrectly their selected distracter was noted. For the middle group, information was simply taken if they got the item correct or incorrect.

To compute the difficulty of a test item Equation 1 was used (Nitko & Brookhart, 2007).

\[
\text{Difficulty (p)} = \frac{\text{number of students choosing the correct answer in the upper, middle and lower group}}{\text{total number of students taking the test}}
\]

Equation 1

Item difficulty ranges from 0.0 to 1.0 where a larger number represents an easier question. Anything above 0.80 is considered to be an easy question and anything below 0.30 is considered to be difficult (Tarrant & Ware, 2012). What falls in between these numbers is considered a medium level question.

When working with multiple choice items it is important to consider the student correctly guessing the answer as opposed to knowing the correct answer. In order to do this there is an optimal difficulty level based on the number of available choices see (Table 5). To compute the optimal difficulty level one can use Equation 2 (Assess Students Item Analysis, 2011).

\[
\text{Optimal Difficulty Level} = \left[ \frac{\text{probability of guessing the correct answer}}{\text{the correct answer}} \right] + \frac{1.00 - \left( \text{probability of guessing the correct answer} \right)}{2}
\]

Equation 2

To compute the discrimination for the multiple choice items Equation 3 was used (Nitko & Brookhart, 2007).
Equation 3

\[
\text{Discrimination (D)} = \left[ \frac{\text{fraction of the upper group that answered correctly}}{} \right] - \left[ \frac{\text{fraction of the lower group that answered correctly}}{} \right]
\]

The item discrimination helps determine how well a test item distinguished between students who understand the material and ones that do not. This value ranges from -1.00 to 1.00. If a question’s discrimination falls into the negative range, more students in the lower group answered the question correctly then those in the upper group and the question then needs serious revisions. The interpretation of the discrimination value if it falls between 0.00 and 1.00 can be observed in Table 5 (Tarrant & Ware, 2012).

When the discrimination value falls too close to 0.00 or to 1.00 it does not help in identifying if the item was effective, because either almost everyone got it right or almost everyone got it wrong.

“Most discrimination values are biased in favor of items with intermediate difficulty levels. That is the maximum discrimination value of an item is related to its difficulty value. Items that all test takers either pass or fail cannot provide any information about individual differences and their discrimination value will always be zero. If half the test takers correctly answer an item and half failed then it is possible for the item’s discrimination value to be 1.0. This does not mean that all items with difficulty values of 0.50 will have a discrimination value of 1.0 but just that the item can conceivably have a discrimination value of 1.0” (Reynolds, Livingston, & Willson, 2009).

Thus, when examining the discrimination value of an item one must also look at it in relation to the item’s difficulty level. The really easy and hard questions may receive
what appears to be a terrible discrimination value but it actually may be close to the maximum possible discrimination value based on the questions difficulty level. See Table 5 for maximum discrimination values.

For the constructed response items it was necessary to compute the average to calculate the difficulty and discrimination values of test items. For clarification purposes equations used to compute the item difficulties and discriminations for constructed response have an asterisk. For $p^*$ see Equation 4 and for $D^*$ see Equation 5 (Nitko & Brookhart, 2007). In order to “maximize the variability and reliability [of constructed response items] the optimal difficulty level is 0.50” (Reynolds, Livingston, & Willson, 2009). It is usually best to select a variety of items so that some have a difficulty value greater than 0.50 and others have a value less than 0.50 (Reynolds, Livingston, & Willson, 2009).

$$\text{Difficulty (p *)} = \frac{\text{average score} - \text{minimum possible score}}{\text{maximum possible score} - \text{minimum possible score}}$$

Equation 4

$$\text{Discrimination (D *)} = \frac{\left[\text{average score of the upper group}\right] - \left[\text{average score of the lower group}\right]}{\text{maximum possible score} - \text{minimum possible score}}$$

Equation 5

To interpret the difficulty and discrimination of each item an interpretation table (see Table 5) was created by combining ideas from a multiple resources (Reynolds, Livingston, & Willson, 2009; Nitko & Brookhart, 2007; Tarrant & Ware, 2012; Assess Students Item Analysis, 2011; Understanding Item Analysis Reports, 2005).
<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>What is Measured</th>
<th>Range of Values</th>
<th>Interpretation</th>
<th>Additional Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difficulty (p)</td>
<td>The proportion of students who answered the item correctly.</td>
<td>0 to 1.00</td>
<td>Low Difficulty: &gt;.80</td>
<td><strong>Optimal Item Difficulty</strong> Choices: Optimal Value:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Medium Difficulty: .30-.80</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High Difficulty: &lt;.30</td>
<td>3</td>
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<td>4</td>
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<td></td>
<td>6</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CR</td>
</tr>
<tr>
<td>Discrimination (D)</td>
<td>The difference in the number of high-achieving and low-achieving students who answered the item correctly</td>
<td>-1.00 to 1.00</td>
<td>Excellent: ≥.40</td>
<td><strong>Maximum Discrimination Values</strong> Difficulty: Max D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Good: .30-.39</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Satisfactory: .15-.29</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Low: &lt; .15</td>
<td>0.80</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.70</td>
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<td>0.60</td>
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<td>0.50</td>
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<td>0.40</td>
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<td>0.30</td>
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<td>0.20</td>
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<td></td>
<td></td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
</tr>
<tr>
<td>Test Reliability</td>
<td>Indicates the homogeneity of test items</td>
<td>0 to 1.00</td>
<td>Excellent: ≥.90</td>
<td><strong>Kuder - Richardson 20:</strong> Multiple Choice Coefficient Alpha: Constructed Response</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Good: .70-.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Satisfactory: .50-.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Low: ≤ .49</td>
<td></td>
</tr>
</tbody>
</table>

Table 5 - Description and Interpretation of Item Analysis
The Kuder-Richardson 20 was computed to determine test reliability for the multiple-choice portion of the test. In this reliability test \( k \) is the number of multiple choice items test items, \( \rho \) is the items difficulty level and \( SD_x \) is the standard deviation of the total score for each individual that are then summed together (Nitko & Brookhart, 2007).

\[
KR20 = \left( \frac{k}{k-1} \right) \left( 1 - \frac{\sum \rho(1 - \rho)}{(SD_x)^2} \right)
\]

Equation 6

For the constructed response items, a coefficient alpha was calculated to determine reliability. Where \( k \) is the number of constructed response test items, \( SD_l \) is the sum of the standard deviation for each test item, and \( SD_x \) is the standard deviation of the total score for each individual in the upper and lower groups and then summed (Nitko & Brookhart, 2007).

\[
Coefficient\ Alpha = \left( \frac{k}{k-1} \right) \left( 1 - \frac{\sum (SD_l)^2}{(SD_x)^2} \right)
\]

Equation 7

Item analysis can also be used to examine the effectiveness of the distracters. As a general rule: “Every distracter should have at least one lower group student choosing it, and more lower group students than upper group students should choose it” (Nitko & Brookhart, 2007). Examining how effective the distracters were and then making appropriate revisions can help increase the items difficulty and discrimination values.

Upon the conclusion of the analysis for each test item a decision was made on whether revisions needed to be made (see Item Development).

Upon the conclusion of revisions the participating teachers received a copy of the revised test items, rubrics and solutions.
CHAPTER 4

ITEM DEVELOPMENT

Many aspects contributed to the design of test items for this project. The first step was to decide on a common core standard and then design an item that fit the standard or at least a portion of the standard. Once the standard had been selected the students’ text book was examined in order to understand the notation and format of the questions that they are use to working with. Then a very rough draft of each item was constructed that attempted to reach a high level of cognitive demand. To help reach these desired higher levels of cognitive demand Bloom’s taxonomy and Cangelosi learning levels were used.

After the initial draft, each item then went through a refining process which included the help of many individuals. The test items were first given to members of my committee for comment and review and then went through a think aloud protocol. After this process they were administered to the students. Based on item analysis results the test items were then revised for the final time.

This chapter will follow a select few problems through the initial construction and revisions that were made. To view all items constructed, see the Appendix. Note that in this section a problem refers to a grouping of questions that test a given concept.

4.1 – Chapter 8

Chapter 8 from their textbook, Integrated Math II: A Common Core Math Program was on trigonometry (Integrated Math II: A Common Core Math Program, Pittsburg). Some of the included concepts were trigonometric ratios of similar triangles, trigonometric functions, complement angle relations and law of sines.
Problem 2

*Initial Construction:*

This set of questions was designed to have students select the trigonometric ratio that would be used for the given situations if they were to solve. The last question requires the student to select one of the given situations and solve.

These questions addressed common core standards G.SRT #8 which states “use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems” and G.SRT #1 which states “use geometric shapes, their measures and their properties to describe objects.”

The first four questions, question one through four, fall under the Cangelosi level and Bloom’s taxonomy level of application, because students are applying trigonometric ratios to given situations. On the last question students are being asked to choose one of the above prompts and use the selected trigonometric ratio to solve, this falls under Bloom’s level of evaluation and Cangelosi’s of algorithmic skill.

---

**Given the following situations choose which trigonometric ratio would prove most useful for solving.**

A. Sine  
B. Cosine  
C. Tangent  
D. Inverse Sine  
E. Inverse Cosine  
F. Inverse

_____1. You and your younger brother are playing on a see-saw (a teeter-totter) at the park. The see-saw is 12 feet long and the middle support is 3 feet off the ground. When you hit the ground what angle does the see-saw make with the middle support?

_____2. You are having a picnic with her friends and all of a sudden a swarm of ants overtakes their food. Abigail realizes that one of the ants is looking up at her from a 36° angle. You are 5 feet tall. How far away is the ant from Abigail?

_____3. You are on vacation with your family in Redwood National Park in California and you want to take a picture of the tallest redwood tree on record which is 379 feet. You are standing 100 feet from the base of the tree. At what angle would you need you camera to be at so he could make sure the top of the tree is in the middle of your picture?
4. In the apartment building near your house a fire gets started on fifth floor which is 50 feet above the ground. The firemen show up and need to rescue someone who also lives on the fifth floor, but their boom truck is broken from trying to rescue a cat from a tree so they must use an old fashion ladder. The firemen set the ladder up with a 75° angle to the ground as determined by safety regulations. How long does the ladder need to be so it reaches the fifth floor and the firemen are able to rescue the individual?

Select one of the above examples and use your trigonometric ratio of choice to solve. 
Explain why you selected to use the trigonometric ratio.

Item after Comments from Committee and Think Alouds (Revision 1):

Based on comments from my committee it was decided that first, the questions needed to be worded in third person and not in second. Second, the statement “explain why you selected to use the trigonometric ratio” was removed because it was a redundant question. It was also decided that students would be required to sketch a picture as part of the answer.

After the think alouds, the only change that was made was in question four. It was decided that too many individuals, including the individual doing the think aloud, would not know what was meant by the phrase ‘boom truck.’ Thus the wording was changed to ‘fire truck.’

Conclusions from Item Analysis:

Question one had a difficulty of 0.148 (really hard) and a discrimination of 0.214 (satisfactory). Since this was the most difficult of the four questions it was decided to have it be the second to last question in this problem. Hopefully this will allow students to have a few less difficult questions to help them be comfortable with the test before they are faced with a more difficult question.

Questions two, three and four had a difficulty ranging from 0.503 - 0.544 which is a medium level question with the optimal difficulty for an item with six choices being...
0.58. The discrimination for these items were in the range of 0.493 - 0.643 which is considered excellent and no changes were made.

Question five had difficulty of 0.493 (medium) and excellent discrimination of 0.595. Thus no revisions were made.

**Item after Final Revisions (Revision 2):**

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<tbody>
<tr>
<td>A. Sine</td>
<td>B. Cosine</td>
</tr>
<tr>
<td>D. Inverse Sine</td>
<td>E. Inverse Cosine</td>
</tr>
</tbody>
</table>

_____ 1. Abigail is having a picnic with her friends and all of a sudden a swarm of ants overtakes their food. Abigail realizes that one of the ants is looking up at her from a 36° angle. She is 5 feet tall. How far away is the ant from Abigail?

_____ 2. Samuel is on vacation with his family in Redwood National Park in California. He wants to take a picture of the tallest redwood tree on record which is 379 feet. He is standing 100 feet from the base of the tree. At what angle would he need his camera to be at so he could make sure the top of the tree was in the picture?

_____ 3. In the apartment building near Sofia’s house a fire gets started on fifth floor which is 50 feet above the ground. The firemen show up and need to rescue Sofia’s friend, Rachel who also lives on the fifth floor, but their fire truck is broken from trying to rescue a cat from a tree so they must use an old fashion ladder. The firemen set the ladder up with a 75° angle to the ground as determined by safety regulations. How long does the ladder need to be so it reaches the fifth floor and the firemen are able to rescue Rachel?

