January 1977

Effects of Soil Heterogeneity on One-Dimensional Infiltration

Charles Courette
Roland W. Jeppson

Follow this and additional works at: https://digitalcommons.usu.edu/water_rep

Part of the Civil and Environmental Engineering Commons, and the Water Resource Management Commons

Recommended Citation
EFFECTS OF SOIL HETEROGENEITY ON
ONE-DIMENSIONAL INFILTRATION

by
Charles Courette
and
Roland W. Jeppson

The work reported herein was supported by The National Science Foundation Through Contract Grant No. ENG 74-24314.

ABSTRACT

A parameter study for heterogeneous infiltration was performed employing a one-dimensional computer program. Soil heterogeneity was defined by linear variations of soil parameters. From the solutions obtained, the instantaneous and accumulative infiltration rates were correlated versus the rate in which the parameters varied in the soil profile. The end result being coaxial graphs from which a relative infiltration rate may be obtain by inputting time and the rates of change of various soil parameters.

Utah Water Research Laboratory
Utah State University
Logan, Utah 84322

December 1977
TABLE OF CONTENTS

BACKGROUND .......................................................... 1
  Statement of Problem .............................................. 1
  Review of Literature .............................................. 1
  Equations for Unsaturated Permeability ....................... 1
  Unsaturated Flow Solutions ...................................... 3

FORMULATION OF THE MATHEMATICAL MODEL ......................... 3
  Unsaturated Properties of Materials ............................ 3
  General Equation .................................................. 3
  Boundary Conditions .............................................. 5
  Finite Difference Formulation ................................... 5

METHODS AND PROCEDURES ........................................... 6
  Defining Heterogeneity ........................................... 6
  Solution Variables Used .......................................... 6
  Data Handling ..................................................... 13

RESULTS AND CONCLUSIONS ........................................ 18
  Discussion of Results ........................................... 18
  Coaxial Graphs ................................................... 22
  Effects of Varying Two Parameters Simultaneously ............. 26

REFERENCES .......................................................... 28

APPENDIX A: DATA SETS FOR REGRESSION ANALYSIS AND RESULTING REGRESSION EQ ................. 29

APPENDIX B: TYPICAL PROGRAM OUTPUT .............................. 35
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>The constant in the quadratic equation which defines the parameter variation in the profile</td>
</tr>
<tr>
<td>AI</td>
<td>Accumulative infiltration rate</td>
</tr>
<tr>
<td>B</td>
<td>The linear coefficient in the quadratic equation which defines the parameter variation in the profile</td>
</tr>
<tr>
<td>BKV</td>
<td>Represents $B$ for the variable $K_v$</td>
</tr>
<tr>
<td>BL</td>
<td>Represents $B$ for the variable $\lambda$</td>
</tr>
<tr>
<td>BPB</td>
<td>Represents $B$ for the variable $P_b$</td>
</tr>
<tr>
<td>BPOR</td>
<td>Represents $B$ for the variable $\eta$</td>
</tr>
<tr>
<td>BSR</td>
<td>Represents $B$ for the variable $S_r$</td>
</tr>
<tr>
<td>C</td>
<td>The quadratic coefficient in the quadratic equation which defines the parameter variation in the profile</td>
</tr>
<tr>
<td>H</td>
<td>Dimensionless depth</td>
</tr>
<tr>
<td>h</td>
<td>Hydraulic head</td>
</tr>
<tr>
<td>I</td>
<td>Instantaneous infiltration rate</td>
</tr>
<tr>
<td>$K$</td>
<td>Hydraulic conductivity</td>
</tr>
<tr>
<td>$K_a$</td>
<td>Saturated hydraulic conductivity at the surface</td>
</tr>
<tr>
<td>$K_e$</td>
<td>$K_{ew}$</td>
</tr>
<tr>
<td>$K_{ew}$</td>
<td>Effective hydraulic conductivity of the wetting phase for unsaturated soil</td>
</tr>
<tr>
<td>$K_r$</td>
<td>$K_{rw}$</td>
</tr>
<tr>
<td>$K_{rw}$</td>
<td>Relative hydraulic conductivity of the wetting phase for unsaturated soil</td>
</tr>
<tr>
<td>$K_v$</td>
<td>Saturated vertical hydraulic conductivity</td>
</tr>
<tr>
<td>L</td>
<td>Characteristic length defined by profile depth</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Bubbling or air entry pressure</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Capillary pressure</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Dimensionless pressure head</td>
</tr>
<tr>
<td>q</td>
<td>Flow rate</td>
</tr>
<tr>
<td>S</td>
<td>Saturation</td>
</tr>
<tr>
<td>$S_e$</td>
<td>Effective saturation</td>
</tr>
<tr>
<td>$S_o$</td>
<td>Surface saturation</td>
</tr>
<tr>
<td>$S_r$</td>
<td>Residual saturation</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Velocity in the $Z$ direction</td>
</tr>
<tr>
<td>$Z$</td>
<td>Vertical Depth</td>
</tr>
<tr>
<td>$z$</td>
<td>Dimensionless depth</td>
</tr>
<tr>
<td>$\Delta \lambda$</td>
<td>$I_{\text{homogeneous}} - I_{\text{heterogeneous}}$</td>
</tr>
<tr>
<td>$\Delta \eta$</td>
<td>$\eta_{\text{homogeneous}} - \eta_{\text{heterogeneous}}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Unit weight of water</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$2 + 3\lambda$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Soil porosity</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Pore size distribution index</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Transformed value of $p_t$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Dimensionless time parameter</td>
</tr>
</tbody>
</table>
BACKGROUND

Statement of Problem

The infiltration process of water into a soil system is a complex problem to solve due to the large number of soil properties which affect the flow. These properties often change spatially in the soil mass and can also change with time due to chemical or biological reactions. Despite this complexity, mathematical models have been developed which are capable of describing this complex process reasonably close, if the soil is relatively homogeneous. Generally, the assumptions used in these models are homogeneous soil and the absence of chemical and biological activity.

The first description of the soil water system was by Darcy (1856). One of Darcy's original assumptions was that the soil be completely saturated. Buckingham (1907) reported that Darcy's equation could be applied equally well to unsaturated soils. Recent investigators have conducted extensive research on unsaturated flow and have arrived at equations relating pressure, saturation and permeability to physical properties of the soil. These equations, in combination with Darcy's Law and the continuity equation, yield differential flow equations describing moisture movement in unsaturated soils.

With the use of the digital computer, the solutions to these flow equations can be obtained rapidly. Even so, because of the number of parameters involved and the large number of possible combinations, an analysis of all possible solutions is not practical.

The purpose of this investigation was to perform a parameter study of infiltration in unsaturated heterogeneous soils. A computer program developed by Jeppson (1976) for heterogeneous unsaturated soils was used to obtain solutions to the differential equation of flow and the effects of the rate of change of the parameters \( \lambda, \eta, \beta, \gamma \) and \( k_w \) as defined by the Brooks-Corey equations and which were used to describe the unsaturated hydraulic properties of the soil were studied. Relationships between the rate of change of the parameter and its effect on infiltration and total water volume adsorbed were developed and coaxial graphs showing their influence on the process are given.

