Stability of Basin-Scale Internal Waves Within the South Arm of the Great Salt Lake

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STABILITY OF BASIN-SCALE INTERNAL WAVES WITHIN THE SOUTH ARM OF THE GREAT SALT LAKE

by

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Abstract

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by

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The fluid circulation patterns, temperature distributions, and density gradients of the South Arm of the Great Salt Lake were modeled using the Estuary, Lake, and Coastal Ocean Model (ELCOM) from the Centre for Water Research at the University of Western Australia. The effort is part of a research study in conjunction with the United States Geological Survey (USGS) and the Utah Water Research Lab located at Utah State University. The model was simulated for several different cases of salinity gradients over different time periods, using temperature and wind data from 2006. The model is then used to identify factors which may provide a transport mechanism of heavy metals such as selenium and mercury from the sediment layers to the upper brine layers where it is introduced into the food chain.

(71 pages)
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Chapter 1

Introduction

The “great” in Great Salt Lake, is certainly there for a reason. The title designates its unusual composition and large magnitude. It is the largest natural lake in the western United States, the fourth largest terminal lake in the world, as well as being three to five times saltier than the ocean. Due to the lake’s hypersaline nature, there are no fish. The largest aquatic life are tiny brine flies and brine shrimp. Yet, despite the lack of wildlife, the lake plays a vital role in the area’s ecosystem as millions of migratory birds call the Great Salt Lake home at some point in their journey.

As the largest remnant of the prehistoric Lake Bonneville, the lake has seen its share of changes through its life. In the last century, human influence on the lake has increased drastically. Of these changes, one stands out above the rest. In 1959, a solid fill railroad causeway was built, dividing the lake into three arms, referred to now as the Northwest Arm, the Northeast Arm, and the South Arm. In the years that have followed the causeway construction, each arm has essentially become a lake of its own.

The Northwest Arm, void of any major tributaries has become far saltier than anywhere else in the lake. The Northeast Arm is the smallest, and has one of the major tributaries, the Bear River. The largest, and for this study, most important is the South Arm. Mixing between the three arms is limited to several 100-foot-long culverts. Depending on the surface elevation of the lake, the amount of water mixing between arms can vary substantially from year to year.

As salinity and density tend to vary from one arm to another, the mixing between arms can be analogous to that of two completely different fluids. For example, the dense brine from the Northwest Arm enters the South Arm and interacts with the lighter brine. The density differences result in the two brine layers remaining separated, with the dense brine
sinking to the bottom and lighter brine rising to the top. The effect is much like putting water and cooking oil in the same container. The boundary formed between these two brine layers is known as a halocline.

In 1997, researchers from the U.S. Environmental Protection Agency identified numerous local point sources for atmospheric deposition into the Great Salt Lake [1]. Subsequent studies measured elevated levels of mercury in the lake. These high levels of mercury and other heavy metals are of serious concern in the Great Salt Lake. These metals enter the lake primarily through atmospheric deposition. After entering the lake, most mercury particles attach to other particles and settle to the bottom of the lake, far away from animal life that may be affected by the toxic metal. In the deep, benthic layers, the mercury particles may be methylated. In its most toxic form as methylmercury, the metal enters the food chain, putting the lake’s many migratory birds at risk, as toxic levels are reached quickly. In 2005, three duck species were listed as advised against human consumption due to high mercury levels: the Northern Shoveler, Common Goldeneye, and Cinnamon Teal [2]. The means by which methylmercury reaches the upper layers of the lake is not well understood. The halocline separating the two brine layers acts to prevent turbulent mixing between the upper and lower brine layers. However, wave propagation on the interface of these two layers results in both horizontal motion and the associated vertical shear along the interface. If the vertical shear is large enough, relative to the stabilizing effect of the density gradient, the shear layer becomes unstable. This instability leads to the formation of short wavelength, high frequency waves along the interface that may break—as a surface wave would—and lead to turbulent mixing. This mixing would present a clear transport mechanism for the methylmercury to reach the upper brine layers from the deep brine layers of the lake. The result of this study should determine if this mechanism is important in the Great Salt Lake, and clarify which parameters are most important to the mixing process.
Chapter 2

Study Overview

2.1 Overview

The brine contained within the South Arm of the Great Salt Lake is constantly trying to reach its equilibrium state. This tendency for equilibrium is hampered by the external forces that the lake is subject to. Everything from wind blowing across the lake’s surface, river inflows, buoyancy currents induced by internal temperature changes, and even the rotation of the earth affect the state of the lake.

These external forces result in motion of the fluid within the lake. The effect of wind blowing across the lake surface produces visible motion, primarily on the lake’s surface. Depending on the velocities of the wind, the resulting waves can be quite large. If these waves become large enough, a whitecap can be seen at the peak of the wave. A white cap is simply the peak of the wave becoming unstable. It is a turbulent mixing of the air and water. Like waves on the surface, waves can also occur within the body of water.

As discussed above, a halocline is an interface between two layers of water with different salinity levels. This interface is quite similar to that of the water-air interface in the fact that it is simply an interface between two fluids of different densities. Waves can develop along this interface, just like they do along the surface. These internal waves are more likely to become unstable due to the small density change from one layer to the other. The Richardson number (Ri) is a dimensionless ratio of the potential energy to kinetic energy in a wave [3]. It can also be thought of as the ratio of the gravitational tendency to remain a stratified flow to the energy supplied by the shear that develops between the layers. The
Richardson number is expressed as

\[ \text{Ri} = \frac{g \left( \frac{du}{dz} \right)}{\rho \left( \frac{du}{dz} \right)^2}. \]  

(2.1)

It is known that for Richardson numbers below a level of 0.25, the flow is unstable [3]. For unstable internal waves, the turbulence levels increase and the mixing between upper and lower brine layers is greatly enhanced. When these waves are large enough to hit land, sediment on the bottom of the lake can be stirred up and quickly transported to the upper brine layers.

Additionally, if the wind blows in a given direction, friction between the air and the water cause the water to move in the general direction of the wind. Eventually the water is forced up against the shore and begins to “set” up. This effect is similar to one taking a basin of water and lifting one side. When the basin is returned to a level position, the surface of the water is no longer level, and hydrostatic pressure will force the water from the deeper side to the shallow side. In most cases, the motion of the water will hit the opposite side of the basin and these waves will begin to oscillate between the sides of the basin. These waves will eventually die out and the surface will remain calm until another forcing event sets the water up again. In the case of the lake, when the wind dies down, the motion is created, as the surface tries to calm itself again. The resulting motions are very long waves, with wavelengths of the same order as the basin dimensions [3]. These waves can be found on both the surface and internally along a halocline and are known commonly as seiches.

2.2 Objectives

This study will primarily focus on the interactions that develop along the halocline found in the South Arm of the Great Salt Lake. The interface along this halocline is subject to basin-scale internal wave motions, including seiching. These internal waves are similar to those which occur on the lake surface. However, the density difference between the upper and lower brine layer is much smaller than the difference between the upper
brine layer and the atmosphere. As a result the displacement of the internal seiching is much larger than typical surface seiches. The density variation through the brine layer is continuous and there are an infinite number of theoretical seiche modes. The vertical motion of the seiche induces a strong horizontal motion of the brine along the halocline. The magnitude of the resulting vertical shear determines if instabilities will develop.

For unstable flows, determined by the Richardson number, the interface becomes unstable in the manner of a Kelvin-Helmholtz instability [3]. This instability causes the formation and breaking of short wavelength, high frequency waves which would result in significantly enhanced mixing of solutes between the upper and lower brine layers. Thus, this work aims to quantify a viable fluid dynamic mechanism for the transport of heavy metals, such as selenium and mercury, from the sediment layers to the upper brine regions of the lake. In addition, it is desired to know which seiche modes exist in the South Arm of the Lake. This information will be determined through numerical simulations forced by external wind events.