_____ 4. Jonathon and his younger brother are playing on a see-saw (a teeter-totter) at the park. The see saw is 12 feet long and the middle support is 3 feet off the ground. When Jonathon hits the ground what angle does the see-saw make with the middle support?

5. Select one of the above examples and write a valid equation that can be used to solve for the unknown. Then solve the equation. (Make sure you include a picture as part of your work).

The following mathematical practice standards were used in this problem: ‘make sense of a problem and preserve in solving,’ ‘reason abstractly and quantitatively,’ ‘model with mathematics,’ and ‘attend to precision.’
Problem 3

Initial Construction:

This problem was constructed for students to understand and “explain the relationship between sine and cosine,” which is common core standard G.SRT #7. This problem addressed a couple of different learning levels. The first question asked students to select the trigonometric ratio they would need to solve the given situation. The information is given to the students abstractly instead of quantitatively to increase the cognitive thinking level of the test question. For the Cangelosi and Bloom learning levels this is an application. The next set of questions, question two, help students comprehend and communicate the relationship between sine and cosine thus it is a comprehension and communication question according to Cangelosi (Cangelosi, 2003). Question two, filling in the table and the follow up question part (a) are on the synthesis level of Bloom’s taxonomy. Then on part (b) students are asked to draw a conclusion which falls under the evaluation level.
1. You are given the hypotenuse and an acute angle measure of a right triangle. You are solving for the length of the opposite leg. What trigonometric ratio would you select to use?
   a) secant
   b) cosecant
   c) inverse cosine
   d) inverse sine

2. Given the right triangle ABC and your understanding of sine and cosine fill in the chart below using the following multiple choice options.

   ![Diagram of right triangle ABC]

<table>
<thead>
<tr>
<th>Reference Angle</th>
<th>Sin</th>
<th>Cos</th>
<th>Csc</th>
<th>Sec</th>
</tr>
</thead>
<tbody>
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<td>A</td>
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<tr>
<td>C</td>
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</tbody>
</table>

   A. \( \frac{a}{b} \)  B. \( \frac{a}{c} \)  C. \( \frac{b}{a} \)
   D. \( \frac{b}{c} \)  E. \( \frac{c}{a} \)  F. \( \frac{c}{b} \)

a) What is the relationship between \( \angle A \) and \( \angle C \)?
   a) Complementary Angles  b) Corresponding Angles  c) Supplementary Angles

b) What conclusions can you draw about the relationship between the trigonometric functions of \( \angle A \) and \( \angle C \)?

Now combine your answer from part (a) and part (b) to write a final statement about the relationship between \( \angle A \) and \( \angle C \).

---

**Item after Comments from Committee and Think Alouds (Revision 1):**

The importance of questions being worded in third person was again brought to my attention from the feedback provided by committee members, thus appropriate changes were made to question one. During the think alouds it was decided that the question would flow better if question two parts (a) and (b) were switched. This would then have students give their conclusions right after finishing the table instead of having to answer another question in between.
a) What conclusions can you draw about the relationship between the trigonometric functions of \( \angle A \) and \( \angle C \)?

b) What is the relationship between \( \angle A \) and \( \angle C \)?
   a) Complementary Angles  
   b) Corresponding Angles  
   c) Supplementary Angles

c) Now combine your answer from part (a) and part (b) to write a final statement about the relationship between \( \angle A \) and \( \angle C \).

Conclusions from Item Analysis:

Question one had a difficulty of 0.436 (medium) with the optimal difficulty for a question with four choices being 0.62. The discrimination was 0.190 (satisfactory). The decision was made to change distracter (d) because it distracted 12 of the 40 in the upper group and nine of forty in the lower group. In the lower group distracter (a) was the most effective. By changing distracter (d) this will hopefully lower the difficulty level slightly and increase the discrimination between the upper and lower group. Distracter (d) started out as the inverse sine with the correct answer to the item being cosecant. One possibility was to reduce the item to three choices; secant, cosecant, and cotangent. But by leaving the inverse options in students have to demonstrate that they know when to use the inverse of a trigonometric function. Thus it was decided to change inverse sine to inverse tangent hoping it would not pull as many students from the upper group.

For question two, the difficulty was 0.919 (really easy) with a discrimination of 0.211 (satisfactory). The first part of the question, which required the students to fill in the table, was intended to help give students some easy points. The summary questions following the table had a medium difficulty level. Part (a) had a difficulty of 0.484 (medium) with the optimal level being .050 and a discrimination of 0.393 (good). Thus no changes were made. Question (b) had a difficulty of 0.497 (medium) with a difficulty of 0.238 (satisfactory). Even though this question was slightly more difficult than the optimal level of 0.67, it was decided this was acceptable because of how easy the table
portion of this question was. Again no changes were made. No analysis was conducted on question (c) because students were asked to combine their answers from previous responses so they could reach a conclusion. Students were given points even if they had gotten previous responses wrong but could correctly combine their previous answers on part (c).

**Item after Final Revisions (Revision 2):**

1. Given the hypotenuse and an acute angle measure of a right triangle. Solve for the length of the opposite leg. What trigonometric ratio should be used?
   - A) secant
   - B) cosecant
   - C) inverse cosine
   - D) inverse tangent

2. Given the right triangle ABC and your understanding of sine and cosine fill in the chart below using the following multiple choice options.

   \[
   \begin{align*}
   A &: \frac{a}{b} & B: \frac{a}{c} & C: \frac{b}{a} \\
   D: \frac{b}{c} & E: \frac{c}{a} & F: \frac{c}{b}
   \end{align*}
   \]

<table>
<thead>
<tr>
<th>Reference Angle</th>
<th>Sin</th>
<th>Cos</th>
<th>Csc</th>
<th>Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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</table>

a) What conclusions can you draw about the relationship between the trigonometric functions of \( \angle A \) and \( \angle C \)?

b) What is the relationship between \( \angle A \) and \( \angle C \)?
   - a) Complementary Angles
   - b) Corresponding Angles
   - c) Supplementary Angles

c) Now combine your answer from part (a) and part (b) to write a final statement about the relationship between \( \angle A \) and \( \angle C \).

The following mathematical practice standards were used in the above problem set: ‘make sense of a problem and preserver in solving,’ ‘reason abstractly and quantitatively,’ and ‘construct viable arguments.’
4.2 – Chapter 9

Chapter 9 was on circles. Some of the included topics were: elements of circles, inscribed angle theorem, arc addition postulate, exterior angle theorems, tangent and secant theorems, and chords.

Problem 1

Initial Construction:

This problem was initially constructed to assess students understanding on chords, diameter, radius, secant and tangent lines. It was written in accordance with the common core standard of G.CO #1 which says, “know precise definition of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notations of point, line, distance along a line, and distance around a circular arc.”

1. Categorize the terms based on those that are lines and those that line segments.

<table>
<thead>
<tr>
<th>a) chord</th>
<th>b) diameter</th>
<th>c) radius</th>
<th>d) secant</th>
<th>e) tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line</td>
<td>Line Segment</td>
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<tr>
<td>Line</td>
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<tr>
<td>Line</td>
<td>Line Segment</td>
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</tr>
</tbody>
</table>

1a. Is there a difference between a chord and a secant? Yes/No

2. Disprove the following statement by drawing an illustration using circle Z “All secants of a circle are parallel.”
Item after Comments from Dr. James Cangelosi (Revision 1):

After clarification from Dr. Cangelosi I realized that this item needed significant modifications. The assumption had been made that a diameter and radius were line segments. This is not true as they are actually dimensions. As a result, having students categorize terms on whether they were lines or line segments would no longer work. It then became necessary to test the standard in a different way. Thus the following item was constructed.

1. Which of the following is not a characteristic of a chord?
   a) A chord is a line segment
   b) A chord has both endpoints on the circle
   c) A chord has two points intersecting the circle

2. Disprove the following statement by drawing an illustration using circle Z
   “All secants of a circle are parallel.”

3. Read each numbered statement and decide which of the following arcs it describes.
   a) $m\widehat{ABC}$
   b) $m\widehat{EFG}$
   c) $m\widehat{ADB}$
   d) $m\widehat{ETH}$
   e) $m\widehat{HGE}$

   ___________ 3. Which of the above is a major arc?
   ___________ 4. Which of the above is a minor arc?
Item after Additional Comments (Revision 2):

Question one was created to fulfill the common core standard of testing student’s knowledge of chords. This question became necessary because the concept of chords was removed from what is now labeled as question three.

After receiving additional feedback on the revised item, I came to the conclusion that the item was still poorly written and would need to undergo additional revisions. Some of those reasons included the fact that a major arc is represented by three points around the circle while a minor arc is represented by two. In the prompt, the minor arcs were also represented by three points on the circle so that they could not as easily be picked out from the major arcs. This deviated from the traditional mathematical practice for describing minor arcs. Also the m at the beginning of each choice was not necessary since the students are not being asked to provide a measurement. To accommodate these changes the following item was constructed.
1. Which of the following is not a characteristic of a chord?
   A) A chord is a line segment
   B) A chord has both endpoints on the circle
   C) A chord has two points intersecting the circle

2. Using the given circle categorize the following:
   A) $\overline{BAC}$
   B) $\angle E$
   C) $\overline{BD}$
   D) $\angle D$
   E) $\overline{AD}$
   F) $\angle C$
   G) $\overline{BAC}$
   H) $\overline{BAD}$
   I) $\angle G$

<table>
<thead>
<tr>
<th>Central Angle</th>
<th>Inscribed Angle</th>
<th>Semicircle</th>
<th>Major Arc</th>
<th>Minor Arc</th>
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</tbody>
</table>

3. Disprove the following statement by drawing an illustration using circle K
   "All secants of a circle are parallel."

---

**Item after Think Alouds (Revision 3):**

At the conclusion of the think alouds it was determined that there were too many things combined into one common list on question two. This made it too easy for think aloud participants to use process of elimination in order to answer the question, not giving a true representation of what the students knew. It was then decided that this question would be changed to a constructed response instead of matching.
1. Which of the following is **not** a characteristic of a chord?
   A) A chord is a line segment
   B) A chord has both endpoints on the circle
   C) A chord has two points intersecting the circle

2. Using the given circle fill in the table providing two examples of each. Your answers should include the use of correct notation.

<table>
<thead>
<tr>
<th>Central Angle</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Inscribed Angle</td>
<td></td>
</tr>
<tr>
<td>Major Arc</td>
<td></td>
</tr>
<tr>
<td>Minor Arc</td>
<td></td>
</tr>
<tr>
<td>Semicircle</td>
<td></td>
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</tbody>
</table>

3. Disprove the following statement by drawing an illustration using circle K
   “All secants of a circle are parallel.”

Once this item was more firmly developed it was then possible to assign learning levels. Since students were being asked to identify something that is not a characteristic of a chord. This first question is an evaluation and construct a concept. Question two, which asks students to fill in the table is a comprehension for Bloom’s level and comprehension and communication under Cangelosi’s learning level because students are “demonstrating they understand the concepts” (Reference Materials, 2014). Question three is evaluation under Bloom’s taxonomy and application under Cangelosi’s learning levels because it is asking students to disprove a statement. Students are being required to apply their knowledge to disprove the statement but also defend the statement through a sketch.
Conclusions from Item Analysis:

For question one the difficulty was 0.926 (easy) where the optimal difficulty level for a question with three choices is 0.67 and the actual discrimination was 0.132 (low). In the research based construction guidelines it recommends avoiding using negative stem based questions (Haladyna, Downing, & Rodriguez, 2002). When negative stem based questions are used it is important for the negative word to be boldface so that it stands out to students. For the purpose of the project I decided I wanted to try using at least one negative stem question. Since this question was found to be ineffective I then revised it to have a positive stem.

Question two had a difficulty of 0.842 (border line easy and medium) with a discrimination of 0.237 (satisfactory). This was intended to be an easier item to give students a few easy points on the test and to help balance the more difficult test items. The only change made was in regards to the labeling of the central angle. The students interpreted point E as the center of the circle and D as an angle. Where the question intended E and D to both label an angle. Due to student’s responses the label D was removed and E was shifted to make it more clearly the label of the center of the circle.

Question three had a difficulty of 0.774 (medium) and a discrimination of 0.342 (good). This item is considered more of an easy medium question, but because of the good discrimination it was decided that no changes would be made.
1. Which of the following is an example of a chord?
   A) An example of a chord is a radius.
   B) An example of a chord is a secant.
   C) An example of a chord is a diameter.

2. Using the given circle fill in the table providing two examples of each. Your answers should include the use of correct notation.

<table>
<thead>
<tr>
<th>Central Angle</th>
<th>Inscribed Angle</th>
<th>Major Arc</th>
<th>Minor Arc</th>
<th>Semicircle</th>
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</table>

3. Disprove the following statement by drawing an illustration using circle K
   “All secants of a circle are parallel.”

The following mathematical practice standards were used: ‘reason abstractly and quantitatively’ and ‘construct viable arguments.’