Review of Literature

Darcy's Law states that the local seepage velocity for fluids is directly proportional to the gradient of the potential or head. In equation form it is written as:

\[ q = -kVh \]

in which \( q \) is the seepage velocity vector, \( k \) is the hydraulic conductivity and \( h \) is the total head and in this report will be restricted to that due to pressure and elevation head. For use in this report the permeability will be used synonymously with hydraulic conductivity as is common in soils engineering. That is, the designation permeability does not mean intrinsic permeability.

Equations for Unsaturated Permeability

For unsaturated flow, the permeability is a function of capillary tension caused by the air-water interface. This tension creates negative pressures in the soil and is referred to as capillary pressures. Functions relating permeability to saturation and capillary pressure have been developed for the flow of two immiscible fluids in porous media. These functions have been brought about mainly by research conducted in the petroleum industry. Burdine (1953) presented the following equation for relative permeability:

\[ K_{rw} = S_e^{2} \left( \frac{S_e}{S_e + \frac{S^\prime}{S_e}} \right) \]

in which \( P_c \) is the capillary pressure, \( S \) is the effective saturation defined by \( S_e = \frac{S - S_r}{1 - S_r} \), \( S_r \) is the residual saturation or sometimes called the irreducible saturation and \( K_{rw} \) is the relative permeability of the wetting phase. \( S_e \) can be found by fitting data best on a log plot of \( S_e \) vs. \( P_c \). As \( P_c \) becomes increasingly negative \( S \) will approach some saturation, \( S_r \). It is assumed here that the permeability is zero at \( S_r \). Relative permeability, \( K_{rw}^{1/2} \), is defined as the ratio of the effective permeability, \( K_e \), to the saturated permeability and wetting phase is defined as the fluid toward which the interface is concave when two immiscible fluids come into contact. For example in an air-water system the water is the wetting phase, whereas in the air-mercury system, the air is the wetting phase. Since the equation was developed in the petroleum industry the two fluids were water and oil. Therefore an expression for the permeability of the non-wetting
phase was also developed. In this study the two fluids are air and water and the non-wetting phase permeability is neglected. Due to the small viscosity and small density of air, this is not a bad assumption as long as the air is free to escape the system. Henceforth in this report the wetting phase permeability will just be referred to as the permeability.

Using the definition of effective saturation,

\[ S_e = \frac{S - S_r}{1 - S_r} \]

the Burdine equation can be rewritten as

\[ K_r = \left( \frac{S_e}{1 - S_r} \right)^2 \]

After analyzing many pressure desaturation curves, Brooks and Corey (1964) empirically determined a relationship between effective saturation and capillary pressure:

\[ S_e = \frac{P_b}{P_c} \lambda \]

in which \( P_b \) is called the bubbling pressure and is a measure of the maximum pore size forming a continuous network of flow channels within the medium. The constant \( \lambda \) is a measure of the pore size distribution where a large \( \lambda \) is associated with uniform pore size and a small \( \lambda \) with a well graded pore size. The graphical representation of this relationship is shown in Figure 1. Upon substituting Equation (5) into Equation (4) and performing the integration the permeability becomes:

\[ K_r = \left( \frac{P_b}{P_c} \right)^{2+3\lambda} \]

in which \( \epsilon = 2 + 3\lambda \).

In applying the theory of Brooks and Corey, the following assumptions were made:

1. \( S_e = S = 1 \) and \( K_r = 1 \), when \( P_c < P_b \)
2. Theory cannot apply for \( S < S_r \)
3. The theory becomes inaccurate at saturations near \( S_r \) because the shape of the pressure saturation curve is very steep at \( S = S_r \)
4. \( K_r \) is zero at \( S_r \)
5. The porous matrix does not undergo any geometrical change as it changes from a saturated state to saturations approaching \( S_r \).

King (1965) developed a hyperbolic function for saturation and relative permeability,

\[ K_r = \sigma \left( \frac{\cosh \left( \frac{P_c}{P_2} \right)^{\gamma} - 1}{\cosh \left( \frac{P_c}{P_2} \right)^{\gamma} + 1} \right) \]

and

\[ S_e = \delta \left( \frac{\cosh \left( \frac{(P/P_c)^{\gamma} + \epsilon} - \gamma \right)}{\cosh \left( \frac{(P/P_c)^{\gamma} + \epsilon} + \gamma \right)} \right) \]

In his paper, King noted the equation developed by Gardner (1958) for permeability, which is

\[ K = \frac{a}{P_c^{\gamma} + b} \]

is dimensionally inconsistent and altered it to

\[ K = \frac{1}{(P_c/P)^{\gamma} + b} \]

He then ran comparison tests of Equations (7) and (10) with lab data. The results showed that both equations fit the data well and they both describe a continuous function that plateaud on the log relative permeability versus log capillary pressure graph near the values of \( K_r = 1 \). This is in contrast to the Brooks-Corey equations which are discontinuous.

![Figure 1. Effective saturation as a function of capillary pressure head for a typical homogeneous soil.](image)
at \( P_c < P_b \) and defines relative permeability as:

\[
K_r = \left( \frac{P_b}{P_c} \right)^e
\]

for \( |P_c| \geq |P_b| \)  \hspace{1cm} (11a)

and \( K_r = 1 \)

for \( |P_c| < |P_b| \)  \hspace{1cm} (11b)

**Unsaturated Flow Solutions**

Using the Brook-Corey relationships along with Darcy’s law and the continuity equation, a differential equation of flow can be developed for the process of unsaturated seepage. The derived differential equation can then be solved by a numerical process thus providing a mathematical description of the flow process.

Previously, most numerical solutions to heterogeneous unsaturated flow handled the heterogeneity of the flow by assuming layered soils. Hanks and Bowers (1962) among others, developed computer solutions which allowed each layer of soil to have different parameters. Although these discontinuities exist as illustrated by the data obtained by Scott and Corey (1961) the use of Darcy’s law in differential form is not justifiable. The reason is, that in order to use Darcy’s law in the differential form a continuous function must exist which is not the case for layered soils with abrupt interfaces. The method used by Hanks and Bowers to calculate permeability was to define moisture diffusity in terms of saturation and then permeability in terms of diffusity.

For the homogeneous case, Jeppson (1974) obtained solutions for both the one-dimensional and the axisymmetric cases. The relationships used for the solutions were the Brooks-Corey equations and a series of curves were obtained showing the effects of certain parameters on infiltrations. Watson (1972) in a one-dimensional solution, also used the Brooks-Corey equations for the description of unsaturated seepage in heterogeneous soils. The soil heterogeneity was defined by varying the permeability as a function of depth. Various cases of varying permeability were studied and graphs were presented for the solutions. These solutions were not, however, compared with physical data. These previous two solutions are in contrast to the one used for this study in that the pore size distribution, saturated permeability, residual saturations, porosity and bubbling pressure in the Brooks-Corey relationships are all able to vary quadratically with depth.