A numerical model will be created to simulate external forcing events. Data collected through the simulation will be processed to calculate the Richardson numbers and identify which seiche modes are present in the flow.

2.3 Limitations

A numerical simulation model will only predict the general circulation patterns and scalar values over time. As such, it is very important to remember that these values will not generally reflect the conditions at any given period of time. Small discrepancies between the actual flow conditions and the model’s solution will exist. These differences may be primarily attributed to the non-linear nature of the Navier-Stokes equations which the model is based. The non-linearities and other reasons for these differences will be discussed in the following chapters.

In addition, the present study is severely limited by a lack of long-term experimental studies on the Great Salt Lake. In an ideal situation, numerical models should be validated in order to determine their accuracy. At the present time there is a lack of sufficient data
to validate the model for the South Arm of the Great Salt Lake. The methods used to
develop this model have been used to analyze many different bodies of water for which
sufficient experimental data is available. The numerical approach has been refined to a
sufficient degree of accuracy. It is hoped that the results from this study will provide a
motivation to perform a long-term experimental study on the lake and will provide insight
on the parameters necessary to base such a study.
Chapter 3
Computational Fluid Dynamics Model

The governing equations for the physics of fluid motion are known as the Navier-Stokes equations. For incompressible flow with constant transport properties, the equations can be expressed as

\[
\frac{DV}{Dt} = -\frac{1}{\rho} \nabla \rho + g + \frac{1}{\rho} \nabla \cdot \tau_{ij}
\]  

(3.1)

where \(\frac{DV}{Dt}\) is the material derivative of the velocity vector.

Due to the non-linearities which exist in the equations, there is a limit to the number of solutions which can be obtained analytically. The solutions which can be determined analytically are often simple geometries, with limiting boundary conditions, which eliminate the said non-linearities. For most fluid flows, like that seen in a complicating lake geometry, these simplifying assumptions are invalid. There are many types of numerical solutions to model fluid flow, ranging from Direct Numerical Simulation (DNS) to the many flavors of the more broad Computational Fluid Dynamics (CFD). DNS is a direct model of the Navier Stokes equations, the results of DNS are said to be exact, yet require extremely fine meshes and time steps to resolve the various processes which occur in viscous fluid flow. CFD has been broken up into four branches. The first branch is known as finite-difference. This method uses the idea that algebraic expressions can be developed for the partial derivatives found in the Navier-Stokes equations at a finite number of discrete mesh points. The second method, finite-element, also makes use of a finite mesh, but actually models the equations between the mesh points. The final two branches are still in development: spectral methods which make use of an approximating function such as the Fourier series, and vortex methods which avoid the Navier-Stokes equations by summing thousands of elementary vortices to construct a model of the flow. Each CFD method has its advantages and disadvantages, but
nearly all of them (with the exception of the vortex methods) require restrictions applied to them. In order to develop a numerical model of the South Arm of the Great Salt Lake, the benefits of each method should be weighed against each other.

The size of the lake immediately removes the possibility of DNS. The computation time and capacity to model such a lake present any solution from converging in a reasonable amount of time. While finite-element has been used in many flow analysis, the solutions tend to be slower and require more computational storage than finite-difference. As a result, a code using a finite difference method will be used to model the South Arm of the Great Salt Lake.

3.1 Modeling of Large Bodies of Water

Modeling a large body of water presents numerous problems that are inherent to the sheer size of the domain. The large physical size of most bodies of water forces the use of large computation domains. This, in turn, forces any model that converges to a solution in a reasonable amount of time to feature a coarse computational grid. These coarse grid solutions can only be taken as a general approximation of the fluid dynamics within the body of water.

In addition, any code which attempts to model a large body of water must also be capable of including the Coriolis forces due to the Earth's rotation, thermal forcing, estuary inflows and outflows, as well as surface wind stress. While most commercial codes allow for input of various forcing functions, many are general codes that are designed to handle a broad range of flows.

The code which was used as the primary software package for this study is the Estuary, Lake, and Coastal Ocean Model (ELCOM) developed at the Centre for Water Research at the University of Western Australia. ELCOM is a three-dimensional, time-dependent model which predicts water flow patterns caused by external forcing functions of air temperature, wind shear, and incident solar radiation.
3.2 ELCOM Background

ELCOM was developed at the Centre for Water Research at the University of Western Australia. The focus for the development of ELCOM was to “produce a numerical model that can reproduce the first-order 3D physical response of a lake to environmental forcing on a coarse grid with low CPU time requirements so that the method can be used to develop greater understanding of the variables which drive mixing dynamics and scalar transport in geophysical systems” [4].

ELCOM is simply a numerical modeling tool which combines a hydrodynamic and a thermodynamic model to simulate the behavior of bodies of water that are subject to environmental forcing. The hydrodynamic model solves the unsteady, viscous, incompressible, Navier-Stokes equations with the hydrostatic assumption employed for the pressure terms.

3.3 Hydrodynamics

There are several equations which comprise the hydrodynamics portion of the ELCOM solver. The following subsections give an overview of the equations utilized by ELCOM. For an in-depth view of ELCOM’s hydrodynamic equations, the reader is referred to the ELCOM Science Manual [5].

3.3.1 Momentum Transport

ELCOM makes use of the Reynolds Averaged Navier Stokes equations, shown in indicial notation in Equation 3.2, to account for momentum transport. When dealing with turbulent flows, the engineer is not generally concerned with resolving the details of the turbulent fluctuations. Time-averaged flow properties are sufficient in most cases. The Navier-Stokes equations can therefore be substituted with time-averaged equations of motion. These equations are known as the Reynolds Averaged Navier-Stokes (RANS) equations. The RANS equations are formed by averaging the unsteady Navier-Stokes equations over a time period that is long relative to sub-grid scale process and small relative to the grid scale processes [5]. The RANS equations allow the the effects of small length scale eddies while
accounting for the larger, mean flow, circulation eddies. In most numerical schemes the
time step used in the iterative process will be the time scale used to average the equations.

\[
\frac{\partial U_\alpha}{\partial t} + U_j \frac{\partial U_\alpha}{\partial x_j} = -g \left\{ \frac{\partial \eta}{\partial x_\alpha} + \frac{1}{\rho_0} \frac{\partial}{\partial x_\alpha} \int_0^\eta \rho' dz \right\} + \frac{\partial}{\partial x_1} \left( \nu_1 \frac{\partial U_\alpha}{\partial x_1} \right) + \\
\frac{\partial}{\partial x_2} \left( \nu_2 \frac{\partial U_\alpha}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left( \nu_3 \frac{\partial U_\alpha}{\partial x_3} \right) - \varepsilon_{\alpha\beta} f U_\beta
\] (3.2)

When solving the unsteady RANS equations for turbulent flows, it is also necessary
to add additional transport equations to model the turbulence. While there are many
turbulence models for various flow conditions, sometimes the simplest models are the best.
ELCOM makes use of the so-called Boussineq approximation to model turbulence. The
Boussinesq approximation was introduced in 1877 by J. Boussinesq. Boussinesq postulated
that the Reynolds stresses are proportional to the mean rates of deformation. The idea is
that the turbulent shear \( (\rho u^i v^j) \) is a gradient diffusion term, which is analogous to molecular
shear [6]:

\[
\tau_t = -\rho u^i v^j = \mu_t \frac{\partial \bar{u}}{\partial \bar{y}}
\] (3.3)

with \( \mu_t \) as the eddy viscosity. This eddy viscosity has the same dimensions as \( \mu \), but
unlike \( \mu \), it is not a fluid property, it varies with geometry and flow conditions. Unlike
many turbulence models, using the Boussinesq approximation adds no additional transport
equations to be solved.

