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Essays on Population Aging and Social Security in the U.S.

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ESSAYS ON POPULATION AGING AND

SOCIAL SECURITY IN THE U.S.

by

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ABSTRACT

Essays on Population Aging and Social Security in the U.S.

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Over the past few decades, falling birth rates and increasing life expectancies have threatened the viability of social security programs all across the Organisation for Economic Co-operation and Development (OECD). In this dissertation, I attempt to shed some light on the extent of the crisis that the social security program in the United States (U.S.) currently faces, and I also recommend one possible reform policy. In the first essay, I provide an alternative estimate of the impact of population aging on the future social security benefits in the U.S., while accounting for the household-level and macroeconomic adjustments to population aging. Using a general equilibrium life-cycle consumption model with endogenous retirement and incomplete private annuity markets, I find that once these adjustments are accounted for, population aging in the U.S. is likely to cause a significantly smaller decline in the future benefits as compared to the commonly reported estimates that suggest a 25-33% decline. I also find that ignoring either the household retirement mechanism or the aggregate factor price adjustment mechanism could lead to a roughly comparable overestimation of the decline in the future retirement benefits. In the second essay, I ask what should be the optimal or welfare-maximizing social security (OASI) tax rate in the U.S. under such demographic developments. I examine this question using a heterogeneous-agent general equilibrium model of life-cycle consumption and labor supply, where social security provides partial insurance against unfavorable efficiency realizations that occur before the agents enter the model. I first calibrate the model such that the current OASI tax rate in the U.S. maximizes social welfare under the current demographics, and then I incorporate empirically reasonable population projections into the calibrated model. Finally, I search for the tax rates that are optimal under such projections. I find that the tax rates that maximize welfare under such projections are about 2 to 5 percentage points higher than the current rate. I also find that a large part of the tax burden of population aging is picked up by the households with relatively favorable efficiency realizations. Finally, the model also predicts that population aging and the optimal tax response may imply a decline in the projected retirement benefits, but of a magnitude smaller than when the tax rate is held unchanged at the current level.

(109 pages)

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CHAPTER 1

INTRODUCTION

All across the Organisation for Economic Co-operation and Development (OECD), social security programs form an important component of national budgets. For example, social security expenditures as percentage of GDP range from 20.9% in Sweden to 8.3% in Australia, with an all-OECD average of roughly 16%.[1] Even though there are significant cross-country differences in the nature of financing, administration and generosity of social security programs, a key determinant of their health is the underlying population structure. Over the past few decades, falling birth rates and increasing life expectancies have threatened the viability of social security programs all across the OECD. In this dissertation, I attempt to shed some light on the extent of the crisis that the social security program in the United States (U.S.) currently faces, and I also recommend one possible reform policy.

In the first essay, I provide an alternative estimate of the impact that population aging in the U.S. is likely to have on the projected retirement benefits in the future. The U.S. Social Security Administration (SSA), among others, reports actuarial estimates of the extent by which future retirement benefits will have to decline given the present unfunded structure of the social security program. However, one simplifying characteristic of these estimates is they do not account for the household-level and macroeconomic adjustments that may be associated with population aging. I argue that accounting for such adjustments is important, as household-level consumption-saving and retirement responses, and the associated factor price adjustments in general equilibrium may to lead to a natural increase in the tax base of the social security program that the conventional estimates overlook. Using a general equilibrium life-cycle consumption model with endogenous retirement and

[1]Source: OECD Historical Statistics.

incomplete private annuity markets, I find that population aging in the U.S. is likely to lead to a much smaller decline in the projected benefits, when compared to the commonly reported estimates. I also find that ignoring either the household-level retirement mechanism or the aggregate factor price adjustment mechanism could lead to a roughly comparable overestimation of the social security crisis.

In the second essay, I adopt a normative perspective and examine the optimal or welfare-maximizing social security reform in the U.S. under the future demographic projections. I construct a heterogeneous-agent general equilibrium model of life-cycle consumption and labor supply, where the source of heterogeneity is a productivity or efficiency realization that occurs before the agents enter the model. In the model, an unfunded social security program provides partial insurance against an unfavorable efficiency realization by paying retirement benefits through a pro-poor rule. I calibrate the model such that the current social security program in the U.S. maximizes welfare under the current demographics (i.e. the optimal tax rate is equal to the current OASI tax rate in the U.S.), and then I introduce empirically reasonable low-cost, intermediate and high-cost demographic shocks using data from the 2009 OASDI Trustees Report. I find that the welfare-maximizing social security tax rates under the future demographic projections are higher than the current rate: 12.5%, 13.9% and 15.5% under the low-cost, intermediate and high-cost shocks respectively. I also find that households with different efficiency realizations respond asymmetrically to the the demographic developments, and that a large part of the tax burden of population aging is picked up by the households with relatively favorable efficiency realizations. Therefore, given that the demographic shocks only have a small impact on the relatively poor households who actually benefit from social security, the model predicts increases in the tax rate that are relatively small compared to several other studies in pension reform under population aging in the U.S.

The results from the two essays broadly suggest that population aging in the U.S. may impose a significantly lesser burden on the social security program once the associated household-level and macroeconomic adjustments are accounted for, and also that the welfare-maximizing tax increases under population aging in the near future are likely to be in the neighborhood of 2 to 5 percentage points. Sensitivity analysis of the results also demonstrates that these findings are not an outcome of the specific model calibrations: I find that the quantitative predictions of the respective models are roughly invariant to the values of several underlying model parameters used in the simulations.

The rest of this dissertation is organized as follows. In Chapter 2, I present an alternative estimate of the decline in the projected retirement benefits in the U.S. under population aging. In Chapter 3, I identify the optimal or welfare-maximizing change in the current social security tax rate under the future demographic in the U.S. Finally, in Chapter 4, I outline some concluding remarks. Within each chapter, I break up the discussion into sections that introduce the specific research question, describe the model being used to answer the question at hand, outline the baseline calibration of the constructed model, and then quantitatively investigate the question. I provide an appendix to each of the two chapters 2 and 3, where I discuss the computational methods used in generating the results. In an additional appendix for Chapter 3, I consider an extension of the basic model by introducing a second role for social security: management of a pollution externality.

CHAPTER 2

IS THE SOCIAL SECURITY CRISIS REALLY AS BAD AS WE THINK?

2.1 Introduction

Mitigating the effect of population aging on unfunded social security programs has been a major policy concern in the developed world over the last few decades. Primarily driven by lower birth rates and higher life expectancies, these demographic developments have significantly strained pension programs all across the OECD. In the U.S., with the current contribution rate, a direct or indirect reduction in the retirement benefits is required to keep the social security program solvent under the projected future demographics. Feldstein (2005) points out that keeping the payroll tax rate unchanged at the current level with the present unfunded structure would require reducing benefits by almost 33% in the year 2075.

One difficulty with using actuarial estimates to measure the social security crisis is that they ignore the household-level and macroeconomic adjustments associated with population aging. There are at least two reasons why accounting for such adjustments is important: first, if incomplete annuity markets prevent households from insuring against the risk of out-living their assets, a higher life expectancy may stimulate private saving, and therefore the aggregate capital stock and the wage rate. Second, a higher life expectancy may directly increase the labor supply because there are more workers alive at any age, and also indirectly because it may induce households to delay retirement. When the factor markets are cleared, these effects may lead to an increase in the tax base of the social security program, which implies that population aging may have a smaller impact on projected social security benefits once these effects are accounted for.[1]

[1] It is useful to note that the Social Security Administration (SSA) uses actuarial estimates to measure the crisis.

Economists have long emphasized the importance of studying pension reform in the U.S. using models that account for these household-level and macroeconomic adjustments. Notable studies in this area, such as Kotlikoff (1997), De Nardi et al. (1999), Nishiyama and Smetters (2005) and Conesa and Garriga (2008a,b), have used large scale applied general equilibrium models to examine different policy responses to the future demographic projections. Moreover, the possible impact of a longer lifespan on a household's retirement choice is also well-known (Sheshinski, 1978). Given these facts, I make two contributions in this paper. First, I provide an alternative estimate of the decline in projected retirement benefits that accounts for these household-level and macroeconomic adjustments. Second, I also show that the declines in the projected retirement benefits are roughly comparable when only either the household retirement mechanism, or the factor price adjustment mechanism is accounted for. This implies that ignoring either of them could lead to biased estimates of the social security crisis in the U.S.

To achieve this, I begin by constructing an applied general equilibrium model with incomplete annuity markets, in which life-cycle permanent income households face mortality risk and optimally choose their consumption-saving paths and retirement ages. In the model, unfunded social security insures households against mortality risk, and the factor markets clear endogenously with perfectly competitive firms choosing capital and labor inputs through profit maximization. Then, I calibrate the model to match some key features of the U.S. economy and I quantitatively examine the decline in the projected retirement benefits associated with population aging.

Understanding the effect of population aging on future social security benefits in the U.S. is important from the perspective of policymaking, as it is a crucial determinant of the increase in the contribution rate that may be needed to prevent the benefits from declining. For example, Feldstein's (2005) estimate of an increase in the Old-Age and Survivors Insurance (OASI) tax rate from the current level of 10.6% to 15.7% in the year 2075 is based on the assumption that population aging would lead to a 33% reduction in future benefits. Given that the current model predicts a smaller reduction in the future benefits, the predicted tax increase required to avert the crisis is also smaller: increasing the social security tax rate from 10.6% to just 13.8% in the current model keeps projected retirement benefits unchanged under population aging.

The rest of the paper is organized as follows: Section 2.2 introduces the applied general equilibrium model, Section 2.4 describes the calibration procedure, Section 2.5 examines the impact of population aging on future benefits, and Section 2.7 concludes.

2.2 The model

Consider a continuous time overlapping generations economy in which households are identical in all respects but their date of birth (τ). The life cycle of a representative household consists of two phases: work from date τ to $\tau + T$ and retirement from date $\tau + T$ to $\tau + \bar{T}$. Households face a finite probability $Q(t-\tau)$ of surviving up to any age $(t-\tau)$, and they cannot insure against mortality risk because of the absence of private annuity markets. Household income over the life cycle consists of wages net of taxes during the work life, social security benefits past the eligibility age of T_{r}, interest income from asset holdings and an accidental bequest from the deceased households. Period utility is a function of consumption (c) as well as the fraction of period time endowment enjoyed in leisure (l), and is given by

$$u(c,l) = \begin{cases} \frac{\left(c^{\eta} l^{1-\eta}\right)^{1-\sigma}}{1-\sigma} & \text{if} \quad \sigma \neq 1 \\ \ln\left(c^{\eta} l^{1-\eta}\right) & \text{if} \quad \sigma = 1 \end{cases} \tag{2.1}$$

where η is the share of consumption in period utility and σ is the Inverse Elasticity of Intertemporal Substitution (IEIS). The population grows exponentially at rate n and there is labor augmenting technological progress at rate g per annum.

I assume that a representative household solves a two-step decision problem: it first solves for the optimal consumption-saving plan corresponding to a given retirement age, and then chooses the retirement age for which the discounted lifetime utility of the optimal consumption plan is maximized. To focus exclusively on the extensive margin of the labor supply decision (i.e. whether or not to supply labor), I follow Ortiz (2009) and others and assume that the household inelastically supplies a fraction $(1 - l_W)$ of its period time endowment to the labor market during the work life. Formally, the optimal retirement age of a household is given by

$$T^* = \arg\max V(T; \cdot) \qquad T \leq \bar{T} \tag{2.2}$$

when $V(T; \cdot)$ is the solution value of the objective functional (for a given retirement age) corresponding to the problem

$$\max \int_{\tau}^{\tau+\bar{T}} \exp\{-\rho(t-\tau)\} Q(t-\tau) \frac{\left\{c(t;\tau)^{\eta} l(t;\tau)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \mathrm{d}t \tag{2.3}$$

subject to

$$c(t;\tau) + \frac{\mathrm{d}a(t;\tau)}{\mathrm{d}t} = ra(t;\tau) + y(t;\tau) + B(t) \tag{2.4}$$

$$y(t;\tau) = (1-\theta)\left(1 - l(t;\tau)\right) w(t) e(t-\tau) + \Theta(t-\tau-T_{\mathrm{r}}) b(t) \tag{2.5}$$

$$l(t;\tau) = \begin{cases} l_W & t \in [\tau, \tau+T] \\ 1 & t \in \left(\tau+T, \tau+\bar{T}\right] \end{cases} \tag{2.6}$$

$$a(\tau;\tau) = a(\tau+\bar{T};\tau) = 0 \tag{2.7}$$

$$l_W \in (0,1)$$

where

$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

is a step function, ρ is the discount rate, $a(t;\tau)$ is the asset holding at date t of a household born at date τ, θ is the social security tax rate, $e(t-\tau)$ is an age-dependent efficiency endowment, $w(t)$ and r are respectively the wage rate and the rate of return, $B(t)$ is the accidental bequest, and $b(t)$ is the social security benefit at date t.

The government balances the social security budget every period, which implies that the expected total tax revenues at date t equal the expected total benefits paid at date t

$$\int_0^{T^*} N(t-s)Q(s)\theta\,(1-l_W)\,w(t)e(s)\,\mathrm{d}s = \int_{T_\mathrm{r}}^{\bar{T}} N(t-s)Q(s)b(t)\,\mathrm{d}s \tag{2.8}$$

where $N(t-s)$ is the size of the cohort born at date $(t-s)$. Equation (2.9) can be rearranged and expressed as

$$b(t) = \theta w(t)\left[\frac{\int_0^{T^*} N(t-s)Q(s)\,(1-l_W)\,e(s)\,\mathrm{d}s}{\int_{T_\mathrm{r}}^{\bar{T}} N(t-s)Q(s)\,\mathrm{d}s}\right] \tag{2.9}$$

where the box-bracketed term is the labor-to-retiree ratio (R^e), which is endogenous in the current model because of the household retirement choice. Also, note from (2.9) that with exponential population growth, the labor-to-retiree ratio is time invariant.

Finally, the assets of the deceased households at date t are uniformly distributed over the living population, which implies that the accidental bequest $B(t)$ at date t satisfies

$$\int_0^{\bar{T}} N(t-s)Q(s)B(t)\,\mathrm{d}s = \int_0^{\bar{T}} N(t-s)Q(s)h(s)\,a(t;t-s)\,\mathrm{d}s \tag{2.10}$$

where

$$h(s) = -\frac{\mathrm{d}}{\mathrm{d}s}\ln Q(s) \tag{2.11}$$

is the hazard rate of dying at age s. Aggregate capital and labor at date t are given by

$$K(t) = \int_0^{\bar{T}} N(t-s)Q(s)\,a(t;t-s)\,\mathrm{d}s \tag{2.12}$$

$$L(t) = \int_0^{T^*} N(t-s)Q(s)\,(1-l_W)\,e(s)\,\mathrm{d}s \tag{2.13}$$

I assume that aggregate production activity can be characterized by a Cobb-Douglas production function with inputs capital, labor, and a stock of technology $A(t)$

$$Y(t) \;=\; K(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha} \tag{2.14}$$

where $A(t) = A(0)e^{gt}$, and α is the share of capital in total income. I also assume that factor markets are perfectly competitive and equilibrate instantaneously, which implies

$$r = MP_K - \delta = \alpha\left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha-1} - \delta \tag{2.15}$$

$$w(t) = MP_L = A(t)(1-\alpha)\left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha} \tag{2.16}$$

where δ is the depreciation rate of physical capital. I define a stationary competitive equilibrium in this framework by (i) a cross-sectional consumption profile $\{c(t;t-s)\}_{s=0}^{\bar{T}}$, a cross-sectional saving profile $\{a(t;t-s)\}_{s=0}^{\bar{T}}$, and an optimal retirement age T^* that solves the household's optimization problem, (ii) an aggregate capital stock $K(t)$, effective labor supply $A(t)L(t)$, and a labor-to-retiree ratio $R^e(t)$ that are consistent with household behavior, (iii) factor prices r and $w(t)$ that clear the respective markets, (iv) retirement benefits $b(t)$ that keep the budget of the social security program balanced, and (v) accidental bequests $B(t)$ that satisfy the bequest-balance condition (2.10). It is useful to note that for this economy, along the steady state growth path aggregate output grows at rate $(n+g)$, the rate of return is time-invariant, and the wages and the accidental bequests grow at rate g.

2.3 Solving the model

As described in the previous section, I assume that a representative household's first-step decision problem is to solve for the optimal consumption and saving plan for any given retirement age, and the second-step is to solve for the retirement age for which the discounted lifetime utility of the optimal consumption plan is maximized. The first-step problem is a standard fixed-endpoint optimal control problem and can be solved using the

Pontryagin Maximum Principle.[2] The present value Hamiltonian for the problem is given by

$$\begin{aligned}\mathcal{H} &= \exp\{-\rho(t-\tau)\} Q(t-\tau)\frac{\left\{c(t;\tau)^{\eta} l(t;\tau)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \\ &\quad +\psi(t;\tau)\left[ra(t;\tau)+y(t;\tau)+B(t)-c(t;\tau)\right] \end{aligned} \tag{2.17}$$

where $\psi(t;\tau)$ is the present value co-state variable. The first-order condition with respect to consumption is given by

$$\frac{\partial \mathcal{H}}{\partial c(t;\tau)} = \exp\{-\rho(t-\tau)\} Q(t-\tau)\eta c(t;\tau)^{\eta(1-\sigma)-1} l(t;\tau)^{(1-\eta)(1-\sigma)} - \psi(t;\tau) = 0 \tag{2.18}$$

and the law of motion of the co-state variable is given by

$$\frac{\mathrm{d}\psi(t;\tau)}{\mathrm{d}t} = -\frac{\partial \mathcal{H}}{\partial a(t;\tau)} = -r\psi(t;\tau) \tag{2.19}$$

The solution to the differential equation (2.19) is given by

$$\psi(t;\tau) = \psi(\tau;\tau)\exp\{-r(t-\tau)\} \tag{2.20}$$

Using (2.20) in (2.18) yields

$$c(t;\tau) = \left[\frac{\psi(\tau;\tau)}{\eta Q(t-\tau) l(t;\tau)^{(1-\eta)(1-\sigma)}}\right]^{\frac{1}{\eta(1-\sigma)-1}} \exp\{g^c(t-\tau)\} \tag{2.21}$$

where $g^c = \frac{r-\rho}{1+\eta(\sigma-1)}$. Also, note that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[a(t;\tau)\exp\{-r(t-\tau)\}\right] = \left[y(t;\tau)+B(t)-c(t;\tau)\right]\exp\{-r(t-\tau)\} \tag{2.22}$$

[2]See Kamien and Schwartz (1992) for a detailed discussion of the Pontryagin Maximum Principle for fixed-endpoint optimal control problems.

Integrating both sides of (2.22) with respect to t yields

$$a(t;\tau)\mathrm{exp}\left\{-r(t-\tau)\right\} = \int^{t} \left[y(s;\tau) + B(s) - c(s;\tau)\right] \exp\left\{-r(s-\tau)\right\} \,\mathrm{d}s + C_1 \tag{2.23}$$

where C_1 is a constant of integration. Finally, using the boundary conditions (2.7) in (2.23) yields

$$\int_{\tau}^{\tau+\bar{T}} \left[y(t;\tau) + B(t) - c(t;\tau)\right] \exp\left\{-r(t-\tau)\right\} \,\mathrm{d}t = 0 \tag{2.24}$$

which is simply the condition that for the life-cycle budget constraint to be satisfied, the present value of consumption should equal the present value of income. Using (2.21) in (2.24) and simplifying, the boundary value $\psi(\tau;\tau)$ can be pinned down as

$$\psi(\tau;\tau) = \left[\frac{\int_{\tau}^{\tau+\bar{T}} \exp\left\{-r(t-\tau)\right\}\left(y(t;\tau) + B(t)\right)\,\mathrm{d}t}{\int_{\tau}^{\tau+\bar{T}} \exp\left\{(g^c - r)(t-\tau)\right\}\left(\eta Q(t-\tau) l(t;\tau)^{(1-\eta)(1-\sigma)}\right)^{\frac{1}{1+\eta(\sigma-1)}}\,\mathrm{d}t}\right]^{\eta(1-\sigma)-1} \tag{2.25}$$

As characterized in equation (2.6), for a given retirement age T the household's labor supply over the life-cycle is given by

$$l(t;\tau) = \begin{cases} l_W & t \in [\tau, \tau+T] \\ 1 & t \in \left(\tau+T, \tau+\bar{T}\right] \end{cases} \tag{2.26}$$

Therefore, using (2.26) in (2.25), and then using (2.25) in (2.21) yields the optimal consumption plan for a given retirement age T. This solves the first-step of the household's decision problem.

To solve the second-step of the household's decision problem (i.e. finding the optimal retirement age), I use the optimal consumption plan derived in the first step in the life-cycle utility function in equation (2.3), and then solve for the retirement age for which the value of life-cycle utility is maximized. However, because of the complex nature of the survival probability function (which is usually estimated using a sixth or seventh order polynomial), equation (2.25) and the value of life-cycle utility for a given retirement age do not have analytic closed form solutions. Therefore, I compute them numerically by approximating the integrals using the trapezoidal method, and then I search over a grid for the optimal retirement age. The detailed computational procedures are discussed in Appendix 2.8.

2.4 Baseline calibration

I parameterize the baseline stationary competitive equilibrium of the model using empirical evidence from various sources. A population growth rate of $n = 1\%$ is consistent with the U.S. demographic history, and I set the rate of technological progress to $g = 1.56\%$, which is the trend growth rate of per-capita income in the postwar U.S. economy (Bullard and Feigenbaum, 2007). I assume that households enter the model at actual age 25, which corresponds to the model age of zero. I obtain the survival probabilities from Feigenbaum's (2008) sextic fit to the mortality data in Arias (2004), which is given by

$$\begin{aligned} \ln Q(s) \quad = \quad & -0.01943039 + \left(-3.055 \times 10^{-4}\right) s + \left(5.998 \times 10^{-6}\right) s^2 \\ & + \left(-3.279 \times 10^{-6}\right) s^3 + \left(-3.055 \times 10^{-8}\right) s^4 + \left(3.188 \times 10^{-9}\right) s^5 \\ & + \left(-5.199 \times 10^{-11}\right) s^6 \end{aligned} \tag{2.27}$$

where s represents household age. The 2001 U.S. Life Tables in Arias (2004) are reported up to actual age 100, so I set the maximum model age to $\bar{T} = 75$. Also, I set the model benefit eligibility age to $T_{\mathrm{r}} = 41$, which corresponds to the current full retirement eligibility age of 66 (for those born between 1943 and 1954) in the U.S. According to the 2001 Current Population Survey (CPS), production workers in the U.S. on an average work for 34 hours per week. Accounting for 8 hours per day as sleep time, this roughly implies that 30.36% of the total weekly time endowment is spent in market work. Using this information, I set the leisure share of total time endowment during work life to $l_W = 0.6964$. I set the social security tax rate to $\theta = 10.6\%$, which is equal to the current combined OASI tax rate in the U.S.[3] As the household's age-dependent efficiency endowment is difficult to observe, I

[3] I abstract from the disability component as it is not used to finance retirement benefits.

use normalized average cross-sectional hourly income data from the 2001 CPS as a proxy for efficiency. The Bureau of Labor statistics (BLS) reports the average hourly earnings of production workers in the discrete age intervals 25-34, 35-44, 45-54, 55-64, and 65 and above. To use this data, I first use piecewise linear interpolation to obtain average hourly earnings for all ages between 25-65, and then normalize the data such that efficiency at actual age 25 is unity. As is well known in the literature, using hourly income to proxy for efficiency can be problematic at higher ages because of the associated sample selection effects. To account for this, I assume that age 65 onwards, efficiency declines at the same rate as it declines before age 65. Finally, I fit a quartic polynomial to the interpolated data, which gives

$$\begin{aligned} \ln e(s) \;=\; & -0.02829952 + (0.02410291)\, s + \left(-6.831 \times 10^{-4}\right) s^2 \\ & + \left(-2.2205 \times 10^{-6}\right) s^3 + \left(5.702 \times 10^{-8}\right) s^4 \end{aligned} \tag{2.28}$$

where s represents household age. The efficiency data from the 2001 CPS are plotted along with the fitted polynomial in Figure 2.1.