Problem 2

Initial Construction:

This problem was designed to help students realize that multiple theorems can be used to find the same piece of information. This problem addresses the common core standard G.MG #1 which states, “use geometric shapes, their measures and their properties to describe objects.” For the Bloom’s taxonomy level this item fell under the analysis category because students are required to break things down in order to come to
the conclusion that the same answer could be reached through multiple methods. For Cangelosi’s learning level this was an application because students were applying their knowledge of the arc addition postulate and the inscribed angle theorem.

<table>
<thead>
<tr>
<th>Arc Addition Postulate</th>
<th>Inscribed Angle Theorem</th>
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</thead>
<tbody>
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</table>

1. Find \( m\angle A \).

2. Show \( m\angle BAC \) is the same when applying both the arc addition postulate and the inscribed angle theorem.

Item after Comments from Committee and Think Aloud (Revision 1):

There was no feedback provided by the committee on this item.

From the think alouds it was decided that there needed to be some mention on part (b) to indicate to the students that they were supposed to show their work in the table provided. Also, to make the problem a little more difficult and require a little more thought the provided angle was changed from being angle B to angle C. To allow the question to flow better, the columns were switched. It made more sense to have the inscribed angle theorem be in the first column since that seems to be the one most individuals use first.
1. Find $m\angle A$.

2. Fill in the table below to show the $m\widehat{AC}$ is the same when applying both the Arc Addition Postulate and Inscribed Angle Theorem.

<table>
<thead>
<tr>
<th>Inscribed Angle Theorem</th>
<th>Arc Addition Postulate</th>
</tr>
</thead>
</table>

**Conclusions from Item Analysis:**

Question one had an item difficulty of 0.748 (medium) with a discrimination of 0.421 (excellent). Even though it is an easier medium level question it did an excellent job of discriminating between students who knew the information and those who did not. It also helped lead students to being able to find the information they needed to complete the next question.

On question two the difficulty was 0.314 (border line medium and hard). This is a harder question but is well balanced with the easier medium level of question one. The discrimination was 0.610 which is very good. The maximum discrimination for the items difficulty level is 0.614 so this question reached its full capacity in regards to its ability to discriminate so no changes were made.

The following mathematical practice standards were used: ‘make sense of a problem and persevere in solving,’ ‘reason abstractly and quantitatively,’ ‘construct viable arguments’ and ‘attend to precision.’
4.3 – Chapter 10

Chapter 10 covered arcs and sectors of circles. Some of the included concepts were inscribed quadrilaterals, arc length and sectors and segments of circles.

Problem 3

Initial Construction:

This problem was designed to help students learn and discover things about arc length but also to combine concepts they had learned previously in this chapter. This problem required students to apply what they had learned without being told what specifically to use when solving the problem.

The following common core standards were used, G.C #5, “derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measures of the angle as constant of proportionality.” And G.MG #1, “use geometric shapes, their measures and their properties to describe objects.”

In terms of Cangelosi’s learning levels the questions for this prompt fall under the application and discover a relationship categories. The problem is designed to help students make discoveries about arc length in the beginning of the problem so that they can answer the later questions which require them to apply what they have learned to specific questions.

For Bloom’s taxonomy there were two different learning levels being addressed. The first question is knowledge. The next few questions (two through 4) are synthesis because the students are compiling ideas in order to draw a conclusion in question five;
thus question five can also be classified as evaluation. The remaining questions are application.

**PLEASE INCLUDE UNITS IN ALL RESPONSES WHERE APPROPRIATE**

A Circular Track:
- The distance around the inside of lane 1 is 400 meters.
- The width of each lane is 2 meters.
- Assume the runners run counter-clockwise as close as they can to the inside of their lane.
- From the start to the finish line for lane 5 is 225°.

1. The circular track could be described as having what pattern of circles? ______________________

2. What is the relationship between the circle that is lane 1 and the circle that is lane 5?
   (A) Circumscribed
   (B) Congruent
   (C) Similar

3. Identify the degree measure from the start to finish line for lane 3? ______________________

4. What conclusions can be made for the degree measure from the start to finish line for all lanes of the track? ________________________________

5. From your conclusion on (4) does this mean that all the lanes will cover the same distance between the start and finish line? Yes or No
   Then explain your reasoning.

6. Find the radius to the inside of lane 1 and lane 5.

7. From the start to finish line how many meters does the runner in lane 5 travel?

8. For the runner in lane 1 to travel the same distance as the runner in lane 5 how many additional degrees would they have to go pass the finish line?

9. As illustrated above it doesn’t work to have individuals start on the same starting line. This is why tracks use what is called a staggered start; this is when everyone starts at a different place so that they all have traveled the same distance by the time they reach the finish line. Find how many meters runner 5 should be in front of the starting line in order for them to have covered the same distance as the runner in lane 1 when they reach the finish line?

<table>
<thead>
<tr>
<th>Radius of Lane 1</th>
<th>Radius of Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

**Item after Comments from Committee and Think Alouds:**

At the conclusion of the think alouds, the following line was added to clarify the original directions provided to the students “The runners all start and finish on the same
line.” There was some confusion over the question in the think alouds since it is common for tracks to have staggered starts. The idea of staggered starts is addressed later in the problem, but I thought it would be wise to clarify this for the students at the beginning of the prompt. Also, on question eight the word “additional” was placed in boldface so that students should not miss exactly what the question was asking.

**Conclusions from Item Analysis:**

Question one had a difficulty of 0.138 (extremely difficult) and a discrimination of 0.105 (low). This was not considered to be an accurate representation of the question due to the fact two of three teachers did not teach the vocabulary word, concentric, and as a consequence, a majority of the students had insufficient knowledge to answer the question correctly. By visual examination of the responses from students who had the one teacher who taught this word this question would fall into the medium difficulty category.

Question two had a difficulty of 0.735 (medium) with a discrimination of 0.211 (satisfactory). All the distracters managed to pull at least a few individuals, so no modifications were made.

Question three had a difficulty of 0.490 (medium), which was very close to the optimal difficulty level of 0.50. The discrimination was excellent at 0.421. The question was originally intended as a fairly easy, one in which students would make the connection that from the start to finish line all the circles have the same degree measure even though they have different circumferences. The results on this question were a surprise because the question turned out to be more difficult and effective than originally planned for.
Question four had a difficulty of 0.547 (medium) with a discrimination of 0.579 (excellent) thus no modifications were made.

Question five (a) had a difficulty of 0.728 (medium) and discrimination of 0.211 (satisfactory). This question has students answer yes or no and then in part (b) they had to explain their answer. On part (b) the difficulty was 0.503 (medium) and the discrimination was .342 (good). A fair portion of the students gave the correct answer on part (a) but when it came to explaining why they selected their answer many could not. This illustrated to me that many students have difficulty expressing their mathematical ideas in words. No changes were made to this question.

The rest of the questions could not be analyzed because the majority of students did not complete the test. This could be because the rest of the questions on this problem required students to use what they have learned in situations where the method of solving was not as obvious and the questions were perhaps too intensive for a testing situation. This item would have been better if I had ended the test after question five and used the rest of the question as a class activity where more structure and support could be given.
10. The circular track could be described as having what pattern of circles? ________________

11. What is the relationship between the circle that is lane 1 and the circle that is lane 5?
   (A) Circumscribed
   (B) Congruent
   (C) Similar

12. Identify the degree measure from the start to finish line for lane 3? ________________

13. What conclusions can be made for the degree measure from the start to finish line for all lanes of the track? ________________________________

14. a. From your conclusion on (4) does this mean that all the lanes will cover the same distance between the start and finish line? Yes or No
   b. Then explain your reasoning.

15. Find the radius to the inside of lane 1 and lane 5.

<table>
<thead>
<tr>
<th>Radius of Lane 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Radius of Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

16. From the start to finish line how many meters does the runner in lane 5 travel?

17. For the runner in lane 1 to travel the same distance as the runner in lane 5 how many additional degrees would they have to go pass the finish line?

18. As illustrated above it doesn’t work to have individuals start on the same starting line. This is why tracks use what is called a staggered start; this is when everyone starts at a different place so that they all have traveled the same distance by the time they reach the finish line. Find how many meters runner 5 should be in front of the starting line in order for them to have covered the same distance as the runner in lane 1 when they reach the finish line?

The mathematical practice standards being used are, ‘make sense of a problem and persevere in solving,’ ‘reason abstractly and quantitatively’ and ‘look for and make use of structure.’
CHAPTER 5

RESULTS

This section will address the project’s objectives and assess the extent to which they have been achieved. These objectives include:

1. Develop open response and multiple choice test items in line with the common core curriculum standards and mathematical practice standards.
2. Construct test items that emphasize the use of higher levels of cognitive demand as determined by Bloom’s taxonomy and Cangelosi’s learning levels.
3. Use research-based test writing methods for multiple-choice and constructed response item development.
4. Perform item analysis to determine each item’s effectiveness in measuring students’ understanding of the content and to assist in the further development of test items.

Objective 1: Develop open response and multiple choice test items in line with the common core curriculum standards and mathematical practice standards.

This objective was reached by using the common core standards as a guideline for writing test items. Many of the objectives were too encompassing to test in a single test item so the item was considered sufficient if it at least covered part of the given standard. In Table 6 the common core standards that were addressed by test items are listed in the left hand column and the chapter test and problem number that addressed the standard are
listed in the right hand column. The table shows that every test item was in line with at least one common core standard.

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Test Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS – G.SRT #6</td>
<td>• Chapter 8 – Problem 1</td>
</tr>
<tr>
<td>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to the definition of trigonometric ratios for acute angles.</td>
<td></td>
</tr>
<tr>
<td>CCSS – G.SRT - #7</td>
<td>• Chapter 8 – Problem 3</td>
</tr>
<tr>
<td>Explain and use the relationship between the sine and cosine.</td>
<td></td>
</tr>
<tr>
<td>CCSS – G.SRT - #8</td>
<td>• Chapter 8 – Problem 2</td>
</tr>
<tr>
<td>Use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems.</td>
<td></td>
</tr>
<tr>
<td>CCSS – G.SRT - #10</td>
<td>• Chapter 8 – Problem 4</td>
</tr>
<tr>
<td>Prove the Law of Sines and Cosines and use them to solve problems.</td>
<td></td>
</tr>
<tr>
<td>CCSS – G.MG - #1</td>
<td>• Chapter 8 – Problem 2</td>
</tr>
<tr>
<td>Use geometric shapes, their measures, and their properties to describe objects.</td>
<td></td>
</tr>
<tr>
<td>CCSS – G.CO #1 – Know precise definition of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</td>
<td>• Chapter 9 – Problem 1</td>
</tr>
<tr>
<td>CCSS – G.C - #2 – Identify and describe relationships among inscribed angles, radii, and chords.</td>
<td>• Chapter 9 – Problem 3</td>
</tr>
<tr>
<td>CCSS – G.C - #3 – Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.</td>
<td>• Chapter 9 – Problem 4</td>
</tr>
<tr>
<td>CCSS – G.C - #5 – Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality.</td>
<td>• Chapter 10 – Problem 1</td>
</tr>
<tr>
<td></td>
<td>• Chapter 10 – Problem 3</td>
</tr>
</tbody>
</table>

Table 6 - Test Items Categorized by Common Core Standard

Once a draft for the test items had been made it was then decided what type of effort would be required of the students taking the test. From these determinations each
item was assigned to one or more mathematical practice standards. This tool of measurement did not prove to be as useful as originally intended. Many of the practice standards were so broad that questions fell into multiple categories as can be seen in Table 7.

<table>
<thead>
<tr>
<th>Common Core Mathematical Practice Standards</th>
<th>Test Items</th>
</tr>
</thead>
</table>
| CCSS – MP1 – Make sense of problems and persevere in solving them. | • Chapter 8 – Problem 1  
• Chapter 8 – Problem 2  
• Chapter 8 – Problem 3  
• Chapter 9 – Problem 2  
• Chapter 9 – Problem 3  
• Chapter 9 – Problem 4  
• Chapter 10 – Problem 1  
• Chapter 10 – Problem 2  
• Chapter 10 – Problem 3 |
| CCSS – MP2 – Reason abstractly and quantitatively. | • Chapter 8 – Problem 1  
• Chapter 8 – Problem 2  
• Chapter 8 – Problem 3  
• Chapter 8 – Problem 4  
• Chapter 9 – Problem 1  
• Chapter 9 – Problem 2  
• Chapter 9 – Problem 3  
• Chapter 9 – Problem 4  
• Chapter 10 – Problem 1  
• Chapter 10 – Problem 2  
• Chapter 10 – Problem 3 |
| CCSS – MP3 – Construct viable arguments and critique the reasoning of others. | • Chapter 8 – Problem 3  
• Chapter 8 – Problem 4  
• Chapter 9 – Problem 1  
• Chapter 9 – Problem 4 |
| CCSS – MP4 – Model with mathematics. | • Chapter 8 – Problem 2 |
| CCSS – MP6 – Attend to precision. | • Chapter 8 – Problem 2  
• Chapter 9 – Problem 2  
• Chapter 10 – Problem 2 |
| CCSS – MP7 – Look for and make use of structure. | • Chapter 10 – Problem 3 |

Table 7 – Test Items Categorized by Mathematical Practice Standard
Objective 2: Construct test items that emphasize the use of higher levels of cognitive demand as determined by Bloom’s taxonomy and Cangelosi’s learning levels.