Additionally, ELCOM neglects the non-hydrostatic pressure terms. This simplifies the
pressure calculation while including the dominate pressure term found in large bodies of
water.

### 3.3.2 Continuity

The mass conservation equation, or continuity equation, is given as

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\] (3.4)
ELCOM is designed for incompressible flows, thus the continuity equation is reduced to its
incompressible form by dropping the time rate of change of density. Shown in indicial form,
the continuity equation used in ELCOM is written as

$$\frac{\partial U_j}{\partial x_j} = 0. \quad (3.5)$$

### 3.3.3 Scalar Transport

For strongly stratified flows, the scalar transport equations are extremely important. The
developers of ELCOM described them as “the most critical piece of the hydrodynamic
numerical algorithm” for such flows [5]. The scalar transport equations are responsible for
calculating the evolution of the density field and internal wave motions. ELCOM uses a
conservative third-order method for scalar transport.

In indicial notation, the transport equation is

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x_j} (CU_j) = \frac{\partial}{\partial x_1} \left( \kappa_1 \frac{\partial C}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left( \kappa_2 \frac{\partial C}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left( \kappa_3 \frac{\partial C}{\partial x_3} \right) + S_c \quad (3.6)$$

where

- $C =$ scalar concentration
- $\kappa_i =$ diffusion components
- $Sc =$ turbulent Schmidt number (or Prandtl number for temperature).

### 3.3.4 Free-Surface

There are several special conditions which are observed at the free surface of large
bodies of water. The first is that waves and other surface motions will cause changes in
the free surface height of the lake. This free surface height can be obtained by integrating
each water column from the bottom of the lake to the free surface height, $\eta$, and then
applying the Reynolds-averaging filter to the kinematic boundary conditions. The equation
for evolution of the free surface is given as

$$\frac{\partial \eta}{\partial t} = - \frac{\partial}{\partial x} \left( \alpha \int_0^n u_\alpha dz \right)$$  \hspace{1cm} (3.7)

Two additional equations are included to account for the wind shear, Equation 3.8, and wind induced momentum, Equation 3.9.

$$\left( u_* \right)_\alpha^2 = C_{10} \frac{\rho_{\text{air}}}{\rho_{\text{water}}} \left( W_{\beta} W_{\beta} \right)^{1/2} W_\alpha$$ \hspace{1cm} (3.8)

$$\frac{\partial U_\alpha}{\partial t} = \frac{\left( U_* \right)_\alpha^2}{h}$$ \hspace{1cm} (3.9)

The values in these equations are defined as

$$\left( u_* \right)_\alpha = \text{wind velocity in } \alpha\text{-direction}$$

$$C_{10} = \text{bulk wind stress coefficient at 10 meters}$$

$$\rho_{\text{air}} = \text{free surface air density}$$

$$\rho_{\text{water}} = \text{free surface water density}$$

$$W_\alpha = \text{wind speed in } \alpha\text{-direction}$$

$$W_{\beta} = \text{wind speed in } \beta\text{-direction}$$

$$h = \text{mixed layer height}$$

### 3.3.5 Boundary Conditions

In order to solve a differential equation, it is required that conditions be known on the boundaries. There are several different boundaries that exist in a lake. The first of these is for the momentum at the free surface boundary.

$$\frac{\partial U_\alpha}{\partial x_3} = 0$$ \hspace{1cm} (3.10)

As seen in the equation, momentum flux is restricted in the z-axis.
For momentum on the lake bottom and sides, the no-slip condition is imposed.

\[ U_i = 0 \]  
\[ (3.11) \]

At any boundary in the domain, the boundaries prevent any scalar flux. Thus the scalar transport boundary condition is given as

\[ \frac{\partial C_\alpha}{\partial x_j} = 0. \]  
\[ (3.12) \]

### 3.4 Thermodynamics

The temperature of a given cell is given with the internal heat energy relation

\[ \Delta T = \frac{\Delta Q}{\rho V c_p}, \]  
\[ (3.13) \]

with

\[ \Delta T = \text{Change in water temperature} \]
\[ \Delta Q = \text{Change in heat energy} \]
\[ \rho = \text{Density of water} \]
\[ V = \text{Volume of water} \]
\[ c_p = \text{Constant pressure specific heat of water} \]

The initial internal temperatures are given to the model at startup as a temperature profile. Surface thermodynamics go here, if used.

### 3.5 Numerical Approach

ELCOM has been adapted from the TRIM approach that was developed by Casulli and Cheng [7]. In addition to the original TRIM approach, there have been several revisions by
Casulli and Cattani [8], Casulli [9], and Gross et al. [10,11], that ELCOM has implemented. While based on TRIM, ELCOM has modifications “for accuracy, scalar conservation, numerical diffusion, and implementation of a mixed-layer turbulence closure” [5]. There are three primary differences between ELCOM and TRIM. ELCOM makes use of:

- a hybrid advection scheme for momentum
- an energy-based mixing model for vertical diffusion
- conservative advection of scalars using a third-order explicit method

Having previously discussed the governing equations that ELCOM is based upon, the numerical approach will be further discussed. ELCOM advances the model solution one time step in the following order [5]:

1. introduction of surface heat transfer in the surface layer
2. mixing of scalar concentrations and momentum using a mixed-layer model
3. introduction of wind energy as a momentum source in the wind-mixed layer
4. solution of the free-surface evolution and velocity field
5. horizontal diffusion of momentum
6. advection of scalars
7. horizontal diffusion of scalars

3.5.1 Surface Thermodynamics

ELCOM models the surface thermodynamics by allowing the user to specify conditions for short-wave radiation, long-wave radiation as well as fluxes between the atmosphere and lake surface due to evaporation and sensible heat. ELCOM’s inputs for these parameters rely on accurate weather data of air temperature, humidity, and cloud cover.

The short-wave radiation is divided into four parts:
• Photosynthetically Active Radiation (PAR) 45%
• Near Infrared (IR) 41%
• Ultra Violet A (UVA) 3.5%
• Ultra Violet B (UVB) 0.5%

Each band of short-wave radiation has a different penetrative depth. While ELCOM allows the user to explicitly specify these values, for this study the defaults will be used. These defaults can be found in the ELCOM Science Manual. The equations for the net shortwave radiation penetrating the water can be written as

\[ Q_{\text{sw}} = Q_{\text{sw(total)}} \left( 1 - r_a^{(\text{sw})} \right) \]  \hspace{1cm} (3.14)

and the short wave albedo of the water surface is

\[ r_a^{(\text{sw})} = \bar{R}_a^{\text{sw}} + a_{(\text{sw})} \sin \left( \frac{2\pi d}{D} + \frac{\pi}{2} \right). \]  \hspace{1cm} (3.15)

where \( \bar{R}_a^{(\text{sw})} = 0.08, a_{(\text{sw})} = 0.02, D \) is the number of days in a year, and \( d \) is the day number of the year. The value \( Q_{\text{sw(total)}} \) is the short wave radiation that reaches the lake surface.