The historically observed value of capital's share in total income in U.S. ranges between 30-40%, so I set $\alpha = 0.35$. Finally, I set $t = 0$ in all the computations and normalize the initial stock of technology and the population to $A(0) = N(0) = 1$.

Once all the observable parameters have been assigned empirically reasonable values, I calibrate the unobservable preference parameters ρ (discount rate), σ (IEIS) and η (share of consumption in period utility) such that the model jointly matches a steady state capital-output ratio of 2.5, an actual retirement age of 64, and a ratio of consumption to income of 75%.[4] Note that with these targets, the depreciation rate that is consistent with the model's steady state is $\delta = 0.0744$.

The values for the unobservable parameters for which the model matches the data targets are $\rho = 0.016$, $\sigma = 3.2$ and $\eta = 0.332$. Under these parameter values, the model generates

[4] The capital-output ratio and the consumption-income ratio targets are consistent with the larger macroeconomic literature, and the retirement age target is also consistent with the U.S. (Gendell, 1999, 2001).

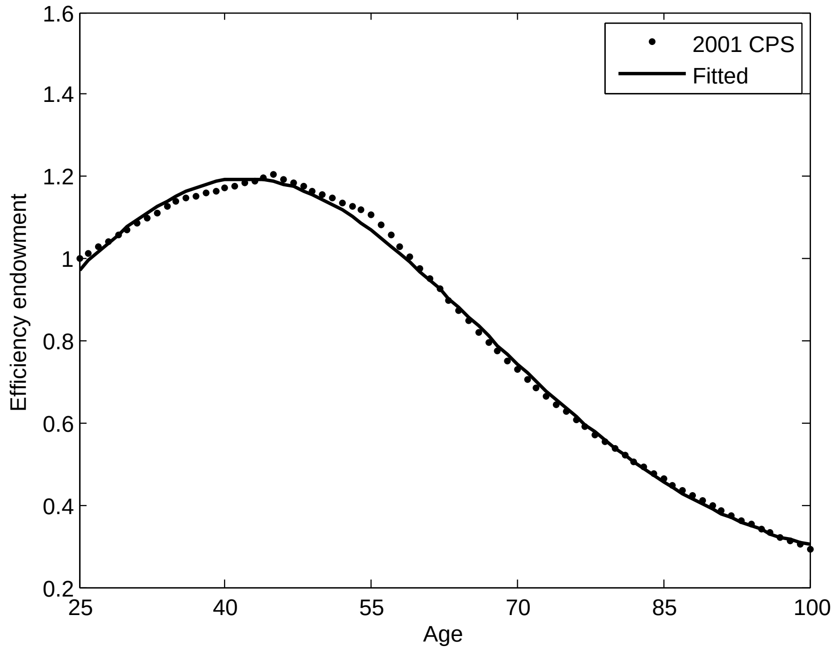

Figure 2.1: Efficiency data from the 2001 CPS along with the fitted quartic polynomial.

a steady state capital-output ratio of $K/Y = 2.495$, an optimal (actual) retirement age of 64.4, and a consumption to income ratio of $C/Y = 75.1\%$. The equilibrium rate of return from the capital stock is $r = 0.0659$, and the accidental bequest is $B = 0.01$.

2.5 The impact of population aging

To understand the quantitative importance of accounting for the household-level and macroeconomic adjustments to population aging, the first step is to construct a future demographic projection that generates a decline in the projected retirement benefits that is consistent with the commonly reported estimates of the social security crisis. More specifically, I construct a demographic scenario such that the decline in model benefits (without the household-level and the macroeconomic adjustments) matches the upper-bound estimate of a 33% decline in benefits in the real world. To accomplish this, I first set the population growth rate to $n = 0.47\%$, and then I change the survival probability distribution of the model such that holding household behavior, factor prices, and the social security tax rate constant at the baseline level, the model predicts a 33% decline in the projected retirement benefits.[5] Specifically, I augment the model survival probabilities with an age-specific increment of the form

$$\mathrm{d}\, Q(s) = \gamma s^{\mu} \tag{2.29}$$

where γ and μ are positive constants to be parameterized, and s represents household age. Therefore, the projected survival probabilities are given by

$$Q_p(s) = Q(s) + \gamma s^{\mu} \tag{2.30}$$

Note that these age-specific increments are consistent with the fact that old-age survivorship in the U.S. has increased at a faster rate than young-age survivorship in the later half of the twentieth century, making the population survival curve more rectangular (Arias, 2004).

With household behavior, factor prices, and the social security tax rate held fixed at the baseline level, parameter values of $\gamma = 0.0001$ and $\mu = 1.553$ lead to a 32.48% decline in the projected retirement benefits. With these values, the model life expectancy increases from 51.31 in the baseline calibration to 53.7 under the future projections, which implies an increase in the actual life expectancy from age 76.31 to age 78.7. It is worth noting that these values are very close to the average life expectancies for the years 2001 and 2075 respectively reported in the 2009 OASDI Trustees Report. The baseline and the projected survival probabilities are plotted together in Figure 2.2.

[5]Note that studies published by the SSA identify smaller declines required in the projected retirement benefits. For example, Goss (2006) reports that with no change in the current social security laws, about 70% of currently scheduled OASI benefits would be payable in 2080 (implying a decline of 30%). In the 2009 Social Security Trustees Report, the long-range actuarial estimates predict that tax revenues under the current laws are sufficient to support expenditures at a level of 74% of scheduled benefits in 2083 (implying a decline of 26%). My results (accounting for the household-level and macroeconomic adjustments to population aging) continue to be quantitatively important when I consider an alternative demographic projection under which the baseline decline in model benefits (without the household-level and macroeconomic adjustments) matches the lower-bound estimate of a 25% decline. This suggests that constructing future demographic projections that are consistent with smaller alternative estimates of the social security crisis simply amounts to a re-scaling of the problem. Therefore, for the remaining part of the discussion I simply focus on the quantitative results generated assuming a baseline decline of 33% in model benefits.

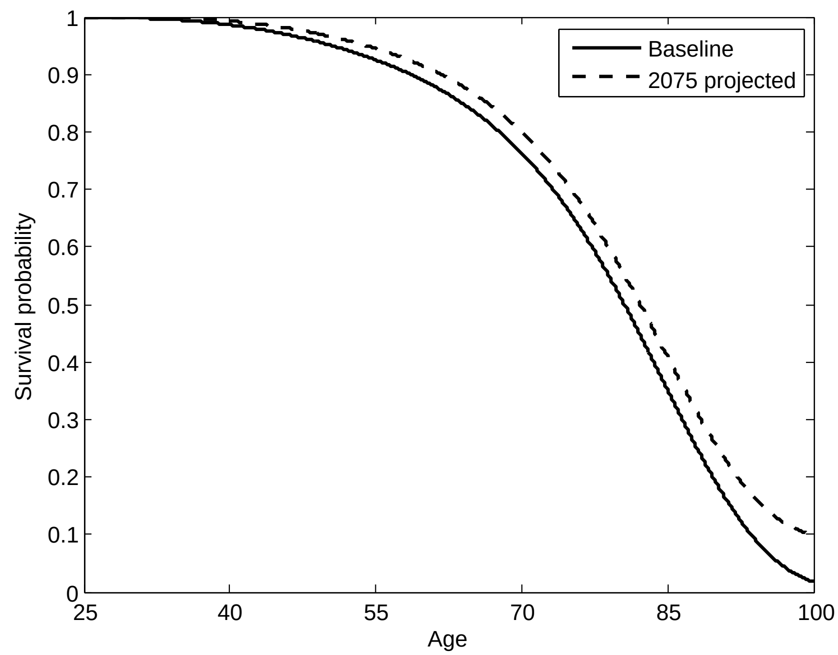

Figure 2.2: Baseline and the projected survival probabilities.

It is also useful to note that with the above demographic projection, holding household behavior and factor prices fixed at the baseline, a payroll tax rate of $\theta = 15.7\%$ restores the projected retirement benefits to their baseline level. This value is consistent with what Feldstein (2005) reports to be necessary to prevent a decline in projected retirement benefits in the year 2075.

The next step of the analysis is to compute the implied decline in the projected retirement benefits when the household-level and macroeconomic adjustments to population aging have been accounted for. To do this, I incorporate the constructed demographic projection in the baseline model and then compute a new stationary competitive equilibrium in which both household consumption-saving and retirement, as well as the factor prices respond to the demographic change. Also, to understand the relative importance of the household retirement and the factor price adjustment mechanisms, I compute projected retirement benefits under two situations: holding retirement fixed at the baseline level, but allowing the consumption-saving and factor price adjustment mechanisms to respond to population aging, and holding the factor prices fixed at the baseline level, but allowing the household retirement mechanism to respond to population aging. I label the four cases as follows:

- Case 1: exogenous retirement, exogenous factor prices,
- Case 2: exogenous retirement, endogenous factor prices,
- Case 3: endogenous retirement, exogenous factor prices, and
- Case 4: endogenous retirement, endogenous factor prices.

The percentage decline in the projected retirement benefit at actual age 70 from the baseline level for all the four cases are reported in Table 2.1. The following facts are clear from the table. First, the impact of population aging on the projected retirement benefits is smallest when both the household-level consumption-saving and retirement responses, as well as the aggregate factor price adjustments are fully accounted for (Case 4). The percentage decline in the projected benefits from the baseline level in this case is roughly 20%, which implies that ignoring these adjustments could lead to an overestimation of the crisis by about 65% ($33/20 = 1.65$). The two intermediate cases also provide some interesting insights into how ignoring either of the two mechanisms leads to an overestimation of the decline in the projected retirement benefits. On the one hand, holding retirement fixed at the baseline level but allowing for the consumption-saving and factor price adjustment mechanisms (Case 2), population aging leads to a roughly 22% decline in the projected benefits. On the other hand, allowing for the household-level retirement response but holding the factor prices

Table 2.1: The impact of population aging on the projected retirement benefits.

	Decline from the baseline
Case 1	32.48%
Case 2	22.23%
Case 3	24.64%
Case 4	19.87%

Table 2.2: Effect of population aging on household behavior and factor prices.

	Baseline	Case 1	Case 2	Case 3	Case 4
Capital stock	42.69	-	52.44	-	52.84
Retirement age (actual)	64.4	-	-	64.6	67.12
Labor supply	10.46	-	11.64	-	12.14
Rate of return	0.0659	-	0.0571	-	0.0601
Wage rate	1.06	-	1.1	-	1.09

fixed at the baseline (Case 3) causes the projected benefits to decline by about 25%.[6]

What is the intuition behind the smallest decline in the projected retirement benefits under Case 4? Table 2.2 shows that when the private annuity markets are incomplete, households respond to a higher life expectancy by saving more, which leads to a higher aggregate capital stock, and also by delaying retirement by almost three years, which increases the effective labor supply (Case 4). However, a larger-than-proportional increase in the capital stock leads to a lower rate of return and a higher wage rate than the baseline, which along with the delayed retirement implies a strictly larger level of aggregate taxable income. This increase in the tax base of the social security program is behind the smallest decline in the projected retirement benefits under Case 4.[7] Experimenting with different social security tax rates also shows that given this decline, increasing the tax rate from 10.6% to 13.8% restores the projected retirement benefits to their baseline level. This increase is significantly smaller than Feldstein's (2005) projection of 15.7%.

2.6 Sensitivity analysis

Given that the extent of the social security crisis identified in the current model is conditional on the set of parameter values used in the baseline calibration, it is important

[6]With a future demographic projection that generates a roughly 25% decline in the baseline benefits holding retirement and the factor prices fixed (Case 1), the respective declines under cases 2, 3 and 4 are 16%, 17% and 15%. Therefore, in this case, the crisis would be overestimated by as much as 67% $(25/15 = 1.67)$.

[7]Note from Table 2.2 that when household retirement is held fixed at the baseline but the consumption-saving and the factor price adjustment mechanisms are allowed (Case 2), the increase in the labor supply documented in the third row is purely due to a higher life expectancy. Also, holding the factor prices fixed at the baseline but allowing for the household retirement mechanism (Case 3) leads to a slight delay in retirement that is documented in the second row of Table 2.2.

to verify whether the quantitative predictions of the model are sensitive to them or not. To do this, in this section I estimate the impact of population aging on the projected retirement benefits under different values of some key model parameters.

Three parameters that are treated as observable in the baseline calibration of the model are capital's share in total income (α), leisure share of total time endowment (l_W) and the age-dependent household efficiency endowment ($e(s)$). The value of capital's share in total income is set to $\alpha = 0.35$ in the initial baseline calibration, but the macroeconomic estimates historically observed in the U.S. range between $30 - 40\%$. Therefore, to verify the sensitivity of the simulation results with respect to α, I first compute new calibrated baseline equilibria of the model under $\alpha = 0.3$ and $\alpha = 0.4$, and then study the impact of population aging on the projected retirement benefits. The unknown preference parameters and target values for the baseline calibrations with $\alpha = 0.3$, 0.35 and 0.4 are reported in Table 2.3. It is clear from the table that calibrated baseline equilibria of the model under $\alpha = 0.3$ and $\alpha = 0.4$ provide reasonable fits to the data targets, with the values for the unknown parameters falling conveniently in the range used in the larger macro-calibration literature.

Once the baseline equilibria under the different α-values have been identified, the next step is to determine the values of the parameters γ and μ, which control the extent by which the baseline retirement benefits decline in response to population aging. As in the initial

Table 2.3: Calibrated baseline equilibria under different values of capital's share in total income.

	$\alpha = 0.3$	$\alpha = 0.35$	$\alpha = 0.4$
Discount rate (ρ)	-0.003	0.016	0.032
IEIS (σ)	3.4	3.2	3.1
Share of consumption in period utility (η)	0.308	0.332	0.357
Optimal (actual) retirement age	64.2	64.4	64.36
Capital-output ratio	2.508	2.495	2.499
Consumption-income ratio	0.749	0.751	0.75
Rate of return	0.0452	0.0659	0.0857
Accidental bequest	0.0066	0.016	0.014

Table 2.4: Values of the population aging parameters under different values of capital's share in total income.

	$\alpha = 0.3$	$\alpha = 0.4$
γ	0.0001	0.0001
μ	1.554	1.553
Actual life expectancy	78.72	78.7
Decline in projected benefits	32.51%	32.51%

baseline, I augment the model survival probabilities by choosing γ and μ such that holding household behavior, factor prices and the social security tax rate fixed, the model predicts a 33% decline in the projected retirement benefits. In Table 2.4, I report the values of these two parameters under $\alpha = 0.3$ and 0.4, and I also report the actual life expectancies associated with them.[8] It is worth noting from the table that with the reported γ- and μ-values, the actual life expectancies of the model households are very close to the SSA's projections for the year 2075. Also, as in the initial baseline, holding household behavior and factor prices fixed, a social security tax rate of $\theta = 15.7\%$ restores the post-population aging retirement benefits to the respective baseline levels.

To identify the quantitative importance of the household-level and macroeconomic adjustments to population aging under different values of capital's share in total income, I compute new stationary competitive equilibria in which both household consumption-saving and retirement, as well as the factor prices respond to the demographic change (Case 4). I also compute the two intermediate cases: holding retirement fixed at the baseline but allowing the consumption-saving and the factor price mechanisms to respond (Case 2), and holding the factor prices fixed at the baseline level but allowing the household retirement mechanism to respond (Case 3). I report the percentage declines in the projected retirement benefits from the respective baselines in Table 2.5. The table reveals a pattern very similar to the one observed under $\alpha = 0.35$: the smallest decline in the projected benefits occurs under Case 4, when all the household-level and macroeconomic adjustments to population aging are accounted for. Moreover, the magnitude of the decline in the projected benefits

[8] As in the initial baseline, I set the population growth rate to $n = 0.47\%$ under both $\alpha = 0.3$ and 0.4.

Table 2.5: The impact of population aging on the projected retirement benefits under different values of capital's share in total income.

	$\alpha = 0.3$	$\alpha = 0.4$
Case 1	32.51%	32.51%
Case 2	22.23%	22.19%
Case 3	23.79%	25.09%
Case 4	19.61%	19.91%

under Case 4 also appears fairly robust: the model predicts declines of roughly 20% from the respective baselines, compared to the commonly reported estimate of 33%. The relative importance of the household retirement and the aggregate factor price adjustment mechanisms also appears to be fairly robust with respect to the value of capital's share in total income used in the simulations: ignoring either of the two mechanisms leads to a roughly comparable decline in the projected retirement benefits under both $\alpha = 0.3$ and 0.4.

Since the model's predictions about the declines in the projected retirement benefits under different values of capital's share in total income are fairly similar, it is natural to expect that the underlying household-level and macroeconomic adjustments to population aging are also similar. To verify this, I report in Table 2.6 the effect of the demographic change on household behavior and factor prices under both $\alpha = 0.3$ and 0.4. It is clear from the table that both the household-level responses and the aggregate factor price adjustments to population aging are fairly robust to the underlying value of capital's share in total income. Under both $\alpha = 0.3$ and 0.4, households respond to a higher life expectancy by increasing their private saving (and therefore aggregate capital and the wage rate) and also by delaying retirement, which leads to an increase in the tax base of the social security program.

It is also useful to note that accounting for both the household-level and macroeconomic adjustments to population aging, the tax increases required to restore projected retirement benefits to their respective baseline levels are also very similar for different α-values. For $\alpha = 0.3$, increasing the tax rate to 13.7% restores projected benefits to their baseline level under Case 4, and for $\alpha = 0.4$ a tax rate of 13.8% achieves this.

Table 2.6: Effect of population aging on household behavior and factor prices under different values of capital's share in total income.

(a) Effect of population aging with $\alpha = 0.3$.

	Baseline	Case 1	Case 2	Case 3	Case 4
Capital stock	38.78	-	48.51	-	48.79
Retirement age (actual)	64.2	-	-	65.12	67.12
Labor supply	10.43	-	11.6	-	12.13
Rate of return	0.0452	-	0.0358	-	0.0388
Wage rate	1.04	-	1.08	-	1.06

(b) Effect of population aging with $\alpha = 0.4$.

	Baseline	Case 1	Case 2	Case 3	Case 4
Capital stock	48.11	-	58.47	-	59.15
Retirement age (actual)	64.36	-	-	64.2	67
Labor supply	10.45	-	11.63	-	12.11
Rate of return	0.0857	-	0.0774	-	0.0801
Wage rate	1.11	-	1.15	-	1.13

The next step is to verify the sensitivity of the simulation results with respect to the leisure share of total time endowment parameter (l_W). According to 2001 CPS data on hours worked by production workers in the U.S., 34 hours per week on an average were spent on market work. Accounting for 8 hours per day as sleep time, this roughly implies that 30.36% of the total weekly time endowment is spent on market work. However, to account for vacations, holidays, weekends and sick days, the value of the leisure share parameter needs to be revised downward. Therefore, to verify the sensitivity of the simulation results with respect to the leisure share of total time endowment parameter (l_W), I compute new calibrated baseline equilibria with the values $l_W = 0.575$ (based on a five-day working week) and $l_W = 0.6458$ (based on a six-day working week), and then re-compute the impact of population aging on the projected retirement benefits. The unknown preference parameters and the associated model-generated target values for the calibrated baseline equilibria with $l_W = 0.575$, 0.6458 and 0.6964 are reported in Table 2.7. It is clear from the table that calibrated baseline equilibria of the model under $l_W = 0.575$ and $l_W = 0.6458$ provide reasonable fits to the data targets, with the values for the unknown parameters falling

Table 2.7: Calibrated baseline equilibria under different values of leisure share in total time endowment.

	$l_W = 0.575$	$l_W = 0.6458$	$l_W = 0.6964$
Discount rate (ρ)	0.012	0.014	0.016
IEIS (σ)	3	3.1	3.2
Share of consumption in period utility (η)	0.434	0.372	0.332
Optimal (actual) retirement age	64.72	64.2	64.4
Capital-output ratio	2.493	2.506	2.495
Consumption-income ratio	0.751	0.749	0.751
Rate of return	0.066	0.0653	0.0659
Accidental bequest	0.0148	0.0118	0.01

Table 2.8: Values of the population aging parameters under different values of leisure share in total time endowment.

	$l_W = 0.575$	$l_W = 0.6458$
γ	0.0001	0.0001
μ	1.554	1.553
Actual life expectancy	78.72	78.7
Decline in projected benefits	32.49%	32.49%

conveniently in the range used in the larger macro-calibration literature.

The values of the population aging parameters γ and μ for which, holding household behavior, factor prices and the social security tax rate fixed at the baseline, the model predicts a 33% decline in the projected retirement benefits under $l_W = 0.575$ and 0.6458 are reported in Table 2.8. Note that the table also reports the actual life expectancies under these parameter values.[9] As before, the actual life expectancies of the model households with these γ- and μ-values are in reasonable agreement with the SSA's projections for the year 2075. Also, with these values, a social security tax rate of $\theta = 15.7\%$ restores the post-population aging retirement benefits to the respective baseline levels when household behavior and factor prices are held fixed.

In Table 2.9, I report the percentage decline in the projected retirement benefits in new stationary competitive equilibria where both household consumption-saving and retirement, as well as the factor prices respond to the demographic change (Case 4). Similar to the

[9] The population growth rate is set to $n = 0.47\%$ under both $l_W = 0.575$ and 0.6458.

Table 2.9: The impact of population aging on the projected retirement benefits under different values of leisure share in total time endowment.

	$l_W = 0.575$	$l_W = 0.6458$
Case 1	32.51%	32.51%
Case 2	22.23%	22.19%
Case 3	23.79%	25.09%
Case 4	19.61%	19.91%

earlier analysis, I also report the two intermediate cases (Cases 2 and 3) under the different l_W-values. The table reveals a pattern very similar to the one observed in the initial baseline. The smallest decline in the projected benefits occurs under Case 4, and the magnitude of the decline is also very similar. Moreover, the declines in benefits are roughly comparable when either of the two mechanisms is not accounted for.

To examine how the demographic change affects household behavior and factor prices under different values of the leisure share parameter, I report in Table 2.10 the values of the total capital stock, retirement age, labor supply and the factor prices in the post-demographic change steady states under both $l_W = 0.575$ and 0.6458. The table shows that

Table 2.10: Effect of population aging on household behavior and factor prices under different values of leisure share of total time endowment.

(a) Effect of population aging with $l_W = 0.575$.

	Baseline	Case 1	Case 2	Case 3	Case 4
Capital stock	59.96	-	74.17	-	74.67
Retirement age (actual)	64.72	-	-	64.96	67.52
Labor supply	14.71	-	16.38	-	17.08
Rate of return	0.066	-	0.0567	-	0.0597
Wage rate	1.06	-	1.1	-	1.09

(b) Effect of population aging with $l_W = 0.6458$.

	Baseline	Case 1	Case 2	Case 3	Case 4
Capital stock	50	-	61.61	-	62.09
Retirement age (actual)	64.2	-	-	64.44	66.96
Labor supply	12.17	-	13.53	-	14.12
Rate of return	0.0653	-	0.0563	-	0.0593
Wage rate	1.07	-	1.1	-	1.09

the household-level responses and the aggregate factor price adjustments to population aging are not sensitive to the underlying l_W-value used in the simulations. Households respond to population aging by increasing their private saving (and therefore aggregate capital and the wage rate) and also by delaying retirement, which leads to an increase in the tax base of the social security program under both $l_W = 0.575$ and 0.6458. Moreover, similar to the experiments with different values of capital's share in total income, the social security tax rate required to restore projected retirement benefits to their respective baseline levels under Case 4 is roughly 13.8% for both $l_W = 0.575$ and 0.6458.

