Maximizing Reliable Crop Production in a Dynamic Stream/Aquifer System

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ABSTRACT

A procedure for planning the optimal spatial distribution of crops to be reliably irrigated by conjunctive use of water resources is presented. The implicitly stochastic procedure utilizes a simulation/optimization model of a stream/aquifer system. The model utilizes linear optimization, hydrologic influence coefficients and time-varying crop water production functions. It appropriately simulates the time variant, interdependent responses of groundwater levels, stream stages and stream/aquifer interflow to groundwater pumping and diversion of river water to nonriparian lands. It determines the temporally and spatially varying distribution of groundwater and diverted river water that should be utilized in order to maximize annual crop yield in a water management district. The diversion of river water to nonriparian land and stream/aquifer interflow are constrained such that effluent from the district through the river satisfies downstream requirements. The model can be used to develop optimal seasonal water use strategies that are in harmony with long-term water use and agricultural development strategies. In that case it represents a suboptimization model applicable for either a period of regional potentiometric surface evolution or a steady-state era. If applied during an era in which the potentiometric surface is maintained at relatively constant elevations, groundwater pumping and recharge are managed such that groundwater levels return to their initial elevations by the end of a one-year simulation period. Thus, if the initial elevations are satisfactory, the optimal strategy is a safe sustained yield conjunctive water management strategy.

INTRODUCTION

The stochastic nature of streamflow is generally accepted and has led to the widespread use of synthetic hydrologic modeling in surface water studies. The random nature of streamflow is an important consideration in an area where crop yield is dependent on applied surface water as well as groundwater. However, the vast majority of modeling efforts that involve systems with stream/aquifer interaction components do not incorporate this stochasticity.

This paper describes an implicitly stochastic optimization (ISO) procedure that couples inflow information (having an associated level of reliability) with a stream/aquifer system model. The purpose of the modeling effort is to develop strategies for groundwater pumping and river water diversion that minimize the reduction in crop yield caused by inadequate water. Such strategies provide valuable guidelines for cropping pattern selection and water management in an irrigation district. Because existing water rights may conflict with regionally optimal strategies, the presented procedure is primarily applicable to developing regions.

Application of the methodology has two components: (a) inflow modeling, and (b) system modeling. In the first, the statistical characteristics of the inflow process and prespecified probability levels establish infall magnitudes for which optimal strategies are to be developed. In the second, the regionally best conjunctive use strategy is determined by an optimization model that adequately represents the dynamic nature of the stream/aquifer system. Optimal strategies are systematically developed for a range of inflows of known probability of occurrence. Because the probability of receiving a specific volume of irrigation water in each cell is then known, these optimal strategies are helpful in assessing where crops should be planted to have the best chance of being irrigated. The methodology is applied to a hypothetical area for illustrative purposes.

PREVIOUS WORK

The estimation of the inflow model from available surface water data has led to a distinct discipline of hydrologic modeling. Jackson (1975) provides a comprehensive and critical discussion of the models developed before 1970. Of the numerous models that are available, linear stochastic models of the inflow process have gained acceptance. Salas et al. (1980) provide a detailed and instructive discussion of this group of models. Thus, methods for finding a process that adequately represents the stochasticity of inflow are well documented.

Many stream/aquifer simulation models have been reported. Maddock (1974), Morel-Seytoux (1975), Illangasekare and Morel-Seytoux (1982) and Danskin and Gorelick (1985) are a few examples. Gorelick (1983)
provides a review of models oriented toward facilitating water management decision-making. Very few of the models address the reliability of the surface water resource and its consequences on irrigated agricultural planning. Furthermore, to our best knowledge no reported optimization models have included simulation of stream/aquifer interflow that is a function of simultaneously existing interdependent variable stream and aquifer head levels.