The Beer-Lambert law is used to determine the depth of penetration, namely

\[ Q(z) = Q_{(\text{sw})} e^{-\eta_a z} \]  \hspace{1cm} (3.16)

with \( z \) being the depth below the water surface, and \( \eta_a \), being the coefficient for each band. ELCOM assumes that all of the radiation energy is converted to heat. For shallow water columns, if the short wave energy propagates to the bottom, the Beer-Lambert law is employed to allow a percentage of the energy to propagate back up through the column.
Long wave radiation is determined from the user input of percent cloud cover, $C$. The net long wave radiation energy is calculated by a slightly modified version of the radiation energy transfer equation,

$$Q_{(lw)} = \left(1 - r_a^{(ls)}\right) \left(1 + 0.17C^2\right) \epsilon_a \sigma T_a^4 - \epsilon_w \sigma T^4$$

where modifications account for atmospheric losses and the air-water interface. The subscript $a$ refers to an atmospheric property, and the subscript $w$ refers to a water property. The two coefficients in the equation, $\epsilon$ and $\sigma$ are the emissivity of the water (taken to be 0.96) and the Stefan-Boltzmann constant respectively ($= 5.67 \cdot 10^{-8} \text{W m}^{-2} \text{K}^{-4}$).

Sensible heat exchange from the lake surface to the atmosphere is given as

$$Q_{(sh)} = C_w \rho_a C_P U_a (T_a - T_w) \Delta t$$

for the period $\Delta t$. The subscripts $a$ and $w$ again denote atmosphere and water surface properties, with $C_w$ being the sensible heat transfer coefficient for water ($= 1.3 \cdot 10^{-3}$), $\rho_a$ the air density, $U_a$ the wind speed at the 10 meter reference point, and $T$ is the air and water temperatures.

Heat flux due to evaporation is modeled as

$$Q_{(lh)} = \min \left(0, 0.622 \frac{P}{C_L \rho_a L_E U_a (e_a - e_w (T_w))} \Delta t \right)$$

where $P$ is the atmospheric pressure, $C_L$ is the latent heat transfer coefficient, $L_E$ the latent heat of evaporation, and $e$ is the vapour pressure of the fluids calculated with the Magnus-Tetens formula.

In addition to these methods of heat exchange, ELCOM also allows for mass flux from rainfall. The change in the mass of the surface layer due to rain is given as

$$\Delta M_{N}^{\text{Rain}} = \rho_{\text{Rain}} d_N d_Y r \Delta t$$
where \( r \) is the rainfall for a given period, \( \Delta t \).

3.5.2 Mixed-Layer Model

Vertical diffusion in ELCOM is modeled using a 3-D mixed layer approach. The mixing events which are of concern tend to be small scale mixing events that are the result of large scale processes. Traditionally, mixing models use the eddy diffusivity terms in the vertical transport equations. While this method works well for stable density gradients, ELCOM is primarily concerned with stratified flows, in which there are two mixing events which occur:

1. Convective mixing of unstable density gradients.
2. Mixing of stable density gradients.

Within each of these mixing events there are four energy terms to be characterized, the turbulent kinetic energy (TKE) available for mixing (TKE\(_A\)), the TKE required for mixing (E\(_\text{req}\)), the TKE dissipated (E\(_\epsilon\)), and the residual mixing energy. The first of the mixing events, decreases potential energy of the fluid and releases TKE and is a source for TKE\(_A\), while the latter increases potential energy while dissipating TKE and is the local E\(_\text{req}\). Essentially, unstable density gradients will create TKE\(_A\), while stable gradients will consume TKE\(_A\). This approach is a 3-D expansion of the mixing energy budgets that were originally developed for 1-D lake modeling.

ELCOM implements the mixing model in the following manner [5]:

1. Calculate wind energy input.
2. For each water column, cycle from the surface cell to the bottom cell.
3. Calculate the TKE generated by shear.
4. Calculate the mixing energy required.
5. Calculate the total energy available if two cells were fully mixed.
6. Calculate the time estimate for total mixing.
7. If unstable, calculate the time estimate based on convective overturn.

8. Calculate mixing friction.

9. If energy remains, mix cells.

10. End water column cycle.

11. Dissipate excess mixing energy.

For the details of each step and the equations associated with those calculations, consult Hodges and Dallimore [5].

### 3.5.3 Wind Energy

In modeling momentum input from the wind, ELCOM steers away from the TRIM approach. ELCOM introduces a wind-mixed layer and a model for distributing the wind momentum over the depth of the layer. The depth of this wind mixed layer is

\[
h_{i,j} = k_b(i,j,k_n) \sum_{m=k_a(i,j,k_n)}^{k_b(i,j,k_n)} \Delta Z_{i,j,m} \tag{3.21}
\]

where \( k_a \) and \( k_b \) are the upper and lower grid indices of the discrete wind-mixed layer for a given water column \((i,j)\), with free-surface cell \( k_n \). Once the depth is known, ELCOM introduces with wind momentum as a uniform distribution over the mixed layer [5].

### 3.5.4 Free Surface Evolution

The free-surface elevation is calculated at each time step and is allowed to move up and down between grid layers. Equation 3.7 is discretized using a backwards-Euler scheme that is first-order accurate in time [5].
3.5.5 Momentum Diffusion

Vertical momentum diffusion is accounted for in the mixed layer model. Horizontal diffusion in ELCOM is discretized using a second-order stencil

\[
D_x (\phi^n_{i,j,k}) = \frac{\nu}{\Delta x^2} \left( \phi^n_{i+1,j,k} - 2\phi^n_{i,j,k} + \phi^n_{i-1,j,k} \right)
\] (3.22)

This simplification differs from the TRIM approach, which applies the discretization at the pathline origin. It was determined that this step adds complexity to the computation without any significant increase in accuracy.

3.5.6 Scalar Advection and Diffusion

ELCOM utilizes a conservative third-order accurate scalar transport model. Using a conservative scalar transport scheme is of vital importance to the accuracy of the model, especially in stratified flow models. Using a non-conservative method would result in rapid dissipation of internal waves in addition to artificial losses of mass and momentum.

Scalar advection is implemented in ELCOM as

\[
C^* = \dot{C} - \Delta t \frac{\partial}{\partial x_j} \left( \dot{C} U_j \right)
\] (3.23)

The above equation is defined over period \( \Delta t \), and for periods when \( \max (CFL_a) > 1 \), ELCOM will use the sub-time step \( \delta t \) and iterate \( m \) times so that \( m\delta t = \Delta t \).

The final step in the scalar transport model is the horizontal diffusion. This diffusion is caused by turbulent motions and is calculated by discretizing the horizontal terms of Equation 3.6. The discretized form becomes

\[
C^n_{i,j,k} = \tilde{C}_{i,j,k} + D_x \left( \tilde{C}_{i,j,k} \right) + D_y \left( \tilde{C}_{i,j,k} \right)
\] (3.24)

where \( D_x \) and \( D_y \) are the finite difference operators for the second derivative.
3.5.7 Limitations

As with any numerical methods, there may be restrictions that need to be placed on the computational domain. These limitations are generally based on grid size. Due to the nature of the geophysical flows for which ELCOM is designed, the computational grids tend to be coarse in the horizontal directions (generally on the order of 100 meters). In the vertical direction they can be refined to be on the order of $1^{-1}$ to 1 meters. As a typical body of water is on the order of 10 to $10^2$ meters. In addition to the spatial grid, the temporal grid is also somewhat flexible. However, once again due to the large spatial grid, it is desired to have a time step on the order of $10^2$ to $10^3$ seconds to prevent long computation times.

The numerical method employed by ELCOM is unconditionally stable for barotropic flow, regardless of the time step used. However, when considering stratified flows, using explicit discretization of the baroclinic terms in the momentum equation causes a time step constraint based on the internal wave Courant Friedrichs Lewy condition (CFL):

$$\left(g'D\right)^{1/2} \frac{\Delta t}{\Delta x} < \sqrt{2}$$

(3.25)

where the left-hand side of this equation is the baroclinic CFL number ($\text{CFL}_{b}$), $g'$ is the reduced gravity due to stratification, effective depth is $D$, and $\sqrt{g'D}$ is approximation of the wave speed of an internal wave.