The coefficients of the age-dependent household efficiency profile ($e(s)$) are also treated as observable in the initial baseline calibration. Given that it is difficult to observe efficiency, the coefficients in (2.28) were estimated from the normalized average cross-sectional hourly wages data from the 2001 CPS. However, the age-dependent component of household efficiency used in several other studies on social security and population aging (De Nardi et al., 1999; Conesa and Garriga, 2008a,b) are estimated from Hansen (1993). Therefore, the next step is to verify the sensitivity of the simulation results using the age-dependent household efficiency data from Hansen (1993). The efficiency units in Hansen (1993) are constructed by taking a weighted sum of the hours worked by each age-sex subgroup using annual data from 1979 to 1987, where the weights reflect the relative productivity of that subgroup. To use this data, I first calculate the average of male and female weights for each age group, and then use piecewise linear interpolation to obtain the weights for all ages between 25 and 65. To control for the sample selection effects beyond age 65, I assume that age 65 onwards, efficiency declines at the same rate as it decline before age 65. Finally, I fit the a quartic polynomial to the interpolated data, which gives

$$\begin{aligned} \ln e(s) &= 0.00271799 + (0.01490187)\, s - \left(1.688 \times 10^{-4}\right) s^2 \\ &\quad - \left(1.4399 \times 10^{-5}\right) s^3 + \left(1.365 \times 10^{-7}\right) s^4 \end{aligned} \tag{2.31}$$

where s is household age. The age-dependent efficiency profiles estimated from the inter-

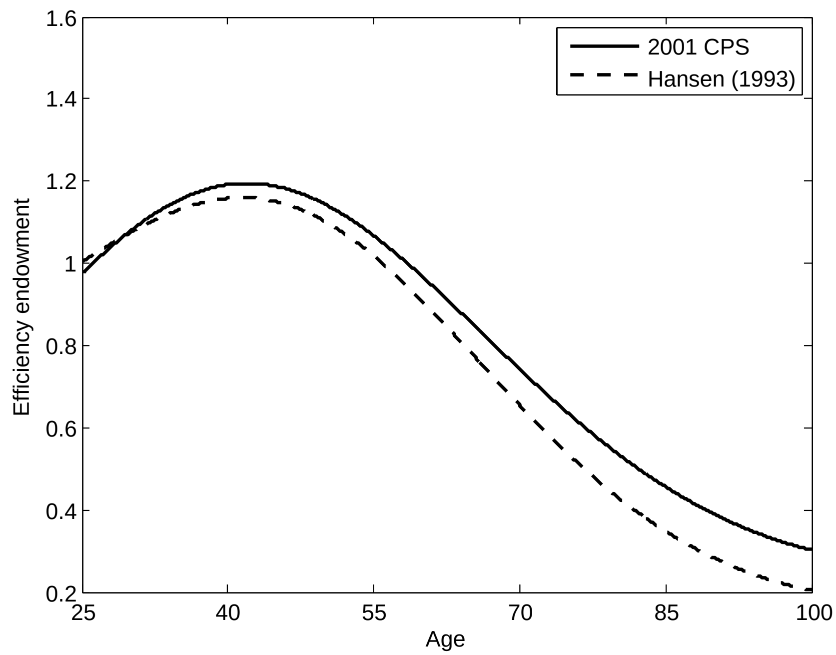

Figure 2.3: The age-dependent household efficiency profiles estimated from the 2001 CPS and Hansen (1993).

polated efficiency data in the 2001 CPS and Hansen (1993) are reported in Figure 2.3.

Next, I compute a new calibrated baseline equilibrium using the efficiency profile from Hansen (1993) and then re-compute the impact of population aging on the projected retirement benefits. The unknown preference parameters and the associated model-generated target values for the calibrated baseline equilibria with the efficiency profile from Hansen (1993) are reported in Table 2.11. The table demonstrates that the calibrated baseline equilibrium of the model with the efficiency profile from Hansen (1993) is very similar to the one in the initial baseline calibration; the values for the unknown parameters are also consistent with the larger macro-calibration literature.

Holding household behavior, factor prices and the social security tax rate fixed at the baseline with the efficiency profile from Hansen (1993), the population aging parameters for which the model predicts a roughly 33% decline in the projected retirement benefits are $\gamma = 0.0001$ and $\mu = 1.553$. With these values, benefits decline by 32.5% and the actual life

Table 2.11: Calibrated baseline equilibria under the different efficiency profiles.

	2001 CPS	Hansen (1993)
Discount rate (ρ)	0.016	0.017
IEIS (σ)	3.2	3.2
Share of consumption in period utility (η)	0.332	0.339
Optimal (actual) retirement age	64.4	64.16
Capital-output ratio	2.495	2.504
Consumption-income ratio	0.751	0.75
Rate of return	0.0659	0.0654
Accidental bequest	0.01	0.0092

Table 2.12: The impact of population aging on the projected retirement benefits under the efficiency profile from Hansen (1993).

	Decline from the baseline
Case 1	32.5%
Case 2	22.38%
Case 3	24.85%
Case 4	20.31%

expectancy predicted by the model increases to 78.7, which is identical to the value in the initial baseline calibration. Also, the benefit-preserving tax rate with household behavior and factor prices fixed at the baseline level is 15.7%.[10]

The percentage decline in the projected retirement benefits under Cases 1, 2, 3 and 4 with the efficiency profile from Hansen (1993) are reported in Table 2.12. The table shows a pattern very similar to the earlier experiments. The smallest decline in the projected benefits occurs under Case 4, and the magnitude of the decline is roughly 20%, compared to the commonly reported estimate of 33%. Moreover, the relative importance of the household retirement and the factor price adjustment mechanisms are fairly similar: the declines in benefits under Cases 2 and 3 are 22.38 and 24.85% respectively.

The effect of population aging on household behavior and factor prices under the efficiency profile from Hansen (1993) is reported in Table 2.13. It is clear from the table that the household-level responses and the aggregate factor price adjustments to popula-

[10] As in the other experiments, I set the population growth rate under the future demographics to $n = 0.47\%$.

Table 2.13: Effect of population aging on household behavior and factor prices under the efficiency profile from Hansen (1993).

	Baseline	Case 1	Case 2	Case 3	Case 4
Capital stock	41.63	-	51.08	-	51.43
Retirement age (actual)	64.16	-	-	64.32	66.64
Labor supply	10.14	-	11.26	-	11.68
Rate of return	0.0654	-	0.0566	-	0.0591
Wage rate	1.07	-	1.1	-	1.09

tion aging are fairly robust with respect to the coefficients of the age-dependent efficiency profile: households respond to population aging by saving more and working longer, which has the effect of increasing aggregate taxable income. Also, similar to the experiments with different values of capital's share in total income, a social security tax rate of roughly 13.8% restores projected retirement benefits to the baseline level when both the household-level and the aggregate factor price adjustments are accounted for.

Sensitivity analysis with respect to the value of capital's share in total income, leisure share in total time endowment and the parameters in the age-dependent household efficiency profile demonstrates that the quantitative predictions of the model are fairly robust. Under all the parameterizations, the model predicts a significantly lower decline in the projected retirement benefits once the household-level and aggregate factor price adjustments to population aging are accounted for. The natural increase in the tax base associated with the demographic change allows the social security program to remain solvent with a relatively smaller impact on the retirement benefits. The only parameter not entertained in the above sensitivity analyses is the economic growth rate. However, re-calibrating the baseline model with a growth rate different from $g = 1.56\%$ would simply lead to slightly different values for the unobservable parameters. The above results show that the quantitative importance of the household-level and the macroeconomic adjustment to population aging are largely insensitive to the unobservable parameter values as long as the model is well-calibrated. Therefore, a different economic growth rate should have no material effect on the results.

One concern with the existing applied general equilibrium model could be that it abstracts from various other taxes that exist in the real world, such as taxes on labor and capital income. In a baseline equilibrium with nonzero labor and capital income taxes, the distortions in the consumption, saving and the retirement decisions at the household level will likely be larger. However, if these tax rates are stationary under population aging, then the distortions are also likely to be largely unchanged in a relative sense. Therefore, the simulation results are not likely to be sensitive to the fact that the current model abstracts from these taxes.

2.7 Conclusions

In this paper, I provide an alternative estimate of the decline in the projected social security benefits under population aging in the U.S. that accounts for the household-level consumption-saving and retirement responses to population aging, as well as the aggregate factor price adjustments. I construct an applied general equilibrium model with endogenous retirement and incomplete annuity markets, calibrate it to some key features of the U.S. economy, and then examine the impact of an empirically consistent future demographic projection on the level of projected retirement benefits. I find that when both the household retirement and the factor price mechanisms are accounted for, the decline in the projected benefits is significantly smaller than what is commonly reported.

2.8 Appendix: Computational methods

In this section I provide a discussion of the computational methods used in the current research, including the development of the required computer codes. Both for its ability to handle complicated symbolic operations and providing a stable programming platform for developing suitable solver algorithms, I choose MATLAB™ version 7.4.0.287 as the main computational software, powered by an Intel Pentium™ T4200 Dual-Core CPU with 3 GB memory.

First, note that given the complex nature of the survival probability function $Q(t-\tau)$, the integrals identified in the various expressions in Section 2.3 do not have analytical closed form solutions. Therefore, I implement the trapezoidal method to approximate these integrals, which uses the idea that an integral is nothing but the limit of a sum. Specifically, I use the approximation

$$\int_a^b f(x)\,\mathrm{d}x \approx \left[0.5 \times \{f(a) + f(b)\} + \sum_{i=1}^{N-1} f(a + i\Delta)\right] \times \Delta \tag{2.32}$$

where $\Delta = (b - a)/N$, which implies that the domain $[a, b]$ has been divided into N equally spaced intervals.

As defined earlier, a stationary competitive equilibrium in the current framework is characterized by a cross-sectional consumption-saving profile and optimal retirement age, an aggregate capital stock, labor supply and a labor-to-retiree ratio, a real rate of return and wage rate, and an accidental bequest that solve the household's optimization problem, satisfy the aggregation conditions, equilibrate the factor markets, balance the social security budget and also satisfy the bequest-balance condition. Therefore, computation of a stationary competitive equilibrium can be broken down into the following two steps:

1. Solve for the household optimum for a given set of factor prices, a given labor-to-retiree ratio, a given accidental bequest and given values for the other model parameters.

2. Using the aggregation conditions, find the factor prices, the labor-to-retiree ratio and the accidental bequest consistent with the household optimum. The general equilibrium is obtained when the factor prices, the labor-to-retiree ratio and the accidental bequest values computed from the household optimum match the given factor prices, the labor-to-retiree ratio and the accidental bequest values that were used to compute the household optimum.

To sequentially accomplish these two steps, I define a contraction mapping algorithm as follows:

- **Step 1:** Set the model parameters to some values and guess some values for the factor prices, the labor-to-retiree ratio and the accidental bequest (label as vector `xin`).

- **Step 2:** Create a grid of retirement ages, compute the optimal consumption plans at each point on the grid using the trapezoidal approximations to the integrals, and then find the age at which the value of discounted life-cycle utility is maximized for the given `xin` vector and the parameter values.

- **Step 3:** Compute the aggregate capital stock, labor supply, the labor-to-retiree ratio and the accidental bequest that are consistent with the optimal retirement ages obtained in **Step 2** using the trapezoidal approximations to the integrals.

- **Step 4:** Compute the market-clearing factor prices consistent with the aggregate capital stock and labor supply obtained in **Step 3**, and store the factor prices, the labor-to-retiree ratio and the accidental bequest in a vector `xout`.

- **Step 5:** Compute the percentage difference between vectors `xin` and `xout`, and store it in a vector `diff`.

- **Step 6:** If the 2-norm of the vector `diff` is greater than some tolerance parameter `Tol`, then update `xin` using the rule `xin = xin.*(1+step)`, where `step` is given by `step = diff/9`, and repeat steps 2 through 5.[11,12]

- **Step 7:** If the 2-norm of the vector `diff` is lesser than some tolerance parameter `Tol`, then terminate the algorithm.

At the end of these steps, the program returns a vector of factor prices, the labor-to-retiree ratio and an accidental bequest that solves the households' optimization problem, clears the factor markets, balances the social security budget and satisfies the bequest-balance condition for a given set of values for the model parameters. Then, calibrating the model to data targets simply involves repeating these steps for different combinations of values for the unobservable parameters and choosing the one that produces model-generated values in reasonable neighborhood of the targets.

[11]For a $n \times 1$ vector $[x_1 x_2 \ldots x_n]'$, the 2-norm is defined as $\sqrt{\sum_{i=1}^{n} x_i^2}$.

[12]This implies that the algorithm updates the guessed vector by a factor that depends on the divergence between the guess and the feedback.

CHAPTER 3

OPTIMAL SOCIAL SECURITY REFORM UNDER POPULATION AGING IN THE U.S.

3.1 Introduction

All across the OECD, lower birth rates and higher life expectancies have threatened the viability of unfunded social security programs. In the U.S., actuaries of the Social Security Administration (SSA) estimate that in the year 2080, a roughly 25-30% decline in the projected retirement benefits is required to keep the current program solvent with the existing contribution rate (Goss, 2006). Also, Feldstein (2005) estimates that to prevent any decline in the projected benefits for the year 2075, the current Old-Age and Survivors Insurance (OASI) tax rate may have to be increased to 16.4%.

In this paper I ask what should be the optimal or welfare-maximizing OASI tax rate in the U.S. under the population developments projected in the future. There are at least two reasons why this question is not trivial. First, even though it is intuitive that a higher tax rate may be required to balance the social security budget under population aging, it is not clear what impact such a strategy would have on social welfare: a higher tax rate can potentially distort individual behavior and equilibrium factor prices in a way that reduces utility. Second, whether or not social security improves welfare depends on which missing market it substitutes. If individuals face uninsurable productivity realizations that occur prior to their entering the labor force, and if social security already insures them against this through a pro-poor retirement benefit rule, then it is not clear whether population aging would create the need for a larger program.[1]

[1] Note that I use the term "optimal" in a relatively narrow sense, as I only consider changes in the OASI tax rate, holding all the other institutional features of the program and its PAYG structure unchanged. For policy experiments along these other dimensions, see studies such as De Nardi et al. (1999), Conesa and Garriga (2008a) and Conesa and Garriga (2008b).

To examine this issue, I begin by constructing a heterogeneous-agent general equilibrium model of life-cycle consumption and labor supply, where the source of heterogeneity is a productivity or efficiency realization that occurs before the agents enter the model. In the model, an unfunded social security program provides partial insurance against an unfavorable efficiency realization by paying retirement benefits through a pro-poor rule. Agents in the model also face mortality risk, against which they cannot insure because of the absence of private annuity markets. I first calibrate the benefit rule to match the degree of redistribution in the U.S. social security program, and then I calibrate the distribution of efficiency such that the current OASI tax rate in the U.S. maximizes social welfare under the current demographics. Then, I introduce empirically reasonable future population projections into the calibrated model, and finally I search for the tax rates that maximize social welfare under those projections.

The baseline equilibrium of the model performs reasonably well in matching some key features of the current U.S. economy, such as the aggregate capital-output ratio, the average fraction of time spent on market work, and the gross benefit replacement rates in the population. Also, in the baseline equilibrium, social security is welfare-improving only for the households with relatively poor efficiency realizations. The relatively efficient households experience welfare losses from social security: their internal rates of return from the program are lower than the market rate of return on capital stock.

The main findings of this paper are as follows. First, I find that the optimal or welfare-maximizing OASI tax rate under the future population projections in the U.S. is about 2 to 5 percentage points higher than the current tax rate. Second, I also find that households with different efficiency realizations respond asymmetrically to the the demographic developments, and that a large part of the tax burden of population aging is picked up by the households with relatively favorable efficiency realizations. Finally, the model also predicts that population aging and the optimal tax response may imply a decline in the projected retirement benefits, but of a magnitude smaller than when the tax rate is held unchanged at the current level.

Among others, three important quantitative studies on social security reform under population aging in the U.S. are De Nardi et al. (1999), Conesa and Garriga (2008a) and Conesa and Garriga (2008b). The current paper complements these studies in two ways. First, I employ a heterogeneous-agent model to study optimal social security reform, where social security provides partial insurance against unfavorable efficiency realizations through a pro-poor retirement benefit rule. This allows me to replicate the degree of redistribution in the U.S. social security program, as the benefit replacement rate of a household in the U.S. is a concave function of work-life income.[2] Second, I only consider equilibria of the model in which the OASI tax rate maximizes social welfare. This allows me to run controlled demographic experiments in which the welfare-maximizing changes in the tax rate predicted by the model can be fully attributed to population aging. To accomplish this, I first calibrate the efficiency distribution in the baseline model such that the current OASI tax rate in the U.S. is exactly optimal, and then I introduce empirically reasonable future population projections. Note that controlling for the optimal program size under the current demographics is crucial, as failing to do so can potentially confound the optimal tax response to population aging.

Beginning with Feldstein (1985), a number of studies have attempted to justify the size of the current social security program in the U.S. on the grounds of its different welfare-improving roles.[3] However, studies in this area have typically focused on computing the optimal or welfare-maximizing social security tax rate under the current U.S. demographics.[4] The current paper also complements these studies, as I compute the optimal OASI tax rate under the future demographic projections in the U.S., conditional on the assumption that the current tax rate is exactly optimal.

The rest of the paper is organized as follows: Section 3.2 introduces the model, Sec-

[2]Huggett and Ventura (1999) investigate social security reform in the U.S. with a two-tier structure using a heterogeneous-agent model with the concave Primary Insurance Amount (PIA) benefits formula used by the SSA.

[3]See, for example studies such as İmrohoroğlu et al. (1995), İmrohoroğlu et al. (2003), Caliendo and Gahramanov (2009) and Findley and Caliendo (2009), among others.

[4]An exception is found in Findley and Caliendo (2009), who find in a robustness analysis that the average tax rate in the presence of short planning horizons under future demographics increases slightly from 11% in their baseline to about 12%.

tion 3.5 describes the baseline calibration, Section 3.6 examines the impact of empirically reasonable future population projections on the calibrated model, and Section 3.8 concludes.

3.2 The model

Consider a continuous time overlapping generations economy with life-cycle permanent income households, where at each instant a new cohort is born and the oldest cohort dies. Cohorts are identical in all respects but their date of birth, but within each cohort there is heterogeneity with respect to household efficiency. The maximum lifespan of a household is $\bar{T}$ years, and the life cycle consists of two phases: work and retirement. During the final $\bar{T} - T_{\mathrm{r}}$ years of life, the household receives social security benefits that are positively related to their work life income. Households face mortality risk against which they cannot insure because of closed private annuity markets, and they derive utility from consumption (c) as well as the fraction of total time endowment enjoyed in leisure (l). They also accumulate a risk-free asset: physical capital. The government runs an unfunded social security program that is financed through taxes on labor income, and the assets of the deceased households at each instant are uniformly distributed over the surviving population in the form of accidental bequests. Perfectly competitive firms produce output using a constant returns to scale Cobb-Douglas production function with constant labor-augmenting technological progress at rate g per annum, and there is no aggregate uncertainty. Finally, population grows at rate n per annum.

3.2.1 Preferences

The period utility function is

$$u(c,l) = \begin{cases} \frac{\left(c^{\eta} l^{1-\eta}\right)^{1-\sigma}}{1-\sigma} & \text{if} \quad \sigma \neq 1 \\ \ln\left(c^{\eta} l^{1-\eta}\right) & \text{if} \quad \sigma = 1 \end{cases} \tag{3.1}$$

where η is the share of consumption in period utility and σ is the Inverse Elasticity of Intertemporal Substitution (IEIS). Expected lifetime utility from the perspective of a household at age $s = 0$ is

$$U = \int_0^{\bar{T}} \exp\{-\rho s\} Q(s) \frac{\left\{c(s)^{\eta} l(s)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \mathrm{d}s \tag{3.2}$$

where ρ is the discount rate and $Q(s)$ is the unconditional probability of surviving to age s. Also, since I define leisure as a fraction of the total time endowment, we have $0 \leq l(s) \leq 1$.

3.2.2 Income

Each household in a given cohort is endowed with an efficiency endowment $\varphi e(s)$, where φ is a realization from a stationary distribution with density $f(\varphi)$ and support $\left[\underline{\varphi}, \overline{\varphi}\right]$ that occurs prior to birth, and $e(s)$ is an age-dependent component that increases early in life, peaks at about middle age, and then declines until death. Households' saving $a(s)$ earns a real rate of return r, and during the work life labor income is taxed at rate θ, which is the OASI tax rate. The tax receipts are used to pay social security benefits to households past the eligibility age of T_{r}. The surviving households also receive an accidental bequest $B(t)$ from the deceased households every period.

3.2.3 Social security

The government runs an unfunded social security program that partially insures households against unfavorable efficiency realizations through a pro-poor benefit rule. The benefit annuity at date t of a household with efficiency φ is $b(t;\varphi)$, which is calculated as follows:

$$b(t;\varphi) = \zeta(\varphi) b(t;\overline{\varphi}) \tag{3.3}$$

$$\zeta(\overline{\varphi}) = 1 \tag{3.4}$$

$$\zeta(\varphi) = \left[\frac{\zeta(\underline{\varphi})\overline{\varphi} - \underline{\varphi}}{\overline{\varphi} - \underline{\varphi}}\right] + \left[\frac{1 - \zeta(\underline{\varphi})}{\overline{\varphi} - \underline{\varphi}}\right] \varphi \tag{3.5}$$

where $b(t;\overline{\varphi})$ is the retirement benefit paid at date t to the households with the highest efficiency realization, $\zeta(\varphi)$ is a linear function of efficiency φ, and $\zeta(\underline{\varphi})$ is a parameter that controls the extent of redistribution in the social security program. As Caliendo and

Gahramanov (2009) have shown, for the social security program to be both pro-poor and positively linked to past income with the benefit rule outlined in (3.3), we must have

$$\underline{\varphi} \leq \zeta(\underline{\varphi}) \leq 1 \tag{3.6}$$

Also, since I only consider equilibria of the model where the OASI tax rate is optimal or maximizes social welfare, θ must satisfy

$$\theta = \arg\max \left\{ \int_{\underline{\varphi}}^{\overline{\varphi}} U(\varphi)\ f(\varphi)\ \mathrm{d}\varphi \right\} \tag{3.7}$$

where $U(\varphi)$ is the ex-ante expected lifetime utility of households with efficiency φ.[5]

3.2.4 Household optimization problem

A household born at date t with the efficiency realization φ faces the following optimization problem

$$\max_{c(s),l(s)} \int_0^{\bar{T}} \exp\{-\rho s\} Q(s) \frac{\left\{c(s)^{\eta} l(s)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma}\, \mathrm{d}s \tag{3.8}$$

subject to

$$c(s) + \frac{\mathrm{d}a(s)}{\mathrm{d}s} = ra(s) + y(s;\varphi) + B(t)\exp\{gs\} \tag{3.9}$$

$$y(s;\varphi) = (1-\theta)\{1-l(s)\}\, w(t)\exp\{gs\}\, \varphi e(s) + \Theta(s - T_{\mathrm{r}}) b(t;\varphi)\exp\{gs\} \tag{3.10}$$

$$0 \leq l(s) \leq 1 \tag{3.11}$$

$$a(0) = a(\bar{T}) = 0 \tag{3.12}$$

[5]Note that this representation of social welfare is essentially utilitarian, which treats utility as a cardinal concept instead of ordinal. However, this is standard in this line of literature, and as Köbberling (2006) points out, cardinal utility is an accepted concept in also a number of other areas, such as decision making under risk, case-based decision theory or discounted utilities for intertemporal evaluations.

where

$$\Theta = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

is a step function. I solve this problem using Pontryagin's Maximum Principle for fixed-endpoint optimal control problems.