MODEL DEVELOPMENT AND ASSUMPTIONS

Governing Equations

The following theory is appropriate for a scenario in which the objective is to maximize crop yield in an irrigation or water management district (Fig. 1). Assume that water supply is inadequate to meet total irrigation requirements. Assume that crop yield can be described as a function of the timed availability of water. Let the result of having unsatisfied water requirements be expressed as a reduction in yield from that which would be obtained if irrigation water needs were completely satisfied. Thus, the objective can be simply restated as minimizing the reduction in crop yield caused by inadequate water supply.

\[
\text{max Yield} = \text{Potential Yield} - \min \text{Reduction in Yield}
\]  \hspace{10mm} [1]

The minimum reduction in yield caused by inadequate water availability during \( K \) time steps in a system consisting of \( J \) cells is expressed as:

\[
\text{min Reduction in Yield} = \sum_{i=1}^{J} y_i \sum_{k=1}^{K} (c_{i,k} u_{i,k} / w_{i,k})
\]  \hspace{10mm} [2]

where

\( y_i \) = the maximum potential annual crop yield from a cell \( i \) assuming that irrigation water needs are completely satisfied throughout the growing season, known, \( M \)

\( u_{i,k} \) = the volume of unsatisfied water needs in cell \( i \) in time step \( k \), unknown, \( L^3 \)

\( w_{i,k} \) = the volume of water (including irrigation and effective precipitation) required in cell \( i \) in time step \( k \) in order to produce the maximum potential yield, known, \( L^3 \)

\( c_{i,k} \) = a dimensionless crop loss coefficient. It equals the proportional reduction in the annual potential yield in cell \( i \) that results from a proportional lack of adequate irrigation water in time step \( k \), known by site-specific studies

\( K \) = the number of time steps in the planning period, known

\( u_{i,k} / w_{i,k} \) = the proportion of water needs in cell \( i \) in time step \( k \) that are unsatisfied

A complete management model requires, in addition to an objective function (equation [2]), the inclusion of pertinent bounds on variables and constraints to assure that physical and institutional limits are appropriately considered and that the hydrologic system is modeled adequately. Assume a study area underlain by an aquifer that is in hydraulic connection with a stream passing through the region. If there are practical or legal limits on how much groundwater and diverted river water can be used to attempt to satisfy water demand, a simple statement of bounds to be considered (assuming discharge to be positive in sign and recharge to be negative) is:

\[
0 \leq u_{i,k} \leq w_{i,k} \text{ for } i = 1 \ldots J, k = 1 \ldots K \ldots \text{ [3]}
\]

\[
0 \leq \theta_{i,k} \leq w_{i,k} \text{ for } i = 1 \ldots J, k = 1 \ldots K \ldots \text{ [4]}
\]

\[
0 \leq r_{i,k} \leq w_{i,k} \text{ for } i = 1 \ldots J, k = 1 \ldots K \ldots \text{ [5]}
\]

\[
s_{i,k} \leq s_{i,k}^U \text{ for } i = 1 \ldots J, k = 1 \ldots K \ldots \text{ [6]}
\]

\[
e_{i,k}^L \leq e_{i,k} \leq e_{i,k}^U \text{ for } i = 1 \ldots J, k = 1 \ldots K \ldots \text{ [7]}
\]

\[
\sigma^L_{m,k} \leq \sigma_{m,k} \leq \sigma^U_{m,k} \text{ for } m \in \mathbb{R}, k = 1 \ldots K \ldots \text{ [8]}
\]

where

\( \theta_{i,k} \) = the groundwater that is pumped from the aquifer and used for irrigation in cell \( i \) in time step \( k \), unknown, \( L^3 \)
\[ r_{i,k} = \text{the river water that is delivered to cell i in time step k and used for irrigation, unknown, } L^3 \]

\[ s_{i,k} = \text{the difference in groundwater level at the center of cell i between the initial level and the level at the end of time step k, unknown, L} \]

\[ s'_{i,k} = \text{the upper bound on acceptable drawdown in cell i by the end of period k, known, L} \]

\[ e_{i,k} = \text{the volume of groundwater that will enter the study area aquifer in cell i and time step k from extensions of the aquifer outside the study area, unknown, L} \]

\[ e'_{i,k} = \text{lower and upper bounds on the volume of groundwater flowing between the aquifer underlying cell i and extensions of the aquifer outside the study area in time step k, known, L} \]

\[ \sigma_{m,k} = \text{the stage of water flowing in the stream in cell m in time step k, unknown, L} \]

\[ \sigma'_{m,k} = \text{lower and upper bounds on acceptable stream stage elevations, known, L} \]

\[ R = \text{a set of cell numbers containing river reaches} \]

In the model presented in this report, each \( w_{i,k} \) is a constant and \( u_{m,k} \), \( r_{i,k} \) and \( \sigma_{m,k} \) are actual variables, permitting equations [3], [5] and [8] to be included within the model exactly as shown above. To reduce computer memory requirements, some of the equations shown above actually exist within the model in a different form. These modifications are described later.