**Unsaturated Properties of Materials**

Although many mathematical expressions have been used to describe relationships between capillary pressure, saturation and hydraulic conductivity in unsaturated flow, the equations used herein will be the ones developed by Brooks and Corey (1964). These equations have been selected primarily because of their simplicity while they also provide results in good agreement with lab data. The largest discrepancy between these equations and actual data for both pressure-saturation and pressure-conductivity relationships is for soil conditions near unit saturations. However, this is not of major concern for many infiltration studies. The parameters in the Brooks-Corey equations are obtained from pressure-saturation plots on log-log paper like that shown in Figure 1. It can be seen from Figure 1 that typical data points vary from the fitted curve only near unit saturations. From the pressure-saturation relationship obtained from this curve a relative permeability-pressure relationship may be defined. The relative permeability can then be used in combination with Darcy’s law and continuity to describe the unsaturated moisture movement in soils.

**General Equation**

In solution to the flow problem presented, a one-dimensional finite difference scheme is used. In formulating the general equation of flow and the boundary conditions the convention of a space coordinate \( Z \) positive upwards, is used in formulating the one-dimensional transient problem. Functions of \( Z \) are:

\[
\begin{align*}
\omega & = \text{velocity in the } Z \text{ direction} \\
\theta & = \text{water content of the soil by volume} \\
\eta & = \text{porosity of the soil}
\end{align*}
\]

Darcy’s law and continuity can be written as:

\[
\begin{align*}
\omega = -K e \frac{\partial (z + \frac{P_c}{\gamma})}{\partial Z} & \hspace{1cm} \text{Darcy's Law (12)} \\
\frac{\partial \omega}{\partial Z} - \frac{\partial \theta}{\partial t} + \frac{\partial (\eta \theta)}{\partial t} & = 0 & \hspace{1cm} \text{Continuity (13)}
\end{align*}
\]

in which:

\[
\begin{align*}
K & = \text{hydraulic conductivity} \\
K_v & = \text{vertical hydraulic conductivity} \\
K_s & = \text{surface hydraulic conductivity} \\
K_r & = \text{relative hydraulic conductivity} \\
P_c & = \text{capillary pressure head}
\end{align*}
\]
Combining Equations (3) and (5) gives a differentiable expression for saturation as:

\[ S = S_x + (1 - S_x)(P_b/P_c)^\lambda \]  

(14)

In the previous equations, the pressure terms have the dimensions of length. For making these equations dimensionless, the following terms are defined by dividing through by a characteristic length, \( L \):

\[ P_t = -P_c/yL \]  

(15)

\[ z = z/L \]  

(16)

\[ \tau = K_b L/L \]  

(17)

Differentiating Equation (14) with respect to \( P_t \) and Equation (12) with respect to \( z \) and substituting them into Equation (13) gives the following equation:

\[ \frac{3}{8Z} (K_V (1 - \frac{3P_t}{8Z})) + \eta \lambda (1 - S_x) \]

\[ \left( \frac{P_b}{yL} \right)^\lambda \left( \frac{1+2\lambda}{P_t} \right) \frac{3P_t}{8Z} = 0 \]  

(18)

Making use of the definition of \( K_x \) in Equation (6), this equation can be expanded and simplified to:

\[ \frac{3}{8Z} \left( \frac{K_V}{V_x} (1 - \frac{3P_t}{8Z}) \right) + \eta \lambda (1 - S_x) \]

\[ \left( \frac{P_b}{yL} \right)^\lambda \left( \frac{1+2\lambda}{P_t} \right) \frac{3P_t}{8Z} = 0 \]  

(19)

The Crank-Nicholson method is used in differencing Equation (19). It has been found however, that because of the sharp wetting front it is difficult to accurately calculate the value of \( P_t \) and its derivatives. This, plus the strong nonlinearity of the equation makes the use of a transformation of the independent variable very helpful. Since the capillary pressures change from highly negative to moderately negative values very rapidly, a transformation is needed to make this change less abrupt. Jeppson (1975) has found for the heterogeneous case that a modified Kirchoff transformation works well. For the homogeneous case the transformation is:

\[ \eta = \eta \left( \frac{P_t}{1 - (1 + 3\lambda_0)\xi} \right) \]

(20)

Substituting Equation (6) for \( K_x \) and integrating, the transformation becomes:

\[ \eta = \left( \frac{P_b}{yL} \right)^{2+3\lambda} \left( \frac{1 - P_t}{1 + 3\lambda} \right) (1 + 3\lambda) \]

(21)

Since for the heterogeneous case \( \lambda \) is a function of depth it is not possible to simply integrate Equation (20). Because of this, a modified Kirchoff transformation is used in which the constant \( (P_b/yL)^{2+3\lambda} \) is dropped and a surface value of \( \lambda_0 \), is used in the transformation yielding:

\[ \xi = \left( \frac{1 - P_t}{1 + 3\lambda_0} \right)^{-1} \]

(22)

and

\[ P_t = \left( 1 - (1 + 3\lambda_0)\xi \right) \]

(23)

In using this transformation the derivatives \( \frac{3P_t}{8Z} \) and \( \frac{3P_t}{8Z} \) must be defined. Expanding the partial derivative of \( P_t \) with respect to \( z \) in terms of \( \xi \) gives:

\[ \frac{3P_t}{8Z} = \frac{3P_t}{8Z} \]

(24)

and

\[ \frac{3P_t}{8Z} = \frac{3P_t}{8Z} \]

(25)

Substituting Equations (24) and (25) into the equation of flow, Equation (19), gives:

\[ \left( \frac{P_t}{1 - (1 + 3\lambda_0)\xi} \right) \left( \frac{3K_V}{V_x} \right) \left( \frac{1 - (1 + 3\lambda_0)\xi}{2} \right) \]

(26)

Equation (26) is the parabolic partial differential equation of flow used in this report to describe one-dimensional unsaturated infiltration.
Boundary Conditions

Since the flow equation derived here is a parabolic partial differential equation, initial and boundary conditions are needed in order to obtain a solution. The initial conditions in this report assume static equilibrium throughout the profile. The surface saturation condition is of two types: surface saturation or an input flux may be specified. Should surface saturation be specified, the values for the first grid point will be known, being a Dirichlet condition, and are not solved for in the solution process. Solving Equation (12) for \( p \) and then substituting the value obtained into Equation (22) the following two expressions for the surface grid point may be obtained:

\[
p_{t_0} = \frac{L}{b} \left( \frac{1}{S_0} - \frac{1}{S_T} \right) \left( 1 + \frac{1}{\lambda_0} \right) \tag{27}
\]

\[
\xi_0 = 1 - \left( \frac{S_0(t) - S_T}{1 - S_T} \right) \left( \frac{\xi}{\xi_b} \right) \tag{28}
\]

Should input flux be specified, the continuity equation used is

\[
-\frac{w}{\text{in}} + \frac{w}{\text{out}} = -\frac{\delta S}{\delta t} \tag{29}
\]

Using previously derived expressions this equation becomes:

\[
\frac{3p}{\lambda_0} \left( \frac{1}{2} \right) + \frac{\lambda}{\lambda_0} \left( \frac{1}{2} \right) \right) \left( \frac{k}{\lambda} + \frac{1}{\lambda_0} \right) \left( \frac{p}{\lambda} + \frac{1}{\lambda_0} \right) = 0 \tag{30}
\]

where \( V(\tau) \) is the surface input flux. The subscript 1 means that the preceding variables will be evaluated at the first grid point below the surface while the \( \lambda \) designates that the variables will be taken midway between this and the surface grid point.