3.2.5 Technology and factor prices

Output is produced using a Cobb-Douglas production function with inputs capital, labor and a stock of technology $A(t)$

$$Y(t) \; = \; K(t)^{\alpha} \left(A(t)L(t)\right)^{1-\alpha} \tag{3.13}$$

where $A(t) = A(0)\exp\{gt\}$, α is the share of capital in total income and $A(0)$ is the initial stock of technology. Factor markets are perfectly competitive and equilibrate instantaneously, which implies

$$r = MP_K - \delta = \alpha \left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha-1} - \delta \tag{3.14}$$

$$w(t) = MP_L = A(t)(1-\alpha)\left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha} \tag{3.15}$$

where δ is the depreciation rate of physical capital and $w(t)$ is the wage rate at time t.

3.2.6 Aggregation

Aggregate capital stock and labor supply in the current model are given by

$$K(t) = \int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\, a(t; t-s, \varphi)\, f(\varphi)\, \mathrm{d}s\, \mathrm{d}\varphi \tag{3.16}$$

$$L(t) = \int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\; \{1 - l(t; t-s, \varphi)\}\, \varphi e(s)\, f(\varphi)\, \mathrm{d}s\, \mathrm{d}\varphi \tag{3.17}$$

where $N(t-s)$ is the size of the cohort born at date $(t-s)$. Also, given that the social security program is unfunded, total taxes collected at any date must be equal to the total benefits paid out, i.e.

$$\int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\theta\left\{1-l(t;t-s,\varphi)\right\} w(t)\varphi e(s)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi$$
$$= \int_{\underline{\varphi}}^{\overline{\varphi}} \int_{T_{\mathrm{r}}}^{\bar{T}} N(t-s)Q(s)b(t;\varphi)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi \tag{3.18}$$

Using (3.3), we can rearrange and express (3.18) as

$$b(t;\overline{\varphi}) = \theta w(t)\left[\frac{\int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\left\{1-l(t;t-s,\varphi)\right\}\varphi e(s)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi}{\int_{\underline{\varphi}}^{\overline{\varphi}} \int_{T_{\mathrm{r}}}^{\bar{T}} N(t-s)Q(s)\,\zeta(\varphi)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi}\right] \tag{3.19}$$

which expresses the retirement benefits paid to the most efficient households in the population as a function of relevant macroeconomic and demographic variables. I define the box-bracketed term in equation (3.19) as the labor-to-retiree ratio and denote it by the symbol $R^e(t)$. Finally, total accidental bequests from the deceased households must satisfy

$$\int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)h(s)a(t;t-s,\varphi)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi$$
$$= \int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)B(t)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi \tag{3.20}$$

where

$$h(s) = -\frac{\mathrm{d}}{\mathrm{d}s}\ln Q(s) \tag{3.21}$$

is the hazard rate of dying between age s and $s+\mathrm{d}s$.

3.2.7 Equilibrium

A Stationary Competitive Equilibrium (SCE) with an optimal OASI tax rate in the current model can be characterized by a collection of

1. cross-sectional consumption programs $\{c(t;t-s,\varphi)\}_{s=0}^{\bar{T}}$, saving programs $\{a(t;t-s,\varphi)\}_{s=0}^{\bar{T}}$ and labor supply programs $\{1-l(t;t-s,\varphi)\}_{s=0}^{\bar{T}}$ for each φ,

2. aggregate capital stock $K(t)$, labor supply $A(t)L(t)$ and labor-to-retiree ratio $R^e(t)$,

3. real rate of return r and wage rate $w(t)$,

4. a tax rate θ, and

5. an accidental bequest $B(t)$

that

1. solves the households' optimization problems,

2. equilibrates the factor markets and balances the social security budget,

3. satisfies the social welfare maximization condition (3.7), and

4. satisfies the bequest balance condition (3.20)

I assume that the model economy is initially at a SCE with an optimal OASI tax rate. It is useful to note that for this economy, along the steady state growth path aggregate output grows at rate $(n+g)$, the real rate of return is time-invariant and wages grow at rate g.

3.3 Solving the model

To solve this model, it is useful to break it into the intra-temporal and the inter-temporal components. The intra-temporal component is

$$\max_{c,l} u = c^{\eta} l^{1-\eta}$$

subject to

$$c + (1-\theta) w \varphi e l = E$$
$$0 \leq l \leq 1$$

Using a monotonic transformation on the utility function, the Lagrangian is given by

$$\mathcal{L} = \eta \ln c + (1-\eta) \ln l - \lambda \left[c + (1-\theta) w \varphi e l - E \right] - \mu (l - 1) \tag{3.22}$$

Given that the choice variables are c and l, the first-order conditions are

$$\frac{\eta}{c} - \lambda = 0 \tag{3.23}$$

$$\frac{1-\eta}{l} - \lambda(1-\theta)w\varphi e - \mu = 0 \tag{3.24}$$

$$c + (1-\theta)w\varphi el - E = 0 \tag{3.25}$$

$$\mu(l-1) = 0,\ \ \mu \geq 0, l \leq 1 \tag{3.26}$$

First, consider the case when $l < 1$, which from (3.26) implies that $\mu = 0$. Then, we have from (3.23), (3.24) and (3.25)

$$\frac{\eta}{c} = \frac{1-\eta}{(1-\theta)w\varphi el} = \lambda \tag{3.27}$$

$$c = \eta E \tag{3.28}$$

$$l = \frac{(1-\eta)E}{(1-\theta)w\varphi e} \tag{3.29}$$

$$\lambda = \frac{1}{E} \tag{3.30}$$

Equations (3.27)-(3.30) are valid when $l < 1$, i.e. when

$$\frac{(1-\eta)E}{(1-\theta)w\varphi e} < 1$$

$$\Rightarrow\ \ E < \frac{(1-\theta)w\varphi e}{1-\eta} \tag{3.31}$$

Now, consider the case when $\mu > 0$, which from (3.26) implies that $l = 1$. Then, we have from (3.23), (3.24) and (3.25)

$$c = E - (1-\theta)w\varphi e \tag{3.32}$$

$$\lambda = \frac{\eta}{E - (1-\theta)w\varphi e} \tag{3.33}$$

$$\mu = (1-\eta) - \frac{\eta}{E-(1-\theta)w\varphi e}(1-\theta)w\varphi e \tag{3.34}$$

For the condition $\mu > 0$ to be satisfied from (3.34), we require

$$\begin{aligned} (1-\eta) - \frac{\eta}{E-(1-\theta)w\varphi e}\theta w\varphi e > 0 \\ \Rightarrow\ E > \frac{(1-\theta)w\varphi e}{1-\eta} \end{aligned} \tag{3.35}$$

which is exactly the opposite of condition (3.31). Therefore, the solution consumption and leisure functions for the intra-temporal problem can be compactly written as

$$c = \begin{cases} \eta E & E < \frac{(1-\theta)w\varphi e}{1-\eta} \\ E-(1-\theta)w\varphi e & E > \frac{(1-\theta)w\varphi e}{1-\eta} \end{cases} \tag{3.36}$$

$$l = \begin{cases} \frac{(1-\eta)E}{(1-\theta)w\varphi e} & E < \frac{(1-\theta)w\varphi e}{1-\eta} \\ 1 & E > \frac{(1-\theta)w\varphi e}{1-\eta} \end{cases} \tag{3.37}$$

Now consider the inter-temporal component

$$\max_{E(s)} \int_0^{\bar{T}} \exp\{-\rho s\} Q(s) V(E(s;\varphi))\,\mathrm{d}s \tag{3.38}$$

subject to

$$E(s;\varphi) + \frac{\mathrm{d}a(s)}{\mathrm{d}s} = ra(s) + y(s;\varphi + B(t)\exp\{gs\} \tag{3.39}$$

$$a(0) = a(\bar{T}) = 0 \tag{3.40}$$

$$b(t;\varphi) = \zeta(\varphi) b(t;\overline{\varphi}) \tag{3.41}$$

where

$$V(E(s;\varphi)) = \frac{\left\{c\,(E(s;\varphi))^{\eta}\, l\,(E(s;\varphi))^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \tag{3.42}$$

is the value function from the intra-temporal component, and $y(s;\varphi)$ is given by (3.10). Then, the Hamiltonian for this problem is given by

$$\begin{aligned}\mathcal{H} &= \exp\{-\rho s\}Q(s)V(E(s;\varphi)) + \psi(s;\varphi)\left[rk(s) + (1-\theta)w(t)\exp\{gs\}\varphi e(s)\right.\\ &\quad \left. + \Theta(s-T_r)b(t;\varphi)\exp\{gs\} + B(t)\exp\{gs\} - E(s;\varphi)\right] \end{aligned} \tag{3.43}$$

The first-order conditions are given by

$$\exp\{-\rho s\}Q(s)V_E(E(s;\varphi)) - \psi(s;\varphi) = 0 \tag{3.44}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\psi(s;\varphi) = -r\psi(s;\varphi) \tag{3.45}$$

The definite solution to (3.45) is given by

$$\psi(s;\varphi) = \psi(0;\varphi)\exp\{-rs\} \tag{3.46}$$

using which in (3.44) gives

$$\exp\{-\rho s\}Q(s)V_E(E(s;\varphi)) = \psi(0;\varphi)\exp\{-rs\} \tag{3.47}$$

Equation (3.47) governs the movement of $E(s)$ (and therefore $c(s)$ and $l(s)$) over the life-cycle. To obtain the closed form solution of $E(s)$ over the life-cycle, we need to rewrite (3.47) with only $E(s)$ on the LHS, i.e. we need to invert $V_E(\cdot)$ w.r.t. $E(s)$. To do that, we must show that $V_E(\cdot)$ is actually continuous and invertible w.r.t. $E(s)$. Now, note from the intra-temporal problem

$$V(E(s;\varphi)) = \begin{cases} \dfrac{\left\{\eta^{\eta}\left(\frac{1-\eta}{(1-\theta)w\varphi e}\right)^{1-\eta}\right\}^{1-\sigma} E^{1-\sigma}}{1-\sigma} & E < \frac{(1-\theta)w\varphi e}{1-\eta} \\ \frac{\{E-(1-\theta)w\varphi e\}^{\eta(1-\sigma)}}{1-\sigma} & E > \frac{(1-\theta)w\varphi e}{1-\eta} \end{cases} \tag{3.48}$$

which implies that

$$V_E(E(s;\varphi)) = \begin{cases} \left\{\eta^\eta \left(\frac{1-\eta}{(1-\theta)w\varphi e}\right)^{1-\eta}\right\}^{1-\sigma} E^{-\sigma} & E < \frac{(1-\theta)w\varphi e}{1-\eta} \\ \eta \left\{E-(1-\theta)w\varphi e\right\}^{\eta(1-\sigma)-1} & E > \frac{(1-\theta)w\varphi e}{1-\eta} \end{cases} \tag{3.49}$$

Note from (3.49) that

$$\lim_{E\uparrow\frac{(1-\theta)w\varphi e}{1-\eta}} V_E(\cdot) = \eta^{\eta(1-\sigma)} \left\{\frac{(1-\theta)w\varphi e}{1-\eta}\right\}^{1+\eta(\sigma-1)} \tag{3.50}$$

$$\lim_{E\downarrow\frac{(1-\theta)w\varphi e}{1-\eta}} V_E(\cdot) = \eta^{\eta(1-\sigma)} \left\{\frac{(1-\theta)w\varphi e}{1-\eta}\right\}^{1+\eta(\sigma-1)} \tag{3.51}$$

Therefore, $V_E(\cdot)$ is continuous in $E(s)$. Also, from (3.49)

$$V_{EE}(E(s;\varphi)) = \begin{cases} -\left\{\eta^\eta \left(\frac{1-\eta}{(1-\theta)w\varphi e}\right)^{1-\eta}\right\}^{1-\sigma} \sigma E^{-\sigma-1} & E < \frac{(1-\theta)w\varphi e}{1-\eta} \\ -\eta\left(1+\eta(\sigma-1)\right) \left\{E-(1-\theta)w\varphi e\right\}^{\eta(1-\sigma)-2} & E > \frac{(1-\theta)w\varphi e}{1-\eta} \end{cases} \tag{3.52}$$

both of which < 0 as long as $E(s) > 0$. Therefore, $V_E(\cdot)$ is continuous and monotonic (strictly decreasing) in $E(s)$, which implies that it is invertible. Let $V_E(\cdot) = x$. Then, from (3.49) we can write

$$x = \begin{cases} \left\{\eta^\eta \left(\frac{1-\eta}{(1-\theta)w\varphi e}\right)^{1-\eta}\right\}^{1-\sigma} E^{-\sigma} & E < \frac{(1-\theta)w\varphi e}{1-\eta} \\ \eta \left\{E-(1-\theta)w\varphi e\right\}^{\eta(1-\sigma)-1} & E > \frac{(1-\theta)w\varphi e}{1-\eta} \end{cases} \tag{3.53}$$

which implies

$$E(s) = \begin{cases} \left[x\left\{\eta^\eta \left(\frac{1-\eta}{(1-\theta)w\varphi e}\right)^{1-\eta}\right\}^{\sigma-1}\right]^{-\frac{1}{\sigma}} & x > x^* \\ \left(\frac{x}{\eta}\right)^{\frac{1}{\eta(1-\sigma)-1}} + (1-\theta)w\varphi e & x < x^* \end{cases} \tag{3.54}$$

where

$$x^* = \eta^{\eta(1-\sigma)} \left(\frac{1-\eta}{(1-\theta)w\varphi e}\right)^{1+\eta(\sigma-1)} \tag{3.55}$$

Also, rearranging (3.47) we get

$$V_E(E(s;\varphi)) = \frac{\psi(0;\varphi)}{Q(s)}\exp\{-(r-\rho)s\} \tag{3.56}$$

Using equation (3.56) in (3.54), we finally have

$$E(s) = \begin{cases} \left[\frac{\psi(0;\varphi)}{Q(s)}\exp\{-(r-\rho)s\}\left\{\eta^\eta\left(\frac{1-\eta}{(1-\theta)w\varphi e}\right)^{1-\eta}\right\}^{\sigma-1}\right]^{-\frac{1}{\sigma}} & \frac{\psi(0;\varphi)}{Q(s)}\exp\{-(r-\rho)s\} > x^* \\ \left(\frac{\frac{\psi(0;\varphi)}{Q(s)}\exp\{-(r-\rho)s\}}{\eta}\right)^{\frac{1}{\eta(1-\sigma)-1}} + (1-\theta)w\varphi e & \frac{\psi(0;\varphi)}{Q(s)}\exp\{-(r-\rho)s\} < x^* \end{cases} \tag{3.57}$$

The evolution of consumption and leisure over the life-cycle can be obtained by applying (3.36) and (3.37) on (3.57). Also, note that

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}s}\left(\exp\{-rs\}\, a(s)\right) &= \exp\{-rs\}\left[\frac{\mathrm{d}a(s)}{\mathrm{d}s} - ra(s)\right] \\ &= \exp\{-rs\}\left[(1-\theta)w(t)\exp\{gs\}\varphi e(s)\right. \\ &\quad \left. +\Theta(s-T_r)b(t;\varphi)\exp\{gs\} + B(t)\exp\{gs\} - E(s)\right] \end{aligned} \tag{3.58}$$

By using the boundary conditions from (3.12), it can be shown that the life-cycle budget constraint is satisfied when

$$\int_0^{\bar{T}} \exp\{-rs\}\left[y(s;\varphi) + B(t)\exp\{gs\}\right]\, \mathrm{d}s = \int_0^{\bar{T}} \exp\{-rs\}\, E(s;\varphi)\, \mathrm{d}s \tag{3.59}$$

which is again the principle that the present value of income over the life-cycle should be equal to the present value of $E(\cdot)$.

Because of the presence of a kink in the labor supply function at the date of retirement, the household's optimization problem described above cannot be solved analytically. Therefore, I rely on numerical methods (described in Appendix 3.9) to solve for the optimal consumption, saving and labor supply profiles over the life-cycle.

3.4 Computational algorithm

To compute the SCE for a given set of model parameters and an OASI tax rate, I use the following algorithm:

- **Step 1:** Guess some values for the factor prices, the labor-to-retiree ratio and the accidental bequest.
- **Step 2:** Solve the households' optimization problems under the values guessed in step 1.
- **Step 3:** Aggregate the household-level optimal choices to obtain the implied total capital stock and labor supply.
- **Step 4:** Compute the factor prices, the labor-to-retiree ratio and the accidental bequest implied by the values obtained in steps 2 and 3.
- **Step 5:** Repeat steps 1-4 until the guessed values in step 1 converge to the implied values in step 4.

Then, to find the optimal tax rate, I repeat steps 1-5 over a grid of tax rates and then choose the value that maximizes social welfare (i.e. satisfies condition (3.7)).

3.5 Baseline calibration

I parameterize the baseline equilibrium of the model using empirical evidence from various sources. A population growth rate of $n = 1\%$ is consistent with the U.S. demographic history, and I set the rate of technological progress to $g = 1.56\%$, which is the trend growth rate of per-capita income in the postwar U.S. economy (Bullard and Feigenbaum, 2007). I assume that households enter the model at actual age 25, which corresponds to the model age of zero. I obtain the survival probabilities from Feigenbaum's (2008) sextic fit to the mortality data in Arias (2004), which is given by

$$\begin{aligned}\ln Q(s) \quad = \quad & -0.01943039 + \left(-3.055 \times 10^{-4}\right) s + \left(5.998 \times 10^{-6}\right) s^2 \\ & + \left(-3.279 \times 10^{-6}\right) s^3 + \left(-3.055 \times 10^{-8}\right) s^4 + \left(3.188 \times 10^{-9}\right) s^5 \\ & + \left(-5.199 \times 10^{-11}\right) s^6 \end{aligned} \tag{3.60}$$

where s is model age. The 2001 U.S. Life Tables in Arias (2004) are reported up to actual age 100, so I set the maximum model age to $\bar{T} = 75$. Also, I set the model benefit eligibility age to $T_{\mathrm{r}} = 41$, which corresponds to the current actual full retirement eligibility age of 66 in the U.S. As the household's age-dependent efficiency endowment $e(s)$ is difficult to observe, I use average cross-sectional hourly income data from the 2001 CPS as a proxy for efficiency. To use this data, I first use piecewise linear interpolation to obtain average hourly earnings for all ages between 25-65, and normalize the data such that earnings at actual age 25 is unity. Then, I fit a quartic polynomial to the interpolated data, which gives

$$\begin{aligned}\ln e(s) \quad = \quad & -3.273 \times 10^{-5} + \left(1.423 \times 10^{-4}\right) s + \left(-3.8696 \times 10^{-5}\right) s^2 \\ & + \left(-1.313 \times 10^{-5}\right) s^3 + \left(6.307 \times 10^{-8}\right) s^4 \end{aligned} \tag{3.61}$$

where s is model age and $s \leq 40$. Beyond actual age 65 (i.e. for $s > 40$), for which data is limited, I use the following quadratic function

$$\ln e(s) = -f_0 - f_1 s - 0.01 s^2 \tag{3.62}$$

and parameterize f_0 and f_1 such that $e(s)$ is continuous and once differentiable at age $s = 40$.[6] The resulting efficiency profile is plotted in Figure 3.1.

The historically observed value of capital's share in total income in U.S. ranges between 30-40%, so I set $\alpha = 0.35$. Also, since I focus only on SCE, I set $t = 0$ in all the computations and normalize the initial stock of technology and the population to $A(0) = N(0) = 1$. I also

[6] The values that satisfy these conditions are $f_0 = 14.7416$ and $f_1 = -0.7643$.

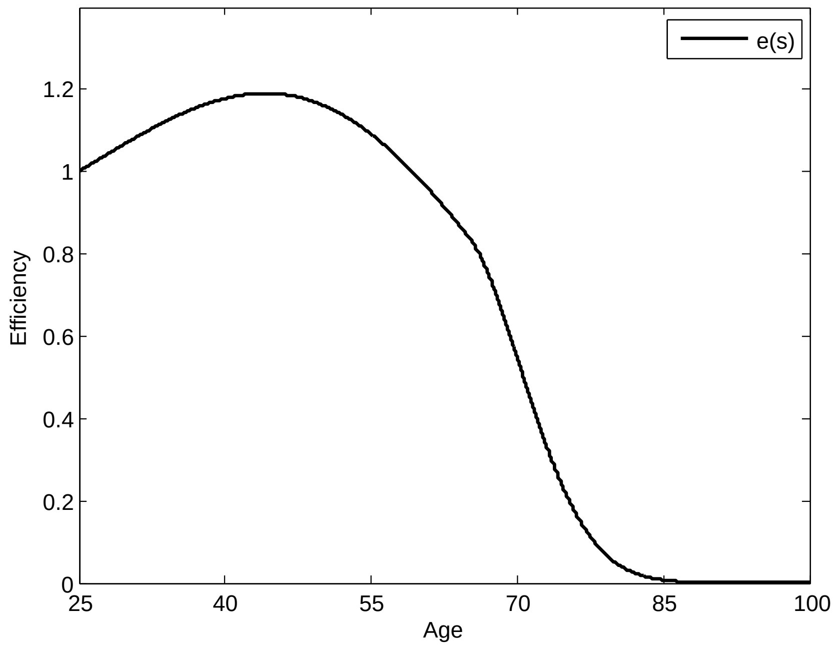

Figure 3.1: Efficiency profile estimated from the 2001 CPS.

assume that the random component of household efficiency (φ) is distributed uniformly within a cohort, and then transform the continuous distribution into a 5-point discrete distribution for computational convenience (i.e. $f(\varphi) = 0.2$).[7] To express efficiency relative to the most efficient households in the cohort (those with $\varphi = \overline{\varphi}$), I specify the lower limit of the support as a fraction of the upper limit, i.e. $\underline{\varphi} = \omega\overline{\varphi}$ with $0 < \omega \leq 1$, and then normalize the upper limit to $\overline{\varphi} = 1$. Note that with this specification, the degree of efficiency heterogeneity within a cohort can be conveniently controlled by simply varying the parameter ω. Finally, I set the depreciation rate to $\delta = 0.0744$.

Once all the observable parameters have been assigned empirically reasonable values, I calibrate the unobservable preference parameters σ (IEIS), ρ (discount rate) and η (share of consumption in period utility), the efficiency heterogeneity parameter ω and the benefit rule parameter $\zeta(\omega)$ (which controls the degree of redistribution in the social security program) such that the model jointly matches the following targets:

[7]Note that the 5-point specification also facilitates reporting model data by income quintiles.

- a steady state capital-output ratio of 3.0,
- an average fraction of time of $34/168 = 0.202$ spent on market work between ages 25-55,
- a replacement rate of 90% for the poorest households in the population, and
- an optimal or welfare-maximizing OASI tax rate of 10.6%.[8]

The capital-output ratio target is consistent with the larger macroeconomic literature. The target for the fraction of time spent on market work is taken from the 2001 CPS, which reports that on an average, production and nonsupervisory employees in the U.S. spend 34 hours per week on market work. The replacement rate target is taken from the Primary Insurance Amount (PIA) benefit formula used by the SSA, which replaces 90% of the average indexed monthly earnings among the poorest income earners in the population. Finally, the current OASI tax rate target allows me to fully control for the optimal program size under the current U.S. demographics.