The maximum allowable time step for a limiting CFL can be calculated as

$$\Delta t < \frac{\text{CFL}\Delta x}{U}$$

(3.26)

The value of $U$ can be taken as either the horizontal water velocity or the internal wave speed. The case of $U$ as the wave propagation speed is more limiting on the time step than water velocity and is on the order of $10^2$ seconds.

Additionally, there are several other time step restrictions that should be concerned when using ELCOM. None of these other limits are as restrictive as the baroclinic stability condition when using grid sizes of the order previously discussed in this section. For further
details the reader is referred to Hodges and Dallimore and Casulli and Cheng [5, 12].
Chapter 4
Model Development

4.1 Overview

Creating the model of the South Arm of the Great Salt Lake required several steps:

1. Mesh Generation
2. Data Collection
3. Input File Generation
4. Model Simulation
5. Post-processing and Analysis

4.2 Mesh Generation

A CFD mesh is a representation of a physical geometry in terms that a computer can understand. The physical geometry for the South Arm of the Great Salt Lake was made available by the USGS [13]. A contour map of the lake’s depths, or bathymetry, was provided by the USGS, as shown in Figure 4.1. A reasonable mesh was determined to be 400 meters in both the $x$ and $y$ directions. From the information in the contour map, a digital representation was obtained. Starting from the Northwest corner of the lake, depth elevations were taken at steps corresponding to the spatial steps in the $x$ and $y$ directions. The digitized bathymetry can be visualized as the surface plot shown in Figure 4.2.

4.3 Data Collection

As previously discussed, there are several external parameters that ELCOM employs to model the flow of the lake. In order to accurately represent them in the model it is
Fig. 4.1: USGS bathymetry map of the South Arm of the Great Salt Lake.

Fig. 4.2: Digitized bathymetry of the South Arm of the Great Salt Lake.
desired to obtain the most accurate conditions as possible. These conditions include weather events such as wind velocity and direction measured ten meters above the lake surface, air temperature, humidity, air pressure, and precipitation. In addition to the weather events, ELCOM allows for inclusion of inflow from any tributary. In this case flow measurements from the Bear, Weber, and Jordan Rivers were included. Initial data on the lake’s water temperature and salinity were also included.

Weather information was taken from MesoWest, “a cooperative project between researchers at the University of Utah, forecasters at the Salt Lake City National Weather Service Office, the NWS Western Region Headquarters, and personnel of participating agencies” [14]. In short, it is a large network of weather stations in the intermountain West region of the United States and a resource for the data those stations collect. Although MesoWest has continued to add weather stations in the region, there are currently only three stations near the lake. All three stations are located on the East side of the lake and when compared to each other for a given date and time, the difference was minuscule. As a result, it was decided that for the weather input data, the data from one location would be sufficient for this model.

The most important weather information supplied to ELCOM is the wind velocity and direction. Since internal waves are the primary investigation of this study, it is desired to have accurate wind forcing. Due to the use of data from one weather station, it will be assumed that the wind will be of uniform strength and direction over the entire surface of the lake. For a smaller lake this would be easily acceptable, but with the magnitude of the Great Salt Lake, it would be desired to eventually repeat this study with more accurate weather data.

4.4 Input File Generation

There are several important input files which must be supplied to ELCOM. Each file has a specific format which must be adhered to. Along with the ELCOM code, the developers supplied example files which were then modified to be representative of the lake. The input files are, in no particular order: a bathymetry file, several boundary conditions files, a
meteorological data file, and a data output file.

4.4.1 Bathymetry File

As previously mentioned, the bathymetry of the lake was digitized in a 2-dimensional grid. This grid must be expanded into the third dimension in order to be processed by ELCOM. With specific information provided in the bathymetry file, ELCOM will expand this information by itself.

ELCOM sees the computational domain as a rectangular prism. This prism is broken up in smaller prisms called cells. The user is allowed to explicitly list the size of each cell and the number of cells in each direction. If one were to look at the prism, each cell could be numbered with three indices starting with the top left cell. The ‘I’ index is the row index and increases as the cells go down, the ‘J’ index is the column index and increases across the page. The ‘K’ index represents the layer and increases from the back towards the front. Again looking at the top of the prism, one could take the two-dimensional depth data collected previously and lay it over the prism, with one data point per cell. The ‘I’, ‘J’, and ‘K’ indices correspond with the $x$, $y$, and $z$-coordinates in the Cartesian System. As the data is applied to the prism, it will be noticed that the depths will prevent each cell from being used. ELCOM recognizes these unused cells as land, and prevents these cells from appearing in any computation, saving computational time and memory when the model is simulated. This step occurs in ELCOM’s preprocessor routine and finalizes the mesh generation previously discussed.

The grid spacing consisted of a uniform 400m in the $x$-direction (North/South) and $y$-direction (East/West), with the origin in the Northwest corner. For most CFD problems, the grid is considered very coarse, yet is normal for geophysical fluids calculations. In the $z$-direction, 63 layers were specified, with clustering centered on the halocline. This clustering should allow ELCOM to better resolve the interactions near the halocline, with which this study is primarily concerned. This layer clustering is shown in Figure 4.3, where the elevation is given as the elevation above sea-level.
Fig. 4.3: Layer thickness at elevations used for sizing the grid. The decrease in the middle corresponds to the average location of the halocline in the lake.

4.4.2 Boundary Conditions

The majority of the boundary conditions are land boundaries which are determined from the bathymetry file. The two rivers that flow into the South Arm of the lake were added as separate boundary conditions, as well as a breach in the causeway separating the Northwest and South Arms. The locations of the inflows were provided directly into the pre-processor and can be shown in Figure 4.4

Readers familiar with the geometry of the Great Salt Lake, may notice that the Jordan River location is much further North than its actual location. The inflow condition was placed on the main culvert on the Antelope Island Causeway, as the region south of the causeway–Farmington Bay–was not included in the model, as it is separated from the main body of the lake. The input file contains flow rates from each inflow at a given point in time. This data was obtained from stations managed by the USGS [15].

4.4.3 Meteorological Data

As previously discussed, all meteorological data were sampled from a weather station operated by MesoWest. The data was collected from the station located on the grounds of
the Salt Lake International Airport, near the south east tip of the lake at 40.778° North and 111.969° West. From the MesoWest website, data from any of their stations can be downloaded for periods of up to 31 days. For each month of the year 2006, the data was downloaded and compiled to provided information for the entire year. This data is in the form of a text file with each line containing a time stamp, air temperature, dew point, sustained wind speed, gust wind speed, wind direction, air quality, air pressure, cloud cover, visibility, solar radiation, and various measures of precipitation.

From the MesoWest data, there are several ELCOM input files which were created to supply the simulation with the needed information. A FORTRAN code was written to read in the MesoWest data and redistribute to create the three necessary meteorological input files.

The first of these inputs is a file containing information about cloud cover and precipitation received. The MesoWest data listed cloud cover as one of four levels: Clear, Partly Cloudy, Mostly Cloudy, and Overcast. ELCOM expects the cloud cover to be in a percentage of cloud cover, so the following relation was developed:

- Clear - 0% cover
Fig. 4.5: Angular histogram showing the dominant wind directions over the lake surface.

Fig. 4.6: Wind velocity measured by MesoWest over the course of 2006.
• Partly Cloudy - 33%
• Mostly Cloudy - 66%
• Overcast - 100%

Precipitation is used in ELCOM simply as a scalar with units of meters per day. Wind affects each simulation in through evaporative cooling in surface thermodynamics and through the addition of wind induced momentum in the wind mixed layer of the lake. The wind input file contains both the wind speed and direction, where the wind speed is the sustained wind for a given period of time.