If one assumes that groundwater and diverted river water (no precipitation is effective in satisfying crop water demand) are the only sources of water, the relationship between groundwater use, water needs, river water use and unmet needs at any cell is:

\[ s_{i,k} + r_{i,k} + u_{i,k} = w_{i,k} \] \[ \text{[9]} \]

Equation [9] maintains the water volume balance at the ground surface (field). Because equation [9] is used, one of equations [3], [4] or [5] is not absolutely essential. However, all are included because the computation time of many optimization algorithms is improved by specifying bounds for the variables, if the bounds are known with certainty, rather than utilizing unbounded variables.

The bounding conditions specified by equations [6] and [7] can be satisfied simultaneously by: (a) replacing the left-hand side (LHS) of equation [6] with a function that describes aquifer response to the hydraulic stimuli of pumping and flow in the river, and (b) converting the recharge bounds specified by equation [7] into drawdown bounds that can be included within the RHS of equation [6]. The following discretized form of the convolution equation (Peralta et al., 1987), is used in the first step. (This expression of head response to pumping and stream-aquifer interflow is similar to an approach taken by Illangasekare and Morel-Seytoux in 1982).

\[ s_{i,j} = \sum_{k=1}^{N} \sum_{j=1}^{J} \beta_{i,j,k} \left( s_{j,k} - h_j \right) \]

\[ - \mu_{i,j,N-k+1} \left( \sigma_{j,k} - h_j \right) \] \[ \text{[10]} \]

where

\[ \beta_{i,j,N-k+1} = \text{a nonnegative-valued linear resolvent influence coefficient that describes the effect on the hydraulic head at cell i in time step N caused by } (q_{i,k} - q_j) \text{. The temporal subscript } N-k+1 \text{ is used merely to insure that the proper } \beta \text{ is utilized in each time step, known, } T/L^2 \]

\[ s_{i,k} = \text{the net vertical hydraulic stimulus in cell j in time step k, not including stream-aquifer interflow. It is the sum of all vertical discharges from the aquifer and recharge to the aquifer from the ground surface, unknown, } L^2/T \]

\[ q_j = \text{the net vertical hydraulic stimulus, not including stream-aquifer interflow, that must occur in each time step in cell j in order for that cell to maintain its initial head. It is calculable using the linearized Boussinesq equation for steady-state two-dimensional flow through porous media (Illangasekare et al., 1984) and does not necessarily represent a steady-state stimulus that is actually occurring initially, } L^2/T \]

\[ \mu_{i,j,N-k+1} = \Gamma \beta_{i,j,N-k+1}, \text{ a dimensionless resolvent influence coefficient. It describes the effect on groundwater levels caused by the stream-aquifer interflow} \]

\[ \Gamma_x = \text{the volumetric reach transmissivity in stream-aquifer cell x for a time step of known duration, } L^2. \text{ Cells without stream-aquifer interflow have a zero reach transmissivity and stream stage} \]

\[ h_j = \text{initial groundwater table elevation in cell j, known, L} \]

Through the use of the \( \beta \) and \( \mu \) influence coefficients, equation [10] maintains the balance of water within the aquifer. Although it may appear in equation [10] that stream/aquifer interflow is based on the difference between stream stage and the initial water table elevation in the aquifer underlying the stream, this is not the case. Drawdown computed by equation [10] actually includes consideration of interflow in the same time step and is based on the difference between stream stage and water table elevations in that time step. Peralta et al. (1987) detail the implicit procedure for resolving influence coefficients and rearranging the discretized convolution integral into the form presented above. The resulting expression requires assuming values of stream stage for each time step and cell. If assumed values of stream stage are not similar enough to optimal values
computed by the optimization model, resolvent influence coefficients should be recomputed. This iterative procedure should be continued until assumed and computed optimal stream stages are sufficiently alike. At that point, the resolvent influence coefficients are accurate and the drawdown, \( s_{i,k} \cdot N \), computed by equation [10] is actually based on all previous and simultaneously existing stream stages. Since, as described later, stream stages are also computed as a function of groundwater pumping and diversion, by the time the influence coefficients are computed with precision, they are completely descriptive of the water table response to stimuli.