The parameter values under which the model reasonably matches the above targets are reported in Table 3.1. Note that discount rates close to zero or even negative are not uncommon in the macro-calibration literature (Huggett, 1996; Bullard and Feigenbaum, 2007; Feigenbaum, 2008) as well as the quantitative public finance literature (Huggett and Ventura, 1999; Conesa and Garriga, 2008a,b), and a share of consumption in period utility around one-sixth is reasonably close to the values used in the general pension reform literature (Kotlikoff, 1997; Huggett and Ventura, 1999; Nishiyama and Smetters,

[8] I compute the replacement rate at date t for a household with efficiency φ surviving to the eligibility age as follows. First, I compute the average indexed pre-tax earnings using the formula

$$AIE(t;\varphi) = \left\{ \int_0^{T^*(\varphi)} \{1 - l(t; t-s, \varphi)\}\, w(t)\varphi e(s)\,\mathrm{d}s \right\} / T^*(\varphi)$$

where $T^*(\varphi)$ is the retirement age of households with efficiency φ, or the age at which labor supply drops to zero. Note that similar to the SSA's calculations, I index past wages to date t in computing the AIE. Then, the I compute the replacement rate using the formula

$$RR(t;\varphi) = b(t;\varphi)/AIE(t;\varphi)$$

Table 3.1: Unobservable parameter values under the baseline calibration.

σ	ρ	η	ω	$\zeta(\omega)$
1.9	-0.0015	0.176	0.2	0.46

Table 3.2: Model performance under the baseline calibration.

	Target	Model
Capital-output ratio	3.0	3.37
Avg. fraction of work time between ages 25-55	0.202	0.205
Replacement rate for the poorest households	0.9	0.896
Social security tax rate	0.106	0.107

2005). Also, note that with leisure in period utility, the relevant IEIS for consumption is $\sigma^c = 1 + \eta(\sigma - 1) = 1.16$, which lies conveniently within the range frequently encountered in the literature.

The factor prices in the baseline equilibrium are $r = 0.0293$ and $w = 1.25$, the labor-to-retiree ratio is $R^e = 0.6386$, and the accidental bequest is $B = 0.0015$. I report the model-generated values for the targets under the baseline calibration in Table 3.2. The baseline equilibrium cross-sectional age-consumption and age-labor hour profiles for the different efficiency groups are reported in Figures 3.2 and 3.3. It is useful to note that the average cross-sectional consumption profile (not plotted in Figure 3.2) exhibits a peak at about age 51, with a ratio of peak to initial consumption of 1.22. Also note that in the figures, 1 corresponds to the lowest efficiency level and 5 corresponds to the highest.

Since households in the current model are life-cycle permanent income consumers, a qualitative measure of their welfare gains from the social security program can be obtained by comparing their internal rates of return (IRR) from the program with the equilibrium rate of return from the capital stock. By definition, the IRR of a household with efficiency φ is the discount rate $\beta(\varphi)$ for which the net present value of the household's social security

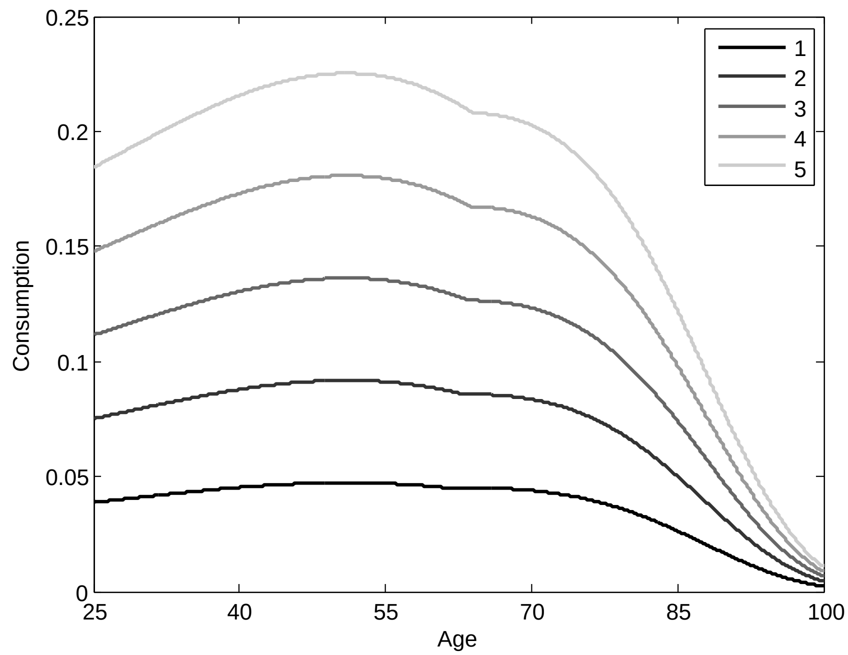

Figure 3.2: Baseline cross-sectional age-consumption profiles by efficiency level.

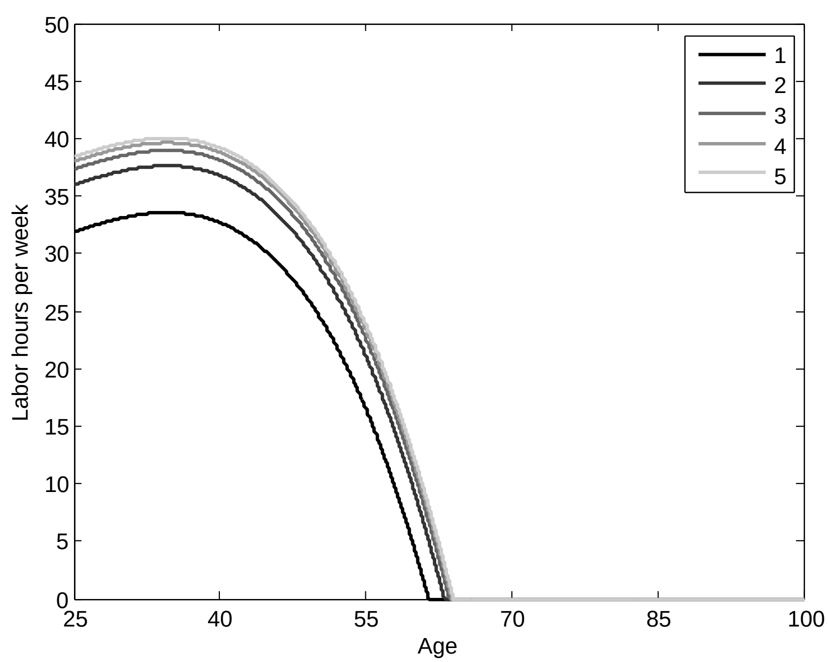

Figure 3.3: Baseline cross-sectional age-labor hour profiles by efficiency level.

wealth equals zero. Mathematically, it is a solution to the following equation

$$\int_0^{\bar{T}} \exp\{(g-\beta(\varphi))s\} Q(s)\theta\{1-l(s;\varphi)\} w(t)\varphi e(s)\,\mathrm{d}s$$
$$= \int_{T_\mathrm{r}}^{\bar{T}} \exp\{(g-\beta(\varphi))s\} Q(s)b(t;\varphi)\,\mathrm{d}s \tag{3.63}$$

where s is model age. Note that the first term in the above equation gives the present value of total benefits received and the second term gives the present value of the total tax payments. Numerically solving equation (3.63) for each efficiency group yields the following IRR distribution under the baseline calibration: $\{0.0479, 0.0323, 0.0256, 0.0218, 0.0192\}$. Comparing these IRRs with the equilibrium rate of return from capital stock shows that the bottom two quintiles of the model population experience a welfare gain from participating in the social security program, and the top three quintiles experience a welfare loss. With $\zeta(\omega) = 0.46$, the social security program partially insures the households against unfavorable efficiency realizations: the benefit annuity of the poorest households is close to half of that of the wealthiest, when the labor income of the former is only one-fifth of the latter (as $\omega = 0.2$). The gross benefit replacement rates in the model population under the baseline calibration are compared to the ones implied by the U.S. PIA benefits formula in Figure 3.4.[9] The optimal retirement age distribution in the baseline calibration is $\{61.48, 63.09, 63.61, 63.86, 64.02\}$, which shows that households with lower efficiency retire earlier than those with higher efficiency.

3.6 Population aging

Population aging in the current model is driven by an increasing life expectancy and a falling population growth rate. The 2009 OASDI Trustees Report published by the SSA contains the long-range social security area population and the average life expectancy at

[9] It may be noted here that even though social security contributions and benefits are capped in the real world, no such cap exists in the model. However, the baseline distribution of income in the model is such that these caps would not bind even if they existed. As Huggett and Ventura (1999) point out, the maximum creditable earnings in the U.S. social security program averaged at 2.47 times the average earnings over the period 1990-94. The ratio of maximum earnings to average earnings in the baseline calibration is 1/0.6=1.67, which is lesser than 2.47.

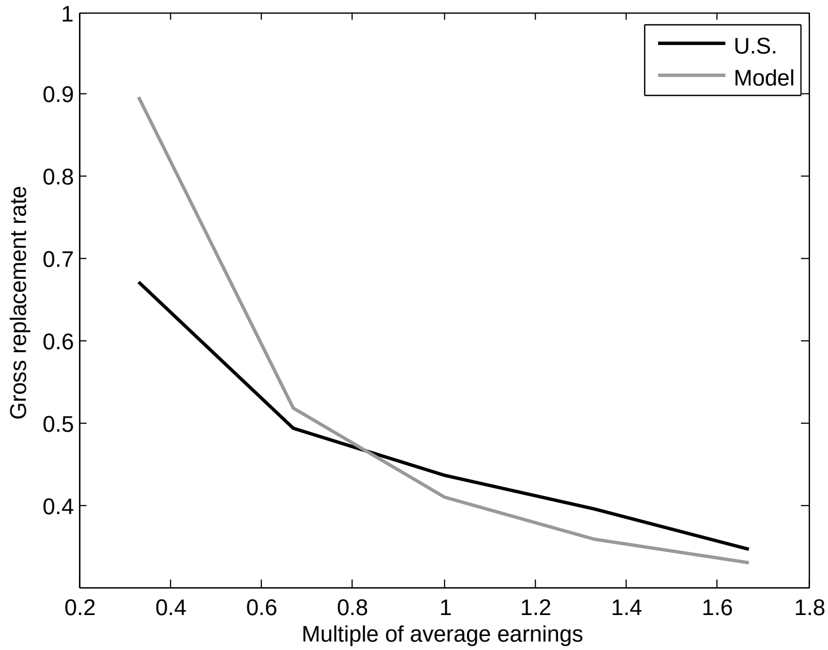

Figure 3.4: Gross replacement rates: Model Vs U.S.

birth projections for 2075 under three plausible sets of assumptions: low-cost, i.e. favorable demographic conditions, intermediate, i.e. best estimates of likely future demographic conditions, and high-cost, i.e. financially disadvantageous demographic conditions. The projections are as follows:

1. **Low-cost:** life expectancy of 80.55, social security area population growth rate of 0.16%,

2. **Intermediate:** life expectancy of 83.95, social security area population growth rate of 0.09%, and

3. **High-cost:** life expectancy of 87.55, social security area population growth rate of 0.01%.

The projections for the growth rate of the social security area population can be taken directly to the model. To use the life expectancy projections, I augment the baseline

Table 3.3: The demographic experiments.

Experiment	$n(\%)$	γ	μ	Life expectancy
Low-cost (1)	0.16	1×10^{-4}	1.5577	80.55
Intermediate (2)	0.09	6×10^{-5}	1.9077	83.95
High-cost (3)	0.01	4×10^{-5}	2.1298	87.55

survival probabilities with an age-specific increment of the form

$$\mathrm{d}\,Q(s) = \gamma s^{\mu} \tag{3.64}$$

where γ and μ are positive constants to be parameterized, and s represents household age. Therefore, the projected survival probabilities are given by

$$Q_{\mathrm{p}}(s) = Q(s) + \gamma s^{\mu} \tag{3.65}$$

Note that these age-specific increments are consistent with the fact that old-age survivorship in the U.S. has increased at a faster rate in the later half of the twentieth century, making the population survival curve more rectangular (Arias, 2004) (see Figure 3.5). Then, I choose values for γ and μ such that the model life expectancies under the augmented survival probabilities match the projections described above. I report the specific demographic experiments in Table 3.3.[10,11] I incorporate these population projections into the baseline model, and then compute a new SCE with an optimal OASI tax rate under each experiment. I report the corresponding values of the social security tax rate and some other relevant variables in Table 3.4.

The results of the three experiments can be summarized as follows. First, the optimal or welfare-maximizing social security tax rate increases from the baseline level by 1.8 percentage points under the low-cost projection, 3.2 percentage points under the intermediate

[10] I hold the maximum lifespan unchanged at $\bar{T} = 75$ under all the three experiments.

[11] Note that in the current model, both the baseline life expectancy and the increases under population aging are symmetric across the different income groups. However, there is evidence that both life expectancy (Attanasio and Emmerson, 2003; De Nardi et al., 2006) and its rate of improvement (Waldron, 2007) are strongly correlated with income.

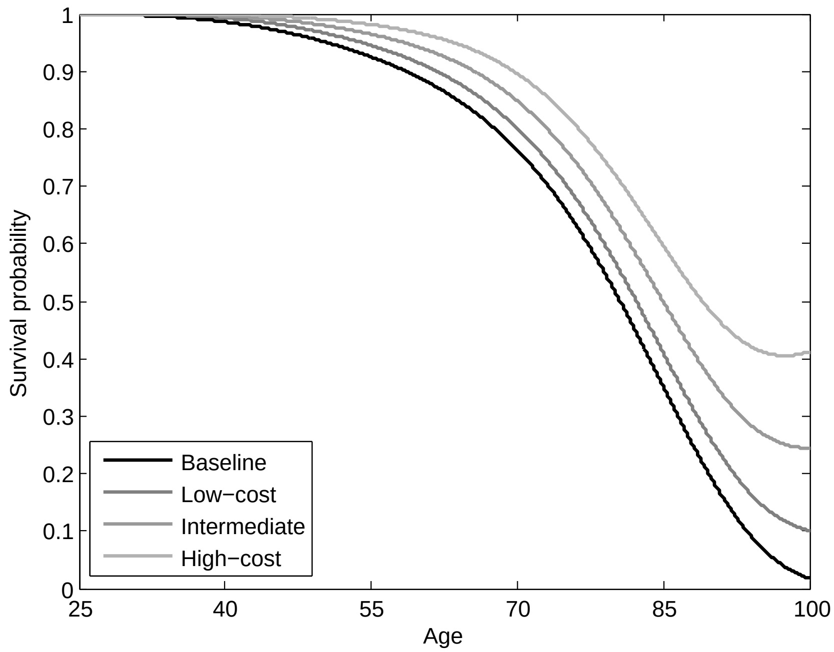

Figure 3.5: Baseline and the projected survival probabilities.

Table 3.4: The effect of population aging on the calibrated model.

Experiment	Baseline	1	2	3
Social security tax rate	0.107	0.125	0.139	0.155
Replacement rate for the poorest households	0.896	0.736	0.706	0.68
Rate of return	0.0293	0.0251	0.0222	0.0194
Wage rate	1.25	1.28	1.3	1.32
Output	7.55	9.2	9.86	10.56
Capital	25.47	32.36	35.75	39.41
Labor	3.92	4.67	4.93	5.2
Capital-output ratio	3.37	3.52	3.62	3.73

projection, and 4.8 percentage points under the high-cost projection. Second, capital accumulation increases and reduces the equilibrium rate of return, with 0.42, 0.71 and 0.99 percentage point declines under experiments 1, 2 and 3, respectively. Finally, the replacement rate for the lowest efficiency group declines to about 74, 71 and 68% under the three experiments, and the output level, labor supply and the capital-output ratio all increase.

It is useful to compare the results in Table 3.4 to those in the other studies on social security reform under population aging in the U.S. De Nardi et al. (1999) consider eight alternative budget-balancing fiscal responses to future demographic shocks, which include keeping the benefits fixed and allowing the tax rates to adjust to the burden of demographic shocks, allowing the benefits to fall by increasing the retirement eligibility age, changing their tax treatment, or changing the benefit formula to allow for a larger dependence of benefits to past income. They find that allowing only the labor income tax rate to adjust to the shock requires it to increase by almost 30 percentage points to keep the benefits unchanged. However, when the benefits are allowed to fall from their baseline level, relatively smaller but still fairly large increases in the tax rate are required (of the order of 13 to 23 percentage points). On the other hand, Conesa and Garriga (2008b) find that the average effective tax rate on labor income (which includes both the regular labor income tax as well as a payroll tax collected to finance social security) actually falls from its initial steady state level of 24.8% to around 22%. In comparison, the current model predicts that the optimal or welfare-maximizing response to population aging is likely to include tax increases ranging from roughly 2 to 5 percentage points. Also, it is important to note that these increases in the tax rate are always smaller than the actuarial projections of the SSA.

Why do the optimal or welfare-maximizing responses to population aging require relatively small increases in the tax rate? To understand this, I examine how population aging affects the extent to which households with different efficiency levels benefit from the social security program. In Table 3.5, I report the households' IRRs from social security under the three experiments, while holding the tax rate fixed at the baseline level. Note that the rates of return reported in Table 3.5 are the equilibrium values in post-population aging

Table 3.5: The effect of population aging on the households' IRRs from social security.

Experiment	$\beta_{\varphi=0.2}$	$\beta_{\varphi=0.4}$	$\beta_{\varphi=0.6}$	$\beta_{\varphi=0.8}$	$\beta_{\varphi=1}$	Rate of return
Baseline	0.0479	0.0323	0.0256	0.0218	0.0192	0.0293
1	0.0406	0.0242	0.0172	0.0132	0.0106	0.0237
2	0.0399	0.0235	0.0165	0.0125	0.0099	0.0199
3	0.0391	0.0227	0.0157	0.0118	0.0092	0.0161

SCE with $\theta = 0.107$. The table shows that population aging negatively impacts the IRRs from social security across all the efficiency levels, but also reveals an interesting asymmetry in the effect: the bottom quintiles of the model population experience smaller declines relative to the top quintiles. Across the baseline calibration and experiment 3, the IRRs for the respective groups decline by roughly 18, 30, 39, 46 and 52%, which implies that the burden of population aging is lesser on the poorer households who actually benefit from social security. Given this fact, it appears intuitive that the optimal or welfare-maximizing response requires relatively small changes in the tax rate.

Why does population aging impose a lesser burden on the poorer households who benefit the most from the social security program? Examination of equation (3.63) reveals that the answer to this question lies in understanding how it affects household labor supply over the life-cycle: a crucial determinant of the IRR. In Table 3.6, I report the expected labor supply over the life-cycle (in efficiency units) for the different efficiency groups under the three experiments, with the tax rate held fixed at the baseline level. Two facts are clear from the table. First, households respond by increasing their labor supply: expected labor supply over the life-cycle is higher than the baseline for all the efficiency groups. Second, the

Table 3.6: The effect of population aging on the households' labor supply over the life-cycle.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	1.25	2.91	4.57	6.23	7.89
1	1.32	3.1	4.89	6.68	8.47
2	1.39	3.27	5.16	7.05	8.94
3	1.47	3.45	5.43	7.41	9.4

labor supply responses are not symmetric: the households with higher efficiency experience larger increases than the households with lower efficiency. Across the baseline calibration and experiment 3, expected labor supply over the life-cycle increases by 17.5 and 18.7% for the bottom two quintiles, and by 19, 19.1 and 19.2% for the top three quintiles of the model population. As the relatively wealthy households supply more labor, they take up a larger tax burden relative to the poorer households, which causes a larger decline in their IRRs.

The life-cycle labor supply responses documented in Table 3.6 are a combined effect of responses both along the intensive margin (i.e. hours worked) as well as the extensive margin (i.e. retirement or the age of exiting the labor force). In Table 3.7, I decompose these changes: Sub-table 3.7a documents the average weekly hours spent on market work between ages 25-55 for the different efficiency groups (the intensive margin), and Sub-table 3.7b documents the actual retirement ages (the extensive margin). Note that in both the sub-tables I report post-population aging equilibrium values with the tax rate held fixed at the initial baseline. Sub-table 3.7a shows that in general, all the efficiency groups experience an increase in their weekly work hours due to population aging (the only exception are the

Table 3.7: Decomposing the labor supply responses along the intensive and the extensive margins.

(a) The intensive margin: average weekly hours spent on market work between ages 25-55.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	29.9	34.07	35.44	36.13	36.54
1	29.75	34.24	35.73	36.47	36.91
2	29.95	34.6	36.14	36.91	37.37
3	30.25	34.92	36.48	37.25	37.72

(b) The extensive margin: actual retirement age.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	61.48	63.09	63.61	63.86	64.02
2	64.37	65.88	66.24	66.41	66.5
3	65.9	66.89	67.17	67.3	67.38
4	66.9	67.67	67.91	68.02	68.09

Table 3.8: Equilibrium social security benefits with the optimal tax response.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	0.0171	0.0342	0.0513	0.0684	0.0855
1	0.014	0.0279	0.0419	0.0558	0.0698
2	0.0136	0.0271	0.0407	0.0543	0.0678
3	0.0133	0.0266	0.0399	0.0532	0.0665

poorest households under the low-cost experiment). Across the baseline and experiment 3, average weekly hours spent on market work increases by roughly 1.2, 2.5, 2.9, 3.1 and 3.2% respectively. With respect to the extensive margin, Sub-table 3.7b shows that the retirement age also increases under population aging. Across the baseline and experiment 3, the age at which labor supply drops to zero for the respective efficiency groups increases by 5.4, 4.6, 4.3, 4.2 and 4.1 years. Note that even with the largest delay in retirement, the smallest increase in the weekly hours for the poorest households leads to the smallest increase in their expected labor supply over the life-cycle.

What is the final impact of population aging and the optimal tax response on the equilibrium social security benefits? In Table 3.8, I report the equilibrium retirement benefits of the households surviving to the eligibility age (T_{r}) under the three demographic experiments, with the OASI tax rate set at the optimal levels identified in Table 3.4. The table shows that population aging always leads to a decline in social security benefits even in the presence of the optimal or welfare-maximizing tax response: benefits fall by roughly 18, 21 and 22% under the three experiments. However, it is important to note that these declines are significantly smaller than what would have occurred if the tax rate were held fixed at the baseline level. Social security benefits in post-population aging equilibria are about 28, 36 and 43% lower under the three demographic experiments with $\theta = 0.107$.

3.7 Sensitivity analysis

The optimal or utility-maximizing social security tax rates under the projected future demographics in the U.S., as outlined in the previous section, are conditional on the set of parameter values used in the baseline calibration. However, as Hansen and Heckman (1996) note, empirical evidence reports not only the point estimates of data targets but also their sampling distributions. This implies that multiple values for the observable parameters (those within a reasonable statistical interval), rather than one, are in agreement with data. In addition, different sets of values for the observable parameters imply different sets of values for the unobservable parameters under which the model matches the data targets. However, as Caliendo and Gahramanov (2009) and Findley and Caliendo (2009) demonstrate, the welfare consequences of social security are crucially dependent on the values that are assigned to these unobservable parameters: different parameterizations that are consistent with the same macro-general equilibrium have very different policy implications for social security. Given these facts, in this section I examine how the results outlined in the previous section are sensitive to the assigned values of some key model parameters.

Two sets of parameters that are treated as observable in the baseline calibration of the model are capital's share in total income (α) and the age-dependent household efficiency endowment ($e(s)$). The value of capital's share in total income is set to $\alpha = 0.35$ in the initial baseline calibration, but the macroeconomic estimates historically observed in the U.S. range between $30 - 40\%$. Therefore, to verify the sensitivity of the simulation results with respect to α, I first compute new calibrated baseline equilibria of the model under $\alpha = 0.3$ and $\alpha = 0.4$, and then re-simulate experiments 1, 2 and 3. The unknown preference parameters and target values for the baseline calibrations with $\alpha = 0.3$, 0.35 and 0.4 are compared in Table 3.9.