It should be noted that for future studies that desire to make use of more weather stations, each of the meteorological input files can be separated into regions for ELCOM to apply the conditions to. Increasing the accuracy of the input files, should result in an improvement to the accuracy of the results.

### 4.4.4 Initial Profile

The final input file is that of the initial profile. Each of the previous input files primarily dealt with external forces acting upon the lake. The initial profile sets the entire lake to contain certain parameters. For the model of the South Arm of the Great Salt Lake, the profile contains two parameters: water temperature, and salinity. In an attempt to better understand the conditions that may increase the instability of the internal waves, it was decided that several different profiles should be measured.

The study will first look at the effect that different vertical profiles of salinity have on the lake. Four different profiles were taken. The first of these featured a sharp salinity gradient with salinity at 120 parts per thousand (ppt) in the upper brine and 240ppt in the deep brine. The second case used a similar profile as the first, with the upper brine salinity of 160ppt and 200ppt in the lower. The third and fourth cases look at two salinity profiles which were measured in the lake, these profiles were measured in August of 1967 and July of 1984 respectively [16]. These four cases will be known in the remainder of this study as Case 1, 2, 3, and 4.
Fig. 4.7: Salinity profiles used to initialize simulations.
Fig. 4.8: Average temperature profiles of the Great Salt Lake throughout the year.
Temperature effects were also looked at addition to the changes in the vertical salinity profile. The temperatures were obtained from unpublished summaries by Gwynn reported by the University of Utah’s Department of Meteorology, and is represented in Figure 4.8 [17]. As illustrated in Figure 4.8, the lake does not present an adequate thermocline to support an internal wave. Without a strong thermocline there is little use in examining this further.

4.4.5 Output File

In general the majority of the desired output parameters are calculated within the ELCOM code. ELCOM allows for specific outputs to be obtained. These outputs contain calculated parameters along a sheet, curtain, profile or over the whole computational domain. Sheet, curtain, and profile are terms used to describe the location of the data. Profiles are one-dimensional in the vertical direction. A profile is specified by its \((x, y)\) location and contains data at each layer of the water column. Curtains and sheets are two-dimensional slices of the data. A sheet can be considered a collection of data along a given layer of the computational grid, or every \((x, y)\) location at a given elevation \((z)\). A curtain is a collection of vertical profiles. These profiles do not need to be adjacent or follow a straight line, but for each curtain used in this study, the curtains contain adjacent \((x, y)\) cells and are arranged in a straight line. ELCOM allows multiple outputs (sheets, curtains, and profiles) from each simulation. For each output group, the user may also select which parameters to store and the frequency of at which they are stored.

For all simulations performed for this study the parameters of the \(u\), \(v\), and \(w\) velocity components, water density, salinity, and water temperature were output. This data was stored for several different locations of the computational domain. These locations can be thought of as measurement ‘stations. The parameters were saved at each time step of the simulation; taking into account the stability requirements of the code the time step was 180 seconds. These stations were numbered from 1 to 11, and will be referenced by their individual number. The locations of these stations are shown in Figure 4.9.

In addition to these stations, data was also recorded along four curtains. These curtains are vertical two-dimensional sheets of data which is analogous to a slice cut out of the lake.
These four curtains spanned the large bay in the center of the South Arm, running across the north, east, south and west sides of the lake. The parameters were stored at every hour of the simulation time. The exact location of these curtains is shown in Figure 4.10.

4.5 Model Simulation

The final file created before running simulations controls the parameters of the simulation. ELCOM allows the user to specify the start date, simulation models to be used, and scalar limits. The code also allows the user to save the simulation process at a given time in the event that the simulation is interrupted. For the four salinity profile cases simulated over the summer months, the simulation was started with input data from June 1, 2006 and was simulated through August 31, 2006. The temperature profile case began at the first of each month and ran until the end of the month.

4.6 Post-processing

ELCOM outputs the data in an unformatted binary file, but provides a means to convert the data to the NetCDF (Network Common Data Form) format. NetCDF is a binary data format designed for large array-oriented data, and is commonly used by researchers in
Fig. 4.10: Locations of curtains within the lake.

Oceanography and atmospheric studies. The format was developed by Glenn Davis, Russ, Rew, Ed Hartnett, John Caron, Steve Emmerson, and Harvey Davies at the Unidata Program Center in Boulder, Colorado. Unidata continues to develop and manage the format, and lists the following attributes of NetCDF on its website [18]:

- **Self-Describing.** A netCDF file includes information about the data it contains.

- **Portable.** A netCDF file can be accessed by computers with different ways of storing integers, characters, and floating-point numbers.

- **Direct-access.** A small subset of a large dataset may be accessed efficiently, without first reading through all the preceding data.

- **Appendable.** Data may be appended to a properly structured netCDF file without copying the dataset or redefining its structure.

- **Sharable.** One writer and multiple readers may simultaneously access the same netCDF file.

- **Archivable.** Access to all earlier forms of netCDF data will be supported by current and future versions of the software.
These characteristics made NetCDF well suited for use as an output for ELCOM-produced data. The use of NetCDF allows for interfacing the data with many software packages. Many extensions have been written for a large variety of software packages. Of these software packages, the author chose to use MATLAB, to compute any additional parameters not calculated within ELCOM. A NetCDF conversion file was written for MATLAB by Paul Spencer and is freely available from the MATLAB File Exchange [19,20].

4.6.1 Richardson Number

The Richardson number can be calculated by Equation 4.1

\[ Ri = -\frac{g \frac{dp}{dz}}{\rho_{avg} \left( \frac{dU}{dz} \right)^2} \]  

(4.1)

where \( \rho \) is the fluid density, \( \rho_{avg} \) is the average density of a given water column, and \( U \) represents the velocity aligned with the flow. For three-dimensional flows, the velocity gradient \( dU/dz \) must include the components from both the \( x \) and \( y \) directions. This gradient can then be calculated as

\[ \left| \frac{dU}{dz} \right| = \sqrt{\left( \frac{du}{dz} \right)^2 + \left( \frac{dv}{dz} \right)^2} \]  

(4.2)

where \( u \) and \( v \) represent the velocities in the \( x \) and \( y \) directions, respectively.

In order to compute the Richardson number for the numerical data produced by ELCOM, the derivatives of Equation 4.1 are computed with a finite difference method. Any method which would be used must be able to handle unequally spaced data. A method was selected which fits a second-order Lagrange interpolating polynomial to each set of three adjacent data points and then analytically differentiates the polynomial [21]. This equation becomes

\[
f'(x) = f(x_{i-1}) \frac{2x - x_i - x_{i-1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})} \\
+ f(x_{i+1}) \frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}. 
\]  

(4.3)
Fig. 4.11: Seiche mode types typically seen with internal wave motions. The upper line represents the free surface, while the lower line represents the internal wave.

with the derivatives accounted for, the remainder of the Richardson number computation is trivial.

4.6.2 Seiche Modes

Internal waves along the halocline of the lake can take on two modes. The mode types are determined by the relationship of the internal wave to the wave along the free surface. Examples of the two mode types are shown in Figure 4.11. The first of these modes is referred to as the external mode. This mode is characterized by the internal wave following the shape of the wave along the free surface. During the internal mode, the internal wave appears to be perfectly out of phase with the free surface wave, that is that as a wave reaches the top of the crest internally, the free surface wave will reach the bottom of the trough [22]. The amplitudes of the free surface wave will be smaller than the internal wave, due the difference in the densities of the fluids on the bottom and top of the interface on which the wave acts. There may also be higher frequency waves acting upon both the internal and free surface waves, however it is the general shape of the mode that is of concern.