Before applying equation [10] to a study area, pertinent hydrogeologic information should be provided. Assume an unconfined aquifer system comprised of internal variable-head cells surrounded entirely by constant-head cells. (Alternatively, one can assume an aquifer that is confined with a completely nonleaky cap in all cells except those containing the main canal.) The only discharges from the aquifer that can occur at internal cells are at pumping wells or at the stream that is in hydraulic connection with the aquifer. Recharge to the aquifer at internal cells can only occur at the stream. No other deep percolation through the soil profile is assumed. Thus \( q_{l,k} \) replaces \( q_{l,k} \cdot m \) in equation [10].

The drawdown constraints in the RHS of equation [6] are useful if it is desirable that groundwater levels in internal cells decline no more than a predetermined distance from initial levels by the end of the planning period. The acceptable decline may be the change in head desired to occur based on a long-term regional water use strategy, for example during an era of evolution toward a target potentiometric surface. Alternatively, the acceptable decline may be very small, thus assuring that groundwater levels are relatively stable over the long term (a sustained-yield scenario). When the purpose of using the constraint is for water levels to be near initial elevations by the end of the planning period, declines during intermediate steps are generally not constrained. The result may be a strategy that causes decline during the first part of the planning period and permits water level recovery during the latter part.

Preventing groundwater levels from deviating too significantly from initial levels during the year permits assuming that transmissivity is relatively constant in time. This assumption helps to justify simulating water table response to hydraulic stimuli via convolution equation [10]. A common rule of thumb is that superposition and linear systems theory are applicable to unconfined aquifer systems as long as changes in transmissivity caused by changing saturated thickness does not exceed ten percent of the initial value (Reilly et al., 1987). Preventing significant changes in water table elevations can satisfy this criterion.

The conditions of equation [7] are important if the aquifer underlying the study area is simulated as being bounded by constant-head cells and if it is necessary that the volume of groundwater entering the study area through the aquifer in these cells must be less than some physically or institutionally-based limit. A physically-based limit is needed for situations in which a "constant-head" cell is not located at a hydrologically infinite source. In such a case, there is a potentially calculable upper limit of groundwater that can enter the study area through such a cell without causing that cell's head to change significantly. An institutionally-based limit is needed if the district is authorized to induce no more than a predetermined rate of recharge along its boundaries. In either situation, the simulated recharge that occurs at a "constant-head" cell in response to a pumping strategy can be calculated from Darcy's Law using the hydraulic gradients between the peripheral cells and adjacent internal cells. Similarly, simulated recharge rates can be forced to adhere to predetermined recharge contraints by imposing limits on groundwater levels in internal cells that are adjacent to constant-head cells (Peralta and Killian, 1985). Such contraints may be imposed during all time steps of the planning period.

In practice, equation [7] is omitted and the value used for the RHS of equation [6] \( s_{l,j} \) is the lesser of: (a) the maximum acceptable decline in groundwater levels from initial water table elevations based on the desire for stable water levels and (b) the maximum possible decline that will not cause recharge constraints to be violated. For each time step this assures that the optimal strategy will not cause unacceptable head declines in internal cells and will not induce unacceptable recharge across peripheral cells.

In practice, because the objective function will attempt to pump as much groundwater as possible to minimize crop yield reduction, there is no need to place a lower bound or nonnegativity constraint on drawdown in equation [6]. Similarly, because the model will attempt to induce as much recharge as possible, and recharge is negative in sign, it is not necessary to impose an upper (positive) bound on recharge shown in equation [7].

Even though equation [8] may be used directly to assure that optimal primary canal depths are acceptable, insuring physical realism in the river requires use of the continuity equation. In this model, continuity is maintained within the canal reach that exists between the centers of each pair of adjacent main canal cells. The following equation, applied to R-I such reaches and K time steps, describes the volume of outflow at the downstream end of the reach between cells \( m \) and \( i \) during time step \( N \).