It is clear from the table that calibrated baseline equilibria of the model under $\alpha = 0.3$ and $\alpha = 0.4$ provide reasonable fits to the data targets, with the values for the unknown parameters falling conveniently in the range used in the larger macro-calibration literature. Baseline cross-sectional age-consumption and age-labor hour profiles for the different efficiency groups under $\alpha = 0.3$ and $\alpha = 0.4$ are reported in Figures 3.6-3.7 and Figures 3.8-3.9. The figures show that both baseline consumption as well as labor hour profile exhibit empirically reasonable shapes.

Numerically solving equation (3.63) for each efficiency group yields the following baseline

Table 3.9: Baseline equilibria under different values of capital's share in total income.

(a) Unobservable parameter values.

	$\alpha = 0.3$	$\alpha = 0.35$	$\alpha = 0.4$
Discount rate (ρ)	0.001	-0.0015	-0.0025
IEIS (σ)	1.8	1.9	1.7
Share of consumption in period utility (η)	0.177	0.176	0.175
Efficiency heterogeneity parameter (ω)	0.16	0.2	0.2
Benefit redistribution parameter ($\zeta(\omega)$)	0.38	0.46	0.48

(b) Model performance.

	$\alpha = 0.3$	$\alpha = 0.35$	$\alpha = 0.4$
Capital-output ratio	2.92	3.37	3.86
Avg. fraction of work time between ages 25-55	0.202	0.205	0.208
Replacement rate for the poorest households	0.93	0.896	0.9
Social security tax rate	0.102	0.107	0.106

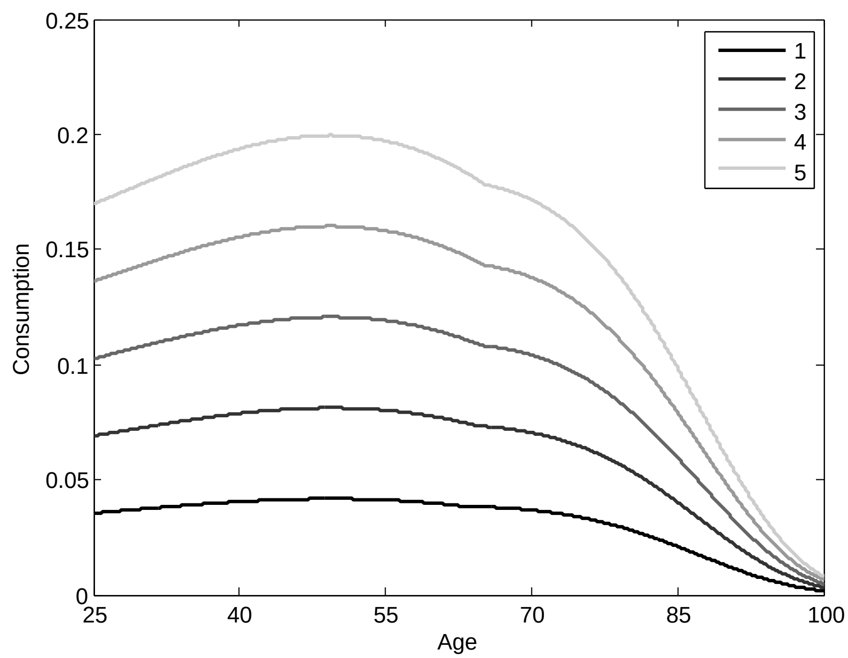

Figure 3.6: Baseline cross-sectional age-consumption profiles under $\alpha = 0.3$.

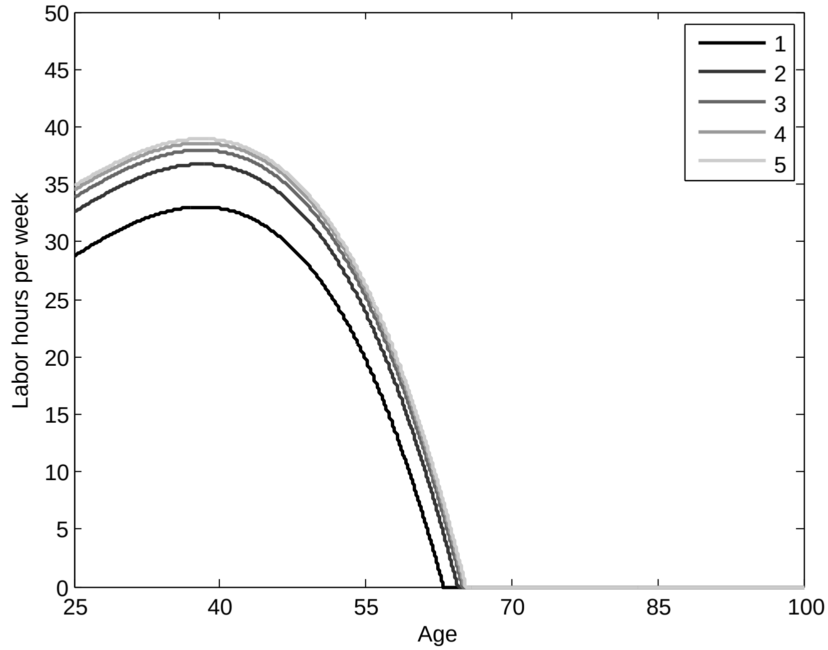

Figure 3.7: Baseline cross-sectional age-labor hour profiles under $\alpha = 0.3$.

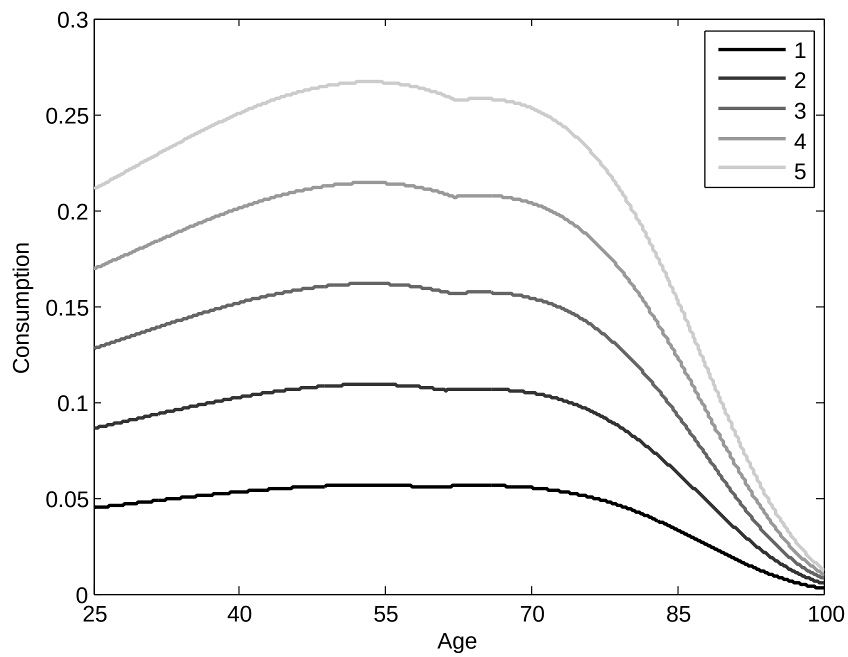

Figure 3.8: Baseline cross-sectional age-consumption profiles under $\alpha = 0.4$.

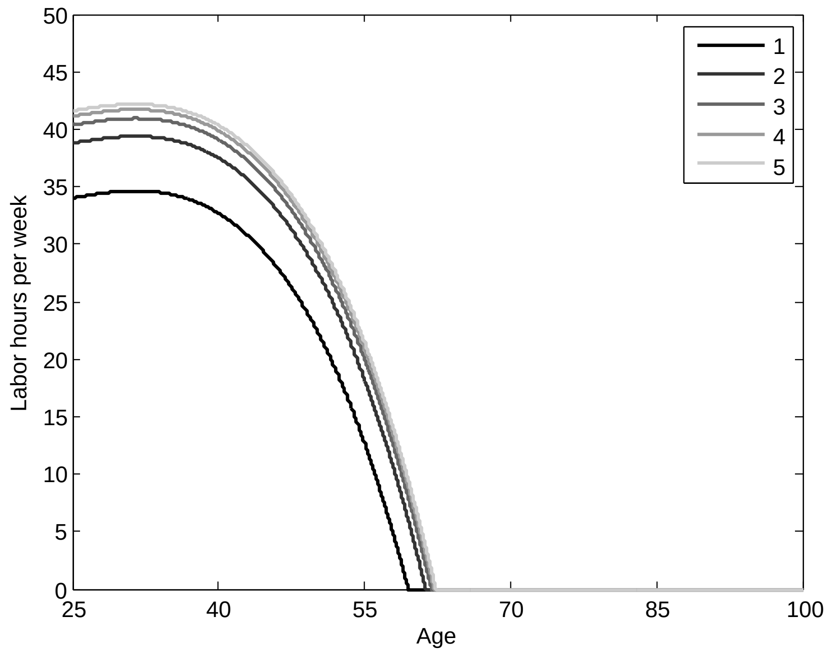

Figure 3.9: Baseline cross-sectional age-labor hour profiles under $\alpha = 0.4$.

household IRR distributions for the two α values: $\{0.0501, 0.0322, 0.0256, 0.0221, 0.0199\}$ and $\{0.0489, 0.0327, 0.0256, 0.0215, 0.0189\}$, respectively. Comparing these IRR distributions with the rates of return from capital stock shows that only the bottom two quintiles of the model population experience welfare gains from participating in the social security program. Therefore, the welfare implications of social security in this model are not sensitive to the α values used in the simulations. The gross benefit replacement rates in the baseline model under the different α values are compared to the ones implied by the U.S. PIA benefits formula in Figures 3.10 and 3.11.

The baseline optimal retirement age distributions under the different values of capital's share in total income are compared in Table 3.10. Also, note that in the table, 1 corresponds to the lowest efficiency level and 5 corresponds to the highest. The table demonstrates that even with different values of capital's share in total income, households with lower efficiency continue to exit the labor force earlier than those with higher efficiency.

The next step is to conduct experiments 1, 2 and 3 on the calibrated model with different

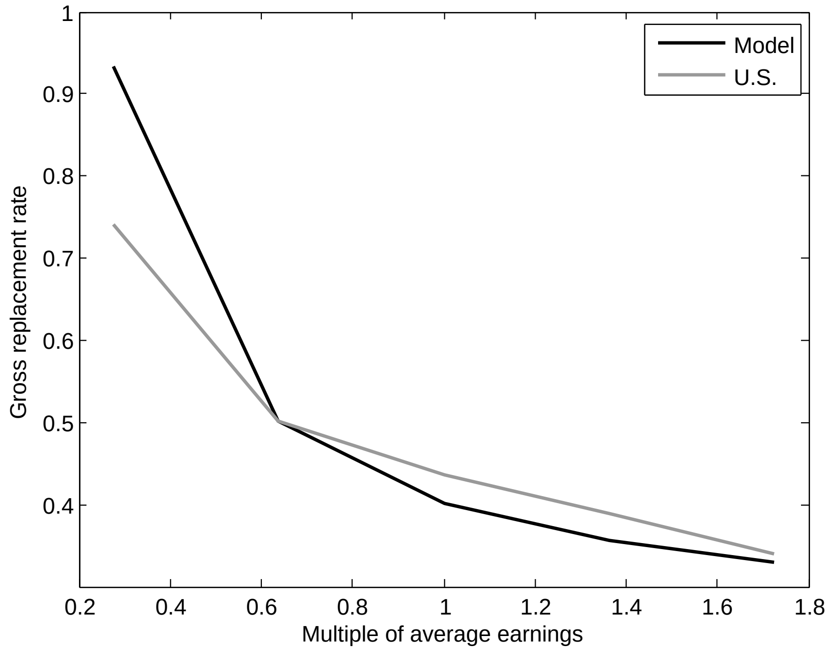

Figure 3.10: Gross replacement rates: Model Vs U.S. under $\alpha = 0.3$.

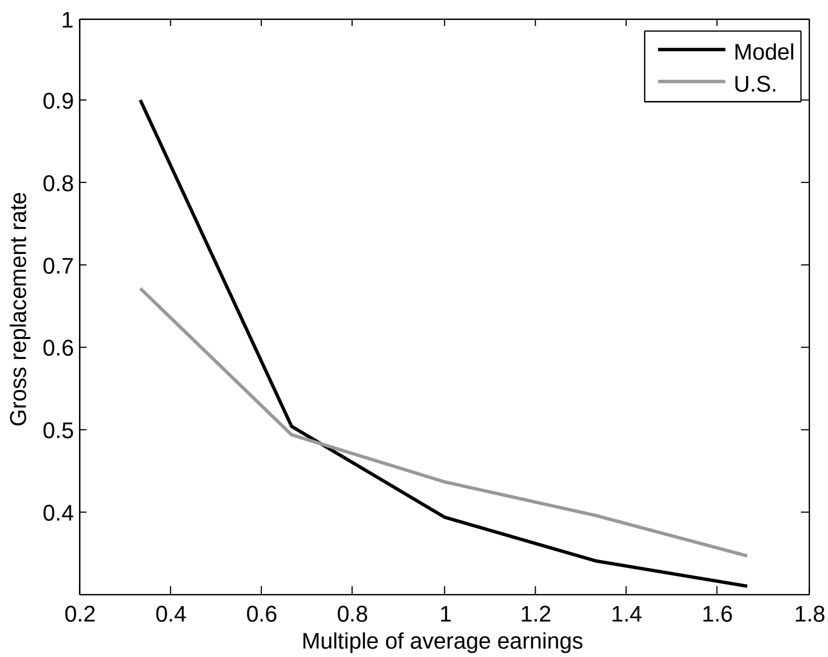

Figure 3.11: Gross replacement rates: Model Vs U.S. under $\alpha = 0.4$.

Table 3.10: Retirement age distributions under different values of capital's share in total income.

Capital's share in income	1	2	3	4	5
0.3	62.75	64.47	64.94	65.15	65.26
0.35	61.48	63.09	63.61	63.86	64.02
0.4	59.52	61.32	61.9	62.18	62.35

Table 3.11: The effect of population aging on the calibrated model.

(a) With capital's share of $\alpha = 0.3$.

Experiment	Baseline	1	2	3
Variable				
Social security tax rate	0.102	0.127	0.141	0.157
Replacement rate for the poorest households	0.93	0.82	0.79	0.76
Rate of return	0.0285	0.0252	0.0223	0.0195
Wage rate	1.11	1.12	1.14	1.15
Output	6.03	7.25	7.72	8.22
Capital	17.57	21.83	23.97	26.27
Labor	3.81	4.52	4.75	5.0
Capital-output ratio	2.92	3.01	3.1	3.19

(b) With capital's share of $\alpha = 0.4$.

Experiment	Baseline	1	2	3
Variable				
Social security tax rate	0.106	0.122	0.136	0.151
Replacement rate for the poorest households	0.9	0.73	0.73	0.67
Rate of return	0.0293	0.0249	0.0221	0.0193
Wage rate	1.48	1.52	1.55	1.58
Output	9.69	11.87	12.8	13.81
Capital	37.38	47.84	53.1	58.95
Labor	3.94	4.69	4.96	5.25
Capital-output ratio	3.86	4.03	4.15	4.27

values of capital's share in total income, and then compute post-population aging SCE with an optimal OASI tax rate. I report the corresponding values of the tax rate and some other relevant variables in Table 3.11. It is clear from the table that the general impact of population aging on the welfare-maximizing social security tax rate and the other endogenous variables is fairly robust across different values for capital's share in total income. With all realistic α values, the model predicts a 2-5 percentage point increase in the social security tax rate under the projected future demographics. Population aging reduces the equilibrium rate of return from capital and increases the equilibrium wage rate, output, capital stock, labor supply and the capital-output ratio under all realistic α values.

Given that the welfare-maximizing changes in the tax rate predicted by the model are fairly robust across different values for capital's share in total income used in the simulation, it is natural to expect that the factors driving such changes are also robust to the different α-values. To verify this, I document in Table 3.12 how the households' IRRs from the social security program are affected by population aging, holding the social security tax rate fixed at the corresponding baseline level identified in Table 3.9. The tables reveal a familiar pattern of the impact of the demographic shocks on the households' IRRs from the social security program. Across the baseline and experiment 3, the IRRs of the five efficiency groups decline by roughly 16, 30, 39, 45 and 51%, respectively, under $\alpha = 0.3$, and by roughly 18, 30, 39, 46 and 53%, respectively, under $\alpha = 0.4$. Therefore, the fact that population aging imposes a lesser burden on the poorer households who are actually

Table 3.12: The effect of population aging on the households' IRRs under different α-values.

(a) With capital's share of $\alpha = 0.3$.

Experiment	$\beta_{\varphi=0.16}$	$\beta_{\varphi=0.37}$	$\beta_{\varphi=0.58}$	$\beta_{\varphi=0.79}$	$\beta_{\varphi=1}$	Rate of return
Baseline	0.0501	0.0322	0.0256	0.0221	0.0199	0.0285
1	0.0433	0.0242	0.0172	0.0135	0.0112	0.0232
2	0.0428	0.0234	0.0165	0.0128	0.0105	0.0195
3	0.0421	0.0226	0.0157	0.0121	0.0098	0.0158

(b) With capital's share of $\alpha = 0.4$.

Experiment	$\beta_{\varphi=0.2}$	$\beta_{\varphi=0.4}$	$\beta_{\varphi=0.6}$	$\beta_{\varphi=0.8}$	$\beta_{\varphi=1}$	Rate of return
Baseline	0.0489	0.0327	0.0256	0.0215	0.0189	0.0293
1	0.0415	0.0246	0.0172	0.013	0.0103	0.0237
2	0.0408	0.0238	0.0165	0.0123	0.0096	0.0201
3	0.0399	0.023	0.0157	0.0116	0.0089	0.0165

Table 3.13: The effect of population aging on the households' labor supply over the life-cycle.

(a) Labor supply responses with $\alpha = 0.3$.

Experiment	$\varphi = 0.16$	$\varphi = 0.37$	$\varphi = 0.58$	$\varphi = 0.79$	$\varphi = 1$
Baseline	0.86	2.33	3.81	5.29	6.76
1	1.01	2.82	4.63	6.43	8.24
2	1.07	2.99	4.92	6.85	8.78
3	1.14	3.18	5.23	7.28	9.33

(b) Labor supply responses with $\alpha = 0.4$.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	1.05	2.49	3.94	5.38	6.82
1	1.26	3.01	4.76	6.52	8.28
2	1.34	3.22	5.1	6.98	8.86
3	1.45	3.46	5.46	7.47	9.48

the beneficiaries of the social security program also appears to be robust to the value of capital's share in total income used in the simulations.

In Table 3.13, I demonstrate that the asymmetry in the labor supply responses from the different efficiency groups is behind the asymmetric declines in the respective IRRs. The table reports expected labor supply over the life-cycle (in efficiency units) for the different efficiency groups under the three experiments, with the social security tax rates held fixed at their respective baseline levels with different α values. The table shows a pattern similar to that observed under the baseline case: households with higher efficiency experience larger delays in retirement than households with lower efficiency under both $\alpha = 0.3$ and 0.4. Across the baseline calibration and experiment 3, life-cycle labor supply for the five efficiency groups increases by roughly 32.3, 36.4, 37.4, 37.8 and 38% respectively under $\alpha = 0.3$, and by roughly 37.6, 38.6, 38.8, 38.9 and 39% respectively under $\alpha = 0.4$. Once again, the relatively larger increases in labor supply experienced by the more efficient households serves to transfer a larger tax burden onto them, which has a larger negative impact on their IRRs from social security.

Decomposing households labor supply responses over the life-cycle under the different α-

Table 3.14: Decomposing the labor supply responses under capital's share of $\alpha = 0.3$.

(a) The intensive margin: average weekly hours spent on market work between ages 25-55.

Experiment	$\varphi = 0.16$	$\varphi = 0.37$	$\varphi = 0.58$	$\varphi = 0.79$	$\varphi = 1$
Baseline	29.34	33.85	35.08	35.65	35.99
1	28.54	33.68	35.09	35.75	36.13
2	28.42	33.81	35.3	35.99	36.39
3	28.41	33.9	35.41	36.12	36.53

(b) The extensive margin: actual retirement age.

Experiment	$\varphi = 0.16$	$\varphi = 0.37$	$\varphi = 0.58$	$\varphi = 0.79$	$\varphi = 1$
Baseline	62.75	64.47	64.94	65.15	65.26
1	65.4	66.64	66.91	67.03	67.1
2	66.5	67.45	67.68	67.79	67.85
3	67.33	68.13	68.34	68.43	68.48

values also reveals a familiar pattern. To see this, consider Table 3.14, in which I report both the average weekly hours spent in market work between ages 25-55 (the intensive margin), as well as the ages at which households exit the labor force (the extensive margin) under a capital's share of $\alpha = 0.3$. Note that similar to the other tables, I hold the social security tax rate held fixed at the baseline level. The table shows that across the baseline and experiment 3, average weekly hours spent on market work for efficiency groups 2 through 5 increases by roughly 0.16, 0.1, 1.3 and 1.5%, respectively. The only exception are the poorest households, whose hours decline under population aging. However, the delays in actual retirement experienced by the different efficiency groups under $\alpha = 0.3$ are very similar. Across the baseline and experiment 3, the age at which labor supply drops to zero for the respective efficiency groups increases by 4.6, 3.7, 3.4, 3.3 and 3.2 years. Note that it is the large delay in retirement for the poorest households that more than compensates for the lower average hours spent on market work between ages 25-55 and generates increased labor supply over the life-cycle. Table 3.15 reports simulation results in the same two categories, but for $\alpha = 0.4$, and shows that household labor supply responses along both the intensive and the extensive margins are fairly robust to the underlying capital's share parameter.

Table 3.15: Decomposing the labor supply responses under capital's share of $\alpha = 0.4$.

(a) The intensive margin: average weekly hours spent on market work between ages 25-55.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	29.64	34.56	36.18	36.98	37.47
1	29.96	35.05	36.72	37.56	38.05
2	30.44	35.61	37.32	38.17	38.68
3	31.01	36.15	37.85	38.7	39.21

(b) The extensive margin: actual retirement age.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	59.52	61.32	61.9	62.18	62.35
1	62.47	64.33	64.93	65.21	65.37
2	64.24	65.87	66.25	66.42	66.52
3	65.82	66.85	67.13	67.27	67.35

Table 3.16: Equilibrium social security benefits with the optimal tax response.

(a) With capital's share of $\alpha = 0.3$.

Experiment	$\varphi = 0.16$	$\varphi = 0.37$	$\varphi = 0.58$	$\varphi = 0.79$	$\varphi = 1$
Baseline	0.0282	0.0397	0.0512	0.0626	0.0741
1	0.0242	0.034	0.0439	0.0538	0.0636
2	0.0233	0.0328	0.0424	0.0519	0.0614
3	0.0227	0.032	0.0413	0.0505	0.0598

(b) With capital's share of $\alpha = 0.4$.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	0.0475	0.0604	0.0732	0.0861	0.099
1	0.0384	0.0488	0.0592	0.0697	0.0801
2	0.0377	0.0479	0.0581	0.0683	0.0784
3	0.0371	0.0471	0.0571	0.0672	0.0772

Finally, I report in Table 3.16 the equilibrium social security benefits at actual age 70 (model age 45) for the different efficiency groups under $\alpha = 0.3$ and 0.4 with the welfare-maximizing tax response. The table demonstrates that the equilibrium social security benefits with the optimal tax response are lower than the baseline level: benefits decline by roughly 14, 17 and 19% under the three experiments with $\alpha = 0.3$. However, these are significantly lower than what would have occurred if the tax rate were held fixed at the baseline level, in which case the decline are about 29, 27 and 44% respectively. Therefore, the effect of population aging and the welfare-maximizing tax response on equilibrium benefits also appears to be fairly robust to the underlying value of the capital's share parameter.