Additionally, there is a more common way of looking at seiche modes. Instead of looking at the wave with respect to the free surface, the area of interest is the mixed region along the halocline. Contours of salinity or density may be used to identify the patterns
existing within this region. A contour on the upper side of the region will be compared with a contour on the lower side. These contours generally fall into one of two modes, denoted by the mode number. The first mode is similar to the external mode from Figure 4.11(a). The contours will rise and fall together in time. The second mode is when these contours will appear to pull away from each other. The second mode can be of particular importance when analyzing stability patterns as when the contours separate the density gradient may become significantly smaller.
Chapter 5
Stability of Internal Waves

5.1 Model Simulation

In order to minimize the overall time it took to complete all simulations, it was decided to make use of the Uinta computing cluster managed by the Center for High Performance Computing at Utah State University. The primary benefit to using Uinta was that multiple jobs could be submitted at the same time. This became important when it was seen that a total of 15 different jobs to be simulated could take between two and three weeks to process on a single machine, not accounting for any machine downtime, or errors that may be present in the input files.

In order to make use of this computing resource, the source code was obtained and re-compiled for use on a 64-bit Linux system with the Intel FORTRAN compiler. All warnings and messages were turned off to improve performance.

Using the Uinta cluster allowed each run to be performed on a separate computing node. A computing node is a single quad-core processor. Each simulation took approximately 14 hours of run time per 30 days of simulation. In an effort to distribute the resources of the cluster, there is a 24-hour limit imposed on any job that is executed by the cluster. For the majority of jobs, this was not an issue; however, for the case of the salinity profiles, each job had to be broken up into a month period. ELCOM allows the user to save a restart file at their preference, so a restart file was saved at the conclusion of each month. A new job would then be started using the data contained in the restart file.

5.2 Measurement Stations

As shown in Figure 4.9, eleven data recording stations were positioned across the lake. Upon reviewing the data, it was realized that only five of the stations are positioned in the
lake in areas that are deep enough to fully resolve the density gradient. These five stations will be the primary source of information presented in this chapter. These locations are the stations numbered 2, 3, 5, 8, and 10. Stations 1, 7, 9, and 11 are positioned over transitional areas, or areas that are deep enough to fully contain the upper brine layer as well as the gradient region between the upper and lower layers. The stations numbered 4 and 6 will be not be used in the remainder of this study; they may prove useful in the future should experimental data be collected to validate the data presented.

5.3 Definitions

5.3.1 Stability

Determining if a profile is stable at a given station is straightforward. The Richardson number is computed at each cell in the vertical profile. The Richardson number at a given time step determines if the profile is stable. From the calculation of the Richardson number, it can be seen that if the velocity gradient remains small, the calculation is mostly dependent on the location of the steepest density gradient. This location roughly corresponds to a point directly separating the upper and lower brine layers. If the maximum value along each profile is 0.25 or greater, the profile is stable. If the profile is stable, buoyancy effects dominate and there is no mixing between the upper and lower brine layers. Examples of stable and unstable profiles can be seen in Figures 5.1 and 5.2. In these figures, the Richardson number at each cell in the profile is shown corresponding to the elevation of the cell. The red dashed line is the stability threshold. Values above the threshold show that the cell is stable, values below the threshold show instability. The maximum Richardson number of the profile is the starred point. It is this value that determines cell stability. It is important to note that these maximum points occur at the same elevation as the interface between the upper and lower brine layers.

5.3.2 Layer Identification

For a given density profile, values of density can be picked out which lie somewhere
Fig. 5.1: Example of a profile with an unstable Richardson number.

Fig. 5.2: Example of a profile featuring a stable Richardson number.
Fig. 5.3: Sample salinity profile illustrating the location of markers used to identify the elevation of the brine layers.

between the main density of a given layer. A marker is selected near the upper layer, near the lower layer, and somewhere in the middle. The elevations of the upper and lower markers form a sort of boundary for the mixing region. The middle marker will serve as an indicator of the internal basin scale waves. The correlation between density and salinity is strong enough that values of salinity can also be selected as markers. Since salinity was used to initialize the simulation runs, it was easier to select values of salinity to serve as markers instead of density.

In addition to resolving these waves, a frequency analysis may then be performed on the waves to determine the frequency and period of these internal waves.

5.4 Internal Wave Seiche Modes

The internal wave seiche modes which are present in the Great Salt Lake are identified by selecting several contours of salinity. Along each profile, three contours were selected. The first is selected to be the value of salinity which is directly between the value of salinity in the upper and lower brine layers. The remaining two contours are selected to be along the gradient on either the upper or lower side of the first contour. The method is essentially
Fig. 5.4: Comparison of elevations of the free surface and internal waves. The scale on the left of each plot corresponds to the elevation of the internal wave, while the scale on the right corresponds to the elevation of the free surface wave.

the same as was described in Figure 5.3. The free surface elevation is also an output from ELCOM and by plotting these two together we can compare the shapes they exhibit. While looking at each of the stations over time, the lake appears to remain in an internal mode for the majority of the period analyzed. Figure 5.4 illustrates the effect of the salinity gradient on the mode type. The strong the density gradient of Case 1 (Figure 5.4, top right), features the internal mode quite clearly. Changes in the salinity gradient and therefore changes to the density gradient may result in changes to the mode. Figure 5.4 also illustrates the long period of these waves. By identifying the peaks of the internal waves at each station over time, it can be seen that these waves have a period of roughly one day, with some higher frequencies visible as well. It is not generally sufficient to determine the mode type by looking at the data. This is especially true as there are several different frequencies present
in both the free surface and along the contour. A correlation coefficient was calculated to help determine the mode type. The coefficients are calculated as

\[ R(i, j) = \frac{C(i, j)}{\sqrt{C(i, i)C(j, j)}} \]  \hspace{1cm} (5.1)

with \( C \) being the covariance matrix. Values of the correlation coefficient range from 1 to \(-1\). The magnitude of the value indicates the strength of the correlation. The sign represents the phase between the two signals, positive being perfectly in phase, negative perfectly out of phase. The sign can then be associated with the mode type. The internal mode will feature a negative sign, external mode feature a positive sign, with the strength of the correlation determined from the magnitude of the coefficient.

As shown in Figure 5.4, the internal mode is the dominate mode. There are several weak coefficient values, which may suggest that the mode may change over time.

As previously mentioned, a more conventional way of illustrating mode is by observing the behavior of the upper and lower contours. Correlation coefficients were also computed to identify the first or second mode which may be present in the wave. In contrast to
Fig. 5.6: Correlation coefficients between the upper and lower contours of salinity in the mixed region.

the free surface modes, the first mode dominates the flow, with a very strong correlation. Figure 5.4 shows the behavior of the contours for two separate cases and stations. The correlation in Figure 5.7(b) indicated that examples of the second mode could be visible. A close examination does appear to show the second mode.

By taking the time series information shown in Figure 5.4 and transforming the information into the frequency domain, the exact frequencies and periods can be identified. Using a signal analysis package a spectral analysis can be performed.

Figure 5.8 illustrates typical spectra of the internal wave. Within the frequency signal there are several low frequency peaks. The signals contain little to no higher frequency information, examples of the power spectral density for the different cases are seen in Figure 5.9.

The lowest of these peaks corresponds to a period of approximately 4 days. There are also frequencies with periods of 35 to 36 hours and 24 hours.

The motions of these internal waves are perhaps best seen by observing the internal waves with the data extracted from the curtains. These curtains show the spatial variation
Fig. 5.7: Upper and lower contours over time.