\[
\begin{align*}
V_{i,N} &= V_{m,N} - V'_{i,m,N} - V''_{i,m,N} - \Delta V_{i,m,N} \ldots \ldots \ldots [11] \\
V &= \text{the volume of river water flowing out of the reach and i,N past the center of cell i in time step N, L}^3 \\
V_{m,N} &= \text{the volume of river water flowing into the reach and past the center of cell m in time step N, L}^3 \\
V_{i,m,N} &= \text{the volume of water that is diverted from main canal between the centers of cells m and i during time step N, L}^3 \\
V'_{i,m,N} &= \text{the volume of water that seeps from main canal to the aquifer between the centers of cells m and i during time step N, L}^3 \\
\Delta V_{i,m,N} &= \text{the change in volume of water in storage in the main canal between the centers of cells m and i that occurred during time step N, L}^3 
\end{align*}
\]

Substituting for the components of equation [11] term by term, without rearranging, yields:
\[ D_i (a_{i,N} - b_i) = D_m (a_{m,N} - b_m) - (d_{i,N} + d_{m,N})/2 \]
\[-\{(\Gamma_i + \Gamma_m) [(a_{i,N} - b_i) - (h_i^0 - b_i) + s_i,N] + (a_{m,N} - b_m) - (h_m^0 - b_m) + s_{m,N}] \}/4\]
\[-\{(a_{i,N} - b_i) - (a_{i,N-1} - b_i) + (a_{m,N} - b_m) \} \] 
\[ (W_i Y_i + W_m Y_m)/4 \] \[ \text{where} \]
\[ D_x \] = the linear stage-volume ratio for the stream at the center of cell x, known, \[ L^2/L \]
\[ b_x \] = the elevation of the bottom of the stream at the center of cell x, (L). Thus, \((j_i - b_i)\) is the depth of water in the stream at that point
\[ d_{x,N} \] = the volume of water diverted from the river through canals in cell x during time step N, unknown
\[ W_s \text{ and } Y_s \] = the width and length of the stream in cell x, known, L

The formulation of the second term in the RHS of equation [12] shows that we assume that half of the water diverted from the river in a cell is diverted upstream of the cell’s center and half is diverted downstream of the center. Note that this ratio may be different for a particular reach, depending on the design of the diversion canal system. The third term in the RHS is simply the average reach transmissivity times the average difference between the river stage and the water table in time step N between cells i and m. Note that many of the stream bottom elevations, \( b \), in the third term may be cancelled.

Since the volume of river water diverted at a particular location does not explicitly exist as a variable in the model as formulated, it must be defined in terms of delivered river water. Assuming no seepage losses from the lateral diversion canals and an appropriate passage time, the total diverted river water equals the total delivered river water for a particular time step. The following assures that a volume balance is maintained in the diversion canals.

\[ \sum_{i=1}^{J} d_{i,N} = \sum_{j=1}^{J} r_{j,N} \] \[ \sum_{j=1}^{J} f_{j,i,N} r_{j,N} \] \[ \sum_{j=1}^{J} f_{i,j,N} r_{j,N} \]

With prior knowledge of the diversion canal system design, the following can be stated.

Substituting the RHS of equation [14] for \( d \) in equation [12], moving unknowns to the left side and leaving knowns on the right yields:

\[ \sum_{i=1}^{J} f_{i,j,N} r_{j,N} = 1.0 \] \[ \sum_{j=1}^{J} f_{j,i,N} r_{j,N} \] \[ \sum_{j=1}^{J} f_{i,j,N} r_{j,N} \]

In this formulation it is assumed that the canal water depth at the influent cell is a known constant during a time step. For simplicity, the following assumptions are also made (changing the model to handle different assumptions is not difficult). Rainfall is insignificant, i.e., during the cropping season, rainfall will cause no runoff to the rivers, no deep percolation to the aquifer and no change in crop yield. (In arid regions irrigation systems are commonly designed with the assumption that rainfall may not contribute to crop development). No deep percolation or return flow will result from irrigation. If one wishes to assume that \( r \) percent of utilized groundwater returns to the aquifer, one would use \((1 - 0.05)\) instead of \( g \) in equation [10]. Conveyance efficiency of the diversion canals (lined) is 100 percent. A nonlinear stage-discharge relation can be represented for the main canal using linearization. An additional assumption is that the time needed to stabilize flow depths within the main canal is small with comparison to time-step size.


**APPLICATION AND RESULTS**

A hypothetical study area (potential water management district) is shown in Fig. 1. It is proposed that water be conveyed in an unlined main canal through the area and that some water be diverted through lined canals for irrigation. The district is underlain by an unconfined, unconsolidated aquifer that extends beyond...
the study area in all directions. As commonly the case, the boundaries of the potential management district do not coincide with hydrologic boundaries.