Computing new calibrated baseline equilibria of the model with different but empirically consistent values of capital's share in total income and re-simulating experiments 1, 2 and 3 demonstrates that the simulation results are fairly robust to the underlying α values. Note that even though using different values for capital's share in total income changes the values for the unknown preference parameters for which the model matches data targets, the quantitative predictions of the model remain largely unchanged.

Another set of parameters treated as observable in the initial simulation results are the coefficients of the age-dependent component of household efficiency ($e(s)$) as specified in (3.61). Given that it is difficult to directly observe efficiency, a standard approach in literature is to use cross-sectional hourly wages as a proxy. Following this approach, the coefficients in (3.61) were estimated from normalized average cross-sectional hourly wages data from the 2001 CPS. However, the age-dependent component of household efficiency used in De Nardi et al. (1999), Conesa and Garriga (2008b) and Conesa and Garriga (2008a) are estimated from Hansen (1993). Therefore, I now re-simulate the baseline results of the current chapter with the age-dependent household efficiency data reported in Hansen (1993). The efficiency units in Hansen (1993) are constructed by taking a weighted sum of the hours worked by each age-sex subgroup using annual data from 1979 to 1987, where the weights reflect the relative productivity of that subgroup. To use this data, I first calculate the average of male and female weights for each age group, and use piecewise linear interpolation to obtain the weights for all ages between 25 and 65. Then, I fit a quartic polynomial to the interpolated data, which gives

$$\begin{aligned} \ln e(s) &= -0.0194 + \left(7.44 \times 10^{-3}\right) s + \left(1.46 \times 10^{-4}\right) s^2 \\ &\quad - \left(4.1 \times 10^{-6}\right) s^3 - \left(1.21 \times 10^{-7}\right) s^4 \end{aligned} \tag{3.66}$$

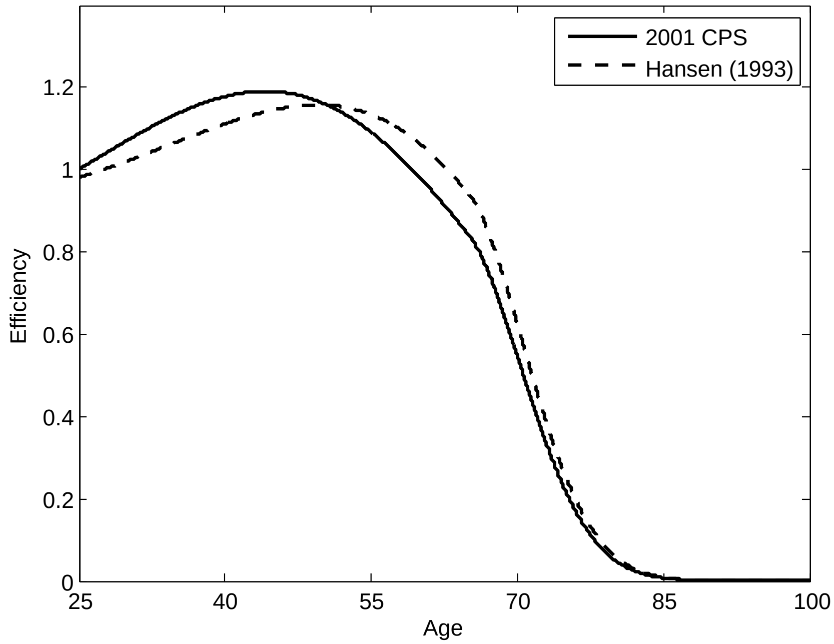

Figure 3.12: Efficiency profiles from the 2001 CPS and Hansen (1993).

where s is model age and $s \leq 40$. Similar to the baseline calibration, beyond actual age 65 (i.e. for $s > 40$), I use the following quadratic function

$$\ln\, e(s) = -f_0 - f_1 s - 0.01s^2 \tag{3.67}$$

and parameterize f_0 and f_1 such that $e(s)$ is continuous and once differentiable at $s = 40$. This yields $f_0 = 14.7984$ and $f_1 = -0.7685$. The age-dependent efficiency profile from Hansen (1993) is compared to the initial profile computed from the 2001 CPS in Figure 3.12.

Following the same approach used in sensitivity analyses with respect to the capital's share in total income, I first compute a new baseline equilibrium with the new age-dependent efficiency profile, and then I re-simulate demographic experiments 1, 2 and 3. The unknown preference parameters and the associated model generated target values for the endogenous variables for the efficiency profile estimated from Hansen (1993) are compared to the initial

Table 3.17: Baseline equilibria with efficiency data from 2001 CPS and Hansen (1993).

(a) Unobservable parameter values.

	2001 CPS	Hansen (1993)
Discount rate (ρ)	-0.0015	-0.004
IEIS (σ)	1.9	1.8
Share of consumption in period utility (η)	0.176	0.175
Efficiency heterogeneity parameter (ω)	0.2	0.18
Benefit redistribution parameter ($\zeta(\omega)$)	0.46	0.4

(b) Model performance.

	2001 CPS	Hansen (1993)
Capital-output ratio	3.38	3.39
Avg. fraction of work time between ages 25-55	0.205	0.203
Replacement rate for the poorest households	0.896	0.924
Social security tax rate	0.107	0.104

baseline calibration in Table 3.17. It is clear from the table that the values for the unknown preference parameters which calibrate the model to data under the efficiency profile estimated from Hansen (1993) are very similar to those under the efficiency profile estimated from 2001 CPS. The baseline cross-sectional age-consumption and age-labor hour profiles are plotted in Figures 3.13 and 3.14, where in the figures 1 corresponds to the lowest efficiency level, and 5 corresponds to the highest. Note that the baseline labor hours under the efficiency profile estimated from Hansen (1993) are not hump-shaped, which is inconsistent with empirical evidence.

The baseline households' IRR distribution with the efficiency profile from Hansen (1993) is as follows: $\{0.0485, 0.0321, 0.0256, 0.0221, 0.0198\}$. As in the initial baseline, the bottom two quintiles of the model population experience a welfare gain from participating in the social security program, as their IRRs are higher than the rate of return on capital stock. The gross benefit replacement rates in the baseline model with the efficiency profile from Hansen (1993) are compared to the ones implied by the U.S. PIA benefits formula in Figure 3.15.

Baseline household retirement behavior under the two different age-dependent efficiency

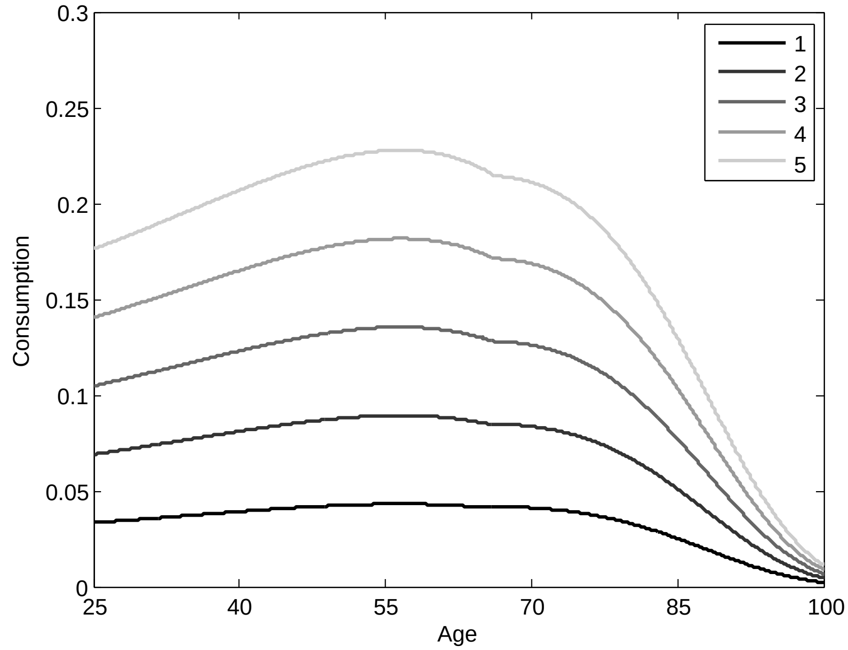

Figure 3.13: Baseline cross-sectional age-consumption profiles under Hansen (1993).

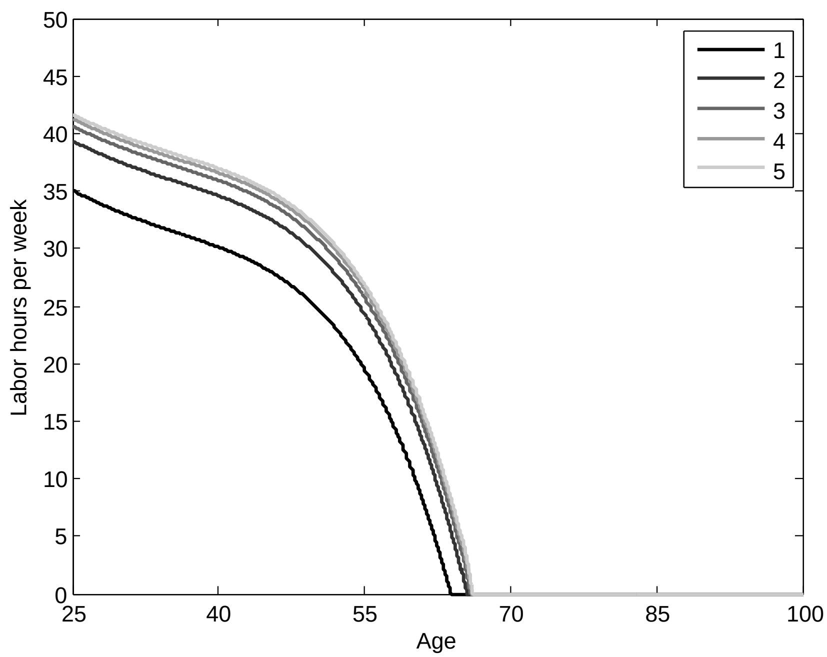

Figure 3.14: Baseline cross-sectional age-labor hour profiles under Hansen (1993).

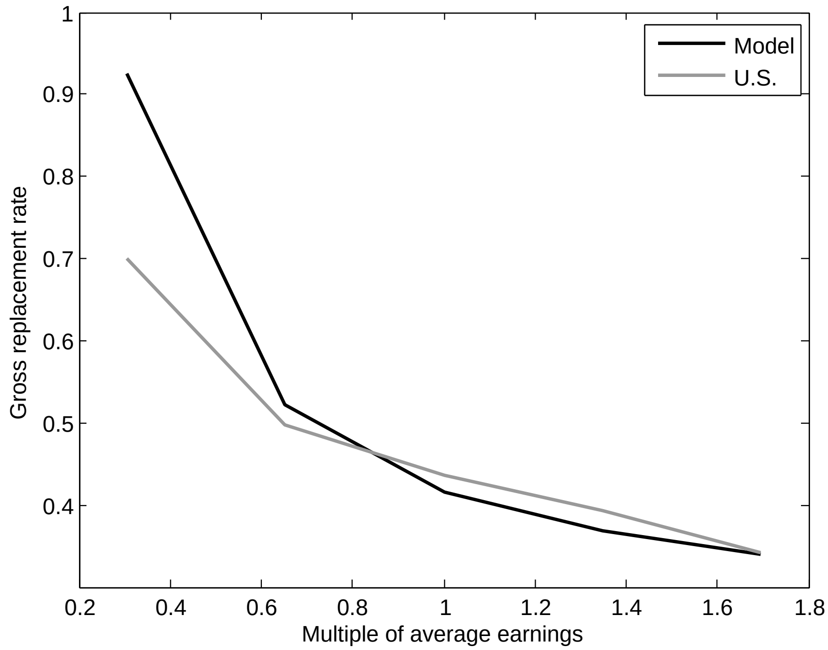

Figure 3.15: Gross replacement rates: Model Vs U.S. under Hansen (1993).

profiles are compared in Table 3.18, in which 1 corresponds to the lowest efficiency level and 5 corresponds to the highest. The table shows that even with the efficiency profile from Hansen (1993), less efficient households continue to exit the labor force before the more efficient households.

Next, I report in Table 3.19 the results of re-simulating demographic experiments 1, 2 and 3 on the calibrated baseline model identified above. Examination of Table 3.19 reveals that the quantitative predictions of the model are fairly robust with respect to the underlying efficiency profile. As in the previous cases, the welfare-maximizing social security tax rates under population aging are 2-5 percentage points higher than the baseline level. The associated general equilibrium adjustments, similar to those in the initial baseline, lead

Table 3.18: Retirement age distributions under the different efficiency profiles.

	1	2	3	4	5
2001 CPS	61.48	63.09	63.61	63.86	64.02
Hansen (1993)	59.52	61.32	61.9	62.18	62.35

Table 3.19: The effect of population aging on the calibrated model.

Experiment	Baseline	1	2	3
Variable				
Social security tax rate	0.104	0.125	0.139	0.156
Replacement rate for the poorest households	0.92	0.78	0.74	0.71
Rate of return	0.0286	0.0249	0.022	0.0193
Wage rate	1.26	1.28	1.3	1.32
Output	7.41	9.08	9.77	10.49
Capital	25.17	31.99	35.47	39.2
Labor	3.83	4.61	4.88	5.16
Capital-output ratio	3.39	3.53	3.63	3.74

to a lower real rate of return, higher wages, output, capital stock and labor supply.

The driving forces behind these optimal tax changes predicted by the model also show a similar pattern. In Table 3.20, I report how the IRRs from the social security program for each efficiency group are affected by population aging, and in Table 3.21 I report the life-cycle labor supply responses from the households (in efficiency units). Note that in both the tables, I hold the social security tax rate fixed at the corresponding baseline level. Table 3.20 shows that the asymmetric impact of population aging on the IRRs of the different efficiency groups is not sensitive to the coefficients of the underlying age-dependent efficiency profile: across the baseline and experiment 3, the IRRs of the five efficiency groups decline by roughly 17, 30, 39, 45 and 51%, respectively. Also, Table 3.21 shows that the asymmetric changes in household labor supply are behind this: across the baseline and experiment 3, labor supply (in efficiency units) of the five efficiency groups increase by about 38, 40.2, 40.8, 41 and 41.2%, respectively.

Table 3.20: The effect of population aging on the households' IRRs with the efficiency profile from Hansen (1993).

Experiment	$\beta_{\varphi=0.18}$	$\beta_{\varphi=0.385}$	$\beta_{\varphi=0.59}$	$\beta_{\varphi=0.795}$	$\beta_{\varphi=1}$	Rate of return
Baseline	0.0485	0.0321	0.0256	0.0221	0.0198	0.0286
1	0.0415	0.024	0.0172	0.0135	0.0111	0.0234
2	0.0409	0.0233	0.0165	0.0128	0.0104	0.197
3	0.0402	0.0225	0.0157	0.0121	0.0097	0.016

Table 3.21: The effect of population aging on the households' labor supply with the efficiency profile from Hansen (1993).

Experiment	$\varphi = 0.18$	$\varphi = 0.385$	$\varphi = 0.59$	$\varphi = 0.795$	$\varphi = 1$
Baseline	0.95	2.39	3.83	5.27	6.72
1	1.14	2.92	4.7	6.49	8.27
2	1.22	3.13	5.04	6.95	8.86
3	1.31	3.35	5.4	7.44	9.48

Decomposing the household labor supply responses along the intensive and the extensive margins with the efficiency profile from Hansen (1993) also reveals a similar pattern (see Table 3.22). As in the earlier cases, both the average labor hours between ages 25-55 and the retirement ages increase under population aging. Across the baseline an experiment 3, the average labor hours for all but lowest efficiency group increase by 0.9, 1.6, 1.9 and 2%, respectively. The labor hours for the lowest efficiency group actually declines by 1.4%. However, large delays in retirement across the baseline and experiment 3 for all the efficiency groups: 8.3, 7.2, 6.8, 6.6 and 6.5 years, ensure that the more efficient households experience larger increases in labor supply over the life-cycle.

Table 3.22: Decomposing the labor supply responses under the efficiency profile from Hansen (1993).

(a) The intensive margin: average weekly hours spent in market work between ages 25-55.

Experiment	$\varphi = 0.18$	$\varphi = 0.385$	$\varphi = 0.59$	$\varphi = 0.795$	$\varphi = 1$
Baseline	29.3	33.79	35.14	35.8	36.18
1	28.82	33.73	35.21	35.94	36.36
2	28.78	33.93	35.49	36.25	36.8
3	28.89	34.11	35.7	36.47	36.92

(b) The extensive margin: actual retirement age.

Experiment	$\varphi = 0.18$	$\varphi = 0.385$	$\varphi = 0.59$	$\varphi = 0.795$	$\varphi = 1$
Baseline	59.52	61.32	61.9	62.18	62.35
1	66.23	67.16	67.41	67.52	67.59
2	67.09	67.88	68.1	68.2	68.26
3	67.82	68.51	68.7	68.8	68.85

Table 3.23: Equilibrium social security benefits with the optimal tax response under Hansen (1993).

Experiment	$\varphi = 0.18$	$\varphi = 0.385$	$\varphi = 0.59$	$\varphi = 0.795$	$\varphi = 1$
Baseline	0.034	0.0468	0.0595	0.0723	0.085
1	0.0287	0.0395	0.0503	0.061	0.0718
2	0.028	0.0385	0.049	0.0595	0.0701
3	0.0277	0.0381	0.0485	0.059	0.0694

Finally, I report in Table 3.23 social security benefits in post-population aging steady states with the welfare-maximizing tax responses. The table reveals a similar pattern, in the sense that benefits decline by 15.5, 17.5 and 18.4% under the three experiments with the optimal tax response. Also, these decline are significantly smaller than what would have occurred if the tax rate were held fixed under population aging: declines of 28, 35.5 and 42%, respectively.

Sensitivity analysis with respect to the parameters in the age-dependent component of household efficiency shows that the quantitative predictions of the model are fairly robust. The welfare-maximizing social security tax rates under population aging are roughly 2-5 percentage points higher than the baseline value even with the efficiency profile estimated from Hansen (1993).

It is worth noting that all the simulation results so far have been based on the assumption that the age at which households start receiving social security benefits (T_{r}) will remain unchanged under population aging. However, the Social Security Administration has been steadily increasing the full retirement age in the U.S. over the past several decades. Currently, the full retirement age for an individual depends on when he/she was born (see Table 3.24). In their applied general equilibrium study on the projected U.S. demographics and social security, De Nardi et al. (1999) consider a fiscal response in which the full retirement age is postponed twice by 2 years each time in 2032 and 2036, eventually raising it to 69. What impact would such a postponement in full retirement eligibility have on the quantitative predictions of the current model? To examine this, I design a new set of computational experiments in which I increase the age at which households start receiving

Table 3.24: Age to receive full social security benefits in the U.S.

Year of birth	Full retirement age
1937 or earlier	65
1938	65 and 2 months
1939	65 and 4 months
1940	65 and 6 months
1941	65 and 8 months
1942	65 and 10 months
1943-54	66
1955	66 and 2 months
1956	66 and 4 months
1957	66 and 6 months
1958	66 and 8 months
1959	66 and 10 months
1960 and later	67

benefits from $T_{\mathrm{r}} = 41$ in the initial baseline to $T_{\mathrm{r}} = 44$ (actual age of 69) under population aging. The effect of this policy on the welfare-maximizing social security tax rate and the other relevant endogenous variables is reported in Table 3.25.

It is clear from the table that the welfare-maximizing changes in the tax rate with postponement of the eligibility age are smaller in magnitude compared to the ones predicted by the baseline model. This should not be surprising, as holding everything else constant, increasing T_{r} would simply lead to higher benefits per retiree. However, these higher benefits would induce the households to increase their labor supply by a smaller magnitude (see Table

Table 3.25: The effect of population aging on the calibrated model with $T_{\mathrm{r}} = 44$.

Experiment	Baseline	1	2	3
Variable				
Social security tax rate	0.107	0.12	0.134	0.149
Replacement rate for the poorest households	0.896	0.83	0.78	0.74
Rate of return	0.0293	0.0256	0.0226	0.0197
Wage rate	1.25	1.28	1.3	1.32
Output	7.55	9.19	9.87	10.6
Capital	25.47	32.17	35.63	39.43
Labor	3.92	4.68	4.94	5.22
Capital-output ratio	3.38	3.5	3.61	3.72

Table 3.26: Percentage change in household labor supply from baseline under population aging.

(a) Without postponement of the eligibility age.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
1	5.41	6.77	7.11	7.27	7.37
2	11.0	12.63	13.04	13.23	13.34
3	17.51	18.69	18.96	19.09	19.16

(b) With postponement of the eligibility age to $T_\mathrm{r} = 44$.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
1	3.1	3.69	3.83	3.9	3.93
2	9.75	10.78	11.02	11.13	11.19
3	17.71	18.42	18.58	18.64	18.68

3.26) relative to the situation without any postponement. This would partially reduce the leisure cost of population aging to the households, and would therefore require smaller welfare-maximizing changes in the social security tax rate.

3.8 Conclusions

In the current chapter I ask what should be the optimal or welfare-maximizing OASI tax rate in the U.S. under the projected future demographics. I construct a heterogeneous-agent general equilibrium model of life-cycle consumption and labor supply, where the source of heterogeneity is a productivity or efficiency realization that occurs before the agents enter the model. In the model, an unfunded social security program provides partial insurance against the unfavorable efficiency realization by paying retirement benefits through a pro-poor rule. I first calibrate the benefit rule to match the degree of redistribution in the U.S. program, and then calibrate the model's efficiency distribution such that the current OASI tax rate in the U.S. is optimal under the current demographics. Then, I introduce empirically reasonable population projections from the 2009 OASDI Trustees Report into the calibrated model, and finally search for the tax rates that maximize social welfare under those projections. I find that the optimal tax rates under the projected future demographics in the U.S. are roughly 2 to 5 percentage points higher than the current rate. I also find that population aging has a smaller impact on the relatively poor households who benefit from social security, as wealthier households respond by supplying more labor and picking up a larger tax burden. Finally, the model also predicts that population aging and the optimal tax response may imply a decline in the projected retirement benefits, but of a magnitude smaller than when the tax rate is held unchanged at the current level.

3.9 Appendix A: Computational methods

In this section I provide a discussion of the computational methods used in this chapter, including the development of the required computer codes. The computational tasks required in the current research can be broadly classified into two categories: symbolic math and numerical methods. The primary symbolic math required includes construction of appropriate algebraic expressions and suitably interfacing them for use with appropriate numerical optimization algorithms, and the numerical methods required include actually solving the interfaced algebraic expressions using contraction mapping algorithms. Both for its ability to handle complicated symbolic operations and providing a stable programming platform for developing suitable solver algorithms, I choose MATLAB™ version 7.4.0.287 as the main computational software, powered by an Intel Pentium™ T4200 Dual-Core CPU with 3 GB memory.

First, note that given the complex nature of the survival probability function $Q(s)$, the integrals identified in the various expressions in Section 2.3 do not have analytical closed form solutions. Therefore, I implement the trapezoidal method to approximate these integrals, which uses the idea that an integral is nothing but the limit of a sum. Specifically, I use the approximation

$$\int_a^b f(x)\,\mathrm{d}x \approx \left[0.5 \times \{f(a) + f(b)\} + \sum_{i=1}^{N-1} f(a + i\Delta)\right] \times \Delta \tag{3.68}$$

where $\Delta = (b - a)/N$, which implies that the domain $[a, b]$ has been divided into N equally spaced intervals.

