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Traveling-Wave Electrophoresis For Microfluidic Separations

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(Received 17 July 2008; published 20 February 2009)

Models and microfluidic experiments are presented of an electrophoretic separation technique in which charged particles whose mobilities exceed a tunable threshold are trapped between the crests of a longitudinal electric wave traveling through a stationary viscous fluid. The wave is created by applying periodic potentials to electrode arrays above and below a microchannel. Predicted average velocities agree with experiments and feature chaotic attractors for intermediate mobilities.

DOI: 10.1103/PhysRevLett.102.076103 PACS numbers: 82.45.–h, 05.45.Ac, 82.40.Bj, 87.15.Tt

Separations of charged substances are important in proteomics, molecular biology, cell biology, genetics, materials synthesis, and bioengineering, and are integral to microfluidic lab-on-a-chip devices that are being developed for rapid clinical and forensic analysis [1]. Over the last 25 years, capillary electrophoresis (CE) has set the standard for high-efficiency separations in solution [2]. This technique employs static, uniform electric fields to separate ions with different charge-to-size ratios into distinct zones for analysis, with zone dispersion limited ultimately by molecular diffusion.

In this Letter, we study an electrophoretic separation technique that differs from CE by trapping ions whose mobilities exceed a tunable threshold between the crests of longitudinal electric field waves traveling through a stationary solution. These waves are created by applying oscillating potentials to interdigitated arrays of stationary electrodes above and below a microfluidic channel (Fig. 1). The trapping threshold depends on the ion mobility, the electrode spacing, and the potential frequency and amplitude, and allows modulation between separative and non-separative transport by simply varying the frequency. Separations by traveling-wave electrophoresis (TWE) (Fig. 2) apply to ions, charged biomolecules, and micron-sized charged particles, and might reduce zone dispersion below the diffusion limit.

Others use interdigitated electrode arrays on a single surface to transport charged species via electrophoresis, imposing static perpendicular gravitational or electric fields to draw particles to the surface [3–5]. Our sandwich architecture precludes such fields by bounding a microfluidic channel by electrode-bearing surfaces above and below. This design allows the use of low applied voltages to avoid unwanted electrochemical effects while keeping the electric field high to achieve rapid separations. Single-surface architectures can also transport charged particles via ac electroosmotic pumping [6,7] and neutral bioparticles via dielectrophoresis [8].

We consider the motion of ions of charge \( q \), hydrodynamic radius \( r \), and velocity \( v \) through a stationary electrically conducting solution of viscosity \( \eta \) and mass density \( \rho \) in response to oscillating electric potentials applied to periodic arrays of electrodes. In contrast with studies of oscillator synchronization [9], TWE potentials are external functions of time. In contrast with electron trapping by plasma waves [10], viscosity is essential to TWE trapping. The 1–32 Hz frequencies of our applied potentials are too small to induce ac electroosmotic pumping, which requires frequencies of the order of 1 kHz [6].

Models presented here include only electric and Stokes drag forces \( \mathbf{F}_E = q \mathbf{E} \) and \( \mathbf{F}_D = -6 \pi \eta r v \) on the ions, ignoring magnetic and gravitational fields, molecular diffusion, charge redistribution, fluid flow, and ionic inertia. Setting \( \mathbf{F}_E + \mathbf{F}_D = 0 \) immediately yields the associated electrophoretic velocity

\[
\mathbf{v} = \mu \mathbf{E}
\]

(1)

with mobility \( \mu = q/6 \pi \eta r \) and electric field \( \mathbf{E} = -\nabla \phi \).

Insight can be drawn from a 1D electric potential [11]

\[
\phi(x, t) = \phi_0 \sin(kx - \omega t)
\]

(2)

with amplitude \( \phi_0 \), wave number \( k = 2 \pi / \lambda \), angular frequency \( \omega = 2 \pi / \tau \), and electric field amplitude \( E_0 = k \phi_0 \). We define a convenient dimensionless measure of the responsiveness of an ion to the wave as

\[
R = \frac{\mu E_0}{c},
\]

(3)

the ratio of the characteristic electrophoretic velocity \( \mu E_0 \) to the wave speed \( c = \omega / k \). Here, \( q > 0 \) and \( R > 0 \) for cations and \( q < 0 \) and \( R < 0 \) for anions.