The bottom boundary condition of the solution will also be a Dirichlet type. Since the saturation changes very rapidly across the wetting front, the solution need not continue more than a few grid points beyond the wetting front. Therefore until the wetting front reaches the bottom of the soil profile the initial conditions of the points ahead of the wetting front will be used as values for the Dirichlet boundary condition. The bottom of the profile is assumed to be a drained layer and after the wetting front has reached this layer no water will drip into it until the bottom reaches a saturation of unity. After the wetting front has reached the bottom of the soil profile but before the high saturations are reached the bottom boundary condition will be:

\[
\frac{3p_b}{\delta Z} = 1 \tag{31}
\]

This can be developed from Darcy's law and the fact that the velocity will be zero through that point. After water drips from the bottom, this bottom condition becomes:

\[
p_t = 0 \tag{32}
\]

Finite Difference Formulation

The Crank-Nicholson Method averages the difference approximation with respect to \( Z \) for the current and advanced time steps. This results in a second order difference with respect to time as well as a second order approximation with respect to \( Z \). The finite difference form of the equation of flow for the interior grid points is:

\[
\xi_j = \frac{1}{2} \left( \xi_{j+1} - \xi_{j-1} \right) - \frac{\xi_{j-1} - \xi_{j+1}}{2 \delta Z} \tag{33}
\]

\[
+ \frac{1}{2} \left( \frac{\xi_{j+1}}{\xi_j} \right) - \frac{1}{2} \left( \frac{\xi_{j-1}}{\xi_j} \right) \tag{34}
\]

\[
e_j = \frac{\lambda n(1 - S_T)}{\delta t (p_L / b) + 2 + \lambda} \tag{35}
\]

and

\[
\xi_j = 1 - \left( 1 + \frac{3 \lambda_0}{\xi} \right) \tag{36}
\]

For the surface condition where a flux is given, the finite difference form of the equation for the top surface point is:
\[ f_0 = \frac{d^{k+1} + e_{1/2} ^{k+1} \left( \frac{1}{2} p_0 ^{k+1} + p_1 ^{k+1} - p_1 ^k \right)}{\text{in which}} \]
\[ d = \frac{p_0 ^{k+1}}{p_0 ^k + p_1 ^k} \left( \frac{1}{2} p_0 ^{k+1} + p_1 ^{k+1} \right)^{(k+1)/2} + d = 0 \quad (37) \]

In writing the appropriate difference equation for each grid point, a system of non-linear equations in \( \xi \) will occur. The Newton-Raphson method is used for the solution of these equations. For this solution process the iterative solution used is:
\[ \xi_{m+1} ^n = \xi_m ^n - J \vec{F} \quad (40) \]
where the superscript \( m \) is the iteration number and \( \vec{F} \) is the solution vector of the linear system of equations
\[ J \vec{X} = \vec{F} \quad (41) \]
where \( J \) is the Jacobian and \( \vec{F} \) is the vector defined by the equation of \( f_i \) applied to each point.

When Equation (40) is solved and the values of \( \xi \) at the \( k \)th + 1 time step are known then the program increments \( \xi \) and the Newton method is again iterated so that values of \( \xi \) may be obtained for the next time step. For the solution process, the initial guess for \( \xi \) in the Newton-Raphson method will be the \( \xi \) for the previous time step incremented by the change in \( \xi \) that occurred at that previous time step.

METHODS AND PROCEDURES

Defining Heterogeneity

In order to model heterogeneous soils, Equation (26) was derived such that each of the parameters: \( \lambda, P_b, S_r, K_v \), and \( n \), were allowed to vary with depth, \( Z \). For convenience, a subroutine in the program defined these variations quadratically with depth by input of the constants \( A, B \) and \( C \) in the equation:
\[ \text{parameter} = A + B(Z) + C(Z)^2 \quad (42) \]
Equation (42) was used to define all of the parameters with depth except \( K_v \). In the program, \( K_v \) is always given a value of 1 at the surface, causing the defining equation for \( K_v \) to be:
\[ \text{Equation (43) gives } K_v \text{ the value of 1 at the surface and at the same time keeps the convention of positive } Z \text{ upwards. The reason for specifying } K_v \text{ as 1 at the surface is to make the saturated permeability terms in Equation (12) equal to } K_a, \text{ the surface saturated permeability.}

Using this method to model heterogeneity, the number of possible variations of the parameters in the profile becomes innumerable. Also, each of the variables can vary separately giving an infinite number of solution possibilities. To reduce the number of solutions to a manageable number for this study, only linear variations were used on each parameter. That is, the value of \( C \) in Equations (42) and (43a) was set equal to zero.

Solution Variables Used

The variations of the parameters studied were determined by establishing a practical range of the parameter, then varying it within that range while keeping the same average value of the parameter in the profile for each solution. The parameter was varied once with the high value at the top of the profile and then again with the low value at the top of the profile. Another solution was obtained varying \( \lambda \) from 1.2 at the top to 0.3 at the bottom of the profile. Intermediate solutions were obtained by narrowing the range while linearly increasing and decreasing the parameter with depth, but keeping the same average value as before. An example is that a second range of \( \lambda \) is 0.5 to 1.0 which has an average of 0.75 as before. The values and ranges used for the five parameters in the Brooks-Corey equations are shown in Table 1.