Ninety-four percent of the test points were attributed to test items in the Bloom’s taxonomy classification levels of application, analysis, synthesis or evaluation. The remaining 6% of the test points were given for test items whose classification was either knowledge or comprehension. Initially, the plan was for the test to be entirely comprised of questions that fell into the three highest levels of Bloom’s taxonomy. Unfortunately, it quickly became apparent that a few application level questions were necessary to facilitate the higher level items. In the end about 62% of the points came from test items that fall under the top three levels. Table 8 shows the number of points and percentage of total points assigned to the different Bloom’s taxonomy levels.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Content</th>
<th>Knowledge</th>
<th>Comprehension</th>
<th>Application</th>
<th>Analysis</th>
<th>Synthesis</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Trigonometric Ratios of Similar Triangles</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Trigonometric Functions</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Complement Angle Relations</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Law of Sines</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>Elements of Circles</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Inscribed Angle Theorem</td>
<td></td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Arc Addition Postulate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>
The Cangelosi learning levels were also used to facilitate achieving higher cognitive levels. The items were constructed using learning levels that provide avenues for students to demonstrate different skills, but also build upon ideas and make new connections during the test itself. In Table 9 one can observe the number of points and percentage of total points assigned to the different Cangelosi learning levels. The level with the most emphasis was application at 39.36% percent of the points then followed by comprehension and communication with 26.60% and discover a relationship with 25.53%.
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Content</th>
<th>Construct a Concept</th>
<th>Discover a Relationship</th>
<th>Simple Knowledge</th>
<th>Comprehension &amp; Communication</th>
<th>Algorithmic Skill</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Trigonometric Ratios of Similar Triangles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Trigonometric Functions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Complement Angle Relations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Law of Sines</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Elements of Circles</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inscribed Angle Theorem</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Arc Addition Postulate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exterior Angle Theorems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tangent &amp; Secant Theorems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chords</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Inscribed Quadrilaterals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Arc Length</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sectors and Segments of Circles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Weight of Taxonomy Level | 2.12% | 25.53% | 1.06% | 26.60% | 5.32% | 39.36% |

Table 9 - Number of points assigned to the Cangelosi learning levels
Objective 3: Use research-based test writing methods for multiple-choice and constructed response item development.

This objective was achieved by using many resources from books and articles that taught about the construction of test items. One article in particular provided a detailed list of guidelines that can be found in the appendix (Haladyna, Downing, & Rodriguez, 2002). Other books and articles used include:


Objective 4: Perform an item analysis to determine the item(s) effectiveness in measuring students understanding of the content and to assist in the revising of test items.

To perform the item analysis it was necessary to create upper and lower groups. For the chapter 8 test this resulted in the upper and lower group each having a sample of 42 and for the chapter 9 and 10 test a sample of 40. Those who did not fall into one of these categories were placed in the middle group. It is important to note that several tests
were not considered in the study because a significant portion was left incomplete. For the chapter 8 test this resulted in the removal of 13 tests thus leaving the overall sample size of 149 and for chapter 9 and 10 the removal of 11 tests with a final sample size of 147.

In Table 11, Table 12 and Table 13 a summary is provided of the item analysis conducted for the chapter 8, 9 and 10 test items. To facilitate in the understanding of these tables a key can be found below in Table 10.

<table>
<thead>
<tr>
<th>Item Type</th>
<th>MC – Multiple Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CR – Constructed Response</td>
</tr>
<tr>
<td></td>
<td>Match - Matching</td>
</tr>
<tr>
<td>Item Difficulty</td>
<td>Scale of 0.00 – 1.00. (0.00 – hard to 1.00 – easy)</td>
</tr>
<tr>
<td>Desired Difficulty</td>
<td>A multiple-choice and matching item is dependent upon the number of choices (see Table 5). For constructed response the desired level is 0.50.</td>
</tr>
<tr>
<td>Interpretation of Item Difficulty</td>
<td>Easy – An easy question</td>
</tr>
<tr>
<td></td>
<td>Med – A medium level question</td>
</tr>
<tr>
<td></td>
<td>Hard – A difficult question</td>
</tr>
<tr>
<td>Item Discrimination</td>
<td>Scale of -1.00 to 1.00. This tells how well a test item did of discriminating between those who knew the material and those who do not. (0.00 – no discrimination to 1.00 – excellent discrimination)</td>
</tr>
<tr>
<td>Maximum Discrimination</td>
<td>Is dependent upon the difficulty level of the item.</td>
</tr>
<tr>
<td>Interpretation</td>
<td>Low – Low discrimination</td>
</tr>
<tr>
<td></td>
<td>Sat – Satisfactory discrimination</td>
</tr>
<tr>
<td></td>
<td>Good – Good discrimination</td>
</tr>
<tr>
<td></td>
<td>Exc – Excellent discrimination.</td>
</tr>
<tr>
<td>Revisions</td>
<td>Yes – Revisions were made</td>
</tr>
<tr>
<td></td>
<td>No – No revisions were made</td>
</tr>
</tbody>
</table>

Table 10 - Key for Item Analysis Tables

As desired, the chapter 8 test started out with some easier items and then gradually worked up to medium and difficult questions. The majority of the test items were of medium difficulty with an item difficulty score near 0.50. There were also very few low discrimination scores, most came in at satisfactory or above. Problem two had
especially excellent discrimination scores. In the end, only about half of the chapter 8 items required any revisions.

<table>
<thead>
<tr>
<th>Test Item</th>
<th>Item Type</th>
<th>Item Difficulty</th>
<th>Desired Difficulty</th>
<th>Item Interpretation</th>
<th>Item Discrimination</th>
<th>Maximum Discrimination</th>
<th>Interpretation</th>
<th>Revisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>Question 1</td>
<td>MC</td>
<td>0.852</td>
<td>0.67</td>
<td>Easy</td>
<td>0.167</td>
<td>0.3</td>
<td>Sat</td>
</tr>
<tr>
<td></td>
<td>Question 2</td>
<td>OR</td>
<td>0.940</td>
<td>0.50</td>
<td>Easy</td>
<td>0.111</td>
<td>0.12</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>Question 2a</td>
<td>OR</td>
<td>0.895</td>
<td>0.50</td>
<td>Easy</td>
<td>0.060</td>
<td>0.22</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>Question 1</td>
<td>Match</td>
<td>0.148</td>
<td>0.58</td>
<td>Hard</td>
<td>0.214</td>
<td>0.3</td>
<td>Sat</td>
</tr>
<tr>
<td></td>
<td>Question 2</td>
<td>Match</td>
<td>0.544</td>
<td>0.58</td>
<td>Med</td>
<td>0.524</td>
<td>0.88</td>
<td>Exc</td>
</tr>
<tr>
<td></td>
<td>Question 3</td>
<td>Match</td>
<td>0.503</td>
<td>0.58</td>
<td>Med</td>
<td>0.643</td>
<td>1.00</td>
<td>Exc</td>
</tr>
<tr>
<td></td>
<td>Question 4</td>
<td>Match</td>
<td>0.537</td>
<td>0.58</td>
<td>Med</td>
<td>0.524</td>
<td>0.91</td>
<td>Exc</td>
</tr>
<tr>
<td></td>
<td>Question 5</td>
<td>OR</td>
<td>0.493</td>
<td>0.50</td>
<td>Med</td>
<td>0.595</td>
<td>0.97</td>
<td>Exc</td>
</tr>
<tr>
<td>Problem 2</td>
<td>Question 1</td>
<td>MC</td>
<td>0.436</td>
<td>0.62</td>
<td>Med</td>
<td>0.190</td>
<td>0.82</td>
<td>Sat</td>
</tr>
<tr>
<td></td>
<td>Question 2</td>
<td>OR</td>
<td>0.919</td>
<td>0.50</td>
<td>Easy</td>
<td>0.211</td>
<td>0.16</td>
<td>Sat</td>
</tr>
<tr>
<td></td>
<td>Question 2a</td>
<td>OR</td>
<td>0.484</td>
<td>0.50</td>
<td>Med</td>
<td>0.393</td>
<td>0.94</td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>Question 2b</td>
<td>MC</td>
<td>0.497</td>
<td>0.67</td>
<td>Med</td>
<td>0.238</td>
<td>0.97</td>
<td>Sat</td>
</tr>
</tbody>
</table>

Chapter 9 test items illustrated more of a balance between easy, medium and hard questions intermixed throughout the test. In contrast the chapter 8 test had most of the easy questions at the beginning. There were a few items with low discrimination values.
which required revisions. In the case of problem 2, questions 1 and 2 were both in excellent range and very close to reaching their maximum discrimination value. About half the items required revisions. This was mostly due to the items difficulty level being easy, which resulted in a very low discrimination value.

<table>
<thead>
<tr>
<th>Test Item</th>
<th>Item Type</th>
<th>Item Difficulty</th>
<th>Desired Difficulty</th>
<th>Interpretation</th>
<th>Item Discrimination</th>
<th>Maximum Discrimination</th>
<th>Interpretation</th>
<th>Revisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question 1</td>
<td>MC</td>
<td>0.926</td>
<td>0.67</td>
<td>Easy</td>
<td>0.132</td>
<td>0.14</td>
<td>Low</td>
<td>Yes</td>
</tr>
<tr>
<td>Question 2</td>
<td>OR</td>
<td>0.842</td>
<td>0.50</td>
<td>Easy</td>
<td>0.237</td>
<td>0.32</td>
<td>Sat</td>
<td>Yes</td>
</tr>
<tr>
<td>Question 3</td>
<td>OR</td>
<td>0.774</td>
<td>0.50</td>
<td>Med</td>
<td>0.342</td>
<td>0.46</td>
<td>Good</td>
<td>No</td>
</tr>
<tr>
<td>Problem 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question 1a</td>
<td>OR</td>
<td>0.748</td>
<td>0.50</td>
<td>Med</td>
<td>0.421</td>
<td>0.44</td>
<td>Exc</td>
<td>No</td>
</tr>
<tr>
<td>Question 1b</td>
<td>OR</td>
<td>0.314</td>
<td>0.50</td>
<td>Hard</td>
<td>0.610</td>
<td>0.62</td>
<td>Exc</td>
<td>No</td>
</tr>
<tr>
<td>Problem 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question 1</td>
<td>MC</td>
<td>0.941</td>
<td>0.67</td>
<td>Easy</td>
<td>0.132</td>
<td>0.12</td>
<td>Low</td>
<td>Yes</td>
</tr>
<tr>
<td>Question 2</td>
<td>MC</td>
<td>0.904</td>
<td>0.67</td>
<td>Easy</td>
<td>0.105</td>
<td>0.2</td>
<td>Low</td>
<td>Yes</td>
</tr>
<tr>
<td>Problem 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question 1</td>
<td>MC</td>
<td>0.654</td>
<td>0.67</td>
<td>Med</td>
<td>0.447</td>
<td>0.65</td>
<td>Exc</td>
<td>No</td>
</tr>
<tr>
<td>Question 2</td>
<td>OR</td>
<td>0.620</td>
<td>0.50</td>
<td>Med</td>
<td>0.553</td>
<td>0.68</td>
<td>Exc</td>
<td>No</td>
</tr>
<tr>
<td>Question 3</td>
<td>OR</td>
<td>0.337</td>
<td>0.50</td>
<td>Hard</td>
<td>0.408</td>
<td>0.63</td>
<td>Exc</td>
<td>Yes</td>
</tr>
<tr>
<td>Question 4</td>
<td>OR</td>
<td>0.436</td>
<td>0.50</td>
<td>Med</td>
<td>0.684</td>
<td>0.82</td>
<td>Exc</td>
<td>No</td>
</tr>
<tr>
<td>Question 5</td>
<td>MC</td>
<td>0.860</td>
<td>0.67</td>
<td>Easy</td>
<td>0.132</td>
<td>0.28</td>
<td>Low</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 12 – Item Analysis – Chapter 9
All of the chapter 10 test items fell under the difficulty range of medium to hard. This was because they were paired with the chapter 9 questions and this chapter’s material tended to be more challenging. On problem 7 questions 3 through 6 all came very close to reaching the optimal difficulty level. In terms of the discrimination values all the questions did great except for problem 7, question 1 because it was a particularly difficult. None of the items required further revisions.