First, note that as defined earlier, a stationary competitive equilibrium with an optimal social security tax rate in the current framework is characterized by a collection of cross-sectional consumption-saving and age-labor hour profiles, an aggregate capital stock, labor supply and a labor-to-retiree ratio, a real rate of return and wage rate, a social security tax rate and an accidental bequest that solves the household's optimization problem, satisfies the aggregation conditions, equilibrates the factor markets, satisfies the social security budget and the bequest-balancing conditions, and maximizes steady state expected life-cycle utility. Therefore, the computational exercise can be broken down into the following steps:

1. Solve for the household optimum for a given set of factor prices, a given labor-to-retiree ratio, a given accidental bequest, a social security tax rate and given values for the other model parameters.

2. Using the aggregation conditions, find the factor prices, the labor-to-retiree ratio and the accidental bequest consistent with the household optimum. The general equilibrium is obtained when the factor prices, the labor-to-retiree ratio and the accidental bequest values computed from the household optimum match the given factor prices, the labor-to-retiree ratio and the accidental bequest values that were used to compute the household optimum.

3. Finally, repeat the above two steps for different social security tax rates until the value that maximizes steady state expected life-cycle utility is found. At the end of this step, we have computed a stationary competitive equilibrium with an optimal social security tax rate for a given set of model parameters.

To sequentially accomplish these three steps, I define a contraction mapping algorithm as follows:

- **Step 1:** Set the social security tax rate and the model parameters to some values and guess some values for the factor prices, the labor-to-retiree ratio and the accidental bequest (label as vector `xin`).

- **Step 2:** Solve the household's utility maximization problem for the given factor prices, the labor-to-retiree ratio and the accidental bequest in the vector `xin`.

- **Step 3:** Compute the aggregate capital stock, labor supply, the labor-to-retiree ratio and the accidental bequest that are consistent with the households' utility maximizing consumption-saving and labor hour profiles obtained in **Step 2**.

- **Step 4:** Compute the market-clearing factor prices consistent with the aggregate capital stock and labor supply obtained in **Step 3**, and store the factor prices and the labor-to-retiree ratio in a vector `xout`.

- **Step 5:** Compute the percentage difference between vectors `xin` and `xout`, and store it in a vector `diff`.

- **Step 6:** If the 2-norm of the vector `diff` is greater than some tolerance parameter `Tol`, then update `xin` using the rule `xin = xin.*(1+step)`, where `step` is given by `step = diff/9`, and repeat steps 2 through 5.[12,13]

- **Step 7:** If the 2-norm of the vector `diff` is lesser than some tolerance parameter `Tol`, then terminate the algorithm.

At the end of these steps, the program returns a vector of factor prices, the labor-to-retiree ratio and an accidental bequest that solves the households' optimization problem, clears the factor markets and satisfies the social security budget and bequest-balancing conditions for a given OASI tax rate and model parameters. Then, to find the tax rate that maximizes welfare, I simply search over a grid of tax rates, repeating the steps 1 through 6 at every point, and finally choose that value at which steady state expected life-cycle utility is maximized. At the end of this step, the model returns a stationary competitive equilibrium with an optimal OASI tax rate for a given set of observable and unobservable parameters of the model. Calibrating the model to data targets simply involves repeating these procedures

[12]For a $n \times 1$ vector $[x_1 x_2 \dots x_n]'$, the 2-norm is defined as $\sqrt{\sum_{i=1}^{n} x_i^2}$.

[13]This implies that the algorithm updates the guessed vector by a factor that depends on the divergence between the guess and the feedback.

for different combinations of values for the unobservable parameters and choosing the one that produces model-generated values in reasonable neighborhood of the targets.

Note that as pointed out earlier, the household's optimization problem cannot be solved analytically because of the presence of an endogenous kink in the labor supply function at the date of retirement. Therefore, to solve the household's problem for a given vector of factor prices, the labor-to-retiree ratio and an accidental bequest, I use the following numerical procedure:

- **Step 1:** Guess a value for $\psi(0;\varphi)$.
- **Step 2:** Use the guess to generate the life-cycle profile for $E(s;\varphi)$ using equation (3.57).
- **Step 3:** Use the $E(s;\varphi)$ profile to pin down consumption and leisure over the life-cycle, which in effect pins down the income profile $y(s;\varphi)$.
- **Step 4:** Update the guessed $\psi(0;\varphi)$ and repeat steps 1-2 continuously until the present value of income is within reasonable tolerance of the present value of $E(s;\varphi)$ (i.e. the life-cycle budget constraint equation (3.59) is satisfied).

3.10 Appendix B: Pollution externality

In this section, I introduce another role for social security in the current general equilibrium model of life-cycle consumption with endogenous labor supply: management of a pollution externality that is positively related to aggregate capital stock. To do this, I simplify the model in two ways. First, I abstract from mortality risk, which reduces the number of equilibrium objects that require to be computed for model equilibrium (specifically, the accidental bequest). Second, I focus on only the extensive margin of a household's labor supply decision (i.e. retirement) and abstract from the labor hour choice (the intensive margin). This allows me to compute analytically closed form solutions for the household's optimization problem without resorting to numerical approximations.

I assume that there is a pollution externality (say "smoke") that increases with aggregate capital stock and reduces household utility. I re-specify the household's period utility function to incorporate the disutility from smoke S

$$u(c, l; S) = \begin{cases} \frac{\left(c^{\eta} l^{1-\eta}\right)^{1-\sigma}}{1-\sigma} - S & \text{if} \quad \sigma \neq 1 \\ \ln\left(c^{\eta} l^{1-\eta}\right) - S & \text{if} \quad \sigma = 1 \end{cases} \tag{3.69}$$

Also, to capture the positive relationship between the volume of smoke and aggregate capital stock, I assume

$$S(t) = \kappa K(t)^{\nu} \tag{3.70}$$

where κ and ν are positive constants. With this modification, life-cycle utility of a representative household (in the absence of mortality risk) changes to

$$\int_0^{\bar{T}} \exp\{-\rho s\} \frac{\left\{c(s;\varphi)^{\eta} l(s;\varphi)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \mathrm{d}s - S(0) \left[\frac{\exp\left\{(\nu(n+g)-\rho)\,\bar{T}\right\} - 1}{\nu(n+g)-\rho}\right]$$

and the maximized value of the objective functional (for a given T) used as the argument in the household's second-step problem changes to

$$V_{\mathrm{P}}(T;\varphi) = V(T;\varphi) - S(0) \left[\frac{\exp\left\{(\nu(n+g)-\rho)\,\bar{T}\right\} - 1}{\nu(n+g)-\rho}\right] \tag{3.71}$$

where the subscript P stands for "Pollution". Note that with this formulation, the household's utility maximization problem remains unchanged (as $S(0)$ is exogenous to the households), but the social planner's problem of choosing the optimal social security tax rate changes to account for the pollution externality. Life-cycle utility with the pollution externality is given by

$$U_{\mathrm{P}} = U - S(0) \left[\frac{\exp\left\{(\nu(n+g)-\rho)\,\bar{T}\right\} - 1}{\nu(n+g)-\rho}\right] \tag{3.72}$$

Table 3.27: Parameter values in the baseline calibration with the pollution externality.

σ	ρ	η	ω	$\zeta(\omega)$	κ	ν
3	-0.003	0.2449	0.15	0.65	0.25	0.3

Therefore, with the pollution externality, the social planner chooses the tax rate that satisfies

$$\theta = \arg\max \{U_{\mathrm{P}}\} = \arg\max \left\{ U - S(0) \left[\frac{\exp\left\{ (\nu(n+g) - \rho)\,\bar{T} \right\} - 1}{\nu(n+g) - \rho} \right] \right\} \tag{3.73}$$

With the introduction of the pollution externality, there are two more parameters κ and ν that must be calibrated. To do this, I choose values for κ and ν such that in the baseline stationary competitive equilibrium with an optimal social security tax rate, pollution causes a loss of 15% in steady state life-cycle utility. The values of the parameters for which the model reasonably matches the targets are reported in Table 3.27. With these parameter values, the model generates a steady state capital-output ratio of 3.2, an average retirement age of 62.48, an optimal social security tax rate of 10.51%, and a replacement rate of 98% for the poorest households. The equilibrium rate of return is $r = 0.0293$, the equilibrium stock of the pollution externality is $S = 1.08$ and it causes a roughly 14% decline in aggregate utility. The baseline distribution of the households' IRRs from social security is $\{0.0579, 0.0326, 0.0203, 0.0104, 0.0023\}$, and comparing these with the rate of return shows that the bottom two efficiency groups experience welfare gains from the social security program, whereas the top three groups suffer welfare losses. The baseline retirement age distribution is $\{62, 62, 62, 62.77, 63.64\}$, which shows that only the top two efficiency groups in the model continue to supply labor beyond the minimum retirement eligibility age.[14]

Once I have identified the baseline calibration of the model with the pollution externality, I examine the effect of the low-cost, intermediate and the high-cost projections (adjusted for the absence of mortality risk) on the welfare-maximizing social security tax rate and some other relevant variables. I report the results in Table 3.28. The table shows that

[14] I also assume that household retirement is synchronous with benefits collection, which is another simplification. Therefore, I restrict retirement to be $\geq T_{\min}$, where $T_{\min} = 37$ is the minimum eligibility age in the model (actual age of 62 in the U.S.).

Table 3.28: The effect of population aging on the calibrated model with the pollution externality.

Experiment	Baseline	1	2	3
Variable				
Social security tax rate	0.1051	0.0827	0.099	0.099
Replacement rate for the poorest households	0.98	0.79	0.8	0.71
Rate of return	0.0293	0.0276	0.0244	0.0201
Wage rate	1.22	1.23	1.25	1.28
Output	41.33	44.55	48.23	53.27
Capital	132.31	144.97	161.63	186.23
Stock of pollutant	1.08	1.11	1.15	1.2
Labor	22.09	23.6	25.15	27.15
Capital-output ratio	3.2	3.25	3.35	3.5
Decline in aggregate utility due to pollution(%)	13.58	13.57	13.83	14.23

with the pollution externality, the optimal response to all the demographic shocks is to reduce the social security tax rate: under the low-cost projection, the model predicts a decline of slightly over 2 percentage points from the baseline, and under the intermediate and the high-cost projections, it predicts a decline of 0.61 percentage points. The effect of population aging and the optimal tax response on the equilibrium real rate of return, the wage rate, output, aggregate capital stock, labor supply and the capital-output ratio are very similar to that observed in the model without the pollution externality. However, the table also shows that with the increase in capital stock, the equilibrium stock of pollutant also increases, and the decline in aggregate utility remains roughly unchanged at about 14%.

The factors that drive these changes in the optimal social security tax rate are virtually unaffected by the introduction of the pollution externality in the model. To see this, consider Tables 3.29 and 3.30, where I respectively report the households' IRRs from the social security program and the optimal (model) retirement ages in the post-population aging steady states with the social security tax rate held fixed at the baseline level of $\theta = 0.1051$. Table 3.29 reveals a very similar pattern of the impact of population aging on the IRRs of

Table 3.29: The effect of population aging on the households' IRRs with the pollution externality.

Experiment	$\beta_{\varphi=0.15}$	$\beta_{\varphi=0.3625}$	$\beta_{\varphi=0.575}$	$\beta_{\varphi=0.7875}$	$\beta_{\varphi=1}$	Rate of return
Baseline	0.0579	0.0326	0.0203	0.0104	0.0023	0.0293
1	0.0559	0.0303	0.0179	0.0072	-0.0011	0.0288
2	0.0548	0.03	0.0141	0.003	-0.0045	0.0246
3	0.0534	0.028	0.0101	0.0001	-0.0067	0.0203

Table 3.30: The effect of population aging on household retirement with the pollution externality.

Experiment	$\varphi = 0.15$	$\varphi = 0.3625$	$\varphi = 0.575$	$\varphi = 0.7875$	$\varphi = 1$	Average
Baseline	62	62	62	62.77	63.64	62.48
1	62	62	62	62.96	63.8	62.55
2	62	62	63.5	64.83	65.55	63.58
3	62	62.64	65.41	66.55	67.17	64.75

the different efficiency groups. Across the baseline and experiment 3, the IRRs of the five efficiency groups decline by 7.8, 14.1, 50.3, 99 and about 400%, respectively. Once more, it is clear that the effect on the households who actually benefit from the social security program is relatively small. Table 3.30 shows that the asymmetric retirement responses from the different efficiency groups are behind these IRRs: across the baseline and experiment 3, the retirement ages of the five quintiles increase by 0, 0.64, 3.41, 3.78 and 3.53 years respectively. The larger delays in retirement experienced by the more efficient households leads to their IRRs experiencing larger declines compared to the less efficient households.

I report in Table 3.31 the effective labor supply, the wage rate and the size of the tax base in post-population aging steady state equilibria (with the social security tax rate held fixed at the baseline level). This helps us understand the impact of population aging on the tax base of the social security program in the presence of the pollution externality. It is clear that even with the pollution externality, the delayed retirement and increased saving mechanisms lead to an expansion of the tax base under population aging.

Finally, it is worth examining the effect that the optimal tax response has on the equilibrium stock of the pollutant in post-population aging equilibria. In Table 3.32, I compare the

Table 3.31: The effect of population aging on the tax base of the social security program with the pollution externality.

Experiment	Baseline	1	2	3
Effective labor	22.09	23.19	25.02	27.01
Wage rate	1.22	1.22	1.25	1.27
Tax base (Wage rate × Effective labor)	26.87	28.27	31.17	34.41

Table 3.32: The effect of population aging on the equilibrium stock of the pollutant.

	1	2	3
Without the optimal tax response	1.1008	1.1469	1.1965
With the optimal tax response	1.1126	1.1495	1.1994

equilibrium stocks of the pollutant under population aging with and without the optimal tax response. It is clear from the table that the optimal tax response to population aging generates a higher equilibrium stock of the pollutant. This should not be surprising, as a lower social security tax rate further encourages private saving and aggregate capital accumulation, and therefore a higher level of the pollution externality. Even though the social planner's response to the demographic shocks in welfare-maximizing, the model predicts that it may come at the cost of a higher equilibrium stock of the pollutant.

To summarize, the household-level consumption-saving and retirement mechanisms, and the aggregate factor price adjustment mechanisms continue to operate in very similar ways with the pollution externality in the model.

CHAPTER 4

CONCLUSIONS

In this dissertation, I attempt to achieve two objectives. First, I provide an alternative estimate of the decline in the projected retirement benefits under population aging in the U.S., while accounting for the household-level and macroeconomic adjustments that may be associated with it. I find that when the household consumption-saving and retirement responses, as well as the aggregate factor price adjustments are accounted for, the decline in projected retirement benefits is significantly smaller than the commonly reported estimates of the social security crisis. This result is driven by the fact that the households respond to a higher life expectancy by delaying retirement and also by saving more, which leads to a natural expansion of the tax base of the social security program through the associated general equilibrium effects. The model also predicts that ignoring either the household retirement mechanism or the aggregate factor price mechanism could lead to a roughly comparable overestimation of the decline in the projected benefits due to population aging in the U.S. Sensitivity analysis with respect to the values of several underlying parameters used in the simulations demonstrates that these results are not calibration-specific.

Second, I examine welfare-maximizing social security reform in the U.S. under the projected future demographics using a general equilibrium model of life-cycle consumption with endogenous retirement and labor efficiency heterogeneity. I find that if the role of the current U.S. social security program is to partially insure households against old-age poverty through a concave benefit-earnings rule, then the population aging projected in the near future will require 2-5 percentage points increase in the current OASI tax rate. Also, the model predicts that under the projected future demographics, the more efficient households are likely to respond by supplying more labor (both in hours per week and retirement age), which will to transfer a larger tax burden away from the groups that actually experience welfare gains from the social security program. Moreover, the model also predicts that population aging and the optimal tax response may imply a decline in the projected retirement benefits, but of a magnitude smaller than when the tax rate is held unchanged at the current level. I also find that these results are fairly robust with respect to the underlying values of the model parameters used in the simulations.

However, it is important to note here that all the above results are based on models that abstract from a number of realistic features such as income uncertainty and borrowing constraints. Also, I ignore the transition paths between pre- and post-population aging steady states and focus on only steady state equilibria. Therefore, one must be cautious before using these recommendations for the purpose of definitive policymaking.

REFERENCES

Arias, E., 2004. United States life tables, 2001. National Vital Statistics Reports 52 (14).

Attanasio, O. P., Emmerson, C., 2003. Mortality, health status, and wealth. Journal of the European Economic Association 1 (4), 821–850.

Bullard, J., Feigenbaum, J., 2007. A leisurely reading of the life-cycle consumption data. Journal of Monetary Economics 54 (8), 2305–2320.

Caliendo, F. N., Gahramanov, E., 2009. Hunting the unobservables for optimal social security: A general equilibrium approach. Public Finance Review 37 (4), 470–502.

Conesa, J. C., Garriga, C., 2008a. Optimal fiscal policy in the design of social security reforms. International Economic Review 49 (1), 291–318.

Conesa, J. C., Garriga, C., 2008b. Optimal response to a transitory demographic shock in social security financing. In: Fenge, R., de Ménil, G., Pestieau, P. (Eds.), Pension Strategies in Europe and the United States. MIT Press, Cambridge, MA.

De Nardi, M., French, E., Jones, J. B., 2006. Differential mortality, uncertain medical expenses, and the saving of elderly singles. Working Paper 12554, National Bureau of Economic Research.

De Nardi, M., İmrohoroğlu, S., Sargent, T. J., 1999. Projected U.S. demographics and social security. Review of Economic Dynamics 2 (3), 575–615.

Feigenbaum, J., 2008. Can mortality risk explain the consumption hump? Journal of Macroeconomics 30 (3), 844–872.

Feldstein, M., 1985. The optimal level of social security benefits. The Quarterly Journal of Economics 100 (2), 303–320.

Feldstein, M., 2005. Structural reform of social security. The Journal of Economic Perspectives 19 (2), 33–55.

Findley, T. S., Caliendo, F. N., 2009. Short horizons, time inconsistency, and optimal social security. International Tax and Public Finance 16 (4), 487–513.

Gendell, M., 1999. Trends in retirement age in four countries, 1965-95. Monthly Labor Review, Bureau of Labor Statistics, U.S. Department of Labor.

Gendell, M., 2001. Retirement age declines again in 1990s. Monthly Labor Review, Bureau of Labor Statistics, U.S. Department of Labor.

Goss, S. C., 2006. The financial outlook for the social security disability insurance program. Social Security Bulletin 66 (3), 47–52.

Hansen, G. D., 1993. The cyclical and secular behaviour of the labour input: Comparing efficiency units and hours worked. Journal of Applied Econometrics 8 (1), 71–80.

Hansen, L. P., Heckman, J. J., 1996. The empirical foundations of calibration. The Journal of Economic Perspectives 10 (1), 87–104.

Huggett, M., 1996. Wealth distribution in life-cycle economies. Journal of Monetary Economics 38 (3), 469–494.

Huggett, M., Ventura, G., 1999. On the distributional effects of social security reform. Review of Economic Dynamics 2 (3), 498–531.

İmrohoroğlu, A., İmrohoroğlu, S., Joines, D. H., 1995. A life cycle analysis of social security. Economic Theory 6 (1), 83–114.

İmrohoroğlu, A., İmrohoroğlu, S., Joines, D. H., 2003. Time-inconsistent preferences and social security. The Quarterly Journal of Economics 118 (2), 745–784.

Kamien, M., Schwartz, N. L., 1992. Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management, 2nd Edition. Elsevier, North Holland.

Köbberling, V., 2006. Strength of preference and cardinal utility. Economic Theory 27, 375–391.

Kotlikoff, L., 1997. Privatizing social security in the United States: Why and how. In: Auerbach, A. J. (Ed.), Ficsal Policy: Lessons From Economic Research. MIT Press, Cambridge, MA.

Nishiyama, S., Smetters, K., 2005. Does social security privatization produce efficiency gains? Working Paper 11622, National Bureau of Economic Research.

Ortiz, J. A., 2009. Social security and retirement across oecd countries. Tech. Rep. 18563, University Library of Munich, Germany.

Sheshinski, E., 1978. A model of social security and retirement decisions. Journal of Public Economics 10 (3), 337–360.

Waldron, H., 2007. Trends in mortality differentials and life expectancy for male social security-covered workers, by average relative earnings. Working Paper 108, Social Security Office of Research, Analysis and Statistics Working Paper Series.

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- **Under Review:**
  - "Is the Social Security Crisis Really as Bad as the SSA's Projections?", Spring 2010 **(Job Market Paper)**
  - "Optimal Pension Reform under Demographic Shocks and Endogenous Retirement", Spring 2010

- **Working Papers:**
  - "Is Smoking a Fiscal Good?" (with James Feigenbaum), Spring 2010
  - "A General Equilibrium Analysis of Pesticide Resistance in a Two-sector Stochastic Neoclassical Growth Model" (with Kenneth S. Lyon), Fall 2008
  - "The Time Frame for Free Trade: A Game-Theoretic Approach", Fall 2007
- **Work in Progress:**
  - "Should Social Security Redistribute Wealth?" (with Frank N. Caliendo), Summer 2010
  - "Social Security and Saving under Endogenous Retirement: Micro and Macro Consequences", Spring 2010

Selected Presentations:

- "Optimal Pension Reform under Demographic Shocks and Endogenous Retirement", Midwest Macroeconomics Meetings, Michigan State University, East Lansing, MI (Summer 2010); QSPS 2010 Summer Workshop, Logan, UT (Summer 2010)
- "Is Smoking a Fiscal Good?" (with James Feigenbaum), Midwest Macroeconomics Meetings, Michigan State University, East Lansing, MI (Summer 2010)
- "Demographic Shocks, Retirement Decisions and Optimal Social Security", Economics Department, Weber State University, Ogden, UT (Fall 2009)
- "Social Security Solvency under Endogenous Retirement: A General Equilibrium Analysis" (with T. Scott Findley), QSPS 2009 Summer Workshop, Logan, UT (Summer 2009); Applied Economics Graduate Seminar, Utah State University, Logan, UT (Fall 2008)
- "A General Equilibrium Analysis of Pesticide Resistance in a Two-sector Stochastic Neoclassical Growth Model" (with Kenneth S. Lyon), Annual Conference of the Re-

source Modeling Association, Juneau, AK (Summer 2009); Applied Economics Graduate Seminar, Utah State University, Logan, UT (Spring 2009)

- "Can Overconfidence Explain the Consumption Hump?: A General Equilibrium Inquiry", Economics Seminar Series, Illinois State University, Normal, IL (Spring 2008)

Computer Skills:

- Numerical simulation: MATLAB, Microsoft Excel
- Statistics and Econometrics: MATLAB, EViews
- Symbolic math: MATLAB, Mathematica
- Document processing: LaTeX, OpenOffice.org

Honors and Awards:

2010	Hemingway Adjunct Faculty Vitality Award, Weber State University
2010	Economics Department Conference Travel Award, Weber State University
2010	College of Agriculture Travel Grant, Utah State University
2006-10	Graduate Assistantship, Utah State University
2006	The University Medal for the first position in M.A. (Economics), Jadavpur University, Kolkata, India
2006	Susanta K. Mitra Memorial Gold-centered Medal for highest marks in Economics, Jadavpur University, Kolkata, India
2006	The Chattagram Parishad (Jadavpur) Award for the first position in M.A. (Economics), Jadavpur University, Kolkata, India

Service:

- Department Representative to the USU Graduate Student Senate, 2007-08