In the solution of the infiltration problem, certain parameters are required and were held constant for each solution. These are:
- Height of profile, \( H = 2.0 \)
Table 1. Variable ranges, averages, and B and A coefficients used in this study for a dimensionless depth of $z$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range(s)</th>
<th>Average</th>
<th>B coefficient</th>
<th>A coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (pore size distribution index)</td>
<td>0.3 - 1.2</td>
<td>0.75</td>
<td>0.45</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>1.2 - 0.3</td>
<td></td>
<td>-0.45</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1.05 - 0.45</td>
<td></td>
<td>-0.30</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>0.5 - 1.0</td>
<td></td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta$ (porosity)</td>
<td>0.45 - 0.35</td>
<td>0.4</td>
<td>-0.05</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>0.35 - 0.45</td>
<td></td>
<td>0.05</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>0.6 - 0.2</td>
<td></td>
<td>-0.2</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.2 - 0.6</td>
<td></td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$S_r$ (residual saturation)</td>
<td>0.15 - 0.1</td>
<td>0.125</td>
<td>-0.025</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.1 - 0.15</td>
<td></td>
<td>0.025</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.2 - 0.05</td>
<td></td>
<td>-0.075</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0.05 - 0.2</td>
<td></td>
<td>0.075</td>
<td>0.05</td>
</tr>
<tr>
<td>$P_b$ (bubbling pressure)</td>
<td>0.1 - 1.2</td>
<td>0.65</td>
<td>0.55</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>1.2 - 0.1</td>
<td></td>
<td>-0.55</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>0.9 - 0.4</td>
<td></td>
<td>-0.25</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>0.4 - 0.9</td>
<td></td>
<td>0.25</td>
<td>0.4</td>
</tr>
<tr>
<td>$K^{1/2}$</td>
<td>1 - 0.1</td>
<td></td>
<td>0.45</td>
<td>-</td>
</tr>
<tr>
<td>(dimensionless saturated hydraulic conductivity)</td>
<td>1.0 - 5.0</td>
<td>-2.0</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

$\frac{1}{K_a}$ was made dimensionless by dividing by the saturated conductivity, $K_a$, at the surface. Therefore the saturated hydraulic conductivity is given by

$$K_0 = K_a K_v$$

in which $K_a$ is a constant.
Figure 2. Solution results giving (a) hydraulic head \((Z + p_t)\) (b) pressure head \((p_t)\) and (c) saturation as a function of depth and time for the cases in which \(\lambda\) varies linearly with depth and the homogeneous condition.
Figure 3. Solution results giving (a) hydraulic head (b) pressure head and (c) saturation as a function of depth and time for the cases in which $\eta$ varies linearly with depth and the homogeneous condition.
Figure 4. Solution results giving (a) hydraulic head (Z + pt) (b) pressure head (pt) and (c) saturation as a function of depth and time for cases in which Pb varies linearly with depth and the homogeneous condition.
Figure 5. Solution results giving (a) hydraulic head (Z + p_L) (b) pressure head (p_L) and (c) saturation as a function of depth and time for the cases in which K_v varies linearly with depth and the homogeneous condition.
Figure 6. Solution results giving (a) hydraulic head (Z + $p_t$) (b) pressure head ($p_t$) and (c) saturation as a function of depth and time for the cases in which $S_r$ varies linearly with depth and the homogeneous condition.
which the saturation curves become tangent, show were studied. The rate of change of the parameters given a constant value of \( \frac{\partial \theta}{\partial t} \) and \( \frac{\partial \psi}{\partial t} \), keeping the problem dimensionally consistent. Having 21 grid points divides the profile into 20 equal increments which then defines \( \Delta z \) as 0.1. Surface saturation was specified at 0.9 to represent the high saturations achieved just below the surface for the case of ponded water. The solutions were elapsed over the time period of \( \tau \) equal 0.0 to 0.5. Time steps of \( \tau = 0.005 \) were used for most solutions. However, for some cases the Newton iteration failed to converge at intermediate time steps for this value of \( \Delta \tau \). For those solutions \( \Delta \tau \) was reduced to 0.0025 to obtain complete solutions to \( \tau = 0.5 \). Complete solutions were obtained for all cases when assigning the smaller dimensionless time interval.

**Data Handling**

Representative results for the solutions are shown in Figures 2-6 for varying \( \lambda, \psi, \eta, S_r \), and \( K_r \). They are shown in contrast to the homogeneous case where all the variables are held constant, showing the curve of saturation and pressure versus depth, the wetting front and the advancement of the wetting front may be clearly seen. It should be noted that the lines to which the saturation curves become tangent, show the initial saturation through the profile. These initial conditions were obtained assuming that at \( \tau = 0.0 \), the soil was under static equilibrium, i.e. no water movement was occurring. As would be expected by inspection of Equation (14) these initial conditions vary with the parameters \( \psi, S_r \), and \( \lambda \). This is brought out clearly by the initial saturations in the profile as shown by these curves.

From observing the pressure head versus depth graphs, Figures 2b, 3b, 4b, 5b, 6b, it is seen that the initial conditions in the profile represent a linear variation with a pressure head of -6.0 at the surface and -4.0 at the bottom of the profile. This is because the capillary pressure at the bottom of the profile was given a constant value of -4.0 and from the equation for total pressure head, 

\[ h = \frac{Z + \psi H}{\gamma} \]

it is seen that an increase in \( Z \) dictates a decrease in \( \psi H/\gamma \).

In this investigation, the effects of the rate of change of the parameters \( \lambda, \eta, \psi, S_r \), and \( K_r \) upon both, the total volume of water absorbed and the instantaneous infiltration rate were studied. The rate of change of the parameter was represented by the value of \( B \) in Equation (42). The volume of water absorbed was discretely determined by assuming an average water content equal to the average at the top and bottom of each grid space. For correlation the volume of water absorbed was divided by \( \tau \) thus giving an overall average infiltration rate up to that time step.

Although the instantaneous infiltration rate was also determined by the computer solution the rates obtained did not plot smoothly. This nonsmoothness was primarily due to not attempting to locate the wetting front intermediately between grid points. The overall approximation to the solution is good but the comparison of values from one time step to the next does not give smooth or continuous data. In order to compensate for this, the instantaneous rates were averaged around the time step desired, using two time steps before and two after.

Typical infiltration curves have been reported on in the past by many researchers and have been described in equation form. In this report however, instead of using the rates from the solution obtained for correlation, the rates were all compared to a homogeneous solution. The homogeneous solution used was the case in which all the parameters were held constant. The values of each of the parameters were the average values, as shown in Table 1, which were used to define the middle of the ranges at which the parameter was varied for the heterogeneous case. The comparison value used was the difference between the homogeneous and the heterogeneous case or:

\[ \Delta I = I_{\text{hom}} - I_{\text{het}} \] (44)

The data was taken from the solutions at time steps of \( \tau \) equal to 0.1, 0.2, 0.3, 0.4 and 0.5. For data analysis scatter diagrams were first drawn to ascertain how the data varied. Typical such diagrams may be found in Figures 7, 8, 9, 10, and 11. The point 0.0, 0.0, in each graph results from the fact that at \( B \) equal to 0.0, the solution is the homogeneous case, meaning \( \Delta I \) is also equal to 0.0. The data was then inputted into a linear regression program to obtain a best fit curve through all of the data points. The regression program fit a curve through data with \( n \) number of independent variables and one dependent variable. The program then solved for linear coefficients for a general equation to fit the data. The equations produced had the form of:

\[ \Delta I = A + B(X_1) + C(X_2) + D(X_3) + \ldots E(X_n) \] (45)

where the coefficients found by the regression program are \( A, B, C, D \) and \( E \). The independent variables are the subscripted \( X \)'s and the dependent variable is \( \Delta I \). The regression program also provided a subroutine for the transformation of the independent variable. That is, the subscripted \( X \)'s could either be the actual value
Figure 7. Scatter diagram for B coefficient describing linear variation of $K_v$ with depth vs. difference between the homogeneous and heterogeneous infiltration rates ($\Delta I$) and time.