<table>
<thead>
<tr>
<th>Test Item</th>
<th>Item Type</th>
<th>Item Difficulty</th>
<th>Desired Difficulty</th>
<th>Interpretation</th>
<th>Item Discrimination</th>
<th>Maximum Discrimination</th>
<th>Interpretation</th>
<th>Revisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 5</td>
<td>Problem 6</td>
<td>Question 1</td>
<td>MC</td>
<td>0.684</td>
<td>0.62</td>
<td>Med</td>
<td>0.368</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Question 1</td>
<td>OR</td>
<td>0.676</td>
<td>0.50</td>
<td>Med</td>
<td>0.412</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Question 2</td>
<td>OR</td>
<td>0.413</td>
<td>0.50</td>
<td>Med</td>
<td>0.559</td>
<td>0.79</td>
</tr>
<tr>
<td>Problem 7</td>
<td>Question 1</td>
<td>OR</td>
<td>0.138</td>
<td>0.50</td>
<td>Hard</td>
<td>0.105</td>
<td>0.26</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>Question 2</td>
<td>MC</td>
<td>0.735</td>
<td>0.67</td>
<td>Med</td>
<td>0.211</td>
<td>0.52</td>
<td>Sat</td>
</tr>
<tr>
<td></td>
<td>Question 3</td>
<td>OR</td>
<td>0.490</td>
<td>0.50</td>
<td>Med</td>
<td>0.421</td>
<td>0.97</td>
<td>Exc</td>
</tr>
<tr>
<td></td>
<td>Question 4</td>
<td>OR</td>
<td>0.547</td>
<td>0.50</td>
<td>Med</td>
<td>0.579</td>
<td>0.76</td>
<td>Exc</td>
</tr>
<tr>
<td></td>
<td>Question 5a</td>
<td>MC</td>
<td>0.728</td>
<td>0.75</td>
<td>Med</td>
<td>0.211</td>
<td>0.56</td>
<td>Sat</td>
</tr>
<tr>
<td></td>
<td>Question 5b</td>
<td>OR</td>
<td>0.503</td>
<td>0.50</td>
<td>Med</td>
<td>0.342</td>
<td>0.91</td>
<td>Good</td>
</tr>
</tbody>
</table>

Table 13 – Item Analysis – Chapter 10
In summary, Table 14 was constructed comparing the overall difficulty and discrimination values for the three chapters. The table breaks down the number of points based on test items difficulty and discrimination.

<table>
<thead>
<tr>
<th>Difficulty &amp; Discrimination Levels</th>
<th>Difficulty Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Easy</td>
</tr>
<tr>
<td>Low</td>
<td>13</td>
</tr>
<tr>
<td>Satisfactory</td>
<td>11</td>
</tr>
<tr>
<td>Good</td>
<td>9</td>
</tr>
<tr>
<td>Excellent</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 14 - Summary of Difficulty and Discrimination Values

In order to make the decision of whether test items needed revision or not the effectiveness of the distracters for the multiple-choice items were also taken into consideration.

For the chapter 8 (Table 15) questions, the majority of the distracters served their purpose. On problem 1, question 1 distracter (b) did not pull any students and so it was necessary to revise this distracter. The next multiple-choice question on the test, problem 2, question 1 all the distracters were successful, with distracter (d) for the upper group and distracter (a) for the lower group pulling more students than the actual answer, illustrating that this was a difficult question. On problem 2, question 3 the majority of the upper group of students answered the question correctly. There were even a few distracters that they did not select. For the lower group of students, all of the distracters
proved to very effective, especially choice (c). Because of the effectiveness of the
distracters for the lower group, it was decided that no changes were necessary for this
question.

<table>
<thead>
<tr>
<th>Problem 1 – Question 1 (correct answer (c))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Group</td>
<td>3</td>
<td>0</td>
<td>39</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>10</td>
<td>0</td>
<td>32</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2 – Question 1 (correct answer (e))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Group</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>17</td>
<td>12</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>13</td>
<td>8</td>
<td>5</td>
<td>8</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2 – Question 2 (correct answer (c))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Group</td>
<td>2</td>
<td>0</td>
<td>37</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>9</td>
<td>5</td>
<td>15</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2 – Question 3 (correct answer (f))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Group</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>35</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>4</td>
<td>7</td>
<td>15</td>
<td>5</td>
<td>2</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2 – Question 4 (correct answer (a))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Group</td>
<td>36</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>14</td>
<td>7</td>
<td>5</td>
<td>9</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 3 – Question 1 (correct answer (b))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Group</td>
<td>2</td>
<td>22</td>
<td>0</td>
<td>12</td>
<td>/</td>
<td>/</td>
<td>3</td>
</tr>
<tr>
<td>Lower Group</td>
<td>11</td>
<td>14</td>
<td>5</td>
<td>9</td>
<td>/</td>
<td>/</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 3 – Question 2b (correct answer (a))</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Group</td>
<td>29</td>
<td>9</td>
<td>4</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>17</td>
<td>18</td>
<td>5</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 15 – Distracter Analysis – Chapter 8

For chapter 9 (Table 16), there are a few noteworthy points to highlight. For
problem 3, question 1 every person in the upper group answered the question correctly
and all but five in the lower group answered it correctly as well. Even though at least one
individual in the lower group selected each of the distracters, it was decided that revisions
needed to be made because of how easy the question was. On problem 4, question 1, distracter (a) was the most successful. Most of the upper group answered the question correctly with only a few individuals selecting distracter (a). In the lower group, distracter (a) pulled more students than the correct answer. This is excellent for the discrimination value and for determining a common mistake among the students.

<table>
<thead>
<tr>
<th>Distracter Analysis – Chapter 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>Problem 1 – Question 1 (correct answer (c))</td>
</tr>
<tr>
<td>Upper Group</td>
</tr>
<tr>
<td>Lower Group</td>
</tr>
<tr>
<td>Problem 3 – Question 1 (correct answer (a))</td>
</tr>
<tr>
<td>Upper Group</td>
</tr>
<tr>
<td>Lower Group</td>
</tr>
<tr>
<td>Problem 3 – Question 2 (correct answer (b))</td>
</tr>
<tr>
<td>Upper Group</td>
</tr>
<tr>
<td>Lower Group</td>
</tr>
<tr>
<td>Problem 4 – Question 1 (correct answer (c))</td>
</tr>
<tr>
<td>Upper Group</td>
</tr>
<tr>
<td>Lower Group</td>
</tr>
<tr>
<td>Problem 4 – Question 5 (correct answer (b))</td>
</tr>
<tr>
<td>Upper Group</td>
</tr>
<tr>
<td>Lower Group</td>
</tr>
</tbody>
</table>

Of the test items for chapter 10 (Table 17) only two of them were multiple-choice. Problem 1, question 1, had excellent distracters. Each distracter pulled at least a few individuals in both the upper and the lower groups. Problem 3, question 2, also had all of the distracters pull at least a few individuals in both groups. However, it should be noted that this question was also the one with the most no responses overall, for this test.
This is probably because it was on the last page and many students did not take the effort or did not have the time to complete this page.

<table>
<thead>
<tr>
<th>Distracter Analysis – Chapter 10</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1 – Question 1 (correct answer (b))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Group</td>
<td>3</td>
<td>32</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>10</td>
<td>18</td>
<td>6</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Problem 3 – Question 2 (correct answer (c))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Group</td>
<td>6</td>
<td>1</td>
<td>31</td>
<td>/</td>
<td>0</td>
</tr>
<tr>
<td>Lower Group</td>
<td>8</td>
<td>4</td>
<td>23</td>
<td>/</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 17 – Distracter Analysis – Chapter 10

As part of the item analysis, a reliability coefficient was computed which included both the Kuder-Richardson 20 (KR20) and coefficient alpha schemes. Since the test was a mixture of multiple-choice and constructed response questions it was necessary to compute separate reliability coefficients. For multiple-choice the Kuder-Richardson 20 was computed and for the constructed response coefficient alpha. As can be viewed in Table 18 the reliability coefficients ranged from low to good (for interpretation index refer to Table 5).

<table>
<thead>
<tr>
<th>Reliability Coefficients</th>
<th>Chapter 8 Test</th>
<th>Chapter 9 &amp; 10 Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>KR20</td>
<td>0.635 Satisfactory</td>
<td>KR20 0.409 Low</td>
</tr>
<tr>
<td>Coefficient Alpha</td>
<td>0.519 Satisfactory</td>
<td>Coefficient Alpha 0.837 Good</td>
</tr>
</tbody>
</table>

Table 18 – Reliability Coefficients

Correlation coefficients were also computed on the student’s percentage for the teacher portion of the test and the study portion of the test.
Figure 3 – A Comparison of Scores – Chapter 8

Figure 4 – A Comparison of Scores – Chapter 9 & 10
For the chapter 8 test (Figure 3) there is a correlation coefficient of 0.63 and for the chapter 9 and 10 test (Figure 4), there is a correlation coefficient of 0.68. Thus we can see that there is a positive relationship between the student’s percentage on the teacher’s portion and their percentage on the study portion. As a general rule students did better on their teacher’s portion of the test. This can be observed by the fact that there are more dots above the $y = x$ line than there are below it.
CHAPTER 6

DISCUSSION & CONCLUSIONS

Over the course of this project there are many things I have learned and plan to implement in future test writing. When I first started I thought it would be pretty easy to throw together some higher cognitive test items so they would be ready for administration to the students. I soon found out this was not the case. It was very difficult to write questions on the cognitive level I desired, especially in the case of multiple-choice questions. I also had to be sure that the test items I constructed could be completed by the students during a testing situation with a time constraint. Fortunately, as I continued to create these test items I found it became easier to construct higher cognitive test items and there were fewer revisions necessary.

In the construction of the test items, I felt it was important that they reached the higher levels of cognition. I also wanted to provide students with opportunity to learn and make discoveries through the questions. The test needed to provide me feedback but also be a learning opportunity for the students.

As I developed the items in alignment with the common core state standards and the mathematical practice standards I realized that these instruments are still in their infancy. The standards are structured but could be more detailed and provide more direction on what the standard entails. I also felt that the mathematical practice standards were too broad. As can be seen in the result section most of my questions fell under at least the first two standards. These standards provide direction but once again are not specific enough.
Once I had selected the common core state standard for which I would base the question, I then needed to ensure the item fell into one of Bloom’s taxonomy levels and one of Cangelosi’s learning levels. The decision was made to use two different scales of measurement because they helped make the items more diverse and cognitively demanding. Bloom’s taxonomy especially helped me focus on the cognitive level while Cangelosi’s levels helped add diversity and a new dimension to the questions.

Once the test items were completed and administered to the students I was then able to collect information on the items difficulty level and discrimination value. Test items that were easier tended to have the lowest discrimination values and thus were not very effective test items but did give students a few easy points. They were also useful ways to start the test to help build students confidence. The most effective test items, and where the bulk of my test items fell, were the medium level questions with excellent discrimination value. I feel like these were the most effective because difficulty wise they were in the middle but still were able to achieve excellent discrimination between the upper and lower groups. The hard questions that had a low or satisfactory discrimination value were of no worth, almost everyone missed the question and they were ineffective in determining who knew the material and who did not. I did like having a couple of hard questions that had excellent discrimination value this way there were at least a few questions that were a challenge to all the students. It is also good to have difficult items on tests to help students develop problem solving skills for more challenging situations.

If was difficult to gauge how effective the reliability values were at helping us understand how reliable of a measure the test was. The sample size of test items was
small and the best way to increase your reliability is to have more test questions. When I
use these test questions again with my own students I will be the one that constructs the
entire test and not just a portion. Thus, I should be able to get a better feel for the
reliability of the questions.

To gain more information about these test questions I would enjoy implementing
them once again now that modifications have been made and see if the revisions made
had the desired effect. I would be able to answer the question; after all the construction
and revision process did these test items become more effective measurements of
students understanding? It would also be nice to interact with the students by conducting
think aloud interviews and gaining feedback on what they thought of the test items.

To extend these items further I intend to submit them to the website, Illustrative
Mathematics, which collects test items that are written for specific common core
standards. Hopefully through their feedback the test items will become even more
effective measurements and be available to more teachers.
Dear Parents,

Ashley Salisbury from the Department of Mathematics and Statistics at Utah State University is conducting a research study to find out more about the cognitive demand level of test items.

By participating in this research students will be asked an additional five to six test items for each chapter being tested on two unit tests. This will include chapters eight, nine and ten. These test items will reflect the material and difficulty that has been addressed in class. The purpose of the study is to observe students responses to the test items and use their responses to improve the test items for further use by classroom teachers.

Research records will be kept confidential, consistent with federal and state regulations. Only the investigator will have access to the data. To protect your privacy, personal, identifiable information will be removed from documents and replaced with a study identifier.

“I certify that the research study has been explained to the individual, by me or a participating teacher, and that the individual understands the nature and purpose, the possible risks and benefits associated with taking part in this research study. Any questions that have been raised have been answered.”