The steady-state solution to the 1D model yields, for \( |R| \approx 1 \), the trapped velocity \( v_t = c \), with trapped cations traveling within potential wells and anions within peaks. For \( |R| < 1 \), ions experience longitudinal oscillations of angular frequency \( \Omega = \omega (1 - R^2)^{1/2} \) but make net forward progress with average velocity [12]

\[
\frac{\bar{v}}{c} = 1 - (1 - R^2)^{1/2}.
\]

(4)
During each oscillation, an ion fails to catch a passing wave and lags one cycle behind the wave, like a surfer failing to catch a passing ocean wave.

Electrodes are needed to sustain traveling waves in viscous solutions, waves that would attenuate otherwise.

Our 2D model accordingly assumes an array of long conducting cylindrical electrodes [Fig. 1(c)] held at synchronized potentials $\Phi_0(t), \Phi_1(t), \Phi_2(t),$ and $\Phi_3(t)$ given by

$$\Phi_i(t) = \phi_0 \sin(kx_i - \omega t),$$

where $x_i = i\lambda/4$ denotes the axial position of electrode $i = 0, 1, 2, 3$ and $\lambda = 2\pi/k$ is the electrode pattern wavelength (Fig. 1). We retain the definition of $R$ given by Eq. (3) and assume that the bulk solution is electrically...
neutral, an improvement upon the non-neutral 1D model. We solve Laplace’s equation to obtain the 2D potential \( \phi(x, y, t) \) within the bulk solution, with the four potentials serving as boundary conditions at the impenetrable electrode surfaces and with impenetrable channel boundaries at \( y = 0 \) and \( y = h \). This solution is a sum of logarithmic potentials for pairs of cylindrical electrodes [13] that includes image electrodes to satisfy the boundary conditions. We integrate the associated 2D nonlinear nonautonomous system of ordinary differential equations,

\[
\dot{x} = v_x(x, y, t), \\
\dot{y} = v_y(x, y, t),
\]

using a fifth-order Runge-Kutta-Fehlberg method with adaptive stepsize control [14], with wavelength \( \lambda = 80 \mu \text{m} \), electrode radius \( a = 1 \mu \text{m} \), and channel height \( h = 15 \mu \text{m} \) pertinent to experiments. The time dependence of the velocity components \( v_x = -\mu \partial \phi / \partial x \) and \( v_y = -\mu \partial \phi / \partial y \) requires a 3D phase space for this system, the minimum dimensionality needed for chaos [15].

Previous studies of 2D nonautonomous systems [16] exclude impenetrable barriers such as our electrodes.

Relative to the 1D model (Fig. 3, dashed trace), the average axial steady-state velocity \( \bar{v}_x \) for the 2D model (Fig. 3, solid trace) exhibits (i) a larger trapping threshold \( R_t = 2.58 \) reflecting incomplete penetration of the potential into the channel and (ii) a nonzero localization threshold \( R_t = 1.10 \) below which low-mobility ions oscillate about electrodes \( (\bar{v}_x = 0) \). The 2D wave traps ions whose mobilities \(|\mu|\) exceed the tunable threshold mobility \( \mu_t \) defined by \( R_t = \mu_t E_0 / c \).

These 2D predictions agree with measurements of the velocities of fluorescein plugs traveling along the TWE channel of Figs. 1(a) and 1(b) with fixed \( \phi_0 = 0.5 \text{ V} \) and variable frequency \( f = 1 / \tau \) (Fig. 3, data points). Plugs containing 20 \( \mu \text{M} \) fluorescein and 10 \( \mu \text{M} \) sodium phosphate were injected electrokinetically into the channel, which was filled with a 10 \( \mu \text{M} \) sodium phosphate running buffer. A six-step stepwise-constant periodic potential that mimics Eq. (5) was then applied to the electrodes. Average plug velocities \( \bar{v}_x \) were determined microscopically by measuring the time of passage of the peak fluorescein intensity between two fixed locations separated by at least 400 \( \mu \text{m} \). Values of \( c \) follow from \( c = f \lambda \) and values of \( R \) from Eq. (3) using the fluorescein mobility \( \mu = -2.8 \times 10^{-4} \text{ cm}^2 / \text{V s} \) determined from CE experiments. Each data point in Fig. 3 is centered at the average of two independent measurements of \( \bar{v}_x / c \), with the ends of the error bars showing these two measurements when they differ by more than the diameter of the plotting symbol.