Figure 8. Scatter diagram for B coefficient describing linear variation of $P_b$ with depth vs. difference between the homogeneous and heterogeneous infiltration rates ($\Delta I$) and time.
Figure 9. Scatter diagram for $B$ coefficient describing linear variation of $n$ with depth vs. difference between the homogeneous and heterogeneous infiltration rates ($\Delta I$) and time.

Figure 10. Scatter diagram for $B$ coefficient describing linear variation of $\lambda$ with depth vs. difference between the homogeneous and heterogeneous infiltration rates ($\Delta I$) and time.
Figure 11. Scatter diagram for B coefficient describing linear variation of Sr with depth vs. difference between the homogeneous and heterogeneous infiltration rates (ΔI) and time.

Figure 12. Pressure head gradient vs. depth and time for the homogeneous case.
the negative and positive data for curve fitting.

The instantaneou infiltration rates were calculated by the computer program by subtracting from the present volume of water in the profile from the previous time step and then dividing by the time elapsed between the two time steps. The accumulative infiltration rates were also calculated by the computer program. They represent the volume of water in the profile at a particular time step minus the initial volume of water in the profile divided by the total elapsed time, $\tau$.

Each of the individual plots on the coaxial graphs are for the $B$ coefficient in Equation (42) which was varied in obtaining the data for the plot. The symbols $BL$, $BPB$, $BSR$, $B$, and $BKv$ on the individual plots correspond to the $B$ coefficients of $\lambda$, $n$, $S_p$, $F_p$, and $K_v$ respectively. In using the graphs, the lower right plot is entered first with a specific dimensionless time parameter, $\tau$. The individual plots are then followed through, as shown by the example line on Figures 18 and 19, selecting the desired $B$ coefficients until arriving at the ordinate scale of the coaxial graph for $\Delta I$ or $\Delta AI$. A caution in using the graphs is that since the data used in developing the graphs were divided into two subsets for positive and negative values, and separate regression analysis were obtained for each subset, the $B$ coefficients used should have values either all negative or zero or all positive or zero. From Equation (45) it is shown that a constant is included in each of the regression equations. Since it is unknown how much the individual data points contributed to the constant the graphs should be used only for the data ranges from which they were made. Thus the restriction of using either all positive or all negative or zero values of $B$ coefficients.

An example is drawn on the graphs for the homogeneous case. A value of $\tau$ is selected, the subsequent $B$ values of $0.0$ are followed through in order to get the appropriate $\Delta I$ or $\Delta AI$. Note that the sign on the $B$ value of $0.0$ on the first graph corresponds to the sign on the $B$ value of $0.0$ used on the last graph. The $\Delta I$ and the $\Delta AI$ obtained for both cases are well within the accuracy of the graphs in giving the value of $0.0$, which is what would be expected for the homogeneous case.

In reading the coaxial graphs it should be noted that a positive $\Delta I$ or $\Delta AI$ results from the infiltration rate for the heterogeneity specified being smaller than for the homogeneous case. For small or negative values of $\Delta I$ or $\Delta AI$ the infiltration rates are much larger and in the case of negative $\Delta I$ or $\Delta AI$ the infiltration rates for the heterogeneous case are greater than that of the homogeneous case.

On the coaxial graph for $\Delta AI$, it can be seen that the negative values of $K_v$ are not plotted. This is because in these solutions water dripped into the drained layer below the profile. The computer program did not take this into account and based its accumulative infiltration rates only using the volume of water present in the profile. This was also done for the instantaneous rates. However, it is fairly easy to calculate the volume of water leaving at one particular time step. From Darcy's law which states:

$$Q = K_v h / 2g$$

(where $h$ is the hydraulic head and $K_v$ is the effective permeability), the volume of water leaving through the bottom may be calculated. The hydraulic head, $h$, was printed out for each grid point by the computer program for each time step making the $h/2g$ easily approximated numerically. $K_v$ is the product of $K_r$ which is defined by Equation (6) and $K_e$ which is an input value. The $Q$ value obtained was then added to the output instantaneous infiltration rate.

In using the coaxial graphs it is understood that when lines are close together it means that the change in $B$ value between the two values listed have little effect on $AI$ or $\Delta AI$ for that parameter. Furthermore for those individual graphs, which make up the coaxial graphs, which are entered horizontally and exited vertically a steep slope indicates little change in $\Delta I$ or $\Delta AI$ with time whereas a small or flat slope indicates a large change in $\Delta I$ or $\Delta AI$ with time. For individual plots which are entered vertically and exited horizontally the opposite is true, that is, a steep slope indicates a large change in $\Delta I$ or $\Delta AI$ with time and a small valued slope indicates a small change in $\Delta I$ or $\Delta AI$ is due to that parameter with time. The discussion of the parameters and their effects on $\Delta I$ and $\Delta AI$ is given below.

$\tau$ - On both of the coaxial graphs, the independent plots containing $\tau$ as their abscissa have small slopes. As was pointed out previously for an individual graph that is entered vertically and exited horizontally a small slope indicates little change with time. These small variations of $\Delta AI$ and $\Delta I$ may be seen from the scatter diagrams, Figures 7 through 11. The relative spacing of the curves are not due to time, but to the parameter which is graphed with time. That is for $\Delta AI$ the spacing of the curves on the first graph are due to the variable $BL$, whereas for $\Delta I$ the spacing is due to the variable $BPB$.

$BL$ - The relative positions of the lines representing the values of the $B$ coefficients of $\lambda$ are different for both the graphs of $\Delta I$ and $\Delta AI$. The accumulative infiltration rate is seen to be larger when all of the values of $\lambda$ are relatively small and no large values of $\lambda$ present. That is, small values of $\Delta AI$ are computed when $\lambda$ is varied between 0.3 and 1.2. For the case of $\Delta I$, the largest instantaneous infiltration rate is...
achieved with a value of BL equal to 0.45. As the wetting front advances, the permeability at the front increases causing an increase in the infiltration rate. This increase in infiltration rate can also be seen in Figure 16 where the infiltration rate is graphed versus time. From these observations, the conclusion is that smaller values of \( \lambda \) correspond to higher infiltration rates. This conclusion is in agreement with the previous discussion of the graphs of saturation and pressures versus depth.