_______________________________
Ashley Salisbury
Principal Investigator
801-928-0183
ashley@salisbury-family.com
A Revised Taxonomy of Multiple Choice Item-Writing Guidelines

<table>
<thead>
<tr>
<th>Content Concerns</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Every item should reflect specific content and a single specific mental behavior, as called for in test specifications (blueprint).</td>
<td></td>
</tr>
<tr>
<td>2. Base each item on important content to learn, avoid trivial content.</td>
<td></td>
</tr>
<tr>
<td>3. Use novel material to test higher level learning.</td>
<td></td>
</tr>
<tr>
<td>4. Keep content of each item independent from content of other items on the test.</td>
<td></td>
</tr>
<tr>
<td>5. Avoid over specific and over general content when writing MC items.</td>
<td></td>
</tr>
<tr>
<td>6. Avoid option-based items.</td>
<td></td>
</tr>
<tr>
<td>7. Avoid trick items.</td>
<td></td>
</tr>
<tr>
<td>8. Keep vocabulary simple for the group of students being tested.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formatting Concerns</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9. Use question, completion, and best answer versions of conventional MC, the alternate choice, true-false (TF), multiple true-false (MTF), matching and context dependent item and item set formats, but AVOID the complex MC format.</td>
<td></td>
</tr>
<tr>
<td>10. Format the item vertically instead of horizontally.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Style Concerns</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11. Edit and proof items.</td>
<td></td>
</tr>
<tr>
<td>12. Use correct grammar, punctuation, capitalization, and spelling</td>
<td></td>
</tr>
<tr>
<td>13. Minimize the amount of reading in each item.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Writing the stem</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14. Ensure that the directions in the stem are very clear.</td>
<td></td>
</tr>
<tr>
<td>15. Include the central idea in the stem instead of the choices.</td>
<td></td>
</tr>
<tr>
<td>16. Avoid window dressing (excessive verbage).</td>
<td></td>
</tr>
<tr>
<td>17. Word the stem positively, avoid negatives such as NOT or EXCEPT. If negative words are used, use cautiously and always ensure the word appears, capitalized and boldface.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Writing the Choices</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18. Develop as many effective choices as you can but research suggests 3 is adequate.</td>
<td></td>
</tr>
<tr>
<td>19. Make sure that only one of these choices is the right answer.</td>
<td></td>
</tr>
<tr>
<td>20. Vary the location of the right answer according to the number of choices.</td>
<td></td>
</tr>
<tr>
<td>21. Place choices in logical or numerical order.</td>
<td></td>
</tr>
<tr>
<td>22. Keep choices independent; choices should not be overlapping.</td>
<td></td>
</tr>
<tr>
<td>23. Keep choices homogenous in content and grammatical structure</td>
<td></td>
</tr>
<tr>
<td>24. Keep the length of choices about equal</td>
<td></td>
</tr>
<tr>
<td>25. <em>None-of-the-above</em> should be used carefully.</td>
<td></td>
</tr>
<tr>
<td>26. Avoid <em>All-of-the-above</em></td>
<td></td>
</tr>
<tr>
<td>27. Phrase choices positively; avoid negatives such as NOT.</td>
<td></td>
</tr>
<tr>
<td>28. Avoid giving clues to the right answer, such as</td>
<td></td>
</tr>
<tr>
<td>a) Specific determiners including always, never, completely, and absolutely.</td>
<td></td>
</tr>
<tr>
<td>b) Clang associations, choices identical to or resembling words in the stem.</td>
<td></td>
</tr>
<tr>
<td>c) Grammatical inconsistencies that cue the test-taker to the correct answer.</td>
<td></td>
</tr>
<tr>
<td>d) Conspicuous correct choice.</td>
<td></td>
</tr>
<tr>
<td>29. Make all distracters plausible</td>
<td></td>
</tr>
<tr>
<td>30. Use typical errors of students to write your distracters</td>
<td></td>
</tr>
<tr>
<td>31. Use humor if it is compatible with the teacher and the learning environment.</td>
<td></td>
</tr>
</tbody>
</table>

(Haladyna, Downing, & Rodriguez, 2002)
Frequency of Scores

Chapter 8 - Frequency of Scores on Study Portion

Scores of Students (out of 30)

Chapter 9 & 10 - Frequency of Scores on Study Portion

Scores of Students (out of 45)
b) Thus one can conclude that _______________triangles have corresponding ratios that are _______________.

Given is the right triangle ABC and right triangle DEF.

1. By inspection of triangle ABC and DEF what can you conclude about their relationship? They are:
   A) Congruent
   B) Disproportional
   C) Similar

2. Find the following ratios using the side lengths of triangle ABC and DEF.

<table>
<thead>
<tr>
<th>Triangle Name</th>
<th>length of side opposite $\angle A$ hypotenuse</th>
<th>length of side adjacent to $\angle A$ hypotenuse</th>
<th>length of side opposite $\angle A$ length of side adjacent to $\angle A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle ABC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle DEF</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) By looking at the table what can be observed about the ratios? (Respond in a complete sentence).

b) Thus one can conclude that ___________triangles have corresponding ratios that are ___________.

answer from (1)  
answer from (2a)
Solution:

Given is the right triangle ABC and right triangle DEF.

1. By inspection of triangle ABC and DEF what can you conclude about their relationship?
   They are:
   A) Congruent
   **B) Similar

2. Find the following ratios using the side lengths of triangle ABC and DEF.

<table>
<thead>
<tr>
<th>Triangle Name</th>
<th>length of side opposite $\triangle A$</th>
<th>length of side adjacent to $\triangle A$</th>
<th>length of side opposite $\triangle A$</th>
<th>length of side adjacent to $\triangle A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle ABC</td>
<td>$\frac{15}{25} = \frac{3}{5}$</td>
<td>$\frac{20}{25} = \frac{4}{5}$</td>
<td>$\frac{15}{20} = \frac{3}{4}$</td>
<td></td>
</tr>
<tr>
<td>Triangle DEF</td>
<td>$\frac{3}{5}$</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{3}{4}$</td>
<td></td>
</tr>
</tbody>
</table>

   a) By looking at the table what can be observed about the ratios?
      (Respond in a complete sentence).
      The corresponding ratios are equivalent (same or equal).

   b) Thus one can conclude that ____similar______ triangles have corresponding ratios that are ____equivalent______.
Chapter 8 – Problem 2

Common Core State Standards:

G.MG #1 – Use geometric shapes, their measures and their properties to describe objects.

G.SRT #8 – Use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems.

Bloom’s Taxonomy – Application, Evaluation

Learning Level – Algorithmic Skill, Application

Given the following situations choose which trigonometric ratio would prove most useful for solving.

A. Sine B. Cosine C. Tangent
D. Inverse Sine E. Inverse Cosine F. Inverse Tangent

1. Abigail is having a picnic with her friends and all of a sudden a swarm of ants overtakes their food. Abigail realizes that one of the ants is looking up at her from a 36° angle. She is 5 feet tall. How far away is the ant from Abigail?

2. Samuel is on vacation with his family in Redwood National Park in California. He wants to take a picture of the tallest redwood tree on record which is 379 feet. He is standing 100 feet from the base of the tree. At what angle would he need his camera to be at so he could make sure the top of the tree was in the picture?

3. In the apartment building near Sofia’s house a fire gets started on fifth floor which is 50 feet above the ground. The firemen show up and need to rescue Sofia’s friend, Rachel who also lives on the fifth floor, but their fire truck is broken from trying to rescue a cat from a tree so they must use an old fashion ladder. The firemen set the ladder up with a 75° angle to the ground as determined by safety regulations. How long does the ladder need to be so it reaches the fifth floor and the firemen are able to rescue Rachel?

4. Jonathon and his younger brother are playing on a see-saw (a teeter-totter) at the park. The see saw is 12 feet long and the middle support is 3 feet off the ground. When Jonathon hits the ground what angle does the see-saw make with the middle support?

5. Select one of the above examples and write a valid equation that can be used to solve for the unknown. Then solve the equation. (Make sure you include a picture as part of your work).
Solution:

Given the following situations choose which trigonometric ratio would prove most useful for solving.

1. 

2. 

3. 

4. 

5. Select one of the above examples and write a valid equation that can be used to solve for the unknown. Then solve the equation. (Make sure you include a picture as part of your work).

1. 

\[ \tan 36^\circ = \frac{b}{x} \]

\[ x = \frac{b}{\tan 36^\circ} \]

\[ x \approx 6.88 \approx 7 \text{ feet} \]

2. 

\[ \tan^{-1} \left( \frac{2722}{500} \right) = x \]

\[ \tan^{-1} \left( \frac{2722}{500} \right) = x \]

\[ x \approx 74.99^\circ \approx 75^\circ \]

3. 

\[ \sin(75^\circ) = \frac{10}{x} \]

\[ x = \frac{10}{\sin 75^\circ} \]

\[ x \approx 51.76 \approx 52 \text{ feet} \]

4. 

\[ \cos^{-1} \left( \frac{5}{12} \right) = x \]

\[ x = 60^\circ \]
Chapter 8 – Problem 3

Common Core State Standards:

G.SRT #7 – Explain and use the relationship between the sine and cosine of complementary angles

Bloom’s Taxonomy – Application, Synthesis and Evaluation

Learning Level – Comprehension and Communication, Application

1. Given the hypotenuse and an acute angle measure of a right triangle. Solve for the length of the opposite leg. What trigonometric ratio should be used?
   A) secant
   B) cosecant
   C) inverse cosine
   D) inverse tangent

2. Given the right triangle ABC and your understanding of sine and cosine fill in the chart below using the following multiple choice options.

<table>
<thead>
<tr>
<th>Reference Angle</th>
<th>Sin</th>
<th>Cos</th>
<th>Csc</th>
<th>Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A. \( \frac{a}{b} \)    B. \( \frac{a}{c} \)    C. \( \frac{b}{a} \)
D. \( \frac{b}{c} \)    E. \( \frac{c}{a} \)    F. \( \frac{c}{b} \)

a) What conclusions can you draw about the relationship between the trigonometric functions of \( \angle A \) and \( \angle C \)?

b) What is the relationship between \( \angle A \) and \( \angle C \)?
   a) Complementary Angles   b) Corresponding Angles   c) Supplementary Angles

c) Now combine your answer from part (a) and part (b) to write a final statement about the relationship between \( \angle A \) and \( \angle C \).
Solution:

1. Given the hypotenuse and an acute angle measure of a right triangle. Solve for the length of the opposite leg. What trigonometric ratio should be used?
   A) secant
   **B) cosecant
   C) inverse cosine
   D) inverse tangent

2. Given the right triangle ABC and your understanding of sine and cosine fill in the chart below using the following multiple choice options.

   A. \( \frac{a}{b} \)    B. \( \frac{a}{c} \)    C. \( \frac{b}{a} \)
   D. \( \frac{b}{c} \)    E. \( \frac{c}{a} \)    F. \( \frac{c}{b} \)

<table>
<thead>
<tr>
<th>Reference Angle</th>
<th>Sin</th>
<th>Cos</th>
<th>Csc</th>
<th>Sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>F</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>C</td>
<td>F</td>
<td>A</td>
<td>D</td>
<td>C</td>
</tr>
</tbody>
</table>

a) What conclusions can you draw about the relationship between the trigonometric functions of \( \angle A \) and \( \angle C \)?

\[
\sin(A) = \cos(C) \\
\sin(C) = \cos(A) \\
csc(A) = \sec(C) \\
sec(A) = \csc(C)
\]

b) What is the relationship between \( \angle A \) and \( \angle C \)?

**A) Complementary Angles**
B) Corresponding Angles
C) Supplementary Angles

c) Now combine your answer from part (a) and part (b) to write a final statement about the relationship between \( \angle A \) and \( \angle C \).

**Example Response 1:**
If two angles are complementary then the sine of one angle will equal the cosine of the other and the secant of one of the angles will equal the cosecant of the other.

**Example Response 2:**
The sine of an angle is equal to the cosine of its complement (or other way around).
The cosecant of an angle is equal to the secant of its complement (or other way around).