Decision-makers (DMs) wish to evaluate the desirability of installing the canal system. Particularly, they wish to develop tentative optimal water allocation strategies for alternative stream inflow stages. Resulting information is valuable in identifying areas that will probably have groundwater or diverted river water available for irrigation. This in turn aids in determining where crops should be planted.

The hydrologic/institutional setting requires that implemented strategies assure that currently existing springtime water levels (Fig. 2) are regained by the beginning of the subsequent spring (i.e., a sustained yield scenario). This is assured via a constraint on final water table elevations. In addition, the strategy should not cause a disruption in regional groundwater flow regimes. Thus, constant-head/restrained-flux cells are used for district boundaries. The entire aquifer system that surrounds the study area is in quasi-steady-state. DMs assume that as long as a selected strategy does not induce more than historic groundwater flow across boundaries, existing potentiometric heads at boundaries will continue to exist over the long-term.

The aquifer is assumed to have an effective porosity of 0.3 and transmissivities computed using saturated thickness and a hydraulic conductivity of 82.3 m/day. Discrete kernels are generated using procedures developed by Verdin et al. (1981) and Peralta et al. (1987). Crop loss coefficients for three-month halves of a growing season are assumed to be 0.32 and 0.62. (Such coefficients are site-specific and not commonly available. Because of uncertainty in these coefficients, it is generally advisable to perform sensitivity analysis—to demonstrate how computed strategies differ depending on the coefficient values). All other data required as constants by the model are assumed.

For the simple example in this paper, assume that upstream water managers can guarantee that the influent stream can be maintained at constant stage during the growing season, although they cannot guarantee what that stage will be. (The model can process time varying influent stream stages, but there must be at least one time step associated with each different stage). Based on historic success in managing upstream reservoirs, DMs assume the population of actual influent depths to be normally distributed. Assume a mean depth of 3.0 m and depths of 3.6 m and 2.4 m for alphas of 0.05 and 0.95 respectively (Fig. 3). (For example, ninety-five percent of the time the influent depth exceeds 2.4 m). Before looking at how the optimization model may be used in agricultural planning, let us examine representative optimal allocation strategies.

Optimal conjunctive allocation strategies are developed for all three depths using the described optimization model. Fig. 4 summarizes optimal production values for each strategy. Production is clearly limited by water availability. Fig. 5 displays seasonal field, canal and aquifer volume balances for the strategies.

Fig. 5 shows that water needs are the same, regardless of strategy. Since unsatisfied demand is so great, crop production is clearly limited by water availability. As canal depth increases, the volume of unsatisfied demand decreases, diverted canal water and pumped groundwater increase. Pumped groundwater increases because of increased flow from stream to aquifer.

Flow into the system increases linearly with canal depth (in accordance with the linear stage/discharge relation). Because of the 0.6 m constraint on minimum acceptable effluent stream depth, the volume of water leaving the system through the canal is the same for all

\[ d = 2.4 \quad 3.0 \quad 3.6 \text{ m} \]

Reduction in crop production due to unsatisfied demand

<table>
<thead>
<tr>
<th></th>
<th>31.0</th>
<th>30.4</th>
<th>29.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total crop production</td>
<td>62.7</td>
<td>63.3</td>
<td>63.9</td>
</tr>
</tbody>
</table>

Fig. 4—Annual crop production consequences of optimal strategy implementation (in 10^4 kgs) for three influent river stages, d.
achieving at least the production computed by the model for him, using the 2.4 m influent stream depth. Fig. 6 contains similar practical guidance for planting practice. It shows the percent confidence users in different cells can have of achieving at least 40% of potential production. Tables analogous to Fig. 6 can be prepared to show the probability of achieving (at least) a greater or lesser proportion of potential production. Forty percent was selected arbitrarily for Fig. 6.