TWE might reduce the dispersion of trapped analyte plugs to below the diffusion limit. At \( f = 1 \text{ Hz} \), a fluorescein plug takes \( t = 50 \text{ s} \) to travel the length of the 4 mm TWE channel. During this time, molecular diffusion would increase the full width at half maximum of the plug by \( 2 \Delta x = (2Dt)^{1/2} = 0.2 \text{ mm} \), where \( D = k_B T \mu / q = 3.5 \times 10^{-6} \text{ cm}^2 / \text{s} \) is the fluorescein diffusivity, \( q = -3.2 \times 10^{-19} \text{ C} \) is its charge, \( k_B \) is Boltzmann’s constant, and \( T = 295 \text{ K} \) is the temperature. Preliminary measurements show that fluorescein plugs of initial width 1.3 mm spread less than 0.2 mm, indicating that TWE might elute bands with little or no dispersion.

Partially trapped ions exhibit (i) periodic and narrow band chaotic attractors with commensurate velocities \( \bar{v}_x / c = 1 / 5 \) and \( 1 / 9 \), (ii) broadband chaotic attractors with incommensurate velocities, and (iii) period-doubling cascades to chaos (Fig. 4). Illustrated in the inset of Fig. 4 are a period-1 attractor, a narrow band chaotic attractor, and a broadband chaotic attractor. Ions on the period-1 attractor follow identical paths between adjacent electrodes, apart from simple translations and reflections, each path including one lag cycle and contacting an electrode at a specific angle, \( \theta = 0.65 \pi \). Ions on the narrow band chaotic attractor follow similar paths and contact electrodes somewhere in the range \( 0.49 < \theta / \pi < 0.88 \).

Ions on the broadband chaotic attractor travel between electrodes in a variety of ways and sample the entire range of angles, \( 0 < \theta < \pi \). Such broadband chaotic attractors exhibit self-similar nested sequences that converge at unstable trapped orbits [17].

Future explorations might include experiments with charged tracer particles and models that include ion diffusion, fluid flow, and charge redistribution. Neglecting diffusion is justified by the large Péclet numbers \( P = \lambda c / 2D \). FIG. 3. Ratio of the steady-state average axial ion velocity \( \bar{v}_x \) to the wave speed \( c \) vs the responsiveness \( R \), from Eq. (4) for the 1D model (dashed trace), for the 2D model (solid trace), and for microfluidic experiments (data points accompanied by numerical values of the frequency \( f \)). In the experiments, ions are trapped for \( f = 1, 1.5, \) and 2 Hz, partially trapped for \( f = 3, 4, \) and 8 Hz, and localized for \( f = 16 \) and 32 Hz. The 2D model captures the localization threshold and underestimates the trapping threshold.
satisfying $10 < P < 300$, though diffusion might refine predictions of the trapping threshold. Our low frequencies $f$ preclude net electroosmotic pumping of fluid along the microchannel axis [6], but might allow localized circulatory electroosmotic flow (EOF) because $\tau = 1/f = 0.1$ s is large compared with the characteristic EOF response time $\tau_{\text{EOF}} = 4h^2\rho/\eta = 0.0003$ s [18]. We intend to study the extent and effects of such flows via models and experiments with uncharged tracer particles.

We gratefully acknowledge support from NSF Grants No. DMR-0647763 and No. EPS-0554328, a WVU Research Corporation PSCoR grant, technical assistance from John Edwards and Kathleen Kelly, and helpful conversations with Mark Koepeke, David Lederman, Earl Scime, Kenneth Showalter, James Vopal, and Arthur Weldon.