**Example Response 3:**
Complementary angles have opposite trig functions that are equal to each other.
Chapter 8 – Problem 4

Common Core State Standards:

G.SRT #10 – Prove the Laws of Sines and Cosines and use them to solve problems

Bloom’s Taxonomy – Synthesis

Learning Level – Comprehension and Communication

Finish showing the construction of the Law of Sines using the triangle ABC.
Where:
\[ \sin A = \frac{h}{c} \Rightarrow h = c \cdot \sin A \]
\[ \sin C = \frac{h}{a} \Rightarrow h = a \cdot \sin C \]
Because both expressions are equal to \( h \)
\[ c \cdot \sin A = a \cdot \sin C \]
By then expressing as a proportion
\[ \frac{\sin A}{a} = \frac{\sin C}{c} \]

(a) Write a ratio that includes length \( k \) for \( \sin B = \_______ \)
(b) Solve for \( k \). \( k = \_______ \)
(c) Write a ratio that includes length \( k \) for \( \sin = \_______ \)
(d) Solve for \( k \). \( k = \_______ \)
(e) What can you conclude about the relationship between your response on part (b) and part (d)?
(f) Express your response to part (e) as a proportion.
(g) What relationship is observed of the proportion \( \frac{\sin A}{a} = \frac{\sin C}{c} \) and the proportion from part (f)?
(h) From your response on part (g) give the Law of Sines: \_______ \_______
Solution:

Finish showing the construction of the Law of Sines using the triangle ABC.

Where:
\[
\sin A = \frac{h}{c} \Rightarrow h = c \cdot \sin A
\]
\[
\sin C = \frac{h}{a} \Rightarrow h = a \cdot \sin C
\]

Because both expressions are equal to \( h \)
\[c \cdot \sin A = a \cdot \sin C\]

By then expressing as a proportion
\[
\frac{\sin A}{a} = \frac{\sin C}{c}
\]

(a) Write a ratio that includes length \( k \) for \( \sin B = \frac{k}{c} \)

(b) Solve for \( k \):
\[k = c \cdot \sin B\]

(c) Write a ratio that includes length \( k \) for \( \sin C = \frac{k}{b} \)

(d) Solve for \( k \):
\[k = b \cdot \sin C\]

(e) What can you conclude about the relationship between response on part (b) and part (d)?

Because both (b) and (d) equal \( k \) we can set them equal to each other.
\[c \cdot \sin B = b \cdot \sin C\]

(f) Express your response to part (e) as a proportion.
\[
\frac{\sin B}{b} = \frac{\sin C}{c}
\]

(g) What relationship is observed of the proportion \( \frac{\sin A}{a} = \frac{\sin C}{c} \) and the proportion from part (f)?

For full points they need to use the transitive property.

Example: Because \( \frac{\sin A}{a} \) and \( \frac{\sin B}{b} \) are equal to \( \frac{\sin C}{c} \) then \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \)

For half points they can make some kind of correct observation.

Example: Sine of an angle over the opposite leg is equal to the sine of another angle over that opposite leg.

(h) From your response on part (g) give the Law of Sines:
\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]
## Chapter 8 – Rubric

### Chapter 8 Test Items – Rubric

#### Problem 1

1. **+2** – Correct (B)  
   +0 – Incorrect Answer

2. **+.5** – For each correctly filled in cell

   +3  +2.5  +2  +1.5  +1  +.5  +0

   a) **+2** – Correct observation of the ratios in the table
   +1 – Partially correct interpretation of the ratios in the table
   +0 – Incorrect interpretation or no response

   b) **+1** – Combined answers from question 1 and part 2b.
      (Even if they are incorrect)
   +0 – Did not combine answers from question 1 and part 2b.

   **PROBLEM 1 TOTAL _____/ 8**

#### Problem 2

1. **+2** – Correct (C)  
   +0 – Incorrect

2. **+2** – Correct (F)  
   +0 – Incorrect

3. **+2** – Correct (A)  
   +0 – Incorrect

4. **+2** – Correct (E)  
   +0 – Incorrect

5. **+3** – Correct answer, valid equation, clean organized work, picture

   +2 – Minimal computation error, no equation or no picture

   +1 – Serious computational error, no work to support answer, include picture

   +0 – Wrong answer based on an incorrect plan, or no response

   **PROBLEM 2 TOTAL _____/ 11**

#### Problem 3

1. **+2** – Correct (B)  
   +0 – Incorrect Answer

2. **+.5** – For each correctly filled in cell

   +4  +3.5  +3  +2.5  +2  +1.5  +1  +.5  +0

   a) **+2** – Draws correct conclusion

   +1 – Partly correct conclusion

   +0 – Incorrect conclusion or no response

   b) **+2** – Answered (A)  
   +0 – Incorrect Answer

   c) **+1** – Combined answers from part a and b to make a final statement.
      (Even if part a and part b are incorrect)

   +0 – Does not make a final state that combines part a and b.

   **PROBLEM 3 TOTAL _____/ 11**

#### Problem 4

1. (a) **+1** – Correct  
   +0 – Incorrect

   (c) **+1** – Correct  
   +0 – Incorrect

   (e) **+1** – Correct  
   +0 – Incorrect

   (g) **+2** – Correct observation  
   +1 – Partial correct observation  
   +0 – Incorrect

   (h) **+1** – Correct  
   +0 – Incorrect

   (b) **+1** – Correct  
   +0 – Incorrect

   (d) **+1** – Correct  
   +0 – Incorrect

   (f) **+1** – Correct  
   +0 – Incorrect

   **PROBLEM 4 TOTAL _____/ 9**

**TOTAL _____/39**
Chapter 9 – Test Items

Chapter 9 – Problem 1

Common Core State Standards:

G.CO #10 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Bloom’s Taxonomy – Comprehension, Evaluation

Learning Level – Construct a Concept, Comprehension and Communication and Application

1. Which of the following is an example of a chord?
   A) Radius
   B) Secant
   C) Diameter

2. Using the given circle fill in the table providing two examples of each. Your answers should include the use of correct notation.

<table>
<thead>
<tr>
<th>Central Angle</th>
<th>Inscribed Angle</th>
<th>Major Arc</th>
<th>Minor Arc</th>
<th>Semicircle</th>
</tr>
</thead>
</table>

3. Disprove the following statement by drawing an illustration using circle K.
   "All secants of a circle are parallel."
Solution:

1. Which of the following is an example of a chord?
   
   A) Radius  
   B) Secant  
   **C) Diameter**

2. Using the given circle fill in the table providing two examples of each. Your answers should include the use of correct notation.

<table>
<thead>
<tr>
<th>Central Angle*</th>
<th>( \angle AEC )</th>
<th>( \angle BEA )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inscribed Angle*</td>
<td>( \angle BCA, \angle FAC, \angle FAE )</td>
<td></td>
</tr>
<tr>
<td>Major Arc*</td>
<td>( \overparen{ACB}, \overparen{AFB}, \overparen{CFA}, \overparen{CBA}, \overparen{FBA}, \overparen{FAC}, \overparen{BAF}, \overparen{BCF} )</td>
<td></td>
</tr>
<tr>
<td>Minor Arc*</td>
<td>( \overparen{AC}, \overparen{CF}, \overparen{FB}, \overparen{BA} )</td>
<td></td>
</tr>
<tr>
<td>Semicircle*</td>
<td>( \overparen{BAC} )</td>
<td>( \overparen{BFC} )</td>
</tr>
</tbody>
</table>

*The reverse notation of any of these arcs is also acceptable.*

3. Disprove the following statement by drawing an illustration using circle K.
   “All secants of a circle are parallel.”

**Student’s answers should include:**
- Two secant lines that are not parallel

**Example Solution:**
Chapter 9 – Problem 2

Common Core State Standards:

G.MG #1 – Use geometric shapes, their measures and their properties to describe objects.

Bloom’s Taxonomy – Analysis

Learning Level – Application

1. Find $m \angle A$.

2. Fill in the table below to show the $m \widehat{AC}$ is the same when applying both the Arc Addition Postulate and the Inscribed Angle Theorem.

<table>
<thead>
<tr>
<th>Inscribed Angle Theorem</th>
<th>Arc Addition Postulate</th>
</tr>
</thead>
</table>
Solution:

1. a) Find $m\angle A$.

$$m\angle A = \frac{1}{2}(86) = 43$$

Inscribed Angle Theorem: The measure of an inscribed angle is half the measure of its intercepted arc

b) Fill in the table below to show the $m\overline{AC}$ is the same when applying both the Arc Addition Postulate and the Inscribed Angle Theorem.

<table>
<thead>
<tr>
<th>Inscribed Angle Theorem</th>
<th>Arc Addition Postulate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sum of the angles will measure to 180.</strong></td>
<td><strong>By using the inscribed angle theorem</strong></td>
</tr>
<tr>
<td>$180 - 70 - 43 = 67$</td>
<td>$m\angle C = \frac{1}{2}(m\overline{AB})$</td>
</tr>
<tr>
<td>$m\angle B = 67$</td>
<td>$m\overline{AB} = 2(70) = 140$</td>
</tr>
<tr>
<td><strong>By the Inscribed Angle Theorem</strong></td>
<td><strong>By the Arc Addition Postulate: The measure of an arc formed by two adjacent arcs is equal to the sum of the measures of the arcs.</strong></td>
</tr>
<tr>
<td>$m\angle B = \frac{1}{2}(m\overline{AC})$</td>
<td><strong>Sum of the arcs equals 360°</strong></td>
</tr>
<tr>
<td>$67 = \frac{1}{2}(m\overline{AC})$</td>
<td>$m\overline{AC} = 360 - (m\overline{AB} + m\overline{BC})$</td>
</tr>
<tr>
<td>$67(2) = 134$</td>
<td>$m\overline{AC} = 360 - (140 + 86)$</td>
</tr>
<tr>
<td>$m\overline{AC} = 134$</td>
<td>$m\overline{AC} = 360 - (226)$</td>
</tr>
</tbody>
</table>

**Students must solve for $m\overline{AC}$ using the Inscribed Angle Theorem**

**Students must solve for $m\overline{AC}$ using the Arc Addition Postulate.**
Chapter 9 – Problem 3

Common Core State Standards:

G.C #2 – Identify and describe relationships among inscribed angles, radii, and chords

Bloom’s Taxonomy – Evaluation

Learning Level – Application

1. In circle K the diameter ST is perpendicular to chord FG. Of the following which will **always** be a correct statement?

(a) \( FR = GR \)
(b) \( RT = KR \)
(c) \( SK = RG \)

2. In circle H chords LM and WV intersect at point K. Of the following which will **always** be a correct statement?

(a) \( LK \cdot WK = MK \cdot VK \)
(b) \( LK \cdot MK = WK \cdot VK \)
(c) \( LK \cdot JK = MK \cdot KH \)

Solution:

1. In circle K the diameter ST is perpendicular to chord FG. Of the following which will **always** be a correct statement?

**(a)** \( FR = GR \) (Diameter-Chord Theorem)
(b) \( RT = KR \)
(c) \( SK = RG \)

2. In circle H chords LM and WV intersect at point K. Of the following which will **always** be a correct statement?

(a) \( LK \cdot WK = MK \cdot VK \)
**(b)** \( LK \cdot MK = WK \cdot VK \) (Segment Chord Theorem)
(c) \( LK \cdot JK = MK \cdot KH \)
Chapter 9 – Problem 4

Common Core State Standards:

G.MG #1 – Use geometric shapes, their measures and their properties to describe objects.

G.C #2 – Identify and describe relationships among inscribed angles, radii, and chords

Bloom’s Taxonomy – Application, Synthesis

Learning Level – Discover a Relationship, Application

**Part 1: Exterior Angles of Circle Theorem**

An angle that is constructed outside of the circle is called an exterior angle.

The Exterior Angle of a Circle Theorem has three cases that can be constructed using secant and tangents.

Case 1)

Case 2)

Case 3)

Using Case 3 answer the following question:

1. From the choices below what information would you need to solve for \( m\angle BFC \)?

   a) \( m\angle AD \) and \( m\angle BC \)
   
   b) \( m\angle ABC \) and \( m\angle ADC \)
   
   c) \( m\angle BDC \) and \( m\angle BC \)

2. What then should be done with the information to solve for \( m\angle BFC \)?

**Part 2: Tangent and Secant Theorems**

We have discussed three theorems that involve secant and tangent lines. They are the Secant Tangent Theorem, the Secant Segment Theorem and the Tangent Segment Theorem. Each of these theorems is represented above by one of the three cases of the Exterior Angle of a Circle Theorem.

3. For case 1 (above), \( FC = 6 \) and \( FB = 4 \). Solve for \( FA \).

4. For case 2 (above), \( AF = 6, BF = 4, \) and \( DF = 8 \). Solve for \( CF \).

5. For case 3 (above), if a line segment \( BC \) was constructed what type of triangle would be \( \triangle BFC \)?

   A) Equilateral Triangle
   
   B) Isosceles Triangle
   
   C) Similar Triangle
Solution:

**Part 1: Exterior Angles of Circle Theorem**

An angle that is constructed outside of the circle is called an exterior angle. The Exterior Angle of a Circle Theorem has three cases that can be constructed using secant and tangents.

![Case 1](image1)

Case 1

![Case 2](image2)

Case 2

![Case 3](image3)

Case 3

1. Using Case 3 answer the following question:

From the choices below what information would you need to solve for \( \angle BFC \)?