(a) July: Case 1-Station 10

(b) July: Case 3-Station 2
of the internal waves, and when seen using an animation technique one can realize the motion that these waves experience. Several frames of such an animation are included and seen in Figures 5.10 and 5.11. Each time step collected along the curtains was one hour. In order to illustrate the lengthy period of the modes, three frames are shown. For the North Curtain the frames are 5 hours apart, starting from the top. The East Curtain plot in Figure 5.11 are spaced 8 hours apart starting from the top.

5.5 Stability Analysis

The focus of this study has been to identify instabilities in the flow of the Great Salt Lake. These instabilities are predicted by calculating the Richardson number at each simulated time step. By simulating the flow over different periods of time and with different salinity gradients, it was hoped that the causes of instabilities could be identified as well.

Figures 5.12 through 5.15 show the results at stations 2, 5, 8, and 10. The dashed line in all the figures indicates the Richardson number threshold of 0.25. Any values below this threshold correspond to an instability in the fluid flow. As indicated by the plots at each of the stations the flow is stable, that is the buoyant affects dominate the flow and mixing
(a) Case 1

(b) Case 2

(c) Case 3

(d) Case 4

Fig. 5.9: Power spectral density.
Fig. 5.10: Contours of salinity (185, 190, and 195 ppt) over the course of one period. View is looking North at the North Curtain for the Case 2 salinity profile.
Fig. 5.11: Contours of salinity (155, 190, and 215 ppt) illustrating the motion of the internal wave over one period. View is looking to the East at the East Curtain for the Case 1 initial salinity profile.
Fig. 5.12: Richardson number computed from the Case 1 salinity profile at various stations for the month of July.
Fig. 5.13: Richardson number computed from the Case 2 salinity profile at various stations for the month of July.
Fig. 5.14: Richardson number computed from the Case 3 salinity profile at various stations for the month of July.
Fig. 5.15: Richardson number computed from the Case 4 salinity profile at various stations for the month of July.
between the layers is limited.

It is seen that the salinity profile does play a large role in determining the value of the Richardson number. For each station in the flow the minimum value of the Richardson number was extract to provide a side by side comparison. These comparison can be seen in the bar graphs of Figure 5.16

Case 2 features the sharp gradient with the narrowest change in salinity between the upper and lower brine layers. Case 3 had a larger change in salinity, but featured a more gradual gradient. These two cases exhibited the lowest Richardson numbers at stations 2, 5, 8, and 10. Thus while no instabilities where seen, the lake has been shown by Loving et al. to have many different salinity profiles over the past 20 years [16]. These other profiles
Fig. 5.17: Richardson number at station 9 as predicted using the Case 2 salinity gradient. may be more susceptible to lower Richardson numbers that indicate instabilities.

Several data stations were placed over areas of the lake that are not deep enough to fully see the lower brine layer. From the data collected, some of these stations indicated Richardson numbers that were below the 0.25 threshold for stability. The deepest of these stations is station 9. Figure 5.17 shows the periods that the station was predicted to be unstable. As seen in the figure, the Richardson number drops below stability threshold around day 206. The same station showed small periods of unstable time for Case 3, but the event was not seen in Case 1 and 4.

The profile of the Richardson number, density, velocity are shown in Figures 5.18 and 5.19. As may be noticed the density profile indicates that only the top edge of the mixed region is in the profile. At this station and time step, both factors affecting the Richardson number appear to be working together to cause the instability. The density gradient appears to be relaxed while there is a sharp increase in both U and V velocity components.

One potential cause for the instability is that only a very small portion of the mixed saline region is found, as the internal wave motion may cause this mixed region to move up and down in elevation. This does present a problem in determining if mixing is occurring
Fig. 5.18: Richardson number at station 9 at unstable point as predicted using the Case 2 salinity gradient.

Fig. 5.19: Velocity and Density profile of station 9 at unstable point as predicted using the Case 2 salinity gradient.
Fig. 5.20: Richardson number at station 10, as predicted with the Case 2 salinity gradient. Between the brine layers or just between the upper brine layer and the mixed region. As similar occurrences were seen at other stations of similar depth, there is a possibility that the breaking of the internal waves could be occurring as the wave approaches the lake bed.

In contrast to the unstable event shown above, a more common profile is featured in Figures 5.20 and 5.21. Stable profiles like these were seen in every case simulated. These stable profile examples appear to be the standard for the salinity gradients tested.

5.6 Affected Areas

While it has been seen that at deeper data stations that the flow is quite stable, there is the question of what happens in the areas of the lake where the mixed brine region makes contact with the lake floor. Figure 5.22 shows the region of the lake that may be affected.

These area estimates are based on the average elevation of the mixed region and should be considered conservative for two reasons. The first is that these elevations are taken slightly inside of the mixed region. The second is that these are from the average elevations. Other salinity profiles which show a larger mixed region will have a greater percentage of the total area of the lake bed.
Fig. 5.21: Velocity and Density profile of station 10, illustrating an example of the two primary components of the Richardson number.
Fig. 5.22: Areas potentially affected by internal waves hitting the lake bed.
Chapter 6

Conclusion

The primary focus of this work has been to determine if turbulent mixing caused by unstable internal waves can be a viable transport mechanism for mercury and other heavy metals from the deep brine layers of the South Arm of the Great Salt Lake to the upper layers. These instabilities may be found by calculating the Richardson number for profiles throughout the lake. Richardson numbers less than $0.25$ are known to be unstable in the manner of the Kelvin-Helmholtz instability. This instability would present itself as a viable transport mechanism for the mercury particles in the lake.

The fluid dynamics of the South Arm of the Great Salt Lake were numerically modeled using ELCOM, developed by the University of Western Australia’s Centre for Water Research. ELCOM is a well-suited model for strongly stratified flows. The numerical model was subject to environmental forcing taken from measured data from summer of 2006. Several different salinity profiles—both fictional and taken from historical data—were tested in an attempt to quantify the conditions which may increase the likelihood of instabilities.

Each case of salinity was simulated for a period of three months. Several locations were selected over the geography of the lake to serve as measurement stations. The variety of station locations provide a look at local phenomenon. The majority of stations provided a look at the entire vertical profile of the lake. Despite using different profiles, the results are fairly uniform across these stations. The Richardson number calculated at these stations are indicative that the internal waves are very stable. That is the buoyancy effects dominate and eliminate the possibility of turbulence mixing as a transport mechanism for mercury between the upper and lower brine layers. It should be remembered that the lack of instabilities only held in all cases for stations with profiles that fully contained the deep brine layer. There were several stations with vertical profiles that did not fully contain the deep brine
layer, but contained only a portion of the interface region. Several instances were recorded for all cases studied where the Richardson number at these stations dropped below the 0.25 threshold for stability. It is possible that just as waves of the ocean break as they approach land a similar event could be occurring with these internal waves. The internal wave could become unstable as it approaches the lake bed. Additionally, the elevation at which the internal wave hits the lake bed may fluctuate. Heavy metals in the deep brine layers may be deposited on the the lake floor when the internal wave reaches its high point. The deposited particles could potentially sit until the wave has lowered. Eventually, conditions may cause the particles to become suspended in the flow again. If this occurs as the internal wave reaches a low point, the particles could potentially be suspended in the upper brine layer. It is evident from the research presented here, that sufficient study was not given to the areas where the internal wave is in contact with the lake bed.

Future work on the Great Salt Lake model should center on two key items. First, it is vital for the lake to have a long term, experimental study performed on the lake. In any numerical model it is important to validate the accuracy of the model. Currently, there is insufficient data to truly validate this model over the length of time that is desired from the model. Second, any additional studies using this model should focus on the areas within the lake where the internal waves come in contact with the lake floor. These locations were the only in the lake to exhibit any instabilities. These unstable conditions were not limited to a certain salinity profile, but were seen in each of the cases studied.
References