**BPOR** - The relative positions between the lines representing the B coefficient of \( n \) are the same for both the coaxial graphs of \( \Delta I \) and \( \Delta AI \). The solutions of BPOR equal to 0.05 and -0.2 have the smallest infiltration rates while the solutions of BPOR equal to -0.05 and 0.2 have the largest infiltration rates. For the solution of BPOR equal to 0.2, the top of the profile has a very high porosity which in turn allows large storage capacity. This large storage capacity enables high infiltration rates at the beginning of the solution. Although the wetting front for the homogeneous case advances faster than the solution in which BPOR equaled 0.2 the latter solution maintained the higher infiltration rates of the two solutions. The advancement of the wetting front for the case of BPOR equal 0.2 slows down as it advances in the profile due to smaller values of \( n \) at the wetting front which inhibit the infiltration process.

**BFB** - In the analysis of the graphs of saturation and pressures with depth, it was seen that solutions with small values of \( P_b \) at the surface show small infiltration rates and slow advancement of the wetting front. This observation can be better understood by inspection of Equation (6) which states that a decrease in \( P_b \) will cause a decrease in \( K_w \). For the solution of BFB equal to -0.55 the surface value of \( P_b \) is 0.1, the infiltration rate is very small and the wetting front essentially does not advance at all. The heterogeneous case defined by varying \( P_b \) with the highest infiltration rates is the solution of BFB equal 0.25. Although this solution does not have the high values of \( P_b \) as in the case of BFB equal to 0.55, the infiltration rate is higher because it does not have values of \( P_b \) which are prohibitive to infiltration lower in the profile.

**BSR** - For the values of the B coefficients of BSR used in this report, little change in \( \Delta I \) or \( \Delta AI \) is seen. This is demonstrated by the close grouping of the curves for BSR shown on the coaxial graphs. This same observation was made by inspection of the saturation and pressure graphs for \( S_r \) where the solutions are well grouped for each time step. Larger infiltration rates are observed for the solutions of \( S_r \) increasing with depth.

**BKV** - From Equation (47) and by observations of the saturation and pressures graphs for BKV, Figures 5a, b and c, it is expected that by increasing the value of \( K_v \) in the profile the infiltration rates would also increase. For the values of BKV positive and less than one, which cause a decreasing \( K_v \) with depth, this reasoning is true. That is, the infiltration rates decrease with decreasing \( K_v \) in the profile. It is witnessed on the coaxial graph for \( \Delta I \), increasing \( K_v \) with depth shows a large infiltration rate for the case of BKV equal to -2.0. The value of BKV equal to -2.0 causing \( K_v \) to vary from a value of 1 at the surface to 5.0 at the bottom of the profile.

The solutions whose values of \( \Delta AI \) and \( \Delta I \) varied very little with time especially the homogeneous case, which gives zero values of \( \Delta AI \) and \( \Delta I \) at all time steps, cause the coaxial graphs to be for the most part time independent. That is, for any set of B coefficients the same values of \( \Delta AI \) and \( \Delta I \) within the accuracy of the graphs will be obtained. This means in using the graphs the values of \( \Delta AI \) and \( \Delta I \) obtained will represent an average or dominant value over the time steps of \( \tau \) equal to 0.0 to 0.5. For a general idea on how the parameters will affect the solution, the coaxial graphs will be useful. However, after consulting the coaxial graphs and obtaining trends of what to expect, the computer program must be used for a detailed analysis. The graphs should prove to be of assistance in infiltration studies in that by consulting them first the number of computer runs needed for a particular study may be decreased.

**Effects of Varying Two Parameters Simultaneously**

The coaxial graphs in this report were developed by varying only one parameter at a time. Therefore, using the graphs for solutions where more than one parameter varies at a time will assume that these parameters act independently of each other. Also, as was pointed out previously, the values of \( \Delta AI \) and \( \Delta I \) will represent average or dominant values because of the time independence of the graphs.

It was not the purpose of this report to study the effects of more than one of the parameters in the Brooks-Corey equations varying at once. However, six solutions of two parameters varying simultaneously have been obtained and the values of \( \Delta AI \) and \( \Delta I \) calculated. These values were compared to the corresponding values obtained from using the coaxial graphs. The solution values and corresponding coaxial graph values are given in Table 2 for comparison. For some of the solutions, the comparisons between the calculated values and the values obtained from the coaxial graphs show no correlation. For other solutions, though, the values obtained from the coaxial graphs show that they could possibly represent an average or a dominant infiltration rate, as described before, when compared to the computer solutions. Further study
Table 2. Comparison of solutions to two linearly varying parameters by the coaxial graphs and computer solution.

<table>
<thead>
<tr>
<th>τ</th>
<th>AI</th>
<th>I</th>
<th>ΔAI</th>
<th>ΔI</th>
<th>ΔAI</th>
<th>ΔI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>λ = .3 + 1.2 B = .45</td>
<td>η = .3 + .4 B = .05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>0.70713</td>
<td>0.5124</td>
<td>0.3039</td>
<td>0.0956</td>
<td>0.475</td>
<td>0.30</td>
</tr>
<tr>
<td>.3</td>
<td>0.580387</td>
<td>0.52972</td>
<td>0.134813</td>
<td>-0.01492</td>
<td>0.47</td>
<td>0.3</td>
</tr>
<tr>
<td>.5</td>
<td>0.568797</td>
<td>0.56726</td>
<td>0.06303</td>
<td>-0.06446</td>
<td>0.475</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>λ = .5 + 1.0 B = .25</td>
<td>P_b = .4 + .9 B = .25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>0.994213</td>
<td>0.54787</td>
<td>0.01689</td>
<td>0.06013</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>.3</td>
<td>0.667371</td>
<td>0.63761</td>
<td>0.05047</td>
<td>0.07719</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>.5</td>
<td>0.567711</td>
<td>0.412135</td>
<td>0.06472</td>
<td>0.9067</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>P_b = .1 + 1.2 B = .55</td>
<td>η = .2 + .6 B = .2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>1.12730</td>
<td>0.57865</td>
<td>-0.1162</td>
<td>0.02935</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>.3</td>
<td>0.667325</td>
<td>0.32095</td>
<td>0.04788</td>
<td>0.19385</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>.5</td>
<td>0.513337</td>
<td>0.255343</td>
<td>0.11849</td>
<td>0.24746</td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>P_b = .4 + .9 B = .25</td>
<td>S = .1 + .15 B = .025</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>0.984502</td>
<td>0.62286</td>
<td>0.02660</td>
<td>-0.01486</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>.3</td>
<td>0.614348</td>
<td>0.390054</td>
<td>0.07385</td>
<td>0.12475</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>.5</td>
<td>0.533769</td>
<td>0.361922</td>
<td>0.09806</td>
<td>0.14088</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>η = .3 + .4 B = .05</td>
<td>K_v = .5 + 1.0 B = .25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>0.989977</td>
<td>0.58477</td>
<td>0.02112</td>
<td>0.02323</td>
<td>0.325</td>
<td>0.32</td>
</tr>
<tr>
<td>.3</td>
<td>0.622991</td>
<td>0.49227</td>
<td>0.02209</td>
<td>0.02253</td>
<td>0.315</td>
<td>0.32</td>
</tr>
<tr>
<td>.5</td>
<td>0.509494</td>
<td>0.479188</td>
<td>0.02335</td>
<td>0.023612</td>
<td>0.325</td>
<td>0.32</td>
</tr>
<tr>
<td>6</td>
<td>P_b = .1 + 1.2 B = .55</td>
<td>K_v = .1 + 1.0 B = .45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>0.938721</td>
<td>0.50762</td>
<td>0.07238</td>
<td>0.10038</td>
<td>0.125</td>
<td>0.3</td>
</tr>
<tr>
<td>.3</td>
<td>0.560752</td>
<td>0.25699</td>
<td>0.15445</td>
<td>0.25781</td>
<td>0.125</td>
<td>0.3</td>
</tr>
<tr>
<td>.5</td>
<td>0.426614</td>
<td>0.203999</td>
<td>0.20522</td>
<td>0.298801</td>
<td>0.125</td>
<td>0.3</td>
</tr>
<tr>
<td>τ</td>
<td>AI_hom</td>
<td>I_hom</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>1.0111</td>
<td>0.608</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.3</td>
<td>0.71520</td>
<td>0.5148</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.5</td>
<td>0.631879</td>
<td>0.5028</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
needs to be done in this area of more than one parameter varying. By obtaining additional computer solutions and checking the values with the coaxial graphs a better idea of independence or interdependence can be explored. That is if the coaxial graphs provide a ballpark value for more than one parameter varying then it is a possibility that those two parameters are independent. However, if the coaxial graphs give values of ΔI or ΔAI that are in complete disagreement with the solutions obtained then possibly a dependence amongst those parameters is present.