(A) \( \angle ADC \) and \( \angle BDC \)

(B) \( \angle ABC \) and \( \angle ADC \)

**C) \( \angle BDC \) and \( \angle BDC \)

**Exterior Angles of a Circle Theorem**

2. What then should be done with the information to solve for \( \angle BFC \)?

**Solution Example 1** \( \angle BFC = \frac{1}{2} (\angle BDC - \angle B) \)

**Solution Example 2** Subtract \( \angle BDC \) from \( \angle BDC \) then divide the answer by 2.

**Part 2: Tangent and Secant Theorems**

We have discussed three theorems that involve secant and tangent lines. They are the Secant Tangent Theorem, the Secant Segment Theorem and the Tangent Segment Theorem. Each of these theorems is represented above by one of the three cases of the Exterior Angle of a Circle Theorem.

3. For case 1 (above), \( FC = 6 \) and \( FB = 4 \). Solve for \( FA \).

Secant Tangent Theorem \( FB \cdot FA = FC^2 \)

\[ 4 \cdot FA = 6^2 \]

\[ FA = \frac{36}{4} = 9 \]

4. For case 2 (above), \( AF = 6, BF = 4, \) and \( DF = 8 \). Solve for \( CF \).

Secant Segment Theorem \( AF \cdot BF = DF \cdot CF \)

\[ 6 \cdot 4 = 8 \cdot CF \]

\[ CF = \frac{24}{8} = 3 \]

5. For case 3 (above), if a line segment BC was constructed what type of triangle would be \( \triangle BFC \)?

(A) Equilateral Triangle

**(B) Isosceles Triangle

(C) Similar Triangle

Tangent Segment Theorem \( BF = CF \)
Chapter 9 – Rubric

Chapter 9 Test Items – Rubric

Problem 1
1) +2 – Correct (C)  +0 – Incorrect Answer
2) +.5 – For each correctly filled in cell
   +5 +4.5 +4 +3.5 +3 +2.5
   +2 +1.5 +1 +.5 +0
3) +2 – Draws two secants lines that are not parallel
   +1 – Draws two secants lines that are parallel or
       Draws two lines that are not secants and are not parallel
   +0 – Incorrect interpretation or no response

Problem 2
1a) +2 – Uses the Inscribed Angle Theorem to find the correct $m\angle$.
    +1 – Uses the Theorem but makes minimal computational error.
    +0 – Wrong answer based on an incorrect plan, or no response
1b) Inscribed Angle Theorem
    +3 – Correct answer, used Inscribed Angle Theorem, clean organized work
    +2 – Minimal computation error
    +1 – Did not use the Theorem, serious computational error or no work to support
    +0 – Wrong answer based on an incorrect plan, or no response

Problem 3
1) +2 – Correct (A)  +0 – Incorrect
2) +2 – Correct (B)  +0 – Incorrect

Problem 4
1) +2 – Correct (C)  +0 – Incorrect Answer
2) +2 – Correct use of the Exterior Angles of a Circle Theorem to find the $m\angle BFC$
    +1 – Partially correct use of Theorem, minimal computational error.
    +0 – Wrong answer based on an incorrect plan, or no response
3) +2 – Correct use of the Secant Tangent Theorem to find $FA$
    +1 – Partially correct use of Theorem, minimal computational error.
    +0 – Wrong answer based on an incorrect plan, or no response
4) +2 – Correct use of the Secant Segment Theorem to find $CF$
    +1 – Partially correct use of Theorem, minimal computational error.
    +0 – Wrong answer based on an incorrect plan, or no response
5) +2 – Correct (B)  +0 – Incorrect Answer

Problem 4 TOTAL ______/ 10

TOTAL ______/31
Chapter 10 – Test Items

Chapter 10 – Problem 1

Common Core State Standards:

G.C #3 – Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle

Bloom’s Taxonomy – Evaluation

Learning Level – Comprehension and Communication

1. Of the following statements which is correct concerning the inscribed quadrilateral?

(A) \( m\angle A + m\angle B = 180^\circ \)
(B) \( m\angle A + m\angle C = 180^\circ \)
(C) \( m\angle BCD = \frac{1}{2} (m\angle A) \)
(D) \( m\angle ADC = m\angle BCA \)

Solution:

1. Of the following statements which is correct concerning the inscribed quadrilateral?

(A) \( m\angle A + m\angle B = 180^\circ \)
** (B) \( m\angle A + m\angle C = 180^\circ \)
(C) \( m\angle BCD = \frac{1}{2} (m\angle A) \)
(D) \( m\angle ADC = m\angle BCA \)
Chapter 10 – Problem 2

Common Core State Standards:

G.MG #1 – Use geometric shapes, their measures and their properties to describe objects.

Bloom’s Taxonomy – Application

Learning Levels - Application

Mrs. Jones has a dry patch on her lawn. In order to conserve water she sets the sprinkler to rotate 90° so only the dry part of the grass gets watered. The only downfall is with the location of the sprinkler it shoots past the edge of the grass onto the road. With the current water pressure the sprinkler sprays water 30 feet.

1. Calculate the area the sprinkler is watering.
2. How much of the road is getting watered?

Solution:

Mrs. Jones has a dry patch on her lawn. In order to conserve water she sets the sprinkler to rotate 90° so only the dry part of the grass gets watered. The only downfall is with the location of the sprinkler it shoots past the edge of the grass onto the road. With the current water pressure the sprinkler sprays water 30 feet.

1. Calculate the area the sprinkler is watering?

\[
\text{Area of Sector} = \frac{\text{measure of angle}}{360} \cdot \pi r^2
\]

\[
= \frac{90}{360} \cdot \pi (30)^2
\]

\[
= 225\pi = 706.86 \text{ ft}^2
\]

The sprinkler is watering 706.86 ft²

2. How much of the road is getting watered?

\[
\text{Area of Segment} = \text{Area of Sector} - \text{Area of Triangle}
\]

\[
= 706.86 - \frac{1}{2} (30)(30)
\]

\[
= 706.86 - 450 = 256.86 \text{ ft}^2
\]

256.86 ft² of the road is getting watered.
Chapter 10 – Problem 3

Common Core State Standards:

- **G.MG #1** – Use geometric shapes, their measures and their properties to describe objects.
- **G.C #5** – Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measures of the angle as constant of proportionality

**Bloom’s Taxonomy** – Knowledge, Synthesis and Evaluation

**Learning Level** – Discover a Relationship, Application

<table>
<thead>
<tr>
<th>PLEASE INCLUDE UNITS IN ALL RESPONSES WHERE APPROPRIATE</th>
</tr>
</thead>
</table>

**A Circular Track:**

- The distance around the inside of lane 1 is 400 meters.
- The width of each lane is 2 meters.
- Assume the runners run counter-clockwise as close as they can to the inside of their lane.
- The runners all start and finish at the same line.
- From the start to the finish line for lane 5 is 225°.

1. The circular track could be described as having what pattern of circles? ________________

2. What is the relationship between the circle that is lane 1 and the circle that is lane 5?
   - (A) Circumscribed
   - (B) Congruent
   - (C) Similar

3. Identify the degree measure from the start to finish line for lane 3? ________________

4. What conclusions can be made for the degree measure from the start to finish line for all lanes of the track? ________________

5. a. From your conclusion on (4) does this mean that all the lanes will cover the same distance between the start and finish line? Yes or No
   b. Then explain your reasoning.

6. Find the radius to the inside of lane 1 and lane 5.
7. From the start to finish line how many meters does the runner in lane 5 travel?

<table>
<thead>
<tr>
<th>Radius of Lane 1</th>
<th>Radius of Lane 5</th>
</tr>
</thead>
</table>

8. For the runner in lane 1 to travel the same distance as the runner in lane 5 how many additional degrees would they have to go pass the finish line?

9. As illustrated above it doesn’t work to have individuals start on the same starting line. This is why tracks use what is called a staggered start; this is when everyone starts at a different place so that they all have traveled the same distance by the time they reach the finish line. Find how many meters runner 5 should be in front of the starting line in order for them to have covered the same distance as the runner in lane 1 when they reach the finish line?

Solution:

A Circular Track:
- The distance around the inside of lane 1 is 400 meters.
- The width of each lane is 2 meters.
- Assume the runners run counter-clockwise as close as they can to the inside of their lane.
- The runners all start and finish at the same line.
- From the start to the finish line for lane 5 is 225°.

1. The circular track could be described as having what pattern of circles? **concentric**

2. What is the relationship between the circle that is lane 1 and the circle that is lane 5?
   - (A) Circumscribed
   - (B) Congruent
   - **(C) Similar**

3. Identify the degree measure from the start to finish line for lane 3? **225°**

4. What conclusions can be made for the degree measure from the start to finish line for all lanes of the track? **From start to finish line all the lanes have the same degree measure**

5. From your conclusion on (4) does this mean that all the lanes will cover the same distance between the start and finish line? Yes or No
   - Yes
   - Explain your reasoning.
   - The degree measure for each lane from start to finish may be the same but the radius for each lane gets bigger which means that the distance between the start to finish line will increase for each lane.

6. Find the radius to the inside of lane 1 and lane 5.
   - \[
   \frac{400}{2\pi r} = \frac{400}{2\pi} = 63.66 \text{ m}
   \]
   - \[
   63.66 + 2(4) = 71.66 \text{ m}
   \]

<table>
<thead>
<tr>
<th>Radius of Lane 1</th>
<th>Radius of Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.66 m</td>
<td>71.66 m</td>
</tr>
</tbody>
</table>

7. From the start to finish line how many meters does the runner in lane 5 travel?

\[
\text{Area Length} = \frac{\text{measure of angle}}{360} \cdot 2\pi r = \frac{225}{360} \cdot 2\pi(71.66) = 281.408 \text{ meters}
\]

Runner 5 runs 281.408 meters.
8. For the runner in lane 1 to travel the same distance as the runner in lane 5 how many \textbf{additional} degrees would they have to go pass the finish line?

\[
281.408 = \frac{x}{360} \cdot 2\pi \cdot 63.33
\]

\[
x = \frac{281.408}{2\pi/63.66} \cdot 360 = 253^\circ
\]

\[
253 - 225 = 28^\circ
\]

9. As illustrated above it doesn’t work to have individuals start on the same starting line. This is why tracks use what is called a staggered start; this is when everyone starts at a different place so that they all have traveled the same distance by the time they reach the finish line.

Find how many meters runner 5 should be in front of the starting line in order for them to have covered the same distance as the runner in lane 1 when they reach the finish line?

\[
Runner \text{ in Lane 1 runs} \quad \frac{225}{360} \cdot 2\pi \cdot 63.66 = 249.99 \text{ meters}
\]

\[
Lane \text{ 5 runner} - Lane \text{ 1 runner} = 281.408 - 249.99 = 31.418
\]

Thus the runner in lane 5 needs to start 31.418 meters in front of the runner in lane 1.
## Chapter 10 – Rubric

### Chapter 10 Test Items – Rubric

#### Problem 5

1) +2 – Correct (B)  
   +0 – Incorrect Answer

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**PROBLEM 5 TOTAL _____/ 2**

#### Problem 6

1) +3 – Found correct area of the sector, clean organized work  
   +2 – Minimal computational error  
   +1 – Serious computational error or no work to support  
   +0 – Wrong answer based on an incorrect plan, or no response

2) +3 – Found correct area of the segment, clean organized work  
   +2 – Minimal computational error  
   +1 – Serious computational error or no work to support  
   +0 – Wrong answer based on an incorrect plan, or no response

---

**PROBLEM 6 TOTAL _____/ 6**

#### Problem 7

1) +1 – Concentric  
   +0 – Incorrect Answer

2) +1 – Correct (C)  
   +0 – Incorrect Answer

3) +1 – 225°  
   +0 – Incorrect Answer

4) +1 – Draws correct conclusion  
   +0 – Incorrect conclusion or no response

5) a) +1 – No  
   +0 – Incorrect Answer

   b) +1 – Draws correct conclusion  
   +0 – Incorrect conclusion or no response

6) Radius of Lane 1:  
   +2 – Find correct radius of lane 1.  
   +1 – Minimal computational error.  
   +0 – Wrong answer based on an incorrect plan

Radius of Lane 5:  
   +1 – Find correct radius of lane 5  
   +0 – Wrong answer based on an incorrect plan

7) +2 – Find correct distance of runner in lane 5.  
   +1 – Minimal computational error.  
   +0 – Wrong answer based on an incorrect plan, or no response

8) +2 – Find the correct additional degrees the runner in lane 1 needs to run.  
   +1 – Minimal computational error  
   +0 – Wrong answer based on an incorrect plan, or no response

9) +3 – Found correct distance, clean organized work  
   +2 – Minimal computation error  
   +1 – Serious computational error or no work to support  
   +0 – Wrong answer based on an incorrect plan, or no response

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**PROBLEM 7 TOTAL _____/ 16**

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**TOTAL FOR CHAPTER 10 _____/24**
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