Since only influent depths with 5, 50 and 95% probabilities are tested, those are the only probabilities that can be displayed in Fig. 6. If optimal conjunctive water use strategies had been developed for 20 different probability levels, Fig. 6 could contain up to 20 different probability values. Furthermore, the results in Fig. 6 would differ for some cells if more optimizations had been performed. For example, one cell currently shows a five percent probability. This cell achieved at least 40% of potential production when influent stage was 3.6 m. (For this case alpha = 0.05 and one is 5% sure of realizing at least 40% of potential production in that cell.). Based on results from the optimization model, that cell might still achieve at least 40% of potential production for the lower influent stage corresponding to an alpha of 0.45. If so, Fig. 6 would show a value of 45 for that cell. Thus, a figure prepared using more optimizations would display, in each cell, the highest tested probability of achieving at least 40% of potential crop production. Once again, the validity of such tables relies on the assumption that, as influent stage increases, allocation volume to a cell never decreases.

The fact that the model considers the time-varying harmful effect of water shortage is illustrated by Fig. 7. This figure is analogous to Fig. 6, except it displays the confidence a user can have in being allocated at least 40% of total water needs. Note that no cell shows a higher probability in Fig. 7 than it shows in Fig. 6. In fact, the percentage of potential production that is produced is always greater than or equal to the percentage of total demand that is supplied. This illustrates that the model times the water shortages to when they do the least harm.

Model results can also be used to determine the spatial distribution of areas that can be assured, to some degree,
cropping patterns based on their attitudes towards risk. Assume an agency plans to allocate water in accordance with the optimization model. Also assume that by the time fields must be planned, prepared or planted, the agency will not yet know how much water can be released from an upstream reservoir—it does not yet know the influent stream stage for the stream-aquifer system. Armed with knowledge of the influent probability distribution function and the optimization model, a planning agency can develop fields and plant in those locations where it will most likely deliver water.

Most simply, an agency can select a particular reliability level (i.e. 80% reliability or alpha of 0.8), and compute the optimal water allocation strategy for the 80% probability influent. It can then plant crop areas in accordance with the volume of water that would be provided to each cell in that strategy. (The agency would need to decide whether it will irrigate selected crop areas fully or based on some proportion of maximum demand). If additional seed and labor is available, the agency can next plant areas that would be irrigated based on a 75% reliability strategy. Continuing in this manner those areas with the greatest probability of receiving water would be planted first. Spatially distributed allocation would be accomplished both optimally and probabilistically to make best use of water and other resources needed for production.

Although the inflow model is described probabilistically, the management model is clearly deterministic. It does not include consideration of uncertain knowledge of aquifer parameters. A model user should conduct sensitivity analysis to evaluate the effect of assumptions on the computed strategies. Assumed hydraulic conductivities could be changed either systematically or randomly to yield a range of optimal strategies for each assumed influent stage. Similarly, the effect of changes in the crop loss coefficients could be evaluated. In this way the water manager would gain a feeling for the robustness of his computed optimal strategies. For example, experience with models for maximizing groundwater use has shown that as hydraulic conductivity is decreased, those areas far from recharge sources or those cells where water table elevations are approaching their lower bounds are the first to suffer a decrease in allocated groundwater use.

**SUMMARY**

Presented is a procedure for evaluating where crops should be planted to maximize reliable crop production in a dynamic stream/aquifer system. The optimal cropping pattern is derived from optimal conjunctive water allocation strategies developed for particular influent stream stage elevations. The probability distribution of the influent stage is assumed known.

The procedure involves using an implicitly stochastic optimization (ISO) model for specific reliability levels. Conceptually, the ISO model consists of an inflow model and a system model. The inflow model represents the random nature of the inflow process and provides influent stream information to the system model. The deterministic system model computes conjunctive water use strategies that maximize crop yield (by minimizing the reduction in yield caused by insufficient water) for each assumed influent stream stage.

(continued on page 1742)
The system model utilizes time-varying linear crop loss coefficients (water production functions). We believe the model to be unique among reported optimization models in that it simulates time-variant, interdependent response of stream stages, groundwater levels, and stream aquifer interflow to groundwater pumping and diversion of river water to nonriparian lands. Through an iterative technique, potentiometric surface response is implicit with respect to water use, stream stage and stream-aquifer interflow.

The ISO model results in alternative strategies that guarantee optimum spatial and temporal distribution of groundwater and river water for selected reliability levels. It is a potentially valuable tool for evaluating future cropping patterns and irrigation water distribution systems. The procedure is most applicable in planning for sustainable agricultural production in less-developed regions. Existing water rights may prevent application of this procedure in many developed areas.

References