For future research, the development of another measure of the infiltration process, such as the movement of the wetting front, could be developed. This new method in conjunction with the coaxial graphs developed in this study could then be used to study the interdependence of the parameters. Possibly the solutions for ΔAI and ΔI could then be shown to be time dependent, giving a more accurate description of the process with time.

REFERENCES


Burdine, N. T. 1953. Relative permeability calculations from pore size distribution data. Transactions, American Institute of Mining and Metallurgical Engineers. 198:71-78.


APPENDIX A

DATA SETS FOR REGRESSION ANALYSIS AND
RESULTING REGRESSION EQ
<table>
<thead>
<tr>
<th>BL</th>
<th>bAV</th>
<th>bSW</th>
<th>bPW</th>
<th>bPB</th>
<th>TAU</th>
<th>DELTA T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.45</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.1</td>
<td>0.123400</td>
</tr>
<tr>
<td>0.35</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.2</td>
<td>0.145600</td>
</tr>
<tr>
<td>0.25</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.3</td>
<td>0.168900</td>
</tr>
<tr>
<td>0.15</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.4</td>
<td>0.192300</td>
</tr>
<tr>
<td>0.05</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.5</td>
<td>0.216900</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.6</td>
<td>0.242700</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.7</td>
<td>0.269600</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.8</td>
<td>0.297400</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.9</td>
<td>0.326200</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>1.0</td>
<td>0.356100</td>
</tr>
</tbody>
</table>

**Data for Instantaneous Infiltration Rate, Negative Case**
<table>
<thead>
<tr>
<th>BL</th>
<th>BKV</th>
<th>BSE</th>
<th>BMB</th>
<th>BBS</th>
<th>TAU</th>
<th>DELTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
<td>0.1</td>
<td>0.332</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
<td>0.2</td>
<td>0.296</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
<td>0.3</td>
<td>0.271</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
<td>0.4</td>
<td>0.288</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
<td>0.5</td>
<td>0.305</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.1</td>
<td>0.298</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.2</td>
<td>0.311</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.3</td>
<td>0.332</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.4</td>
<td>0.369</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.5</td>
<td>0.429</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.6</td>
<td>0.529</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.7</td>
<td>0.676</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.8</td>
<td>0.882</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>0.9</td>
<td>0.989</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.20</td>
<td>1.0</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**DATA FOR INSTANTANEOUS INFILTRATION RATE, POSITIVE CASE**
<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Stored Ave</th>
<th>( \text{ERR} )</th>
<th>( \text{NRIf} )</th>
<th>( \text{VALUE} )</th>
<th>( \text{NRIf} )</th>
<th>( \text{VALUE} )</th>
<th>( \text{NRIf} )</th>
<th>( \text{VALUE} )</th>
<th>( \text{NRIf} )</th>
<th>( \text{VALUE} )</th>
<th>( \text{NRIf} )</th>
<th>( \text{VALUE} )</th>
<th>( \text{NRIf} )</th>
<th>( \text{VALUE} )</th>
<th>( \text{NRIf} )</th>
<th>( \text{VALUE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

VALUES OF THE PRESSURE FOR TIME = .31133E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 1.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 2.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 3.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 4.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 5.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 6.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 7.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 8.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 9.0000E+02

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 1.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 2.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 3.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 4.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 5.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 6.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 7.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 8.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 9.0000E+03

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 1.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 2.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 3.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 4.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 5.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 6.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 7.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 8.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 9.0000E+04

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 1.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 2.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 3.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 4.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 5.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 6.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 7.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 8.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 9.0000E+05

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 1.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 2.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 3.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 4.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 5.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 6.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 7.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 8.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 9.0000E+06

VALUES OF THE SATURATION TAU = 0.000

VALUES OF THE PRESSURE FOR TIME = 1.0000E+07

VALUES OF THE SATURATION TAU = 0.000
<table>
<thead>
<tr>
<th>VALUES OF SATURATION TAU =</th>
<th>0.0050</th>
<th>0.0100</th>
<th>0.0150</th>
<th>0.0200</th>
<th>0.0250</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOL. OF WATER ABSORBED =</td>
<td>0.1607</td>
<td>0.1607</td>
<td>0.1607</td>
<td>0.1607</td>
<td>0.1607</td>
</tr>
<tr>
<td>VALUES FOR HYDRAULIC HEAD TAU =</td>
<td>0.4900</td>
<td>0.4900</td>
<td>0.4900</td>
<td>0.4900</td>
<td>0.4900</td>
</tr>
</tbody>
</table>

VALUES OF THE PRESSURE FOR TIME: 0.5000E+00

VALUES OF THE PRESSURE FOR TIME: 1.0000E+00

VALUES OF THE PRESSURE FOR TIME: 1.5000E+00

VALUES OF THE PRESSURE FOR TIME: 2.0000E+00

VALUES OF THE PRESSURE FOR TIME: 2.5000E+00

VALUES OF THE PRESSURE FOR TIME: 3.0000E+00

VALUES OF THE PRESSURE FOR TIME: 3.5000E+00

VALUES OF THE PRESSURE FOR TIME: 4.0000E+00